Tri-complex Rough Neutrosophic Similarity Measure and its Application in Multi-Attribute Decision Making

Kalyan Mondal[1], Surapati Pramanik[2*]

[1]Department of Mathematics, West Bengal State University, Barasat, North 24 Paraganas, Berunanpukuria, P.O. Malikapur, North 24 Parganas, Pincode: 700126, West Bengal, India
Email:kalyanmathematic@gmail.com

[2]Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, PO-Narayanpur, and District: North 24 Parganas, Pin Code: 743126, West Bengal, India
*Corresponding author's email: sura_pati@yahoo.co.in

Abstract

This paper presents multi-attribute decision making based on tri-complex rough neutrosophic similarity measure with rough neutrosophic attribute values. The concept of rough neutrosophic set is a powerful mathematical tool to deal with incomplete, indeterminate and inconsistent information. The ratings of all alternatives are expressed in terms of the upper and lower approximation operators and the pair of neutrosophic sets which are characterized by truth-membership degree, indeterminacy-membership degree, and falsity-membership degree. We define a function based on tri-complex number system to determine the degree of similarity between rough neutrosophic sets. The approach of using tri-complex number system in formulating the similarity measure in rough neutrosophic environment is new. Finally, a numerical example demonstrates the applicability of the proposed approach.

Keyword

tri-complex rough neutrosophic similarity measure, rough neutrosophic set, MCDM problem, approximation operator.

1 Introduction

The concept of rough neutrosophic set is grounded by Broumi et al. [1], [2] in 2014. It is derived by hybridizing the concepts of rough set proposed by Pawlak [3] and neutrosophic set originated by Smarandache [4, 5]. Neutrosophic sets and rough sets are both capable of dealing with uncertainty and partial information. Wang et al. [6] introduced single valued neutrosophic set (SVNS) in 2010 to deal with real world problems.

Rough neutrosophic set is the generalization of rough fuzzy sets [7], [8] and rough intuitionistic fuzzy sets [9]. Mondal and Pramanik [10] applied the concept of rough neutrosophic set in multi-attribute decision making based on grey relational analysis in 2015. Mondal and Pramanik [11] also studied cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis in 2015. The same authors [12] proposed multi attribute decision making using rough accuracy score function, and also proposed cotangent similarity measure under rough neutrosophic environment [13]. The same authors [14] further proposed some similarity measures namely Dice and Jaccard similarity measures in rough neutrosophic environment. Olariu [15] introduced the concept of hypercomplex numbers and studied some of its properties in 2002, then studied exponential and trigonometric form, the concept of analyticity, contour integration and residue. Mandal and Basu [16] studied hyper-complex similarity measure for SVNS and presented application in decision making. No studies have been made on multi-attribute decision making using tri-complex rough neutrosophic environment.

In this paper, we develop rough tri-complex neutrosophic multi-attribute decision making based on rough tri-complex neutrosophic similarity function (RTNSF). RNSs are represented as a tri-complex number. The distance measured between so transformed tri-complex numbers produce the similarity value. Section 2 presents preliminaries of neutrosophic sets and rough neutrosophic sets. Section 3 describes some basic ideas of tri-complex number. Section 4 presents tri-complex similarity measures in rough neutrosophic environment. Section 5 is devoted to present multi attribute decision-making method based on rough tri-complex neutrosophic similarity function. Section 6 presents a numerical example of the proposed approach. Section 7 presents comparison with existing rough neutrosophic similarity measures. Finally, section 8 presents concluding remarks and scope of future research.

2 Neutrosophic Preliminaries

Definition 2.1 [4, 5]

Let U be an universe of discourse. Then the neutrosophic set A can be presented in the form:

$$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in U\},$$

where the functions T, I, F: $U \rightarrow\ ]^-0,1^+[$ represent respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in U$ to the set P satisfying the following the condition:

$$^{-}0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^{+}$$

Wang et al. [6] mentioned that the neutrosophic set assumes the value from real standard or non-standard subsets of $]^{-}0, 1^{+}[$ based on philosophical point of view. So instead of $]^{-}0, 1^{+}[$ Wang et al. [6] consider the interval [0, 1] for technical applications, because $]^{-}0, 1^{+}[$ is difficult to apply in the real applications such as scientific and engineering problems. For two neutrosophic sets (NSs), $A_{NS} = \{<x: T_A(x), I_A(x), F_A(x)> \mid x \in X\}$ and $B_{NS} = \{<x, T_B(x), I_B(x), F_B(x)> \mid x \in X\}$ the two relations are defined as follows:

(1) $A_{NS} \subseteq B_{NS}$ if and only if $T_A(x) \le T_B(x), I_A(x) \ge I_B(x), F_A(x) \ge F_B(x)$

