On the hybrid mean value of the Smarandache double factorial function and the approximate pseudo-Smarandache function

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Abstract: The elementary and the analysis methods were used to study the properties of Smarandache double factorial function and the approximate pseudo-Smarandache function. Some hybrid mean properties and some asymptotic formulas were obtained which helped to promote the relevant research work of classical arithmetical function.

Key words: Smarandache double factorial function; approximate pseudo-Smarandache function; composite function; mean value

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1.1 Smarandache double factorial function

\[ \text{sdf}(n) = \min \{ m \mid m!! = kn \} \]

where \( m!! \) is the double factorial of \( m \), defined as:

\[ m!! = \begin{cases} 1 \cdot 3 \cdot 5 \cdots m & \text{if } m \text{ is odd} \\ 2 \cdot 4 \cdot 6 \cdots m & \text{if } m \text{ is even} \end{cases} \]

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$$\sum_{n=2}^{\infty} Sdf(n) = \frac{5\pi^2}{48} \frac{x^2}{\ln x} + \sum_{r=2}^{\infty} \frac{c_r \cdot x^2}{\ln^2 x} + O\left(\frac{x^2}{\ln^3 x}\right).$$

3 Smarandache\, U_*(n) \, U(n)

$$U_*(n) = \min \left\{ m \in \mathbb{N}^*: n \leq m + \frac{1}{2} m(m - 1) \cdot (r - 2) \mid r \in \mathbb{N}, r \geq 3 \right\};$$

$$U(n) = \max \left\{ m \in \mathbb{N}^*: n \geq m + \frac{1}{2} m(m - 1) \cdot (r - 2) \mid r \in \mathbb{N}, r \geq 3 \right\}.$$
关于Smarandache双阶乘函数与近似伪Smarandache函数的混合均值

\[ \sum_{i=0}^{n} Sdf(Z(n)) = \frac{5\pi^2}{18} \frac{(2x)^{\frac{x}{2}}}{\ln \sqrt{2x}} + O\left( \frac{x^{\frac{x}{2}}}{\ln \sqrt{x}} \right) \]

\[ \sum_{i=0}^{n} Sdf(Z(n)) = \frac{5\pi^2}{18} \frac{(2x)^{\frac{x}{2}}}{\ln \sqrt{2x}} + O\left( \frac{x^{\frac{x}{2}}}{\ln \sqrt{x}} \right) \]

\[ \sum_{i=0}^{n} Sdf(U(n)) = \frac{5\pi^2}{144} \frac{(2x)^{\frac{x}{2}}}{\ln \sqrt{2x}} + \sum_{i=2}^{n} \frac{f_i \cdot (2x)^{\frac{x}{2}}}{\ln^i \sqrt{2x}} + O\left( \frac{x^{\frac{x}{2}}}{\ln^{i+1} \sqrt{x}} \right) \]

\[ \sum_{i=0}^{n} Sdf(U(n)) = \frac{5\pi^2}{144} \frac{(2x)^{\frac{x}{2}}}{\ln \sqrt{2x}} + \sum_{i=2}^{n} \frac{h_i \cdot (2x)^{\frac{x}{2}}}{\ln^i \sqrt{2x}} + O\left( \frac{x^{\frac{x}{2}}}{\ln^{i+1} \sqrt{x}} \right) \]

\[ f_i, h_i = 2 \cdot 3 \cdot \ldots \cdot (i) \]

\[ \pi(x) = \sum_{p \leq x} \phi(p) \leq x, \quad n = 1 \cdot \pi(x) = \pi(x) \]

\[ \pi(x) = \sum_{p \leq x} \frac{a(p) \cdot x}{\ln^2 x} + O\left( \frac{x}{\ln^{i+1} x} \right) \]

\[ c_i = (i - 1) \cdot \left( \frac{\pi(i)}{i(i-1)} \right) \]

\[ \sum_{n=1}^{\pi(x)} a(n) f(n) = A(x) f(x) - A(y) f(y) - \int_{r}^{x} A(t) f'(t) dt \]

\[ P(n) \equiv n \equiv \quad | \quad 0 < y < x \]

\[ \sum_{n=1}^{\pi(x)} a(n) f(n) = A(x) f(x) - A(y) f(y) - \int_{r}^{x} A(t) f'(t) dt \]

\[ (1) \quad P > \sqrt{n} \quad n = 1 \mod{2} \quad Sdf(n) = P(n) \]

\[ (2) \quad P > \sqrt{n} \quad n = 0 \mod{2} \quad Sdf(n) = 2P(n) \]

\[ (3) \quad P > \sqrt{n} \quad Sdf(n) \leq \sqrt{n} \ln n \]

\[ (4) \quad P > \sqrt{n} \quad n = p_1^{n_1} p_2^{n_2} \cdots p_i^{n_i} \]

\[ (5) \quad 0 < p_1^{n_1} p_2^{n_2} \cdots p_i^{n_i} \leq \alpha \cdot n_1 \cdot \alpha \cdot \alpha \cdots \alpha \]

\[ n = 1 \mod{2} \quad Sdf(2n) = 2 \max \left\{ Sdf(p_i^{n_i}) \right\} \]

