Algorithms for neutrosophic soft decision making based on EDAS and new similarity measure

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Abstract. This paper presents two novel single-valued neutrosophic soft set (SVNSS) methods. First, we initiate a new axiomatic definition of single-valued neutrosophic simlarity measure, which is expressed by single-valued neutrosophic number (SVNN) that will reduce the information loss and remain more original information. Then, the objective weights of various parameters are determined via grey system theory. Combining objective weights with subjective weights, we present the combined weights, which can reflect both the subjective considerations of the decision maker and the objective information. Later, we present two algorithms to solve decision making problem based on Evaluation based on Distance from Average Solution (EDAS) and similarity measure. Finally, the effectiveness and feasibility of approaches are demonstrated by a numerical example.

Keywords: single-valued neutrosophic soft set, similarity measure, SVNN, Combined weighed, EDAS

1. Introduction

Soft set theory, initiated by Molodtsov [1], is free from the inadequacy of the parameterized tools of those theories such as probability theory, fuzzy set theory [2], rough set theory [3], and interval mathematics [4]. The study of hybrid models combing soft sets with other mathematical structures are an important research topic. Maji et al. [5] firstly proposed fuzzy soft sets, a more generalized notion combining fuzzy sets and soft sets. Alcantud [6] proposed a novel algorithm for fuzzy soft set based decision making from multiobserver input parameter data set. Later, he discussed the formal relationships among soft sets, fuzzy sets, and their extensions in [7]. Yang et al. [8] developed the concept of interval-valued fuzzy soft sets. Meanwhile, Peng and Yang applied interval-valued fuzzy soft sets in clustering analysis [9] and decision making [10]. Peng et al.[11] presented Pythagorean fuzzy soft sets, and discussed their operations. Yang et al. [12] proposed the multi-fuzzy soft sets and successfully applied them to decision making, meanwhile they extended the multi-fuzzy soft sets to that of bipolar multi-fuzzy soft sets [13] which can describe the parameter more accurately and precisely. Wang et al. [14] initiated the hesitant fuzzy soft sets by integrating hesitant fuzzy set [15] with soft set model, and presented an algorithm to solve decision making problems. Peng and Yang [16] further proposed interval-valued hesitant fuzzy soft set, presented an algorithm, and discussed its calculation complexity with others algorithms.

Smarandache [17] initially presented the concept of a neutrosophic set from a philosophical point of view. A neutrosophic set is characterized by a truth-membership degree, an indeterminacy-membership degree, and a falsity-membership degree. It generalizes the concept of the classic set, fuzzy set [2], interval-valued fuzzy set [4], paraconsistent set [17], and tautological set [17]. From scientific or engineer-

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ing point of view, the neutrosophic set and set-theoretic operators need to be specified. Otherwise, it will be difficult to apply in the real applications. Hence, Wang et al. [18] proposed a single valued neutrosophic set (SVNS) and provided the set-theoretic operators and various properties of SVNSs. At present, SVNSs have attracted widely attention and made some achievements. Ye [19,20] proposed a multi-attribute decision making (MADM) method using the correlation coefficient under single-valued neutrosophic environment. Ye [21,22] further developed clustering method and decision making methods by similarity measures of SVNS. Meanwhile, Ye [23] presented cross entropy measures of SVNS and applied them to decision making. Biswas et al. [24] extended the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) method for multi-attribute single-valued neutrosophic decision-making problem. Sahin and Kucuk [25] defined a subsethood measure for SVNS, and applied to MADM. Yang et al. [26] introduced single valued neutrosophic relations and discussed their properties. Huang [27] developed a new distance measure of SVNSs, and applied them to clustering analysis and MADM. Liu [28] proposed aggregation operators based on Archimedean t-conorm and t-norm for SVNS and also gave an application in MADM. Li et al. [29] extended the Heronian mean to SVNS, and proposed some Heronian mean operators for SVNS.

Combining the advantages of the instance of neutrosophic set [17] with soft set [1], Maji [30] proposed the single-valued neutrosophic soft set. Şahin and Küçük [31] presented the definition of similarity and entropy in single-valued neutrosophic soft environment. Mukherjee and Sarkar [32] defined some similarity measures of single-valued neutrosophic soft sets, and applied them to real life problems. Deli and Broumi [33] defined some new operators and soft matrix based on single-valued neutrosophic soft sets. Alkhazaleh [34] introduced time-neutrosophic soft set and studied some of its properties in detail. Al-Quran and Hassan [35] presented the neutosophic vague soft expert set theory and discussed their properties in detail.

Evaluation based on Distance from Average Solution (EDAS), originally proposed by Ghorabaee et al. [36], is a new MADM method for inventory ABC classification. It is a very useful when we have some conflicting attributes. In the compromise MADM methods such as VIKOR and TOPSIS [37], the best alternative is obtained by computing the distance from ideal and nadir solutions. The desirable alternative has lower distance from ideal solution and higher distance from nadir solution in these MADM methods. Ghorabaee et al. [38] extended the EDAS method to supplier selection. To the best of our knowledge, however, the study of the MADM problem based on EDAS method have not been reported in the existing academic literature. Hence, it is an interesting research topic to apply the EDAS in MADM to rank and determine the best alternative under single-valued neutrosophic soft environment. Through a comparison analysis of the given methods, their objective evaluation is carried out, and the method which maintains consistency of its results is chosen.

In order to compute the similarity measure of two SVNSs, we propose a new axiomatic definition of similarity measure, which takes in the form of SVNN. Comparing with the existing literature [21,22,31,32], our similarity measure can remain more original decision information.

Considering that different sets of attribute weights will influence the ranking results of alternatives, we develop a novel method to determine the criteria weights by combining the subjective factors with the objective ones. This model is different from the existing methods, which can be divided into two categories: one is the subjective weighting methods and the other is the objective weighting methods, which can be computed by grey system theory [39]. The subjective weighting methods pay much attention to the preference information of the decision maker [19-21,23,27-30], while they neglect the objective information. The objective weighting methods do not take into account the preference of the decision maker, in particular, these methods fail to take into account the risk attitude of the decision maker [22,24]. The characteristic of our weighting model can reflect both the subjective considerations of the decision maker and the objective information. Consequently, combining subjective weights with objective weights, we provide a combined model to determine attribute weights.