(2) $A_{NS} = B_{NS}$ if and only if $T_A(x) = T_B(x), I_A(x) = I_B(x), F_A(x) = F_B(x)$

2.2 Single valued neutrosophic sets

Definition 2.2 [6]

Assume that X be a space of points (objects) with generic elements in X denoted by x. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity membership function $F_A(x)$, for each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. When X is continuous, a SVNS A can be written as follows:

$$A = \int_x \frac{<T_A(x), I_A(x), F_A(x)>}{x} : x \in X$$

When X is discrete, a SVNS A can be written as follows:

$$A = \sum_{i=1}^{n} \frac{<T_A(x_i), I_A(x_i), F_A(x_i)>}{x_i} : x_i \in X$$

For two SVNSs , $A_{SVNS} = \{<x: T_A(x), I_A(x), F_A(x)> \mid x \in X\}$ and $B_{SVNS} = \{<x, T_B(x), I_B(x), F_B(x)> \mid x \in X\}$ the two relations are defined as follows:

(1) $A_{SVNS} \subseteq B_{SVNS}$
if and only if $T_A(x) \le T_B(x), I_A(x) \ge I_B(x), F_A(x) \ge F_B(x)$

(2) $A_{SVNS} = B_{SVNS}$
if and only if $T_A(x) = T_Q(x), I_A(x) = I_B(x), F_A(x) = F_B(x)$ for any $x \in X$.

2.3 Rough neutrosophic set

Definition 2.2.1 [1], [2]

Let Z be a non-null set and R be an equivalence relation on Z. Let A be neutrosophic set in Z with the membership function T_A, indeterminacy function I_A and non-membership function F_A. The lower and the upper approximations of A in the approximation (*Z, R*) denoted by $\underline{N}(A)$ and $\overline{N}(A)$ are respectively defined as follows:

$$\underline{N}(A)=\left\langle <x, T_{\underline{N}(A)}(x), I_{\underline{N}(A)}(x), F_{\underline{N}(A)}(x)>/z\in[x]_R, x\in Z\right\rangle \quad (1)$$

$$\overline{N}(A)=\left\langle <x, T_{\overline{N}(A)}(x), I_{\overline{N}(A)}(x), F_{\overline{N}(A)}(x)>/z\in[x]_R, x\in Z\right\rangle \quad (2)$$

where, $T_{\underline{N}(A)}(x) = \wedge_z \in [x]_R\ T_A(z)$,

$I_{\underline{N}(A)}(x) = \wedge_z \in [x]_R\ I_A(z)$, $F_{\underline{N}(A)}(x) = \wedge_z \in [x]_R\ F_A(z)$,

$T_{\overline{N}(A)}(x) = \vee_z \in [x]_R\ T_A(z)$, $I_{\overline{N}(A)}(x) = \vee_z \in [x]_R\ T_A(z)$,

$F_{\overline{N}(A)}(x) = \vee_z \in [x]_R\ I_A(z)$

So, $0 \le T_{\underline{N}(A)}(x) + I_{\underline{N}(A)}(x) + F_{\underline{N}(A)}(x) \le 3$ and $0 \le T_{\overline{N}(A)}(x) + I_{\overline{N}(A)}(x) + F_{\overline{N}(A)}(x) \le 3$ hold good. Here $\vee$ and $\wedge$ denote “max” and “min’’ operators respectively. $T_A(z)$, $I_A(z)$ and $F_A(z)$ are the membership, indeterminacy and non-membership of *z* with respect to A. $\underline{N}(A)$ and $\overline{N}(A)$ are two neutrosophic sets in *Z*.

Thus, NS mappings $\underline{N}$, $\overline{N}$: $N(Z) \rightarrow N(Z)$ are respectively referred to as the lower and upper rough NS approximation operators, and the pair $(\underline{N}(A), \overline{N}(A))$ is called the rough neutrosophic set in (*Z*, R).

Based on the above mentioned definition, it is observed that $\underline{N}(A)$ and $\overline{N}(A)$ have constant membership on the equivalence classes of *R*, if $\underline{N}(A) = \overline{N}(A)$; i.e. $T_{\underline{N}(A)}(x) = T_{\overline{N}(A)}(x)$, $I_{\underline{N}(A)}(x) = I_{\overline{N}(A)}(x)$, $F_{\underline{N}(A)}(x) = F_{\overline{N}(A)}(x)$.

For any x belongs to *Z,* P is said to be a definable neutrosophic set in the approximation (*Z, R*). Obviously, zero neutrosophic set (0_N) and unit neutrosophic sets (1_N) are definable neutrosophic sets.

