Rough Neutrosophic Hyper-complex set and its Application to Multi-attribute Decision Making

Kalyan Mondal[1] Surapati Pramanik[2], Florentin Smarandache[3]

[1]Department of Mathematics, Jadavpur University, West Bengal, India

Email: kalyanmathematic@gmail.com

[2]Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, PO-Narayanpur, and District: North 24 Parganas, Pin Code: 743126, West Bengal, India

Email: sura_pati@yahoo.co.in

[4]Department of Mathematics, University of New Mexico, 705 Gurley Avenue, Gallup, NM 87301, USA.

E-mail: fsmarandache@gmail.com

Abstract

This paper presents multi-attribute decision making based on rough neutrosophic hyper-complex sets with rough neutrosophic hyper-complex attribute values. The concept of neutrosophic hyper-complex set is a powerful mathematical tool to deal with incomplete, indeterminate and inconsistent information. We extend the concept of neutrosophic hyper-complex set to rough neutrosophic hyper-complex set. The ratings of all alternatives have been expressed in terms of the upper and lower approximations and the pair of neutrosophic hyper-complex sets which are characterized by two hyper-complex functions and an indeterminacy component. We also define cosine function based on rough neutrosophic hyper-complex set to determine the degree of similarity between rough neutrosophic hyper-complex sets. We establish new decision making approach based on rough neutrosphic hyper-complex set. Finally, a numerical example has been furnished to demonstrate the applicability of the proposed approach.

Keyword

Neutrosophic set, Rough neutrosophic set, Rough neutrosophic hyper-complex set, Cosine function, Decision making.

1. Introduction

The concept of rough neutrosophic set has been introduced by Broumi et al. [1, 2]. It has been derived as a combination of the concepts of rough set proposed by Z. Pawlak [3] and neutrosophic set introduced by F. Smarandache [4, 5]. Rough sets and neutrosophic sets are both capable of dealing with partial information and uncertainty. To deal with real world problems, Wang et al. [6] introduced single valued netrosophic sets (SVNSs).

Recently, Mondal and Pramanik proposed a few decision making models in rough neutrosophic environment. Mondal and Pramanik [7] applied the concept of grey relational analysis to rough neutrosophic multi-attribte decision making problems. Pramanik and Mondal [8] studied cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. Mondal and Pramanik [9] proposed multi attribute decision making approach using rough accuracy score function. Pramanik and Mondal [10] also proposed cotangent similarity measure under rough neutrosophic sets. The same authors [11] further studied some similarity measures namely Dice similarity measure [12] and Jaccard similarity measure [12] in rough neutrosophic environment.

Rough neutrosophic hyper-complex set is the generalization of rough neutrosophic set [1, 2] and neutrosophic hyper-complex sets [13]. S. Olariu [14] introduced the concept of hyper-complex number and studied some of its properties. Mandal and Basu [15] studied hyper-complex similarity measure for SVNS and its application in decision making. Mondal and Pramanik [16] studied tri-complex rough neutrosophic similarity measure and presented an application in multi-attribute decision making.

In this paper, we have defined rough neutrosophic hyper-complex set and rough neutrosophic hyper-complex cosine function (RNHCF). We have also proposed a multi-attribute decision making approach in rough neutrosophic hyper-complex environment.

Rest of the paper is organized in the following way. Section 2 presents preliminaries of neutrosophic sets, single valued neutrosophic sets and some basic ideas of hyper-complex sets. Section 3 gives the definition of rough neutrosophic hyper-complex sets. Section 4 gives the definition of rough neutrosophic hyper-complex cosine function. Section 5 is devoted to present multi attribute decision-making method based on rough neutrosophic hyper-complex cosine function. Section 6 presents a numerical example of the proposed approach. Finally section 7 presents concluding remarks and scope of future research.

2. Neutrosophic Preliminaries

Neutrosophic set is derived from neutrosophy [4].

2.1 Neutrosophic set

Definition 2.1[4, 5]

Let U be a universe of discourse. Then a neutrosophic set A can be presented in the form:

$$A = \{< x:T_A(x), I_A(x), F_A(x)>, x \in U\}, \quad (1)$$

where the functions T, I, F: U→ $]^-0,1^+[$ represent respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in U$ to the set Asatisfying the following the condition.

$$^-0 \leq \sup T_A(x)+ \sup I_A(x)+ \sup F_A(x) \leq 3^+ \quad (2)$$

Wang et al. [6] mentioned that the neutrosophic set assumes the values from the real standard or non-standard subsets of $]^-0, 1^+[$ based on philosophical point of view. So instead of $]^-0, 1^+[$ Wang et al. [6] consider the interval [0, 1] for technical applications, because $]^-0, 1^+[$ is difficult to apply in the real applications such as scientific and engineering problems. For two netrosophic sets (NSs),

$$A_{NS} = \{<x: T_A(x), I_A(x), F_A(x)> \mid x \in X\} \quad (3)$$

And

$$B_{NS} = \{< x, T_B(x), I_B(x), F_B(x)> \mid x \in X\}, \quad (4)$$

the two relations are defined as follows:

(1) $A_{NS} \subseteq B_{NS}$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$