The remainder of this paper is organized as follows: In Section 2, we review some fundamental conceptions of neutrosophic sets, single-valued neutrosophic sets, soft set and single-valued neutrosophic soft sets. In Section 3, a new axiomatic definition of single-valued neutrosophic similarity measure is presented. In Section 4, two single-valued neutrosophic soft decision making approaches based on EDAS and similarity measure are shown. In Section 5, a numerical example is given to illustrate the proposed methods. The paper is concluded in Section 6.

2. Preliminaries

2.1. Neutrosophic set

Neutrosophic set is a portion of neutrosophy, which researches the origin, and domain of neutralities, as well as their interactions with diverse ideational scope [17], and is a convincing general formal framework, which extends the presented sets [2,4] from philosophical point. Smarandache [17] introduced the definition of neutrosophic set as follows:

Definition 1. [3] Let X be a universe of discourse, with a class of elements in X denoted by x. A neutrosophic set B in X is summarized by a truth-membership function $T_B(x)$, an indeterminacy-membership function $I_B(x)$, and a falsity-membership function $F_B(x)$. The functions $T_B(x), I_B(x)$, and $F_B(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is $T_B(x) : X \to]0^-, 1^+[$, $I_B(x) : X \to]0^-, 1^+[$, and $F_B(x) : X \to]0^-, 1^+[$.

There is restriction on the sum of $T_B(x), I_B(x)$, and $F_B(x)$, so $0^- \leq \sup T_B(x) + \sup I_B(x) + \sup F_B(x) \leq 3^+$.

As mentioned above, it is hard to apply the neutrosophic set to solve some real problems. Hence, Wang et al. [18] presented SVNS, which is a subclass of the neutrosophic set and mentioned the definition as follows:

Definition 2. [18] Let X be a universe of discourse, with a class of elements in X denoted by x. A single-valued neutrosophic set N in X is summarized by a truth-membership function $T_N(x)$, an indeterminacy-membership function $I_N(x)$, and a falsity-membership function $F_N(x)$. Then a SVNS N can be denoted as follows:

$$N = \{< x, T_N(x), I_N(x), F_N(x) > \mid x \in X\}, \tag{1}$$

where $T_N(x), I_N(x), F_N(x) \in [0,1]$ for $\forall x \in X$. Meanwhile, the sum of $T_N(x), I_N(x)$, and $F_N(x)$ fulfills the condition $0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3$. For a SVNS N in X, the triplet $(T_N(x), I_N(x), F_N(x))$ is called single-valued neutrosophic number (SVNN). For convenience, we can simply use $x = (T_x, I_x, F_x)$ to represent a SVNN as an element in the SVNS N.

Generally speaking, two special values are taken into consideration, i.e., SVNN 0 and 1; One can deem as 0 for (0,0,1) or (0, 1, 1), and as 1 for (1,0,0) or (1,1,0) relying on the some applications. However, if we think about 0 as the worst value and 1 as the best value, we can set 0 as (0,0,1) and 1 as (1,0,0).

Definition 3. [18] Let $x = (T_x, I_x, F_x)$ and $y = (T_y, I_y, F_y)$ be two SVNNs, then operations can be defined as follows:

(1) $x^c = (F_x, 1 - I_x, T_x)$;
(2) $x \bigcup y = (\max\{T_x, T_y\}, \min\{I_x, I_y\}, \min\{F_x, F_y\})$;
(3) $x \bigcap y = (\min\{T_x, T_y\}, \max\{I_x, I_y\}, \max\{F_x, F_y\})$;
(4) $x \oplus y = (T_x + T_y - T_x * T_y, I_x * I_y, F_x * F_y)$;
(5) $x \otimes y = (T_x * T_y, I_x + I_y - I_x * I_y, F_x + F_y - F_x * F_y)$;
(6) $\lambda x = (1 - (1 - T_x)^\lambda, (I_x)^\lambda, (F_x)^\lambda), \lambda > 0$;
(7) $x^\lambda = ((T_x)^\lambda, 1 - (1 - I_x)^\lambda, 1 - (1 - F_x)^\lambda), \lambda > 0$.

For comparing two SVNNs, Peng et al. [40] introduced a similarity measure method for a SVNN.

Definition 4. [40] Let $x = (T_x, I_x, F_x)$ be a SVNN, then the score function $s(x)$ is defined as follows:

$$s(x) = \frac{2}{3} + \frac{T_x}{3} - \frac{I_x}{3} - \frac{F_x}{3}. \tag{2}$$

It measures the hamming similarity between $x = (T_x, I_x, F_x)$ and the ideal solution (1, 0, 0) for the comparison of SVNNs.

Definition 5. [30] A pair $(\widetilde{F}, A)$ is called a single-valued neutrosophic soft set over U, where $\widetilde{F}$ is a mapping given by $\widetilde{F} : A \to \widetilde{P}(U)$.

In other words, the soft set is not a kind of set, but a parameterized family of subsets of the set U. For any parameter $e \in A$, $\widetilde{F}(e)$ may be considered as the set of e-approximate elements of the single-valued neutrosophic soft set $(\widetilde{F}, A)$. Let $\widetilde{F}(e)(x)$ denote the membership value that object x holds parameter e, then $\widetilde{F}(e)$ can be written as a single-valued neutrosophic set that $\widetilde{F}(e) = \{x/\widetilde{F}(e)(x) \mid x \in U\} = \{x/(T_{\widetilde{F}}(e)(x), I_{\widetilde{F}}(e)(x), F_{\widetilde{F}}(e)(x)) \mid x \in U\}$.