Definition 2.2.2 [1], [2]

Let N(A) = ($\underline{N}(A), \overline{N}(A)$) is a rough neutrosophic set in (*Z,* R). The rough complement of *N*(A) is denoted by $\sim N(A) = (\underline{N}(A)^c, \overline{N}(A)^c)$, where $\underline{N}(A)^c$, $\overline{N}(A)^c$ are the complements of neutrosophic sets of $\underline{N}(A), \overline{N}(A)$ respectively.

$$\underline{N}(A)^c=\left\langle <x, F_{\underline{N}(A)}(x), 1-I_{\underline{N}(A)}(x), T_{\underline{N}(A)}(x)>/, x\in Z\right\rangle, \text{ and}$$

$$\overline{N}(A)^c=\left\langle <x, F_{\underline{N}(A)}(x), 1-I_{\overline{N}(A)}(x), T_{\overline{N}(A)}(x)>/, x\in Z\right\rangle \quad (3)$$

Definition 2.2.3 [1], [2]

Let $N(A)$ and $N(B)$ are two rough neutrosophic sets respectively in *Z*, then the following definitions hold good:

$$N(A) = N(B) \Leftrightarrow \underline{N}(A) = \underline{N}(B) \wedge \overline{N}(A) = \overline{N}(B)$$

$$N(A) \subseteq N(B) \Leftrightarrow \underline{N}(A) \subseteq \underline{N}(B) \wedge \overline{N}(A) \subseteq \overline{N}(B)$$

$$N(A) \cup N(B) = < \underline{N}(A) \cup \underline{N}(B), \overline{N}(A) \cup \overline{N}(B) >$$

$$N(A) \cap N(B) = < \underline{N}(A) \cap \underline{N}(B), \overline{N}(A) \cap \overline{N}(B) >$$

$$N(A) + N(B) = < \underline{N}(A) + \underline{N}(B), \overline{N}(A) + \overline{N}(B) >$$

$$N(A) \,.\, N(B) = < \underline{N}(A) \,.\, \underline{N}(B), \overline{N}(A) \,.\, \overline{N}(B) >$$

If A, B, C are the rough neutrosophic sets in (*Z*, R), then the following propositions are stated from definitions

Proposition 1 [1], [2]

1. $\sim A(\sim A) = A$
2. $A \cup B = B \cup A, A \cap B = B \cap A$
3. $(A \cup B) \cup C = A \cup (B \cup C), (A \cap B) \cap C = A \cap (B \cap C)$
4. $(A \cup B) \cap C = (A \cup B) \cap (A \cup C), (A \cap B) \cup C = (A \cap B) \cup (A \cap C)$

Proposition 2 [1], [2]

De Morgan's Laws are satisfied for rough neutrosophic sets N(A) and N(B)

1. $\sim (N(A) \cup N(B)) = (\sim N(A)) \cap (\sim N(B))$
2. $\sim (N(A) \cap N(B)) = (\sim N(A)) \cup (\sim N(B))$

For proof of the proposition, see [1], [2].

Proposition 3 [1], [2]:

If A and B are two neutrosophic sets in *U* such that $A \subseteq B$, then $N(A) \subseteq N(B)$

1. $N(A \cap B) \subseteq N(A) \cap N(B)$
2. $N(A \cup B) \supseteq N(A) \cup N(B)$

For proof of the proposition, see [1], [2].

Proposition 4 [1], [2]:

1. $\underline{N}(A) = \sim \overline{N}(\sim A)$
2. $\overline{N}(A) = \sim \underline{N}(\sim A)$
3. $\underline{N}(A) \subseteq \overline{N}(A)$

For proof of the proposition, see [1], [2].

3 Basic concept of Tri-complex number in three dimension

Olariu [15] described a system of hypercomplex numbers in three dimensions, where multiplication is associative and commutative. Hypercomplex numbers can be expressed in exponential and trigonometric forms and for which the concepts of analytic tri-complex function, contour integration and residue are well defined. Olariu [15] introduced the concept of tri-complex numbers which is expressed in the form $u = x + h_1 y + h_2 z$, the variables x, y, and z being real numbers. The multiplication rules [15] for the complex units h_1, h_2 are given by $h_1^2 = h_2$, $h_2^2 = h_1$, $1.\ h_1 = h_1$, $1.\ h_2 = h_2$, $h_1.\ h_2 = 1$. Geometrically, tri-complex number u is expressed by the point D(x, y, z). Assume that O be the origin of the x, y, z axes, T be the trisector line $x = y = z$ of the positive octant. Also, let L be the plane $x + y + z = 0$ passing through the origin O and perpendicular to T. The tricomplex number u can be expressed as the projection p of the segment OD along the line T, by the distance δ from D to the line T, and by the azimuthal angle ϕ in the plane L (see *Fig. 1* below).