(2) $A_{NS} = B_{NS}$ if and only if $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$

2. 2 Single valued neutrosophic sets (SVNS)

Definition 2.2 [6]

Assume that X is a space of points (objects) with generic elements in X denoted by x. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity membership function $F_A(x)$, for each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. When X is continuous, a SVNS A can be written as follows:

$$A = \int_x \frac{< T_A(x), I_A(x), F_A(x) >}{x} : x \in X. \quad (5)$$

When X is discrete, a SVNS A can be written as:

$$A = \Sigma_{i=1}^{n} \frac{< T_A(x_i), I_A(x_i), F_A(x_i) >}{x_i} : x_i \in X \quad (6)$$

For two SVNSs ,

$$A_{SVNS} = \{<x: T_A(x), I_A(x), F_A(x)> \mid x \in X\} \quad (7)$$

and

$$B_{SVNS} = \{<x, T_B(x), I_B(x), F_B(x)> \mid x \in X\}, \quad (8)$$

the two relations are defined as follows:

(i) $A_{SVNS} \subseteq B_{SVNS}$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$

(ii) $A_{SVNS} = B_{SVNS}$ if and only if $T_A(x) = T_Q(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$ for any $x \in X$

2.3. Basic concept of Hyper-complex number of dimension n [13]

The hyper-complex number of dimension n (or n-complex number) was defined by S. Olariu [13] as a number of the form:

$$u = h_0 x_o + h_1 x_1 + h_2 x_2 + \ldots + h_{n-1} x_{n-1}$$
$$= h_0 x_o + h_1 x_1 + h_2 x_2 + \ldots + h_{n-1} x_{n-1} \quad (9)$$

where $n \ge 2$, and the variables $x_0, x_1, x_2, \ldots, x_{n-1}$ are real numbers, while $h_1, h_2, \ldots, h_{n-1}$ are the complex units, $h_o = 1$, and they are multiplied as follows:

$$h_j h_k = h_{j+k} \text{ if } 0 \le j+k \le n-1, \text{ and } h_j h_k = h_{j+k-n} \text{ if } n \le j+k \le 2n-2. \quad (10)$$

The above complex unit multiplication formulas can be written in a simpler form as:

$$h_j h_k = h_{j+k} \pmod n \quad (11)$$

where mod n means modulo n. For example, if n = 5, then

$$h_3 h_4 = h_{3+4} (\mathrm{mod}\ 5) = h_7 (\mathrm{mod} 5) = h_2. \quad (12)$$

The formula(11) allows us to multiply many complex units at once, as follows:

$$h_{j1} h_{j2} \ldots h_{jp} = h_{j1+j2+\ldots+jp} \pmod n, \text{ for } p \ge 1. \quad (13)$$

The Neutrosophic hyper-complex number of dimension n [12] which is a number and it can be written of the form:

$$u + vI \quad (14)$$

where u and v are n-complex numbers and I is the indeterminacy.

3. Rough Neutrosophic Hyper-complex Set in Dimension n

Definition 3.1

Let Z be a non-null set and R be an equivalence relation on Z. Let A be a neutrosophic hyper-complex set of dimension n (or neutrosophic n-complex number), and its elements of the form u+vI, where u and v are n-complex numbers and I is the indeterminacy. The lower and the upper approximations of A in the approximation space (Z, R) denoted by $\underline{N}(A)$ and $\overline{N}(A)$ are respectively defined as follows:

$$\underline{N}(A) = \left\langle < x, [u+vI]_{\underline{N}(A)}(x) > / z \in [x]_R , x \in Z \right\rangle \quad (15)$$

$$\overline{N}(A) = \left\langle < x, [u+vI]_{\overline{N}(A)}(x) > / z \in [x]_R , x \in Z \right\rangle \quad (16)$$

where,

$$[u+vI]_{\underline{N}(A)}(x) = \wedge_z \in [x]_R \, [u+vI]_A(z), \quad (17)$$

$$[u+vI]_{\overline{N}(A)}(x) = \vee_z \in [x]_R \, [u+vI]_A(z) \quad (18)$$

So, $[u+vI]_{\underline{N}(A)}(x)$ and $[u+vI]_{\overline{N}(A)}(x)$ are neutrosophic hyper-complex numbers of dimension n. Here $\vee$ and $\wedge$ denote 'max' and 'min' operators respectively. $[u+vI]_A(z)$ and $[u+vI]_A(z)$ are the neutrosophic hyper-complex sets of dimension n of z with respect to A. $\underline{N}(A)$ and $\overline{N}(A)$ are two neutrosophic hyper-complex sets of dimension n in Z.

Thus, NS mappings $\underline{N}$, $\overline{N}$: N(Z) $\rightarrow$ N(Z) are respectively referred to as the lower and upper rough neutrosophic hyper-complex approximation operators, and the pair $(\underline{N}(A), \overline{N}(A))$ is called the rough neutrosophic hyper-complex set in (Z, R).

Based on the above mentioned definition, it is observed that $\underline{N}(A)$ and $\overline{N}(A)$ have constant membership on the equivalence clases of R, if $\underline{N}(A) = \overline{N}(A)$; i.e. $[u+vI]_{\underline{N}(A)}(x)$ = $[u+vI]_{\overline{N}(A)}(x)$.

