A New Similarity Measure on *npn*-Soft Set Theory and Its Application

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ABSTRACT

In this paper, we give a new similarity measure on npn-soft set theory which is the extension of correlation measure of neutrosophic refined sets. By using the similarity measure we propose a new method for decision making problem. Finally, we give an example for diagnosis of diseases could be improved by incorporating clinical results and other competing diagnosis in npn-soft environment..

Keywords: Soft set, neutrosophic set, *npn*-soft set, similarity measure, correlation measure decision making,

1 INTRODUCTION

Theory of fuzzy set [13] and intuistionistic fuzzy set [1,2] are used as efficiently diverse types of uncertainties. Then, neutrosophic set theory introduced by Smarandache [7,8] which is the generalization of the classical sets, conventional fuzzy sets and intuitionistic fuzzy sets. Wang et al. [11] proposed single valued neutrosophic sets as a example of neutrosophic sets. Also same authors defined interval valued neutrosophic sets [12] which is generalization of neutrosophic sets and interval fuzzy sets [9].
In 1999, soft set theory was proposed by Molodstov [6] to supply an alternative for fuzzy theory. This structures ware used medical diagnose, decision making, control theory. After the introduction of soft set and neutrosophic set many scholars have done a lot of good researches in these fields [3-5]. In recently, Deli [4] defined the notion of npn-soft set and operations to make more functional the definitions on soft sets. In this study, we presented a new similarity measure on npn-soft set theory which is the extension of correlation measure of neutrosophic refined sets [3]. By using the similarity measure we propose a new a decision making method and an application on the method in medical diagnosis.

2 PRELIMİNARY

İn this section, we explain some required definitions for neutrosophic sets and npn-soft sets [7, 19]

Definition 2.1. [13] Let E be a universe. Then a fuzzy set X over E is defined by

$$X = \{(\mu_X(x)/x) : x \in E\}$$

where, $\mu_X(x)$ is called membership function of X and defined by $\mu_X(x) : E \to [0,1]$. For each $x \in E$, the value $\mu_X(x)$ represents the degree of x belonging to the fuzzy set X.

Definition 2.2. [1] Let E be a universe. An intuitionistic fuzzy set I on E can be defined as follows:

$$I = \{\langle x, \mu_I(x), \nu_I(x)\rangle : x \in X\}$$

where, $\mu_I(x) : E \to [0,1]$ and $\nu_I(x) : E \to [0,1]$ such that $0 < \mu_I(x) + \nu_I(x) < 1$ for any $x \in E$.

Definition 2.3. [7,8] Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic set (N-set) A in U is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A, and a falsity- membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $[0,1]$. It can be written as

$$A = \{\langle u, T_A(x), I_A(x), F_A(x)\rangle : x \in E, T_A(x), I_A(x), F_A(x) \in [0,1]\}$$

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$

Definition 2.4. [6] Let U be an initial universe, $P(U)$ be the power set of U, E be a set of all parameters and $X \subseteq E$. Then a soft set F_x over U is a set defined by a function representing a mapping F_x: $E \to P(U)$ such that $F_x(x) = \emptyset \ if \ x \notin X$. Here, f_X is called approximate function of the soft set F_X, and the value $f_X(x)$ is a set called *x-element* of the soft set for all $x \in E$. It is worth noting that the sets is worth noting that the sets $f_X(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. Thus, a soft set over U can be represented by the set of ordered pairs

$$F_X = \{(x, f_X(x)) : x \in E, f_X(x) \in P(U)\}$$

Definition 2.5. [4] Let U be a universe, 0 :7 ;be the set of all neutrosophic sets on 7Æ′be a set of parameters that are describe the elements of 7and -be a neutrosophic set over ′ Then, a neutrosophic parameterized neutrosophic soft set (*npn*-soft set) #over 7is a set defined by a set valued function Ƀ representing a mapping

Ƀ ª- 7 0 :7 ;

where f_A is called approximate function of the *npn*-soft set A. For $x \in E$, the set $f_A(x)$ is called x-approximation of the *npn*-soft set A which may be arbitrary, some of them may be empty and some may have a nonempty intersection. It can be written a set of ordered pairs,

$$A = \{(< x, T_A(x), I_A(x), F_A(x) >, \{< u, T_{f_{A(x)}}(u), I_{f_{A(x)}}(u), F_{f_{A(x)}}(u) >: x \in E\}$$

where $T_A(x), I_A(x), F_A(x), T_{f_{A(x)}}(u), I_{f_{A(x)}}(u), F_{f_{A(x)}}(u) \in [0,1]$.