Example 1. Let $U = \{x_1, x_2, x_3\}$ and $A = \{e_1, e_2, e_3\}$. Let $(\widetilde{F}, A)$ be a single-valued neutrosophic soft set over U, defined as follows:

$\widetilde{F}(e_1) = \{x_1/(0.3, 0.4, 0.7), x_2/(0.3, 0.5, 0.6), x_3/(0.4, 0.4, 0.6)\}$,
$\widetilde{F}(e_2) = \{x_1/(0.7, 0.4, 0.8), x_2/(0.6, 0.4, 0.6), x_3/(0.3, 0.5, 0.7)\}$,
$\widetilde{F}(e_3) = \{x_1/(0.5, 0.4, 0.6), x_2/(0.3, 0.6, 0.7), x_3/(0.3, 0.4, 0.8)\}$.

Then, $(\widetilde{F}, A)$ is described by the following Table 1.

Table 1
The single-valued neutrosophic soft set $(\widetilde{F},A)$

	e_1	e_2	e_3
x_1	(0.3,0.4,0.7)	(0.7,0.4,0.8)	(0.5,0.4,0.6)
x_2	(0.3,0.5,0.6)	(0.6,0.4,0.6)	(0.3,0.6,0.7)
x_3	(0.4,0.4,0.6)	(0.3,0.5,0.7)	(0.3,0.4,0.8)

Definition 6. [30] Let $(\widetilde{F},A)$ and $(\widetilde{G},B)$ be two single-valued neutrosophic soft sets over the common universe U. $(\widetilde{F},A)$ is said to be single-valued neutrosophic soft subset of $(\widetilde{G},B)$ if $A \subset B$, $T_{\widetilde{F}}(e)(x) \le T_{\widetilde{G}}(e)(x), I_{\widetilde{F}}(e)(x) \le I_{\widetilde{G}}(e)(x), F_{\widetilde{F}}(e)(x) \ge F_{\widetilde{G}}(e)(x)$, $\forall e \in A, x \in U$. We denote it by $(\widetilde{F},A) \subseteq (\widetilde{G},B)$.

3. A new single-valued neutrosophic similarity measure

Definition 7. Let A_1,A_2 and A_3 be three SVNSs on X. A similarity measure $D^{\Delta}(A_1,A_2)$ is a mapping $S^{\Delta}:$ SVNS$(X) \times$ SVNS$(X) \rightarrow$ SVNN, possessing the following properties:

(1) $S^{\Delta}(A_1,A_2)$ is a SVNN;
(2) $S^{\Delta}(A_1,A_2) = (1,0,0)$, iff $A_1 = A_2$;
(3) $S^{\Delta}(A_1,A_2) = S^{\Delta}(A_2,A_1)$;
(4) If $A_1 \subseteq A_2 \subseteq A_3$, then $S^{\Delta}(A_1,A_2) \supseteq S^{\Delta}(A_1,A_3)$ and $S^{\Delta}(A_2,A_3) \supseteq S^{\Delta}(A_1,A_3)$.

Theorem 1. Let A_i and A_k be two SVNSs, then $S^{\Delta}(A_i,A_k)$ is a similarity measure.

$$S^{\Delta}(A_i,A_k) = \Big(\min\{L(A_i,A_k),M(A_i,A_k),R(A_i,A_k)\}, \min\{L(A_i,A_k),W(A_i,A_k),R(A_i,A_k)\}, 1-\max\{L(A_i,A_k),M(A_i,A_k),R(A_i,A_k)\}\Big), \tag{3}$$

where $L(A_i,A_k) = \frac{\sum_{j=1}^{n} w_j \min\{T_{ij},T_{kj}\}}{\sum_{j=1}^{n} w_j \max\{T_{ij},T_{kj}\}}$,

$M(A_i,A_k) = \frac{\sum_{j=1}^{n} w_j \min\{1-I_{ij},1-I_{kj}\}}{\sum_{j=1}^{n} w_j \max\{1-I_{ij},1-I_{kj}\}}$,

$W(A_i,A_k) = 1-\frac{\sum_{j=1}^{n} w_j \min\{1-I_{ij},1-I_{kj}\}}{\sum_{j=1}^{n} w_j \max\{1-I_{ij},1-I_{kj}\}}$,

$R(A_i,A_k) = \frac{\sum_{j=1}^{n} w_j \min\{1-F_{ij},1-F_{kj}\}}{\sum_{j=1}^{n} w_j \max\{1-F_{ij},1-F_{kj}\}}$ and w_j is the weight of jth SVNN.

Proof. In order for $S^{\Delta}(A_i,A_k)$ to be qualified as a sensible similarity measure for SVNSs, it must satisfy the (1)-(4) of axiomatic requirements.

(1) Since $0 \le L(A_i,A_k) = \frac{\sum_{j=1}^{n} w_j \min\{T_{ij},T_{kj}\}}{\sum_{j=1}^{n} w_j \max\{T_{ij},T_{kj}\}} \le 1$,

$0 \le M(A_i,A_k) = \frac{\sum_{j=1}^{n} w_j \min\{1-I_{ij},1-I_{kj}\}}{\sum_{j=1}^{n} w_j \max\{1-I_{ij},1-I_{kj}\}} \le 1$,

$0 \le W(A_i,A_k) = 1-\frac{\sum_{j=1}^{n} w_j \min\{1-I_{ij},1-I_{kj}\}}{\sum_{j=1}^{n} w_j \max\{1-I_{ij},1-I_{kj}\}} \le 1$,

$0 \le R(A_i,A_k) = \frac{\sum_{j=1}^{n} w_j \min\{1-F_{ij},1-F_{kj}\}}{\sum_{j=1}^{n} w_j \max\{1-F_{ij},1-F_{kj}\}} \le 1$,

therefore,
$0 \le \min\{L(A_i,A_k),M(A_i,A_k),R(A_i,A_k)\} \le 1$,
$0 \le \min\{L(A_i,A_k),W(A_i,A_k),R(A_i,A_k)\} \le 1$,
$0 \le 1-\max\{L(A_i,A_k),M(A_i,A_k),R(A_i,A_k)\} \le 1$.