Definition 3.2

Let N(A) = ($\underline{N}(A), \overline{N}(A)$) be a rough neutrosophic hyper-complex set in (Z, R). The rough complement of N(A) is denoted by $\sim N(A) = (\underline{N}(A)^c, \overline{N}(A)^c)$, where $\underline{N}(A)^c$ and $\overline{N}(A)^c$ are the complements of neutrosophic hyper-complex set of $\underline{N}(A)$ and $\overline{N}(A)$ respectively.

$$\underline{N}(A)^c = \left\langle < x, [u+v(1-I)]_{\underline{N}(A)}(x) > / , x \in Z \right\rangle, \quad (19)$$

and

$$\overline{N}(A)^c = \left\langle < x, [u+v(1-I)]_{\overline{N}(A)}(x) > / , x \in Z \right\rangle \quad (20)$$

Definition 3.3

Let N(A) and N(B) are two rough neutrosophic hyper-complex sets respectively in Z, then the following definitions hold:

$$N(A) = N(B) \Leftrightarrow \underline{N}(A) = \underline{N}(B) \wedge \overline{N}(A) = \overline{N}(B) \quad (21)$$

$$N(A) \subseteq N(B) \Leftrightarrow \underline{N}(A) \subseteq \underline{N}(B) \wedge \overline{N}(A) \subseteq \overline{N}(B) \quad (22)$$

$$N(A) \cup N(B) = < \underline{N}(A) \cup \underline{N}(B), \overline{N}(A) \cup \overline{N}(B) > \quad (23)$$

$$N(A) \cap N(B) = < \underline{N}(A) \cap \underline{N}(B), \overline{N}(A) \cap \overline{N}(B) > \quad (24)$$

If A, B, C are the rough neutrosophic hyper-complex sets in (Z, R), then the following propositions are stated from definitions

Proposition 1

I. $\sim(\sim A) = A$ (25)

II. $\underline{N}(A) \subseteq \overline{N}(B)$ (26)

III. $<\sim(\underline{N}(A) \cup \underline{N}(B)) = \sim(\underline{N}(A)) \cap \sim(\underline{N}(B))$ (27)

IV. $<\sim(\underline{N}(A) \cap \underline{N}(B)) = \sim(\underline{N}(A)) \cup \sim(\underline{N}(B))$ (28)

V. $<\sim(\overline{N}(A) \cup \overline{N}(B)) = \sim(\overline{N}(A)) \cap \sim(\overline{N}(B))$ (29)

VI. $<\sim(\overline{N}(A) \cap \overline{N}(B)) = \sim(\overline{N}(A)) \cup \sim(\overline{N}(B))$ (30)

Proofs I:

If $N(A) = [\underline{N}(A), \overline{N}(A)]$ is a rough neutrosophic hyper-complex set in (Z, R), the complement of N(A) is the rough neutrosophic hyper-complex set defined as follows.

$$\underline{N}(A)^c = \left\langle < x, [u + v(1-I)]_{\underline{N}(A)}(x) > /, x \in Z \right\rangle, \quad (31)$$

and

$$\overline{N}(A)^c = \left\langle < x, [u + v(1-I)]_{\overline{N}(A)}(x) > /, x \in Z \right\rangle \quad (32)$$

From these definitions, we can write:

$$\sim(\sim A) = A. \quad (33)$$

Proof II:

The lower and the upper approximations of A in the approximation space (Z, R) denoted by $\underline{N}(A)$ and $\overline{N}(A)$ are respectively defined as follows:

$$\underline{N}(A)^c = \left\langle < x, [u + v(1-I)]_{\underline{N}(A)}(x) > /, x \in Z \right\rangle, \quad (34)$$

and

$$\overline{N}(A)^c = \left\langle < x, [u + v(1-I)]_{\overline{N}(A)}(x) > /, x \in Z \right\rangle \quad (35)$$

where,

$$[u + vI]_{\underline{N}(A)}(x) = \wedge_z \in [x]_R \, [u + vI]_A(z), \quad (36)$$

$$[u + vI]_{\overline{N}(A)}(x) = \vee_z \in [x]_R \, [u + vI]_A(z) \quad (37)$$

So,

$$\underline{N}(A) \subseteq \overline{N}(A) \tag{38}$$

Proof III:

Consider:

$$x \in \sim(\underline{N}(A) \cup \underline{N}(B))$$

$$\Rightarrow x \in \sim \underline{N}(A) \text{ and } x \in \sim \underline{N}(B)$$

$$\Rightarrow x \in \sim(\underline{N}(A)) \cap \sim(\underline{N}(B))$$

$$\Rightarrow x \in \sim(\underline{N}(A)) \cap \sim(\underline{N}(B)) \tag{39}$$

$$\Rightarrow \sim(\underline{N}(A) \cup \underline{N}(B)) \subseteq \sim((\underline{N}(A)) \cap \sim(\underline{N}(B))).$$