Definition 2.6. [4] A, A_1 and A_2 be two *npn*- soft sets. Then,

i. The union of A_1 and A_2 is denoted by $A_3 = A_1 \cup A_2$ and is defined by

$$A_3 = \{\left(< x, T_{A_3}(x), I_{A_3}(x), F_{A_3}(x) >, \left\{< u, T_{A_{3(x)}}(u), I_{A_{3(x)}}(u), F_{A_{3(x)}}(u) >: u \in U\right\}\right): x \in E\}$$

where $T_{A_3}(x) = s(T_{A_1}(x), T_{A_2}(x))$, $I_{A_3}(x) = t(I_{A_1}(x), I_{A_2}(x))$, $F_{A_3}(x) = t(F_{A_1}(x), F_A(x))$, $T_{A_{3(x)}}(u) = s(T_{f_{A_1(x)}}(u), T_{f_{A_2(x)}}(u))$, $I_{A_{3(x)}}(u) = t(I_{f_{A_1(x)}}(u), (I_{f_{A_2(x)}}(u))$ and

$$F_{A_{3(x)}}(u) = t(F_{f_{A_1(x)}}(u), F_{f_{A_2(x)}}(u))$$

ii. The intersection of a_1 and A_2 is denoted by $A_4 = A_1 \cap A_2$ and is defined by

$$A_4 = \{\left(< x, T_{A_4}(x), I_{A_4}(x), F_{A_4}(x) >, \left\{< u, T_{A_{4(x)}}(u), I_{A_{4(x)}}(u), F_{A_{4(x)}}(u) >: u \in U\right\}\right): x \in E\}$$

where $T_{A_4}(x) = t(T_{A_1}(x), T_{A_2}(x))$, $I_{A_4}(x) = s(I_{A_1}(x), I_{A_2}(x))$, $F_A(x) = s(F_{A_1}(x), F_{A_2}(x))$, $T_{A_{4(x)}}(u) = t\left(T_{f_{A_1(x)}}(u), T_{f_{A_2(x)}}(u)\right)$, $I_{A_{4(x)}}(u) = s(I_{f_{A_1(x)}}(u), (I_{f_{A_2(x)}}(u))$ and

$$F_{A_{4(x)}}(u) = s(F_{f_{A_1(x)}}(u), F_{f_{A_2(x)}}(u))$$

iii. The complement of an *npn*-soft set N denoted by N^c and is denoted by

$$A^c = \{(< x, F_A(x), 1 - I_A(x), T_A(x) >, \left\{< u, F_{f_{A(x)}}(u), 1 - I_{f_{A(x)}}(u), T_{f_{A(x)}}(u) >: x \in E\right\}.$$

3 A new similarity measure on npn-soft sets

İn this section, we define a new similarity measure of be two *npn*- soft sets over U which is the extension of correlation measure of neutrosophic refined sets [3] to *npn*- soft sets.

Definition 3.1. Let A and B be two *npn*- soft sets over U as follws;

$$A = \{< (T_i^1, I_i^1, F_i^1)/x_i, \left\{\left(T_{e_i}^1(u_j), I_{e_i}^1(u_j), F_{e_i}^1(u_j)\right)/u_j : u_j \in U\right\}, x \in X >\}$$

$$B = \{< (T_i^2, I_i^2, F_i^2)/x_i, \left\{\left(T_{e_i}^2(u_j), I_{e_i}^2(u_j), F_{e_i}^2(u_j)\right)/u_j : u_j \in U\right\}, x \in X >\}$$

Then, correlation measure between A and B is given by;

$$\acute{S}(A, B) = \frac{C(A,B)}{\sqrt{C(A,A)}\sqrt{C(B,B)}}$$

where.

$$C(A,A) = \frac{1}{9mn}\sum_{i=1}^{n}(\sum_{j=1}^{m}\left({T_i^1}^2 + {I_i^1}^2 + {F_i^1}^2\right).({T_{ij}^1}^2 + {I_{ij}^1}^2 + {F_{ij}^1}^2))$$

$$C(A,B) = \frac{1}{9mn}\sum_{i=1}^{n}(\sum_{j=1}^{m}(T_i^1T_j^2 + I_i^1I_j^2 + F_i^1F_j^2)(T_{ij}^1T_{ij}^2 + I_{ij}^1I_{ij}^2 + F_{ij}^1F_{ij}^2))$$

$$C(B,B) = \frac{1}{9mn}\sum_{i=1}^{n}(\sum_{j=1}^{m}\left({T_i^2}^2 + {I_i^2}^2 + {F_i^2}^2\right).({T_{ij}^2}^2 + {I_{ij}^2}^2 + {F_{ij}^2}^2))$$