Furthermore,

$0 \le \min\{L(A_i,A_k),M(A_i,A_k),R(A_i,A_k)\} + \min\{L(A_i,A_k),W(A_i,A_k),R(A_i,A_k)\} + 1-\max\{L(A_i,A_k),M(A_i,A_k),R(A_i,A_k)\} \le 3$.
Consequently, $S^{\Delta}(A_i,A_k)$ is a SVNN.

(2)
① Necessity:
Since $S^{\Delta}(A_i,A_k) = (1,0,0)$, we have
$\min\{L(A_i,A_k),M(A_i,A_k),R(A_i,A_k)\} = 1$,
$\min\{L(A_i,A_k),W(A_i,A_k),R(A_i,A_k)\} = 0$,
$\max\{L(A_i,A_k),M(A_i,A_k),R(A_i,A_k)\} = 1$.
It means that $L(A_i,A_k) = M(A_i,A_k) = R(A_i,A_k) = 1, W(A_i,A_k) = 0$.
Furthermore,

$L(A_i,A_k) = \frac{\sum_{j=1}^{n} w_j \min\{T_{ij},T_{kj}\}}{\sum_{j=1}^{n} w_j \max\{T_{ij},T_{kj}\}} = 1$,

$M(A_i,A_k) = \frac{\sum_{j=1}^{n} w_j \min\{1-I_{ij},1-I_{kj}\}}{\sum_{j=1}^{n} w_j \max\{1-I_{ij},1-I_{kj}\}} = 1$,

$$W(A_i,A_k) = 1 - \frac{\sum_{j=1}^{n} w_j \min\{1-I_{ij},1-I_{kj}\}}{\sum_{j=1}^{n} w_j \max\{1-I_{ij},1-I_{kj}\}} = 0,$$

$$R(A_i,A_k) = \frac{\sum_{j=1}^{n} w_j \min\{1-F_{ij},1-F_{kj}\}}{\sum_{j=1}^{n} w_j \max\{1-F_{ij},1-F_{kj}\}} = 1.$$

Based on the randomicity of w_j, we can have $T_{ij} = T_{kj}, I_{ij} = I_{kj}, F_{ij} = F_{kj}$, i.e., $A_i = A_k$.

② Sufficiency:

Since $A_i = A_k$, we have $T_{ij} = T_{kj}, I_{ij} = I_{kj}, F_{ij} = F_{kj}$. Furthermore, $\sum_{j=1}^{n} w_j \min\{T_{ij},T_{kj}\} = \sum_{j=1}^{n} w_j \max\{T_{ij},T_{kj}\}$, $\sum_{j=1}^{n} w_j \min\{1-I_{ij},1-I_{kj}\} = \sum_{j=1}^{n} w_j \max\{1-I_{ij},1-I_{kj}\}$, $\sum_{j=1}^{n} w_j \min\{1-F_{ij},1-F_{kj}\} = \sum_{j=1}^{n} w_j \max\{1-F_{ij},1-F_{kj}\}$.

Consequently, $S^{\Delta}(A_i,A_k) = (1,0,0)$.

(3) It is obvious.

(4) If $A_1 \subseteq A_2 \subseteq A_3$, then $\forall j, T_{1j} \le T_{2j} \le T_{3j}, I_{1j} \ge I_{2j} \ge I_{3j}$ and $F_{1j} \ge F_{2j} \ge F_{3j}$.

Hence,

$$L(A_1,A_3) = \frac{\sum_{j=1}^{n} w_j \min\{T_{1j},T_{3j}\}}{\sum_{j=1}^{n} w_j \max\{T_{1j},T_{3j}\}} = \frac{\sum_{j=1}^{n} w_j T_{1j}}{\sum_{j=1}^{n} w_j T_{3j}} \le \frac{\sum_{j=1}^{n} w_j T_{1j}}{\sum_{j=1}^{n} w_j T_{2j}} = \frac{\sum_{j=1}^{n} w_j \min\{T_{1j},T_{2j}\}}{\sum_{j=1}^{n} w_j \max\{T_{1j},T_{2j}\}} = L(A_1,A_2),$$

$$M(A_1,A_3) = \frac{\sum_{j=1}^{n} w_j \min\{1-I_{1j},1-I_{3j}\}}{\sum_{j=1}^{n} w_j \max\{1-I_{1j},1-I_{3j}\}} = \frac{\sum_{j=1}^{n} w_j (1-I_{1j})}{\sum_{j=1}^{n} w_j (1-I_{3j})} \le \frac{\sum_{j=1}^{n} w_j (1-I_{1j})}{\sum_{j=1}^{n} w_j (1-I_{2j})} = \frac{\sum_{j=1}^{n} w_j \min\{1-I_{1j},1-I_{2j}\}}{\sum_{j=1}^{n} w_j \max\{1-I_{1j},1-I_{2j}\}} = M(A_1,A_2),$$

$$W(A_1,A_3) = 1 - \frac{\sum_{j=1}^{n} w_j \min\{1-I_{1j},1-I_{3j}\}}{\sum_{j=1}^{n} w_j \max\{1-I_{1j},1-I_{3j}\}}$$
$$= 1 - \frac{\sum_{j=1}^{n} w_j (1-I_{1j})}{\sum_{j=1}^{n} w_j (1-I_{3j})} \le 1 - \frac{\sum_{j=1}^{n} w_j (1-I_{1j})}{\sum_{j=1}^{n} w_j (1-I_{2j})}$$
$$= 1 - \frac{\sum_{j=1}^{n} w_j \min\{1-I_{1j},1-I_{2j}\}}{\sum_{j=1}^{n} w_j \max\{1-I_{1j},1-I_{2j}\}} = W(A_1,A_2),$$

$$R(A_1,A_3) = \frac{\sum_{j=1}^{n} w_j \min\{1-F_{1j},1-F_{3j}\}}{\sum_{j=1}^{n} w_j \max\{1-F_{1j},1-F_{3j}\}} = \frac{\sum_{j=1}^{n} w_j (1-F_{1j})}{\sum_{j=1}^{n} w_j (1-F_{3j})} \le \frac{\sum_{j=1}^{n} w_j (1-F_{1j})}{\sum_{j=1}^{n} w_j (1-F_{2j})} = \frac{\sum_{j=1}^{n} w_j \min\{1-F_{1j},1-F_{2j}\}}{\sum_{j=1}^{n} w_j \max\{1-F_{1j},1-F_{2j}\}} = R(A_1,A_2).$$

