

**Marius Coman**

# **SEQUENCES OF PRIMES**

**OBTAINED BY**

**THE METHOD OF CONCATENATION**

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**(COLLECTED PAPERS)**

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## INTRODUCTION

The definition of “concatenation” in mathematics is, according to Wikipedia, “the joining of two numbers by their numerals. That is, the concatenation of 69 and 420 is 69420”. Though the method of concatenation is widely considered as a part of so called “recreational mathematics”, in fact this method can often lead to very “serious” results, and even more than that, to really amazing results. This is the purpose of this book: to show that this method, unfairly neglected, can be a powerful tool in number theory. In particular, as revealed by the title, I used the method of concatenation in this book to obtain possible infinite sequences of primes.

Part One of this book, “Primes in Smarandache concatenated sequences and Smarandache-Coman sequences”, contains 12 papers on various sequences of primes that are distinguished among the terms of the well known Smarandache concatenated sequences (as, for instance, the prime terms in *Smarandache concatenated odd sequence*, defined as the sequence obtained through the concatenation of the first  $n$  odd numbers, or the terms obtained concatenating two primes of the form  $6k + 1$  or of the form  $6k - 1$  in *Smarandache prime-partial-digital sequence*, defined as the sequence of prime numbers which admit a deconcatenation into a set of primes), but also on “Smarandache-Coman sequences of primes”, defined by the author as “all sequences of primes obtained from the terms of Smarandache sequences using any arithmetical operation”; the SC sequences presented in this book are related, of course, to concatenation, but in three different ways: the S sequence is obtained by the method of concatenation but the operation applied on its terms is some other arithmetical operation (e.g. the case of SC sequence of primes obtained through a recurrence relation between the terms of *Smarandache consecutive numbers sequence*, which itself is defined through concatenation), the S sequence is not obtained by the method of concatenation but the operation applied on its terms is concatenation (e.g. the case of the SC sequence of primes obtained concatenating with digit 1 the terms of *Smarandache proper divisor products sequence*), or both S sequence and SC sequence are using the method of concatenation (e.g. the case of the SC sequence of primes obtained concatenating with the digit 1 the terms of *Smarandache reverse sequence*, defined as the sequence obtained through the concatenation of the first  $n$  positive integers in reverse order).

Part Two of this book, “Sequences of primes obtained by the method of concatenation”, brings together 51 articles which aim, using the mentioned method, to highlight sequences of numbers that are rich in primes or are liable to lead to large primes. The method of concatenation is applied to different classes of numbers, e.g. squares of primes, Poulet numbers, triangular numbers, reversible primes, twin primes, repdigits, factorials, primorials, in order to obtain sequences, possible infinite, of primes. Beside other particular conjectures stated in these papers, a generally applicable conjecture states that for any arithmetic progression  $a + b*k$ , where at least one of  $a$  and  $b$  is different than 1, that also satisfies the conditions imposed by the Dirichlet’s Theorem ( $a$  and  $b$  are positive coprime integers) is true that the sequence obtained by the consecutive concatenation of the terms  $a + b*k$  has an infinity of prime terms. If this conjecture were true, the fact that the *Smarandache consecutive numbers sequence* 1, 12, 123, 1234 (...) could have not any prime term (thus far there is no prime number known in this sequence, though there have been checked the first about 40000 terms) would be even more amazing. Part Two of this book also contains a paper which lists a number of 33 sequences of primes obtained by the method of concatenation, sequences presented and analyzed in more detail in my previous papers, gathered together in five books of collected papers: “Two hundred conjectures and one hundred and fifty open problems on Fermat pseudoprimes”, “Two hundred and thirteen conjectures on primes”, “Conjectures on primes and Fermat pseudoprimes, many based on Smarandache function”, “Sequences of integers, conjectures and new arithmetical tools”, “Formulas and polynomials which generate primes and Fermat pseudoprimes”.

## SUMMARY

### **Part One. Primes in Smarandache concatenated sequences and Smarandache-Coman sequences**

1. Sets of primes distinguished among the terms of twenty Smarandache concatenated sequences
2. Four conjectures on the Smarandache prime partial digital sequence
3. Poulet numbers in Smarandache prime partial digital sequence and a possible infinite set of primes
4. Conjecture on an infinity of subsequences of primes in Smarandache prime partial digital sequence
5. Two conjectures on Smarandache's proper divisor products sequence
6. Conjecture on the primes  $S(n) + S(n + 1) - 1$  where  $S(n)$  is a term in Smarandache-Wellin sequence
7. Conjecture on the primes  $S(n + 1) + S(n) - 1$  where  $S(n)$  is a term in the concatenated odd sequence
8. Four Smarandache type sequences obtained concatenating numbers of the form  $6k - 1$  respectively  $6k + 1$
9. The definition of Smarandache reconcatenated sequences and six such sequences
10. On terms of consecutive numbers sequence concatenated both to the left and to the right with same prime
11. Primes obtained concatenating with 1 to the left the terms of three Smarandache sequences
12. Primes obtained concatenating to the right with 1 the terms of concatenated n-th powers sequences

### **Part Two. Sequences of primes obtained by the method of concatenation**

1. A list of 33 sequences of primes obtained by the method of concatenation
2. Sixteen sequences of primes obtained by concatenation from  $p - 1$  respectively  $p + 1$  where  $p$  prime
3. Four conjectures on the numbers  $2*(p*q*r) \pm 1$  where  $p$  and  $q = p + 6$  and  $r = q + 6$  are odd numbers
4. Two conjectures involving the numbers obtained concatenating repeatedly odd multiples of 3 with 111
5. Conjecture involving the numbers obtained concatenating the square of a prime  $p$  with  $p$  then with 1
6. Two conjectures involving the numbers obtained concatenating a prime  $p$  with 9 then with  $p$  itself
7. Conjecture on the numbers obtained concatenating two primes  $p$  and  $q$  where  $q - p + 1$  also prime
8. Four conjectures on the numbers  $p$ ,  $2*p - 1$ ,  $3*p - 10$  and  $n*p - n + 1$  where  $p$  prime
9. Three conjecture on the numbers obtained concatenating  $p^2$  with  $(p^2 + 1)/2$ ,  $p + 12$ ,  $p^2 + 12$
10. Three conjecture on the primes obtained concatenating  $p$  with  $(p - 1)/2$  respectively with  $(p + 1)/2$  where  $p$  prime

11. Primes obtained concatenating a prime  $p$  to the left with 3 and to the right with a square of prime  $q^2$
12. Primes obtained concatenating a square of prime to the left with  $24k + 4$  and to the right with 3
13. Conjecture on the infinity of primes obtained concatenating a prime  $p$  with  $p + 30k$
14. Conjecture on a set of primes obtained by a formula involving reversible primes and concatenation
15. Primes obtained concatenating even numbers  $n$  with 0 then with  $n + 2$  then again with 0 then with  $n + 5$
16. Primes of the form  $(6k - 1) \text{c} [(6k + 1)] \text{c} [(6k - 1)]$  and  $(6k + 1) \text{c} [(6k - 1)] \text{c} [(6k + 1)]$  where “c[“ means “concatenated to”
17. Primes obtained concatenating  $p - 1$  with  $q^2$  where  $p$  and  $q$  are primes or Poulet numbers
18. Primes obtained concatenating two primes with the same digital root respectively digital sum
19. Primes obtained concatenating the prime factors of composite numbers
20. On the numbers  $(n + 1) \cdot p - n \cdot q$  where  $p$  and  $q$  primes,  $p$  having the group of its last digits equal to  $q$
21. Four conjectures on the primes  $p^2 + 18m$  and  $q^2 - 18n$  between the squares  $p^2$ ,  $q^2$  of a pair of twin primes
22. Three sequences obtained concatenating  $P - 1$  with 1 and 11 respectively  $P + 1$  with 11 where  $P$  Poulet numbers
23. Two conjectures on the quintets of numbers  $(p, p + 10, p + 30, p + 40, p + 60)$
24. Two conjectures on the quintets of numbers  $(p, p + 20, p + 30, p + 50, p + 80)$
25. Three conjectures on the primes obtained concatenating  $30k$  with  $30k + p$  where  $p$  prime
26. An unusual conjecture on primes involving concatenation and repunits
27. Primes obtained concatenating  $p$  repeatedly with 6 then with  $q$  where  $(p, q)$  are sexy primes
28. Primes of the form  $p \text{c} [x] \text{c} [q] \text{c} [y] \text{c} [r]$  where  $p, q, r$  consecutive primes,  $q - p = x$  and  $r - q = y$
29. Conjecture on the pairs of primes  $(p, q = p + k)$  involving concatenation
30. Conjecture on the consecutive concatenation of the numbers  $n \cdot k + 1$  where  $k$  multiple of 3
31. Primes obtained concatenating  $p$  prime with  $p + 2$  and  $p + 6$  respectively with  $p + 4$  and  $p + 6$
32. Four conjectures on the triplets  $[p, p + 2, p + 8]$  and  $[p, p + 6, p + 8]$  where  $p$  prime
33. Four conjectures on the triplets  $[p, p + 4, p + 10]$  and  $[p, p + 6, p + 10]$  where  $p$  prime
34. Conjectures on  $(q + 2) \text{c} [n] \text{c} [q]$  and  $(q - 4) \text{c} [n] \text{c} [q]$  where  $n$  is equal to  $1 \text{c} [2] \text{c} [\dots] \text{c} [p]$  and  $p, q$  are primes
35. Conjectures on  $q \text{c} [n] \text{c} [(q + 6)]$  and  $(q + 6) \text{c} [n] \text{c} [q]$  where  $n$  is equal to  $1 \text{c} [2] \text{c} [\dots] \text{c} [p]$  and  $p, q$  are primes
36. Primes obtained concatenating  $p - 1$  with 3 where  $p$  prime of the form  $30k + 17$
37. Primes obtained concatenating with 1 to the right the triangular numbers
38. Primes obtained concatenating with 1 to the left the terms of two back concatenated “multiples of 3” sequences
39. Primes obtained concatenating with 1 to the left the terms of three back concatenated “powers of 3” sequences

40. Conjecture on an infinity of Poulet numbers which are also triangular numbers
41. Conjecture on an infinity of numbers  $(30*k + 7)*(60*k + 13)$  which admit a deconcatenation in two primes
42. Primes obtained concatenating to the right with 1 the partial sums of repdigits
43. Primes obtained concatenating  $n$  consecutive numbers and then the resulting number with 1
44. Conjecture on the consecutive concatenation of the terms of an arithmetic progression
45. Two conjectures on the primes which admit deconcatenation in two primes, involving multiples of 30
46. Four conjectures on the numbers  $p \pm 1$  concatenated with 1 where  $p$  primes of the form  $30*k + 11$
47. Primes obtained deconcatenating with 1 or with 01 the Poulet numbers of the form  $30*k + 1$  or  $300*k + 1$
48. Primes obtained deconcatenating with a group of  $k$  digits of 0 the factorial numbers then adding or subtracting 1
49. Primes obtained deconcatenating with a group of  $k$  digits of 0 the fibonorial numbers then adding or subtracting 1
50. Primes obtained concatenating two consecutive primorial numbers then adding or subtracting 1
51. Primes obtained concatenating  $30*p$  with  $30*q$  then adding or subtracting 1, where  $p$  and  $q = p + 6$  primes

**Annex A. Summary of the methods we have used in order to obtain Smarandache-Coman sequences**

**Annex B. List of few Smarandache concatenated sequences liable to lead to Smarandache-Coman sequences**

## Part One.

### Primes in Smarandache concatenated sequences and Smarandache-Coman sequences

#### 1. Sets of primes distinguished among the terms of twenty Smarandache concatenated sequences

**Abstract.** In this paper I list a number of 20 Smarandache concatenated sequences (for other lists and analyses on these sequences see “Smarandache Sequences” on Wolfram MathWorld and “The math encyclopedia of Smarandache type notions”, Educational Publishing, 2013) and I highlight the sets of primes distinguished among the terms of these sequences, but also I list 25 “sets of primes which can be obtained from the terms of Smarandache sequences using any arithmetical operation” (I named such primes Smarandache-Coman sequences of primes, see my previous papers “Fourteen Smarandache-Coman sequences of primes” and “Seven Smarandache-Coman sequences of primes”).

##### I.

##### The consecutive numbers sequence

$S(n)$  is defined as the sequence obtained through the concatenation of the first  $n$  positive integers.

The first ten terms of the sequence (A007908 in OEIS):

1, 12, 123, 1234, 12345, 123456, 1234567, 12345678, 123456789, 12345678910.

There is not even a prime known among the terms of this sequence, though there have been checked the first about 40 thousand terms.

##### 1. Smarandache-Coman sequence

I shall use the notation  $a(n)$  for a term of a Smarandache concatenated sequence and  $b(n)$  for a term of a Smarandache-Coman sequence.

$SC(n)$  is defined as follows:  $b(n) = a(n+1) - a(n) - 2$  if the last digit of the term  $a(n+1)$  is even and  $b(n) = a(n+1) - a(n) + 2$  if the last digit of the term  $a(n+1)$  is odd.

The first seven terms of  $SC(n)$  (primes by definition):

113, 1109, 11113, 111109, 111111113, 12222222119, 122222221210099.

##### 2. Smarandache-Coman sequence

$SC(n)$  is defined as follows:  $b(n) = a(n)1$ , i.e. the terms of the Smarandache sequence concatenated to the right with the number 1.

The first six terms of  $SC(n)$  (primes by definition):

11, 1231, 1234567891, 12345678910111, 123456789101112131415161,  
12345678910111213141516171819202122232425261.

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.



## II.

### The reverse sequence

$S(n)$  is defined as the sequence obtained through the concatenation of the first  $n$  positive integers, in reverse order.

The first ten terms of the sequence (A000422 in OEIS):

1, 21, 321, 4321, 54321, 654321, 7654321, 87654321, 987654321, 10987654321.

The primes appear very rare among the terms of this sequence: until now there are only two known, corresponding to  $n = 82$  (a number having 155 digits) și  $n = 37765$  (a number having 177719 digits).

#### 3. Smarandache-Coman sequence

$SC(n)$  is defined as follows:  $b(n) = a(n)1$ , i.e. the terms of the Smarandache sequence concatenated to the right with the number 1.

The first five terms of  $SC(n)$  (primes by definition):

11, 211, 876543211, 9876543211, 222120191817161514131211109876543211.

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

## III.

### The concatenated odd sequence

$S(n)$  is defined as the sequence obtained through the concatenation of the first  $n$  odd numbers.

The first ten terms of the sequence (A019519 in OEIS):

1, 13, 135, 1357, 13579, 1357911, 135791113, 13579111315, 1357911131517, 135791113151719.

The terms of this sequence are primes for the following values of  $n$ : 2, 10, 16, 34, 49, 2570 (the term corresponding to  $n = 2570$  is a number with 9725 digits); there is no other prime term known though where checked the first about 26 thousand terms of this sequence. F.S. (Florentin Smarandache) conjectured that there exist an infinity of prime terms of this sequence.

#### 4. Smarandache-Coman sequence

$SC(n)$  is defined as follows:  $b(n) = a(n+1) + a(n) - S(a(n+1)) - S(a(n)) + 2$ , where  $S(a(n))$  is the sum of the digits of the term  $a(n)$ .

The first five terms of  $SC(n)$  (primes by definition):

11, 137, 14897, 1371431, 13714902317.

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

## 5. Smarandache-Coman sequence

$SC(n)$  is defined as follows:  $b(n) = a(n)1$ , i.e. the terms of the Smarandache sequence concatenated to the right with the number 1.

The first six terms of  $SC(n)$  (primes by definition):

11, 131, 13579111315171, 1357911131517191, 13579111315171921231,  
13579111315171921232527291.

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

### IV.

#### The concatenated even sequence

$S(n)$  is defined as the sequence obtained through the concatenation of the first  $n$  even numbers.

The first ten terms of the sequence (A019520 in OEIS):

2, 24, 246, 2468, 246810, 24681012, 2468101214, 246810121416, 24681012141618,  
2468101214161820.

The terms of this sequence can't be, obviously, primes. In the case of this sequence might be studied the primality of the numbers obtained through the division of its terms by 2 (for instance,  $a(7)/2 = 1234050607$  is prime).

It might be also interesting to study the primality of the numbers obtained through the division of its terms by 4 or 6.

The sequence of the primes  $a(n)/4$ : 3, 617 (...)  
obtained for  $n = 2, 4$  (...)

The sequence of the primes  $a(n)/6$ : 41, 41135020236030337 (...), obtained for  $n = 3, 11$   
(...)

## 6. Smarandache-Coman sequence

$SC(n)$  is defined as follows:  $b(n) = a(n+1) + a(n) - S(a(n+1)) - S(a(n)) + 1$ , where  $S(a(n))$  is the sum of the consecutive even numbers which form the term  $a(n)$ ; for instance,  $S(246810) = 2 + 4 + 6 + 8 + 10 = 30$ .

The first four terms of  $SC(n)$  (primes by definition):

1 19, 2683, 249229, 2492782129.

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

### V.

#### The concatenated prime sequence

$S(n)$  is defined as the sequence obtained through the concatenation of the first  $n$  primes.

The first ten terms of the sequence (A019518 in OEIS):

2, 23, 235, 2357, 235711, 23571113, 2357111317, 235711131719, 23571113171923, 2357111317192329.

The terms of this sequence are known as Smarandache-Wellin numbers. Also, the Smarandache-Wellin numbers which are primes are named Smarandache-Wellin primes. The first three such numbers are 2, 23 și 2357; the fourth is a number with 355 digits and there are known only 8 such primes. The corresponding values of  $n$  are 1, 2, 4, 128, 174, 342, 435, 1429. F.S. conjectured that there exist an infinity of prime terms of this sequence.

### 7. *Smarandache-Coman sequence*

$SC(n)$  is defined as follows:  $b(n) = 3a(n)$ , i.e. the terms of the Smarandache sequence concatenated to the left with the number 3.

The first three terms of  $SC(n)$  (primes by definition):

3235711, 323571113171923, 32357111317192329313741.

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

## VI.

### **The back concatenated prime sequence**

$S(n)$  is defined as the sequence obtained through the concatenation of the first  $n$  primes, in reverse order.

The first ten terms of the sequence (A038394 in OEIS):

2, 32, 532, 7532, 117532, 13117532, 1713117532, 191713117532, 23191713117532, 2923191713117532.

The terms of this sequence can't be, obviously, primes. In the case of this sequence might be studied the primality of the numbers obtained through the division of its terms by 4.

The sequence of the primes  $a(n)/4$ : 29383, 47928279383 (...) obtained for  $n = 5, 8$  (...)

## VII.

### **The back concatenated odd sequence**

$S(n)$  is defined as the sequence obtained through the concatenation of the first  $n$  odd numbers, in reverse order.

The first ten terms of the sequence (A038395 in OEIS):

1, 31, 531, 7531, 97531, 1197531, 131197531, 15131197531, 1715131197531, 1917151311975311.

### 8. *Smarandache-Coman sequence*

$SC(n)$  is defined as follows:  $b(n) = 2*a(n) - 1$ .

The first four terms of  $SC(n)$  (primes by definition):  
61, 1061, 15061, 262395061.

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

### VIII.

#### The back concatenated even sequence

$S(n)$  is defined as the sequence obtained through the concatenation of the first  $n$  even numbers, in reverse order.

The first ten terms of the sequence (A038396 in OEIS):  
2, 42, 642, 8642, 108642, 12108642, 1412108642, 161412108642, 18161412108642, 2018161412108642.

#### 9. Smarandache-Coman sequence

$SC(n)$  is defined as follows:  $b(n) = a(n) - 1$ .

The first four terms of  $SC(n)$  (primes by definition):  
41, 641, 8641, 18161412108641.

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

### IX.

#### The concatenated square sequence

$S(n)$  is defined as the sequence obtained through the concatenation of the first  $n$  squares.

The first ten terms of the sequence (A019521 in OEIS):  
1, 14, 149, 14916, 1491625, 149162536, 14916253649, 1491625364964, 149162536496481, 149162536496481100.

The third term, the number 149, is the only prime from the first about 26 thousand terms of this sequence.

### X.

#### The concatenated odd square sequence

$S(n)$  is defined as the sequence obtained through the concatenation of the first  $n$  odd squares.

The first ten terms of the sequence:  
1, 19, 1925, 192549, 19254981, 19254981121, 19254981121169, 19254981121169225, 19254981121169225289, 19254981121169225289361.

### *10. Smarandache-Coman sequence*

SC(n) is defined as follows:  $b(n) = 2 \cdot a(n) + 1$ .

The first three terms of SC(n) (primes by definition):  
3851, 38509963, 38509962242338451,  
obtained for  $n = 3, 5, 8$ .

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

## **XI.**

### **The concatenated even square sequence**

S(n) is defined as the sequence obtained through the concatenation of the first  $n$  even squares.

The first ten terms of the sequence:  
4, 416, 41636, 4163664, 4163664100, 4163664100144, 4163664100144196,  
4163664100144196256, 4163664100144196256324, 4163664100144196256324400.

### *11. Smarandache-Coman sequence*

SC(n) is defined as follows:  $b(n) = a(n) + 5$ .

The first three terms of SC(n) (primes by definition):  
421, 41641, 4163669,  
obtained for  $n = 2, 3, 4$ .

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

### *12. Smarandache-Coman sequence*

SC(n) is defined as follows:  $b(n) = a(n)(a(n+1) + 1)$ , i.e.  $b(n)$  is obtained concatenating the term  $a(n)$  with the value of the term  $a(n+1)$  added to 1.

The first three terms of SC(n) (primes by definition):  
41641637, 41636641001444163664100144197,  
obtained for  $n = 2, 6$ .

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

## **XII.**

### **The concatenated cubic sequence**

S(n) is defined as the sequence obtained through the concatenation of the first  $n$  cubes.

The first ten terms of the sequence (A019521 in OEIS):  
1, 18, 1827, 182764, 182764125, 182764125216, 182764125216343,  
182764125216343512, 182764125216343512729, 1827641252163435127291000.

There were not found prime terms of this sequence, though there were checked the first about 22 terms.

### **XIII.**

#### **The concatenated triangular numbers sequence**

$S(n)$  is defined as the sequence obtained through the concatenation of the first  $n$  triangular numbers [the triangular numbers are a subset of the polygonal numbers constructed with the formula  $T(n) = (n*(n + 1))/2 = 1 + 2 + 3 + \dots + n$ ].

*The first ten terms of the sequence (A078795 in OEIS):*

1, 13, 136, 13610, 1361015, 136101521, 13610152128, 1361015212836, 136101521283645, 13610152128364555.

The only two known primes from this sequence (among the first about 5000 terms) are 13 and 136101521.

#### *13. Smarandache-Coman sequence*

$SC(n)$  is defined as follows:  $b(n) = a(n)9$ , i.e. the terms of the Smarandache sequence concatenated to the right with 9.

The first three terms of  $SC(n)$  (primes by definition):

19, 139, 13610159,  
obtained for  $n = 1, 2, 5$ .

### **XIV.**

#### **The “ $n$ concatenated $n$ times” sequence**

$S(n)$  is defined as the sequence obtained concatenating  $n$  times the number  $n$ .

*The first ten terms of the sequence (A000461 in OEIS):*

1, 22, 333, 4444, 55555, 666666, 7777777, 88888888, 999999999, 101010101010101010.

There is no term of this sequence which can be prime, all terms of the sequence being repdigit numbers, therefore multiples of repunit numbers.

#### *14. Smarandache-Coman sequence*

$SC(n)$  is defined as follows:  $b(n) = 2*a(n)- 1$

The first four terms of  $SC(n)$  (primes by definition):

43, 8887, 111109, 1333331,  
obtained for  $n = 2, 4, 5, 6$ .

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

### 15. Smarandache-Coman sequence

SC(n) is defined as follows:  $b(n) = a(n+1) - a(n)$ .

The first three terms of SC(n) (primes by definition):

311, 4111, 611111,

obtained for  $n = 2, 3, 5$ .

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

### XV.

#### The permutation sequence

S(n) is defined as the sequence of numbers obtained through concatenation and permutation in the following way:  $13...(2n - 3)(2n - 1)(2n)(2n - 2)(2n - 4)...42$ .

The first seven terms of the sequence (A007943 in OEIS):

12, 1342, 135642, 13578642, 13579108642, 135791112108642, 1357911131412108642, 13579111315161412108642.

There is obviously no term of this sequence which can be prime. In the case of this sequence is studied the primality of the numbers obtained through the division of its terms by 2: 6, 671, 67821, 6789321 (...), or the primality of the numbers of the form  $13...(2n - 3)(2n - 1)(2n)(2n - 2)(2n - 4)...42 \pm 1$ .

It might be also interesting to study the primality of the numbers obtained through the division of its terms by 6.

The sequence of the primes  $a(n)/6$ :

2, 22631852018107, 226318521902018107,  
obtained for  $n = 1, 6, 7$ .

### XVI.

#### The pierced chain sequence

The sequence obtained in the following way: the first term is 101 and every next term is obtained through concatenation of the previous term with the group of digits 0101.

The first six terms of the sequence (A031982 in OEIS):

101, 1010101, 10101010101, 101010101010101, 1010101010101010101, 101010101010101010101.

There are no primes obtained through the division of the terms of the sequence by 101 (it is proved).

### XVII.

#### The $n2*n$ sequence

The sequence obtained concatenating  $n$  with  $2*n$ .

The first fifteen terms of the sequence (A019550 in OEIS):

12, 24, 36, 48, 510, 612, 714, 816, 918, 1020, 1122, 1224, 1326, 1428, 1530.

Because obviously every term of this sequence  $a(n)$  is divisible by  $6 \cdot n$ , in the case of this sequence is studied the primality of the numbers  $a(n)/6 \cdot n$ . It is conjectured that this sequence contains infinitely many primes.

#### *16. Smarandache-Coman sequence*

SC(n) is defined as follows:  $b(n) = m(n)/6 + 1$ , where  $m(n)$  is the number obtained concatenating  $a(n)$  with  $a(n+1)$  then with  $a(n+2)$ .

The first three terms of SC(n) (primes by definition):

20407, 40609, 102119137,

obtained for  $n = 1, 2, 6$ .

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

#### *17. Smarandache-Coman sequence*

SC(n) is defined as follows:  $b(n) = m - 1$ , where  $m(n)$  is the number obtained concatenating  $a(n)$  with  $a(n+1)$ .

The first three terms of SC(n) (primes by definition):

1223, 510611, 612713, 9181019, 14281529,

obtained for  $n = 1, 5, 6, 9, 14$ .

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

#### *18. Smarandache-Coman sequence*

SC(n) is defined as follows:  $b(n)$  are obtained concatenating to the right the terms  $a(n)$  with 1.

The first nine terms of SC(n) (primes by definition):

241, 5101, 6121, 8161, 9181, 12241, 14281, 17341, 19381.

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

#### *19. Smarandache-Coman sequence*

SC(n) is defined as follows:  $b(n)$  are obtained concatenating both to the left and to the right the terms  $a(n)$  with 1.

The first eight terms of SC(n) (primes by definition):

1361, 1481, 15101, 19181, 112241, 114281, 115301, 118361.

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.



## XVIII.

### The $nn^2$ sequence

The sequence obtained concatenating  $n$  with  $n^2$ .

The first fifteen terms of the sequence (A053061 in OEIS):

11, 24, 39, 416, 525, 636, 749, 864, 981, 10100, 11121, 12144, 13169, 14196, 15225.

The sequence  $a(n)/n$  is called the reduced Smarandache  $nn^2$  sequence.

The first fifteen terms of the reduced Smarandache  $nn^2$  sequence (A061082 in OEIS):

11, 12, 13, 104, 105, 106, 107, 108, 109, 1010, 1011, 1012, 1013, 1014, 1015.

It is conjectured that there are infinitely many primes in the reduced Smarandache  $nn^2$  sequence.

The sequence  $a(n)$  obtained concatenating the numbers  $n$  and  $n^m$  is called the Smarandache  $nn^m$  sequence and, for any value of  $m$ , contains only one prime, the number 11.

The sequence  $a(n)/n$  is called the reduced Smarandache  $nn^m$  sequence.

#### *20. Smarandache-Coman sequence*

SC(n) is defined as follows:  $b(n) = a(n) + n + 1$ .

The first ten terms of SC(n) (primes by definition):

13, 43, 421, 643, 757, 991, 10111, 12157, 15241, 13183.

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

#### *21. Smarandache-Coman sequence*

SC(n) is defined as follows:  $b(n)$  are obtained concatenating to the right the terms  $a(n)$  with 1.

The first seven terms of SC(n) (primes by definition):

241, 6361, 8641, 9181, 111211, 121441, 298411.

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

#### *22. Smarandache-Coman sequence*

SC(n) is defined as follows:  $b(n)$  are obtained concatenating to the right the terms  $a(n)$  with 11.

The first eight terms of SC(n) (primes by definition):

2411, 3911, 41611, 52511, 63611, 1419611, 1522511, 1728911.

I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

### *23. Smarandache-Coman sequence*

SC(n) is defined as follows: b(n) are obtained concatenating both to the left and to the right the terms a(n) with 1.

The first six terms of SC(n) (primes by definition):  
16361, 17491, 111211, 1183241, 1266761, 1287841.

I conjecture that there exist an infinity of terms b(n) which are primes.

### **XIX.**

#### **The nk\*n generalized sequence**

The n-th term of the sequence a(n) is obtained concatenating all of the numbers n, 2\*n, 3\*n,..., n\*n.

The first eight terms of the sequence (A053062 in OEIS):  
1, 24, 369, 481216, 510152025, 61218243036, 7142128354249, 816243240485664.

### *24. Smarandache-Coman sequence*

SC(n) is defined as follows:  $b(n) = 2*a(n) - 1$ .

The first five terms of SC(n) (primes by definition):  
47, 962431, 1020304049, 14284256708497, 1632486480971327.

I conjecture that there exist an infinity of terms b(n) which are primes.

### **XX.**

#### **The breakup prime sequence**

The n-th term of the sequence is defined as the smallest positive integer which, by concatenation with all previous terms, forms a prime.

The first nine terms of the sequence (A048549 in OEIS):  
2, 23, 233, 2333, 23333, 2333321, 233332117, 2333321173, 233332117313.

### *25. Smarandache-Coman sequence*

SC(n) is defined as follows:  $b(n) = a(n) + a(n+1) + 1$ .

The first two terms of SC(n) (primes by definition):  
257, 25667 (...)

I conjecture that there exist an infinity of terms b(n) which are primes.

## 2. Four conjectures on the Smarandache prime partial digital sequence

**Abstract.** In this paper I make the following four conjectures on the *Smarandache prime-partial-digital sequence* defined as the sequence of prime numbers which admit a deconcatenation into a set of primes: (I) there exist an infinity of primes  $p$  obtained concatenating two primes  $m$  and  $n$ , both of the form  $6^*k + 1$ , such that  $n = m^*h - h + 1$ , where  $h$  positive integer; (II) there exist an infinity of primes  $p$  obtained concatenating two primes  $m$  and  $n$ , both of the form  $6^*k - 1$ , such that  $n = m^*h + h - 1$ , where  $h$  positive integer; (III) there exist an infinity of primes  $p$  obtained concatenating two primes  $m$  and  $n$ , both of the form  $6^*k + 1$ , such that  $n + m - 1$  is prime or power of prime; (IV) there exist an infinity of primes  $p$  obtained concatenating two primes  $m$  and  $n$ , both of the form  $6^*k - 1$ , such that  $n - m + 1$  is prime or power of prime. Note that almost all from the first 65 primes obtained from  $m = 6^*x + 1$ , prime, concatenated with  $n = 6^*y + 1$ , prime (exceptions: 3779, 4373, 6173, 6719, 6779), and all the first 65 primes obtained from  $m = 6^*x - 1$ , prime, concatenated with  $n = 6^*y - 1$ , prime, belong to one of the 4 sequences considered by the conjectures above.

The *Smarandache prime-partial-digital sequence* (see A019549 in OEIS):

: 23, 37, 53, 73, 113, 137, 173, 193, 197, 211, 223, 227, 229, 233, 241, 257, 271, 277, 283, 293, 311, 313, 317, 331, 337, 347, 353, 359, 367, 373, 379, 383, 389, 397, 433, 523, 541, 547, 557, 571, 577, 593, 613, 617, 673, 677, 719, 727, 733, 743, 757, 761, 773, 797, 977 (...)

### Conjecture 1:

There exist an infinity of primes  $p$  obtained concatenating two primes  $m$  and  $n$ , both of the form  $6^*k + 1$ , such that  $n = m^*h - h + 1$ , where  $h$  positive integer.

Note that all primes  $n$  larger than 7 of the form  $6^*k + 1$  can be written as  $7^*h - h + 1$ , where  $h$  positive integer, so all the primes obtained concatenating a prime of the form  $6^*k + 1$  with 7 is term of this sequence.

The sequence of primes  $p$ :

: 137, 197, 317, 617, 677, 719, 743, 761, 773, 797, 977, 1097, 1277, 1361 ( $61 = 13^*5 - 5 + 1$ ), 1373 ( $73 = 13^*6 - 6 + 1$ ), 1637, 1973 ( $73 = 19^*4 - 4 + 1$ ), 1997, 2237, 2297, 2417, 2777, 2837, 3167, 3677, 3719 ( $37 = 19^*2 - 2 + 1$ ), 3797, 4217, 4337, 4637, 5237, 5477, 5717, 5897, 6113 ( $61 = 13^*5 - 5 + 1$ ), 6131 ( $61 = 31^*2 - 2 + 1$ ), 6197, 6317, 6917, 7151, 7229, 7283, 7331, 7349 (...)

Example of larger  $p$ :

:  $p = 499943$  where  $4999 = 43^*119 - 119 + 1$ .

### Conjecture 2:

There exist an infinity of primes  $p$  obtained concatenating two primes  $m$  and  $n$ , both of the form  $6^*k - 1$ , such that  $n = m^*h + h - 1$ , where  $h$  positive integer.

Note that all primes  $n$  larger than 5 of the form  $6*k - 1$  can be written as  $5*h + h - 1$ , where  $h$  positive integer, so all the primes obtained concatenating a prime of the form  $6*k - 1$  with 5 is term of this sequence.

The sequence of primes  $p$ :

: 541, 547, 571, 1123 ( $23 = 11*2 + 2 - 1$ ), 1171 ( $71 = 11*6 + 6 - 1$ ), 1753 ( $53 = 17*3 + 3 - 1$ ), 1789 ( $89 = 17*5 + 5 - 1$ ), 2311 ( $23 = 11*2 + 2 - 1$ ), 2371 ( $71 = 23*3 + 3 - 1$ ), 4723 ( $47 = 23*2 + 2 - 1$ ), 5101, 5107, 5113, 5167, 5179, 5197, 5227, 5233, 5347, 5419, 5431, 5443, 5449, 5479, 5503, 5521, 5557, 5641, 5647, 5653, 5659, 5683, 5701, 5743, 5821, 5827, 5839, 5857, 5881, 5953 (...)

### Conjecture 3:

There exist an infinity of primes  $p$  obtained concatenating two primes  $m$  and  $n$ , both of the form  $6*k + 1$ , such that  $n + m - 1$  is prime or power of prime.

The sequence of primes  $p$ :

: 137 ( $13 + 7 + 1 = 19$ ), 197 ( $19 + 7 - 1 = 25 = 5^2$ ), 317 ( $31 + 7 - 1 = 37$ ), 617 ( $61 + 7 - 1 = 67$ ), 677 ( $67 + 7 - 1 = 73$ ), 719 ( $71 + 9 - 1 = 79$ ), 743 ( $7 + 43 - 1 = 49 = 7^2$ ), 761 ( $7 + 67 - 1 = 73$ ), 773 ( $7 + 73 - 1 = 79$ ), 797 ( $7 + 97 - 1 = 103$ ), 977 ( $97 + 7 - 1 = 103$ ), 1319 ( $13 + 9 - 1 = 31$ ), 1361 ( $13 + 61 - 1 = 73$ ), 1367 ( $13 + 67 - 1 = 79$ ), 1637 ( $163 + 7 - 1 = 169 = 13^2$ ), 1913 ( $19 + 13 - 1 = 31$ ), 1931 ( $19 + 31 - 1 = 49 = 7^2$ ), 1979 ( $19 + 79 - 1 = 97$ ), 2237 ( $223 + 7 - 1 = 229$ ), 2777 ( $277 + 7 - 1 = 283$ ), 2837 ( $283 + 7 - 1 = 289 = 17^2$ ), 3119 ( $31 + 19 - 1 = 49 = 7^2$ ), 3167 ( $31 + 67 - 1 = 97$ ), 3677 ( $367 + 7 - 1 = 373$ ), 3761 ( $37 + 61 - 1 = 97$ ), 3767 ( $37 + 67 - 1 = 103$ ), 4397 ( $43 + 97 - 1 = 139$ ), 5237 ( $523 + 7 - 1 = 529 = 23^2$ ), 5717 ( $571 + 7 - 1 = 577$ ), 6113 ( $61 + 13 - 1 = 73$ ), 6143 ( $61 + 43 - 1 = 103$ ), 6197 ( $61 + 97 - 1 = 157$ ), 6737 ( $67 + 37 - 1 = 103$ ), 6761 ( $67 + 61 - 1 = 127$ ), 7283 ( $7 + 283 - 1 = 289 = 17^2$ ), 7331 ( $73 + 31 - 1 = 103$  or  $7 + 331 - 1 = 337$ ), 7349 ( $349 - 7 + 1 = 343 = 7^3$ ) (...)

Example of larger  $p$ :

:  $p = 499979$  where  $4999 + 79 - 1 = 5077$ , prime.

### Conjecture 4:

There exist an infinity of primes  $p$  obtained concatenating two primes  $m$  and  $n$ , both of the form  $6*k - 1$ , such that  $n - m + 1$  is prime or power of prime.

The sequence of primes  $p$ :

: 541 ( $41 - 5 + 1 = 37$ ), 547 ( $47 - 5 + 1 = 43$ ), 571 ( $71 - 5 + 1 = 67$ ), 1117 ( $17 - 11 + 1 = 7$ ), 1123 ( $23 - 11 + 1 = 13$ ), 1129 ( $29 - 11 + 1 = 19$ ), 1153 ( $53 - 11 + 1 = 43$ ), 1171 ( $71 - 11 + 1 = 61$ ), 1723 ( $23 - 17 + 1 = 7$ ), 1741 ( $41 - 17 + 1 = 25 = 5^2$ ), 1747 ( $47 - 17 + 1 = 31$ ), 1753 ( $53 - 17 + 1 = 37$ ), 1759 ( $59 - 17 + 1 = 43$ ), 1783 ( $83 - 17 + 1 = 67$ ), 1789 ( $89 - 17 + 1 = 73$ ), 2311 ( $23 - 11 + 1 = 13$ ), 2341 ( $41 - 23 + 1 = 19$ ), 2347 ( $47 - 23 + 1 = 25 = 5^2$ ), 2371 ( $71 - 23 + 1 = 49 = 7^2$ ),

2383 ( $83 - 23 + 1 = 61$ ), 2389 ( $89 - 23 + 1 = 67$ ), 2971 ( $71 - 29 + 1 = 43$ ), 4111 ( $41 - 11 + 1 = 31$ ), 4129 ( $41 - 29 + 1 = 13$ ), 4153 ( $53 - 41 + 1 = 13$ ), 4159 ( $59 - 41 + 1 = 19$ ), 4723 ( $47 - 23 + 1 = 25 = 5^2$ ), 4729 ( $47 - 29 + 1 = 19$ ), 4759 ( $59 - 47 + 1 = 13$ ), 4783 ( $83 - 47 + 1 = 37$ ), 4789 ( $89 - 47 + 1 = 43$ ), 5101 ( $101 - 5 + 1 = 97$ ), 5107 ( $107 - 5 + 1 = 103$ ), 5113 ( $113 - 5 + 1 = 109$ ), 5167 ( $167 - 5 + 1 = 163$ ), 5179 ( $179 - 5 + 1 = 173$ ), 5197 ( $197 - 5 + 1 = 193$ ), 5227 ( $227 - 5 + 1 = 223$ ), 5233 ( $233 - 5 + 1 = 227$ ), 5323 ( $53 - 23 + 1 = 31$ ), 5347 ( $53 - 47 + 1 = 7$ ), 5647 ( $647 - 5 + 1 = 643$ ), 5743 ( $743 - 5 + 1 = 739$ ), 5827 ( $827 - 5 + 1 = 823$ ), 5857 ( $857 - 5 + 1 = 853$ ), 5881 ( $881 - 5 + 1 = 877$ ), 5923 ( $59 - 23 + 1 = 37$ ), 7129 ( $71 - 29 + 1 = 43$ ), 7159 ( $71 - 59 + 1 = 13$ ) (...)

