关于 Smarandache 对偶函数的相关均值

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摘要 对任意正整数 $n$, Smarandache LCM对偶函数是满足 $[1, 2, ..., k] | n$ 的最小正整数，其中 $[1, 2, ..., k]$ 代表 $1, 2, ..., k$ 的最小公倍数。用初等方法研究 $\frac{S_n}{n}$，并给一个有趣的渐近公式。

关键词 Smarandache LCM 对偶函数 均值 渐近公式

1 Introduction and result

For any positive integer $n$, the famous F. Smarandache LCM function $S(n)$ is defined as the smallest positive integer $k$ such that $n | [1, 2, ..., k]$. This means $S(n) = \min\{k : k \in N, n | [1, 2, ..., k]\}$.

About the elementary properties of $S(n)$, many authors had studied it and obtained some interesting results; see references [2] and [3]. For example, Murthy proved that if $p$ is prime then $S(p) = S(n)$, where $S(n) = \min\{m : m | n, m \in N\}$ is the F. Smarandache function. Simultaneously, Murthy also proposed the following problem:

\[ S(n) = \min\{k : k \in N, n | [1, 2, ..., k]\} \]

Zhongtian Lü proved that for any real number $x > 1$ and fixed positive integer $k$, we have the asymptotic formula

\[ \sum_{n \leq x} \frac{S(n)}{n} = \pi^2 \frac{\pi}{12} \ln x + \frac{\pi}{2} \ln \frac{x}{\pi} + O \left( \frac{x}{\ln^2 x} \right) \]

Where $C = 2, 3, 5$ is a computable constant.

Chengliang Tian defined the F. Smarandache LCM dual function $S^*(n)$ as follows:

\[ S^*(n) = \max\{k : k \in N, n | [1, 2, ..., k]\} \]

For example, $S^*(2) = 1$, $S^*(3) = 2$, $S^*(5) = 1$, $S^*(7) = 2$, $S^*(11) = 1$.
SL^r(8) = 2, SL^r(9) = 4, SL^r(1) = 2 \ldots. Obviously, if \( n \) is an odd number, then \( SL^r(\frac{n}{h}) = 1 \). If \( n \) is an even number, the \( SL^r(\frac{n}{h}) \geq 2 \). About the mean value properties of \( SL^r(\frac{n}{h}) \), Chengliang Tian\(^4\) proved that for any real number \( x \geq 1 \), we have the asymptotic formula

\[
\sum_{n \leq x} SL^r(\frac{n}{h}) = cx + o\left(\frac{\ln^3 x}{\ln x}\right).
\]

Where \( c = \sum_{s=1}^{\infty} \sum_p \left( \frac{p-1}{p} \left( \frac{p-1}{p-1} \right) \right) \) is a constant.

The main purpose of this paper is to use elementary methods to study the mean value properties of \( SL^r(\frac{n}{h}) \) and give an interesting asymptotic formula for it. That is we shall proved the following conclusion:

**Theorem** For any real number \( x \geq 2 \), we have the asymptotic formula

\[
\sum_{n \leq x} SL^r(\frac{n}{h}) = \frac{1}{x} \sum_{n \leq x} SL^r(\frac{n}{h}) + \int_1^x \frac{1}{t} \left( \sum_{n \leq t} SL^r(\frac{n}{h}) \right) \, dt = c + o\left(\frac{\ln^3 x}{\ln x}\right) + \int_1^x \frac{1}{t} \left( c + o\left(\frac{\ln^3 x}{\ln x}\right) \right) \, dt + \int_1^x \frac{1}{t} \left( c + o\left(\frac{\ln^3 x}{\ln x}\right) \right) \, dt + \int_1^x c + o\left(\frac{\ln^3 x}{\ln x}\right) \, dt + o\left(\frac{\ln^3 x}{\ln x}\right) \, dt = c + o\left(\frac{\ln^3 x}{\ln x}\right).
\]

Where \( c = \sum_{s=1}^{\infty} \sum_p \left( \frac{p-1}{p} \left( \frac{p-1}{p-1} \right) \right) \) is a constant.

This completes the proof of Lemma 1.

**Lemma 2** For any real number \( x \geq 1 \), we have the asymptotic formula

\[
\sum_{n \leq x} SL^r(\frac{n}{h}) = cx + o\left(\ln^3 x\right).
\]

Where \( c = \sum_{s=1}^{\infty} \sum_p \left( \frac{p-1}{p} \left( \frac{p-1}{p-1} \right) \right) \) is a constant.

Proof See reference [5].

3 Proof of the theorem

In this section, we shall complete the proof of the theorem. First applying the Abel's summation\(^6\), we may have

\[
\sum_{n \leq x} SL^r(\frac{n}{h}) = \frac{1}{x} \sum_{n \leq x} SL^r(\frac{n}{h}) + \int_1^x \frac{1}{t} \left( \sum_{n \leq t} SL^r(\frac{n}{h}) \right) \, dt = c + o\left(\frac{\ln^3 x}{\ln x}\right) + \int_1^x \frac{1}{t} \left( c + o\left(\frac{\ln^3 x}{\ln x}\right) \right) \, dt = c + o\left(\frac{\ln^3 x}{\ln x}\right) + \int_1^x c + o\left(\frac{\ln^3 x}{\ln x}\right) \, dt + o\left(\frac{\ln^3 x}{\ln x}\right) \, dt = c + o\left(\frac{\ln^3 x}{\ln x}\right).
\]

Where \( c = \sum_{s=1}^{\infty} \sum_p \left( \frac{p-1}{p} \left( \frac{p-1}{p-1} \right) \right) \) is a constant.

This completes the proof of Theorem.

References

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Mean Value Involving the Smarandache LCM Dual Function

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[Abstract] For any positive integer \( n \), the F. Smarandache LCM dual function \( S_L(n) \) is defined as the greatest positive integer \( k \) such that \( 1, 2, \ldots, k \) divides \( n \). The main purpose is to use elementary methods to study the mean value property of \( \frac{S_L(n)}{n} \) and give an interesting asymptotic formula for it.

[Keywords] F. Smarandache LCM function, mean value, asymptotic formula.

Survey on Research Dynamic Voltage Restorer

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[Abstract] Dynamic Voltage Restorer (DVR) is a series compensating device. It is the most economical and effective means to solve the power quality problems, especially the voltage sags, because of its good dynamic characteristics and high cost effective characteristic. The DVR's function, topology and its feature are mainly introduced, analyzed the research actuality and the existing problems of DVRs, and finally discussed the DVR's development direction and trends.

[Keywords] power quality, voltage sag, dynamic voltage restorer (DVR).