\[ n = 0 \mod{2} \quad 2n = 2^{n_1} n_2 \quad n_1 = 1 \mod{2} \quad n_2 = 1 \mod{2} \quad Sdf(n) \leq \max \left\{ Sdf(2n) \cdot Sdf(n_1) \right\} \]

\[ u_i(n) = \frac{1}{2} \left( 2^{m+1} + m(m+1) \cdot (r-2) \right) \]

\[ \sqrt{2(r-2)n} + O(1)^{1+\delta} \]
4.2 定理的证明

我们来完成定理的证明

在 $\sum_{n=1}^{\infty} Sdf(U, (n)) \geq \frac{1}{2} (2m + m(m-1) \cdot (r-2)) \forall n > 1$，我们有

$$
\frac{1}{2} (2m + m(m-1) \cdot (r-2)) \leq \frac{1}{2} (2m + m(m-1) \cdot (r-2)) + 1
$$

则有

$$
\frac{1}{2} (2m + m(m-1) \cdot (r-2)) + \frac{1}{2} (2m + m(m-1) \cdot (r-2)) + (r-2)m
$$

又令 $u_j = \frac{1}{2} (2m + m(m-1) \cdot (r-2)) + j$，则有

$$
u_j = \frac{1}{2} (2m + m(m-1) \cdot (r-2)) + j = 0 \text{ 时 } u_j = \frac{1}{2} (2m + m(m-1) \cdot (r-2)) + (r-2)m
$$

因此，对于任意正整数 $n \geq 1$，我们有 $U, (n) = m \cdot m \cdot m$

并且

$$
\frac{1}{2} (2m + M(M-1) \cdot (r-2)) \leq \frac{1}{2} (2m + M(M+1) \cdot (r-2))
$$

则有

$$
\frac{1}{2} (2m + M(M-1) \cdot (r-2)) \leq \frac{1}{2} (2m + M(M+1) \cdot (r-2)) + O(1) \text{ 和 lnM = } \frac{1}{2} \text{ lnx} + O(1)
$$

并且

$$
\sum_{n=1}^{\infty} Sdf(U, (n)) = \sum_{t=1}^{M-1} \left( \sum_{2 \leq M+M(M-1) \cdot (r-2) < t < M+M(M+1) \cdot (r-2)} Sdf(U, (n)) \right) + \sum_{t=1}^{M-1} \left( \sum_{2 \leq M+M(M-1) \cdot (r-2) < t < M+M(M+1) \cdot (r-2)} Sdf(U, (n)) \right)
$$

因此

$$
A(x) = \sum_{n=1}^{\infty} Sdf(n) \text{ 和 } M \cdot Sdf(M) = (r-2) \sum_{t=0}^{M-1} tSdf(t) + O(M \cdot M \cdot M)
$$

并且

$$
B(t) = MSdf(M) - \int_{1}^{M} A(t) dt =
$$

$$
M \left( \frac{5\pi^2}{48} \frac{M^2}{\ln M} + \sum_{i=1}^{\frac{M}{2}} \frac{c_i \cdot M^2}{\ln^2 M} + O \left( \frac{M^2}{\ln^{4+i} M} \right) \right) - \int_{1}^{M} \left( \frac{5\pi^2}{48} \frac{t^2}{\ln t} + \sum_{i=1}^{\frac{M}{2}} \frac{c_i \cdot t^2}{\ln^2 t} + O \left( \frac{t^2}{\ln^{4+i} t} \right) \right) dt =
$$

$$
\frac{5\pi^2}{48} \frac{M^2}{\ln M} + \sum_{i=1}^{\frac{M}{2}} \frac{c_i \cdot M^2}{\ln^2 M} + O \left( \frac{M^2}{\ln^{4+i} M} \right) - \frac{1}{3} \left( \frac{5\pi^2}{48} \frac{M^2}{\ln M} + \sum_{i=1}^{\frac{M}{2}} \frac{c_i \cdot M^2}{\ln^2 M} + O \left( \frac{M^2}{\ln^{4+i} M} \right) \right) =
$$

$$
\frac{5\pi^2}{72} \frac{M^2}{\ln M} + \frac{1}{3} \sum_{i=1}^{\frac{M}{2}} \frac{c_i \cdot M^2}{\ln M} + O \left( \frac{M^2}{\ln^{4+i} M} \right) =
$$

$$
\frac{5\pi^2}{72} \left( 2x \right)^{\frac{3}{2}} \ln \left( 2x \right)^{\frac{3}{2}} + \frac{1}{3} \sum_{i=1}^{\frac{M}{2}} \frac{c_i \cdot (2x)^{\frac{3}{2}}}{\ln^2 \left( 2x \right)} + O \left( \frac{x^{\frac{3}{2}}}{\ln^{4+i} \left( 2x \right)} \right).
$$
\[
\sum_{k=0}^n Sdf(U_*(n)) = \frac{5\pi^2}{72} \frac{(2x)^\frac{3}{2}}{\ln\sqrt{2x}} + \sum_{i=2}^n \frac{d_i \cdot (2x)^\frac{3}{2}}{\ln^{i-1}\sqrt{2x}} + O\left(\frac{\pi^2}{\ln^{2n+1}x}\right).
\]

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