Example of larger p:

:  $p = 499711$  where  $4997 - 11 + 1 = 4987$ , prime.

**Note:**

Almost all from the first 65 primes obtained from  $m = 6*x + 1$ , prime, concatenated with  $n = 6*y + 1$ , prime (exceptions: 3779, 4373, 6173, 6719, 6779), and all the first 65 primes obtained from  $m = 6*x - 1$ , prime, concatenated with  $n = 6*y - 1$ , prime, belong to one of the 4 sequences considered by the conjectures above.

**Note:**

Up to the number 7349 there are 65 primes obtained concatenated two primes of the form  $6*k + 1$  and 65 primes obtained concatenated two primes of the form  $6*k - 1$ !

### 3. Poulet numbers in Smarandache prime partial digital sequence and a possible infinite set of primes

**Abstract.** Though the well known Fermat's conjecture on the diophantine equation  $x^n + y^n = z^n$  is named "Fermat's big theorem", in fact probably much more important for number theory is what is called "Fermat's little theorem" which was the most important step up to that time in order to discover a primality criterion. This exceptional criterion of primality still has its exceptions: Fermat pseudoprimes, numbers which "behave" like primes though they are no primes; but they are still a class of numbers at least as interesting as the class of primes. Among Fermat pseudoprimes two classes of numbers are particularly distinguished: Poulet numbers (relative Fermat pseudoprimes) and Carmichael numbers (absolute Fermat pseudoprimes). The initial aim of this paper was only to see which Poulet numbers can be obtained concatenating primes (or, in other words, whichever admit a deconcatenation in prime numbers) but, inspired by a characteristic of a subset of Poulet numbers, I also made the following conjecture: there exist an infinity of primes  $p$  obtained concatenating to the right a prime  $q$  having the sum of the digits  $s(q)$  equal to a multiple of 5 with 3.

The *Smarandache prime-partial-digital sequence* (see A019549 in OEIS):

: 23, 37, 53, 73, 113, 137, 173, 193, 197, 211, 223, 227, 229, 233, 241, 257, 271, 277, 283, 293, 311, 313, 317, 331, 337, 347, 353, 359, 367, 373, 379, 383, 389, 397, 433, 523, 541, 547, 557, 571, 577, 593, 613, 617, 673, 677, 719, 727, 733, 743, 757, 761, 773, 797, 977 (...)

The *Fermat pseudoprimes to base two (Poulet numbers) sequence* (see A001567 in OEIS):

: 23, 37, 53, 73, 113, 137, 173, 193, 197, 211, 223, 227, 229, 233, 241, 257, 271, 277, 283, 293, 311, 313, 317, 331, 337, 347, 353, 359, 367, 373, 379, 383, 389, 397, 433, 523, 541, 547, 557, 571, 577, 593, 613, 617, 673, 677, 719, 727, 733, 743, 757, 761, 773, 797, 977 (...)

#### Conjecture:

There exist an infinity of Poulet numbers  $P$  which admit a deconcatenation in prime numbers.

The sequence of such Poulet numbers  $P$ :

:  $P = 341$  (3 and 41 are primes);  
 :  $P = 561$  (5 and 61 are primes);  
 :  $P = 1729$  (17 and 29 are primes);  
 :  $P = 2701$  (2 and 701 are primes);  
 :  $P = 2821$  (2 and 821 are primes);  
 :  $P = 3277$  (3 and 277 are primes, also 3 and 2 and 7 and 7 are primes);  
 :  $P = 4371$  (43 and 71 are primes);  
 :  $P = 8911$  (89 and 11 are primes);  
 :  $P = 13741$  (137 and 41 are primes, also 13 and 7 and 41 are primes);  
 :  $P = 13747$  (137 and 47 are primes, also 13 and 7 and 47 are primes);

: P = 23001 (2 and 3001 are primes);  
 : P = 25761 (257 and 61 are primes, also 2 and 5 and 761 are primes);  
 : P = 29341 (293 and 41 are primes, also 2 and 9341 are primes);  
 : P = 33153 (331 and 53 are primes);  
 : P = 35333 (3533 and 3 are primes, also 353 and 3 and 3 or 3 and 53 and 3 and 3  
 or 3 and 5 and 3 and 3 and 3);  
 : P = 49141 (491 and 41 are primes);  
 : P = 63973 (6397 and 3 are primes);  
 : p = 83333 (83 and 3 and 3 and 3 are primes);  
 : p = 88357 (883 and 5 and 7 are primes);  
 : P = 101101 (101 is prime);  
 : P = 137149 (137 and 149 are primes, also 13 and 7 and 149 are primes);  
 : P = 149281 (149 and 281 are primes);  
 : P = 157641 (157 and 641 are primes);  
 : P = 172081 (17 and 2081 are primes);  
 : P = 196093 (19609 and 3 are primes);  
 : P = 212421 (2 and 12421 are primes);  
 : P = 215749 (2 and 15749 are primes);  
 : P = 220729 (2207 and 29 are primes);  
 : P = 226801 (2 and 26801 are primes);  
 : p = 233017 (2 and 3301 and 7 are primes);  
 : p = 253241 (2 and 53 and 241 are primes);  
 : P = 264773 (2647 and 73 are primes, also 2647 and 7 and 3 are primes);  
 : P = 272251 (2 and 72251 are primes, also 2 and 7 and 2251 are primes, also 2 and  
 7 and 2 and 251);  
 : p = 276013 (2 and 7 and 601 and 3 are primes);  
 : p = 289941 (2 and 89 and 941 are primes);  
 : P = 294271 (29 and 4271 are primes);  
 : P = 294409 (29 and 4409 are primes);  
 : p = 314821 (3 and 14821 are primes);  
 : p = 318361 (31 and 83 and 61 are primes);  
 : p = 323713 (32371 and 3 are primes, also 3 and 2 and 3 and 7 and 13);  
 : p = 334153 (3 and 3 and 41 and 53 are primes)  
 : P = 387731 (3877 and 31 are primes);  
 : P = 401401 (401 is prime);  
 (note that 101101 is also a Poulet number, and 101 is also prime);  
 : p = 423793 (42379 and 3 are primes);  
 : P = 443719 (443 and 719 are primes, also 443 and 7 and 19);  
 : P = 476971 (47 and 6971 are primes);  
 : P = 481573 (48157 and 3 are primes);  
 : P = 486737 (48673 and 7 are primes);  
 : P = 493697 (49369 and 7 are primes);  
 (...)

**Note:**

The Poulet number 1729 (Hardy-Ramanujan number), can be deconcatenated in 17 and 29, primes, and also the Poulet number 137149 can be deconcatenated in two primes:  $137 = 17 + 120$  and  $149 = 29 + 120$ ; the numbers obtained concatenating the primes of the form  $17 + 120*k$  with primes with the form  $29 + 120*k$  might be interesting to study; one

other Poulet number which can be deconcatenated in two numbers  $x$  and  $y$  such that  $x - y = 12$  is 253241 but this time 253 is not prime but a semiprime; also  $1729 = 7 \cdot 13 \cdot 19$  and  $137149 = 67 \cdot 23 \cdot 89$  and the numbers of the form  $(60 \cdot k + 7) \cdot (10 \cdot k + 13) \cdot (70 \cdot k + 19)$  might be interesting to study.

**Note:**

An other interesting thing is that the Poulet numbers 63973, 196093 and 481573, beside the fact that are obtained from primes of the form  $6 \cdot k + 1$  concatenated to the right with 3, have in common the sum of their digits, *i.e.* 28.

**Conjecture:**

There exist an infinity of primes  $p$  obtained concatenating to the right a prime  $q$  having the sum of the digits  $s(q)$  equal to a multiple of 5 with 3.

The sequence of such primes  $p$  for  $s(q) = 5$ :

- :  $P = 53$  ( $q = 5$  prime);
- :  $p = 233$  ( $q = 23$  prime and  $2 + 3 = 5$ );
- :  $P = 4013$  ( $q = 401$  prime and  $4 + 0 + 1 = 5$ );
- :  $P = 10133$  ( $q = 1013$  prime and  $1 + 0 + 1 + 3 = 5$ );
- :  $P = 10313$  ( $q = 1031$  prime and  $1 + 0 + 3 + 1 = 5$ ); (...)

The sequence of such primes  $p$  for  $s(q) = 10$ :

- :  $P = 193$  ( $q = 19$  prime and  $1 + 9 = 10$ );
- :  $P = 373$  ( $q = 37$  prime and  $3 + 7 = 10$ );
- :  $P = 733$  ( $q = 73$  prime and  $7 + 3 = 10$ );
- :  $P = 1093$  ( $q = 109$  prime and  $1 + 0 + 9 = 10$ );
- :  $P = 2713$  ( $q = 271$  prime and  $2 + 7 + 1 = 10$ );
- :  $P = 5233$  ( $q = 523$  prime and  $5 + 2 + 3 = 10$ );
- :  $P = 5413$  ( $q = 541$  prime and  $5 + 4 + 1 = 10$ );
- :  $P = 6133$  ( $q = 613$  prime and  $6 + 1 + 3 = 10$ );
- :  $P = 10093$  ( $q = 1009$  prime and  $1 + 0 + 0 + 9 = 10$ );
- :  $P = 11173$  ( $q = 1117$  prime and  $1 + 1 + 1 + 7 = 10$ );
- (...)
- :  $P = 1043113$  ( $q = 104311$  prime  $s(q) = 10$ ); (...)

The sequence of such primes  $p$  for  $s(q) = 20$ :

- :  $P = 4793$  ( $q = 479$  prime and  $4 + 7 + 9 = 20$ );
- :  $P = 5693$  ( $q = 569$  prime and  $5 + 6 + 9 = 20$ );
- :  $P = 9293$  ( $q = 929$  prime and  $9 + 2 + 9 = 20$ );
- :  $P = 9473$  ( $q = 947$  prime and  $9 + 4 + 7 = 20$ );
- :  $P = 9833$  ( $q = 983$  prime and  $9 + 8 + 3 = 20$ );
- :  $P = 12893$  ( $q = 1289$  prime and  $1 + 2 + 8 + 9 = 20$ );
- (...)
- :  $P = 1047173$  ( $q = 104717$  prime  $s(q) = 20$ ); (...)



The sequence of such primes  $p$  for  $s(q) = 25$ :

- :  $P = 9973$  ( $q = 997$  prime and  $9 + 9 + 7 = 25$ );
- :  $P = 16993$  ( $q = 1699$  prime and  $1 + 6 + 9 + 9 = 25$ );
- :  $P = 18793$  ( $q = 1879$  prime and  $1 + 8 + 7 + 9 = 25$ );
- :  $P = 19873$  ( $q = 1987$  prime and  $1 + 9 + 8 + 7 = 25$ );
- (...)
- :  $P = 1036873$  ( $q = 103687$  prime  $s(q) = 25$ ); (...)

#### 4. Conjecture on an infinity of subsequences of primes in Smarandache prime partial digital sequence

**Abstract.** In this paper I make the following conjecture on an infinity of subsequences of primes in *Smarandache prime-partial-digital sequence*, defined as the sequence of prime numbers which admit a deconcatenation into a set of primes: for any prime  $p$  which admits a deconcatenation in  $k$  primes larger than 3 is true that there exist a number of  $k$  sequences of primes  $P_1, P_2, \dots, P_k$ , each one having an infinity of prime terms which also admit a deconcatenation in prime numbers, obtained replacing a prime  $q$  in  $p$  with primes having the same digital root as  $q$  (example: for the prime 547 there exist an infinite sequence of primes obtained replacing 5 with primes having the digital root equal to 5 (2347, 13147, 14947, ...) and also an infinite sequence of primes obtained replacing 47 with primes having the digital root equal to 2 (5101, 5227, 5281,...)).

#### Conjecture:

For any prime  $p$  which admits a deconcatenation in  $k$  primes larger than 3 is true that there exist a number of  $k$  sequences of primes  $P_1, P_2, \dots, P_k$ , each one having an infinity of prime terms which also admit a deconcatenation in prime numbers, obtained replacing a prime  $q$  in  $p$  with primes having the same digital root as  $q$  (example: for the prime 547 there exist an infinite sequence of primes obtained replacing 5 with primes having the digital root equal to 5 (2347, 13147, 14947, ...) and also an infinite sequence of primes obtained replacing 47 with primes having the digital root equal to 2 (5101, 5227, 5281,...)).

Note: the operator “\” it will be used with the meaning “concatenated to”.

#### The sequences P1 and P2

for the first two primes  $p$  which admit a deconcatenation in 2 primes

The sequence P1 for 137 (13\7), obtained replacing 13 with primes having  $dr = 4$ :

: 317, 677, 2297, 2837, 4637, 5717, 7517, 7877 (...)

The sequence P2 for 137 (13\7), obtained replacing 7 with primes having  $dr = 7$ :

: 1361, 13151, 13241, 13311, 13331, 13367, 13421 (...)

\*

The sequence P1 for 197 (19\7), obtained replacing 19 with primes having  $dr = 1$ :

: 1097, 1277, 1637, 1997, 3797, 4337, 4877, 5237 (...)

The sequence P2 for 197 (19\7), obtained replacing 7 with primes having  $dr = 7$ :

: 1979, 1997, 19421, 19457, 19709, 19727, 19853 (...)

### The sequences P1, P2 and P3

for the first two primes  $p$  which admit a deconcatenation in 3 primes

The sequence P1 for 577 ( $= 5\|7\|7$ ), obtained replacing 5 with primes having  $dr = 5$ :

: 2377, 4177, 13177, 23977, 31177, 38377, 40177 (...)

The sequence P2 for 577 ( $= 5\|7\|7$ ), obtained replacing (the first) 7 with primes having  $dr = 5$ :

: 5437, 51517, 52237, 54217, 54577, 56197, 56737 (...)

The sequence P3 for 577 ( $= 5\|7\|7$ ), obtained replacing (the second) 7 with primes having  $dr = 7$ :

: 5743, 5779, 57223, 57241, 57331, 57349, 57367 (...)

\*

The sequence P1 for 757 ( $= 7\|5\|7$ ), obtained replacing (the first) 7 with primes having  $dr = 7$ :

: 4357, 31357, 42157, 45757, 70957, 103357, 106957 (...)

The sequence P2 for 757 ( $= 7\|5\|7$ ), obtained replacing 5 with primes having  $dr = 5$ :

: 7237, 7417, 71317, 72577, 72937, 73477, 74017 (...)

The sequence P3 for 757 ( $= 7\|5\|7$ ), obtained replacing (the second) 7 with primes having  $dr = 7$ :

: 7561, 75223, 75277, 75367, 75619, 75709, 75853, 75997 (...)

## 5. Two conjectures on Smarandache's proper divisor products sequence

**Abstract.** In this paper I make the following two conjectures on the *Smarandache's proper divisor products sequence* where a term  $P(n)$  of the sequence is defined as the product of the proper divisors of  $n$ : (1) there exist an infinity of numbers  $n$  divisible by 3 such that the number obtained concatenating the value of  $P(n)$  to the right with 1 is prime; (2) there exist an infinity of numbers  $n$  divisible by 3 such that the number obtained concatenating the value of  $P(n)$  to the right with 1 is semiprime  $p*q$  with the property that  $q - p + 1$  is prime.

The *Smarandache's proper divisor products sequence* (see A007956 in OEIS):

: 1, 1, 1, 2, 1, 6, 1, 8, 3, 10, 1, 144, 1, 14, 15, 64, 1, 324, 1, 400, 21, 22, 1, 13824, 5, 26, 27, 784, 1, 27000, 1, 1024, 33, 34, 35, 279936, 1, 38, 39, 64000, 1, 74088, 1, 1936, 2025, 46, 1, 5308416, 7, 2500, 51, 2704, 1, 157464, 55, 175616, 57, 58, 1, 777600000, 1, 62, 3969, 32768, 65 (...)

### Conjecture 1:

There exist an infinity of numbers  $n$  divisible by 3 such that the number  $m$  obtained concatenating the value of  $P(n)$  to the right with 1 is prime.

The sequence of primes  $m$ :

:  $m = 11$ , prime, for  $(n, P(n)) = (3, 1)$ ;  
:  $m = 61$ , prime, for  $(n, P(n)) = (6, 6)$ ;  
:  $m = 31$ , prime, for  $(n, P(n)) = (9, 3)$ ;  
:  $m = 151$ , prime, for  $(n, P(n)) = (15, 15)$ ;  
:  $m = 211$ , prime, for  $(n, P(n)) = (21, 21)$ ;  
:  $m = 138241$ , prime, for  $(n, P(n)) = (24, 13824)$ ;  
:  $m = 271$ , prime, for  $(n, P(n)) = (27, 27)$ ;  
:  $m = 270001$ , prime, for  $(n, P(n)) = (30, 27000)$ ;  
:  $m = 331$ , prime, for  $(n, P(n)) = (33, 33)$ ;  
:  $m = 2799361$ , prime, for  $(n, P(n)) = (36, 279936)$ ;  
:  $m = 571$ , prime, for  $(n, P(n)) = (57, 57)$ ;  
:  $m = 418121194241$ , prime, for  $(n, P(n)) = (84, 41812119424)$ ;  
:  $m = 98011$ , prime, for  $(n, P(n)) = (99, 9801)$ ;  
:  $m = 14815441$ , prime, for  $(n, P(n)) = (114, 1481544)$ ;  
:  $m = 1231$ , prime, for  $(n, P(n)) = (123, 123)$ ;  
:  $m = 1291$ , prime, for  $(n, P(n)) = (129, 129)$ ;  
:  $m = 216091$ , prime, for  $(n, P(n)) = (147, 21609)$ ;  
:  $m = 44921251$ , prime, for  $(n, P(n)) = (165, 4492125)$ ;  
:  $m = 52680241$ , prime, for  $(n, P(n)) = (174, 5268024)$ ;  
:  $m = 1831$ , prime, for  $(n, P(n)) = (183, 183)$ ;  
:  $m = 2011$ , prime, for  $(n, P(n)) = (201, 201)$ ;  
:  $m = 2131$ , prime, for  $(n, P(n)) = (213, 213)$ ;  
:  $m = 109410481$ , prime, for  $(n, P(n)) = (222, 10941048)$ ;  
:  $m = 2371$ , prime, for  $(n, P(n)) = (237, 237)$ ;  
:  $m = 148869361$ , prime, for  $(n, P(n)) = (246, 14886936)$ ;  
:  $m = 171735121$ , prime, for  $(n, P(n)) = (258, 17173512)$ ;

:  $m = 2671$ , prime, for  $(n, P(n)) = (267, 267)$ ;  
 :  $m = 203464171$ , prime, for  $(n, P(n)) = (273, 20346417)$ ;  
 :  $m = 321574321$ , prime, for  $(n, P(n)) = (318, 32157432)$ ;  
 :  $m = 3271$ , prime, for  $(n, P(n)) = (327, 327)$ ;  
 :  $m = 3391$ , prime, for  $(n, P(n)) = (339, 339)$ ;  
 :  $m = 1317691$ , prime, for  $(n, P(n)) = (363, 131769)$ ;  
 :  $m = 11026624842058533121$ , prime, for  $(n, P(n)) = (378, 1102662484205853312)$ ;  
 :  $m = 13723100667900000001$ , prime, for  $(n, P(n)) = (390, 1372310066790000000)$ ;  
 :  $m = 3931$ , prime, for  $(n, P(n)) = (393, 393)$ ;  
 :  $m = 635211991$ , prime, for  $(n, P(n)) = (399, 63521199)$ ;  
 :  $m = 269042006251$ , prime, for  $(n, P(n)) = (405, 26904200625)$ ;  
 :  $m = 4111$ , prime, for  $(n, P(n)) = (411, 411)$ ;  
 :  $m = 773087761$ , prime, for  $(n, P(n)) = (426, 77308776)$ ;  
 :  $m = 840276721$ , prime, for  $(n, P(n)) = (438, 84027672)$ ;  
 :  $m = 967025791$ , prime, for  $(n, P(n)) = (459, 96702579)$ ;  
 (...)

Examples of larger m:

:  $m = 1006131972417915371063869440000000000000001$ , prime, for  $(n, P(n)) = (720, 100613197241791537106386944000000000000000)$ ;  
 :  $m = 895338254258716445109900000000000001$ , prime, for  $(n, P(n)) = (990, 89533825425871644510990000000000000)$ .

## Conjecture 2:

There exist an infinity of numbers  $n$  divisible by 3 such that the number  $m$  obtained concatenating the value of  $P(n)$  to the right with 1 is semiprime  $m = p \cdot q$  with the property that  $q - p + 1$  is prime.

The sequence of semiprimes  $m$ :

:  $m = 3241 = 7 \cdot 463$  (where  $463 - 7 + 1 = 457$ , prime), for  $(n, P(n)) = (18, 324)$ ;  
 :  $m = 391 = 17 \cdot 23$  (where  $23 - 17 + 1 = 7$ , prime), for  $(n, P(n)) = (39, 39)$ ;  
 :  $m = 20251 = 7 \cdot 2893$  (where  $2893 - 7 + 1 = 2887$ , prime), for  $(n, P(n)) = (45, 2025)$ ;  
 :  $m = 511 = 7 \cdot 73$  (where  $73 - 7 + 1 = 67$ , prime), for  $(n, P(n)) = (51, 51)$ ;  
 :  $m = 59049000001 = 215161 \cdot 274441$  (where  $274441 - 215161 + 1 = 59281$ , prime), for  $(n, P(n)) = (90, 5904900000)$ ;  
 :  $m = 10612081 = 37 \cdot 286813$  (where  $286813 - 37 + 1 = 286777$ , prime), for  $(n, P(n)) = (102, 1061208)$ ;  
 :  $m = 136891 = 367 \cdot 373$  (where  $373 - 367 + 1 = 7$ , prime), for  $(n, P(n)) = (117, 13689)$ ;  
 :  $m = 1411 = 17 \cdot 83$  (where  $83 - 17 + 1 = 67$ , prime), for  $(n, P(n)) = (141, 141)$ ;  
 :  $m = 1591 = 37 \cdot 43$  (where  $43 - 37 + 1 = 7$ , prime), for  $(n, P(n)) = (159, 159)$ ;  
 :  $m = 2191 = 7 \cdot 313$  (where  $313 - 7 + 1 = 307$ , prime), for  $(n, P(n)) = (219, 219)$ ;  
 :  $m = 2491 = 47 \cdot 53$  (where  $53 - 47 + 1 = 7$ , prime), for  $(n, P(n)) = (249, 249)$ ;  
 :  $m = 1046035320300000001 = 37 \cdot 28271224872972973$  (where  $28271224872972973 - 37 + 1 = 28271224872972937$ , prime), for  $(n, P(n)) = (270, 104603532030000000)$ ;

:  $m = 291 = 41 \cdot 71$  (where  $71 - 41 + 1 = 31$ , prime), for  $(n, P(n)) = (291, 291)$ ;  
 :  $m = 261980731 = 3037 \cdot 86263$  (where  $86263 - 3037 + 1 = 83227$ , prime), for  $(n, P(n)) = (297, 26198073)$ ;  
 :  $m = 3091 = 11 \cdot 281$  (where  $281 - 11 + 1 = 271$ , prime), for  $(n, P(n)) = (309, 309)$ ;  
 :  $m = 454992931 = 331 \cdot 1374601$  (where  $1374601 - 331 + 1 = 1374271$ , prime),  
 for  $(n, P(n)) = (357, 45499293)$ ;  
 :  $m = 3811 = 37 \cdot 103$  (where  $103 - 37 + 1 = 67$ , prime), for  $(n, P(n)) = (381, 381)$ ;  
 :  $m = 649648081 = 17 \cdot 38214593$  (where  $38214593 - 17 + 1 = 38214577$ , prime),  
 for  $(n, P(n)) = (402, 64964808)$ ;  
 :  $m = 2275291 = 139 \cdot 16369$  (where  $16369 - 139 + 1 = 16231$ , prime), for  $(n, P(n)) = (477, 227529)$ ;  
 :  $m = 1126785871 = 421 \cdot 2676451$  (where  $2676451 - 421 + 1 = 2676031$ , prime),  
 for  $(n, P(n)) = (477, 112678587)$ ;  
 :  $m = 4891 = 67 \cdot 73$  (where  $73 - 67 + 1 = 7$ , prime), for  $(n, P(n)) = (489, 489)$ ;  
 (...)

## 6. Conjecture on the primes $S(n) + S(n + 1) - 1$ where $S(n)$ is a term in Smarandache-Wellin sequence

**Abstract.** In this paper I make the following conjecture: There exist an infinity of primes  $S(n) + S(n + 1) - 1$ , where  $S(n)$  is a term in Smarandache-Wellin sequence (which is defined as the sequence obtained through the concatenation of the first  $n$  primes).

### Conjecture :

There exist an infinity of primes  $S(n) + S(n + 1) - 1$ , where  $S(n)$  is a term in Smarandache-Wellin sequence (which is defined as the sequence obtained through the concatenation of the first  $n$  primes). I will name this primes “Smarandache-Wellin-Marius primes” or SWM.

#### The Smarandache-Wellin numbers:

(A019518 in OEIS)

: 2, 23, 235, 2357, 235711, 23571113, 2357111317, 235711131719, 23571113171923, 2357111317192329, 235711131719232931, 23571113171923293137, 2357111317192329313741, 235711131719232931374143, 23571113171923293137414347 (...)

Note: the Smarandache-Wellin numbers which are primes are named Smarandache-Wellin primes. The first three such numbers are 2, 23 și 2357; the fourth is a number with 355 digits and there are known only 8 such primes. The 8 known values of  $n$  for which through the concatenation of the first  $n$  primes we obtain a prime number are 1, 2, 4, 128, 174, 342, 435, 1429. The computer programs not yet found, until  $n = 10^4$ , another such a prime. Florentin Smarandache conjectured that there exist an infinity of prime terms of this sequence.

#### The Smarandache-Wellin-Marius primes:

(A019518 in OEIS)

:  $SWM1 = S(2) + S(3) - 1 = 23 + 235 - 1 = 257$ ;  
 :  $SWM2 = S(3) + S(4) - 1 = 235 + 2357 - 1 = 2591$ ;  
 :  $SWM3 = S(5) + S(6) - 1 = 235711 + 23571113 - 1 = 23806823$ ;  
 :  $SWM4 = S(11) + S(12) - 1 = 235711131719232931 + 23571113171923293137 - 1 = 23806824303642526067$ ;  
 :  $SWM5 = S(12) + S(13) - 1 = 23571113171923293137 + 2357111317192329313741 - 1 = 23806824303642526068877$ ;  
 (...)

## 7. Conjecture on the primes $S(n + 1) + S(n) - 1$ where $S(n)$ is a term in the concatenated odd sequence

**Abstract.** In this paper I make the following conjecture: There exist an infinity of primes  $S(n+1) + S(n) - 1$ , where  $S(n)$  is a term in Smarandache concatenated odd sequence (which is defined as the sequence obtained through the concatenation of the first  $n$  odd primes).

### Conjecture :

There exist an infinity of primes  $S(n+1) + S(n) - 1$ , where  $S(n)$  is a term in Smarandache concatenated odd sequence (which is defined as the sequence obtained through the concatenation of the first  $n$  odd primes).

**The concatenated odd sequence:**  
(A089933 in OEIS)

: 3, 35, 357, 35711, 3571113, 357111317, 35711131719, 3571113171923,  
357111317192329, 35711131719232931, 3571113171923293137,  
357111317192329313741, 35711131719232931374143,  
3571113171923293137414347 (...)

Note: Florentin Smarandache conjectured that there exist an infinity of prime terms of this sequence. The terms of this sequence are primes for the following values of  $n$ : 2, 10, 16, 34, 49, 2570 (the term corresponding to  $n = 2570$  is a number with 9725 digits); there is no other prime term known though where checked the first about 26 thousand terms of this sequence.

**The primes of the form  $P = S(n+1) + S(n) - 1$ :**

: P1 = 37 =  $S(2) + S(1) - 1 = 35 + 35 - 1$ ;  
: P2 = 36067 =  $S(4) + S(3) - 1 = 35711 + 357 - 1$ ;  
: P3 = 360682429 =  $S(6) + S(5) - 1 = 357111317 + 3571113 - 1$ ;  
: P4 = 360682430364251 =  $S(9) + S(8) - 1 =$   
357111317192329 + 3571113171923 - 1;  
: P5 = 36068243036425260687883 =  $S(14) + S(13) - 1 =$   
35711131719232931374143 + 357111317192329313741 - 1;  
: P6 = 360682430364252606878849099 =  $S(16) + S(15) - 1 =$   
357111317192329313741434753 + 3571113171923293137414347 - 1;  
: P7 = 3606824303642526068788491011321293943  
=  $S(21) + S(20) - 1 = 3571113171923293137414347535961677173 +$   
35711131719232931374143475359616771 - 1;  
(...)

Note that there also exist primes of the form  $Q = S(n+1) - S(n) + 1$ ; I conjecture that there exist an infinity of such primes too:



$$\begin{aligned}
: & \quad Q1 &= 3535403 = S(4) - S(3) + 1 = 3571113 - 35711 + 1; \\
: & \quad Q2 &= 35354020402040603 = S(10) - S(9) + 1 = 35711131719232931 - \\
& & \quad 357111317192329 + 1; \\
: & \quad Q3 &= 3535402040204060207 = S(11) - S(10) + 1 = 3571113171923293137 - \\
& & \quad 35711131719232931 + 1; \\
& & \quad (\dots)
\end{aligned}$$

## 8. Four Smarandache type sequences obtained concatenating numbers of the form $6k - 1$ respectively $6k + 1$

**Abstract.** In this paper I present the following four Smarandache type sequences: (I) The sequence of numbers obtained concatenating the positive integers of the form  $6^*k - 1$ ; (II) The sequence of numbers obtained concatenating the primes of the form  $6^*k - 1$ ; (III) The sequence of numbers obtained concatenating the positive integers of the form  $6^*k + 1$ ; (IV) The sequence of numbers obtained concatenating the primes of the form  $6^*k + 1$ .

### Sequence 1 :

Numbers obtained concatenating the positive integers of the form  $6^*k - 1$ .

: 511, 51117, 5111723, 511172329, 51117232935, 5111723293541,  
 511172329354147, 51117232935414753, 5111723293541475359,  
 511172329354147535965, 51117232935414753596571,  
 5111723293541475359657173  
 (...)

Prime terms in this sequence:

:  $a(3) = 5111723$ ;  
 :  $a(6) = 5111723293541$ ;  
 (...)

Question: does this sequence contain an infinity of prime terms?

### Sequence 2 :

Numbers obtained concatenating the primes of the form  $6^*k - 1$ .

: 511, 51117, 5111723, 511172329, 51117232941, 5111723294147,  
 511172329414753, 51117232941475359, 5111723294147535971,  
 511172329414753597183, 51117232941475359718389,  
 51117232941475359718389101  
 (...)

Primes in this sequence:

:  $a(3) = 5111723$ ;  
 :  $a(10) = 511172329414753597183$ ;  
 :  $a(24) = 51117232941475359718389101107113131137149167173179191197227$   
 233;  
 (...)

Question: does this sequence contain an infinity of prime terms? Is just a coincidence that the first three prime terms of this sequence end in a prime of the form  $30^*k + 23$  (23, 83, respectively 233)?

Question: note that  $a(2) + a(3) + a(4) - 2 = 51117 + 5111723 + 511172329 - 2 = 516335167$ , which is a prime number; does exist an infinity of such primes of the form  $a(n) + a(n+1) + a(n+2) - 2$ ?

### Sequence 3 :

Numbers obtained concatenating the positive integers of the form  $6^*k + 1$ .

: 1713, 171319, 17131925, 1713192531, 171319253137, 17131925313743,  
 1713192531374349, 171319253137434955, 17131925313743495561,  
 1713192531374349556167, 171319253137434955616773,  
 17131925313743495561677379  
 (...)

Prime terms in this sequence:

:  $a(10) = 17131925313743495561$ ;  
 :  $a(12) = 171319253137434955616773$ ;  
 (...)

Question: does this sequence contain an infinity of prime terms?

### Sequence 4 :

Numbers obtained concatenating the primes of the form  $6^*k + 1$ .

: 713, 71319, 7131931, 713193137, 71319313743, 7131931374361,  
 713193137436167, 71319313743616773, 7131931374361677379,  
 713193137436167737997, 713193137436167737997103,  
 713193137436167737997103109  
 (...)

Question: does this sequence contain an infinity of prime terms?

Question: note that  $a(1) + a(2) + a(3) - 2 = 713 + 71319 + 7131931 - 2 = 7203961$ , which is a prime number; also  $a(3) + a(4) + a(5) - 2 = 7131931 + 713193137 + 71319313743 - 2 = 72039638809$ , which is a prime number; does exist an infinity of such primes of the form  $a(n) + a(n+1) + a(n+2) - 2$ ?

## 9. The definition of Smarandache reconcatenated sequences and six such sequences

**Abstract.** In this paper I define a *Smarandache reconcatenated sequence*  $Sr(n)$  as “the sequence obtained from the terms of a Smarandache concatenated sequence  $S(n)$ , terms for which was applied the operation of consecutive concatenation” and I present six such sequences. Example: for *Smarandache consecutive numbers sequence* (1, 12, 123, 1234, 12345...), the *Smarandache reconcatenated consecutive numbers sequence* has the terms: 1, 112, 112123, 1121231234, 112123123412345...). According to the same pattern, we can define back reconcatenated sequences (the terms of the *Smarandache back reconcatenated consecutive numbers sequence*, noted  $Sbr(n)$ , are 1, 121, 123121, 1234123121...).

### Definition:

A *Smarandache reconcatenated sequence*  $Sr(n)$  is “the sequence obtained from the terms of a Smarandache concatenated sequence  $S(n)$ , terms for which was applied the operation of consecutive concatenation”. Example: for *Smarandache consecutive numbers sequence* (1, 12, 123, 1234, 12345...), the *Smarandache reconcatenated consecutive numbers sequence* has the terms: 1, 112, 112123, 1121231234, 112123123412345...). According to the same pattern, we can define back reconcatenated sequences (the terms of the *Smarandache back reconcatenated consecutive numbers sequence*, noted  $Sbr(n)$ , are 1, 121, 123121, 1234123121...).

### I.

#### The reconcatenated back concatenated odd prime sequence

$S(n)$  is defined as the sequence obtained through the concatenation of the first  $n$  primes, in reverse order, having the terms (A092447 in OEIS): 3, 53, 753, 11753, 1311753, 171311753, 19171311753, 2319171311753, 292319171311753, 31292319171311753 (...).

The terms of  $Sr(n)$ : 3, 353, 353753, 35375311753, 353753117531311753, 353753117531311753171311753 (...)

The terms  $Sr(2) = 353$  and  $Sr(4) = 35375311753$  are primes. The question is: are there infinitely many primes in this sequence?

### II.

#### The reconcatenated back concatenated odd sequence

$S(n)$  is defined as the sequence obtained through the concatenation of the first  $n$  odd numbers, in reverse order, having the terms (A038395 in OEIS): 1, 31, 531, 7531, 97531, 1197531, 131197531, 15131197531, 1715131197531, 1917151311975311, 211917151311975311 (...)

Because sometimes are obtained interesting results not considering the initial term of a Smarandache concatenated sequence (e.g., in this sequence, considering as the first term the

number 31, the back concatenation of the second odd number, 3, with the first, 1, and not the initial term, 1) we will proceed this way reconcatenating this sequence. To distinguish such sequences from the standard ones, we will note them  $S+(n)$ , having in this case the terms 31, 531, 7531, 97531 (...), respectively  $Sr+(n)$ .

The terms of  $Sr+(n)$ : 31531, 315317531, 31531753197531, 315317531975311197531, 315317531975311197531131197531, 31531753197531119753113119753115131197531 (...)

The terms  $Sr+(2) = 315317531$  and  $Sr+(6) = 31531753197531119753113119753115131197531$  are primes. Note that  $Sr+(6)$  is a prime with 41 digits! The question is: are there infinitely many primes in this sequence?

### III.

#### The reconcatenated reverse sequence

$S(n)$  is defined as the sequence obtained through the concatenation of the first  $n$  positive integers, in reverse order, having the terms (A000422 in OEIS): 1, 21, 321, 4321, 54321, 654321, 7654321, 87654321, 987654321, 10987654321 (...)

We will reconcatenate the sequence  $S+(n)$ , accordingly to the definition from the previously treated sequence, having in this case the terms 21, 321, 4321, 543211, 654321 (...).

The terms of  $Sr+(n)$ : 21321, 213214321, 21321432154321, 21321432154321654321, 213214321543216543217654321, 21321432154321654321765432187654321, 21321432154321654321765432187654321987654321 (...)

The terms  $Sr+(2) = 21321432154321654321765432187654321987654321$  and  $Sr+(7) = 21321432154321654321765432187654321987654321$  are primes. Note that  $Sr+(7)$  is a prime with 44 digits! The question is: are there infinitely many primes in this sequence?

### IV.

#### The reconcatenated back concatenated square sequence

$S(n)$  is defined as the sequence obtained through the concatenation of the squares of the first  $n$  positive integers, in reverse order, having the terms (A038397 in OEIS): 1, 41, 941, 16941, 2516941, 362516941, 49362516941 (...)

We will reconcatenate the sequence  $S+(n)$ , accordingly to the definition from the previously treated two sequences, having in this case the terms 41, 941, 16941, 2516941 (...).

The terms of  $Sr+(n)$ : 41941, 4194116941, 41941169412516941, 41941169412516941362516941, 4194116941251694136251694149362516941, 41941169412516941362516941493625169416449362516941 (...)

The term  $Sr+(6)$  is a semiprime having 50 digits! The question is: are there infinitely many primes in this sequence?



## 10. On terms of consecutive numbers sequence concatenated both to the left and to the right with same prime

**Abstract.** In this paper I make the following conjecture: For any term  $S(n)$  of the *Smarandache consecutive numbers sequence* (1, 12, 123, 1234, 12345, 123456, 1234567...) there exist an infinity of primes  $p$  such that the number  $q$  obtained concatenating  $S(n)$  both to the left and to the right with  $p$  is prime.

### Conjecture:

For any term  $S(n)$  of the Smarandache consecutive numbers sequence (1, 12, 123, 1234, 12345, 123456, 1234567...) there exist an infinity of primes  $p$  such that the number  $q$  obtained concatenating  $S(n)$  both to the left and to the right with  $p$  is prime.

#### The sequence of primes $q$ for $S(n) = 1$ :

: 313, 17117, 29129, 41141, 47147, 59159, 71171, 89189, 1131113, 1311131, 1371137, 2391239, 2631263, 3591359, 3891389, 4431443, 4611461, 4671467, 5091509 (...)

#### The sequence of primes $q$ for $S(n) = 12$ :

: 7127, 111211, 131213, 231223, 371237, 411241, 531253, 591259, 10112101, 17912179, 22912229, 24112241, 29312293, 30712307, 31112311, 4191249, 47912479 (...)

#### The sequence of primes $q$ for $S(n) = 123$ :

: 71237, 3112331, 5312353, 6112361, 6712367, 8912389, 9712397, 103123103, 131123131, 151123151, 167123167, 173123173, 193123193, 211123211, 227123227, 241123241, 251123251, 271123271, 307123307, 311123311, 313123313, 379123379, 389123389, 421123421, 449123449 (...)

#### The sequence of primes $q$ for $S(n) = 1234$ :

: 312343, 71123471, 1491234149, 2271234227, 2511234251, 3531234353 (...)