Here, ϕ is the angle between the projection of D on the plane L and the straight line which is the intersection of the plane L and the plane determined by line T and x axis. ϕ satisfied the relation $0 \leq \phi \leq 2\pi$. The amplitude ψ of a tri-complex number is defined as $\psi = (x^3+y^3+z^3 - 3xyz)^{1/3}$. The polar angle θ of OD with respect to the tri-sector line T is presented as $tan\theta = \delta/p$. θ satisfies the inequality $0 \leq \theta \leq 2\pi$. The distance d from D to the origin is obtained as $d^2 = x^2 + y^2 + z^2$. The division $1/(x + h_1 y + h_2 z)$ is possible if $\psi \neq 0$.

The product of two tri-complex numbers is equal to zero if both numbers are equal to zero, or if one of the tri-complex numbers lies in the plane L and the other on the T line. The tri-complex number $u = x + h_1 y + h_2 z$ can be represented by the point D having coordinates *(x, y, z).* The projection $p = OQ$ of the line OD on the tri-sector line $x = y = z$, which has the unit tangent $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$, *is* $p = \frac{1}{\sqrt{3}}(x+y+z)$.

The distance $\delta = DQ$ from D to the tri-sector line $x = y = z$, measured as the distance from the point $D(x, y, z)$ to the point Q of coordinates $\left(\frac{x+y+z}{3}, \frac{x+y+z}{3}, \frac{x+y+z}{3}\right)$, is $\delta^2 = \frac{2}{3}\left(x^2+y^2+z^2-xy-yz-zx\right)$.

The plane through the *point* D and perpendicular to the tri-sector line T intersects the x-axis at point A of coordinates $(x + y + z, 0, 0)$, the y-axis at point B of coordinates $(0, x + y + z, 0)$, and the z-axis at point C of coordinates $(0, 0, x + y + z)$. The expression of ϕ in terms of *x, y, z* can be obtained in a system of coordinates defined by the unit vectors as follows:

$$\zeta_1 = \frac{1}{\sqrt{6}}(2,-1,-1),\ \zeta_2 = \frac{1}{\sqrt{2}}(0,-1,-1),\ \zeta_3 = \frac{1}{\sqrt{3}}(1, 1, 1).$$

The relation between the coordinates of D in the systems $(\zeta_1, \zeta_2, \zeta_3)$ and *x, y, z* can be presented as follows:

$$\begin{bmatrix}\zeta_1\\ \zeta_2\\ \zeta_3\end{bmatrix} = \begin{bmatrix}\frac{2}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{6}}\\ 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} \tag{4}$$

$$[\zeta_1, \zeta_2, \zeta_3] = \left(\frac{1}{\sqrt{6}}(2x-y-z), -\frac{1}{\sqrt{2}}(y+z), \frac{1}{\sqrt{3}}(x+y+z)\right) \tag{5}$$

Also, $\cos\varphi = \dfrac{2x-y-z}{2(x^2+y^2+z^2-xy-yz-zx)}$ (6)

$\sin\varphi = \dfrac{\sqrt{3}(y-z)}{2(x^2+y^2+z^2-xy-yz-zx)}$ (7)

The angle θ between the line OD and the tri-sector line T is given by $tan\theta = \frac{\delta}{p}$.

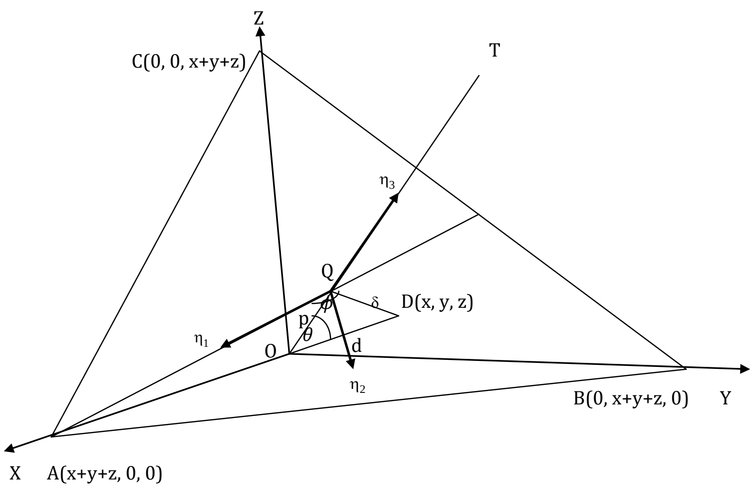

Figure 1. Tri-complex number.