Again, consider:

$$y \in \sim((\underline{N}(A)) \cap \sim(\underline{N}(B)))$$

$$\Rightarrow y \in \sim \underline{N}(A) \text{ or } y \in \sim \underline{N}(B)$$

$$\Rightarrow y \in \Rightarrow \sim(\underline{N}(A) \cup \underline{N}(B))$$

$$\Rightarrow \sim(\underline{N}(A) \cup \underline{N}(B)) \supseteq \sim((\underline{N}(A)) \cap \sim(\underline{N}(B))). \tag{40}$$

Hence,

$$\sim(\underline{N}(A) \cup \underline{N}(B)) = \sim((\underline{N}(A)) \cap \sim(\underline{N}(B))) \tag{41}$$

Proof IV:

Consider:

$$x \in \sim(\underline{N}(A) \cap \underline{N}(B))$$

$$\Rightarrow x \in \sim \underline{N}(A) \text{ or } x \in \sim \underline{N}(B)$$

$$\Rightarrow x \in \sim(\underline{N}(A)) \cup \sim(\underline{N}(B))$$

$$\Rightarrow x \in \sim(\underline{N}(A)) \cup \sim(\underline{N}(B))$$

$$\Rightarrow \sim(\underline{N}(A) \cap \underline{N}(B)) \subseteq \sim((\underline{N}(A)) \cup \sim(\underline{N}(B))) \tag{42}$$

Again, consider:

$$y \in \sim((\underline{N}(A)) \cup \sim(\underline{N}(B)))$$

$$\Rightarrow y \in \sim \underline{N}(A) \text{ and } y \in \sim \underline{N}(B)$$

$$\Rightarrow y \in \sim(\underline{N}(A) \cap \underline{N}(B))$$

$$\Rightarrow \sim(\underline{N}(A) \cap \underline{N}(B)) \supseteq \sim((\underline{N}(A)) \cup \sim(\underline{N}(B))) \tag{43}$$

Hence,

$$\sim\left(\underline{N}(A)\cap\underline{N}(B)\right)=\sim\left(\left(\underline{N}(A)\right)\cup\sim\left(\underline{N}(B)\right)\right) \tag{44}$$

Proof V:

Consider:

$$x\in\sim\left(\overline{N}(A)\cup\overline{N}(B)\right)$$

$$\Rightarrow x\in\sim\overline{N}(A) \text{ and } x\in\sim\overline{N}(B)$$

$$\Rightarrow x\in\sim\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)$$

$$\Rightarrow x\in\sim\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)$$

$$\Rightarrow\sim\left(\overline{N}(A)\cup\overline{N}(B)\right)\subseteq\sim\left(\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)\right) \tag{45}$$

Again, consider:

$$y\in\sim\left(\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)\right)$$

$$\Rightarrow y\in\sim\overline{N}(A) \text{ or } y\in\sim\overline{N}(B)$$

$$\Rightarrow y\in\sim\left(\overline{N}(A)\cup\overline{N}(B)\right)$$

$$\Rightarrow\sim\left(\overline{N}(A)\cup\overline{N}(B)\right)\supseteq\sim\left(\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)\right) \tag{46}$$

Hence,

$$\sim\left(\overline{N}(A)\cup\overline{N}(B)\right)=\sim\left(\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)\right) \tag{47}$$

Proof VI:

Consider:

$$x\in\sim\left(\overline{N}(A)\cap\overline{N}(B)\right)$$

$$\Rightarrow x\in\sim\overline{N}(A) \text{ or } x\in\sim\overline{N}(B)$$

$$\Rightarrow x\in\sim\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)$$

$$\Rightarrow x\in\sim\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)$$

$$\Rightarrow\sim\left(\overline{N}(A)\cap\overline{N}(B)\right)\subseteq\sim\left(\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)\right) \tag{48}$$

Again, consider:

$$y\in\sim\left(\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)\right)$$

$$\Rightarrow y\in\sim\overline{N}(A) \text{ and } y\in\sim\overline{N}(B)$$

$$\Rightarrow y\in\sim\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)$$

$$\Rightarrow\sim\left(\overline{N}(A)\cap\overline{N}(B)\right)\supseteq\sim\left(\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)\right) \tag{49}$$

Hence,

$$\sim\left(\overline{N}(A)\cap\overline{N}(B)\right)=\sim\left(\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)\right) \quad (50)$$

Proposition 2:

I. $\sim[N(A)\cup N(B)] = (\sim N(A))\cap(\sim N(B))$ (51)

II. $\sim[N(A)\cap N(B)] = (\sim N(A))\cup(\sim N(B))$ (52)

Proof I:

$\sim[N(A)\cup N(B)]$

$= \sim < \underline{N}(A)\cup\underline{N}(B),\ \overline{N}(A)\cup\overline{N}(B) >$

$= <\sim\left(\underline{N}(P)\cap\underline{N}(Q)\right),\ \sim\left(\overline{N}(P)\cap\overline{N}(Q)\right)>$

$= (\sim N(A))\cap(\sim N(B))$ (53)

Proof II:

$\sim[N(A)\cap N(B)]$

$= \sim < \underline{N}(A)\cap\underline{N}(B),\ \overline{N}(A)\cap\overline{N}(B) >$

$= <\sim\left(\underline{N}(A)\cup\underline{N}(B)\right),\ \sim\left(\overline{N}(A)\cup\overline{N}(B)\right)>$

$= (\sim N(A))\cup(\sim N(B))$ (54)

4. Rough neutrosophic hyper-complex cosine function (RNHCF)

The cosine similarity measure is calculated as the inner product of two vectors divided by the product of their lengths. It is the cosine of the angle between the vector representations of two rough neutrosophic hyper-complex sets. The cosine similarity measure is a fundamental measure used in information technology. Now, a new cosine function between rough neutrosophic hyper-complex sets is proposed as follows.