Example 3.2. Let $\boldsymbol{E} = \{\boldsymbol{x_1}\}$ and $\boldsymbol{U} = \{\boldsymbol{u_1}, \boldsymbol{u_2}\}$ be a parameter set and universal set, respectively. Then, A= $\{(< \boldsymbol{x_1}.(\mathbf{0.5}, \mathbf{0.3}, \mathbf{0.1}) >, \{< \boldsymbol{u_1}, (\mathbf{0.4}, \mathbf{0.5}, \mathbf{0.1}) >, < \boldsymbol{u_2}, (\mathbf{0.1}, \mathbf{0.2}, \mathbf{0.3}) >\})\}$

and $\boldsymbol{B} = \{(< \boldsymbol{x_1}.(\mathbf{0.3}, \mathbf{0.2}, \mathbf{0.0}) >, \{< \boldsymbol{u_1}, (\mathbf{0.1}, \mathbf{0.5}, \mathbf{0.1}) >, < \boldsymbol{u_2}, (\mathbf{0.4}, \mathbf{0.8}, \mathbf{0.9}) >\})\}$ be two npn-soft set U. Now we calculate the similarity A and B as;

$$\acute{S}(A, B) = \frac{C(A,B)}{\sqrt{C(A,A)}\sqrt{C(B,B)}} = 0{,}759429$$

where $C(A,A)$ =0,010306, $C(B,B)$ =0,013578 and $C(A,B)$ =0,008983.

Definition 3.3. (Its adopted from [14]) Let A and B be two *npn*-soft sets over U. Then, A and B are said to be α −similar, denoted as $A \approx^{\alpha} B$, ıf and only ıf $\acute{S}(A, B) \geq \alpha$ for $\alpha \in (0,1)$. We call the two two *npn*-soft sets significantly similar if $\acute{S}(A, B) > \frac{1}{2}$.

4 Decision making method on npn-soft sets

In this section, we construct a decision making method by using similarity measure of two *npn*- soft sets. This algorithm can be given as the following that;

Algorithm:

Step 1. Constructs an *npn*- soft set A over U based on an expert,

Step 2. Constructs an npn- soft set B over U based on a responsible person for the problem,

Step 3. Calculate the similarity measure $\acute{S}(A\ ,B)$ of A and B,

Example 4.1. (Its adopted from [3, 14]) Let assume our universal set consists of two elements as $\boldsymbol{U}=\{\boldsymbol{u_1},\boldsymbol{u_2}\}$. These elements indicate cancer and not cancer respectively. For $\boldsymbol{E}=\{\boldsymbol{x_1},\boldsymbol{x_2},\boldsymbol{x_3}\}$ where $\boldsymbol{x_1}=$ *headache*, $\boldsymbol{x_2}=$ *cough*, $\boldsymbol{x_3}=$ *throat pain*. Then,

Step 1**:** Constructs an npn-soft set *A* over *U* for cancer and this can be prepared with the help of a medical person as;

$$A=\{(< x_1,(0.5,0.3,0.1)>,\{< u_1,(0.4,0.5,0.1)>,< u_2,(0.1,0.2,0.3)>\}),(< x_2,(0.1,0.3,0.5)>\{< u_1,(0.2,0.5,0.8)>,< u_2,(0.8,0.5,0.1)>\}),(< x_3,(0.0,0.2,0.5)>,\{< u_1,(0.2,0.5,0.8)>,< u_2,(0.5,0.3,0.2)>\})\}$$

Step 2**:** Constructs an npn-soft set *B* over *U* based on data of ill person as;

$$B=\{(< x_1,(0.3,0.2,0.0)>,\{< u_1,(0.1,0.5,0.1)>,< u_2,(0.4,0.8,0.9)>\}),(< x_2,(0.4,0.2,0.3)>\{< u_1,(0.1,0.6,0.8)>,< u_2,(0.2,0.1,0.8)>\}),(< x_3,(0.3,0.7,0.2)>,\{< u_1,(0.1,0.3,0.1)>,< u_2,(0.4,0.1,0.3)>\})\}$$

Step 3 **:** Calculate the similarity measure of A and B as;

$$\acute{S}(A\ ,B)=0.54925$$

Since $\acute{S}(A\ ,B)>\frac{1}{2}$ for the npn-soft set, A and B are significantly similar. Therefore, we conclude that the person is possibly suffering from cancer

5 CONCLUSION

In this paper, we define a similarity measure on npn-soft sets. Then, we proposed a decision making method on the *npn*-soft set theory and provided an example that demonstrated that this method can be successfully worked. This method can be developed more detailed future to solve uncertainly problems.

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