#### The sequence of primes $q$ for $S(n) = 12345$ :

: 111234511, 311234531, 371234537, 531234553, 711234571, 11312345113, 13112345131, 15112345151, 15712345157, 17912345179, 19312345193, 24112345241, 31112345311, 34712345347, 35312345353, 38912345389, 40112345401, 44312345443, 44912345449, 47912345479, 49912345499 (...)

#### The sequence of primes $q$ for $S(n) = 123456$ :

: 71234567, 1912345619, 311234563, 4712345647, 7312345673, 127123456127, 131123456131, 157123456157, 167123456167, 179123456179, 181123456181,

193123456193, 227123456227, 229123456229, 233123456233, 281123456281,  
313123456313, 317123456317, 353123456353, 359123456359, 409123456409,  
421123456421, 443123456443, 449123456449, 487123456487 (...)

**The sequence of primes  $q$  for  $S(n) = 1234567$ :**

: 71123456771, 1311234567131, 1371234567137, 1491234567149,  
2571234567257 (...)

**The sequence of primes  $q$  for  $S(n) = 12345678$ :**

: 611234567861, 831234567883, 10712345678107, 18112345678181,  
21112345678211, 29312345678293, 34712345678347, 35912345678359,  
37912345678379, 38912345678389, 40912345678409, 45712345678457,  
49112345678491, 49912345678499 (...)

**The sequence of primes  $q$  for  $S(n) = 123456789$ :**

: 1712345678917, 6712345678967, 227123456789227, 281123456789281,  
353123456789353, 409123456789409 (...)

**Note:** the numbers from sequences above covers all possibilities up to  $S(n) = 123456789$   
and  $p \leq 509$ .

**The least prime  $q$  for  $S(n) = 12345678910$ :**

: 831234567891083, for  $p = 83$ .

**The least prime  $q$  for  $S(n) = 123456789101112131415161718$ :**

: 71234567891011121314151617187, for  $p = 7$ .



## 11. Primes obtained concatenating with 1 to the left the terms of three Smarandache sequences

**Abstract.** In this paper I state the following three conjectures: (I) There exist an infinity of primes  $p$  obtained concatenating to the left with 1 the terms of the *Smarandache reverse sequence*; (II) There exist an infinity of primes  $p$  obtained concatenating to the left with 1 the terms of the *Smarandache back concatenated odd sequence*; (III) There exist an infinity of primes  $p$  obtained concatenating to the left with 1 the terms of the *Smarandache back concatenated square sequence*.

### Conjecture I:

There exist an infinity of primes  $p$  obtained concatenating to the left with 1 the terms of the *Smarandache reverse sequence* (defined as the sequence obtained through the concatenation of the first  $n$  positive integers, in reverse order).

**The Smarandache reverse sequence** (A000422 in OEIS):

: 1, 21, 321, 4321, 54321, 654321, 7654321, 87654321, 987654321, 10987654321 (...)

**The sequence of primes  $p$ :**

: 11, 1321, 14321, 154321, 113121110987654321, 11413121110987654321 (...)

### Conjecture II:

There exist an infinity of primes  $p$  obtained concatenating to the left with 1 the terms of the *Smarandache back concatenated odd sequence* (defined as the sequence obtained through the concatenation of the first  $n$  odd numbers, in reverse order).

**The Smarandache back concatenated odd sequence** (A038395 in OEIS):

: 1, 31, 531, 7531, 97531, 1197531, 131197531, 15131197531, 1715131197531, 1917151311975311 (...)

**The sequence of primes  $p$ :**

: 131, 1531, 151494745434139373533312927252321191715131197531 (...)

### Conjecture III:

There exist an infinity of primes  $p$  obtained concatenating to the left with 1 the terms of the *Smarandache back concatenated square sequence* (defined as the sequence obtained through the concatenation of the first  $n$  squares, in reverse order).

**The Smarandache back concatenated square sequence** (A038397 in OEIS):

: 1, 41, 941, 16941, 2516941, 362516941, 49362516941, 6449362516941, 816449362516941, 100816449362516941 (...)

**The sequence of primes  $p$ :**

: 11, 12516941, 16449362516941, 1100816449362516941 (...)

## 12. Primes obtained concatenating to the right with 1 the terms of concatenated n-th powers sequences

**Abstract.** In this paper I state the following conjecture: for any positive integer  $n > 1$  there exist a sequence having an infinity of prime terms  $p$ , obtained concatenating to the right with 1 the terms of the sequence of concatenated n-th powers. For  $n = 2$  the primes  $p$  are obtained concatenating with 1 to the right the terms of the *Smarandache concatenated squares sequence*; for  $n = 3$  the primes  $p$  are obtained concatenating with 1 to the right the terms of the *Smarandache concatenated cubic sequence*.

### Conjecture:

For any positive integer  $n > 1$  there exist a sequence having an infinity of prime terms  $p$ , obtained concatenating to the right with 1 the terms of the sequence of concatenated n-th powers. For  $n = 2$  the primes  $p$  are obtained concatenating with 1 to the right the terms of the *Smarandache concatenated squares sequence* (defined as the sequence obtained through the concatenation of the first  $m$  squares); for  $n = 3$  the primes  $p$  are obtained concatenating with 1 to the right the terms of the *Smarandache concatenated cubic sequence* (defined as the sequence obtained through the concatenation of the first  $m$  cubes); for  $n = 4$  the primes  $p$  are obtained concatenating with 1 to the right the terms of the *concatenated 4<sup>th</sup> powers sequence* and so on.

**The concatenated squares sequence** (A019521 in OEIS):

: 1, 14, 149, 14916, 1491625, 149162536, 14916253649, 1491625364964,  
149162536496481, 149162536496481100, 149162536496481100121,  
149162536496481100121144, (...)

The sequence of primes  $p$ :

: 11, 149161, 1491625361, 1491625364964811001,  
1491625364964811001211441691961 (...)

**The concatenated cubic sequence** (A019522 in OEIS):

: 1, 18, 1827, 182764, 182764125, 182764125216, 182764125216343,  
182764125216343512, 182764125216343512729,  
1827641252163435127291000 (...)

The sequence of primes  $p$ :

: 11, 181, 1827641251, 1827641252161, 1827641252163435127291,  
1491625364964811001211441691961 (...)

**The concatenated 4<sup>th</sup> powers sequence**  
(see A000583 in OEIS for the fourth powers):

: 1, 116, 11681, 11681256, 11681256625, 116812566251296,  
1168125662512962401, 11681256625129624014096,  
116812566251296240140966561 (...)

The sequence of primes p:

: 11, 1168125662512961 (...)

**The concatenated 5<sup>th</sup> powers sequence**

(see A000584 in OEIS for the fifth powers):

: 1, 132, 132243, 1322431024, 13224310243125, 132243102431257776,  
13224310243125777616807, 1322431024312577761680732768 (...)

The sequence of primes p:

: 11, 1321, 132243102431257776168071, 132243102431257776168071  
(...)

**The concatenated 6<sup>th</sup> powers sequence**

(see A001014 in OEIS for the sixth powers):

: 1, 164, 164729, 1647294096, 164729409615625, 16472940961562546656,  
16472940961562546656117649, 16472940961562546656117649262144 (...)

The sequence of primes p:

: 11, 1647294096156251 (...)

## **Part Two.**

### **Sequences of primes obtained by the method of concatenation**

#### **1. A list of 33 sequences of primes obtained by the method of concatenation**

**Abstract.** In this paper I list a number of 33 sequences of primes obtained by the method of concatenation; some of these sequences are presented and analyzed in more detail in my previous papers, gathered together in five books of collected papers: “Two hundred conjectures and one hundred and fifty open problems on Fermat pseudoprimes”, “Two hundred and thirteen conjectures on primes”, “Conjectures on primes and Fermat pseudoprimes, many based on Smarandache function”, “Sequences of integers, conjectures and new arithmetical tools”, “Formulas and polynomials which generate primes and Fermat pseudoprimes”.

##### **Sequence 1**

Conjecture: there exist an infinity of primes  $p$  obtained concatenating the square of a prime  $q$ , to the right, with the group of digits 0001.

The sequence of primes  $p$ :

: 490001, 2890001, 8410001, 18490001, 22090001, 50410001, 114490001, 171610001, 193210001, 327610001, 364810001 (...), obtained for  $q = 7, 17, 29, 43, 47, 71, 107, 131, 139, 181, 191$  (...)

##### **Sequence 2**

Conjecture: there exist an infinity of primes  $p = n/q$ , where  $n$  is the number obtained concatenating the square of a prime  $q$ , to the right, with  $q$  (example:  $p = 497/7 = 71$ ):

The sequence of primes  $p$ :

: 31, 71, 1301, 1901, 3701, 6101, 6701, 7901, 103001, 109001, 181001 (...), obtained for  $q = 3, 7, 13, 19, 37, 61, 67, 79, 103, 109, 181$  (...)

##### **Sequence 3**

Conjecture: there exist an infinity of primes  $p = n/q$ , where  $n$  is the number obtained concatenating the square of a prime  $q$ , to the left, with  $q$  (example:  $p = 749/7 = 107$ ):

The sequence of primes  $p$ :

: 13, 107, 1013, 1019, 10037, 10061, 10067, 10079, 100103, 100109 (...), obtained for  $q = 3, 7, 13, 19, 37, 61, 67, 79, 103, 109$  (...)

##### **Sequence 4**

Conjecture: there exist an infinity of pairs of primes  $(p_1, p_2)$  defined as follows:  $p_1 = m/q$ , where  $m$  is the number obtained concatenating the square of a prime  $q$ , to the left, with  $q$ , and  $p_2 = n/q$ , where  $n$  is the number obtained concatenating the square of the prime  $q$ , to the right, with  $q$ .

The sequence of pairs of primes (p, q):

: (13, 31), (107, 71), (1013, 1301), (1019, 1901), (10037, 3701), (10061, 6101), (10067, 6701), (10079, 7901), (100103, 103001), (100109, 109001) (...), obtained for  $q = 3, 7, 13, 19, 37, 61, 67, 79, 103, 109$  (...)

### Sequence 5

Conjecture: there exist, for any  $k$  positive integer,  $k > 1$ , an infinity of primes  $p = n/q$ , where  $n$  is the number obtained concatenating a prime  $q$  raised to power  $k$ , to the left, with  $q$  (example: for  $k = 3$ ,  $p = 7343/7 = 1049$ ):

The sequence of primes  $p$  for  $k = 3$ :

: 109, 1049, 10169, 101681 (...), obtained for  $q = 3, 7, 13, 41$  (...)

The sequence of primes  $p$  for  $k = 4$ :

: 127, 10343, 102197 (...), obtained for  $q = 3, 7, 13$  (...)

### Sequence 6

Conjecture: there exist, for any  $k$  positive integer,  $k > 1$ , an infinity of primes  $p = n/q$ , where  $n$  is the number obtained concatenating a prime  $q$  raised to power  $k$ , to the right, with  $q$  (example: for  $k = 3$ ,  $p = 3437/7 = 491$ ):

The sequence of primes  $p$  for  $k = 3$ :

: 491, 12101, 16901, 52901, 184901, 220901 (...), obtained for  $q = 7, 11, 13, 23, 43, 47$  (...)

The sequence of primes  $p$  for  $k = 4$ :

: 271 (...), obtained for  $q = 3$  (...)

### Sequence 7

Conjecture: there exist, for any  $k$  positive integer,  $k > 1$ , an infinity of pairs of primes ( $p_1, p_2$ ) defined as follows:  $p_1 = m/q$ , where  $m$  is the number obtained concatenating the square of a prime  $q$  raised to power  $k$ , to the left, with  $q$ , and  $p_2 = n/q$ , where  $n$  is the number obtained concatenating the square of the prime  $q$  raised to power  $k$ , to the right, with  $q$ .

The sequence of pairs of primes (p, q) for  $k = 3$ :

: (1049, 491), (10169, 16901) (...), obtained for  $q = 7, 13$  (...)

The sequence of pairs of primes (p, q) for  $k = 4$ :

: (127, 271) (...), obtained for  $q = 3$  (...)

### Sequence 8

Primes  $p$  obtained through successive concatenation of the numbers  $q, R(q), q, R(q)$  and  $q$ , where  $q$  is an emirp (prime whose reversal is a different prime) and  $R(q)$  its reversal:

The sequence of primes  $q$ :  
 : 1331133113, 9779977997, 769967769967769,  
 15111151151111511511 (...), obtained for  $q = 13, 97, 769, 1511$  (...)

### Sequence 9

Primes  $p$  obtained through successive concatenation of a prime  $q$  with its reversal, not necessarily prime,  $R(q)$ :

The least prime  $p$  obtained for the following primes  $q$ :  
 : for  $q = 13$ ,  $p = 1331133113$  is prime;  
 : for  $q = 19$ ,  $p = 19911991199119$  is prime;  
 : for  $q = 29$ ,  $p = 2992299229$  is prime;  
 : for  $q = 31$ ,  $p = 31133113311331$  is prime;  
 : for  $q = 43$ ,  $p = 4334433443$  is prime;  
 : for  $q = 97$ ,  $p = 9779977997$  is prime;  
 : for  $q = 127$ ,  $p = 127721127721127$  is prime.

Note the following patterns:  
 (we will note with “[ $c$ ]” the operation “concatenate to”)

:  $p = q[c[R(q)]c[q]c[R(q)]c[q]$ ;  
 :  $p = q[c[R(q)]c[q]c[R(q)]c[q]c[R(q)]c[q]$ .

### Sequence 10

Primes  $p$  obtained through successive concatenation of the numbers  $q_1, q_2, q_1, q_2$  and  $q_1$ , where  $q_1$  and  $q_2$  are primes such that  $q_1 - q_2$  or  $q_2 - q_1$  is multiple of 18 (note that in the case of reversible primes  $n - R(n)$  or  $R(n) - n$  is multiple of 18). I conjecture that there exist an infinity of primes  $p$ .

The sequence of pairs of primes  $(q_1, q_2)$ :

: for  $(q_1, q_2) = (13, 31)$ ,  $p = 1331133113$  is prime;  
 : for  $(q_1, q_2) = (29, 47)$ ,  $p = 2947294729$  is prime;  
 : for  $(q_1, q_2) = (53, 71)$ ,  $p = 5371537153$  is prime;  
 : for  $(q_1, q_2) = (79, 61)$ ,  $p = 7961796179$  is prime;  
 : for  $(q_1, q_2) = (89, 71)$ ,  $p = 8971897189$  is prime;  
 : for  $(q_1, q_2) = (97, 79)$ ,  $p = 9779977997$  is prime (...)

### Sequence 11

Primes  $p$  obtained through successive concatenation of the numbers  $q_1, q_2, q_1, q_2, q_1, q_2$  and  $q_1$ , where  $q_1$  and  $q_2$  are primes such that  $q_1 - q_2$  or  $q_2 - q_1$  is multiple of 18. I conjecture that there exist an infinity of primes  $p$ .

The sequence of pairs of primes  $(q_1, q_2)$ :

: for  $(q_1, q_2) = (13, 31)$ ,  $p = 31133113311331$  prime;  
 : for  $(q_1, q_2) = (41, 23)$ ,  $p = 41234123412341$  prime (...)

### Sequence 12

Primes  $p = n/11$ , where  $n$  is obtained concatenating a prime  $q$  with its reversal, not necessarily prime,  $R(q)$ :

The sequence of primes  $p$ :

: 181, 283, 647, 727, 9391, 9791, 10301, 12721, 14341, 14851 (...), obtained for  $q = 19, 31, 71, 79, 103, 109, 113, 139, 157, 163$  (...)

### Sequence 13

Conjecture: there exist an infinity of primes  $q$  obtained concatenating to the left with 1 the square of a Poulet number  $P$ :

The sequence of primes  $q$ :

:  $q = 1116281, 12989441, 17295401, 119105641, 121911761, 171927361, 1163865601, 1188815081, 1195468361, 1907274641$  (...), obtained for  $P = 341, 1729, 2701, 4371, 4681, 8481, 12801, 13741, 13981, 30121$  (...)

### Sequence 14

Conjecture: there exist an infinity of primes  $q$  obtained concatenating to the left with 9 the square of a Poulet number  $P$ :

The sequence of primes  $q$ :

:  $q = 9116281, 97958041, 919088161, 921911761, 969239041, 9209989081$  (...), obtained for  $P = 341, 2821, 4369, 4681, 8321, 14491$  (...)

### Sequence 15

Conjecture: there exist an infinity of primes  $p$  obtained concatenating a prime  $q$  to the left with  $n$  where  $n$  has the sum of its digits  $s(n)$  equal to  $q$ .

Primes  $p$  that belong to this sequence:

:  $p = 9413$ , where  $(q, n) = (13, 94)$  and  $s(n) = 13 = q$ ;  
:  $p = 9817$ , where  $(q, n) = (17, 98)$  and  $s(n) = 17 = q$ ;  
:  $p = 99119$ , where  $(q, n) = (19, 991)$  and  $s(n) = 19 = q$ ;  
:  $p = 19919$ , where  $(q, n) = (19, 199)$  and  $s(n) = 19 = q$ ;  
:  $p = 95923$ , where  $(q, n) = (23, 959)$  and  $s(n) = 23 = q$ ;  
:  $p = 999431$ , where  $(q, n) = (31, 9994)$  and  $s(n) = 31 = q$ ;  
:  $p = 949931$ , where  $(q, n) = (31, 9499)$  and  $s(n) = 23 = q$ .

### Sequence 16

Conjecture: there exist an infinity of primes  $p$  obtained concatenating a prime  $q$  to the left with  $n$  where  $n$  has the sum of its digits  $s(n)$  equal to the sum of the digits  $s(q)$  of  $q$ .

Primes  $p$  that belong to this sequence:

- :  $p = 211$ , where  $(q, n) = (11, 2)$  and  $s(q) = s(n) = 2$ ;
- :  $p = 2011$ , where  $(q, n) = (11, 20)$  and  $s(q) = s(n) = 2$ ;
- :  $p = 20011$ , where  $(q, n) = (11, 200)$  and  $s(q) = s(n) = 2$ ;
- :  $p = 431$ , where  $(q, n) = (31, 4)$  and  $s(q) = s(n) = 4$ ;
- :  $p = 4013$ , where  $(q, n) = (13, 40)$  and  $s(q) = s(n) = 4$ ;
- :  $p = 22013$ , where  $(q, n) = (13, 220)$  and  $s(q) = s(n) = 4$ .

### **Sequence 17**

Conjecture: there exist an infinity of primes  $p$  obtained concatenating a prime  $q$  to the left with  $n$  where  $n$  has the digital root  $d(n)$  equal to the digital root  $d(q)$  of  $q$ .

Primes  $p$  that belong to this sequence:

- :  $p = 1019$ , where  $(q, n) = (19, 10)$  and  $d(q) = d(n) = 1$ ;
- :  $p = 100019$ , where  $(q, n) = (19, 1000)$  and  $d(q) = d(n) = 1$ ;
- :  $p = 1129$ , where  $(q, n) = (29, 11)$  and  $d(q) = d(n) = 2$ .

### **Sequence 18**

Conjecture: there exist an infinity of primes  $p$  obtained concatenating a prime  $q$  to the right with  $n$  where  $n$  has the sum of its digits  $s(n)$  equal to  $q$ .

Primes  $p$  that belong to this sequence:

- :  $p = 523$ , where  $(q, n) = (5, 23)$  and  $s(n) = 5 = q$ ;
- :  $p = 761$ , where  $(q, n) = (7, 61)$  and  $s(n) = 7 = q$ ;
- :  $p = 1367$ , where  $(q, n) = (13, 67)$  and  $s(n) = 13 = q$ ;
- :  $p = 11821$ , where  $(q, n) = (11, 821)$  and  $s(n) = 11 = q$ .

### **Sequence 19**

Conjecture: there exist an infinity of primes  $p$  obtained concatenating a prime  $q$  to the right with  $n$  where  $n$  has the sum of its digits  $s(n)$  equal to the sum of the digits  $s(q)$  of  $q$ .

Primes  $p$  that belong to this sequence:

- :  $p = 2341$ , where  $(q, n) = (23, 41)$  and  $s(q) = s(n) = 5$ ;
- :  $p = 200341$ , where  $(q, n) = (2003, 41)$  and  $s(q) = s(n) = 5$ .

### **Sequence 20**

Conjecture: there exist an infinity of primes  $p$  obtained concatenating a prime  $q$  to the right with  $n$  where  $n$  has the digital root  $d(n)$  equal to the digital root  $d(q)$  of  $q$ .

Primes  $p$  that belong to this sequence:

- :  $p = 29100109$ , where  $(q, n) = (29, 100109)$  and  $s(q) = s(n) = 2$ ;
- :  $p = 20000911000009$ , where  $(q, n) = (200009, 11000009)$  and  $s(q) = s(n) = 2$ .



### Sequence 21

Conjecture: for any prime  $p$  greater than or equal to 7 there exist  $n$ , a power of 2, such that, concatenating to the left  $p$  with  $n$ , the number resulted is a prime.

The sequence of the primes obtained, for  $p \geq 7$ :

47, 211, 1613, 3217, 419, 223, 229, 431, 1637, 241, 443, 1638447, 853, 859, 461, 467, 271, 6473, 479, 283, 12889, 1697, 8101, 16103, 2048107, 64109, 2113, 4127, 2131 (...)

The corresponding sequence of the exponents of 2 for which a prime is obtained:

2, 1, 4, 5, 2, 1, 1, 2, 4, 1, 2, 14, 3, 3, 2, 2, 1, 6, 2, 1, 7, 4, 3, 4, 11, 6, 1, 2, 1 (...)

### Sequence 22

Conjecture: for any odd prime  $p$  there exist  $n$ , a power of 2, such that, subtracting one from the number resulted concatenating to the right  $p$  with  $n$ , is obtained a prime.

The sequence of the primes obtained, for odd primes  $p$ :

31, 53, 71, 113, 131, 173, 191, 233, 293, 311, 373, 41257, 431, 47262143, 531023, 593, 613, 673, 71257 (...)

The corresponding sequence of the exponents of 2 for which a prime is obtained:

1, 2, 1, 2, 1, 2, 1, 2, 2, 1, 2, 8, 1, 18, 10, 2, 2, 2, 8 (...)

### Sequence 23

Conjecture: for any odd prime  $p$  there exist  $n$ , a power of 2, such that, adding one to the number resulted concatenating to the right  $p$  with  $n$ , is obtained a prime.

The sequence of the primes obtained, for odd primes  $p$ :

317, 53, 73, 113, 139, 173, 193, 233, 293, 313, 373, 419, 479, 5333, 613, 673, 719, 733, 7933, 839, 163, 8933 (...)

The corresponding sequence of the exponents of 2 for which a prime is obtained:

4, 1, 1, 1, 3, 1, 1, 1, 1, 1, 1, 3, 5, 1, 1, 3, 1, 5, 3, 1, 5 (...)

### Sequence 24

Conjecture: there exist an infinity of primes  $q$  of the form  $p^2 + 4320$ , where  $p$  is prime.

The sequence of the primes  $q$ :

: 4339, 4441, 5281, 5689, 6529, 7129, 9649, 12241, 13729, 14929, 21481 (...)

obtained for  $p = 11, 19, 31, 37, 53, 73, 89, 97, 103, 131 (\dots)$

### Sequence 25

Conjecture: there exist an infinity of primes  $p$  obtained concatenating an odd power of 2, i.e.  $2^k$ , where  $k$  odd number, both to the left and to the right with 1.

The sequence of the primes  $p$ :

: 181, 1321, 15121, 1335544321, 121474836481, 1351843720888321,  
194447329657392904273921, 1405648192073033408478945025720321,  
125961484292674138142652481646100481,  
1425352958651173079329218259289710264321,  
16805647338418769269267492148635364229121 (...)

obtained for  $k = 3, 5, 9, 25, 31, 45, 73, 105, 111, 125, 129 (\dots)$

### Sequence 26

Conjecture: there exist an infinity of primes  $p$  obtained concatenating to the right a number  $n$  of the form  $6 \cdot k + 1$  with the group of digits 081.

The sequence of the pairs  $[n, p]$ :

: [19, 19081]; [31, 31081]; [97, 97081]; [49, 49081]; [85, 85081]; [91, 91081];  
[121, 121081]; [127, 127081]; [157, 157081]; [175, 175081]; [181, 181081];  
[187, 187081]; [199, 199081]; [205, 205081]; [217, 217081]; [229, 229081];  
[241, 241081 =  $491^2$ ]; [253, 253081]; [259, 259081 =  $509^2$ ]; [295, 295081];  
[313, 313081]; [325, 325081]; [331, 331081]; [337, 337081]; [343, 343081];  
[349, 349081]; [379, 379081]; [385, 385081]; [409, 409081]; [421, 421081];  
[427, 427081]; [439, 439081]; [475, 475081]; [517, 517081]; [559, 559081];  
[577, 577081]; [563, 563081]; [569, 569081]; [595, 595081]; [607, 607081]...

### Sequence 27

Conjecture: there exist an infinity of primes  $p$  obtained concatenating to the right a multiple of 30,  $m$ , with a power of prime,  $n$ .

Triplets  $[m, n, p]$  that belong to this sequence:

: [30, 49, 3049]; [30, 169, 30169]; [30, 529, 30529]; [30, 841, 30841]; [30, 1681, 301681];  
[30, 4489, 304489]; [30, 5329, 305329]; [60, 169, 60169]; [60, 289, 60289];  
[60, 961, 60961]; [60, 1849, 601849]; [60, 5329, 605329]; [60, 6241, 606241];  
[60, 7921, 607921]; [90, 49, 9049]; [90, 121, 90121]; [90, 289, 90289];  
[90, 529, 90529]; [90, 841, 90841]; [90, 4489, 904489]; [90, 5329, 905329]; [90, 9409, 909409];  
[120, 49, 12049]; [120, 121, 120121]; [150, 169, 150169]; [180, 49, 18049];  
[180, 289, 180289]; [210, 361, 210361]; [240, 49, 24049]; [270, 121, 270121];  
[300, 961, 300961]; [330, 49, 33049]...

### Sequence 28

Conjecture: there exist an infinity of primes  $p$  obtained concatenating to the right the numbers  $q^2 - 1$ , where  $q$  are primes of the form  $6k - 1$ , with the digit 1.

The sequence of the primes  $p$ :

: 241, 1201, 5281, 28081, 68881, 79201, 102001, 127681, 278881, 299281,  
320401, 364801, 388081 (...) (...)  
obtained for  $q = 5, 11, 23, 53, 83, 89, 101, 113, 167, 173, 179, 191, 197 (...)$

### Sequence 29

Observation: taking a number having just even digits and concatenating it three times with itself and then to the right with the digit 1 seems that are great chances to obtain a number with very few prime factors and (I conjecture that an infinite sequence of) primes  $p$ .

The sequence of primes  $p$ :

: 2221, 4441, 6661, 2424241, 2828281, 4040401, 4242421, 6262621, 6868681,  
8282821, 2002002001, 2242242241, 2422422421, 2482482481, 2602602601,  
2622622621, 2642642641, 4044044041, 4424424421, 4824824821, 6226226221,  
6266266261, 6486486481, 6646646641, 6666666661, 6846846841, 8448448441,  
8648648641, 2004200420041, 2024202420241, 2042204220421 (...)

### Sequence 30

Conjecture: there exist an infinity of primes  $p$  obtained concatenating a number of the form  $6k + 1$  with its reversal then with 1.

The sequence of primes  $p$ :

: 23321, 29921, 35531, 41141, 47741, 59951, 71171, 1255211, 1311311, 1855811,  
1911911, 2033021, 2099021, 2155121, 2277221, 2333321, 2511521, 2699621,  
2755721, 2999921 (...)

Note the chain of five consecutive primes (23321, 29921, 35531, 41141, 47741) obtained for five consecutive numbers of the form  $6k + 1$  (23, 29, 35, 41, 47).

Few larger primes  $p$ :

: 1046399364011, 1046811864011, 1047233274011.

### Sequence 31

Conjecture: there exist an infinity of primes  $p$  obtained concatenating a square of a prime  $q^2$ ,  $q$  greater than 5, to the left, with a number of the form  $6k$ .

The sequence of primes  $p$  for  $q^2 = 49$ :

: 1849, 3049, 5449, 9049, 9649 (...), obtained for  $k = 3, 5, 9, 15, 16 (...)$

The sequence of primes  $p$  for  $q^2 = 121$ :

: 6121, 18121, 24121, 48121, 54121, 78121, 84121, 90121 (...), obtained for  $k = 1, 3, 4, 8, 9, 13, 14, 15$  (...)

The sequence of primes  $p$  for  $q^2 = 169$ :

: 18169, 24169, 30169, 42169, 60169, 66169, 72169 (...), obtained for  $k = 3, 4, 5, 7, 10, 11, 12$  (...)

The sequence of primes  $p$  for  $q^2 = 289$ :

: 12289, 18289, 60289, 90289, 96289 (...), obtained for  $k = 2, 3, 10, 15, 16$  (...)

Few larger primes  $p$ :

: 12010963555849, for  $q^2 = 104707^2$  and  $k = 20$ ;

: 121096433521, for  $q^2 = 104711^2$  and  $k = 2$ ;

: 1810965650089, for  $q^2 = 104717^2$  and  $k = 3$ ;

: 6610966906729, for  $q^2 = 104723^2$  and  $k = 11$ ;

: 1810968163441, for  $q^2 = 104729^2$  and  $k = 3$ .

### Sequence 32

Conjecture: there exist an infinity of primes  $p$  obtained concatenating a square of a number  $n$  of the form  $6*k + 1$  with the square of the number  $2*n - 1$ .

The sequence of primes  $p$ :

: 49169 ( $49 = 7^2$  and  $169 = 13^2$  where  $13 = 2*7 - 1$ );

: 3611369 ( $361 = 19^2$  and  $1369 = 37^2$  where  $37 = 2*19 - 1$ );

: 6252401 ( $625 = 25^2$  and  $2401 = 49^2$  where  $49 = 2*25 - 1$ );

: 13695329 ( $1369 = 37^2$  and  $5329 = 73^2$  where  $73 = 2*37 - 1$ );

: 372114641 ( $3721 = 61^2$  and  $14641 = 121^2$  where  $121 = 2*61 - 1$ );

: 624124649 ( $6241 = 79^2$  and  $24649 = 157^2$  where  $157 = 2*79 - 1$ ); (...)

### Sequence 33

Conjecture: for any  $n$  of the form  $3*k + 1$  there exist an infinity of primes  $q$  such that the numbers  $p_1 = n \backslash (n + 1) \backslash p$  and  $p_2 = (n + 1) \backslash n \backslash p$  are both primes. The operator “ $\backslash$ ” is used with the meaning “concatenated to”.

: for  $n = 1$ , there exist the following pairs of  $(P_1, P_2)$ :

(1213, 2113), (1229, 2129), (1231, 2131), (1237, 2137), (1279, 2179), (12101, 21101), (12107, 21107), (12149, 21149)...

: for  $n = 4$ , there exist the following pairs of  $(P_1, P_2)$ :

(457, 547), (4513, 5413), (4517, 5417), (4519, 5419), (4583, 5483), (45139, 54139), (45181, 54181)...

: for  $n = 7$ , there exist the following pairs of  $(P_1, P_2)$ :

(787, 877), (7841, 8741), (7853, 8753), (7879, 8779), (7883, 8787), (78179, 87179)...

## 2. Sixteen sequences of primes obtained by concatenation from $p - 1$ respectively $p + 1$ where $p$ prime

**Abstract.** In this paper I make the following four conjectures: (I) there exist, for any prime  $p$  having the value of the last digit  $d$  equal to 1, respectively to 3, 7 or 9, an infinity of primes obtained concatenating  $p - 1$  with the value of  $d$ ; (II) there exist, for any prime  $p$  having the value of the last digit  $d$  equal to 1, respectively to 3, 7 or 9, an infinity of primes obtained concatenating twice  $p - 1$  with the value of  $d$ ; (III) there exist, for any prime  $p$  having the value of the last digit  $d$  equal to 1, respectively to 3, 7 or 9, an infinity of primes obtained concatenating  $p + 1$  with the value of  $d$ ; (II) there exist, for any prime  $p$  having the value of the last digit  $d$  equal to 1, respectively to 3, 7 or 9, an infinity of primes obtained concatenating twice  $p + 1$  with the value of  $d$ .

### Conjecture 1:

There exist, for any prime  $p$  having the value of the last digit  $d$  equal to 1, respectively to 3, 7 or 9, an infinity of primes  $q$  obtained concatenating  $p - 1$  with the value of  $d$ .

The sequence of  $q$  obtained from  $p$  having last digit 1:

: 101, 401, 601, 701, 1301, 1801, 1901, 2801, 3301 (...) 9824504101, 9824511601, 9824513201 (...)  
obtained for  $p = 11, 41, 61, 71, 181, 191, 281, 311$  (...) 982450411, 982451161, 982451321 (...)

The sequence of  $q$  obtained from  $p$  having last digit 3:

: 223, 523, 823, 1123, 1723, 3823, 4423, 5023, 5623 (...) 9824490523, 9824491123, 9824502223 (...)  
obtained for  $p = 23, 53, 83, 113, 173, 383, 443, 503$  (...) 9824490523, 982449113, 982450223 (...)

The sequence of  $q$  obtained from  $p$  having last digit 7:

: 167, 367, 467, 967, 1367, 1567, 1667, 2267, 2567 (...) 9824507867, 9824507867, 9824514967 (...)  
obtained for  $p = 17, 37, 47, 97, 137, 157, 167, 227, 257$  (...) 982450787, 982450787, 982451497 (...)

The sequence of  $q$  obtained from  $p$  having last digit 9:

: 1489, 1789, 2389, 2689, 3889, 4789, 5689, 8089, 8389 (...) 9824490589, 9824494489, 9824511589 (...)  
obtained for  $p = 149, 179, 239, 269, 389, 479, 569, 809, 839$  (...) 982449059, 982449449, 982451159 (...)

### Conjecture 2:

There exist, for any prime  $p$  having the value of the last digit  $d$  equal to 1, respectively to 3, 7 or 9, an infinity of primes  $q$  obtained concatenating twice  $p - 1$  with the value of  $d$ .

The sequence of  $q$  obtained from  $p$  having last digit 1:

: 3011, 6011, 21011, 54011, 69011, 75011, 120011, 129011 (...) 982448491,  
98244912011, 98244996011 (...)  
obtained for  $p = 31, 61, 211, 541, 691, 751, 1201, 1291$  (...) 98244849011,  
982449121, 982449961 (...)

The sequence of  $q$  obtained from  $p$  having last digit 3:

: 5233, 8233, 59233, 95233, 110233, 119233, 158233, 161233 (...) 98244998233,  
98245022233, 98245091233 (...)  
obtained for  $p = 53, 83, 593, 953, 1103, 1193, 1583, 1613$  (...) 98244998233,  
982450223, 982450913 (...)

The sequence of  $q$  obtained from  $p$  having last digit 7:

: 3677, 9677, 30677, 45677, 48677, 108677, 123677, 156677 (...) 98245149677  
(...)  
obtained for  $p = 37, 97, 307, 457, 487, 1087, 1237, 1567$  (...) 982451497 (...)

The sequence of  $q$  obtained from  $p$  having last digit 9:

: 23899, 35899, 65899, 71899, 92899, 110899, 125899, 131899 (...) 98244887899,  
98244986899, 98245064899 (...)  
obtained for  $p = 239, 359, 659, 719, 929, 1109, 1259, 1319$  (...) 982448879,  
982449869, 982450649 (...)

### Conjecture 3:

There exist, for any prime  $p$  having the value of the last digit  $d$  equal to 1, respectively to 3, 7 or 9, an infinity of primes  $q$  obtained concatenating  $p + 1$  with the value of  $d$ .

The sequence of  $q$  obtained from  $p$  having last digit 1:

: 421, 1021, 1321, 2521, 3121, 4021, 4621, 6421, 7621 (...) 9824501521 (...)  
obtained for  $p = 41, 101, 131, 251, 311, 401, 461, 641, 761$  (...) 982450151 (...)

The sequence of  $q$  obtained from  $p$  having last digit 3:

: 443, 743, 2243, 2843, 4643, 6143, 8243, 8543, 10343 (...) 9824491343,  
9824495543, 9824510243 (...)  
obtained for  $p = 43, 73, 223, 283, 463, 613, 823, 853, 1033$  (...) 982449133,  
982449553, 982451023 (...)

The sequence of  $q$  obtained from  $p$  having last digit 7:

: 487, 1087, 1987, 2287, 3187, 8287, 8887, 9787, 10987 (...) 9824494387,  
9824497087, 9824510887 (...)  
obtained for  $p = 47, 107, 197, 227, 317, 827, 887, 977, 1097$  (...) 982449437,  
982449707, 982451087 (...)

The sequence of  $q$  obtained from  $p$  having last digit 9:

: 809, 1109, 1409, 2309, 4409, 5009, 7109, 8609, 9209 (...) 9824491109,  
9824500409 (...)  
obtained for  $p = 79, 109, 139, 229, 439, 499, 709, 859, 919$  (...) 98244919,  
982450039 (...)

#### Conjecture 4:

There exist, for any prime  $p$  having the value of the last digit  $d$  equal to 1, respectively to 3, 7 or 9, an infinity of primes  $q$  obtained concatenating twice  $p + 1$  with the value of  $d$ .

The sequence of  $q$  obtained from  $p$  having last digit 1:

: 4211, 6211, 7211, 10211, 18211, 19211, 21211, 27211 (...) 98245087211,  
98245119211, 98245123211, 98244957211 (...)  
obtained for  $p = 41, 61, 71, 101, 181, 191, 211, 271$  (...) 982450871, 982451191,  
982451231, 982449571 (...)

The sequence of  $q$  obtained from  $p$  having last digit 3:

: 1433, 7433, 10433, 16433, 19433, 22433, 28433, 52433 (...) 98244895433,  
98244994433 (...)  
obtained for  $p = 13, 73, 103, 163, 193, 283, 523$  (...) 982448953, 982449943 (...)

The sequence of  $q$  obtained from  $p$  having last digit 7:

: 1877, 3877, 4877, 13877, 15877, 22877, 34877, 36877 (...) 98245018877,  
98245020877, 98245069877 (...)  
obtained for  $p = 17, 37, 47, 137, 157, 227, 347, 367$  (...) 982450187, 982450207,  
982450697 (...)

The sequence of  $q$  obtained from  $p$  having last digit 9:

: 2099, 23099, 35099, 62099, 74099, 107099, 128099, 146099 (...) 98245007099,  
98245118099 (...)  
obtained for  $p = 19, 229, 349, 619, 739, 1069, 1279, 1459$  (...) 982450069,  
982451179 (...)

### 3. Four conjectures on the numbers $2^*(p*q*r) \pm 1$ where $p$ and $q = p + 6$ and $r = q + 6$ are odd numbers

**Abstract.** In this paper I make the following four conjectures: (I) there exist an infinity of primes of the form  $2^*(p*q*r) + 1$ , where  $p, q = p + 6, r = q + 6$  are odd numbers of the form  $6*k - 1$ ; (II) there exist an infinity of semiprimes  $m*n$  of the form  $2^*(p*q*r) + 1$ , where  $p, q = p + 6, r = q + 6$  are odd numbers of the form  $6*k - 1$ , semiprimes having the property that  $n - m + 1$  is prime; (III) there exist an infinity of primes of the form  $2^*(p*q*r) - 1$ , where  $p, q = p + 6, r = q + 6$  are odd numbers of the form  $6*k + 1$ ; (IV) there exist an infinity of semiprimes  $m*n$  of the form  $2^*(p*q*r) + 1$ , where  $p, q = p + 6, r = q + 6$  are odd numbers of the form  $6*k + 1$ , semiprimes having the property that  $n - m + 1$  is prime.

#### Conjecture 1:

There exist an infinity of primes  $m$  of the form  $2^*(p*q*r) + 1$ , where  $p, q = p + 6, r = q + 6$  are odd numbers of the form  $6*k - 1$ .