Tri-complex variables p, d, θ, and ϕ for the tri-complex number x+ $h_1 y$ + $h_2 z$, represented by the point $D(x, y, z)$. The angle ϕ is shown in the plane parallel to L, passing through D, which intersects the tri-sector line T at Q. The orthogonal axes: η_1, η_2, η_3 intersect at the origin Q. The axis $Q\eta_1$ is parallel to the axis $O\eta_1$, the axis $Q\eta_2$ is parallel to the axis $O\eta_2$ and the axis $Q\eta_3$ is parallel to the axis $O\eta_3$, so that, in the plane ABC, the angle ϕ is measured from the line QA.

4 Tri-complex similarity measure in RNS

From the basic concept of Tri-complex number we have the following relations.

$$tan\theta = \frac{\delta}{p} = \frac{\sqrt{(x-y)^2+(y-z)^2+(z-x)^2}}{x+y+z} \quad (8)$$

where, $\delta^2 = \frac{2}{3}\left(x^2+y^2+z^2-xy-yz-zx\right)$ and $p = \frac{1}{\sqrt{3}}(x+y+z)$.

$$\cos\varphi = \frac{2x-y-z}{2(x^2+y^2+z^2-xy-yz-zx)}$$

$$\sin\varphi = \frac{\sqrt{3}(y-z)}{2(x^2+y^2+z^2-xy-yz-zx)}$$

This implies, $\tan\varphi = \frac{\sqrt{3}(y-z)}{(2x-y-z)}$ (9)

We now define a function for similarity measure between rough neutrosophic set (RNSs). The function satisfies the basic properties of similarity measure method in tri-complex system. The rough tri-complex similarity function is defined as follows (see definition 1).

Definition 1:

Let A=< $\left\langle (\underline{T}_A(x_i), \underline{I}_A(x_i), \underline{F}_A(x_i)), (\overline{T}_A(x_i), \overline{I}_A(x_i), \overline{F}_A(x_i)) \right\rangle$ > and

B = < $\left\langle (\underline{T}_B(x_i), \underline{I}_B(x_i), \underline{F}_B(x_i)), (\overline{T}_B(x_i), \overline{I}_B(x_i), \overline{F}_B(x_i)) \right\rangle$ > be two rough neutrosophic numbers in X ={x_i: i = 1, 2, ..., n}.

Also let, $tan\theta_1 = \frac{\sqrt{(\delta T_A(x_i) - \delta I_A(x_i))^2 + (\delta I_A(x_i) - \delta F_A(x_i))^2 + (\delta F_A(x_i) - \delta T_A(x_i))^2}}{\delta T_A(x_i) + \delta I_A(x_i) + \delta F_A(x_i)}$

$$tan\theta_2 = \frac{\sqrt{(\delta T_B(x_i) - \delta I_B(x_i))^2 + (\delta I_B(x_i) - \delta F_B(x_i))^2 + (\delta F_B(x_i) - \delta T_B(x_i))^2}}{\delta T_B(x_i) + \delta I_B(x_i) + \delta F_B(x_i)}$$

$$\tan\varphi_1 = \frac{\sqrt{3}\left[\delta I_A(x_i) - \delta F_A(x_i)\right]}{2\delta T_A(x_i) - \delta I_A(x_i) - \delta F_A(x_i)}$$

$$\tan\varphi_2 = \frac{\sqrt{3}\left[\delta I_B(x_i) - \delta F_B(x_i)\right]}{2\delta T_B(x_i) - \delta I_B(x_i) - \delta F_B(x_i)}.$$

Taking, $tan\theta_1 = \Delta_{\theta_1}$, $tan\theta_2 = \Delta_{\theta_2}$, $\tan\varphi_1 = \Delta_{\phi_1}$, $\tan\varphi_2 = \Delta_{\phi_2}$, the rough tri-complex neutrosophic similarity function (RTNSF) between two neutrosophic sets A and B is defined as follows:

$S_{RTNSF}(A, B) =$

$$\frac{1}{2}\left[\frac{\left(1+\nabla_{\theta_1}\nabla_{\theta_2}\right)^2}{1+\nabla_{\theta_1}^2+\nabla_{\theta_2}^2+\nabla_{\theta_1}^2\nabla_{\theta_2}^2}+\frac{\left(1+\nabla_{\phi_1}\nabla_{\phi_2}\right)^2}{1+\nabla_{\phi_1}^2+\nabla_{\phi_2}^2+\nabla_{\phi_1}^2\nabla_{\phi_2}^2}\right] \tag{10}$$

where,

$$\delta T_A(x_i) = \left(\frac{\underline{T}_A(x_i)+\overline{T}_A(x_i)}{2}\right),$$

$$\delta T_B(x_i) = \left(\frac{\underline{T}_B(x_i)+\overline{T}_B(x_i)}{2}\right),$$

$$\delta I_A(x_i) = \left(\frac{\underline{I}_A(x_i)+\overline{I}_A(x_i)}{2}\right),$$

$$\delta I_B(x_i) = \left(\frac{\underline{I}_B(x_i)+\overline{I}_B(x_i)}{2}\right),$$

$$\delta F_A(x_i) = \left(\frac{\underline{F}_A(x_i)+\overline{F}_A(x_i)}{2}\right),$$

$$\delta F_B(x_i) = \left(\frac{\underline{F}_B(x_i)+\overline{F}_B(x_i)}{2}\right).$$