Definition 4.1

Assume that there are two rough neutrosophic hyper-complex sets

$$A=\left\langle [u+vI]_{\underline{N}(A)}(x), [u+vI]_{\overline{N}(A)}(x)\right\rangle \quad (55)$$

and

$$B=\left\langle [u+vI]_{\underline{N}(B)}(x), [u+vI]_{\overline{N}(B)}(x)\right\rangle \quad (56)$$

in $X = \{x_1, x_2, \ldots, x_n\}$.

Then rough neutrosophic hyper-complex cosine function between two sets A and B is defined as follows:

$C_{RNHCF}(A,B) =$

$$\frac{1}{n}\sum_{i=1}^{n}\frac{\Delta u_A(x_i).\Delta u_B(x_i)+\Delta v_A(x_i).\Delta v_B(x_i)+\Delta I_A(x_i).\Delta I_B(x_i)}{\sqrt{(\Delta u_A(x_i))^2+(\Delta v_A(x_i))^2+(\Delta I_A(x_i))^2}\sqrt{(\Delta u_B(x_i))^2+(\Delta v_B(x_i))^2+(\Delta I_B(x_i))^2}} \quad (57)$$

where,

$$\Delta u_A(x_i) = 0.5.\left|u_{\underline{N}(A)(x_i)}+u_{\overline{N}(A)(x_i)}\right|, \quad (58)$$

$$\Delta u_B(x_i) = 0.5.\left|u_{\underline{N}(B)(x_i)}+u_{\overline{N}(B)(x_i)}\right|, \quad (59)$$

$$\Delta v_A(x_i) = 0.5.\left|v_{\underline{N}(A)(x_i)} + v_{\overline{N}(A)(x_i)}\right|, \quad (60)$$

$$\Delta v_B(x_i) = 0.5.\left|v_{\underline{N}(B)(x_i)} + v_{\overline{N}(B)(x_i)}\right|, \quad (61)$$

$$\Delta I_A(x_i) = 0.5.\left|I_{\underline{N}(A)(x_i)} + I_{\overline{N}(A)(x_i)}\right|, \quad (62)$$

$$\Delta I_B(x_i) = 0.5.\left|I_{\underline{N}(B)(x_i)} + I_{\overline{N}(B)(x_i)}\right|. \quad (63)$$

Proposition 3:
Let A and B be rough neutrosophic sets, then:

I. $0 \le C_{RNHCF}(A,B) \le 1$ (64)

II. $C_{RNHCF}(A,B) = C_{RNHCF}(B,A)$ (65)

III. $C_{RNHCF}(A, B) = 1$, if and only if A = B (66)

IV. If C is a RNHCF in Y and $A \subset B \subset C$ then, $C_{RNHCF}(A, C) \le C_{RNHCF}(A, B)$, and $C_{RNHCF}(A, C) \le C_{RNHCF}(B,C)$. (67)

Proofs :
I. It is obvious because all positive values of cosine function are within 0 and 1
II. It is obvious that the proposition is true.
III. When A = B, then obviously $C_{RNHCF}(A, B) = 1$. On the other hand if $C_{RNHCF}(A, B) =1$ then, $\Delta T_A(x_i) = \Delta T_B(x_i)$, $\Delta I_A(x_i) = \Delta I_B(x_i)$, $\Delta F_A(x_i) = \Delta F_B(x_i)$ ie,
This implies that A = B.
IV. If $A \subset B \subset C$, then we can write

$$u_{\underline{N}(A)}(x_i) \le u_{\underline{N}(B)}(x_i) \le u_{\underline{N}(C)}(x_i), \quad (68)$$

$$u_{\overline{N}(A)}(x_i) \le u_{\overline{N}(B)}(x_i) \le u_{\overline{N}(C)}(x_i), \quad (69)$$

$$v_{\underline{N}(A)}(x_i) \le v_{\underline{N}(B)}(x_i) \le v_{\underline{N}(C)}(x_i), \quad (70)$$

$$v_{\overline{N}(A)}(x_i) \le v_{\overline{N}(B)}(x_i) \le v_{\overline{N}(C)}(x_i), \quad (71)$$

$$I_{\underline{N}(A)}(x_i) \ge I_{\underline{N}(B)}(x_i) \ge I_{\underline{N}(C)}(x_i), \quad (72)$$

$$I_{\overline{N}(A)}(x_i) \ge I_{\overline{N}(B)}(x_i) \ge I_{\overline{N}(C)}(x_i) \quad (73)$$

The cosine function is decreasing function within the interval $\left[0, \frac{\pi}{2}\right]$. Hence we can write $C_{RNHCF}(A, C) \le C_{RNHCF}(A, B)$, and $C_{RNHCF}(A, C) \le C_{RNHCF}(B, C)$.