The sequence of primes  $m$ :

- :  $m = 1871$ , for  $(p, q, r) = (5, 11, 17)$ ;
- :  $m = 22679$ , for  $(p, q, r) = (17, 23, 29)$ ;
- :  $m = 46691$ , for  $(p, q, r) = (23, 29, 35)$ ;
- :  $m = 83231$ , for  $(p, q, r) = (29, 35, 41)$ ;
- :  $m = 1403531$ , for  $(p, q, r) = (83, 89, 95)$ ;
- :  $m = 2442383$ , for  $(p, q, r) = (101, 107, 113)$ ;
- :  $m = 2877659$ , for  $(p, q, r) = (107, 113, 119)$ ;
- :  $m = 3361751$ , for  $(p, q, r) = (113, 119, 125)$ ;
- :  $m = 4486751$ , for  $(p, q, r) = (125, 131, 137)$ ;
- :  $m = 5132843$ , for  $(p, q, r) = (131, 137, 143)$ ;
- :  $m = 6605171$ , for  $(p, q, r) = (143, 149, 155)$ ;
- :  $m = 7436591$ , for  $(p, q, r) = (149, 155, 161)$ ;
- :  $m = 10342979$ , for  $(p, q, r) = (167, 173, 179)$ ;
- :  $m = 11457791$ , for  $(p, q, r) = (173, 179, 185)$ ;
- :  $m = 15276563$ , for  $(p, q, r) = (191, 197, 203)$ ;
- (...)

#### Conjecture 2:

There exist an infinity of semiprimes  $m*n$  of the form  $2^*(p*q*r) + 1$ , where  $p, q = p + 6, r = q + 6$  are odd numbers of the form  $6*k - 1$ , semiprimes having the property that  $n - m + 1$  is prime.

The sequence of semiprimes  $m*n$ :

- :  $m*n = 8603 = 7*1229$  where  $1229 - 7 + 1 = 1223$ , prime, for  $(p, q, r) = (11, 17, 23)$ ;



- :  $m*n = 406511 = 7*58073$  where  $58073 - 7 + 1 = 58067$ , prime, for  $(p, q, r) = (53, 59, 65)$ ;
- :  $m*n = 18243611 = 139*131249$  where  $131249 - 139 + 1 = 131111$ , prime, for  $(p, q, r) = (203, 209, 215)$ ;
- (...)

### Conjecture 3:

There exist an infinity of primes  $m$  of the form  $2*(p*q*r) - 1$ , where  $p, q = p + 6, r = q + 6$  are odd numbers of the form  $6*k + 1$ .

The sequence of primes  $m$ :

- :  $m = 3457$ , for  $(p, q, r) = (7, 13, 19)$ ;
- :  $m = 57349$ , for  $(p, q, r) = (25, 31, 37)$ ;
- :  $m = 98641$ , for  $(p, q, r) = (31, 37, 43)$ ;
- :  $m = 449569$ , for  $(p, q, r) = (65, 71, 73)$ ;
- :  $m = 1222129$ , for  $(p, q, r) = (79, 85, 91)$ ;
- :  $m = 2178037$ , for  $(p, q, r) = (97, 103, 109)$ ;
- :  $m = 4087621$ , for  $(p, q, r) = (121, 127, 133)$ ;
- :  $m = 2178037$ , for  $(p, q, r) = (139, 145, 151)$ ;
- :  $m = 8649757$ , for  $(p, q, r) = (157, 163, 169)$ ;
- (...)

### Conjecture 4:

There exist an infinity of semiprimes  $m*n$  of the form  $2*(p*q*r) - 1$ , where  $p, q = p + 6, r = q + 6$  are odd numbers of the form  $6*k + 1$ , semiprimes having the property that  $n - m + 1$  is prime.

The sequence of semiprimes  $m*n$ :

- :  $m*n = 12349 = 53*233$  where  $233 - 53 + 1 = 181$ , prime, for  $(p, q, r) = (13, 19, 25)$ ;
- :  $m*n = 596701 = 7*85243$  where  $85243 - 7 + 1 = 85237$ , prime, for  $(p, q, r) = (61, 67, 73)$ ;
- :  $m*n = 772777 = 23*33599$  where  $33599 - 23 + 1 = 33577$ , prime, for  $(p, q, r) = (67, 73, 79)$ ;
- :  $m*n = 18121 = 193*18313$  where  $18313 - 193 + 1 = 33577$ , prime, for  $(p, q, r) = (115, 121, 127)$ ;
- :  $m*n = 7728481 = 149*51869$  where  $51869 - 149 + 1 = 51721$ , prime, for  $(p, q, r) = (151, 157, 163)$ ;
- :  $m*n = 9641449 = 107*90107$  where  $90107 - 107 + 1 = 90001$ , prime, for  $(p, q, r) = (163, 169, 175)$ ;
- :  $m*n = 10706149 = 1481*7229$  where  $7229 - 1481 + 1 = 5749$ , prime, for  $(p, q, r) = (169, 175, 181)$ ; (...)

#### 4. Two conjectures involving the numbers obtained concatenating repeatedly odd multiples of 3 with 111

**Abstract.** In this paper I make the following two conjectures: (I) there exist an infinity of primes obtained concatenating, once or repeatedly, an odd multiple  $n$  of 3 with 111, then raising the number obtained to the power 2, adding to it  $n$  and subtracting 1 (Examples:  $3111^2 + 3 - 1 = 9678323$ , prime;  $27111111^2 + 27 - 1 = 735012339654347$ , prime); (II) there exist an infinity of semiprimes obtained concatenating, once or repeatedly, an odd multiple  $n$  of 3 with 111, then raising the number obtained to the power 2, adding to it  $n$  and subtracting 1.

##### Conjecture 1:

There exist an infinity of primes  $p$  obtained concatenating, once or repeatedly, an odd multiple  $n$  of 3 with 111, then raising the number obtained to the power 2, adding to it  $n$  and subtracting 1.

Examples:

- :  $p = 3111^2 + 3 - 1 = 9678323$ , prime;
- :  $p = 27111111^2 + 27 - 1 = 735012339654347$ , prime.

The sequence of primes  $p$  when  $n$  is concatenated with 111:

- :  $p = 3111^2 + 3 - 1 = 9678323$ , for  $n = 3$ ;
- :  $p = 27111^2 + 27 - 1 = 9678323$ , for  $n = 9$ ;
- :  $p = 51111^2 + 27 - 1 = 2612334371$ , for  $n = 51$ ;
- :  $p = 57111^2 + 57 - 1 = 3261666377$ , for  $n = 57$ ;
- :  $p = 87111^2 + 87 - 1 = 7588326407$ , for  $n = 87$ ;
- :  $p = 129111^2 + 129 - 1 = 16669650449$ , for  $n = 129$ ;
- (...)

The sequence of primes  $p$  when  $n$  is concatenated with 111111:

- :  $p = 9111111^2 + 9 - 1 = 83012343654329$ , for  $n = 9$ ;
- :  $p = 21111111^2 + 21 - 1 = 445679007654341$ , for  $n = 21$ ;
- :  $p = 27111111^2 + 27 - 1 = 735012339654347$ , for  $n = 27$ ;
- :  $p = 81111111^2 + 81 - 1 = 6579012327654401$ , for  $n = 81$ ;
- :  $p = 87111111^2 + 87 - 1 = 7588345659654407$ , for  $n = 87$ ;
- (...)

The sequence of primes  $p$  when  $n$  is concatenated with 111111111:

- :  $p = 93111111111^2 + 93 - 1 = 329928072999695115409$ , for  $n = 93$ ;
- (...)

The sequence of primes  $p$  when  $n$  is concatenated with 111111111111:

- :  $p = 911111111111^2 + 9 - 1 = 83012345679010320987654329$ , for  $n = 9$ ;
- :  $p = 2711111111111^2 + 27 - 1 = 735012345679006320987654347$ , for  $n = 27$ ;

$$: \quad p = 5711111111111111^2 + 57 - 1 = 3261679012345666320987654377, \text{ for } n = 57;$$

(...)

The sequence of primes p when n is concatenated with 11111111111111:

$$: \quad p = 5111111111111111^2 + 51 - 1 = 2612345679012345667654320987654371, \\ \text{for } n = 51;$$

(...)

**Conjecture 2:**

There exist an infinity of primes p obtained concatenating, once or repeatedly, an odd multiple n of 3 with 111, then raising the number obtained to the power 2, adding to it n and subtracting 1.

The sequence of semiprimes p when n is concatenated with 111:

$$: \quad p = 9111^2 + 9 - 1 = 83010329 = 2003*41443, \text{ for } n = 9;$$

$$: \quad p = 39111^2 + 39 - 1 = 1529670359 = 7*218524337, \text{ for } n = 39;$$

$$: \quad p = 69111^2 + 69 - 1 = 4776330389 = 71*67272259, \text{ for } n = 69;$$

$$: \quad p = 81111^2 + 81 - 1 = 6578994401 = 7*939856343, \text{ for } n = 81;$$

$$: \quad p = 93111^2 + 93 - 1 = 8669658413 = 13*666896801, \text{ for } n = 93;$$

$$: \quad p = 141111^2 + 141 - 1 = 19912314461 = 141*1531716497, \text{ for } n = 141;$$

(...)

The sequence of semiprimes p when n is concatenated with 111111:

$$: \quad p = 15111111^2 + 15 - 1 = 228345675654335 = 5* 45669135130867, \text{ for } n = 5;$$

$$: \quad p = 75111111^2 + 75 - 1 = 5641678995654395 = 5* 1128335799130879, \text{ for } n = 75;$$

$$: \quad p = 93111111^2 + 93 - 1 = 8669678991654413 = 151* 57415092659963, \text{ for } n = 93;$$

$$: \quad p = 99111111^2 + 99 - 1 = 9823012323654419 = 212633*46197026443, \text{ for } n = 99;$$

(...)

The sequence of semiprimes p when n is concatenated with 111111111:

$$: \quad p = 3111111111^2 + 3 - 1 = 9679012344987654323 = 40889963*236708757721, \\ \text{for } n = 3;$$

$$: \quad p = 9111111111^2 + 9 - 1 = 83012345676987654329 = 601*138123703289496929, \text{ for } n = 9;$$

$$: \quad p = 51111111111^2 + 51 - 1 = 2612345679000987654371 = 20641*126561003778934531, \text{ for } n = 51;$$

$$: \quad p = 69111111111^2 + 69 - 1 = 4776345678996987654389 = 4013*126561003778934531, \text{ for } n = 69;$$

$$: \quad p = 87111111111^2 + 87 - 1 = 7588345678992987654407 = 23*329928072999695115409, \text{ for } n = 87;$$

$$: \quad p = 111111111111^2 + 111 - 1 = 12345679012320987654431 = 11*1122334455665544332221, \text{ for } n = 111;$$

$$\begin{aligned}
& : \quad p = 1171111111111111^2 + 117 - 1 = 13715012345652987654437 = \\
& \quad 38593*355375647025444709, \text{ for } n = 117; \\
& : \quad p = 1351111111111111^2 + 135 - 1 = 18255012345648987654455 = \\
& \quad 5*3651002469129797530891, \text{ for } n = 135; \\
& (\dots)
\end{aligned}$$

The sequence of semiprimes p when n is concatenated with 111111111111:

$$\begin{aligned}
& : \quad p = 6311111111111111^2 + 63 - 1 = 3983012345678998320987654383 = \\
& \quad 484181*8226287990811284046643, \text{ for } n = 63; \\
& : \quad p = 9311111111111111^2 + 93 - 1 = 8669679012345658320987654413 = \\
& \quad 6029*1437996187153036709402497, \text{ for } n = 93; \\
& (\dots)
\end{aligned}$$

The sequence of semiprimes p when n is concatenated with 11111111111111:

$$\begin{aligned}
& : \quad p = 9111111111111111^2 + 9 - 1 = 83012345679012343654320987654329 = \\
& \quad 61051*1359721309708478872652716379, \text{ for } n = 9; \\
& (\dots)
\end{aligned}$$

The sequence of semiprimes p when n is concatenated with 111111111111111:

$$\begin{aligned}
& : \quad p & = & 911111111111111111^2 & + & 9 & - & 1 & = & \\
& & 9679012345679012344987654320987654323 & & & & & & = & \\
& & 11*879910213243546576817059483726150393, \text{ for } n = 9; & & & & & & & \\
& : \quad p & = & 271111111111111111^2 & + & 27 & - & 1 & = & \\
& & 735012345679012345672987654320987654347 & & & & & & = & \\
& & 754109*974676533072821496193504724543783, \text{ for } n = 27; & & & & & & & \\
& : \quad p & = & 571111111111111111^2 & + & 57 & - & 1 & = & \\
& & 3261679012345679012332987654320987654377 & & & & & & = & \\
& & 18843063578219*173097065602214822339973883, \text{ for } n = 63; & & & & & & & \\
& (\dots)
\end{aligned}$$

The sequence of semiprimes p when n is concatenated with 11111111111111111:

$$\begin{aligned}
& : \quad p & = & 311111111111111111^2 & + & 3 & - & 1 & = & \\
& & 9679012345679012345678320987654320987654323 & & & & & & = & 313* \\
& & 30923362126770007494179939257681536701771, \text{ for } n = 3; & & & & & & & \\
& (\dots)
\end{aligned}$$

## 5. Conjecture involving the numbers obtained concatenating the square of a prime $p$ with $p$ then with 1

**Abstract.** In this paper I make the following conjecture: there exist an infinity of primes obtained concatenating the square of a prime  $p$  with  $p$  then with 1 and then subtracting 2 from the resulting number (example:  $127^2 = 16129$  and the number  $161291271 - 2 = 161291269$  is prime)

### Conjecture:

There exist an infinity of primes  $q$  obtained concatenating the square of a prime  $p$  with  $p$  then with 1 and then subtracting 2 from the resulting number.

Example:

:  $127^2 = 16129$  and the number  $161291271 - 2 = 161291269$  is prime.

The sequence of primes  $q$ :

:  $q = 929$  for  $(p, p^2 = 3, 9)$ ;  
:  $q = 2549$  for  $(p, p^2 = 5, 25)$ ;  
:  $q = 4969$  for  $(p, p^2 = 7, 49)$ ;  
:  $q = 169129$  for  $(p, p^2 = 13, 169)$ ;  
:  $q = 289169$  for  $(p, p^2 = 17, 289)$ ;  
:  $q = 529229$  for  $(p, p^2 = 23, 529)$ ;  
:  $q = 841289$  for  $(p, p^2 = 29, 841)$ ;  
:  $q = 1369369$  for  $(p, p^2 = 37, 1369)$ ;  
:  $q = 2809529$  for  $(p, p^2 = 53, 2809)$ ;  
:  $q = 5041709$  for  $(p, p^2 = 71, 5041)$ ;  
:  $q = 7921889$  for  $(p, p^2 = 89, 7921)$ ;  
:  $q = 127691129$  for  $(p, p^2 = 113, 12769)$ ;  
:  $q = 161291269$  for  $(p, p^2 = 127, 16129)$ ;  
:  $q = 176611309$  for  $(p, p^2 = 131, 17161)$ ;  
:  $q = 187691369$  for  $(p, p^2 = 137, 18769)$ ;  
:  $q = 445212109$  for  $(p, p^2 = 211, 44521)$ ;  
:  $q = 515292269$  for  $(p, p^2 = 227, 51529)$ ;  
:  $q = 524412289$  for  $(p, p^2 = 229, 52441)$ ;  
:  $q = 979693129$  for  $(p, p^2 = 313, 97969)$ ;  
:  $q = 1772414209$  for  $(p, p^2 = 421, 177241)$ ;  
:  $q = 1857614309$  for  $(p, p^2 = 431, 185761)$ ;  
:  $q = 2143694629$  for  $(p, p^2 = 463, 214369)$ ;  
(...)

Note the chain of four primes 127691129, 161291269, 176611309, 187691369 obtained for four consecutive primes  $p$  (113, 127, 131, 137).

## 6. Two conjectures involving the numbers obtained concatenating a prime p with 9 then with p itself

**Abstract.** In this paper I make the following two conjectures: (I) there exist an infinity of primes  $q$  obtained concatenating a prime  $p$  with 9 then with  $p$  itself (example:  $p = 104593$  is prime and  $q = 1045939104593$  is also prime); (II) there exist an infinity of primes  $q$  obtained concatenating a prime  $p$  of the form  $6*k - 1$  with 9 then with  $p$  itself and subtracting 2 (example:  $p = 104471$  is prime and  $q = 1044719104471 - 2 = 1044719104469$  is also prime).

### Conjecture 1:

There exist an infinity of primes  $q$  obtained concatenating a prime  $p$  with 9 then with  $p$  itself.

Example:

:  $p = 104593$  is prime and  $q = 1045939104593$  is also prime.

The sequence of primes  $q$ :

: 797, 13913, 19919, 47947, 61961, 67967, 71971, 73973, 79979, 1079107, 1379137, 1499149, 1739173, 1799179, 2119211, 2299229, 2699269, 2779277, 2819281, 2839283, 3799379 (...) 1042439104243, 104399, 1043839104383, 1043999104399, 1045939104593, 1045939104593 (...) obtained for  $p = 7, 13, 1941, 47, 61, 67, 71, 73, 79, 107, 137, 149, 173, 179, 211, 229, 269, 277, 281, 283, 379$  (...) 104243, 104383, 104399, 104593, 104347 (...)

### Conjecture 2:

There exist an infinity of primes  $q$  obtained concatenating a prime  $p$  of the form  $6*k - 1$  with 9 then with  $p$  itself and subtracting 2.

Example:

:  $p = 104471$  is prime and  $q = 1044719104471 - 2 = 1044719104469$  is also prime.

The sequence of primes  $q$ :

: 11909, 299027, 53951, 101999, 1799177, 2399237, 2939291, 3119309, 3599357 (...) 1043999104397, 1044719104469, 1046519104649, 1046939104691, 1047119104709 (...) obtained for  $p = 11, 29, 53, 101, 179, 197, 239, 293, 311, 359$  (...) 104399, 104471, 104651, 104693, 104711 (...)

## 7. Conjecture on the numbers obtained concatenating two primes $p$ and $q$ where $q - p + 1$ also prime

**Abstract.** In this paper I make the following conjecture: there exist, for any  $m$  prime of the form  $6 \cdot k + 1$ , an infinity of primes  $n$  obtained concatenating a prime  $p$  with a prime  $q$  where  $q - p + 1 = m$  (example: for  $m = 457$ , prime, we have  $q - p + 1 = 457$  for  $[p, q] = [11, 467]$ , both primes, and the number  $n = 11467$  is prime).

### Conjecture 1:

There exist, for any  $m$  prime of the form  $6 \cdot k + 1$ , an infinity of primes  $n$  obtained concatenating a prime  $p$  with a prime  $q$  where  $q - p + 1 = m$ .

Example:

: for  $m = 457$ , prime, we have  $q - p + 1 = 457$  for  $[p, q] = [11, 467]$ , both primes, and the number  $n = 11467$  is prime.

The sequence of primes  $n$  for  $m = 7$ :

: 1117, 1723, 3137, 8389, 97103, 101107, 151157, 157163, 223229, 227233, 233239, 251257, 257263, 263269, 271277 (...) 104473104479, 104723104729 (...)  
obtained for  $[p, q] = [11, 17], [17, 23], [31, 37], [83, 89], [97, 103], [101, 107], [151, 157], [157, 163], [223, 229], [227, 233], [233, 239], [251, 257], [257, 263], [263, 269], [271, 277] (...) [104473, 104479], [104723, 104729] (...)$

Note the chain of six primes (223229, 227233, 233239, 251257, 257263, 263269) obtained for six consecutive pairs of primes  $[p, q = p + 6]$ .

The sequence of primes  $n$  for  $m = 13$ :

: 719, 1123, 1931, 4153, 4759, 6173, 6779, 89101, 101113, 127139, 151163, 181193, 199211, 239251, 251263, 269281, 239251, 251263, 269281, 337349, 347359 (...)  
obtained for  $[p, q] = [7, 19], [11, 23], [19, 31], [41, 53], [47, 59], [61, 73], [67, 79], [89, 101], [101, 113], [127, 139], [151, 163], [181, 193], [199, 211], [239, 251], [251, 263], [269, 281], [239, 251], [251, 263], [269, 281], [337, 349], [347, 359] (...)$

The sequence of primes  $n$  for  $m = 19$ :

: 1129, 2341, 4159, 83101, 89107, 113131, 131149, 163181, 173191, 181199, 211229, 223241, 233251 (...)  
obtained for  $[p, q] = [11, 29], [23, 41], [41, 59], [83, 101], [89, 107], [113, 131], [131, 149], [163, 181], [173, 191], [181, 199], [211, 229], [223, 241], [233, 251] (...)$

The sequence of primes  $n$  for  $m = 31$ :

: 1747, 3767, 97127, 107137, 109139, 127157, 163193, 167197, 181211, 211241, 227257, 241271 (...) 104113104119, 104693104723 (...)  
obtained for  $[p, q] = [17, 47], [37, 67], [97, 127], [107, 137], [109, 139], [127, 157], [163, 193], [167, 197], [181, 211], [211, 241], [227, 257], [241, 271] (...)$   
 $[104113, 104119], [104693, 104723] (...)$

The sequence of primes  $n$  for  $m = 601$ :

: 7607, 13613, 41641, 53653, 59659, 73673, 101701, 127727, 139739, 173773, 211811 (...)  
obtained for  $[p, q] = [7, 607], [13, 613], [41, 641], [53, 653], [59, 659], [73, 673], [101, 701], [127, 727], [139, 739], [173, 773], [211, 811] (...)$

The sequence of primes  $n$  for  $m = 1723$ :

: 111733, 791801, 1391861 (...)  
obtained for  $[p, q] = [11, 1733], [79, 1801], [139, 1861] (...)$

The sequence of primes  $n$  for  $m = 3001$ :

: 113011, 613061, 10930109, 22330223, 26930269 (...)  
obtained for  $[p, q] = [11, 3011], [61, 3061], [109, 30109], [223, 30223], [269, 30269] (...)$

The sequence of primes  $n$  for  $m = 9001$ :

: 299029, 679067, 1379137, 1739173, 2779277, 2819281 (...)  
obtained for  $[p, q] = [29, 9029], [67, 9067], [137, 9137], [173, 9173], [277, 9277], [281, 9281] (...)$

The sequence of primes  $n$  for  $m = 90001$ :

: 1190011, 2390023 (...)  
obtained for  $[p, q] = [11, 90011], [23, 90023] (...)$



## 8. Four conjectures on the numbers $p$ , $2*p - 1$ , $3*p - 10$ and $n*p - n + 1$ where $p$ prime

**Abstract.** In this paper I make the following four conjectures: (I) there exist an infinity of primes  $p$  such that  $3*p - 10$  is also prime; (II) there exist an infinity of triplets of primes  $(p, 2*p - 1, 3*p - 10)$ ; (III) there exist an infinity of primes  $q$  obtained concatenating a prime  $p$  to the right with  $2*p - 1$  and to the left with 3 (example: for  $p = 11$ ,  $q = 31121$ , prime); (IV) there exist, for any  $n$  positive integer,  $n > 1$ , an infinity of primes  $q$  obtained concatenating a prime  $p$  to the right with  $n*p - n + 1$  and to the left with 3 (examples: for  $n = 5$  and  $p = 19$ ,  $q = 31991$ , prime; for  $n = 8$  and  $p = 13$ ,  $q = 31397$ , prime).

### Conjecture 1:

There exist an infinity of primes  $p$  such that  $q = 3*p - 10$  is also prime.

Primes  $p$  such that  $3*p - 10$  is also prime (see A230227 in OEIS):

: 5, 7, 11, 13, 17, 19, 23, 31, 37, 41, 47, 53, 59, 61, 67, 79, 83, 89, 97, 101, 107, 109, 131, 137, 151, 157, 163, 167, 173, 191, 193, 199, 223, 229, 251, 257, 269, 277, 283, 307, 313, 317, 331, 347, 353, 367, 373, 397, 401, 409 (...)

The sequence of primes  $q$ :

: 5, 11, 23, 29, 41, 47, 59, 83, 101, 113, 131, 149 (...)

### Conjecture 2:

There exist an infinity of triplets of primes  $(p, q = 2*p - 1, r = 3*p - 10)$ . Note that  $p, p > 3$ , can be only of the form  $6*k + 1$ .

Primes  $p$  such that  $2*p - 1$  is also prime (see A005382 in OEIS):

: 2, 3, 7, 19, 31, 37, 79, 97, 139, 157, 199, 211, 229, 271, 307, 331, 337, 367, 379, 439, 499, 547, 577, 601, 607, 619, 661, 691, 727, 811, 829, 877, 937, 967, 997, 1009, 1069, 1171, 1237, 1279, 1297, 1399, 1429, 1459, 1531, 1609, 1627, 1657, 1759, 1867, 2011 (...)

The sequence of triplets  $(p, q, r)$ :

: (7, 13, 11), (19, 37, 47), (31, 61, 83), (37, 73, 101), (79, 157, 227), (97, 193, 281), (157, 313, 461), (99, 397, 587), (229, 457, 677), (307, 613, 911), (331, 661, 983), (367, 733, 1091), (439, 877, 1307), (499, 997, 1487) (...)

Note the chain of six triplets obtained for six consecutive primes  $p$  of the form  $6*k + 1$  for which  $2*p - 1$  is prime (7, 19, 31, 37, 79, 97).

Note that for 14 primes  $p$  for which  $2*p - 1$  is also prime from the first 20 such primes the number  $3*p - 10$  is also prime.

### Conjecture 3:

There exist an infinity of primes  $q$  obtained concatenating a prime  $p$  to the right with  $2 \cdot p - 1$  and to the left with 3 (example: for  $p = 11$ ,  $q = 31121$ , prime).

The sequence of primes  $q$ :

: 359, 31121, 32957, 33161, 33773, 371141, 379157, 3127253, 3157313, 3191381 (...)

### Conjecture 4:

There exist, for any  $n$  positive integer,  $n > 1$ , an infinity of primes  $q$  obtained concatenating a prime  $p$  to the right with  $m = n \cdot p - n + 1$  and to the left with 3 (examples: for  $n = 5$  and  $p = 19$ ,  $q = 31991$ , prime; for  $n = 8$  and  $p = 13$ ,  $q = 31397$ , prime).

The sequence of primes  $p$  for  $n = 3$  ( $m = 3 \cdot p - 2$ ):  
(Note that  $p$  can be only of the form  $6 \cdot k + 1$ )

: 3719, 31337, 31957, 33191, 343127, 397289 (...),  
obtained for  $p = 7, 13, 19, 31, 43, 97$  (...)

The sequence of primes  $p$  for  $n = 4$  ( $m = 4 \cdot p - 3$ ):  
(Note that  $p$  can be only of the form  $6 \cdot k + 1$ )

: 3517, 31973, 343169, 361241, 371281, 3103409 (...),  
obtained for  $p = 5, 19, 43, 61, 71, 103$  (...)

The sequence of primes  $p$  for  $n = 5$  ( $m = 5 \cdot p - 4$ ):

: 31151, 31991, 359291, 373361, 379391, 3109541 (...),  
obtained for  $p = 11, 19, 59, 73, 79, 109$  (...)

The sequence of primes  $p$  for  $n = 6$  ( $m = 6 \cdot p - 5$ ):  
(Note that  $p$  can be only of the form  $6 \cdot k + 1$ )

: 337217, 343253, 367397, 3109649 (...),  
obtained for  $p = 37, 43, 67, 109$  (...)

The sequence of primes  $p$  for  $n = 7$  ( $m = 7 \cdot p - 6$ ):

: 3529, 319127, 341281, 359407, 361421, 371491, 397673, 3107743, 3131911,  
3139967 (...),  
obtained for  $p = 5, 19, 41, 59, 61, 71, 97, 107, 131, 139$  (...)

## 9. Three conjecture on the numbers obtained concatenating $p^2$ with $(p^2 + 1)/2$ , $p + 12$ , $p^2 + 12$

**Abstract.** In this paper I make the following three conjectures on squares of primes: (I) there exist an infinity of primes  $q$  obtained concatenating to the left a square of a prime  $p^2$  with the number  $(p^2 + 1)/2$  (example: for  $p = 17$ ,  $p^2 = 289$  and  $q$  is the number obtained concatenating 289 to the left with  $(p^2 + 1)/2 = 145$ , i.e.  $q = 145289$ , prime); (II) there exist an infinity of primes  $q$  obtained concatenating to the left a square of a prime  $p^2$  with the number  $p + 12$  (example: for  $p = 7$ ,  $p^2 = 49$  and  $q = 1949$ , prime); (III) there exist an infinity of primes  $q$  obtained concatenating to the left a square of a prime  $p^2$  with the number  $p^2 + 12$  (example: for  $p = 11$ ,  $p^2 = 121$  and  $q = 133121$ , prime).

### Conjecture 1:

There exist an infinity of primes  $q$  obtained concatenating to the left a square of a prime  $p^2$  with the number  $(p^2 + 1)/2$  (example: for  $p = 17$ ,  $p^2 = 289$  and  $q$  is the number obtained concatenating 289 to the left with  $(p^2 + 1)/2 = 145$ , i.e.  $q = 145289$ , prime).

The sequence of primes  $q$ :

:	q = 2549	for p = 7;	:	q = 61121	for p = 11;
:	q = 145289	for p = 17;	:	q = 181361	for p = 19;
:	q = 8411681	for p = 41;	:	q = 14052809	for p = 53;
:	q = 26655329	for p = 73; (...)			

### Conjecture 2:

There exist an infinity of primes  $q$  obtained concatenating to the left a square of a prime  $p^2$  with the number  $p + 12$  (example: for  $p = 7$ ,  $p^2 = 49$  and  $q = 1949$ , prime). Note that  $p$  can be only of the form  $6*k + 1$  (otherwise the number obtained is divisible by 3).

The sequence of primes  $q$ :

:	q = 1949	for p = 7;	:	q = 25169	for p = 13;
:	q = 43961	for p = 31;	:	q = 551849	for p = 43;
:	q = 1099409	for p = 97; (...)			

### Conjecture 3:

There exist an infinity of primes  $q$  obtained concatenating to the left a square of a prime  $p^2$  with the number  $p^2 + 12$  (example: for  $p = 11$ ,  $p^2 = 121$  and  $q = 133121$ , prime).

The sequence of primes  $q$ :

:	q = 133121	for p = 11;	:	q = 373361	for p = 19;
:	q = 541529	for p = 23;	:	q = 37333721	for p = 61;
:	q = 94219409	for p = 97;	:	q = 1021310201	for p = 101;
:	q = 1278112769	for p = 113; (...)			

## 10. Three conjecture on the primes obtained concatenating $p$ with $(p - 1)/2$ respectively with $(p + 1)/2$ where $p$ prime

**Abstract.** In this paper I make the following three conjectures on primes: (I) there exist an infinity of primes  $q$  obtained concatenating to the left a prime  $p$  with the number  $(p - 1)/2$  (example: for  $p = 23$ ,  $q$  is the number obtained concatenating 23 to the left with  $(p - 1)/2 = 11$ , i.e.  $q = 1123$ , prime); (II) there exist an infinity of primes  $q$  obtained concatenating to the left a prime  $p$  with the number  $(p + 1)/2$  (example: for  $p = 41$ ,  $q$  is the number obtained concatenating 41 to the left with  $(p + 1)/2 = 21$ , i.e.  $q = 2141$ , prime); (III) there exist an infinity of pairs of primes  $(q_1, q_2)$  where  $q_1$  is obtained concatenating to the left a prime  $p$  with the number  $(p - 1)/2$  and  $q_2$  is obtained concatenating to the left the same prime  $p$  with the number  $(p + 1)/2$ .

### Conjecture 1:

There exist an infinity of primes  $q$  obtained concatenating to the left a prime  $p$  with the number  $(p - 1)/2$  (example: for  $p = 23$ ,  $q$  is the number obtained concatenating 23 to the left with  $(p - 1)/2 = 11$ , i.e.  $q = 1123$ , prime).

The sequence of primes  $q$ :

:	$q = 37$ for $p = 7$ ;	:	$q = 613$ for $p = 13$ ;
:	$q = 919$ for $p = 19$ ;	:	$q = 1123$ for $p = 23$ ;
:	$q = 1429$ for $p = 29$ ;	:	$q = 1531$ for $p = 31$ ;
:	$q = 2143$ for $p = 43$ ;	:	$q = 2347$ for $p = 47$ ;
:	$q = 3061$ for $p = 61$ ;	:	$q = 3571$ for $p = 71$ ;
:	$q = 3673$ for $p = 73$ ;	:	$q = 50101$ for $p = 101$ ;
:	$q = 56113$ for $p = 113$ ;	:	$q = 63127$ for $p = 127$ ;
:	$q = 74149$ for $p = 149$ ;	:	(...)

### Conjecture 2:

There exist an infinity of primes  $q$  obtained concatenating to the left a prime  $p$  with the number  $(p + 1)/2$  (example: for  $p = 41$ ,  $q$  is the number obtained concatenating 41 to the left with  $(p + 1)/2 = 21$ , i.e.  $q = 2141$ , prime).

The sequence of primes  $q$ :

:	$q = 47$ for $p = 7$ ;	:	$q = 1019$ for $p = 19$ ;
:	$q = 1223$ for $p = 23$ ;	:	$q = 2141$ for $p = 41$ ;
:	$q = 2243$ for $p = 19$ ;	:	$q = 2447$ for $p = 47$ ;
:	$q = 2753$ for $p = 53$ ;	:	$q = 2753$ for $p = 53$ ;
:	$q = 3467$ for $p = 67$ ;	:	$q = 3671$ for $p = 71$ ;
:	$q = 4079$ for $p = 79$ ;	:	$q = 4283$ for $p = 83$ ;
:	$q = 52103$ for $p = 103$ ;	:	$q = 55109$ for $p = 109$ ;
:	$q = 70139$ for $p = 139$ ;	:	(...)
:	$q = 52362104723$ for $p = 104723$ ;	:	(...)

**Conjecture 3:**

There exist an infinity of pairs of primes  $(q_1, q_2)$  where  $q_1$  is obtained concatenating to the left a prime  $p$  with the number  $(p - 1)/2$  and  $q_2$  is obtained concatenating to the left the same prime  $p$  with the number  $(p + 1)/2$ .

The sequence of pairs of primes  $(q_1, q_2)$ :

- :  $(q_1, q_2) = (37, 47)$  for  $p = 7$ ;
- :  $(q_1, q_2) = (919, 1019)$  for  $p = 19$ ;
- :  $(q_1, q_2) = (1123, 1223)$  for  $p = 23$ ;
- :  $(q_1, q_2) = (2143, 2243)$  for  $p = 43$ ;
- :  $(q_1, q_2) = (2347, 2447)$  for  $p = 47$ ;
- :  $(q_1, q_2) = (3571, 3671)$  for  $p = 71$ ;
- (...)

**Observation:**

Note the pairs of twin primes  $(41, 43)$  and  $(71, 73)$  and the corresponding pairs of twin primes  $(2141, 2143)$  and  $(3671, 3673)$  obtained with the formula above.

## 11. Primes obtained concatenating a prime $p$ to the left with 3 and to the right with a square of prime $q^2$

**Abstract.** In this paper I make the following conjecture: for any prime  $p$  of the form  $6k + 1$  there exist an infinity of primes  $n$  obtained concatenating  $p$  to the left with 3 and to the right with a square of prime  $q^2$  (examples: for  $p = 13$ , the numbers  $n = 313289, 313961, 3131369$  – obtained for  $q = 17, 31, 37$  – are primes).

### Conjecture:

For any prime  $p$  of the form  $6k + 1$  there exist an infinity of primes  $n$  obtained concatenating  $p$  to the left with 3 and to the right with a square of prime  $q^2$  (examples: for  $p = 13$ , the numbers  $n = 313289, 313961, 3131369$  – obtained for  $q = 17, 31, 37$  – are primes).

The sequence of squares of primes (A001248 in OEIS):

: 4, 9, 25, 49, 121, 169, 289, 361, 529, 841, 961, 1369, 1681, 1849, 2209, 2809, 3481, 3721, 4489, 5041, 5329, 6241, 6889, 7921, 9409, 10201, 10609, 11449, 11881, 12769, 16129, 17161, 18769, 19321, 22201, 22801, 24649, 26569, 27889, 29929, 32041, 32761, 36481, 37249, 38809, 39601 (...)

The sequence of primes  $n$  for  $p = 7$  (up to  $q = 149$ ):

:  $n = 37361$  for  $q = 19$ ;  
:  $n = 37529$  for  $q = 23$ ;  
:  $n = 372809$  for  $q = 53$ ;  
:  $n = 373721$  for  $q = 61$ ;  
:  $n = 376241$  for  $q = 79$ ;  
:  $n = 376889$  for  $q = 83$ ;  
:  $n = 3711881$  for  $q = 109$ ;  
:  $n = 3712769$  for  $q = 113$ ;  
:  $n = 3719321$  for  $q = 139$ .

The sequence of primes  $n$  for  $p = 13$  (up to  $q = 149$ ):

:  $n = 313289$  for  $q = 17$ ;  
:  $n = 313961$  for  $q = 31$ ;  
:  $n = 3131369$  for  $q = 37$ ;  
:  $n = 3133721$  for  $q = 61$ ;  
:  $n = 3135329$  for  $q = 73$ ;  
:  $n = 31311881$  for  $q = 109$ ;  
:  $n = 31312769$  for  $q = 113$ ;  
:  $n = 31322201$  for  $q = 149$ .

The sequence of primes  $n$  for  $p = 19$  (up to  $q = 149$ ):

:  $n = 319169$  for  $q = 13$ ;  
:  $n = 319289$  for  $q = 17$ ;  
:  $n = 3191681$  for  $q = 19$ ;

:  $n = 3191849$  for  $q = 43$ ;  
 :  $n = 3192809$  for  $q = 53$ ;  
 :  $n = 31910201$  for  $q = 101$ ;  
 :  $n = 31910609$  for  $q = 103$ ;  
 :  $n = 31911881$  for  $q = 109$ ;  
 :  $n = 31917161$  for  $q = 131$ ;  
 :  $n = 31922201$  for  $q = 149$ .

The sequence of primes  $n$  for  $p = 31$  (up to  $q = 149$ ):

:  $n = 33149$  for  $q = 7$ ;  
 :  $n = 331841$  for  $q = 29$ ;  
 :  $n = 3311849$  for  $q = 43$ ;  
 :  $n = 3312209$  for  $q = 47$ ;  
 :  $n = 3313481$  for  $q = 59$ ;  
 :  $n = 3315041$  for  $q = 31$ ;  
 :  $n = 33116129$  for  $q = 127$ .

The sequence of primes  $n$  for  $p = 991$  (up to  $q = 149$ ):

:  $n = 399149$  for  $q = 7$ ;  
 :  $n = 3991289$  for  $q = 17$ ;  
 :  $n = 3991961$  for  $q = 31$ ;  
 :  $n = 39911681$  for  $q = 41$ ;  
 :  $n = 39913721$  for  $q = 61$ ;  
 :  $n = 39916241$  for  $q = 79$ ;  
 :  $n = 39919409$  for  $q = 97$ .

The sequence of primes  $n$  for  $p = 997$  (up to  $q = 149$ ):

:  $n = 3997361$  for  $q = 19$ ;  
 :  $n = 39972209$  for  $q = 47$ ;  
 :  $n = 39973721$  for  $q = 61$ ;  
 :  $n = 39974489$  for  $q = 67$ ;  
 :  $n = 399710609$  for  $q = 103$ ;  
 :  $n = 399711881$  for  $q = 109$ ;  
 :  $n = 399718769$  for  $q = 137$ .

The sequence of primes  $n$  for  $p = 104701$  (up to  $q = 149$ ):

:  $n = 3104701361$  for  $q = 19$ ;  
 :  $n = 310470111449$  for  $q = 107$ ;  
 :  $n = 310470118769$  for  $q = 137$ ;  
 :  $n = 310470119321$  for  $q = 139$ .

The sequence of primes  $n$  for  $p = 104707$  (up to  $q = 149$ ):

:  $n = 3104707121$  for  $q = 7$ ;  
 :  $n = 31047074489$  for  $q = 67$ ;  
 :  $n = 31047077921$  for  $q = 89$ .