Also, $[\delta T_A(x), \delta I_A(x), \delta F_A(x)] \neq [0, 0, 0]$ and $[\delta T_B(x), \delta I_B(x), \delta F_B(x)] \neq [0, 0, 0]$, $i = 1, 2, \ldots, n$.

The proposed rough neutrosophic operator satisfies the following conditions of similarity measures.

P1. $0 \leq S_{RTNSF}(A, B) \leq 1$

P2. $S_{RTNSF}(A, B) = S_{RTNSF}(B, A)$

P3. $S_{RTNSF}(A, B) = 1$ if $A = B$

Proof:

P1. Since $2\nabla_{\theta_1}\nabla_{\theta_2} \leq \nabla_{\theta_1}^2 + \nabla_{\theta_2}^2$ and $2\nabla_{\varphi_1}\nabla_{\varphi_2} \leq \nabla_{\varphi_1}^2 + \nabla_{\varphi_2}^2$ so it is obvious that $0 \leq S_{RTNSF}(A, B) \leq 1$

P2. Obviously, $S_{RTNSF}(A, B) = S_{RTNSF}(B, A)$

P3. When A = B then, $\nabla_{\theta_1} = \nabla_{\theta_2}$ and $\nabla_{\varphi_1} = \nabla_{\varphi_2}$ so, $S_{RTNSF}(A, B) = (1/2) \times (1+1) = 1$.

When, $S_{RTNSF}(A, B) = 1$ then, $2\nabla_{\theta_1}\nabla_{\theta_2} = \nabla_{\theta_1}^2 + \nabla_{\theta_2}^2$ and $2\nabla_{\varphi_1}\nabla_{\varphi_2} = \nabla_{\varphi_1}^2 + \nabla_{\varphi_2}^2$. It is possible when $\nabla_{\theta_1} = \nabla_{\theta_2}$ and $\nabla_{\varphi_1} = \nabla_{\varphi_2}$. This implies that A = B.

Alternative proof:

Assume that

$$H(A, B) = \frac{1}{2}\left[\frac{1}{1+\tan^2(\alpha_1 - \alpha_2)} + \frac{1}{1+\tan^2(\beta_1 - \beta_2)}\right]$$

$$= \frac{1}{2}\left[\frac{(1+\tan\alpha_1\tan\alpha_2)^2}{1+\tan^2\alpha_1 + \tan^2\alpha_2 + \tan^2\alpha_1\tan^2\alpha_2} + \frac{(1+\tan\beta_1\tan\beta_2)^2}{1+\tan^2\beta_1 + \tan^2\beta_2 + \tan^2\beta_1\tan^2\beta_2}\right]$$

Taking, $\tan\alpha_1 = \nabla_{\theta_1}$, $\tan\alpha_2 = \nabla_{\theta_2}$, $\tan\beta_1 = \nabla_{\varphi_1}$, $\tan\beta_2 = \nabla_{\varphi_2}$, then,

$$H(A, B) = S_{RTNSF}(A, B).$$

The function H(A, B) obviously satisfies the following conditions.

P1. $0 \leq H(A, B) \leq 1$ (obvious)

P2. H(A, B) = H(B, A) (obvious)

P3. When A = B then $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$ then, H(A, B) = 1.

Conversely, if H(A, B) = 1 then obviously, $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$.

This implies that A = B.

5 Decision making procedure under rough tri-complex neutrosophic similarity measure

In this section, we apply rough tri-complex similarity measures between RNSs to the multi-criteria decision making problem. Let $A = A_1, A_2, \ldots, A_m$ be a set of alternatives and $C = C_1, C_2, \ldots, C_n$ be a set of attributes.

The proposed decision making method is described using the following steps.