Furthermore,

$\min\{L(A_1,A_2),M(A_1,A_2),R(A_1,A_2)\} \ge \min\{L(A_1,A_3),M(A_1,A_3),R(A_1,A_3)\}$,

$\min\{L(A_1,A_2),W(A_1,A_2),R(A_1,A_2)\} \ge \min\{L(A_1,A_3),W(A_1,A_3),R(A_1,A_3)\}$,

$1-\max\{L(A_1,A_2),M(A_1,A_2),R(A_1,A_2)\} \le 1-\max\{L(A_1,A_2),M(A_1,A_2),R(A_1,A_2)\}$.

Consequently, $S^{\Delta}(A_1,A_2) \supseteq S^{\Delta}(A_1,A_3)$.

Similarly, $S^{\Delta}(A_2,A_3) \supseteq S^{\Delta}(A_1,A_3)$. This completes the proof.

Especially, for any two SVNNs x_1 and x_2, the similarity measure between $x_1 = (T_1,I_1,F_1)$ and $x_2 = (T_2,I_2,F_2)$ is defined as follows:

$$S^{\Delta}(x_1,x_2) = \left(\min\left\{\frac{\min\{T_1,T_2\}}{\max\{T_1,T_2\}}, \frac{\min\{1-I_1,1-I_2\}}{\max\{1-I_1,1-I_2\}}, \frac{\min\{1-F_1,1-F_2\}}{\max\{1-F_1,1-F_2\}}\right\}, \min\left\{\frac{\min\{T_1,T_2\}}{\max\{T_1,T_2\}}, 1-\frac{\min\{1-I_1,1-I_2\}}{\max\{1-I_1,1-I_2\}}, \frac{\min\{1-F_1,1-F_2\}}{\max\{1-F_1,1-F_2\}}\right\}, 1-\max\left\{\frac{\min\{T_1,T_2\}}{\max\{T_1,T_2\}}, \frac{\min\{1-I_1,1-I_2\}}{\max\{1-I_1,1-I_2\}}, \frac{\min\{1-F_1,1-F_2\}}{\max\{1-F_1,1-F_2\}}\right\}\right). \tag{4}$$

If $x_1 = x_2$, then $S^{\Delta}(x_1,x_2) = (1,0,0)$, i.e., the similarity is the biggest; if $x_1 = (0,0,1), x_2 = (1,0,0)$ or $x_1 = (1,0,0), x_2 = (0,0,1)$, then $S^{\Delta}(x_1,x_2) = (0,0,1)$, i.e., the similarity is the smallest.

4. Two algorithms for single-valued neutrosophic soft decision making

4.1. Problem description

Let $U = \{x_1, x_2, \cdots, x_m\}$ be a finite set of m alternatives, $E = \{e_1, e_2, \cdots, e_n\}$ be a set of n parameters, and the weight of parameter e_i is w_i, $w_i \in [0,1], \sum_{i=1}^{n} w_i = 1$. $(\widetilde{F}, E)$ is single-valued neutrosophic soft set can be expressed in Table 2. $\widetilde{F}(e_j)(x_i) = (T_{\widetilde{F}}(e_j)(x_i), I_{\widetilde{F}}(e_j)(x_i), F_{\widetilde{F}}(e_j)(x_i))$ represents the possible SVNN of the ith alternative x_i satisfying the jth parameter e_j which is given by the decision maker.

Table 2
Tabular representation of $(\widetilde{F}, E)$

	e_1	e_2	$\cdots$	e_n
x_1	$\widetilde{F}(e_1)(x_1)$	$\widetilde{F}(e_2)(x_1)$	$\cdots$	$\widetilde{F}(e_n)(x_1)$
x_2	$\widetilde{F}(e_1)(x_2)$	$\widetilde{F}(e_2)(x_2)$	$\cdots$	$\widetilde{F}(e_n)(x_2)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
x_m	$\widetilde{F}(e_1)(x_m)$	$\widetilde{F}(e_2)(x_m)$	$\cdots$	$\widetilde{F}(e_n)(x_m)$

In the following, we will apply the EDAS and similarity measure methods to SVNSSs.

4.2. The method of computing the combined weights

For a decision maker problem, considering that different sets of weights will influence the ranking results of alternatives, we develop a novel method to obtain the weights by combining the subjective elements with the objective ones. This model is different from the existing methods, which can be divided into two tactics: one is the subjective weighting determine methods and the other is the objective weighting methods, which can be computed by grey system theory [39]. The subjective weighting methods focus on the preference information of the decision maker, while they ignore the objective information. The objective weighting methods do not take the preference of the decision maker into account, that is to say, these methods fail to take the risk attitude of the decision maker into account. The feature of our weighting model can show both the subjective information and the objective information. Hence, combining subjective weights with objective weights, we provide a combined model to determine attribute weights.

4.2.1. Determining the objective weights: the grey system method

The grey system theory [39] is an excellent tool to deal with small sample and poor information. For a decision maker problem, the alternatives and attributes are generally small which accord with the condition of grey system theory, so we take the method of grey system into consideration.

Theoretically, if an attribute information with respect to other attribute more matches in the average information of the attribute, the attribute contains more information for decision making, the greater the weight. Based on this idea, we propose a grey relational analysis method to determine the attribute weights.