## 12. Primes obtained concatenating a square of prime to the left with $24*k + 4$ and to the right with 3

**Abstract.** In this paper I make the following conjecture: for any  $k$  positive integer there exist an infinity of primes  $p$  obtained concatenating the square of a prime  $q^2$  to the left with  $n = 24*k + 4$  and to the right with 3 (examples: for  $k = 2$ ,  $n = 52$  and the numbers  $p = 521693, 528413, 5213693$  – obtained for  $q = 13, 29, 37$  – are primes).

### Conjecture:

For any  $k$  positive integer there exist an infinity of primes  $p$  obtained concatenating the square of a prime  $q^2$  to the left with  $n = 24*k + 4$  and to the right with 3 (examples: for  $k = 2$ ,  $n = 52$  and the numbers  $p = 521693, 528413, 5213693$  – obtained for  $q = 13, 29, 37$  – are primes).

The sequence of squares of primes (A001248 in OEIS):

: 4, 9, 25, 49, 121, 169, 289, 361, 529, 841, 961, 1369, 1681, 1849, 2209, 2809, 3481, 3721, 4489, 5041, 5329, 6241, 6889, 7921, 9409, 10201, 10609, 11449, 11881, 12769, 16129, 17161, 18769, 19321, 22201, 22801, 24649, 26569, 27889, 29929, 32041, 32761, 36481, 37249, 38809, 39601 (...)

The sequence of  $p$  for  $k = 0$ ,  $n = 4$  (up to  $q = 149$ ):

:  $p = 4493$  for  $q = 7$ ;  
:  $p = 41213$  for  $q = 11$ ;  
:  $p = 43613$  for  $q = 19$ ;  
:  $p = 45293$  for  $q = 23$ ;  
:  $p = 48413$  for  $q = 29$ ;  
:  $p = 49613$  for  $q = 31$ ;  
:  $p = 418493$  for  $q = 43$ ;  
:  $p = 428093$  for  $q = 53$ ;  
:  $p = 434813$  for  $q = 59$ ;  
:  $p = 444893$  for  $q = 67$ ;  
:  $p = 450413$  for  $q = 71$ ;  
:  $p = 453293$  for  $q = 73$ ;  
:  $p = 468893$  for  $q = 83$ ;  
:  $p = 494093$  for  $q = 97$ ;  
:  $p = 4222013$  for  $q = 149$ .

Two larger values of  $p$ : 4109669067293 (for  $q = 104723$ ) and 4109681634413 (for  $q = 104729$ ).

The sequence of  $p$  for  $k = 1$ ,  $n = 28$  (up to  $q = 149$ ):

:  $p = 28493$  for  $q = 7$ ;  
:  $p = 288413$  for  $q = 29$ ;  
:  $p = 2813693$  for  $q = 37$ ;



:  $p = 2822093$  for  $q = 47$ ;  
 :  $p = 2834813$  for  $q = 59$ ;  
 :  $p = 2862413$  for  $q = 79$ ;  
 :  $p = 2879213$  for  $q = 89$ ;  
 :  $p = 28106093$  for  $q = 103$ ;  
 :  $p = 28118813$  for  $q = 109$ ;  
 :  $p = 28187693$  for  $q = 137$ .

The sequence of  $p$  for  $k = 3$ ,  $n = 76$  (up to  $q = 149$ ):

:  $p = 76493$  for  $q = 7$ ;  
 :  $p = 761213$  for  $q = 11$ ;  
 :  $p = 762893$  for  $q = 17$ ;  
 :  $p = 763613$  for  $q = 19$ ;  
 :  $p = 765293$  for  $q = 23$ ;  
 :  $p = 7622093$  for  $q = 47$ ;  
 :  $p = 7637213$  for  $q = 61$ ;  
 :  $p = 7650413$  for  $q = 71$ ;  
 :  $p = 7653293$  for  $q = 73$ ;  
 :  $p = 7668893$  for  $q = 83$ ;  
 :  $p = 7679213$  for  $q = 89$ ;  
 :  $p = 76118813$  for  $q = 109$ ;  
 :  $p = 76193213$  for  $q = 139$ .

The sequence of  $p$  for  $k = 4$ ,  $n = 100$  (up to  $q = 149$ ):

:  $p = 100493$  for  $q = 7$ ;  
 :  $p = 1002893$  for  $q = 17$ ;  
 :  $p = 1005293$  for  $q = 23$ ;  
 :  $p = 10013693$  for  $q = 37$ ;  
 :  $p = 10028093$  for  $q = 53$ ;  
 :  $p = 10037213$  for  $q = 61$ ;  
 :  $p = 10044893$  for  $q = 67$ ;  
 :  $p = 10050413$  for  $q = 71$ ;  
 :  $p = 10062413$  for  $q = 79$ ;  
 :  $p = 100106093$  for  $q = 103$ ;  
 :  $p = 100187693$  for  $q = 137$ ;  
 :  $p = 100222013$  for  $q = 149$ .

The sequence of  $p$  for  $k = 5$ ,  $n = 124$  (up to  $q = 149$ ):

:  $p = 124493$  for  $q = 7$ ;  
 :  $p = 1242893$  for  $q = 17$ ;  
 :  $p = 1248413$  for  $q = 29$ ;  
 :  $p = 12418493$  for  $q = 43$ ;  
 :  $p = 12437213$  for  $q = 61$ ;  
 :  $p = 12444893$  for  $q = 67$ ;  
 :  $p = 12453293$  for  $q = 73$ ;  
 :  $p = 12462413$  for  $q = 79$ ;  
 :  $p = 12479213$  for  $q = 89$ .

### 13. Conjecture on the infinity of primes obtained concatenating a prime $p$ with $p + 30*k$

**Abstract.** In this paper I make the following conjecture: for any  $p$  prime,  $p > 5$ , there exist an infinity of  $k$  positive integers such that the number  $q$  obtained concatenating to the right  $p$  with  $p + 30*k$  is prime (examples: for  $p = 13$ , the least  $k$  for which  $q$  is prime is 2 because 1373 is prime; for  $p = 104729$ , the least  $k$  for which  $q$  is prime is 3 because 104729104819 is prime). It is notable the small values of  $k$  for which primes  $q$  are obtained, even in the case of primes  $p$  having 20 digits, so this formula could be a way to easily find, starting from a prime  $p$ , a prime  $q$  having twice as many digits!

#### Conjecture:

For any  $p$  prime,  $p > 5$ , there exist an infinity of  $k$  positive integers such that the number  $q$  obtained concatenating to the right  $p$  with  $p + 30*k$  is prime (examples: for  $p = 13$ , the least  $k$  for which  $q$  is prime is 2 because 1373 is prime; for  $p = 104729$ , the least  $k$  for which  $q$  is prime is 3 because 104729104819 is prime).

The sequence of the least  $k$  for which  $q$  is prime:  
(for  $p \geq 7$ )

: for  $p = 7$ ,  $q = 797$  is prime, so  $k = 3$ ;  
: for  $p = 11$ ,  $q = 1171$  is prime, so  $k = 2$ ;  
: for  $p = 13$ ,  $q = 1373$  is prime, so  $k = 2$ ;  
: for  $p = 17$ ,  $q = 1747$  is prime, so  $k = 1$ ;  
: for  $p = 23$ ,  $q = 2383$  is prime, so  $k = 3$ ;  
: for  $p = 29$ ,  $q = 29179$  is prime, so  $k = 5$ ;  
: for  $p = 31$ ,  $q = 3191$  is prime, so  $k = 2$ ;  
: for  $p = 37$ ,  $q = 3767$  is prime, so  $k = 1$ ;  
: for  $p = 41$ ,  $q = 41131$  is prime, so  $k = 3$ ;  
: for  $p = 43$ ,  $q = 4373$  is prime, so  $k = 1$ ;  
: for  $p = 47$ ,  $q = 47137$  is prime, so  $k = 3$ ;  
: for  $p = 53$ ,  $q = 53113$  is prime, so  $k = 2$ ;  
: for  $p = 59$ ,  $q = 59119$  is prime, so  $k = 2$ ;  
: for  $p = 61$ ,  $q = 61121$  is prime, so  $k = 2$ ;  
: for  $p = 67$ ,  $q = 67157$  is prime, so  $k = 3$ ;  
: for  $p = 71$ ,  $q = 71161$  is prime, so  $k = 3$ ;  
: for  $p = 73$ ,  $q = 73133$  is prime, so  $k = 2$ ;  
: for  $p = 79$ ,  $q = 79139$  is prime, so  $k = 2$ ;  
: for  $p = 83$ ,  $q = 83203$  is prime, so  $k = 4$ ;  
: for  $p = 89$ ,  $q = 89119$  is prime, so  $k = 1$ ;  
: for  $p = 97$ ,  $q = 97127$  is prime, so  $k = 1$ ;  
: for  $p = 101$ ,  $q = 101161$  is prime, so  $k = 2$ ;  
: for  $p = 103$ ,  $q = 103133$  is prime, so  $k = 1$ ;  
: for  $p = 107$ ,  $q = 107137$  is prime, so  $k = 1$ ;  
: for  $p = 109$ ,  $q = 109139$  is prime, so  $k = 1$ ;  
: for  $p = 113$ ,  $q = 113143$  is prime, so  $k = 1$ ;  
: for  $p = 127$ ,  $q = 127157$  is prime, so  $k = 1$ ;

[note the chain of 5 primes  $q$  (103133, 107137, 109139, 113143, 127157) obtained for  $k = 1$  from 5 consecutive primes  $p$ ]

- (...)
- : for  $p = 104651$ ,  $q = 104651104771$  is prime, so  $k = 4$ ;
- : for  $p = 104659$ ,  $q = 104659104749$  is prime, so  $k = 3$ ;
- : for  $p = 104677$ ,  $q = 104677104737$  is prime, so  $k = 2$ ;
- : for  $p = 104681$ ,  $q = 104681104831$  is prime, so  $k = 5$ ;
- : for  $p = 104683$ ,  $q = 104683104833$  is prime, so  $k = 5$ ;
- : for  $p = 104693$ ,  $q = 104693104723$  is prime, so  $k = 1$ ;
- : for  $p = 104701$ ,  $q = 104701104821$  is prime, so  $k = 4$ ;
- : for  $p = 104707$ ,  $q = 104707104797$  is prime, so  $k = 3$ ;
- : for  $p = 104711$ ,  $q = 104711104921$  is prime, so  $k = 7$ ;
- : for  $p = 104717$ ,  $q = 104717104837$  is prime, so  $k = 4$ ;
- : for  $p = 104723$ ,  $q = 104723104753$  is prime, so  $k = 1$ ;
- : for  $p = 104729$ ,  $q = 104729104819$  is prime, so  $k = 3$ ;
- (...)
- : for  $p = 982451501$ ,  $q = 982451501982451561$  is prime, so  $k = 2$ ;
- : for  $p = 982451549$ ,  $q = 982451549982451609$  is prime, so  $k = 2$ ;
- : for  $p = 982451567$ ,  $q = 982451567982451597$  is prime, so  $k = 1$ ;
- (...)

The value of the least  $k$  for 5 random 20 digit primes  $p$ :

- : for  $p = 48112959837082048697$ ,  
 $q = 4811295983708204869748112959837082049237$ , prime, so  $k = 18$ ;
- : for  $p = 54673257461630679457$ ,  
 $q = 5467325746163067945754673257461630680777$ , prime, so  $k = 44$ ;
- : for  $p = 29497513910652490397$ ,  
 $q = 2949751391065249039729497513910652490847$ , prime, so  $k = 15$ ;
- : for  $p = 12764787846358441471$ ,  
 $q = 1276478784635844147112764787846358441741$ , prime, so  $k = 9$ ;
- : for  $p = 71755440315342536873$ ,  
 $q = 7175544031534253687371755440315342537023$ , prime, so  $k = 5$ .

Note the small value of  $k$  for which first prime  $q$  is obtained, even in the case of primes  $p$  having 20 digits! This formula could be a way to easily find, starting from a prime  $p$ , a prime  $q$  having twice as many digits!

## 14. Conjecture on a set of primes obtained by a formula involving reversible primes and concatenation

**Abstract.** In this paper I make the following conjecture: there exist an infinity of primes  $q = 2*n - 1$ , where  $n$  is the sum of a reversible prime  $p$  of the form  $6*k + 1$  concatenated to the left with 1 and its reversal, also concatenated to the left with 1 (example: for  $p = 13$ ,  $n = 113 + 131 = 244$  and  $q = 244*2 - 1 = 487$ , prime).

### Conjecture:

There exist an infinity of primes  $q = 2*n - 1$ , where  $n$  is the sum of a reversible prime  $p$  of the form  $6*k + 1$  concatenated to the left with 1 and its reversal, also concatenated to the left with 1 (example: for  $p = 13$ ,  $n = 113 + 131 = 244$  and  $q = 244*2 - 1 = 487$ , prime).

The sequence of reversible primes (A007500 in OEIS):

: 2, 3, 5, 7, 11, 13, 17, 31, 37, 71, 73, 79, 97, 101, 107, 113, 131, 149, 151, 157, 167, 179, 181, 191, 199, 311, 313, 337, 347, 353, 359, 373, 383, 389, 701, 709, 727, 733, 739, 743, 751, 757, 761, 769, 787, 797, 907, 919, 929, 937, 941, 953, 967, 971, 983, 991, 1009, 1021 (...)

The sequence of the primes  $q$ :

: for  $p = 7$ ,  $n = 17 + 17 = 34$  and  $q = 67$ ;  
 : for  $p = 13$ ,  $n = 113 + 131 = 244$  and  $q = 487$ ;  
 : for  $p = 37$ ,  $n = 137 + 173 = 310$  and  $q = 619$ ;  
 : for  $p = 79$ ,  $n = 179 + 197 = 376$  and  $q = 751$ ;  
 : for  $p = 151$ ,  $n = 1151 + 1151 = 2302$  and  $q = 4603$ ;

[note the chain of five primes obtained for five consecutive reversible primes of the form  $6*k + 1$ ]

: for  $p = 181$ ,  $n = 1181 + 1181 = 2362$  and  $q = 4723$ ;  
 : for  $p = 199$ ,  $n = 1199 + 1991 = 3190$  and  $q = 6379$ ;  
 : for  $p = 727$ ,  $n = 1727 + 1727 = 3454$  and  $q = 6907$ ;  
 : for  $p = 739$ ,  $n = 1739 + 1937 = 3676$  and  $q = 7351$ ;  
 : for  $p = 757$ ,  $n = 1757 + 1757 = 3514$  and  $q = 7027$ ;  
 : for  $p = 1231$ ,  $n = 11231 + 11321 = 24442$ ,  $q = 48883$ ;  
 : for  $p = 1249$ ,  $n = 11249 + 19421 = 30670$ ,  $q = 61339$ ;

[an interesting number, though not prime, is obtained for  $p = 1381$ :  $(11381 + 11831)*2 - 1 = 43*1381$ ]

: for  $p = 1399$ ,  $n = 11399 + 19931 = 31330$ ,  $q = 62659$ ;  
 : for  $p = 1429$ ,  $n = 11429 + 19241 = 30670$ ,  $q = 61339$ ;  
 : for  $p = 1669$ ,  $n = 11669 + 19661 = 31330$ ,  $q = 62659$ ;

[note that for  $p = 1399$  and  $p = 1669$  we have the same value of  $q$ , i.e. 62659]

- : for  $p = 1753$ ,  $n = 11753 + 13571 = 25324$ ,  $q = 50647$ ;
- : for  $p = 1933$ ,  $n = 11933 + 13391 = 25324$ ,  $q = 50647$ ;

[note that for  $p = 1753$  and  $p = 1933$  we have the same value of  $q$ , i.e. 50647]

- : for  $p = 3067$ ,  $n = 11933 + 13391 = 30670$ ,  $q = 61339$ ;

[note that for  $p = 1429$  and  $p = 3067$  we have the same value of  $q$ , i.e. 61339]

- : for  $p = 3163$ ,  $n = 13163 + 13613 = 26776$ ,  $q = 53551$ ;
- : for  $p = 3169$ ,  $n = 13169 + 19613 = 32782$ ,  $q = 65563$ ;
- : for  $p = 3343$ ,  $n = 13343 + 13433 = 26776$ ,  $q = 53551$ ;

[note that for  $p = 3163$  and  $p = 3343$  we have the same value of  $q$ , i.e. 53551]

(...)

## 15. Primes obtained concatenating even numbers $n$ with 0 then with $n + 2$ then again with 0 then with $n + 5$

**Abstract.** In this paper I make the following three conjectures: (I) there exist an infinity of primes  $p$  obtained concatenating even numbers  $n$  with 0 then with  $n + 2$ , then again with 0, then with  $n + 5$  (example: for  $n = 44$ , the number  $p = 44046049$  is prime). It is notable that are found chains with 4 primes  $p$  obtained for 4 consecutive even numbers  $n$  (example: 17201740177, 17401760177, 17601780181, 17801800183, obtained for 172, 174, 176, 178); (II) there exist an infinity of pairs of primes  $(p, q)$  obtained applying on two consecutive even numbers  $(m, n)$  the method of concatenation showed in the conjecture above (note that  $q - p = 20202$ ; 2002002; 200020002 and so on); (III) there exist, for any  $k$  positive integer, an infinity of primes  $q = p + n$ , where  $p$  is prime and  $n$  is the number obtained concatenating 2 with a number of  $k$  digits of 0 then with 2 then again with the same number of  $k$  digits of 0 then again with 2.

### Conjecture 1:

There exist an infinity of primes  $p$  obtained concatenating even numbers  $n$  with 0 then with  $n + 2$ , then again with 0, then with  $n + 5$  (example: for  $n = 44$ , the number  $p = 44046049$  is prime).

The sequence of primes  $p$ :

- : for  $n = 2$ ,  $p = 20407$  is prime;
- : for  $n = 4$ ,  $p = 40609$  is prime;
- : for  $n = 6$ ,  $p = 608011$  is prime;
- : for  $n = 12$ ,  $p = 12014017$  is prime;
- : for  $n = 16$ ,  $p = 16018021$  is prime;
- : for  $n = 24$ ,  $p = 24026029$  is prime;
- : for  $n = 26$ ,  $p = 26028031$  is prime;
- : for  $n = 28$ ,  $p = 28030033$  is prime;

[note the chain of three primes  $p$  (24026029, 26028031, 28030033) obtained for three consecutive even numbers  $n$  (24, 26, 28)]

- : for  $n = 42$ ,  $p = 42044047$  is prime;
- : for  $n = 44$ ,  $p = 44046049$  is prime;
- : for  $n = 58$ ,  $p = 58060063$  is prime;
- : for  $n = 66$ ,  $p = 66068071$  is prime;
- : for  $n = 78$ ,  $p = 78080083$  is prime;
- : for  $n = 108$ ,  $p = 10801100113$  is prime;
- : for  $n = 112$ ,  $p = 11201140117$  is prime;
- : for  $n = 114$ ,  $p = 11401160119$  is prime;
- : for  $n = 172$ ,  $p = 17201740177$  is prime;
- : for  $n = 174$ ,  $p = 17401760179$  is prime;
- : for  $n = 176$ ,  $p = 17601780181$  is prime;
- : for  $n = 178$ ,  $p = 17801800183$  is prime;

[note the chain of four primes  $p$  (17201740177, 17401760177, 17601780181, 17801800183) obtained for four consecutive even numbers  $n$  (172, 174, 176, 178)]

- : for  $n = 186$ ,  $p = 18601880191$  is prime;
- : for  $n = 204$ ,  $p = 20402060209$  is prime;
- : for  $n = 218$ ,  $p = 21802200223$  is prime;
- : for  $n = 232$ ,  $p = 23202340237$  is prime;
- : for  $n = 234$ ,  $p = 23402360239$  is prime;
- : for  $n = 242$ ,  $p = 24202440247$  is prime;
- : for  $n = 252$ ,  $p = 25202540257$  is prime;
- : for  $n = 276$ ,  $p = 27602780281$  is prime;
- : for  $n = 282$ ,  $p = 28202840287$  is prime;
- : for  $n = 284$ ,  $p = 28402860289$  is prime;
- : for  $n = 292$ ,  $p = 29202940297$  is prime;
- : for  $n = 306$ ,  $p = 30603080311$  is prime;
- : for  $n = 328$ ,  $p = 32803300333$  is prime;
- : for  $n = 352$ ,  $p = 35203540357$  is prime;
- : for  $n = 372$ ,  $p = 37203740377$  is prime;
- : for  $n = 376$ ,  $p = 37603780381$  is prime;
- : for  $n = 382$ ,  $p = 38203840387$  is prime;
- (...)
- : for  $n = 1008$ ,  $p = 10080101001013$  is prime;
- : for  $n = 1018$ ,  $p = 10180102001023$  is prime;
- : for  $n = 1026$ ,  $p = 10260102801031$  is prime;
- : for  $n = 1046$ ,  $p = 10460104801051$  is prime;
- (...)
- : for  $n = 10000002$ ,  $p = 10000002010000004010000007$ ;
- (...)
- : for  $n = 100000000000002$ ,
- $p = 100000000000002010000000000004010000000000007$ .
- (...)
- : for  $n = 1000000000000000002$ ,
- $p = 1000000000000000002010000000000000000401000000000000000000000000000007$ .

### Conjecture 2:

There exist an infinity of pairs of primes  $(p, q)$  obtained applying on two consecutive even numbers  $(m, n)$  the method of concatenation showed in the conjecture above (note that  $q - p = 20202; 2002002; 200020002$  and so on).

The sequence of pairs of primes  $(p, q)$ :

- :  $(p, q) = (20407, 40609)$  for  $(m, n) = (2, 4)$ ;
- :  $(p, q) = (40609, 608011)$  for  $(m, n) = (4, 6)$ ;
- :  $(p, q) = (24026029, 26028031)$  for  $(m, n) = (24, 26)$ ;
- :  $(p, q) = (26028031, 28030033)$  for  $(m, n) = (26, 28)$ ;
- :  $(p, q) = (42044047, 44046049)$  for  $(m, n) = (42, 44)$ ;
- :  $(p, q) = (11201140117, 11401160119)$  for  $(112, 114)$ ;
- :  $(p, q) = (17201740177, 17401760179)$  for  $(172, 174)$ ;

- :  $(p, q) = (17201740177, 17601780181)$  for  $(174, 176)$ ;
- :  $(p, q) = (17601780181, 17801800183)$  for  $(176, 178)$ ;
- :  $(p, q) = (23202340237, 23402360239)$  for  $(232, 234)$ ;
- :  $(p, q) = (28202840287, 28402860289)$  for  $(282, 284)$ ;
- (...)

**Conjecture 3:**

There exist, for any  $k$  positive integer, an infinity of primes  $q = p + n$ , where  $p$  is prime and  $n$  is the number obtained concatenating 2 with a number of  $k$  digits of 0 then with 2 then again with the same number of  $k$  digits of 0 then again with 2.

The sequence of pairs of primes  $(p, p + 20202)$ :

- :  $(17, 20219), (29, 20231), (31, 20233),$   
 $(47, 20249), (59, 20261), (67, 20269) (...)$

The sequence of pairs of primes  $(p, p + 2002002)$ :

- :  $(7, 2002009), (17, 2002019), (59, 2002061) (...)$

The sequence of pairs of primes  $(p, p + 200020002)$ :

- :  $(29, 200020031), (31, 200020033), (41, 200020043),$   
 $(67, 200020069) (...)$

The sequence of pairs of primes  $(p, p + 20000200002)$ :

- :  $(41, 2000020004309) (...)$

The sequence of pairs of primes  $(p, p + 2000002000002)$ :

- :  $(61, 2000002000063) (...)$

The sequence of pairs of primes  $(p, p + 200000020000002)$ :

- :  $(59, 200000020000061) (...)$

The sequence of pairs of primes  $(p, p + 20000000200000002)$ :

- :  $(19, 20000000200000021) (...)$

The sequence of pairs of primes  $(p, p + 2000000002000000002)$ :

- :  $(67, 2000000002000000069) (...)$



## 16. Primes of the form $(6k - 1)]c[(6k + 1)]c[(6k - 1)$ and $(6k + 1)]c[(6k - 1)]c[(6k + 1)$ where “]c[“ means “concatenated to”

**Abstract.** In this paper I make the following four conjectures: (I) there exist an infinity of primes  $p$  of the form  $(6^*k - 1)]c[(6^*k + 1)]c[(6^*k - 1)$ , where “]c[“ means “concatenated to” (example: for  $k = 4$ , the number  $p = 232523$  is prime); (II) there exist an infinity of primes  $q$  of the form  $(6^*k + 1)]c[(6^*k - 1)]c[(6^*k + 1)$  (example: for  $k = 2$ , the number  $p = 131113$  is prime); (III) there exist an infinity of pairs of primes  $(p, q) = ((6^*k - 1)]c[(6^*k + 1)]c[(6^*k - 1), ((6^*k + 1)]c[(6^*k - 1)]c[(6^*k + 1))$ ; example: for  $k = 5$ ,  $(p, q) = (293129, 32931)$ ; note that, for such a pair  $(p, q)$ ,  $q - p = 19802$ ;  $1998002$ ;  $199980002$  and so on; (IV) there exist, for any  $h$  positive integer, an infinity of primes  $q = p + m$ , where  $p$  is prime and  $m$  is the number obtained concatenating 1 with a number of  $h$  digits of 9 then with 8 then with the same number of  $h$  digits of 0 then with 2.

### Conjecture 1:

There exist an infinity of primes  $p$  of the form  $(6^*k - 1)]c[(6^*k + 1)]c[(6^*k - 1)$ , where “]c[“ means “concatenated to” (example: for  $k = 4$ , the number  $p = 232523$  is prime).

The sequence of primes  $p$ :

- : for  $k = 3$ ,  $p = 171917$  is prime;
- : for  $k = 4$ ,  $p = 232523$  is prime;
- : for  $k = 5$ ,  $p = 293129$  is prime;

[note the chain of three primes  $p$  (171917, 232523, 293129) obtained for three consecutive  $k$  (3, 4, 5)]

- : for  $k = 10$ ,  $p = 615961$  is prime;
- : for  $k = 13$ ,  $p = 777977$  is prime;
- : for  $k = 14$ ,  $p = 838583$  is prime;
- : for  $k = 15$ ,  $p = 899189$  is prime;

[note the chain of three primes  $p$  (777977, 838583, 899189) obtained for three consecutive  $k$  (13, 14, 15)]

- : for  $k = 30$ ,  $p = 179181179$  is prime;
- : for  $k = 37$ ,  $p = 221223221$  is prime;
- (...)
- : for  $k = 1000000000000000$ ,  
 $p = 5999999999999999600000000000000015999999999999999$   
 is prime;
- (...)

### Conjecture 2:

There exist an infinity of primes  $q$  of the form  $(6^*k + 1)]c[(6^*k - 1)]c[(6^*k + 1)$  (example: for  $k = 2$ , the number  $p = 131113$  is prime).

The sequence of primes p:

: for k = 1, p = 757 is prime;  
: for k = 2, p = 131113 is prime;  
: for k = 5, p = 312931 is prime;  
: for k = 8, p = 494749 is prime;  
: for k = 21, p = 127125127 is prime;  
: for k = 23, p = 139137139 is prime;  
: for k = 24, p = 151149151 is prime;  
: for k = 28, p = 169167169 is prime;  
: for k = 36, p = 217215217 is prime; (...)  
: for k = 100000000,  
p = 60000000015999999996000000001 is prime;  
(...)

### Conjecture 3:

There exist an infinity of pairs of primes  $(p, q) = ((6^k - 1)c[(6^k + 1)]c[(6^k - 1), ((6^k + 1)]c[(6^k - 1)]c[(6^k + 1))$ ; example: for k = 5,  $(p, q) = (293129, 32931)$ ; note that, for such a pair  $(p, q)$ ,  $q - p = 19802$ ; 1998002; 199980002 and so on.

The sequence of pairs of primes  $(p, q)$ :

:  $(p, q) = (293129, 32931)$  for k = 5; (...)

### Conjecture 4:

There exist, for any h positive integer, an infinity of primes  $q = p + m$ , where p is prime and m is the number obtained concatenating 1 with a number of h digits of 9 then with 8 then with the same number of h digits of 0 then with 2 ( $m = 19802, 1998002, 199980002$  and so on).

The sequence of pairs of primes  $(p, p + 19802)$ :

: (11, 19813), (17, 19819), (41, 19843), (59, 19861), (89, 19891) (...)

The sequence of pairs of primes  $(p, p + 1998002)$ :

: (17, 1998019), (47, 1998049) (...)

The sequence of pairs of primes  $(p, p + 199980002)$ :

: (5, 199980007), (11, 199980013), (29, 199980031) (41, 199980043), (47, 199980049) (...)

The sequence of pairs of primes  $(p, p + 19999800002)$ :

: (41, 19999800043), (47, 19999800049), (59, 19999800061) (...)

The sequence of pairs of primes  $(p, p + 1999998000002)$ :

: (59, 1999998000061) (...)

The sequence of pairs of primes  $(p, p + 199999989999992)$ :

: (41, 199999990000033) (...)

## 17. Primes obtained concatenating $p - 1$ with $q^2$ where $p$ and $q$ are primes or Poulet numbers

**Abstract.** In this paper I make the following eight conjectures: (Ia) for any  $p$  prime,  $p > 3$ , there exist an infinity of primes  $q$  such that the number  $n$  obtained concatenating  $p - 1$  to the right with  $q^2$  is prime; (Ib) there exist an infinity of terms in any of the sequences above (for any  $p$ ) such that  $r = (p - 1) * q^2 + 1$  is prime; (IIa) for any  $q$  prime,  $q > 3$ , there exist an infinity of primes  $p$  such that the number  $n$  obtained concatenating  $q^2$  to the left with  $p - 1$  is prime; (IIb) there exist an infinity of terms in any of the sequences above (for any  $q$ ) such that  $r = (p - 1) * q^2 + 1$  is prime; (IIIa) for any Poulet number  $P$ , not divisible by 3, there exist an infinity of primes  $q$  such that the number  $n$  obtained concatenating  $P - 1$  to the right with  $q^2$  is prime; (IIIb) there exist an infinity of terms in any of the sequences above (for any  $P$ ) such that  $r = (P - 1) * q^2 + 1$  is prime; (IVa) for any Poulet number  $Q$ , not divisible by 3 or 5, there exist an infinity of primes  $p$  such that the number  $n$  obtained concatenating  $Q^2$  to the left with  $p - 1$  is prime; (IVb) there exist an infinity of terms in any of the sequences above (for any  $Q$ ) such that  $r = (p - 1) * Q^2 + 1$  is prime.

### Conjecture 1a:

For any  $p$  prime,  $p > 3$ , there exist an infinity of primes  $q$  such that the number  $n$  obtained concatenating  $p - 1$  to the right with  $q^2$  is prime (example: for  $p = 5$ , the number  $n = 449$  obtained for  $q = 7$  is prime).

The sequence of primes  $n$  for  $p = 5$ :

: 449, 4289, 41681, 41849, 42209, 43481, 43721, 45329, 46889, 49409 (...)  
obtained for  $q = 7, 17, 41, 59, 61, 79, 83, 97$  (...)

The sequence of primes  $n$  for  $p = 7$ :

: 6121, 6361, 6529, 6841, 6961, 61681, 66889 (...)  
obtained for  $q = 7, 19, 23, 29, 31, 41, 83$  (...)  
[note the chain of four primes  $n$  (6361, 6529, 6841, 6961) obtained for four consecutive primes  $q$  (19, 23, 29, 31)]

The sequence of primes  $n$  for  $p = 11$ :

: 1049, 10169, 10289, 10529, 101681 (...)  
obtained for  $q = 7, 13, 17, 23, 41$  (...)

The sequence of primes  $n$  for  $p = 13$ :

: 1249, 12289, 12841, 121369, 122209, 124489, 125329, 126241, 127921 (...)  
obtained for  $q = 7, 17, 29, 37, 47, 67, 73, 79, 89$  (...)

The sequence of primes  $n$  for  $p = 17$ :

: 16361, 16529, 162209, 163481, 165041, 169409 (...)  
obtained for  $q = 19, 23, 47, 59, 71, 97$  (...)

### Conjecture 1b:

There exist an infinity of terms in any of the sequences above (for any  $p$ ) such that  $r = (p - 1) \cdot q^2 + 1$  is prime.

The sequence of primes  $r$  for  $p = 5$ :

: 197 ( $= 4 \cdot 49 + 1$ ), 8837 ( $= 4 \cdot 2209 + 1$ ), 21317 ( $= 4 \cdot 5329 + 1$ )...

The sequence of primes  $r$  for  $p = 7$ :

: 727 ( $= 6 \cdot 121 + 1$ ), 41047 ( $= 6 \cdot 841 + 1$ )...

The sequence of primes  $r$  for  $p = 11$ :

: 491 ( $= 10 \cdot 49 + 1$ ), 16811 ( $= 10 \cdot 1681 + 1$ )...

The sequence of primes  $r$  for  $p = 13$ :

: 3469 ( $= 12 \cdot 289 + 1$ ), 10093 ( $= 12 \cdot 841 + 1$ ), 63949 ( $= 12 \cdot 5329 + 1$ )...

The sequence of primes  $r$  for  $p = 17$ :

: 55697 ( $= 16 \cdot 3481 + 1$ ), 80657 ( $= 16 \cdot 5041 + 1$ )...

### Conjecture 2a:

For any  $q$  prime,  $q > 3$ , there exist an infinity of primes  $p$  such that the number  $n$  obtained concatenating  $q^2$  to the left with  $p - 1$  is prime;

The sequence of primes  $n$  for  $q^2 = 7^2 = 49$ :

: 449, 1049, 1249, 3049, 4049, 4649, 5849, 8849, 9649 (...), obtained for  $p = 5, 11, 13, 31, 41, 47, 59, 89, 97$  (...)

The sequence of primes  $n$  for  $q^2 = 11^2 = 121$ :

: 6121, 18121, 52121, 70121, 78121 (...), obtained for  $p = 7, 19, 53, 71, 79$  (...)

The sequence of primes  $n$  for  $q^2 = 13^2 = 169$ :

: 10169, 18169, 30169, 40169, 42169, 58169, 60169, 66169, 72169, 88169 (...), obtained for  $p = 11, 19, 31, 41, 43, 59, 61, 67, 73, 89$  (...)

The sequence of  $n$  for  $q^2 = 104729^2 = 10968163441$ :

: 1810968163441, 4010968163441, 5210968163441, 7810968163441, 8810968163441 (...), obtained for  $p = 19, 41, 53, 79, 89$  (...)

**Conjecture 2b:**

There exist an infinity of terms in any of the sequences above (for any  $q$ ) such that  $r = (p - 1) \cdot q^2 + 1$  is prime.

The sequence of primes  $r$  for  $q^2 = 7^2 = 49$ :

: 197, 491, 1471 ( $= 30 \cdot 49 + 1$ ), 2843 ( $= 58 \cdot 49 + 1$ )...

The sequence of primes  $r$  for  $q^2 = 11^2 = 121$ :

: 727, 2179 ( $= 18 \cdot 121 + 1$ ), 9439 ( $= 70 \cdot 121 + 1$ )...

The sequence of primes  $r$  for  $q^2 = 13^2 = 169$ :

: 6761 ( $= 40 \cdot 169 + 1$ ), 9803 ( $= 58 \cdot 169 + 1$ ), 10141 ( $= 60 \cdot 169 + 1$ )...

**Conjecture 3a:**

For any Poulet number  $P$ , not divisible by 3, there exist an infinity of primes  $q$  such that the number  $n$  obtained concatenating  $P - 1$  to the right with  $q^2$  is prime.

The sequence of primes  $n$  for  $P = 341$ :

: 34049, 340169, 3409409 (...), obtained for  $q = 11, 13, 97$  (...)

The sequence of primes  $n$  for  $P = 1105$ :

: 1104289, 11041369, 11042209, 11043481, 11044489, 11046241, 11047921 (...),  
obtained for  $q = 17, 37, 47, 59, 67, 79, 89$  (...)

The sequence of primes  $n$  for  $P = 1387$ :

: 1386361, 13861369, 13862809, 13867921 (...), obtained for  $q = 19, 37, 53, 89$  (...)

The sequence of primes  $n$  for  $P = 1729$ :

: 172849, 1728121, 1728361, 17281681, 17283481, 17286889, 17289409 (...),  
obtained for  $q = 7, 11, 19, 41, 59, 83, 97$  (...)

**Conjecture 3b:**

There exist an infinity of terms in any of the sequences above (for any  $P$ ) such that  $r = (P - 1) \cdot q^2 + 1$  is prime.

The sequence of primes  $r$  for  $P = 341$ :

: 16661 ( $= 340 \cdot 49 + 1$ ), 3199061 ( $= 340 \cdot 9409 + 1$ )...

The sequence of primes  $r$  for  $P = 1105$ :

: 319057 (= 1104\*289 + 1)...

The sequence of primes r for P = 1387:

: 14871781 (= 1104\*7921 + 1)...

The sequence of primes r for P = 1729:

: 84673 (= 1728\*49 + 1), 209089 (= 1728\*121 + 1), 6015169 (= 1728\*3481 + 1)...

#### **Conjecture 4a:**

For any Poulet number Q, not divisible by 3 or 5, there exist an infinity of primes p such that the number n obtained concatenating Q<sup>2</sup> to the left with p - 1 is prime.

The sequence of primes n for Q<sup>2</sup> = 341<sup>2</sup> = 116281:

: 6116281, 18116281, 40116281, 42116281, 58116281, 60116281, 72116281, 78116281 (...), obtained for p = 7, 19, 41, 43, 59, 61, 73, 79 (...)

The sequence of primes n for Q<sup>2</sup> = 1387<sup>2</sup> = 1923769:

: 181923769, 281923769, 881923769 (...), obtained for p = 19, 23, 89 (...)

The sequence of primes n for Q<sup>2</sup> = 1729<sup>2</sup> = 2989441:

: 62989441, 162989441, 222989441, 722989441, 822989441 (...), obtained for p = 7, 17, 23, 72, 82 (...)

#### **Conjecture 4b:**

There exist an infinity of terms in any of the sequences above (for any Q) such that r = (p - 1)\*Q<sup>2</sup> + 1 is prime.

The sequence of primes r for Q<sup>2</sup> = 341<sup>2</sup> = 116281:

: 697687 (= 6\*116281 + 1), 6744299 (= 58\*116281 + 1), 6976861 (= 6\*116281 + 1), 8372233 (= 72\*116281 + 1)...

The sequence of primes r for Q<sup>2</sup> = 1387<sup>2</sup> = 1923769:

: 169291673 (= 88\*1923769 + 1)...

The sequence of primes r for Q<sup>2</sup> = 1729<sup>2</sup> = 2989441:

: 65767703 (= 22\*2989441 + 1)...

## 18. Primes obtained concatenating two primes with the same digital root respectively digital sum

**Abstract.** In this paper I make the following three conjectures: (I) for any  $k$  having one of the values 1, 2, 4, 5, 7 or 8, there exist an infinity of primes obtained concatenating two primes that both have the digital root equal to  $k$ ; (II) for any  $n$  positive integer, not divisible by 3,  $n \geq 4$ , there exist primes obtained concatenating two primes that both have the digital sum equal to  $n$ ; (III) there exist an infinity of values of  $n$ , positive integer, for which exist an infinity of primes obtained concatenating two primes that both have the digital sum equal to  $n$ .

### Conjecture 1:

For any  $k$  having one of the values 1, 2, 4, 5, 7 or 8, there exist an infinity of primes  $p$  obtained concatenating two primes  $q_1$  and  $q_2$  that both have the digital root equal to  $k$ .

Note: the operator “\” it will be used with the meaning “concatenated to”.

The sequence of  $p$  when  $dr(q_1) = dr(q_2) = 1$ :

: 1973 (19\73), 3719 (37\19), 10937 (109\37), 10973 (109\73), 19163 (19\163),  
19181 (19\181)...

The sequence of  $p$  when  $dr(q_1) = dr(q_2) = 2$ :

: 1129 (11\29), 8311 (83\11), 8329 (83\29), 10111 (101\11), 11173 (11\173)...