If we consider the weight of each element x_i, a weighted rough neutrosophic hyper-complex cosine function (WRNHCF) between two sets A and B can be defined as follows:

$C_{WRNHCF}(A,B) =$

$$\sum_{i=1}^{n} W_i \frac{\Delta u_A(x_i).\Delta u_B(x_i) + \Delta v_A(x_i).\Delta v_B(x_i) + \Delta I_A(x_i).\Delta I_B(x_i)}{\sqrt{(\Delta u_A(x_i))^2 + (\Delta v_A(x_i))^2 + (\Delta I_A(x_i))^2}\sqrt{(\Delta u_B(x_i))^2 + (\Delta v_B(x_i))^2 + (\Delta I_B(x_i))^2}} \quad (74)$$

where,

$$\Delta u_A(x_i) = 0.5.\left|u_{\underline{N}(A)(x_i)} + u_{\overline{N}(A)(x_i)}\right|, \quad (75)$$

$$\Delta u_B(x_i) = 0.5.\left|u_{\underline{N}(B)(x_i)} + u_{\overline{N}(B)(x_i)}\right|, \quad (76)$$

$$\Delta v_A(x_i) = 0.5.\left|v_{\underline{N}(A)(x_i)} + v_{\overline{N}(A)(x_i)}\right|, \quad (77)$$

$$\Delta v_B(x_i) = 0.5.\left|v_{\underline{N}(B)(x_i)} + v_{\overline{N}(B)(x_i)}\right|, \quad (78)$$

$$\Delta I_A(x_i) = 0.5.\left|I_{\underline{N}(A)(x_i)} + I_{\overline{N}(A)(x_i)}\right|, \quad (79)$$

$$\Delta I_B(x_i) = 0.5.\left| I_{\underline{N}(B)(x_i)} + I_{\overline{N}(B)(x_i)} \right|. \quad (80)$$

$W_i \in [0,1]$, i = 1, 2,..., n and $\sum_{i=1}^{n} W_i = 1$.

If we take $W_i = \frac{1}{n}$, i = 1, 2,..., n, then:

$$C_{WRNHCF}(A, B) = C_{RNHCF}(A, B) \quad (81)$$

The weighted rough neutrosophic hyper-complex cosine function (WRNHCF) between two rough neutrosophic hyper-complex sets A and B also satisfies the following properties:

I. $0 \le C_{WRNHCF}(A, B) \le 1$ (82)

II. $C_{WRNHCF}(A, B) = C_{WRNHCF}(B, A)$ (83)

III. $C_{WRNHCF}(A, B) = 1$, if and only if A = B (84)

IV. If C is a WRNHCF in Y and $A \subset B \subset C$ then, $C_{WRNHCF}(A, C) \le C_{WRNHCF}(A, B)$, and $C_{WRNHCF}(A, C) \le C_{WRNHCF}(B, C)$ (85)

5. Decision making procedure based on rough hyper-complex neutrosophic function

In this section, we apply rough neutrosophic hyper-complex cosine function to the multi-attribute decision making problem. Let $A_1, A_2, \ldots, A_m$ be a set of alternatives and $C_1, C_2, \ldots, C_n$ be a set of attributes.

The proposed multi attribute decision making approach is described using the following steps.

Step1: **Construction of the decision matrix with rough neutrosophic hyper-complex numbers**

The decision maker considers a decision matrix with respect to m alternatives and n attributes in terms of rough neutrosophic hyper-complex numbers as follows.

Table1: Rough neutrosophic hyper-complex decision matrix

$$DM = \left\langle \underline{dm}_{ij}, \overline{dm}_{ij} \right\rangle_{m \times n} =$$

	C_1	C_2	$\cdots$	C_n
A_1	$\langle \underline{dm}_{11}, \overline{dm}_{11} \rangle$	$\langle \underline{dm}_{12}, \overline{dm}_{12} \rangle$	...	$\langle \underline{dm}_{1n}, \overline{dm}_{1n} \rangle$
A_2	$\langle \underline{dm}_{21}, \overline{dm}_{21} \rangle$	$\langle \underline{dm}_{22}, \overline{dm}_{22} \rangle$	...	$\langle \underline{dm}_{2n}, \overline{dm}_{2n} \rangle$
.	...	...	...	...
.	...	...	...	...
A_m	$\langle \underline{dm}_{m1}, \overline{dm}_{m1} \rangle$	$\langle \underline{dm}_{m2}, \overline{dm}_{m2} \rangle$	...	$\langle \underline{dm}_{mn}, \overline{dm}_{mn} \rangle$

(86)

Here $\langle \underline{dm}_{ij}, \overline{dm}_{ij} \rangle$ is the rough neutrosophic hyper-complex number according to the i-th alternative and the j-th attribute.