Definition 8. Suppose $R = (r_{ij})_{m\times n} (i = 1,2,\cdots,m; j = 1,2,\cdots,n)$ be a single-valued neutrosophic matrix. And $S = (s_{ij})_{m\times n} (i = 1,2,\cdots,m; j = 1,2,\cdots,n)$ is the score function s (Eq.(2)) of R. Let $\bar{s}_i = \frac{1}{n}\sum_{j=1}^{n} s_{ij}$, then the attribute weight ω_j is defined as follows:

$$\omega_j = \frac{1 - \frac{1}{m}\left(\sum_{i=1}^{m}(o_{ij}^{q})\right)^{\frac{1}{q}}}{n - \frac{1}{m}\sum_{j=1}^{n}\left(\sum_{i=1}^{m}(o_{ij}^{q})\right)^{\frac{1}{q}}}, \tag{5}$$

where $o_{ij} = \frac{\min_i |s_{ij}-\bar{s}_i| + \xi \max_i |s_{ij}-\bar{s}_i|}{|s_{ij}-\bar{s}_i| + \xi \max_i |s_{ij}-\bar{s}_i|}$ is grey mean relational degree, in general, we set $\xi = 0.5$.

In order to improve the effective resolution, this paper uses Euclidean distance instead of Hamming distance, i.e., $q = 2$.

4.2.2. Determining the combined weights: the non-linear weighted comprehensive method

Suppose that the vector of the subjective weight, given by the decision makers directly, is $w = \{w_1, w_2, \cdots, w_n\}$, where $\sum_{j=1}^{n} w_j = 1, 0 \le w_j \le 1$. The vector of the objective weight, computed by Eq.(11) directly, is $\omega = \{\omega_1, \omega_2, \cdots, \omega_n\}$, where $\sum_{j=1}^{n} \omega_j = 1, 0 \le \omega_j \le 1$.

Therefore, the vector of the combined weight $\varpi = \{\varpi_1, \varpi_2, \cdots, \varpi_n\}$ can be defined as follows:

$$\varpi_j = \frac{w_j * \omega_j}{\sum_{j=1}^{n} w_j * \omega_j}, \tag{6}$$

where $\sum_{j=1}^{n} \varpi_j = 1, 0 \leq \varpi_j \leq 1$.

The objective weight and subjective weight are aggregated by non-linear weighted comprehensive method. According to the multiplier effect, the larger the value of the subjective weight and objective weight are, the larger the combined weight is, or vice versa. At the same time, we can obtain that the Eq.(6) overcomes the limitation of only considering either subjective or objective factor influence. The advantage of Eq.(6) is that the attribute weights and rankings of alternatives can show both the subjective information and the objective information.

4.3. The method of EDAS

In this section, an extended version of the EDAS method is proposed to deal with decision making problems in the single-valued neutrosophic soft environment. Therefore, the concepts and arithmetic operations of the SVNN are utilized for extending the EDAS method.

Algorithm 1: EDAS

Step 1. Identify the alternatives and parameters, and obtain the single-valued neutrosophic soft set $(\widetilde{F}, E)$ which is shown in Table 2.

Step 2. Normalize the single-valued neutrosophic soft set $(\widetilde{F}, E)$ into $(\widetilde{F}', E)$ by Eq. (7).

$$\widetilde{F}'(e_j)(x_i) = \begin{cases} \left(T_{\widetilde{F}}(e_j)(x_i), I_{\widetilde{F}}(e_j)(x_i), F_{\widetilde{F}}(e_j)(x_i)\right), & e_j \in B, \\ \left(F_{\widetilde{F}}(e_j)(x_i), 1 - I_{\widetilde{F}}(e_j)(x_i), T_{\widetilde{F}}(e_j)(x_i)\right), & e_j \in C, \end{cases} \tag{7}$$

where B is benefit parameter set and C is cost parameter set.

Step 3. Compute the combined weights by Eq.(6).

Step 4. Determine the average solution according to all parameters, shown as follows:

$$AV = (AV_j)_{1 \times n}, \tag{8}$$

where

$$\begin{aligned} AV_j &= \frac{1}{m} \bigoplus_{i=1}^{m} \widetilde{F}(e_j)(x_i) \\ &= \frac{1}{m} \left(1 - \prod_{i=1}^{m}(1 - T_{\widetilde{F}}(e_j)(x_i)), \prod_{i=1}^{m} I_{\widetilde{F}}(e_j)(x_i), \prod_{i=1}^{m} F_{\widetilde{F}}(e_j)(x_i)\right) \\ &= \left(1 - (\prod_{i=1}^{m}(1 - T_{\widetilde{F}}(e_j)(x_i)))^{\frac{1}{m}}, (\prod_{i=1}^{m} I_{\widetilde{F}}(e_j)(x_i))^{\frac{1}{m}}, (\prod_{i=1}^{m} F_{\widetilde{F}}(e_j)(x_i))^{\frac{1}{m}}\right). \end{aligned} \tag{9}$$

Step 5. Calculate the positive distance from average (PDA) with $PDA = (P_{ij})_{m \times n}$ and the negative distance from average (NDA) with $NDA = (N_{ij})_{m \times n}$ matrixes according to the type of parameters, shown as follows:

$$P_{ij} = \begin{cases} \frac{\max\{0, s(\widetilde{F}(e_j)(x_i)) - s(AV_j)\}}{s(AV_j)}, & e_j \in B, \\ \frac{\max\{0, s(AV_j) - s(\widetilde{F}(e_j)(x_i))\}}{s(AV_j)}, & e_j \in C, \end{cases} \tag{10}$$

$$N_{ij} = \begin{cases} \frac{\max\{0, s(AV_j) - s(\widetilde{F}(e_j)(x_i))\}}{s(AV_j)}, & e_j \in B, \\ \frac{\max\{0, s(\widetilde{F}(e_j)(x_i)) - s(AV_j)\}}{s(AV_j)}, & e_j \in C, \end{cases} \tag{11}$$

where $s(AV_j)$ and $s(\widetilde{F}(e_j)(x_i))$ are score function of AV_j and $\widetilde{F}(e_j)(x_i)$, respectively.