The sequence of  $p$  when  $dr(q_1) = dr(q_2) = 4$ :

: 1367 (13\67), 3167 (31\67), 10313 (103\13), 10331 (103\31), 13103  
(13\103)...

The sequence of  $p$  when  $dr(q_1) = dr(q_2) = 5$ :

: 523 (5\23), 541 (5\41), 2341 (23\41), 4159 (41\59), 5113 (5\113), 5167  
(5\167), 5923 (59\23)...

The sequence of  $p$  when  $dr(q_1) = dr(q_2) = 7$ :

: 617 (61\7), 743 (7\43), 761 (7\61), 797 (7\97 or 79\7)...

The sequence of  $p$  when  $dr(q_1) = dr(q_2) = 8$ :

: 1753 (17\53), 1789 (17\89), 8971 (89\71), 10753 (107\53), 10771 (107\71),  
10789 (107\89), 17107 (17\107)...

### Conjecture 2:

For any  $n$  positive integer not divisible by 3,  $n \geq 4$ , there exist primes  $p$  obtained concatenating two primes  $q_1$  and  $q_2$  that both have the digital sum equal to  $n$ .

The least prime  $p$  obtained for the following values of  $n$ :

: for  $n = 4$ ,  $p = 10313 (103\13)$  is prime;  
: for  $n = 5$ ,  $p = 523 (5\23)$  is prime;

- : for  $n = 7$ ,  $p = 617$  ( $61 \parallel 7$ ) is prime;
- : for  $n = 8$ ,  $p = 1753$  ( $17 \parallel 53$ ) is prime;
- : for  $n = 10$ ,  $p = 3719$  ( $37 \parallel 19$ ) is prime;
- : for  $n = 11$ ,  $p = 4729$  ( $47 \parallel 29$ ) is prime;
- : for  $n = 13$ ,  $p = 19339$  ( $193 \parallel 139$ ) is prime;
- : for  $n = 14$ ,  $p = 59149$  ( $59 \parallel 149$ ) is prime;
- : for  $n = 16$ ,  $p = 27779$  ( $277 \parallel 79$ ) is prime;
- : for  $n = 17$ ,  $p = 89269$  ( $89 \parallel 269$ ) is prime;
- : for  $n = 19$ ,  $p = 199379$  ( $199 \parallel 379$ ) is prime;
- : for  $n = 20$ ,  $p = 389479$  ( $389 \parallel 479$ ) is prime;
- : for  $n = 22$ ,  $p = 499787$  ( $499 \parallel 787$ ) is prime;
- : for  $n = 23$ ,  $p = 797887$  ( $797 \parallel 887$ ) is prime.

Random primes  $p$  when  $s(q_1) = s(q_2) = 4$ :

- :  $p = 10000031000000000003$ ,  
where  $q_1 = 1000003$  and  $q_2 = 1000000000003$ ;
- :  $p = 10000000000000000003103$ ,  
where  $q_1 = 10000000000000000003$  and  $q_2 = 103$ ;
- :  $p = 100000000000000000031003$ ,  
where  $q_1 = 10000000000000000003$  and  $q_2 = 1003$ .

### Conjecture 3:

There exist an infinity of values of  $n$ , positive integer, for which exist an infinity of primes obtained concatenating two primes that both have the digital sum equal to  $n$ .



## 19. Primes obtained concatenating the prime factors of composite numbers

**Abstract.** In this paper I make the following conjecture: Let's consider the primes  $p$  obtained from composite numbers in the following way: concatenating the prime factors of a composite number  $n$  (example: for  $31941 = 3*3*3*7*13*13$ , the concatenation of its prime factors is 33371313) is obtained either a prime (in which case this prime is  $p$ ), either a composite; if is obtained a composite, is reiterated the operation until is obtained a prime (in which case this prime is  $p$ ). I conjecture that there exist such prime  $p$  for every composite number.

### Conjecture:

Let's consider the primes  $p$  obtained from composite numbers in the following way: concatenating the prime factors of a composite number  $n$  (example: for  $31941 = 3*3*3*7*13*13$ , the concatenation of its prime factors is 33371313) is obtained either a prime (in which case this prime is  $p$ ), either a composite; if is obtained a composite, is reiterated the operation until is obtained a prime (in which case this prime is  $p$ ). I conjecture that there exist such prime  $p$  for every composite number.

The sequence of primes obtained by this method:

- : for  $n = 4 = 2*2$ ,  $22 = 2*11$  and  $p = 211$ , prime;
- : for  $n = 6 = 2*3$ ,  $p = 23$ , prime;
- : for  $n = 8 = 2*2*2$ ,  $222 = 2*3*37$ ,  $2337 = 3*19*41$ ,  $31941 = 3*3*3*7*13*13$ ,  $33371313 = 3*11123771 = 7*149*317*941$ ,  $7149317941 = 229*31219729 = 11*2084656339$ ,  $112084656339 = 3*347*911*118189$ ,  $3347911118189 = 11*613*496501723$ ,  $11613496501723 = 97*130517*917327$ ,  $97130517917327 = 53*1832651281459$ ,  $531832651281459 = 3*3*3*11*139*653*3863*5107$  and  $p = 3331113965338635107$ , prime;
- : for  $n = 9 = 3*3$ ,  $33 = 3*11$  and  $p = 311$ , prime;
- : for  $n = 10 = 2*5$ ,  $25 = 5*5$ ,  $55 = 5*11$ ,  $511 = 7*73$  and  $p = 773$ , prime;
- : for  $n = 12 = 3*4$ ,  $34 = 2*17$ ,  $217 = 7*31$ ,  $731 = 17*43$ ,  $1743 = 3*7*83$ ,  $3783 = 3*13*97$  and  $p = 31397$ , prime;
- : for  $n = 14 = 2*7$ ,  $27 = 3*3*3$ ,  $333 = 3*3*37$ ,  $3337 = 47*71$ ,  $4771 = 13*367$  and  $p = 13367$ , prime;
- : for  $n = 15 = 3*5$ ,  $35 = 5*7$ ,  $57 = 3*19$ ,  $319 = 11*29$  and  $1129 = p$ , prime;
- : for  $n = 18 = 2*3*3$ ,  $p = 233$ , prime;
- : for  $n = 20 = 2*2*5$ ,  $225 = 3*3*5*5$ ,  $3355 = 5*11*61$ ,  $51161 = 11*4651$ ,  $114651 = 3*3*12739$ ,  $3312739 = 17*194867$ ,  $17194867 = 19*41*22073$ ,  $194122073 = 709*273797$ ,  $709273797 = 3*97*137*17791$ ,  $39713717791 = 11*3610337981$ ,  $113610337981 = 7*3391*4786213$ ,  $733914786213 = 3*3*3*3*7*23*31*1815403$ ,  $3333723311815403 = 13*17*23*655857429041$ ,  $131723655857429041 = 7*7*2688237874641409$ ,  $772688237874641409 = 3*31*8308475676071413$  and  $p = 3318308475676071413$ , prime;
- : for  $n = 21 = 3*7$ ,  $p = 37$ , prime;
- : for  $p = 22$ ,  $p = 211$ , prime (see above  $n = 4$ );
- : for  $n = 24$ ,  $p = 2*2*2*3$ ,  $2223 = 3*3*13*19$  and  $p = 331319$ , prime;

: for  $n = 25$ ,  $p = 773$ , prime (see above  $n = 10$ );  
 : for  $n = 26 = 2 \cdot 13$ ,  $213 = 3 \cdot 71$ ,  $371 = 7 \cdot 53$ ,  $753 = 3 \cdot 251$  and  $p = 3251$ , prime;  
 : for  $n = 27$ ,  $p = 13367$ , prime (see above  $n = 14$ );  
 : for  $n = 28 = 2 \cdot 2 \cdot 7$ ,  $p = 227$ , prime;  
 : for  $n = 30 = 2 \cdot 3 \cdot 5$ ,  $235 = 5 \cdot 47$  and  $p = 547$ , prime;  
 : for  $n = 32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ ,  $22222 = 2 \cdot 41 \cdot 271$  and  $p = 241271$ , prime;  
 : for  $n = 33 = 3 \cdot 11$ ,  $p = 311$ , prime;  
 : for  $n = 34$ ,  $p = 31397$  (see above  $n = 12$ )  
 : for  $n = 35$ ,  $p = 1129$ , prime (see above  $n = 15$ );  
 : for  $n = 36 = 2 \cdot 2 \cdot 3 \cdot 3$ ,  $2233 = 7 \cdot 11 \cdot 29$  and  $p = 71129$ , prime;  
 : for  $n = 38 = 2 \cdot 19$ ,  $219 = 3 \cdot 73$  and  $p = 373$ , prime;  
 : for  $n = 39 = 3 \cdot 13$ ,  $p = 313$ , prime;  
 (...)

So the sequence of these primes is:

: 211, 23, 3331113965338635107, 311, 773, 31397, 13367, 1129, 233,  
 3318308475676071413, 37, 211, 331319, 773, 3251, 13367, 227, 547, 241271,  
 311, 31397, 1129, 71129, 373, 313 (...)

corresponding to the numbers 4, 6, 8, 9, 10, 12, 14, 15, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38, 39 (...)

Note that  $p$  is the same, 211, for  $n = 4$  and  $n = 22$ ; 773, for  $n = 10$  and  $n = 25$ ; 13367, for  $n = 14$  and  $n = 27$ ; 31397, for  $n = 12$  and  $n = 34$ .

## 20. On the numbers $(n + 1)*p - n*q$ where $p$ and $q$ primes, $p$ having the group of its last digits equal to $q$

**Abstract.** In this paper I make the following two conjectures: (I) For any prime  $p$ ,  $p > 5$ , there exist a pair of primes  $(q_1, q_2)$ , both having the group of their last digits equal to  $p$ , and a positive integer  $n$ , such that  $p = (n + 1)*q_1 - n*q_2$  (examples: for  $p = 11$ , there exist the primes  $q_1 = 211$  and  $q_2 = 311$  and also the number  $n = 2$  such that  $11 = 3*211 - 2*311$ ; for  $p = 29$ , there exist the primes  $q_1 = 829$  and  $q_2 = 929$  and also the number  $n = 8$  such that  $29 = 9*829 - 8*929$ ); (II) For any  $q_1$  prime,  $q_1 > 5$ , and any  $n$  non-null positive integer, there exist an infinity of primes  $q_2$ , having the group of their last digits equal to  $q_1$ , such that  $p = (n + 1)*q_2 - n*q_1$  is prime; (III) For any  $q_1$  prime,  $q_1 > 5$ , and any  $q_2$  prime having the group of its last digits equal to  $q_1$ , there exist an infinity of positive integers  $n$  such that  $p = (n + 1)*q_2 - n*q_1$  is prime.

### Conjecture 1:

For any prime  $p$ ,  $p > 5$ , there exist a pair of primes  $(q_1, q_2)$ , both having the group of their last digits equal to  $p$ , and a positive integer  $n$ , such that  $p = (n + 1)*q_1 - n*q_2$  (examples: for  $p = 11$ , there exist the primes  $q_1 = 211$  and  $q_2 = 311$  and also the number  $n = 2$  such that  $11 = 3*211 - 2*311$ ; for  $p = 29$ , there exist the primes  $q_1 = 829$  and  $q_2 = 929$  and also the number  $n = 8$  such that  $29 = 9*829 - 8*929$ ).

The pairs of primes  $(q_1, q_2)$  for  $p > 5$ :

- : for  $p = 7$ ,  $(q_1, q_2) = (37, 47)$  because  $4*37 - 3*47 = 7$ ; :
- : for  $p = 11$ ,  $(q_1, q_2) = (211, 311)$  because  $3*211 - 2*311 = 7$ ;
- : for  $p = 13$ ,  $(q_1, q_2) = (1013, 1213)$  because  $6*1013 - 5*1213 = 13$ ;
- : for  $p = 17$ ,  $(q_1, q_2) = (1117, 1217)$  because  $12*1117 - 11*1217 = 17$ ;
- : for  $p = 19$ ,  $(q_1, q_2) = (419, 619)$  because  $3*419 - 2*619 = 19$ ;
- : for  $p = 23$ ,  $(q_1, q_2) = (1123, 1223)$  because  $12*1123 - 11*1223 = 23$ ;
- : for  $p = 29$ ,  $(q_1, q_2) = (829, 929)$  because  $9*829 - 8*929 = 29$ ;
- : for  $p = 31$ ,  $(q_1, q_2) = (331, 431)$  because  $4*331 - 3*431 = 19$ ;
- (...)

### Conjecture 2:

For any  $q_1$  prime,  $q_1 > 5$ , and any  $n$  non-null positive integer, there exist an infinity of primes  $q_2$ , having the group of their last digits equal to  $q_1$ , such that  $p = (n + 1)*q_2 - n*q_1$  is prime.

The sequence of primes  $p$  for  $q_1 = 7$ ,  $n = 1$ :

- :  $67 (= 2*37 - 1*7)$ ,  $127 (= 2*67 - 1*7)$ ,  $307 (= 2*157 - 7)$ ..., corresponding to  $q_2 = 37, 67, 157$  (...)

The sequence of primes  $p$  for  $q_1 = 7$ ,  $n = 2$ :

:  $37 (= 3 \cdot 17 - 2 \cdot 7)$ ,  $97 (= 3 \cdot 37 - 2 \cdot 7)$ ,  $127 (= 3 \cdot 47 - 2 \cdot 7)$ ..., corresponding to  $q_2 = 17, 37, 47 (\dots)$

The sequence of primes  $p$  for  $q_1 = 7$ ,  $n = 4$ :

:  $47 (= 4 \cdot 17 - 3 \cdot 7)$ ,  $127 (= 4 \cdot 37 - 3 \cdot 7)$ ,  $167 (= 4 \cdot 47 - 3 \cdot 7)$ , corresponding to  $q_2 = 17, 37, 47 (\dots)$

\*

The sequence of primes  $p$  for  $q_1 = 11$ ,  $n = 1$ :

:  $1811 (= 2 \cdot 911 - 1 \cdot 11)$ ,  $3011 (= 2 \cdot 1511 - 1 \cdot 11)$ ,  $4211 (= 2 \cdot 2111 - 1 \cdot 11)$ ..., corresponding to  $q_2 = 911, 1511, 2111 (\dots)$

The sequence of primes  $p$  for  $q_1 = 11$ ,  $n = 2$ :

:  $911 (= 3 \cdot 311 - 2 \cdot 11)$ ,  $2411 (= 3 \cdot 811 - 2 \cdot 11)$ ,  $2711 (= 3 \cdot 911 - 2 \cdot 11)$ ..., corresponding to  $q_2 = 311, 811, 911 (\dots)$

The sequence of primes  $p$  for  $q_1 = 11$ ,  $n = 3$ :

:  $811 (= 4 \cdot 211 - 3 \cdot 11)$ ,  $6011 (= 4 \cdot 1511 - 3 \cdot 11)$ ,  $7211 (= 4 \cdot 1811 - 3 \cdot 11)$ , corresponding to  $q_2 = 211, 1511, 1811 (\dots)$

\*

The sequence of primes  $p$  for  $q_1 = 97$ ,  $n = 3$ :

:  $3697 (= 4 \cdot 997 - 3 \cdot 97)$ ,  $27697 (= 4 \cdot 6997 - 3 \cdot 97)$ ,  $55697 (= 4 \cdot 13997 - 3 \cdot 97)$ ,  $79697 (= 4 \cdot 19997 - 3 \cdot 97)$ ,  $87697 (= 4 \cdot 21997 - 3 \cdot 97)$ ..., corresponding to  $q_2 = 997, 6997, 13997, 21997 (\dots)$

### Conjecture 3:

For any  $q_1$  prime,  $q_1 > 5$ , and any  $q_2$  prime having the group of its last digits equal to  $q_1$ , there exist an infinity of positive integers  $n$  such that  $p = (n + 1) \cdot q_2 - n \cdot q_1$  is prime.

The sequence of primes  $p$  for  $(q_1, q_2) = (11, 211)$ :

:  $811 (= 4 \cdot 211 - 3 \cdot 11)$ ,  $1811 (= 9 \cdot 211 - 8 \cdot 11)$ ,  $2011 (= 10 \cdot 211 - 9 \cdot 11)$ ,  $2411 (= 12 \cdot 211 - 11 \cdot 11)$ ... corresponding to  $n = 3, 8, 9, 11 (\dots)$

## 21. Four conjectures on the primes $p^2 + 18*m$ and $q^2 - 18*n$ between the squares $p^2$ , $q^2$ of a pair of twin primes

**Abstract.** In this paper I make the following four conjectures: (I) there exist always a prime of the form  $p^2 + 18*m$  between the squares  $p^2$  and  $q^2$  of a pair of twin primes  $[p, q = p + 2]$ , beside the pair  $[3, 5]$ ; examples: for  $[p^2, q^2] = [11^2, 13^2] = [121, 169]$  there exist the primes  $139 = 121 + 1*18$  and  $157 = 121 + 2*18$ ; for  $[p^2, q^2] = [17^2, 19^2] = [289, 361]$  there exist the prime  $307 = 289 + 1*18$ ; (II) there exist always a prime of the form  $q^2 - 18*n$  between the squares  $p^2$  and  $q^2$  of a pair of twin primes  $[p, q = p + 2]$ , beside the pair  $[3, 5]$ ; examples: for  $[p^2, q^2] = [11^2, 13^2] = [121, 169]$  there exist the prime  $151 = 169 - 1*18$ ; for  $[p^2, q^2] = [17^2, 19^2] = [289, 361]$  there exist the prime  $307 = 361 - 3*18$ ; (III) there exist an infinity of  $r$  primes of the form  $p^2 + 18*m$  or  $q^2 - 18*n$  between the squares  $p^2$  and  $q^2$  of a pair of twin primes  $[p, q = p + 2]$  such that the number obtained concatenating  $p^2$  to the right with  $r$  is prime; example:  $121139$  is prime; (IV) there exist an infinity of  $r$  primes of the form  $p^2 + 18*m$  or  $q^2 - 18*n$  between the squares  $p^2$  and  $q^2$  of a pair of twin primes  $[p, q = p + 2]$  such that the number obtained concatenating  $q^2$  to the left with  $r$  is prime; example:  $139169$  is prime. Of course, the conjectures (III) and (IV) imply that there exist an infinity of pairs of twin primes.

### Conjecture 1:

There exist always a prime of the form  $p^2 + 18*m$  between the squares  $p^2$  and  $q^2$  of a pair of twin primes  $[p, q = p + 2]$ , beside the pair  $[3, 5]$ .

#### Verifying the conjecture:

(for the first  $n$  pairs of twin primes beside  $[3, 5]$ )

- : for  $[p^2, q^2] = [5^2, 7^2] = [25, 49]$  there exist the prime  $43 = 25 + 1*18$ ;
- : for  $[p^2, q^2] = [11^2, 13^2] = [121, 169]$  there exist the prime  $139 = 121 + 1*18$  and  $157 = 121 + 2*18$ ;
- : for  $[p^2, q^2] = [17^2, 19^2] = [289, 361]$  there exist the prime  $307 = 289 + 1*18$ ;
- : for  $[p^2, q^2] = [29^2, 31^2] = [841, 961]$  there exist the primes  $859 = 841 + 1*18$  and  $877 = 841 + 2*18$ ;
- : for  $[p^2, q^2] = [41^2, 43^2] = [1681, 1849]$  there exist the primes  $1699 = 1681 + 1*18$  and  $1753 = 1681 + 4*18$  and  $1789 = 1681 + 6*18$ ;
- : for  $[p^2, q^2] = [59^2, 61^2] = [3481, 3721]$  there exist the primes  $3499 = 3481 + 1*18$  and  $3517 = 3481 + 2*18$  and  $3571 = 3481 + 5*18$  and  $3607 = 3481 + 7*18$  and  $3643 = 3481 + 9*18$  and  $3697 = 3481 + 12*18$ .

### Conjecture 2:

There exist always a prime of the form  $q^2 - 18*n$  between the squares  $p^2$  and  $q^2$  of a pair of twin primes  $[p, q = p + 2]$ , beside the pair  $[3, 5]$ .

#### Verifying the conjecture:

(for the first  $n$  pairs of twin primes beside  $[3, 5]$ )

- : for  $[p^2, q^2] = [5^2, 7^2] = [25, 49]$  there exist the prime  $31 = 49 - 1 \cdot 18$ ;
- : for  $[p^2, q^2] = [11^2, 13^2] = [121, 169]$  there exist the prime  $151 = 169 - 1 \cdot 18$ ;
- : for  $[p^2, q^2] = [17^2, 19^2] = [289, 361]$  there exist the prime  $307 = 361 - 3 \cdot 18$ ;
- : for  $[p^2, q^2] = [29^2, 31^2] = [841, 961]$  there exist the primes  $907 = 961 - 3 \cdot 18$  and  $853 = 961 - 6 \cdot 18$ ;
- : for  $[p^2, q^2] = [41^2, 43^2] = [1681, 1849]$  there exist the primes  $1831 = 1849 - 1 \cdot 18$  and  $1777 = 1849 - 4 \cdot 18$  and  $1759 = 1849 - 5 \cdot 18$  and  $1741 = 1849 - 6 \cdot 18$  and  $1723 = 1849 - 7 \cdot 18$ ;
- : for  $[p^2, q^2] = [59^2, 61^2] = [3481, 3721]$  there exist the primes  $3631 = 3721 - 5 \cdot 18$  and  $3613 = 3721 - 6 \cdot 18$  and  $3599 = 3721 - 9 \cdot 18$  and  $3541 = 3721 - 10 \cdot 18$ .

### Conjecture 3:

There exist an infinity of  $r$  primes of the form  $p^2 + 18 \cdot m$  or  $q^2 - 18 \cdot n$  between the squares  $p^2$  and  $q^2$  of a pair of twin primes  $[p, q = p + 2]$  such that the number obtained concatenating  $p^2$  to the right with  $r$  is prime.

#### The sequence of such primes:

- : 2531, obtained for  $[p^2, r, q^2] = [25, 43, 49]$ ;
- : 2543, obtained for  $[p^2, r, q^2] = [25, 43, 49]$ ;
- : 121139, obtained for  $[p^2, r, q^2] = [121, 139, 169]$ ;
- : 121151, obtained for  $[p^2, r, q^2] = [121, 151, 169]$ ;
- : 121157, obtained for  $[p^2, r, q^2] = [121, 157, 169]$ ;
- : 841859, obtained for  $[p^2, r, q^2] = [841, 859, 961]$ ;
- : 16811741, obtained for  $[p^2, r, q^2] = [1681, 1741, 1849]$ ;
- : 16811831, obtained for  $[p^2, r, q^2] = [1681, 1831, 1849]$ ;
- : 34813607, obtained for  $[p^2, r, q^2] = [3481, 3607, 3721]$ ;
- : 34813613, obtained for  $[p^2, r, q^2] = [3481, 3613, 3721]$ ;
- : 34813631, obtained for  $[p^2, r, q^2] = [3481, 3631, 3721]$ ;
- (...)

### Conjecture 4:

There exist an infinity of  $r$  primes of the form  $p^2 + 18 \cdot m$  or  $q^2 - 18 \cdot n$  between the squares  $p^2$  and  $q^2$  of a pair of twin primes  $[p, q = p + 2]$  such that the number obtained concatenating  $q^2$  to the left with  $r$  is prime.

#### The sequence of such primes:

- : 4349, obtained for  $[p^2, r, q^2] = [25, 43, 49]$ ;
- : 139169, obtained for  $[p^2, r, q^2] = [121, 139, 169]$ ;
- : 151169, obtained for  $[p^2, r, q^2] = [121, 151, 169]$ ;
- : 307361, obtained for  $[p^2, r, q^2] = [289, 307, 361]$ ;
- : 35713721, obtained for  $[p^2, r, q^2] = [3481, 3571, 3721]$ ;
- (..)

## 22. Three sequences obtained concatenating $P - 1$ with 1 and 11 respectively $P + 1$ with 11 where $P$ Poulet numbers

**Abstract.** In this paper I make the following three conjectures: (I) there exist an infinity of primes obtained concatenating the number  $P - 1$  to the right with 1, where  $P$  is a Poulet number; (II) there exist an infinity of primes obtained concatenating the number  $P - 1$  to the right with 11, where  $P$  is a Poulet number; (III) there exist an infinity of primes obtained concatenating the number  $P + 1$  to the right with 11, where  $P$  is a Poulet number.

The *Poulet numbers sequence* (see A001567 in OEIS):

: 341, 561, 645, 1105, 1387, 1729, 1905, 2047, 2465, 2701, 2821, 3277, 4033, 4369, 4371, 4681, 5461, 6601, 7957, 8321, 8481, 8911, 10261, 10585, 11305, 12801, 13741, 13747, 13981, 14491, 15709, 15841, 16705, 18705, 18721, 19951, 23001, 23377, 25761, 29341, 30121, 30889, 31417, 31609, 31621, 33153, 34945, 35333, 39865, 41041 (...)

### Conjecture 1:

There exist an infinity of primes  $q$  obtained concatenating the number  $P - 1$  to the right with 1, where  $P$  is a Poulet number.

The sequence of primes  $q$ :

: 28201, 89101, 54601, 79561, 89101, 113041, 139801, 157081, 199501, 314161, 316201, 353321 (...)

A set of consecutive larger primes  $q$ :

:	9837795900601,	obtained for the 100937 <sup>th</sup> Poulet number;
:	9839260242001,	obtained for the 100943 <sup>th</sup> Poulet number;
:	9842341233001,	obtained for the 100953 <sup>th</sup> Poulet number;
:	9843747924001,	obtained for the 100959 <sup>th</sup> Poulet number;
:	9846582537601,	obtained for the 100973 <sup>th</sup> Poulet number;
:	9846698538601,	obtained for the 100975 <sup>th</sup> Poulet number;
:	9849620410561,	obtained for the 100992 <sup>th</sup> Poulet number;
:	9850167756001,	obtained for the 100996 <sup>th</sup> Poulet number;
:	9853866288001,	obtained for the 101006 <sup>th</sup> Poulet number.

### Conjecture 2:

There exist an infinity of primes  $q$  obtained concatenating the number  $P - 1$  to the right with 11, where  $P$  is a Poulet number.

The sequence of primes  $q$ :

: 282011, 436811, 437011, 468011, 1398011, 1670411, 1870411, 1995011, 3160811, 3162011, 3315211 (...)

A set of consecutive larger primes  $q$ :

: 98411198086811, obtained for the 100950<sup>th</sup> Poulet number;  
: 98458094700011, obtained for the 100971<sup>th</sup> Poulet number.

### Conjecture 3:

There exist an infinity of primes  $q$  obtained concatenating the number  $P + 1$  to the right with 11, where  $P$  is a Poulet number.

The sequence of primes  $q$ :

: 34211, 246611, 437011, 546211, 832211, 1374211, 1398211, 1449211, 1882211,  
1995211, 3012211, 3141811, 3986611 (...)

A set of consecutive larger primes  $q$ :

: 98362065254611, obtained for the 100933<sup>th</sup> Poulet number;  
: 98426744346211, obtained for the 100954<sup>th</sup> Poulet number;  
: 98427318504211, obtained for the 100955<sup>th</sup> Poulet number;  
: 98428953546211, obtained for the 100956<sup>th</sup> Poulet number;  
: 98470940178811, obtained for the 100978<sup>th</sup> Poulet number;  
: 98475388070611, obtained for the 100980<sup>th</sup> Poulet number;  
: 98483254330211, obtained for the 100986<sup>th</sup> Poulet number.

Note the set of three primes obtained for three consecutive Poulet numbers:  
98426744346211, 98427318504211, 98428953546211.



## 23. Two conjectures on the quintets of numbers ( $p, p + 10, p + 30, p + 40, p + 60$ )

**Abstract.** In this paper I make the following two conjectures: (I) there exist an infinity of quintets of primes ( $p, p + 10, p + 30, p + 40, p + 60$ ), where  $p$  is a prime of the form  $6*k + 1$ ; (II) there exist an infinity of primes of the form  $p \backslash (p + 10) \backslash (p + 30) \backslash (p + 40) \backslash (p + 60)$ , where  $p$  is a number of the form  $6*k + 1$ . I used the operator “ $\backslash$ ” with the meaning “concatenated to”.

### Conjecture 1 :

There exist an infinity of quintets of primes ( $p, p + 10, p + 30, p + 40, p + 60$ ), where  $p$  is a prime of the form  $6*k + 1$ .

#### The sequence of quintets of primes

( $p, p + 10, p + 30, p + 40, p + 60$ ):

: (7, 17, 37, 47, 67);  
 : (13, 23, 43, 53, 73);  
 : (43, 53, 73, 83, 103);  
 : (97, 107, 127, 137, 157);  
 : (379, 389, 409, 419, 439);  
 : (547, 557, 577, 587, 607);  
 (...)

Note that among the numbers obtained this way there exist quintets formed by four primes and a square of prime or quintets formed by four primes and a semiprime.

### Conjecture 2 :

There exist an infinity of primes of the form  $p \backslash (p + 10) \backslash (p + 30) \backslash (p + 40) \backslash (p + 60)$ , where  $p$  is a number of the form  $6*k + 1$ . I used the operator “ $\backslash$ ” with the meaning “concatenated to”.

#### The sequence of primes

$p \backslash (p + 10) \backslash (p + 30) \backslash (p + 40) \backslash (p + 60)$ :

: 717374767, 1929495979, 3747677797, 133143163173193, 139149169179199,  
 151161181191211, 241251271281301, 253263283293313, 271281301311331,  
 277287307317337, 343353373383403, 391401421431451, 427437457467487,  
 463473493503523, 487497517527547, 553563583593613 (...)

Note that many of the numbers obtained this way are semiprimes.

## 24. Two conjectures on the quintets of numbers ( $p, p + 20, p + 30, p + 50, p + 80$ )

**Abstract.** In this paper I make the following two conjectures: (I) there exist an infinity of quintets of primes ( $p, p + 20, p + 30, p + 50, p + 80$ ), where  $p$  is a prime of the form  $6^*k - 1$ ; (II) there exist an infinity of primes of the form  $p \backslash (p + 20) \backslash (p + 30) \backslash (p + 50) \backslash (p + 80)$ , where  $p$  is a number of the form  $6^*k - 1$ . I used the operator “ $\backslash$ ” with the meaning “concatenated to”.

### Conjecture 1 :

There exist an infinity of quintets of primes ( $p, p + 20, p + 30, p + 50, p + 80$ ), where  $p$  is a prime of the form  $6^*k - 1$ .

#### The sequence of quintets of primes

$(p, p + 20, p + 30, p + 50, p + 80)$

: (23, 43, 53, 73, 103);  
 (59, 79, 89, 109, 139);  
 (317, 337, 347, 367, 397);  
 (359, 379, 389, 409, 439);  
 (389, 409, 419, 439, 469);  
 (...)

Note that among the numbers obtained this way there exist quintets formed by four primes and a square of prime or quintets formed by four primes and a semiprime.

### Conjecture 2 :

There exist an infinity of primes of the form  $p \backslash (p + 20) \backslash (p + 30) \backslash (p + 50) \backslash (p + 80)$ , where  $p$  is a number of the form  $6^*k - 1$ . I used the operator “ $\backslash$ ” with the meaning “concatenated to”.

#### The sequence of primes

$p \backslash (p + 20) \backslash (p + 30) \backslash (p + 50) \backslash (p + 80)$

: 1131416191, 113133143163193, 137157167187217, 161181191211241,  
 239259269289319, 263283293313343, 377397407427457, 473493503523553,  
 479499509529559  
 (...)

Note that many of the numbers obtained this way are semiprimes.

## 25. Three conjectures on the primes obtained concatenating $30*k$ with $30*k + p$ where $p$ prime

**Abstract.** In this paper I make the following three conjectures: (I) there exist positive integers  $k$  such that the number  $(30*k)\|(30*k + p)$  is prime for an infinity of primes  $p$ . I used the operator “ $\|$ ” with the meaning “concatenated to”; (II) there exist primes  $p$  such that the number  $(30*k)\|(30*k + p)$  is prime for an infinity of values of  $k$ ; (III) there exist an infinity of primes of the form  $(30*k)\|(30*k + 1)$ , where  $k$  positive integer.

**Note:** I use, in this paper, the operator “ $\|$ ” with the meaning “concatenated to”.

### Conjecture 1:

There exist positive integers  $k$  such that the number  $(30*k)\|(30*k + p)$  is prime for an infinity of primes  $p$ . I conjecture that such a  $k$  is 1.

#### The sequence of primes $q$ for $k = 1$ :

: 3037, 3041, 3049, 3061, 3067, 3083, 3089, 30103, 30109, 30113, 30119, 30133, 30137, 30139, 30161, 30169, 30181, 30187, 30197, 30203 (...),

obtained for  $p = 7, 11, 19, 31, 37, 53, 59, 73, 79, 83, 89, 103, 107, 109, 131, 139, 151, 157, 167, 173$  (...)

Note the chain of four primes obtained for four consecutive values of  $k$  (73, 79, 83, 89).

### Conjecture 2:

There exist exist primes  $p$  such that the number  $q = (30*k)\|(30*k + p)$  is prime for an infinity of values of  $k$ . I conjecture that such a prime is 23.

#### The sequence of primes $q$ for $p = 23$ :

: 210233, 240263, 300323, 390413, 450473, 480503, 600623, 660683, 720743 (...),

obtained for 7, 8, 10, 13, 15, 16, 20, 22, 24 (...)

#### The sequence of primes $q$ for $p = 23$ and $k = 4*h$ :

: 240263, 480503, 600623, 720743, 840863, 960983 (...),

obtained for  $h = 2, 4, 5, 6, 7, 8$  (...)

Note the chain of five primes obtained for five consecutive values of  $k$  (5, 6, 7, 8, 9).

### Conjecture 3:

There exist an infinity of primes of the form  $q = (30*k) \parallel (30*k + 1)$ , where  $k$  positive integer (this is a subsequence of the sequence “primes formed concatenating  $n$  with  $n + 1$ ” (A030458 in OEIS).

#### The sequence of primes $q$ :

: 9091, 120121, 150151, 180181, 270271, 300301, 330331, 390391, 420421, 450451, 540541, 600601, 660661, 840841, 870871, 930931, 960961 (...),

obtained for  $k = 3, 4, 5, 6, 9, 10, 11, 13, 14, 15, 17$  (...)

Note the chain of four primes obtained for four consecutive values of  $k$  (3, 4, 5, 6).

It might be also true (I conjecture that it is) that for any  $k$  non-null positive integer there exist an infinity of  $h$  such that  $q = (30*k) \parallel (30*h + 1)$ , where  $h$  positive integer, is prime. Such primes  $q$ , for  $k = 1$ , for example, are: 3061, 30181, 30211, 30241, 30271, 30391, 30631 (...)

## 26. An unusual conjecture on primes involving concatenation and repunits

**Abstract.** In this paper I make the following conjecture: for any  $k$  positive integer there exist an infinity of primes  $p$  such that the number  $q$ , obtained concatenating  $(p - k)$  with  $p$  then, repeatedly  $k$  times, with the digit 1, is prime. Examples: for  $k = 1$ , there exist  $p = 19$  such that  $q = 18191$  is prime; for  $k = 2$ , there exist  $p = 5$  such that  $q = 3511$  is prime; for  $k = 3$ , there exist  $p = 7$  such that  $q = 47111$  is prime; for  $k = 4$ , there exist  $p = 37$  such that  $q = 33371111$  is prime; for  $k = 5$ , there exist  $p = 11$  such that  $q = 61111111$  is prime; for  $k = 6$ , there exist  $p = 17$  such that  $q = 1117111111$  is prime.

### Conjecture :

For any  $k$  positive integer there exist an infinity of primes  $p$  such that the number  $q$ , obtained concatenating  $(p - k)$  with  $p$  then, repeatedly  $k$  times, with the digit 1, is prime. Examples: for  $k = 1$ , there exist  $p = 19$  such that  $q = 18191$  is prime; for  $k = 2$ , there exist  $p = 5$  such that  $q = 3511$  is prime; for  $k = 3$ , there exist  $p = 7$  such that  $q = 47111$  is prime; for  $k = 4$ , there exist  $p = 37$  such that  $q = 33371111$  is prime; for  $k = 5$ , there exist  $p = 11$  such that  $q = 61111111$  is prime; for  $k = 6$ , there exist  $p = 17$  such that  $q = 1117111111$  is prime.

### The sequence of primes $q$ for $k = 1$ :

: 10111, 18191, 46471, 60611, 78791 (...)  
obtained for  $p = 11, 19, 47, 61, 79$  (...)

### The sequence of primes $q$ for $k = 2$ :

: 3511, 5711, 272911, 353711, 414311, 454711, 515311, 697111, 777911, 10510711, 11111311, 14915111, 16516711, 17717911, 17918111, 18919111, 19719911 (...)  
obtained for  $p = 5, 7, 29, 37, 43, 47, 53, 71, 79, 107, 113, 151, 167, 179, 181, 189, 199$  (...)

### The sequence of primes $q$ for $k = 3$ :

: 25111, 47111, 3841111, 4043111, 5659111, 8083111, 8689111, 100103111, 104107111, 106109111, 176179111, 178181111, 190193111 (...)  
obtained for  $p = 5, 7, 41, 43, 59, 83, 89, 103, 107, 109, 179, 181, 193$  (...)

### The sequence of primes $q$ for $k = 4$ :

: 33371111, 39431111, 57611111 (...)  
obtained for  $p = 37, 43, 61$  (...)

### The sequence of primes $q$ for $k = 5$ :

: 61111111, 485311111, 66711111, 747911111, 9610111111, 10811311111, 13213711111, 17618111111, 19419911111 (...)  
obtained for  $p = 11, 53, 67, 79, 101, 113, 137, 181, 199$  (...)

**The sequence of primes q for k = 6:**

: 11171111111, 23291111111, 1011071111111, 1331391111111 (...)  
obtained for p = 17, 29, 107, 139 (...)

**The sequence of primes q for k = 7:**

: 647111111111, 909711111111, 1021091111111, 1241311111111, 1841911111111,  
19019711111111 (...)  
obtained for p = 71, 97, 109, 131, 191, 197 (...)

**The sequence of primes q for k = 8:**

: 31111111111, 233111111111, 455311111111, 818911111111, 9510311111111,  
12313111111111, 14915711111111, 16517311111111, 18519311111111,  
191199111111111 (...)  
obtained for p = 11, 31, 53, 89, 103, 131, 157, 173, 193, 199 (...)

**The sequence of primes q for k = 9:**

: 2231111111111, 6473111111111, 92101111111111, 158167111111111 (...)  
obtained for p = 31, 73, 101, 167(...)

**The sequence of primes q for k = 10:**

: 9191111111111, 21311111111111, 49591111111111, 911011111111111,  
147157111111111, 1831931111111111, 1871971111111111,  
1891991111111111 (...)  
obtained for p = 19, 31, 59, 101, 157, 193, 197, 199 (...)

Note: all the possible primes q are listed above, for k up to 10 and p up to 199.

## 27. Primes obtained concatenating $p$ repeatedly with 6 then with $q$ where $(p, q)$ are sexy primes

**Abstract.** In this paper I make the following conjecture: for any  $k$  non-null positive integer there exist an infinity of pairs of sexy primes  $(p, q = p + 6)$  such that the number  $r$  formed concatenating  $p$ , repeatedly  $k$  times, with the digit 6 then with  $q$  is prime. Examples: for  $k = 1$  there exist  $(p, q) = (11, 17)$  such that the number  $r = 11617$  is prime; for  $k = 2$  there exist the pair  $(p, q) = (31, 37)$  such that the number  $r = 316637$  is prime.

### Conjecture :

For any  $k$  non-null positive integer there exist an infinity of pairs of sexy primes  $(p, q = p + 6)$  such that the number  $r$  formed concatenating  $p$ , repeatedly  $k$  times, with the digit 6 then with  $q$  is prime. Examples: for  $k = 1$  there exist  $(p, q) = (11, 17)$  such that the number  $r = 11617$  is prime; for  $k = 2$  there exist the pair  $(p, q) = (31, 37)$  such that the number  $r = 316637$  is prime.