Step2: **Determination of the weights of the attributes**

Assume that the weight of the attribute C_j (j = 1, 2, ... , n) considered by the decision-maker be w_j (j = 1, 2, ... , n) such that $\forall w_j \in [0, 1]$ (j = 1, 2, ..., n) and $\sum_{j=1}^{n} w_j = 1$.

Step 3: **Determination of the benefit type attribute and cost type attribute**

Generally, the evaluation of attributes can be categorized into two types: benefit attribute and cost attribute. Let K be a set of benefit attributes and M be a set of cost attributes. In the proposed decision-making approach, an ideal alternative can be identified by using a maximum operator for the benefit attribute and a minimum operator for the cost attribute to determine the best value of each criterion among all alternatives. Therefore, we define an ideal alternative as follows.

$$A^* = \{C_1^*, C_2^*, \dots, C_m^*\}. \quad (87)$$

Benefit attribute:

$$C_j^* = \left[\max_i u_{C_j}^{(A_i)}, \max_i v_{C_j}^{(A_i)}, \min_i I_{C_j}^{(A_i)} \right] \quad (88)$$

Cost attribute:

$$C_j^* = \left[\min_i T_{C_j}^{(A_i)}, \min_i I_{C_j}^{(A_i)}, \max_i F_{C_j}^{(A_i)} \right] \quad (89)$$

where,

$$u_{C_j}^{(A_i)} = 0.5.\left|\left(u_{C_j}\right)_{\underline{N}(A_i)} + \left(u_{C_j}\right)_{\overline{N}(A_i)}\right|, \quad (90)$$

$$v_{C_j}^{(A_i)} = 0.5.\left|\left(v_{C_j}\right)_{\underline{N}(A_i)} + \left(v_{C_j}\right)_{\overline{N}(A_i)}\right|, \quad (91)$$

and

$$I_{C_j}^{(A_i)} = 0.5.\left|\left(I_{C_j}\right)_{\underline{N}(A_i)} + \left(I_{C_j}\right)_{\overline{N}(A_i)}\right|. \quad (92)$$

Step4: **Determination of the over all weighted rough hyper-complex neutrosophic cosine function (WRNHCF)of the alternatives**

Weighted rough neutrosophic hyper-complex cosine function is given as follows.

$$C_{WRNHCF}(A, B) = \sum_{j=1}^{n} W_j C_{WRNHCF}(A, B) \quad (93)$$

Step5: **Ranking the alternatives**

Using the weighted rough hyper-complex neutrosophic cosine function between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be easily selected with the highest similarity value.

Step6: End

6. Numerical Example

Assume that a decision maker (an adult man/woman who eligible to marrage) intends to select the most suitable life partner for arrange marrage from the three initially chosen candidates (S_1, S_2, S_3) by considering five attributes namely: physical and mental health C_1, education and job C_2, management power C_3, family background C_4, risk factor C_5. Based on the proposed approach discussed in section 5, the considered problem has been solved using the following steps:

Step1: **Construction of the decision matrix with rough neutrosophic hyper-complex numbers**

The decision maker considers a decision matrix with respect to three alternatives and five attributes in terms of rough neutrosophic hyper-complex numbers shown in the Table 2.

Table2. Decision matrix with rough neutrosophic hyper-complex number

$$DM = \left\langle \underline{dm}_{ij}, \overline{dm}_{ij} \right\rangle_{3\times 5} =$$

$$\begin{array}{c|ccccc}
 & C_1 & C_2 & C_3 & C_4 & C_5 \\ \hline
A_1 & \left\langle \begin{matrix}(i+0.6(1+i)),\\(2i+0.4(2+i))\end{matrix}\right\rangle & \left\langle \begin{matrix}((1+i)+0.65(2i)),\\((1+2i)+0.55(3i))\end{matrix}\right\rangle & \left\langle \begin{matrix}((1+i)+0.4(2+i)),\\((1+2i)+0.2(2+3i))\end{matrix}\right\rangle & \left\langle \begin{matrix}(4i+0.55(1+i)),\\((4+i)+0.45(2+i))\end{matrix}\right\rangle & \left\langle \begin{matrix}(3i+0.78(2+3i)),\\((1+3i)+0.72(3+3i))\end{matrix}\right\rangle \\
A_2 & \left\langle \begin{matrix}(i+0.6(1+2i)),\\(3i+0.5(1+3i))\end{matrix}\right\rangle & \left\langle \begin{matrix}((1+i)+0.55(i)),\\((1+2i)+0.45(3i))\end{matrix}\right\rangle & \left\langle \begin{matrix}(2i+0.3(2+i)),\\((2+i)+0.2(1+3i))\end{matrix}\right\rangle & \left\langle \begin{matrix}(i+0.52(2+3i)),\\(2i+0.48(4+3i))\end{matrix}\right\rangle & \left\langle \begin{matrix}((1+i)+0.82(2+i)),\\(2i+0.78(4+3i))\end{matrix}\right\rangle \\
A_3 & \left\langle \begin{matrix}(2i+0.5(1+i)),\\(3i+0.4(1+3i))\end{matrix}\right\rangle & \left\langle \begin{matrix}((2+i)+0.69(5i)),\\((2+i)+0.51(6i))\end{matrix}\right\rangle & \left\langle \begin{matrix}(i+0.6(1+i)),\\(2i+0.4(3+2i))\end{matrix}\right\rangle & \left\langle \begin{matrix}((1+i)+0.48(3+4i)),\\((1+2i)+0.42(5+3i))\end{matrix}\right\rangle & \left\langle \begin{matrix}((1+i)+0.9(i)),\\((1+2i)+0.7(2+3i))\end{matrix}\right\rangle
\end{array} \quad (94)$$

Where, $i=\sqrt{-1}$

Step 2: Determination of the weights of the attributes

The weight vectors considered by the decision maker are 0.25, 0.20, 0.25, 0.10, and 0.20 respectively.