Step 6. Determine the weighted sum of PDA and NDA for all alternatives, shown as follows:

$$SP_i = \sum_{j=1}^{n} w_j P_{ij}, \tag{12}$$

$$SN_i = \sum_{j=1}^{n} w_j N_{ij}. \tag{13}$$

Step 7. Normalize the values of SP_i and SN_i for all alternatives, shown as follows:

$$NSP_i = \frac{SP_i}{\max_i\{SP_i\}} \tag{14}$$

$$NSN_i = 1 - \frac{SN_i}{\max_i\{SN_i\}} \tag{15}$$

Step 8. Calculate the appraisal score $AS_i (i = 1, 2, \cdots, m)$ for all alternatives, shown as follows:

$$AS_i = \frac{1}{2}(NSP_i + NSN_i), \tag{16}$$

where $0 \leq AS_i \leq 1$.

Step 9. Rank the alternatives according to the decreasing values of AS_i. The alternative with the highest AS_i is the best choice among the candidate alternatives.

4.4. The method of similarity measure

In this section, we introduce a method for the decision making problem by the proposed similarity measure between SVNSs. The concept of ideal point has been applied to help determine the best alternative in the decision process. Although the ideal alternative does not exist in practical problems, it does offer a useful theoretical construct against which to appraise alternatives. Therefore, we define the ideal alternative x^* as the SVNN $x_j^* = (T^*, I^*, F^*) = (1,0,0)$ for $\forall j$.

Hence, by applying Eq.(3), the proposed similarity measure S^{Δ} between an alternative x_i and the ideal alternative x^* represented by the SVNSs is defined by

$$\begin{aligned} S^{\Delta}(x_i,x^*) = \\ \Big(\min\{L(x_i,x^*), M(x_i,x^*), R(x_i,x^*)\}, \\ \min\{L(x_i,x^*), W(x_i,x^*), R(x_i,x^*)\}, \\ 1-\max\{L(x_i,x^*), M(x_i,x^*), R(x_i,x^*)\}\Big), \end{aligned} \tag{17}$$

where

$$L(x_i,x^*) = \frac{\sum_{j=1}^{n} \varpi_j \min\{T_{\widetilde{F}}(e_j)(x_i),1\}}{\sum_{j=1}^{n} \varpi_j \max\{T_{\widetilde{F}}(e_j)(x_i),1\}} = \sum_{j=1}^{n} \varpi_j T_{\widetilde{F}}(e_j)(x_i),$$

$$M(x_i,x^*) = \frac{\sum_{j=1}^{n} \varpi_j \min\{1-I_{\widetilde{F}}(e_j)(x_i),1\}}{\sum_{j=1}^{n} \varpi_j \max\{1-I_{\widetilde{F}}(e_j)(x_i),1\}} = 1-\sum_{j=1}^{n} \varpi_j I_{\widetilde{F}}(e_j)(x_i),$$

$$W(x_i,x^*) = 1-\frac{\sum_{j=1}^{n} \varpi_j \min\{1-I_{\widetilde{F}}(e_j)(x_i),1\}}{\sum_{j=1}^{n} \varpi_j \max\{1-I_{\widetilde{F}}(e_j)(x_i),1\}} = \sum_{j=1}^{n} \varpi_j I_{\widetilde{F}}(e_j)(x_i),$$

$$R(x_i,x^*) = \frac{\sum_{j=1}^{n} \varpi_j \min\{1-F_{\widetilde{F}}(e_j)(x_i),1\}}{\sum_{j=1}^{n} \varpi_j \max\{1-F_{\widetilde{F}}(e_j)(x_i),1\}} = 1-\sum_{j=1}^{n} \varpi_j F_{\widetilde{F}}(e_j)(x_i).$$

Algorithm 2: similarity measure

Steps 1-3. Similarly to Steps 1-3 in Algorithm 1.
Step 4. Calculate the similarity measure $S(x_i,x^*)(i=1,2,\cdots,m)$ by Eq.(17).
Step 5. Compute the each alternative of score function $s(S(x_i,x^*))$ by Eq.(2).
Step 6. Rank the alternatives by $s(S(x_i,x^*))(i=1,2,\cdots,m)$. The most desired alternative is the one with the biggest value of x_i.

5. A numerical example

An internet company wants to select a software development project to invest. Suppose that there are four software development projects: e-commerce development project, game development project, browser development project and web development project. The company selects three parameters to evaluate the four software development projects.

Let $U=\{x_1,x_2,x_3,x_4\}$ is the set of software development projects under consideration, $E=\{e_1,e_2,e_3\}$ is the set of parameters, e_1 stands for economic feasibility, e_2 stands for technological feasibility, e_3 stands for staff feasibility. e_1 and e_2 are benefit parameters, e_3 is cost parameter. The weight vector of the parameters is given as $w=(0.4,0.1,0.5)^T$. The tabular is presented in Table 3.

In what follows, we utilize the algorithms proposed above for software development projects selection with single-valued neutrosophic soft information.

Table 3
Tabular representation of $(\widetilde{F},E)$

	e_1	e_2	e_3
x_1	$(0.5,0.4,0.7)$	$(0.7,0.5,0.1)$	$(0.6,0.6,0.3)$
x_2	$(0.6,0.5,0.6)$	$(0.6,0.2,0.2)$	$(0.5,0.4,0.4)$
x_3	$(0.7,0.3,0.5)$	$(0.7,0.2,0.1)$	$(0.7,0.5,0.4)$
x_4	$(0.6,0.4,0.5)$	$(0.7,0.4,0.2)$	$(0.5,0.6,0.4)$

Algorithm 1: EDAS

Step 1. Identify the alternatives and parameters, and obtain the single-valued neutrosophic soft set $(\widetilde{F},E)$ which is shown in Table 3.
Step 2. Normalize the single-valued neutrosophic soft set $(\widetilde{F},E)$ into $(\widetilde{F}',E)$ by Eq. (7), which is shown in Table 4.
Step 3. Compute the combined weights by Eq.(6) as follows:
$\varpi_1=0.4078, \varpi_2=0.1017, \varpi_3=0.4905$.