### The sequence of primes $p$ : (A023201 in OEIS)

: 5, 7, 11, 13, 17, 23, 31, 37, 41, 47, 53, 61, 67, 73, 83, 97, 101, 103, 107, 131, 151, 157, 167, 173, 191, 193, 223, 227, 233, 251, 257, 263, 271, 277, 307, 311, 331, 347, 353, 367, 373, 383, 433, 443, 457, 461, 503, 541, 557, 563, 571, 587, 593, 601, 607, 613, 641, 647, 653 (...)

### The sequence of primes $r$ for $k = 1$ :

: 11617, 13619, 17623, 23629, 37643, 41647, 47653, 61667, 73679, 83689, 976103, 1036109, 1076113, 1516157, 1676173, 1733179, 2276233, 2516257, 2576263, 3076313, 3111317, 3676373, 3836389, 4336439, 5576563, 5876593, 6076613, 6136619, 6536659 (...)

### The sequence of primes $r$ for $k = 2$ :

: 56611, 316637, 736679, 9766103, 13166137, 15166157, 19166197, 25166257, 27166277, 58766593, 59366599, 65366659 (...)

### The sequence of primes $r$ for $k = 3$ :

: 1166617, 2366629, 3166637, 41647, 97666103, 107666113, 151666157, 167666173, 1736179, 191666197, 233666239, 251666257, 263666269, 271666277, 307666313, 347666353, 383666389, 43366666439, 587666593, 641666647, 647666653, 659666659 (...)

### The sequence of primes $r$ for $k = 4$ :

: 11666617, 83666689, 1036666109, 2336666239, 2636666269, 3116666317, 3476666353, 3536666359, 3676666373, 5716666577, 6416666647 (...)

**The sequence of primes r for k = 5:**

: 116666617, 236666629, 616666667, 676666673, 16766666173, 22366666229,  
25166666257, 26366666269, 33166666337, 34766666353, 36766666373,  
55766666563, 58766666593 (...)

**The sequence of primes r for k = 6:**

: 2366666629, 3166666637, 3766666643, 107666666113, 191666666197,  
193666666199, 223666666229, 233666666239, 251666666257, 257666666263,  
311666666317, 347666666353, 383666666389, 457666666463, 557666666563,  
587666666593, 593666666599 (...)

**The sequence of primes r for k = 7:**

: 11666666617, 47666666653, 61666666667, 1016666666107, 1076666666113,  
1916666666197, 2276666666233, 2516666666257, 5936666666599 (...)

**The sequence of primes r for k = 8:**

: 116666666617, 316666666637, 9766666666103, 45766666666463,  
60766666666613, 65366666666659 (...)

**The sequence of primes r for k = 9:**

: 97666666666103, 223666666666229, 263666666269, 3536666666359,  
647666666666653 (...)

**The sequence of primes r for k = 10:**

: 1516666666666157, 1736666666666179, 6476666666666653 (...)

**The sequence of primes r for k = 11:**

: 136666666666619, 17366666666666179, 36766666666666373,  
46166666666666467, 61366666666666619 (...)

**The sequence of primes r for k = 12:**

: 2366666666666629, 7366666666666679, 101666666666666107,  
311666666666666317, 60766666666666613, 64166666666666647 (...)

Note: all the possible primes r are listed above, for k up to 12 and (p, q) up to 653.



**28. Primes of the form  $p]c[x]c[q]c[y]c[r$  where  $p, q, r$  consecutive primes,  
 $q - p = x$  and  $r - q = y$**

**Abstract.** In this paper I make the following two conjectures: (I) there exist an infinity of triplets of consecutive primes  $[p, q, r]$  such that the number obtained concatenating  $p$  with  $x$  then with  $q$  then with  $y$  then with  $r$ , where  $x$  is the gap between  $p$  and  $q$  and  $y$  the gap between  $q$  and  $r$ , is prime. In other words, if we use the operator “]c[“ with the meaning “concatenating to”,  $p]c[x]c[q]c[y]c[r$  is prime for an infinity of triplets  $[p, q, r]$ . Example: for  $[p, q, r, x, y] = [11, 13, 17, 2, 4]$  the number 11213417 is prime; (II) for any pair of consecutive primes  $[p, q]$ ,  $p \geq 7$ , there exist an infinity of primes  $r$  such that the number  $n = p]c[x]c[q]c[y]c[r$  is prime, where  $x$  is the gap between  $p$  and  $q$  and  $y$  the gap between  $q$  and  $r$ . Example: for  $[p, q] = [13, 17]$  there exist  $r = 61$  such that  $n = 134174461$  is prime ( $x = 4$  and  $y = 44$ ).

**Conjecture 1:**

There exist an infinity of triplets of consecutive primes  $[p, q, r]$  such that the number  $n$  obtained concatenating  $p$  with  $x$  then with  $q$  then with  $y$  then with  $r$ , where  $x$  is the gap between  $p$  and  $q$  and  $y$  the gap between  $q$  and  $r$ , is prime. In other words, if we use the operator “]c[“ with the meaning “concatenating to”,  $p]c[x]c[q]c[y]c[r$  is prime for an infinity of triplets  $[p, q, r]$ . Example: for  $[p, q, r, x, y] = [11, 13, 17, 2, 4]$  the number 11213417 is prime.

**The sequence of primes  $n$ :**

: 527411, 7411213, 11213417, 61667471, 73679483, 10121034107, 10721094113, 139101492151, 181101912193, 23362392241, 26362692271, 313431714331, 38983974401, 409104192421, 46124634467 (...)

obtained for  $[p, q, r] = [5, 7, 11], [7, 11, 13], [11, 13, 17], [61, 67, 71], [73, 79, 83], [101, 103, 107], [107, 109, 113], [139, 149, 151], [181, 191, 193], [233, 239, 241], [263, 269, 271], [313, 317, 331], [389, 397, 401], [409, 419, 421], [461, 463, 467]...$

respectively for  $[x, y] = [2, 4], [4, 2], [2, 4], [6, 4], [6, 4], [2, 4], [2, 4], [10, 2], [10, 2], [6, 2], [6, 2], [4, 14], [8, 4], [10, 2], [2, 4]...$

**Conjecture 2:**

For any pair of consecutive primes  $[p, q]$ ,  $p \geq 7$ , there exist an infinity of primes  $r$  such that the number  $n = p]c[x]c[q]c[y]c[r$  is prime, where  $x$  is the gap between  $p$  and  $q$  and  $y$  the gap between  $q$  and  $r$ . Example: for  $[p, q] = [13, 17]$  there exist  $r = 61$  such that  $n = 134174461$  is prime ( $x = 4$  and  $y = 44$ ).

**The sequence of primes  $n$  for  $[p, q] = [7, 11]$ :**

: 7411213, 741198109, 7411128139, 7411146157 (...)

obtained for  $r = 13, 109, 139, 157\dots$

respectively for  $[x, y] = [4, 2], [4, 98], [4, 128], [4, 146]\dots$

**The sequence of primes  $n$  for  $[p, q] = [11, 13]$ :**

: 11213417, 112131629, 112133447, 112134053 (...)

obtained for  $r = 17, 29, 47, 53\dots$

respectively for  $[x, y] = [2, 4], [2, 16], [2, 34], [2, 40]\dots$

**The sequence of primes  $n$  for  $[p, q] = [13, 17]$ :**

: 134174461, 134175673, 134176279, 134178097 (...)

obtained for  $r = 61, 73, 79, 97\dots$

respectively for  $[x, y] = [4, 44], [4, 56], [4, 62], [4, 80]\dots$

**The least prime  $n$  for the next few pairs of  $[p, q]$ :**

:  $n = 172193453$  and  $r = 53$  for  $[p, q] = [17, 19]$ ;  
:  $n = 1942380103$  and  $r = 103$  for  $[p, q] = [19, 23]$ ;  
:  $n = 236293867$  and  $r = 67$  for  $[p, q] = [23, 29]$ ;  
:  $n = 29231136167$  and  $r = 167$  for  $[p, q] = [29, 31]$ ;  
:  $n = 31637226263$  and  $r = 263$  for  $[p, q] = [31, 37]$ ;  
:  $n = 376431659$  and  $r = 59$  for  $[p, q] = [37, 43]$ ;  
:  $n = 434472067$  and  $r = 67$  for  $[p, q] = [43, 47]$ ;  
:  $n = 53659867$  and  $r = 67$  for  $[p, q] = [53, 59]$ ;  
:  $n = 59261166227$  and  $r = 227$  for  $[p, q] = [59, 61]$ ;  
:  $n = 61667471$  and  $r = 71$  for  $[p, q] = [61, 67]$ ;  
:  $n = 6747168139$  and  $r = 139$  for  $[p, q] = [67, 71]$ ;  
:  $n = 712731083$  and  $r = 83$  for  $[p, q] = [71, 73]$ .

## 29. Conjecture on the pairs of primes $(p, q = p + k)$ involving concatenation

**Abstract.** In this paper I make the following conjecture: there exist an infinity of pairs of primes  $(p, q)$ , where  $q - p = k$ , for any even number  $k$ , such that the number obtained concatenating  $p$  with  $k$  then with  $q$  is prime. Note that is not necessary, as is stipulated in the Polignac's Conjecture, for the primes  $p$  and  $q$  to be consecutive (though, for the particular cases  $k = 2$  and  $k = 4$ , of course that  $p$  and  $q$  are consecutive, which means that the conjecture above can be regarded as well as a stronger statement than the Twin primes Conjecture).

### Conjecture:

There exist an infinity of pairs of primes  $(p, q)$ , where  $q - p = k$ , for any even number  $k$ , such that the number  $r$  obtained concatenating  $p$  with  $k$  then with  $q$  is prime. Note that is not necessary, as is stipulated in the Polignac's Conjecture, for the primes  $p$  and  $q$  to be consecutive (though, for the particular cases  $k = 2$  and  $k = 4$ , of course that  $p$  and  $q$  are consecutive, which means that the conjecture above can be regarded as well as a stronger statement than the Twin primes Conjecture).

### The sequence of primes $r$ for $k = 2$ :

: 11213, 29231, 41243, 1012103, 1372139, 1912193, 3112313, 3472349, 4312433, 6172619, 6412643 (...)

obtained for  $[p, q] = [11, 13], [29, 31], [41, 43], [101, 103], [137, 139], [191, 193], [311, 313], [347, 349], [431, 433], [617, 619], [641, 643]...$

(see A001359 in OEIS for the pairs of "twin primes"  $[p, q = p + 2]$ )

### The sequence of primes $r$ for $k = 4$ :

: 7411, 19423, 37441, 1634167, 2234227, 4574461, 6134617, 6434647, 7574761, 8594863, 9074911 (...)

obtained for  $[p, q] = [7, 11], [19, 23], [37, 41], [163, 167], [223, 227], [457, 461], [613, 617], [643, 647], [757, 761], [859, 863], [907, 911]...$

(see A046132 in OEIS for the pairs of "cousin primes"  $[p, q = p + 4]$ )

### The sequence of primes $r$ for $k = 6$ :

: 17623, 23629, 37643, 41647, 47653, 61667, 73679, 83689, 976103, 1036109, 1076113, 1516157, 1676173, 1736179, 2276233(...)

obtained for  $[p, q] = [11, 17], [13, 19], [17, 23], [23, 29], [37, 43], [41, 47], [47, 53], [61, 67], [73, 79], [83, 89], [97, 103], [103, 109], [107, 113], [151, 157], [167, 173], [173, 179], [227, 233]...$

(see A023201 in OEIS for the pairs of "sexy primes"  $[p, q = p + 6]$ )

**The sequence of primes  $r$  for  $k = 8$ :**

: 5813, 23831, 29837, 53861, 71879, 89897, 1018109, 1318139, 2638271, 2698277, 5698577, 7018709 (...)

obtained for  $[p, q] = [5, 13], [23, 31], [29, 37], [53, 61], [71, 79], [89, 97], [101, 109], [131, 139], [263, 271], [269, 277], [569, 577], [701, 709]$ ...

(see A023202 in OEIS for the pairs of primes  $[p, q = p + 8]$ )

**The sequence of primes  $r$  for  $k = 10$ :**

: 131023, 311041, 611071, 1213101223, 1471101481, 1489191499, 1867101877 (...)

obtained for  $[p, q] = [13, 23], [31, 41], [61, 71], [1213, 1223], [1471, 1481], [1489, 1499], [1867, 1877]$ ...

(see A023203 in OEIS for the pairs of primes  $[p, q = p + 10]$ )

**The sequence of primes  $r$  for  $k = 12$ :**

: 51217, 191231, 411253, 471259, 591271, 1021121033, 1091121103, 1117121129 (...)

obtained for  $[p, q] = [5, 17], [19, 31], [41, 53], [47, 59], [59, 71], [1021, 1031], [1091, 1103], [1117, 1129]$ ...

(see A046133 in OEIS for the pairs of primes  $[p, q = p + 12]$ )

**The sequence of primes  $r$  for  $k = 14$ :**

: 51419, 291443, 1019141033, 1187141201, 1223141237, 1283141297, 1367141381 (...)

obtained for  $[p, q] = [5, 19], [29, 43], [1019, 1033], [1187, 1201], [1223, 1237], [1283, 1297], [1367, 1381]$ ...

(see A153417 in OEIS for the pairs of primes  $[p, q = p + 14]$ )

**The sequence of primes  $r$  for  $k = 16$ :**

: 431659, 1291161307, 1693161709, 2671162687, 2713162729, 2887162903 (...)

obtained for  $[p, q] = [43, 59], [1291, 1307], [1693, 1709], [2671, 2687], [2713, 2729], [2887, 2903]$ ...

(see A049488 in OEIS for the pairs of primes  $[p, q = p + 16]$ )

**The sequence of primes  $r$  for  $k = 18$ :**

: 111829, 191837, 231841, 531871, 611879, 711889, 791897, 1013181031,  
1021181039, 1051181069 (...)

obtained for  $[p, q] = [11, 29], [19, 37], [23, 41], [53, 71], [61, 79], [71, 89], [79, 97], [1013, 1031], [1021, 1039], [1051, 1069]$ ...

(see A153418 in OEIS for the pairs of primes  $[p, q = p + )$ )

Note: the examples above, i.e.  $k = 2, 4, 6, 8, 10, 12, 14, 16, 18$ , covers any possible digital root of  $k$ : 1, 2, 3, 4, 5, 6, 7, 8 or 9.

### 30. Conjecture on the consecutive concatenation of the numbers $n^*k + 1$ where $k$ multiple of 3

**Abstract.** In this paper I make the following conjecture: for any  $k$  multiple of 3, the sequence obtained by the consecutive concatenation of the numbers  $n^*k + 1$ , where  $n$  positive integer, has an infinity of prime terms. Examples: for  $k = 3$ , the sequence 1, 14, 147, 14710 (...) has the prime terms 14710131619, 14710131619222528313437 (...); for  $k = 6$ , the sequence 1, 7, 13, 19 (...) has the prime terms 17, 17131925313743495561 (...).

#### Conjecture:

For any  $k$  multiple of 3, the sequence obtained by the consecutive concatenation of the numbers  $n^*k + 1$ , where  $n$  positive integer, has an infinity of prime terms. Examples: for  $k = 3$ , the sequence 1, 14, 147, 14710 (...) has the prime terms 14710131619, 14710131619222528313437 (...); for  $k = 6$ , the sequence 1, 7, 13, 19 (...) has the prime terms 17, 17131925313743495561 (...).

#### The sequence of primes for $k = 3$ :

: 14710131619, 14710131619222528313437 (...)

#### The sequence of primes for $k = 6$ :

: 17, 17131925313743495561, 171319253137434955616773 (...)

#### The sequence of primes for $k = 12$ :

: 11325374961738597109121133145157169181193205217,  
11325374961738597109121133145157169181193205217229241253 (...)

#### The sequence of primes for $k = 24$ :

: 125497397, 125497397121145169 (...)

#### The sequence of primes for $k = 30$ :

: 131 (...)

#### The sequence of primes for $k = 33$ :

: 13467100133 (...)

#### The sequence of primes for $k = 36$ :

: 13773109145181217253289325361397433469505541577613649 (...)

#### The sequence of primes for $k = 60$ :

: 161121181241301361,  
161121181241301361421481541601661721781841901961102110811141 (...)

**The sequence of primes for  $k = 90$ :**

: 191, 191181271, 191181271361, 1911812713614515416317218119019911081  
(...)

**The sequence of primes for  $k = 120$ :**

: 1121241361841 (...)

**The sequence of primes for  $k = 150$ :**

: 1151, 1151301451 (...)

**The sequence of primes for  $k = 180$ :**

: 1181361541721901108112611441162118011981216123412521  
2701 (...)

**The sequence of primes for  $k = 300$ :**

: 1301, 1301601901120115011801 (...)

**The sequence of primes for  $k = 900$ :**

: 1901, 190118012701360145015401 (...)

### 31. Primes obtained concatenating $p$ prime with $p + 2$ and $p + 6$ respectively with $p + 4$ and $p + 6$

**Abstract.** The triplets of primes  $[p, p + 2, p + 6]$  and  $[p, p + 4, p + 6]$  have already been studied: Hardy and Wright conjectured that there exist an infinity of such triplets. In this paper I make the following two conjectures on the triplets  $[p, p + 2, p + 6]$  and  $[p, p + 4, p + 6]$ , but only  $p$  is required to be prime: (I) there exist an infinity of primes  $q$  obtained concatenating a prime  $p$  with  $p + 2$  then with  $p + 6$ ; example: for  $p = 11$ , the number  $q = 111317$  is prime; (II) there exist an infinity of primes  $q$  obtained concatenating a prime  $p$  with  $p + 4$  then with  $p + 6$ ; example: for  $p = 241$ , the number  $q = 241245247$  is prime.

#### Conjecture I:

There exist an infinity of primes  $q$  obtained concatenating a prime  $p$  with  $p + 2$  then with  $p + 6$ ; example: for  $p = 11$ , the number  $q = 111317$  is prime.

#### The sequence of primes $q$ :

: 5711, 111317, 131519, 171923, 373943, 414347, 616367, 838589, 9799103, 103105109, 151153157, 167169173, 173175179, 223225229, 331333337, 593595599, 631633637, 653655659, 673675679, 701703707, 727729733, 751753757, 761763767, 797799803, 877879883, 9979991003 (...),

obtained for  $p = 5, 11, 13, 17, 37, 41, 83, 167, 173, 223, 331, 593, 631, 653, 673, 701, 727, 751, 761, 797, 877, 997$  (...)

#### Conjecture II:

There exist an infinity of primes  $q$  obtained concatenating a prime  $p$  with  $p + 4$  then with  $p + 6$ ; example: for  $p = 241$ , the number  $q = 241245247$  is prime.

#### The sequence of primes $q$ :

: 137141143, 241245247, 283287289, 293297299, 307311313, 311315317, 431435437, 461465467, 503507509, 521525527, 547551553, 577581583, 587591593, 617621623, 673677679, 701705707, 787791793, 821825827, 857861863, 881885887, 937941943, 983987989, 101310171019 (...)

obtained for  $p = 137, 241, 283, 293, 307, 311, 431, 461, 521, 547, 577, 587, 617, 673, 701, 787, 821, 857, 881, 937, 983, 1013$  (...)



## 32. Four conjectures on the triplets $[p, p + 2, p + 8]$ and $[p, p + 6, p + 8]$ where $p$ prime

**Abstract.** In this paper I make the following four conjectures on the triplets  $[p, p + 2, p + 8]$  and  $[p, p + 6, p + 8]$ : (I) there exist an infinity of triplets of primes of the form  $[p, p + 2, p + 8]$ ; (II) there exist an infinity of triplets of primes of the form  $[p, p + 6, p + 8]$ ; (III) there exist an infinity of primes  $q$  obtained concatenating a prime  $p$  with  $p + 2$  then with  $p + 8$  (only  $p$  is necessary prime); (IV) there exist an infinity of primes  $q$  obtained concatenating a prime  $p$  with  $p + 6$  then with  $p + 8$  (only  $p$  is necessary prime).

### Conjecture I:

There exist an infinity of triplets of primes of the form  $[p, p + 2, p + 8]$ . Obviously  $p$  has the form  $6^*k - 1$ .

**Such triplets of primes are:**

: [5, 7, 13], [11, 13, 19], [29, 31, 37], [59, 61, 67], [71, 73, 79], [101, 103, 109],  
[149, 151, 157], [191, 193, 199], [269, 271, 277]...  
(see A046134 in OEIS)

### Conjecture II:

There exist an infinity of triplets of primes of the form  $[p, p + 6, p + 8]$ . Obviously  $p$  has the form  $6^*k - 1$ .

**Such triplets of primes are:**

: [5, 11, 13], [11, 17, 19], [23, 29, 31], [53, 59, 61], [101, 107, 109], [131, 137, 139],  
[173, 179, 181], [191, 197, 199], [233, 239, 241], [263, 269, 271]...  
(see A046138 in OEIS)

### Conjecture III:

There exist an infinity of primes  $q$  obtained concatenating a prime  $p$  with  $p + 2$  then with  $p + 8$  (only  $p$  is necessary prime).

**The sequence of primes  $q$ :**

: 101103109, 103105111, 179181187, 199201207, 263265271, 283285291,  
311313319, 349351357, 353355361 (...)

### Conjecture IV:

There exist an infinity of primes  $q$  obtained concatenating a prime  $p$  with  $p + 6$  then with  $p + 8$  (only  $p$  is necessary prime).

**The sequence of primes  $q$ :**

: 313739, 616769, 737981, 838991, 109115117, 239245247, 263269271,  
223229231, 281287289, 389395397 (...)

### 33. Four conjectures on the triplets $[p, p + 4, p + 10]$ and $[p, p + 6, p + 10]$ where $p$ prime

**Abstract.** In this paper I make the following four conjectures on the triplets  $[p, p + 4, p + 10]$  and  $[p, p + 6, p + 10]$ : (I) there exist an infinity of triplets of primes of the form  $[p, p + 4, p + 10]$ ; (II) there exist an infinity of triplets of primes of the form  $[p, p + 6, p + 10]$ ; (III) there exist an infinity of primes  $q$  obtained concatenating a prime  $p$  with  $p + 4$  then with  $p + 10$  (only  $p$  is necessary prime); (IV) there exist an infinity of primes  $q$  obtained concatenating a prime  $p$  with  $p + 6$  then with  $p + 10$  (only  $p$  is necessary prime).

#### Conjecture I:

There exist an infinity of triplets of primes of the form  $[p, p + 4, p + 10]$ . Obviously  $p$  has the form  $6^*k + 1$ .

#### Such triplets of primes are:

: [7, 11, 17], [13, 17, 23], [19, 23, 29], [37, 41, 47], [43, 47, 53], [79, 83, 89], [97, 101, 107], [103, 107, 113], [127, 133, 137], [163, 167, 173], [223, 227, 233], [229, 233, 239], [307, 311, 317], [349, 353, 359], [379, 383, 389]...  
(see A046136 in OEIS)

#### Conjecture II:

There exist an infinity of triplets of primes of the form  $[p, p + 6, p + 10]$ . Obviously  $p$  has the form  $6^*k + 1$ .

#### Such triplets of primes are:

: [7, 13, 17], [13, 19, 23], [31, 37, 41], [37, 43, 47], [61, 67, 71], [73, 79, 83], [97, 103, 107], [103, 109, 113], [157, 163, 167], [223, 229, 233], [271, 277, 281], [307, 313, 317], [373, 379, 383]...(see A046139 in OEIS)

#### Conjecture III:

There exist an infinity of primes  $q$  obtained concatenating a prime  $p$  with  $p + 4$  then with  $p + 10$  (only  $p$  is necessary prime).

#### The sequence of primes $q$ :

: 111521, 172127, 293339, 596369, 677177, 717581, 103107113, 109113119, 139143149, 151155161, 157161167, 179183189, 229233239, 233237243, 251255261, 373377383 (...)

#### Conjecture IV:

There exist an infinity of primes  $q$  obtained concatenating a prime  $p$  with  $p + 6$  then with  $p + 10$  (only  $p$  is necessary prime).

#### The sequence of primes $q$ :

: 71317, 111721, 192529, 313741, 374347, 596569, 838993, 107113117, 127133137, 131137141, 163169173, 211217221, 251257261, 257263267, 281287291, 307313317, 331337341, 349355359 (...)

### 34. Conjectures on $(q + 2)[c][n][c][q$ and $(q - 4)[c][n][c][q$ where $n$ is equal to $1][c][2][c][\dots][c][p$ and $p, q$ are primes

**Abstract.** In this paper I make the following two conjectures: (I) let  $n$  be a number obtained concatenating the positive integers from 1 to  $p$ , where  $p$  prime of the form  $6^*k - 1$  (e.g.  $n = 12345$  for  $p = 5$ ); there exist an infinity of primes  $q$  of the form  $6^*h + 1$  such that the number  $r$  obtained concatenating  $q + 2$  with  $n$  then with  $q$  is prime (e.g. for  $n = 12345$  there exist  $q = 19$  such that  $r = 211234519$  is prime); (II) let  $n$  be a number obtained concatenating the positive integers from 1 to  $p$ , where  $p$  prime of the form  $6^*k - 1$ ; there exist an infinity of primes  $q$  of the form  $6^*h + 1$  such that the number  $r$  obtained concatenating  $q - 4$  with  $n$  then with  $q$  is prime (e.g. for  $n = 12345$  there exist  $q = 37$  such that  $r = 331234537$  is prime). I use the operator “[c]” with the meaning “concatenated to”.

#### Conjecture I:

Let  $n$  be a number obtained concatenating the positive integers from 1 to  $p$ , where  $p$  prime of the form  $6^*k - 1$  (e.g.  $n = 12345$  for  $p = 5$ ); there exist an infinity of primes  $q$  of the form  $6^*h + 1$  such that the number  $r$  obtained concatenating  $q + 2$  with  $n$  then with  $q$  is prime (e.g. for  $n = 12345$  there exist  $q = 19$  such that  $r = 211234519$  is prime).

#### The sequence of primes $r$ for $n = 12345$ :

: 211234519, 691234567, 991234597, 21312345211, 27912345277, 41112345409, 43512345433, 44112345439, 46512345463, 63312345631, 67512345673, 71112345709, 90912345907, 96912345967, 99312345991 (...)

obtained for  $q = 19, 67, 97, 211, 277, 409, 433, 439, 463, 631, 673, 709, 907, 969, 991$  (...)

#### The sequence of primes $r$ for $n = 1234567891011$ :

: 912345678910117, 1411234567891011139, 3151234567891011313, 3811234567891011379, 5491234567891011547, 6091234567891011607, (...)

obtained for  $q = 7, 139, 313, 379, 547, 607$  (...)

#### The least $r$ for $n = 1234567891011121314151617$ :

: 1051234567891011121314151617103.

#### Conjecture II:

Let  $n$  be a number obtained concatenating the positive integers from 1 to  $p$ , where  $p$  prime of the form  $6^*k - 1$ ; there exist an infinity of primes  $q$  of the form  $6^*h + 1$  such that the number  $r$  obtained concatenating  $q - 4$  with  $n$  then with  $q$  is prime (e.g. for  $n = 12345$  there exist  $q = 37$  such that  $r = 331234537$  is prime).

**The sequence of primes r for n = 12345:**

: 331234537, 931234597, 17712345181, 19512345199, 30312345307,  
36312345367, 36912345373, 40512345409, 41712345421, 45312345457,  
57312345577, 82512345829, 84912345853, 87312345877, 87912345883 (...)

obtained for  $q = 37, 97, 181, 199, 307, 367, 373, 409, 421, 457, 577, 829, 853,$   
 $877, 883 (...)$

**The sequence of primes r for n = 1234567891011:**

: 1231234567891011127, 1471234567891011151, 1951234567891011199,  
3271234567891011331, 4171234567891011421, 6571234567891011661,  
8191234567891011823, 9631234567891011967 (...)

obtained for  $q = 127, 151, 199, 331, 421, 661, 823, 967 (...)$

**Note:**

A wider statement would not require for the number  $q$  to be prime but number of the form  $6*k + 1$ ; in this case, the sequences from Conjecture 1 would contain also the numbers:

24912345247, 25512345253, 45312345451, 48312345481, 90312345901, 90912345907,  
93123456789101191, 5131234567891011511, 5851234567891011583,  
6811234567891011679, 8431234567891011841, 8731234567891011871 (...)

and the sequences from Conjecture 2 would contain also the numbers:

871234591, 18312345187, 25512345259, 47712345481, 63312345637, 90912345913,  
1831234567891011187, 2551234567891011259, 3991234567891011403,  
6991234567891011703, 8431234567891011847, 9391234567891011943 (...).

**Observation:**

Note the remarkable symmetry between the sequences from the two conjectures: up to  $q = 1000$ , for  $n = 12345$ , 15 primes  $q$  in the sequence from the first conjecture, 15 primes  $q$  in the sequence from the second conjecture; for  $n = 1234567891011$ , 6 primes  $q$  in the sequence from the first conjecture, 8 primes  $q$  in the sequence from the second conjecture; for non-primes satisfying the statements, for  $n = 12345$ , 6 in the sequence from the first conjecture, 6 in the sequence from the second conjecture; for  $n = 123456789101112$ , 6 in the sequence from the first conjecture, 6 in the sequence from the second conjecture.

### 35. Conjectures on $q|c[n]c[(q + 6)]$ and $(q + 6)|c[n]c[q]$ where $n$ is equal to $1|c[2]c[...]|c[p]$ and $p, q$ are primes

**Abstract.** In this paper I make the following four conjectures: (I) let  $n$  be a number obtained concatenating the positive integers from 1 to  $p$ , where  $p$  prime of the form  $6^*k - 1$ ; there exist an infinity of primes  $q$  of the form  $6^*h + 1$  such that the number  $r$  obtained concatenating  $q$  with  $n$  then with  $q + 6$  is prime; (II) let  $n$  be defined as in Conjecture 1; there exist an infinity of primes  $q$  of the form  $6^*h + 1$  such that the number  $r$  obtained concatenating  $q + 6$  with  $n$  then with  $q$  is prime; (III) let  $n$  be a number obtained concatenating the positive integers from 1 to  $p$ , where  $p$  prime of the form  $6^*k + 1$ ; there exist an infinity of primes  $q$  of the form  $6^*h - 1$  such that the number  $r$  obtained concatenating  $q$  with  $n$  then with  $q + 6$  is prime; (IV) let  $n$  be defined as in Conjecture 3; there exist an infinity of primes  $q$  of the form  $6^*h - 1$  such that the number  $r$  obtained concatenating  $q + 6$  with  $n$  then with  $q$  is prime.

#### Conjecture I:

Let  $n$  be a number obtained concatenating the positive integers from 1 to  $p$ , where  $p$  prime of the form  $6^*k - 1$ ; there exist an infinity of primes  $q$  of the form  $6^*h + 1$  such that the number  $r$  obtained concatenating  $q$  with  $n$  then with  $q + 6$  is prime.

**The least  $r$  for  $n = 12345$ :**

: 731234579, for  $q = 73$ .

**The least  $r$  for  $n = 1234567891011$ :**

: 1271234567891011133, for  $q = 127$ .

**The least  $r$  for  $n = 1234567891011121314151617$ :**

: 2111234567891011121314151617217, for  $q = 211$ .

#### Conjecture II:

Let  $n$  be defined as in Conjecture 1; there exist an infinity of primes  $q$  of the form  $6^*h + 1$  such that the number  $r$  obtained concatenating  $q + 6$  with  $n$  then with  $q$  is prime.

**The least  $r$  for  $n = 12345$ :**

: 251234519, for  $q = 19$ .

**The least  $r$  for  $n = 1234567891011$ :**

: 1312345678910117, for  $q = 7$ .

**The least  $r$  for  $n = 1234567891011121314151617$ :**

: 25123456789101112131415161719, for  $q = 19$ .

### Conjecture III:

Let  $n$  be a number obtained concatenating the positive integers from 1 to  $p$ , where  $p$  prime of the form  $6k + 1$ ; there exist an infinity of primes  $q$  of the form  $6h - 1$  such that the number  $r$  obtained concatenating  $q$  with  $n$  then with  $q + 6$  is prime.

**The least  $r$  for  $n = 1234567$ :**

: 41123456747, for  $q = 41$ .

**The least  $r$  for  $n = 12345678910111213$ :**

: 231234567891011121329, for  $q = 23$ .

**The least  $r$  for  $n = 12345678910111213141516171819$ :**

: 711234567891011121314151617181977, for  $q = 71$ .

### Conjecture IV:

Let  $n$  be defined as in Conjecture 3; there exist an infinity of primes  $q$  of the form  $6h - 1$  such that the number  $r$  obtained concatenating  $q + 6$  with  $n$  then with  $q$  is prime.

**The least  $r$  for  $n = 1234567$ :**

: 17123456711, for  $q = 11$ .

**The least  $r$  for  $n = 12345678910111213$ :**

: 771234567891011121371, for  $q = 71$ .

**The least  $r$  for  $n = 12345678910111213141516171819$ :**

: 291234567891011121314151617181923, for  $q = 23$ .

### 36. Primes obtained concatenating $p - 1$ with 3 where $p$ prime of the form $30*k + 17$

**Abstract.** In this paper I state the following conjecture: Let  $p$  be a prime of the form  $30*k + 17$ ; then there exist an infinity of primes  $q$  obtained concatenating  $p - 1$  with 3; example: 677, 797, 827, 857, 887, 947 are primes (succesive primes of the form  $30*k + 17$ ) and the numbers 6763, 7963, 8263, 8563, 8863, 9463 are also primes. As an incidental observation, many of the semiprimes  $x*y$  obtained in the way defined have one of the following two properties: (i)  $y - x + 1$  is a prime of the form  $13 + 30*k$ ; (ii)  $y - x + 1$  is a prime of the form  $19 + 30*k$ .

#### Conjecture:

Let  $p$  be a prime of the form  $30*k + 17$ ; then there exist an infinity of primes  $q$  obtained concatenating  $p - 1$  with 3; example: 677, 797, 827, 857, 887, 947 are primes (succesive primes of the form  $30*k + 17$ ) and the numbers 6763, 7963, 8263, 8563, 8863, 9463 are also primes.

#### Primes of the form $30*k + 17$ :

(Sequence A039949 in OEIS)

: 17, 47, 107, 137, 167, 197, 227, 257, 317, 347, 467, 557, 587, 617, 647, 677, 797, 827, 857, 887, 947, 977, 1097, 1187, 1217, 1277, 1307, 1367, 1427, 1487, 1607, 1637, 1667, 1697, 1787, 1847, 1877, 1907, 1997, 2027, 2087, 2207, 2237, 2267, 2297, 2357, 2417, 2447, 2477, 2657, 26863, 2777, 2837, 2897, 2927, 2957, 3137, 3167, 3257, 3347, 3407, 3467, 3527 (...)

#### The sequence of primes $q$ :

: 163, 463, 1063, 1663, 3163, 3463, 4663, 5563, 6163, 6763, 7963, 8263, 8563, 8863, 9463, 11863, 12163, 12763, 13063, 16063, 16363, 16963, 17863, 18763, 19963, 22063, 22663, 22963, 23563, 24163, 24763, 26863, 27763, 31663, 32563 (...)

#### Observation:

Many of the numbers obtained concatenating  $p - 1$  with 3 are semiprimes: 1363, 1963, 2263, 2563, 6463, 9763, 10963, 13663, 14263, 14863, 16663, 18463, 19063, 20263, 24463, 26563, 28363, 28963, 29263, 31363, 33463, 34063, 34663, 35263 (...).

Some of these semiprimes  $x*y$  have one of the following two properties:

(i)  $y - x + 1$  is a prime of the form  $13 + 30*k$ :

: 2263 = 31\*73 and 73 - 3 + 1 = 43;  
 : 2563 = 11\*233 and 233 - 11 + 1 = 223;  
 : 14263 = 17\*839 and 839 - 17 + 1 = 823;  
 : 18463 = 37\*499 and 499 - 37 + 1 = 463;

:  $19063 = 11 \cdot 1733$  and  $1733 - 11 + 1 = 1723$ ;  
 :  $20863 = 31 \cdot 673$  and  $1733 - 11 + 1 = 643$ ;  
 :  $22663 = 131 \cdot 173$  and  $173 - 131 + 1 = 43$ ;  
 :  $24463 = 17 \cdot 1439$  and  $173 - 131 + 1 = 1423$ ;  
 :  $26563 = 101 \cdot 263$  and  $263 - 101 + 1 = 163$ .

(ii)  $y - x + 1$  is a prime of the form  $19 + 30 \cdot k$ :

:  $1363 = 29 \cdot 47$  and  $47 - 29 + 1 = 19$ ;  
 :  $1963 = 13 \cdot 151$  and  $151 - 13 + 1 = 139$ ;  
 :  $9963 = 13 \cdot 751$  and  $751 - 13 + 1 = 739$ ;  
 :  $13663 = 13 \cdot 1051$  and  $1051 - 13 + 1 = 1039$ ;  
 :  $14863 = 89 \cdot 167$  and  $167 - 89 + 1 = 79$ ;  
 :  $16663 = 19 \cdot 877$  and  $877 - 19 + 1 = 859$ ;  
 :  $18763 = 29 \cdot 647$  and  $647 - 29 + 1 = 619$ .  
 :  $20263 = 23 \cdot 881$  and  $881 - 23 + 1 = 859$ ;  
 :  $28363 = 113 \cdot 251$  and  $251 - 113 + 1 = 139$ ;  
 :  $29263 = 13 \cdot 2251$  and  $2251 - 13 + 1 = 2239$ ;  
 :  $33463 = 109 \cdot 307$  and  $307 - 109 + 1 = 199$ ;  
 :  $34063 = 109 \cdot 307$  and  $307 - 109 + 1 = 1459$ ;  
 :  $35263 = 179 \cdot 197$  and  $197 - 179 + 1 = 19$ .



### 37. Primes obtained concatenating with 1 to the right the triangular numbers

**Abstract.** In this paper I state the following conjecture: There exist an infinity of primes  $p$  obtained concatenating to the right with 1 the triangular numbers.

#### Conjecture:

There exist an infinity of primes  $p$  obtained concatenating to the right with 1 the triangular numbers.

Note: the formula of triangular numbers is  $T(n) = n*(n + 1)/2 = 1 + 2 + 3 + \dots + n$ .

#### The triangular numbers $T(n)$ :

(Sequence A000217 in OEIS)

: 0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210, 231, 253, 276, 300, 325, 351, 378, 406, 435, 465, 496, 528, 561, 595, 630, 666, 703, 741, 780, 820, 861, 903, 946, 990, 1035, 1081, 1128, 1176, 1225, 1275, 1326, 1378, 1431 (...) 4498050211, 4498350151 (...)

#### The sequence of primes $p$ :

: 11, 31, 61, 101, 151, 211, 281, 661, 911, 1051, 1201, 1361, 1531, 1901, 2311, 2531, 3001, 3251, 3511, 4651, 5281, 6301, 6661, 7411, 9461, 9901, 12251, 13781 (...) 4498050211, 4498350151 (...)

#### Observations:

: Note the chain of 7 primes (11, 31, 61, 101, 151, 211, 281) obtained from 7 consecutive triangular numbers (1, 3, 6, 10, 15, 21, 28), also the chain of 5 primes (91, 105, 120, 136, 153) obtained from 5 consecutive triangular numbers (91, 105, 120, 136, 153).

Note that many of the numbers obtained by this method are semiprimes  $x*y$  with the property that  $x$  and  $y$  have the same last digit (some of them have also the property that  $y - x + 1$  is prime).

#### Examples:

: 11761 = 19\*619 (and 619 - 19 + 1 = 601);  
: 12751 = 41\*311 (and 311 - 41 + 1 = 271);  
: 13261 = 89\*149 (and 149 - 89 + 1 = 61);  
: 14311 = 11\*1301 (and 1301 - 11 + 1 = 1291);  
: 4498650101 = 11\*408968191;  
: 4498950061 = 29\*155136209;  
: 4499250031 = 701\*6418331;  
: 4499850001 = 5309\*847589.