Step 3: Determination of the benefit attribute and cost attribute

Here four benefit type attributes are C_1, C_2, C_3, C_4 and one cost type attribute is C_5. Using equations (12) and (13) we calculate A^* as follows.

A^* = [(5.00, 2.69, 0.45), (4.47, 5.50, 0.50), (3.60, 2.83, 0.25), (6.40, 5.30, 0.45), (3.16, 2.24, 0.80)]

Step 4: Determination of the over all weighted rough hyper-complex neutrosophic similarity function (WRHNSF)of the alternatives

We calculate weighted rough neutrosophic hyper-complex similarity values as follows.

$S_{WRHCF}(A_1, A^*) = 0.9622$

$S_{WRHCF}(A_2, A^*) = 0.9404$

$S_{WRHCF}(A_3, A^*) = 0.9942$

Step 5: Ranking the alternatives

Ranking of the alternatives is prepared based on the descending order of similarity measures. Highest value reflects the best alternative.
Here,

$$S_{WRHCF}(A_3, A^*) \succ S_{WRHCF}(A_1, A^*) \succ S_{WRHCF}(A_2, A^*) \quad (95)$$

Hence, the decision maker must choose the candidate A_3 as the best alternative for arrange marriage.
Step 6: End

7 Conclusion

In this paper, we have proposed rough neutrosophic hyper-complex set and rough neutrosophic hyper-complex cosine function and proved some of their basic properties. We have also proposed rough neutrosophic hyper-complex similarity measure based multi-attribute decision making approach. We have presented an application, namely selection of best candidate for arrange marriage for indian context. The concept presented in this paper can be applied for other multiple attribute decision making problems in rough neutrosophic hyper-complex environment.

References

[1] S. Broumi, F. Smarandache, M. Dhar, Rough neutrosophic sets, ''Italian journal of pure and applied mathematics,'' 32 (2014), 493-502.
[2] S. Broumi, F. Smarandache, M. Dhar, Rough neutrosophic sets, ''Neutrosophic Sets and Systems,'' 3 (2014), 60-66.
[3] Z. Pawlak, Rough sets, ''International Journal of Information and Computer Sciences,'' 11(5) (1982), 341-356.
[4] F. Smarandache, A unifying field in logics, neutrosophy: neutrosophic probability, set and logic, American Research Press, Rehoboth, 1998.
[5] F. Smarandache, Neutrosophic set – a generalization of intuitionistic fuzzy sets, "International Journal of Pure and Applied Mathematics", 24(3) (2005), 287-297.
[6] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman, Single valued neutrosophic sets, ''Multispace and Multistructure,'' 4 (2010), 410–413.
[7] K. Mondal, S. Pramanik, Rough neutrosophic multi-attribute decision-making based on grey relational analysis, ''Neutrosophic Sets and Systems,'' 7(2015), 8-17.
[8] S. Pramanik, K. Mondal, Cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis, ''Global Journal of Advanced Research,'' 2(1) (2015), 212-220.
[9] K. Mondal, S. Pramanik, Rough neutrosophic multi-attribute decision-making based on rough accuracy score function, "Neutrosophic Sets and Systems,'' 8(2015), 16-22.
[10] S. Pramanik, K. Mondal, Cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis, ''Journal of New Theory,'' 4(2015), 90-102.
[11] S. Pramanik, K. Mondal, Some rough neutrosophic similarity measure and their application to multi attribute decision making, ''Global Journal of Engineering Science and Research Management,'' 2(7) (2015), 61-74.
[12] S. Pramanik, P. Biswas, B. C. Giri, Hybrid vector similarity measure and its application to multi-attribute decision making under neutrosophic environment, ''Neural Computing and Application'', 2014,1–14. doi:10.1007/s00521-015-2125-3.
[13] F. Smarandache, Symbolic neutrosophic theory, ''EuropaNova asbl Clos du Parnasse, 3E 1000, Bruxelles Belgium, 2015.
[14] S. Olariu, Complex numbers in n dimensions, Elsevier publication, North-Holland, 2002, 195-205.
[15] K. Mandal, K. Basu. Hyper-complex neutrosophic similarity measure & its application in multi-criteria decision making problem, ''Neutrosophic Sets and Systems,'' 9 (2015), 6-12.
[16] K. Mondal, S. Pramanik, Tri-complex rough neutrosophic similarity measure and its application in multiAttribute decision making, Critical Review, 11 (2015), 26-40.