Table 4
The normalized single-valued neutrosophic soft set $(\widetilde{F}',E)$

	e_1	e_2	e_3
x_1	$(0.5,0.4,0.7)$	$(0.7,0.5,0.1)$	$(0.3,0.4,0.6)$
x_2	$(0.6,0.5,0.6)$	$(0.6,0.2,0.2)$	$(0.4,0.6,0.5)$
x_3	$(0.7,0.3,0.5)$	$(0.7,0.2,0.1)$	$(0.4,0.5,0.7)$
x_4	$(0.6,0.4,0.5)$	$(0.7,0.4,0.2)$	$(0.4,0.4,0.5)$

Step 4. Determine the average solution according to all parameters by Eq.(9), shown as follows:

$AV_1 = (0.6064, 0.3936, 0.5692)$,

$AV_2 = (0.6776, 0.2991, 0.1414)$,

$AV_3 = (0.5838, 0.5180, 0.3722)$.

Step 5. Calculate the positive distance from average $PDA = (P_{ij})_{4\times3}$ and the negative distance from average $NDA = (N_{ij})_{4\times3}$ matrixes by Eqs.(10) and (11), shown as follows:

$$PDA = (P_{ij})_{4\times3} \begin{pmatrix} 0.0000 & 0.0000 & 0.0038 \\ 0.0000 & 0.0000 & 0.0038 \\ 0.1560 & 0.0728 & 0.0628 \\ 0.0343 & 0.0000 & 0.0000 \end{pmatrix},$$

$$NDA = (N_{ij})_{4\times3} \begin{pmatrix} 0.1482 & 0.0613 & 0.0000 \\ 0.0873 & 0.0166 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0613 & 0.1143 \end{pmatrix}.$$

Step 6. Determine the weighted sum of PDA and NDA for all alternatives by Eqs.(12) and (13), respectively, shown as follows:

$SP_1 = 0.0019, SP_2 = 0.0019, SP_3 = 0.1019, SP_4 = 0.0140$,

$NP_1 = 0.0667, NP_2 = 0.0373, NP_3 = 0.0000, NP_4 = 0.0623$.

Step 7. Normalize the values of SP_i and SN_i for all alternatives by Eqs. (14) and (15), respectively, shown as follows:

$NSP_1 = 0.0183, NSP_2 = 0.0183$,

$NSP_3 = 1.0000, NSP_4 = 0.1375$,

$NSN_1 = -0.0702, NSN_2 = 0.4011$,

$NSN_3 = 1.0000, NSN_4 = 0.0000$.

Step 8. Calculate the appraisal score $AS_i (i = 1,2,3,4)$ for all alternatives by Eq. (16), shown as follows:

$AS_1 = -0.0260, AS_2 = 0.2097, AS_3 = 1.0000, AS_4 = 0.0687$.

Step 9. Rank the alternatives according to the decreasing values of AS_i as follows:

$AS_3 \succ AS_2 \succ AS_4 \succ AS_1$.

Obviously, amongst them x_3 is the best alternative.

Algorithm 2: similarity measure

Steps 1-3. Similarly to Steps 1-3 in Algorithm 1.

Step 4. Calculate the similarity measure $S(x_i, x^*)(i = 1,2,3,4)$ by Eq.(17), shown as follows:

$S(x_1, x^*) = (0.4223, 0.4102, 0.4101)$,

$S(x_2, x^*) = (0.4815, 0.5019, 0.4897)$,

$S(x_3, x^*) = (0.5529, 0.3879, 0.3879)$,

$S(x_4, x^*) = (0.4695, 0.4000, 0.4000)$.

Step 5. Compute the each alternative of score function $s(S(A_i, A^*))$ by Eq.(2), shown as follows:

$s(S(x_1, x^*)) = 0.533993, s(S(x_2, x^*)) = 0.496602$,

$s(S(x_3, x^*)) = 0.592335, s(S(x_4, x^*)) = 0.556487$.

Step 6. Rank the alternatives by $s(S(x_i, x^*))(i = 1,2,3,4)$ as follows:

$x_3 \succ x_4 \succ x_1 \succ x_2$.

Obviously, amongst them x_3 is the best alternative.

By means of the Algorithms 1 and 2, we can find that the final results are the same, i.e., x_3 is the most desirable investment alternative. Hence, the two approaches proposed above are effective and feasible.

In the following, we give some comparisons of Algorithm 1 and Algorithm 2.

(1) Comparison of computational complexity

We know that Algorithm 1 will be consumed more computational complexity than Algorithm 2, especially in Step 4. So if we take the computational complexity into consideration, the Algorithm 2 is given priority to make decision.

(2) Comparison of discrimination

Comparing the results in Algorithm 1 with Algorithm 2, we can find that the results of Algorithm 2 are quite close and vary from 0.496602 to 0.592335. These result of decision values cannot clearly distinguish, in other words, the results derived from Algorithm 2 are not very convincing (or at least not applicable). On the contrary, the Algorithm 1 has a clearly distinguish. So if we take the discrimination into consideration, the Algorithm 1 is given priority to make decision.

6. Conclusion and remarks

The major contributions in this paper can be summarized as follows:

(1) We construct a new axiomatic definition of single-valued neutrosophic similarity measure and give a similarity formula. Comparing with the existing literature [21,22,31,32], it can reduce the information miss and remain more original information.

(2) A novel single-valued neutrosophic soft approach in multi-criteria decision making based on EDAS is explored, which has not been reported in the existing literature. The approach doesn't need to calculate the ideal and the nadir solution.

(3) A novel single-valued neutrosophic soft approach in multi-criteria decision making based on similarity measure, which can reduce the information loss and remain more original information.

(4) The subjective weighting methods pay much attention to the preference information of the decision

maker [19-21,23,27-30], while they neglect the objective information. The objective weighting methods do not take into account the preference of the decision maker, in particular, these methods fail to take into account the risk attitude of the decision maker [22,24]. The characteristic of our weighting model can reflect both the subjective considerations of the decision maker and the objective information.

In the future, we shall apply more advanced theories into single-valued neutrosophic soft set and solve more decision making problems.

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