Step 1: Construction of the decision matrix with rough neutrosophic number

The decision maker considers a decision matrix with respect to *m* alternatives and *n* attributes in terms of rough neutrosophic numbers as follows.

$$D = \left\langle \underline{d}_{ij}, \overline{d}_{ij} \right\rangle_{m\times n} = \qquad (11)$$

	C_1	C_2	$\cdots$	C_n
A_1	$\left\langle \underline{d}_{11}, \overline{d}_{11} \right\rangle$	$\left\langle \underline{d}_{12}, \overline{d}_{12} \right\rangle$	...	$\left\langle \underline{d}_{1n}, \overline{d}_{1n} \right\rangle$
A_2	$\left\langle \underline{d}_{21}, \overline{d}_{21} \right\rangle$	$\left\langle \underline{d}_{22}, \overline{d}_{22} \right\rangle$	...	$\left\langle \underline{d}_{2n}, \overline{d}_{2n} \right\rangle$
.	...	...	...	...
.	...	...	...	...
A_m	$\left\langle \underline{d}_{m1}, \overline{d}_{m1} \right\rangle$	$\left\langle \underline{d}_{m2}, \overline{d}_{m2} \right\rangle$	...	$\left\langle \underline{d}_{mn}, \overline{d}_{mn} \right\rangle$

Table1. Rough neutrosophic decision matrix.

Here $\left\langle \underline{d}_{ij}, \overline{d}_{ij} \right\rangle$ is the rough neutrosophic number according to the i-th alternative and the j-th attribute.

Step 2: Determination of the weights of attribute

Assume that the weight of the attributes C (j = 1, 2, ... , n) considered by the decision-maker be w_j ((j = 1, 2, ... , n)) such that $\forall w_j \in [0, 1]$ (j = 1, 2, ..., n) and $\sum_{j=1}^{n} w_j = 1$.

Step 3: Determination of the benefit type attribute and cost type attribute

Generally, the evaluation attribute can be categorized into two types: benefit attribute and cost attribute. Let K be a set of benefit attribute and M be a set of cost attribute. In the proposed decision-making method, an ideal alternative can be identified by using a maximum operator for the benefit attribute and a minimum operator for the cost attribute to determine the best value of each criterion among all alternatives. Therefore, we define an ideal alternative as follows:

$$A^* = \{C_1^*, C_2^*, \ldots, C_m^*\},$$

where benefit attribute

$$C_j^* = \left[\max_i T_{C_j}^{(A_i)}, \min_i I_{C_j}^{(A_i)}, \min_i F_{C_j}^{(A_i)} \right]$$

and the cost attribute

$$C_j^* = \left[\min_i T_{C_j}^{(A_i)}, \max_i I_{C_j}^{(A_i)}, \max_i F_{C_j}^{(A_i)} \right]$$

Step 4: Determination of the overall weighted rough tri-complex neutrosophic similarity function (WRTNSF) of the alternatives

We define weighted rough tri-complex neutrosophic similarity function as follows.

$$S_{WRTNSF}(A, B) = \sum_{j=1}^{n} w_j S_{WRTNSF}(A, B) \quad (12)$$

Properties:

This weighted rough tri-complex neutrosophic operator satisfies the following conditions of similarity measures.

P1. $0 \leq S_{WRTNSF}(A, B) \leq 1$

P2. $S_{WRTNSF}(A, B) = S_{WRTNSF}(B, A)$

P3. $S_{WRTNSF}(A, B) = 1$ if $A = B$

Proofs:

P1. Since $2D_{\theta_1}D_{\theta_2} \leq D_{\theta_1}^2 + D_{\theta_2}^2$ and $2D_{\phi_1}D_{\phi_2} \leq D_{\phi_1}^2 + D_{\phi_2}^2$ and $\sum_{j=1}^{n} w_j = 1$, so it is obvious that $0 \leq S_{WRTNSF}(A, B) \leq 1$

P2. Obviously, $S_{WRTNSF}(A, B) = S_{WRTNSF}(B, A)$

P3. When A = B then, $D_{\theta_1} = D_{\theta_2}$ and $D_{\phi_1} = D_{\phi_2}$ so, $S_{WRTNSF}(A, B) = \sum_{j=1}^{n} w_j = 1$.

When, $S_{WRTNSF}(A, B) = 1$ then, $2D_{\theta_1}D_{\theta_2} = D_{\theta_1}^2 + D_{\theta_2}^2$ and $2D_{\phi_1}D_{\phi_2} = D_{\phi_1}^2 + D_{\phi_2}^2$. It is possible when $D_{\theta_1} = D_{\theta_2}$ and $D_{\phi_1} = D_{\phi_2}$. Again, $\sum_{j=1}^{n} w_j = 1$. This implies that A = B.

Step 5: Ranking the alternatives

Using the weighted rough tri-complex neutrosophic similarity measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be easily selected with the highest similarity value.

Step 6: End.

6 Numerical Example

Let us assume that a decision maker intends to select the most suitable smartphone for rough use from the four initially chosen smartphones (S_1, S_2, S_3) by considering four attributes namely: features C_1, reasonable price C_2, customer care C_3, risk factor C_4. Based on the proposed approach discussed in section 5, the considered problem is solved using the following steps:

Step 1: Construction of decision matrix with rough neutrosophic numbers

The decision maker considers a decision matrix with respect to three alternatives and four attributes in terms of rough neutrosophic numbers as follows (see the *Table 2*).

$d_S = \left\langle \underline{N}(P), \overline{N}(P) \right\rangle_{3\times 4} =$

	C_1	C_2	C_3	C_4
A_1	$\left\langle (0.6,0.3,0.3), (0.8,0.1,0.1) \right\rangle$	$\left\langle (0.6,0.4,0.4), (0.8,0.2,0.2) \right\rangle$	$\left\langle (0.6,0.4,0.4), (0.8,0.2,0.2) \right\rangle$	$\left\langle (0.7,0.4,0.4), (0.9,0.2,0.2) \right\rangle$
A_2	$\left\langle (0.7,0.3,0.3), (0.9,0.1,0.3) \right\rangle$	$\left\langle (0.6,0.3,0.3), (0.8,0.3,0.3) \right\rangle$	$\left\langle (0.6,0.2,0.2), (0.8,0.4,0.2) \right\rangle$	$\left\langle (0.7,0.3,0.3), (0.9,0.3,0.3) \right\rangle$
A_3	$\left\langle (0.6,0.2,0.2), (0.8,0.0,0.2) \right\rangle$	$\left\langle (0.7,0.3,0.3), (0.9,0.1,0.1) \right\rangle$	$\left\langle (0.7,0.4,0.6), (0.9,0.2,0.4) \right\rangle$	$\left\langle (0.6,0.3,0.2), (0.8,0.1,0.2) \right\rangle$

Table 2. Decision matrix with rough neutrosophic number.

Step 2: Determination of the weights of the attributes

The weight vectors considered by the decision maker are 0.30, 0.30, 0.30 and 0.10 respectively.

Step 3: Determination of the benefit attribute and cost attribute

Here three benefit types attributes C_1, C_2, C_3 and one cost type attribute C_4.

$$A^* = [(0.8, 0.1, 0.2), (0.8, 0.2, 0.2), (0.8, 0.3, 0.3), (0.0.7, 0.3,0.3)]$$

Step 4: Determination of the overall weighted rough tri-complex neutrosophic similarity function (WRHNSF)of the alternatives

We calculate weighted rough tri-complex neutrosophic similarity values as follows.

$$S_{WRTNSF}(A_1, A^*) = 0.99554$$

$$S_{WRTNSF}(A_2, A^*) = 0.99253$$

$$S_{WRTNSF}(A_3, A^*) = 0.99799$$

Step 5: Ranking the alternatives

Ranking the alternatives is prepared based on the descending order of similarity measures. Highest value reflects the best alternative. Here,

$$S_{WRTNSF}(A_3, A^*) \prec S_{WRTNSF}(A_1, A^*) \prec S_{WRTNSF}(A_2, A^*).$$

Hence, the Smartphone A_3 is the best alternative for rough use.

Step 6: End.

7 Comparison with other similarity measures

We compare our result to other existing rough neutrosophic similarity measures as follows.

Rough neutrosophic similarity measure	*Measure value*	*Ranking order*
Weighted rough Cosine similarity measure	$C_{WRNS}(A_1, A^*) = 0.99260$ $C_{WRNS}(A_2, A^*) = 0.99083$ $C_{WRNS}(A_3, A^*) = 0.99482$	$A_3 \succ A_1 \succ A_2$
Weighted rough Dice similarity measure	$D_{WRNS}(A_1, A^*) = 0.98606$ $D_{WRNS}(A_2, A^*) = 0.98559$ $D_{WRNS}(A_3, A^*) = 0.98926$	$A_3 \succ A_1 \succ A_2$
Weighted rough Jaccard similarity measure	$J_{WRNS}(A_1, A^*) = 0.97856$ $J_{WRNS}(A_2, A^*) = 0.97772$ $J_{WRNS}(A_3, A^*) = 0.97891$	$A_3 \succ A_1 \succ A_2$
Weighted rough Tri-complex similarity measure	$S_{WRTNSF}(A_1, A^*) = 0.99554$ $S_{WRTNSF}(A_2, A^*) = 0.99253$ $S_{WRTNSF}(A_3, A^*) = 0.99799$	$A_3 \succ A_1 \succ A_2$

Table 3. Comparison with other existing rough neutrosophic similarity measures.

8 Conclusion

In this paper, we have proposed rough tri-complex similarity measure based multi-attribute decision making of rough neutrosophic environment and proved some of its basic properties. We have presented an application, namely selection of best smart-phone for rough use. We have also presented comparison with other existing rough neutrosophic similarity measures. In this paper, predefined weights of the decision makers have been considered. The proposed approach can be extended for generalized hypercomplex system with weighting scheme.

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