### 38. Primes obtained concatenating with 1 to the left the terms of two back concatenated “multiples of 3” sequences

**Abstract.** In this paper I state the following two conjectures: (I) there exist an infinity of primes  $p$  obtained concatenating to the left with 1 the terms of *back concatenated “multiples of 3” sequence* (defined as the sequence obtained through the concatenation of multiples of 3, in reverse order); such prime is, for example, 13330272421181512963; (II) there exist an infinity of primes  $p$  obtained concatenating to the left with 1 the terms of *back concatenated “odd multiples of 3” sequence* (defined as the sequence obtained through the concatenation of odd multiples of 3, in reverse order); such prime is, for example, 145393327211593.

#### Conjecture 1:

There exist an infinity of primes  $p$  obtained concatenating to the left with 1 the terms of *back concatenated “multiples of 3” sequence* (defined as the sequence obtained through the concatenation of multiples of 3, in reverse order); such prime is, for example, 13330272421181512963.

#### The back concatenated “multiples of 3” sequence:

: 3, 63, 963, 12963, 1512963, 181512963, 21181512963, 2421181512963, 272421181512963, 30272421181512963 (...) and

The sequence of primes  $p$ :

: 163, 13330272421181512963,  
19996939087848178757269666360575451484542393633302724211815  
12963,  
11081051029996939087848178757269666360575451484542393633302  
72421181512963 (...)

#### Conjecture 2:

There exist an infinity of primes  $p$  obtained concatenating to the left with 1 the terms of *back concatenated “odd multiples of 3” sequence* (defined as the sequence obtained through the concatenation of odd multiples of 3, in reverse order); such prime is, for example, 145393327211593.

#### The back concatenated “odd multiples of 3” sequence:

: 3, 93, 1593, 211593, 27211593, 3327211593, 393327211593, 45393327211593,  
5145393327211593 (...) and

The sequence of primes  $p$ :

: 193, 11593, 1211593, 145393327211593 (...)

### 39. Primes obtained concatenating with 1 to the left the terms of three back concatenated “powers of 3” sequences

**Abstract.** In this paper I state the following conjecture three conjectures: (I) there exist an infinity of primes  $p$  obtained concatenating to the left with 1 the terms of *back concatenated “powers of 3” sequence* (defined as the sequence obtained through the concatenation of powers of 3, in reverse order); such prime is, for example, 1243812793; (II) there exist an infinity of primes  $p$  obtained concatenating to the left with 1 the terms of *back concatenated “odd powers of 3” sequence* (defined as the sequence obtained through the concatenation of odd powers of 3, in reverse order); such prime is, for example, 1243273; (III) there exist an infinity of primes  $p$  obtained concatenating to the left with 1 the terms of *back concatenated “even powers of 3” sequence* (defined as the sequence obtained through the concatenation of even powers of 3, in reverse order); such prime is, for example, 14782969531441590496561729819.

#### Conjecture 1:

There exist an infinity of primes  $p$  obtained concatenating to the left with 1 the terms of *back concatenated “powers of 3” sequence* (defined as the sequence obtained through the concatenation of powers of 3, in reverse order); such prime is, for example, 1243812793.

#### The back concatenated “powers of 3” sequence:

: 3, 93, 2793, 812793, 243812793, 729243812793, 2187729243812793, 65612187729243812793 (...) and

The sequence of primes  $p$ :

: 13, 193, 1812793, 1243812793 (...)

#### Conjecture 2:

There exist an infinity of primes  $p$  obtained concatenating to the left with 1 the terms of *back concatenated “odd powers of 3” sequence* (defined as the sequence obtained through the concatenation of odd powers of 3, in reverse order); such prime is, for example, 1243273.

#### The back concatenated “odd powers of 3” sequence:

: 3, 273, 243273, 2187243273, 218724327319683, 177147218724327319683, 1594323177147218724327319683 (...) and

The sequence of primes  $p$ :

: 13, 1243273, 12187243273 (...)

**Conjecture 3:**

There exist an infinity of primes  $p$  obtained concatenating to the left with 1 the terms of *back concatenated “even powers of 3” sequence* (defined as the sequence obtained through the concatenation of even powers of 3, in reverse order); such prime is, for example, 14782969531441590496561729819.

**The back concatenated “even powers of 3” sequence:**

: 9, 819, 729819, 6561729819, 6561729819, 590496561729819, 531441590496561729819, 4782969531441590496561729819 (...) and

The sequence of primes  $p$ :

: 19, 14782969531441590496561729819 (...)

## 40. Conjecture on an infinity of Poulet numbers which are also triangular numbers

**Abstract.** I was studying the sequences of primes obtained applying concatenation to some well known classes of numbers, when I discovered that the second Poulet number, 561 (also the first Carmichael number, also a very interesting number – I wrote a paper dedicated to some of its properties), is also a triangular number. Continuing to look, I found, up to the triangular number  $T(817)$ , if we note  $T(n) = n*(n + 1)/2 = 1 + 2 + \dots + n$ , fifteen Poulet numbers. In this paper I state the conjecture that there exist an infinity of Poulet numbers which are also triangular numbers.

### Conjecture:

There exist an infinity of Poulet numbers which are also triangular numbers.

### The triangular numbers $T(n)$ :

(Sequence A000217 in OEIS)

: 0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210, 231, 253, 276, 300, 325, 351, 378, 406, 435, 465, 496, 528, 561, 595, 630, 666, 703, 741, 780, 820, 861, 903 (...)

### The Poulet numbers:

(Sequence A001567 in OEIS)

: 341, 561, 645, 1105, 1387, 1729, 1905, 2047, 2465, 2701, 2821, 3277, 4033, 4369, 4371, 4681, 5461, 6601, 7957, 8321, 8481, 8911, 10261, 10585, 11305, 12801, 13741, 13747, 13981, 14491, 15709 (...)

### The sequence of Poulet numbers which are also triangular numbers:

: 561 ( $T(33)$ ), 2701 ( $T(73)$ ), 4371 ( $T(93)$ ), 8911 ( $T(133)$ ), 33153 ( $T(257)$ ), 41041 ( $T(286)$ ), 49141 ( $T(313)$ ), 93961 ( $T(433)$ ), 104653 ( $T(457)$ ), 115921 ( $T(481)$ ), 157641 ( $T(561)$ ), 226801 ( $T(673)$ ), 289941 ( $T(761)$ ), 314821 ( $T(793)$ ), 334153 ( $T(817)$ )...

### Observations:

Note the interesting fact that  $T(561) = 157641$  is a Poulet number! Question: are there other Poulet numbers  $p$  such that  $T(p)$  is also a Poulet number? Note the numbers obtained when these Poulet, also triangular numbers, are concatenated to the right with 1: there are primes among them (27011, 43711, 410411, 939611, 3341531) or semiprimes  $x*y$  having the property that  $x$  and  $y$  have the same last digit (1 or 9) and sometimes the property that  $x + y - 1$  is prime (5611 = 31\*181 and 3 + 181 - 1 = 211, prime; 89111 = 11\*8101 and 11 + 8101 - 1 = 8111, prime; 331531 = 19\*17449 and 19 + 17449 - 1 = 17467, prime; 491411 = 59\*8329 and 59 + 8329 - 1 = 8387, prime; 1046531 = 139\*7529; 1159211 = 359\*3229; 2899411 = 1601\*1811). Note that many of these Poulet, also triangular numbers, admit a deconcatenation in two primes: 561 (5 and 61), 2701 (2 and 701), 4371 (43 and 71), 8911 (89 and 11), 33153 (331 and 53), 49141 (491 and 41), 157641 (157 and 641), 226801 (2 and 26801), 314821 (3 and 14821).

## 41. Conjecture on an infinity of numbers $(30*k + 7)*(60*k + 13)$ which admit a deconcatenation in two primes

**Abstract.** In this paper I state the following conjecture: there exist an infinity of numbers  $q = (30*k + 7)*(60*k + 13)$  which admit a deconcatenation in two primes  $p_1$  and  $p_2$ . Examples: for  $k = 2$ ,  $q = 67*133 = 8911$  which can be deconcatenated in  $p_1 = 89$  and  $p_2 = 11$ ; for  $k = 5$ ,  $q = 157*313 = 49141$  which can be deconcatenated in  $p_1 = 491$  and  $p_2 = 41$ .

### Conjecture:

There exist an infinity of numbers  $q = (30*k + 7)*(60*k + 13)$  which admit a deconcatenation in two primes  $p_1$  and  $p_2$ . Examples: for  $k = 2$ ,  $q = 67*133 = 8911$  which can be deconcatenated in  $p_1 = 89$  and  $p_2 = 11$ ; for  $k = 5$ ,  $q = 157*313 = 49141$  which can be deconcatenated in  $p_1 = 491$  and  $p_2 = 41$ .

### The sequence of numbers $q$ :

:  $q = 2701$ , for  $k = 1$ ;  $p_1 = 2$  and  $p_2 = 701$ ;  
:  $q = 8911$ , for  $k = 2$ ;  $p_1 = 89$  and  $p_2 = 11$ ;  
:  $q = 32131$ , for  $k = 4$ ;  $p_1 = 3$  and  $p_2 = 2131$ ;  
:  $q = 49141$ , for  $k = 5$ ;  $p_1 = 491$  and  $p_2 = 41$ ;  
:  $q = 121771$ , for  $k = 8$ ;  $p_1 = 1217$  and  $p_2 = 71$ ;  
:  $q = 473851$ , for  $k = 16$ ;  $p_1 = 47$  and  $p_2 = 3851$ ;  
:  $q = 534061$ , for  $k = 17$ ;  $p_1 = 5$  and  $p_2 = 34061$ ;  
:  $q = 597871$ , for  $k = 18$ ;  $p_1 = 5$  and  $p_2 = 97871$ ;  
:  $q = 1145341$ , for  $k = 25$ ;  $p_1 = 11$  and  $p_2 = 45341$ ;  
:  $q = 1433971$ , for  $k = 28$ ;  $p_1 = 1433$  and  $p_2 = 971$ ;  
:  $q = 1755001$ , for  $k = 31$ ;  $p_1 = 17$  and  $p_2 = 55001$ ;  
:  $q = 2362051$ , for  $k = 36$ ;  $p_1 = 2$  and  $p_2 = 362051$ ;  
:  $q = 2912491$ , for  $k = 40$ ;  $p_1 = 29$  and  $p_2 = 12491$ ;  
:  $q = 3209311$ , for  $k = 42$ ;  $p_1 = 3209$  and  $p_2 = 311$ ;  
:  $q = 4186171$ , for  $k = 48$ ;  $p_1 = 41$  and  $p_2 = 86171$ ;  
:  $q = 4723201$ , for  $k = 51$ ;  $p_1 = 47$  and  $p_2 = 23201$ ;  
:  $q = 5099221$ , for  $k = 53$ ;  $p_1 = 509$  and  $p_2 = 9221$ ;  
:  $q = 5292631$ , for  $k = 54$ ;  $p_1 = 5$  and  $p_2 = 292631$ ;  
:  $q = 8876791$ , for  $k = 70$ ;  $p_1 = 887$  and  $p_2 = 6791$ ;  
:  $q = 11297881$ , for  $k = 79$ ;  $p_1 = 2$  and  $p_2 = 297881$  (...)

### Few larger numbers $q$ :

:  $q = 1793615671$ , for  $k = 998$ ;  $p_1 = 179$  and  $p_2 = 3615671$ ;  
:  $q = 1797211081$ , for  $k = 999$ ;  $p_1 = 179$  and  $p_2 = 7211081$ ;  
:  $q = 179936105671$ , for  $k = 9998$ ;  $p_1 = 17$  and  $p_2 = 9936105671$ ;  
:  $q = 179972101081$ , for  $k = 9999$ ;  $p_1 = 17$  and  $p_2 = 9972101081$ ;  
:  $q = 17999936100005671$ , for  $k = 999997$ ;  $p_1 = 179999$  and  $p_2 = 900100013861$ ;  
:  $q = 17999936100005671$ , for  $k = 999998$ ;  $p_1 = 179999$  and  $p_2 = 36100005671$ .

## 42. Primes obtained concatenating to the right with 1 the partial sums of repdigits

**Abstract.** In this paper I state the following conjecture: For any digit from 1 to 9 there exist a sequence with an infinity of prime terms obtained concatenating to the right with 1 the partial sums of the repdigits. Examples: for repunit numbers 1, 11, 111 (...), concatenating the sum  $S(3) = 1 + 11 + 111 = 123$  to the right with 1 is obtained 1231, prime; for repdigit numbers 3, 33, 333, 3333 (...), concatenating the sum  $S(4) = 3 + 33 + 333 + 3333 = 3702$  to the right with 1 is obtained 37021, prime.

### Conjecture:

For any digit from 1 to 9 there exist a sequence with an infinity of prime terms  $p$  obtained concatenating to the right with 1 the partial sums of the repdigits. Examples: for repunit numbers 1, 11, 111 (...), concatenating the sum  $S(3) = 1 + 11 + 111 = 123$  to the right with 1 is obtained 1231, prime; for repdigit numbers 3, 33, 333, 3333 (...), concatenating the sum  $S(4) = 3 + 33 + 333 + 3333 = 3702$  to the right with 1 is obtained 37021, prime.

#### Primes $p$ for the sums of the numbers $(10^n - 1)/9$ :

(see sequence A014824 in OEIS for the partial sums of repunits)

: 11, 1231, 1234567891, 123456790123441, 12345679012345661,  
1234567901234567901234567901201 (...)

#### Primes $p$ for the sums of the numbers $2*(10^n - 1)/9$ :

(see sequence A099669 in OEIS for the partial sums of these repdigits)

: 241, 2469135802469101 (...)

#### Primes $p$ for the sums of the numbers $3*(10^n - 1)/9$ :

(see sequence A099670 in OEIS for the partial sums of these repdigits)

: 31, 3691, 37021, 370370370370321, 3703703703703651, 370370370370370311,  
3703703703703703641 (...)

#### Primes $p$ for the sums of the numbers $4*(10^n - 1)/9$ :

(see sequence A099671 in OEIS for the partial sums of these repdigits)

: 41, 4938271561, 49382716001, 493827160441 (...)

#### Primes $p$ for the sums of the numbers $5*(10^n - 1)/9$ :

(see sequence A099672 in OEIS for the partial sums of these repdigits)

: 601, 6151, 6172801 (...)

#### Primes $p$ for the sums of the numbers $6*(10^n - 1)/9$ :

(see sequence A099673 in OEIS for the partial sums of these repdigits)

: 61 (...)

**Primes  $p$  for the sums of the numbers  $7 \cdot (10^n - 1)/9$ :**

(see sequence A099674 in OEIS for the partial sums of these repdigits)

: 71, 86381, 864151, 86419691 (...)

**Primes  $p$  for the sums of the numbers  $8 \cdot (10^n - 1)/9$ :**

(see sequence A099675 in OEIS for the partial sums of these repdigits)

: 6172801 (...)

**Primes  $p$  for the sums of the numbers  $9 \cdot (10^n - 1)/9$ :**

(see sequence A099676 in OEIS for the partial sums of these repdigits)

: 11071, 11111111110971, 1111111111110941, 111111111111111110881,  
1111111111111111110871 (...)



### 43. Primes obtained concatenating $n$ consecutive numbers and then the resulting number with 1

**Abstract.** In this paper I state the following conjecture: For any positive integer  $n$ ,  $n > 1$ , there exist a sequence having an infinity of prime terms obtained concatenating  $n$  consecutive numbers and then the resulting number, to the right, with 1. Examples: for  $n = 2$ , the sequence obtained this way contains the primes 10111, 15161, 18191, 21221 (...); for  $n = 9$ , the sequence obtained this way contains the primes 1234567891, 910111213141516171, 2021222324252627281, 2930313233343536371 (...).

#### Conjecture:

For any positive integer  $n$ ,  $n > 1$ , there exist a sequence having an infinity of prime terms  $p$  obtained concatenating  $n$  consecutive numbers and then the resulting number, to the right, with 1. Examples: for  $n = 2$ , the sequence obtained this way contains the primes 10111, 15161, 18191, 21221 (...); for  $n = 9$ , the sequence obtained this way contains the primes 1234567891, 910111213141516171, 2021222324252627281, 2930313233343536371 (...).

#### The sequence of primes $p$ for $n = 2$ :

: 10111, 15161, 18191, 21221, 24251, 25261, 27281, 31321, 34351, 43441, 46471, 48491, 51521, 60611, 76771, 78791, 79801, 87881, 90911, 91921, 93941 (...)

#### The sequence of primes $p$ for $n = 3$ :

: 1231, 2341, 4561, 6781, 89101, 1112131, 1213141, 1718191, 1920211, 2122231, 2425261, 2829301, 3132331, 3334351, 3435361, 3536371, 3839401, 3940411 (...)

#### The sequence of primes $p$ for $n = 4$ :

: 67891, 789101, 222324251, 363738391, 434445461, 464748491, 495051521, 555657581, 585960611, 646566671, 697071721, 707172731, 798081821 (...)

#### The sequence of primes $p$ for $n = 5$ :

: 9101112131, 15161718191, 21222324251, 36373839401, 43444546471, 46474849501 (...)

#### The sequence of primes $p$ for $n = 6$ :

: 3456781, 4567891, 56789101 (...)

#### The sequence of primes $p$ for $n = 7$ :

: 91011121314151, 181920212223241, 313233343536371 (...)

**The sequence of primes p for n = 8:**

: 45678910111, 15161718192021221, 37383940414243441 (...)

**The sequence of primes p for n = 9:**

: 1234567891, 910111213141516171, 2021222324252627281,  
2930313233343536371, 3839404142434445461 (...)

**The least prime p for the following values of n:**

:	p = 303132333435363738391,	for n = 10;
:	p = 12345678910111,	for n = 11;
:	p = 939495969798991001011021031041,	for n = 12;
:	p = 91011121314151617181920211,	for n = 13;
:	p = 39404142434445464748495051521,	for n = 14;
:	p = 3334353637383940414243444546471,	for n = 15;
:	p = 123456789101112131415161,	for n = 16;
:	p = 25262728293031323334353637383940411,	for n = 17;
:	p = 2223242526272829303132333435363738391,	for n = 18;
:	p = 161718192021222324252627282930313233341,	for n = 19.

**Observation:**

An interesting question is also the following one: if m is a non-null positive integer, are there an infinity of primes q obtained concatenating m with consecutive numbers and then, the resulting number, with 1? Example: is, for m = 1, the sequence of primes 1231, 1234567891, 12345678910111 (...) infinite? Or, for m = 2, the sequence 2341, 23456789101112131415161718192021222324251 (...)? Or, for any non-null positive integer m?

**The sequence of the least primes q for m starting from 1:**

: 1231, 2341, 3456781, 4561, 6781, 789101, 89101, 9101112131, 10111, 1112131,  
1213141 (...)

#### 44. Conjecture on the consecutive concatenation of the terms of an arithmetic progression

**Abstract.** In this paper I make the following conjecture: for any arithmetic progression  $a + b \cdot k$ , where at least one of  $a$  and  $b$  is different than 1, that also satisfies the conditions imposed by the Dirichlet's Theorem ( $a$  and  $b$  are positive coprime integers) is true that the sequence obtained by the consecutive concatenation of the terms  $a + b \cdot k$  has an infinity of prime terms. Example: for  $[a, b] = [7, 11]$ , the sequence obtained by consecutive concatenation of 7, 18, 29, 40, 51, 62, 73 (...) has the prime terms 718294051, 7182940516273 (...). If this conjecture were true, the fact that the *Smarandache consecutive numbers sequence* 1, 12, 123, 1234, 12345 (...) could have not any prime term (thus far there is no prime number known in this sequence, though there have been checked the first about 40 thousand terms) would be even more amazing.

#### Conjecture:

For any arithmetic progression  $a + b \cdot k$ , where at least one of  $a$  and  $b$  is different than 1, that also satisfies the conditions imposed by the Dirichlet's Theorem ( $a$  and  $b$  are positive coprime integers) is true that the sequence obtained by the consecutive concatenation of the terms  $a + b \cdot k$  has an infinity of prime terms.

#### Example:

For  $[a, b] = [7, 11]$ , the sequence obtained by consecutive concatenation of 7, 18, 29, 40, 51, 62, 73 (...) has the prime terms 718294051, 7182940516273 (...).

#### Note:

If this conjecture were true, the fact that the *Smarandache consecutive numbers sequence* 1, 12, 123, 1234, 12345 (...) could have not any prime term (thus far there is no prime number known in this sequence, though there have been checked the first about 40 thousand terms) would be even more amazing.

#### The sequence of primes for $[a, b] = [2, 1]$ :

: 2, 23, 23456789, 23456789101112131415161718192021222324252627 (...)

#### The sequence of primes for $[a, b] = [3, 1]$ :

: 3, 345678910111213141516171819 (...)

#### The sequence of primes for $[a, b] = [4, 1]$ :

: 4567, 45678910111213 (...)

#### The sequence of primes for $[a, b] = [5, 1]$ :

: 5, 567891011121314151617 (...)

**The sequence of primes for  $[a, b] = [6, 1]$ :**

: 67, 678910111213 (...)

**The sequence of primes for  $[a, b] = [25, 1]$ :**

: 25262728293031323334353637383940414243444546474849 (...)

Note that 25 and 49 are both squares of primes (twins); question: are there any other pairs of squares of primes  $[p^2, q^2]$  having the property that concatenating the numbers  $p^2, p^2 + 1, \dots, q^2$  is obtained a prime?

**The sequence of primes for  $[a, b] = [1, 2]$ :**

: 13, 135791113151719, 135791113151719212325272931 (...)

This sequence is known as *Smarandache concatenated odd sequence* and Florentin Smarandache conjectured that there exist an infinity of prime terms of this sequence. The terms of this sequence are primes for the following values of  $n$ : 2, 10, 16, 34, 49, 2570 (the term corresponding to  $n = 2570$  is a number with 9725 digits); there is no other prime term known though where checked the first about 26 thousand terms of this sequence.

**The sequence of primes for  $[a, b] = [1, 3]$ :**

: 14710131619, 14710131619222528313437 (...)

In my previous paper “Conjecture on the consecutive concatenation of the numbers  $nk+1$  where  $k$  multiple of 3” I already conjectured that this sequence has an infinity of prime terms.

**The sequence of primes for  $[a, b] = [1, 4]$ :**

: 159131721252933373145495357616569737781 (...)

**The sequence of primes for  $[a, b] = [1, 20]$ :**

: 121416181101121141161181 (...)

**The sequence of primes for  $[a, b] = [3, 2]$ :**

: 357911131517192123252729 (...)

**The sequence of primes for  $[a, b] = [5, 2]$ :**

: 57911131517, 57911131517192123252729313335373941 (...)

**The sequence of primes for  $[a, b] = [17, 16]$ :**

: 1733496581, 173349658197113, 173349658197113129145161177,  
173349658197113129145161177193209 (...)

**The sequence of primes for [a, b] = [19, 30]:**

: 1949, 194979109139169199229259289319 (...)

**The sequence of primes for [a, b] = [23, 30]:**

: 23, 235383113143173203233263293323,  
235383113143173203233263293323353383 (...)

**The sequence of primes for [a, b] = [61, 30]:**

: 61, 6191121151181, 6191121151181211241271301331 (...)

**The sequence of primes for [a, b] = [1729, 30]:**

: 17291759, 1729175917891819184918791909 (...)

**The sequence of primes for [a, b] = [1729, 60]:**

: 1729178918491909 (...)

**The sequence of primes for [a, b] = [1729, 90]:**

: 17291819, 1729181919091999 (...)

**The sequence of primes for [a, b] = [341, 90]:**

: 341431521611, 341431521611701 (...)

**The sequence of primes for [a, b] = [341, 340]:**

: 341681 (...)

**The sequence of primes for [a, b] = [561, 560]:**

: 5611121 (...)

**The sequence of primes for [a, b] = [1729, 1728]:**

: 17293457 (...)

Note that from a Poulet number P (see above 341, 561, 1729) is obtained often a prime Q when is concatenated with  $2*P - 1$  (it is also the case for P = 1387 (Q = 13872773), P = 3277 (Q = 32776553) etc.

## 45. Two conjectures on the primes which admit deconcatenation in two primes, involving multiples of 30

**Abstract.** In this paper I state the following two conjectures: (I) If  $p$  is a prime which admits deconcatenation in two primes  $p_1$  and  $p_2$ , both of the form  $6^*k - 1$ , then there exist an infinity of primes  $q$  obtained concatenating  $q_1$  with  $q_2$ , where  $q_1 = 30^*n - p_1$ ,  $q_2 = 30^*n - p_2$  and  $n$  positive integer; (II) If  $p$  is a prime which admits deconcatenation in two primes  $p_1$  and  $p_2$ , both of the form  $6^*k + 1$ , then there exist an infinity of primes  $q$  obtained concatenating  $q_1$  with  $q_2$ , where  $q_1 = 30^*n + p_1$ ,  $q_2 = 30^*n + p_2$  and  $n$  positive integer.

### Conjecture 1:

If  $p$  is a prime which admits deconcatenation in two primes  $p_1$  and  $p_2$ , both of the form  $6^*k - 1$ , then there exist an infinity of primes  $q$  obtained concatenating  $q_1$  with  $q_2$ , where  $q_1 = 30^*n - p_1$ ,  $q_2 = 30^*n - p_2$  and  $n$  positive integer.

**The sequence of  $q$  for  $p = 523$  ( $[p_1, p_2] = [5, 23]$ ):**

:  $q = 257$ , for  $n = 1$  and  $[q_1, q_2] = [25, 7]$ ;  
:  $q = 11597$ , for  $n = 4$  and  $[q_1, q_2] = [115, 97]$ ;  
:  $q = 205187$ , for  $n = 7$  and  $[q_1, q_2] = [205, 187]$  (...)

**The sequence of  $q$  for  $p = 541$  ( $[p_1, p_2] = [5, 41]$ ):**

:  $q = 5519$ , for  $n = 2$  and  $[q_1, q_2] = [55, 19]$ ;  
:  $q = 11579$ , for  $n = 4$  and  $[q_1, q_2] = [115, 79]$ ;  
:  $q = 145109$ , for  $n = 5$  and  $[q_1, q_2] = [145, 109]$  (...)

**The sequence of  $q$  for  $p = 1117$  ( $[p_1, p_2] = [11, 17]$ ):**

:  $q = 1913$ , for  $n = 1$  and  $[q_1, q_2] = [19, 13]$ ;  
:  $q = 4943$ , for  $n = 2$  and  $[q_1, q_2] = [49, 43]$ ;  
:  $q = 109103$ , for  $n = 4$  and  $[q_1, q_2] = [109, 103]$  (...)

**The sequence of  $q$  for  $p = 1123$  ( $[p_1, p_2] = [11, 23]$ ):**

:  $q = 197$ , for  $n = 1$  and  $[q_1, q_2] = [19, 7]$ ;  
:  $q = 4937$ , for  $n = 2$  and  $[q_1, q_2] = [49, 37]$ ;  
:  $q = 349337$ , for  $n = 12$  and  $[q_1, q_2] = [349, 337]$  (...)

### Observation:

The conjecture above seems to apply as well to Poulet numbers which admit the mentioned deconcatenation.

**The sequence of q for p = 49141 ([p1, p2] = [491, 41]):**

- : q = 19469, for n = 17 and [q1, q2] = [19, 469];
- : q = 49499, for n = 18 and [q1, q2] = [49, 499];
- : q = 139589, for n = 21 and [q1, q2] = [139, 589] (...)

**The sequence of q for p = 1729 ([p1, p2] = [17, 29]):**

- : q = 131, for n = 1 and [q1, q2] = [13, 1];
- : q = 10391, for n = 4 and [q1, q2] = [103, 91];
- : q = 133121, for n = 5 and [q1, q2] = [133, 121];
- : q = 163151, for n = 6 and [q1, q2] = [163, 151];
- : q = 193181, for n = 7 and [q1, q2] = [193, 181];
- : q = 223211, for n = 8 and [q1, q2] = [223, 211] (...)

Note the chain of five successive primes (10391, 133121, 163151, 193181, 223211) obtained for n from 4 to 8.

**Conjecture 2:**

If p is a prime which admits deconcatenation in two primes p1 and p2, both of the form  $6*k + 1$ , then there exist an infinity of primes q obtained concatenating q1 with q2, where  $q1 = 30*n + p1$ ,  $q2 = 30*n + p2$  and n positive integer.

**The sequence of q for p = 719 ([p1, p2] = [7, 19]):**

- : q = 6779, for n = 2 and [q1, q2] = [67, 79];
- : q = 127139, for n = 4 and [q1, q2] = [127, 139];
- : q = 217229, for n = 7 and [q1, q2] = [217, 229] (...)

**The sequence of q for p = 743 ([p1, p2] = [7, 43]):**

- : q = 67103, for n = 2 and [q1, q2] = [67, 103];
- : q = 127163, for n = 4 and [q1, q2] = [127, 163];
- : q = 187223, for n = 6 and [q1, q2] = [187, 223] (...)

**The sequence of q for p = 137 ([p1, p2] = [13, 7]):**

- : q = 4337, for n = 1 and [q1, q2] = [43, 37];
- : q = 223217, for n = 7 and [q1, q2] = [223, 217];
- : q = 253247, for n = 8 and [q1, q2] = [253, 247] (...)

**The sequence of q for p = 1319 ([p1, p2] = [13, 19]):**

- : q = 4349, for n = 1 and [q1, q2] = [43, 49];
- : q = 163169, for n = 5 and [q1, q2] = [163, 169];
- : q = 223229, for n = 7 and [q1, q2] = [223, 229] (...)

## 46. Four conjectures on the numbers $p \pm 1$ concatenated with 1 where $p$ primes of the form $30*k + 11$

**Abstract.** In this paper I state the following four conjectures. Let  $q$  be the number obtained concatenating to the right with 1 the numbers  $p - 1$ , where  $p$  primes of the form  $30*k + 11$ ; then: (I) there exist an infinity of primes  $q$ ; (II) there exist an infinity of semiprimes  $q = q_1*q_2$ , such that  $q_2 + q_1 - 1$  is prime. Let  $q$  be the number obtained concatenating to the right with 1 the numbers  $p + 1$ , where  $p$  primes of the form  $30*k + 11$ ; then: (III) there exist an infinity of primes  $q$ ; (IV) there exist an infinity of semiprimes  $q = q_1*q_2$ , such that  $q_2 - q_1 + 1$  is prime.

### Conjecture 1:

There exist an infinity of primes  $q$  obtained concatenating to the right with 1 the numbers  $p - 1$ , where  $p$  primes of the form  $30*k + 11$ .

#### The sequence of primes $q$ :

: 101, 401, 701, 1301, 1901, 2801, 4001, 7001, 10301, 10601, 11801, 13001,  
15101, 16001, 18701, 19001, 19301, 21101, 21401, 23801, 25301  
(...)

### Conjecture 2:

There exist an infinity of semiprimes  $q_1*q_2$  obtained concatenating to the right with 1 the numbers  $p - 1$ , where  $p$  primes of the form  $30*k + 11$ , such that  $q_2 + q_1 - 1$  is prime.

#### The sequence of semiprimes $q_1*q_2$ :

: 2501 = 41\*61, where  $61 + 41 - 1 = 101$ , prime;  
: 3101 = 7\*443, where  $443 + 7 - 1 = 449$ , prime;  
: 4601 = 43\*107, where  $107 + 43 - 1 = 149$ , prime;  
: 7601 = 11\*691, where  $691 + 1 - 1 = 701$ , prime;  
: 8201 = 59\*139, where  $139 + 59 - 1 = 197$ , prime;  
: 9701 = 89\*109, where  $109 + 89 - 1 = 197$ , prime;  
: 17201 = 103\*167, where  $167 + 103 - 1 = 269$ , prime;  
: 18101 = 23\*787, where  $787 + 23 - 1 = 809$ , prime;  
: 23501 = 71\*331, where  $331 + 71 - 1 = 401$ , prime;  
: 24401 = 13\*1877, where  $331 + 71 - 1 = 1889$ , prime;  
(...)  
: 9617567101 = 2521\*3814981, where  $3814981 + 2521 - 1 = 3817501$ , prime;  
(...)

### Conjecture 3:

There exist an infinity of primes  $q$  obtained concatenating to the right with 1 the numbers  $p + 1$ , where  $p$  primes of the form  $30*k + 11$ .





## **47. Primes obtained deconcatenating with 1 or with 01 the Poulet numbers of the form $30*k + 1$ or $300*k + 1$**

**Abstract.** In this paper I state the following three conjectures: (I) There exist an infinity of primes  $q$  obtained deconcatenating to the right with 1 the Poulet numbers of the form  $30*k + 1$  then subtracting 1 (example: from  $P = 997465414921$  is obtained  $q = 99746541491$ ); (II) There exist an infinity of primes  $q$  obtained deconcatenating to the right with 1 the Poulet numbers of the form  $30*k + 1$  then adding 1 (example: from  $P = 996881835961$  is obtained  $q = 99688183597$ ); (III) There exist an infinity of primes  $q$  obtained deconcatenating to the right with 01 the Poulet numbers of the form  $300*k + 1$  then subtracting 1 (example: from  $P = 999666754801$  is obtained  $q = 9996667547$ ).

### **Conjecture 1:**

There exist an infinity of primes  $q$  obtained deconcatenating to the right with 1 the Poulet numbers of the form  $30*k + 1$  then subtracting 1 (example: from  $P = 997465414921$  is obtained  $q = 99746541491$ ).

**The sequence of primes  $q$ :**

: 269, 281, 467, 659 (...) 99768766931, 99787971653, 99797119967, 99801898811, 99852315947, 99930863801, 99985731071, 99746541491 (...),  
obtained from  $P = 2701, 2821, 4681, 6601$  (...) 997687669321, 997879716541, 997971199681, 998018988121, 998523159481, 999308638021, 999857310721, 997465414921 (...)

### **Conjecture 2:**

There exist an infinity of primes  $q$  obtained deconcatenating to the right with 1 the Poulet numbers of the form  $30*k + 1$  then adding 1 (example: from  $P = 996881835961$  is obtained  $q = 99688183597$ ).

**The sequence of primes  $q$ :**

: 271, 283, 547, 671 (...) 99754190929, 99804924937, 99895342933, 99940765783, 99986301829, 99703138417, 99702731311, 99688183597 (...),  
obtained from  $P = 2701, 2821, 5461, 6601$  (...) 997541909281, 998049249361, 998953429321, 999407657821, 999863018281, 997031384161, 997027313101, 996881835961 (...)

### **Conjecture 3:**

There exist an infinity of primes  $q$  obtained deconcatenating to the right with 01 the Poulet numbers of the form  $300*k + 1$  then subtracting 1 (example: from  $P = 999666754801$  is obtained  $q = 9996667547$ ).

**The sequence of primes  $q$ :**

: 2039 (...) 9968306903, 9978038783, 9979667011, 9988536011, 9996667547 (...),  
obtained from  $P = 204001$  (...) 996830690401, 997803878401, 997966701201, 998853601201, 999666754801 (...)

## **48. Primes obtained deconcatenating with a group of k digits of 0 the factorial numbers then adding or subtracting 1**

**Abstract.** In this paper I state the following two conjectures: (I) For any k non-null positive integer there exist a sequence having an infinity of prime terms obtained deconcatenating to the right with a group with k digits of 0 the factorial numbers and adding 1 to the resulted number; (II) for any k non-null positive integer there exist a sequence having an infinity of prime terms obtained deconcatenating to the right with a group with k digits of 0 the factorial numbers and subtracting 1 from the resulted number. It is worth noting the pair of twin primes having 49 digits each obtained for k = 9: (5502622159812088949850305428800254892961651752959, 5502622159812088949850305428800254892961651752961).

**The sequence of factorial numbers  $n! = 1*2*3*...*n$  (A000142 in OEIS):**

: 1, 1, 2, 6, 24, 120, 720, 5040, 40320, 362880, 3628800, 39916800, 479001600, 6227020800, 87178291200, 1307674368000, 20922789888000, 355687428096000, 6402373705728000, 121645100408832000, 2432902008176640000, 51090942171709440000, 1124000727777607680000 (...)

### **Conjecture I:**

For any k non-null positive integer there exist a sequence having an infinity of prime terms p obtained deconcatenating to the right with a group with k digits of 0 the factorial numbers and adding 1 to the resulted number.

**The sequence of primes p for k = 1:**

: 13, 73, 3991681, 2585201673888497664001, 40329146112660563558400001, 1376375309122634504631597958158090240000001 (...)

**The sequence of primes p for k = 2:**

: 62270207, 871782911, 24329020081766401 (...)

**The least prime p for the following values of k:**

: 1307674369, for k = 3;  
 : 243290200817663, for k = 4;  
 : 2652528598121910586363084801, for k = 5;  
 : 37199332678990121746799944815083521, for k = 7;  
 : 8159152832478977343456112695961158942721, for k = 8;  
 : 5502622159812088949850305428800254892961651752961, for k = 9.

### **Conjecture II:**

For any k non-null positive integer there exist a sequence having an infinity of prime terms p obtained deconcatenating to the right with a group with k digits of 0 the factorial numbers and subtracting 1 from the resulted number.

**The sequence of primes p for k = 1:**

: 11, 71, 503, 622702079, 35568742809599, 3048883446117136050150399999  
(...)

**The sequence of primes p for k = 2:**

: 871782913, 3556874280959, 64023737057279, 510909421717094399,  
86833176188118864955181944012799999 (...)

**The least prime p for the following values of k:**

: 121645100408831, for k = 3;  
: 243290200817663, for k = 4;  
: 4032914611266056355841, for k = 5;  
: 304888344611713860501503, for k = 6;  
: 52302261746660111176000722410007429119, for k = 7;  
: 137637530912263450463159795815809023, for k = 8.  
: 5502622159812088949850305428800254892961651752959, for k = 9.

**Observation:**

It is worth noting the pair of twin primes having 49 digits each obtained for k = 9:  
(5502622159812088949850305428800254892961651752959,  
5502622159812088949850305428800254892961651752961).

## 49. Primes obtained deconcatenating with a group of k digits of 0 the fibonorial numbers then adding or subtracting 1

**Abstract.** In this paper I state the following two conjectures: (I) For any k positive integer there exist a sequence having an infinity of prime terms obtained deconcatenating to the right with a group with k digits of 0 the fibonorial numbers and adding 1 to the resulted number; (II) for any k non-null positive integer there exist a sequence having an infinity of prime terms obtained deconcatenating to the right with a group with k digits of 0 the fibonorial numbers and subtracting 1 from the resulted number. It is known that fibonorial numbers are defined as the products of nonzero Fibonacci numbers.

**The sequence of fibonorial numbers** (A003266 in OEIS):

: 1, 1, 2, 6, 30, 240, 3120, 65520, 2227680, 122522400, 10904493600,  
1570247078400, 365867569267200, 137932073613734400,  
84138564904377984000, 83044763560621070208000,  
132622487406311849122176000, 342696507457909818131702784000 (...)

### Conjecture I:

For any k positive integer there exist a sequence having an infinity of prime terms p obtained deconcatenating to the right with a group with k digits of 0 the fibonorial numbers and adding 1 to the resulted number.

**The sequence of primes p for k = 0:**

: 2, 3, 7, 31, 241, 3121, 65521, 137932073613734399 (...)

**The sequence of primes p for k = 1:**

: 313, 6553, 1879127177606120717127879344567470740879360001 (...)

**The sequence of primes p for k = 2:**

: 841385649043779841 (...)

**The sequence of primes p for k = 3:**

: 1879127177606120717127879344567470740879361 (...)

### Conjecture II:

For any k positive integer there exist a sequence having an infinity of prime terms p obtained deconcatenating to the right with a group with k digits of 0 the fibonorial numbers and subtracting 1 from the resulted number.

**The sequence of primes p for k = 0:**

: 5, 29, 239, 3119, 65519, 84138564904377983999 (...)

**The sequence of primes  $p$  for  $k = 1$ :**

: 2, 23, 311, 6551, 13793207361373439 (...)

**The sequence of primes  $p$  for  $k = 2$ :**

: 1225223, 3658675692671 (...)

**The sequence of primes  $p$  for  $k = 3$ :**

: 53850147528658601390733638377270009021379819519 (...)

## 50. Primes obtained concatenating two consecutive primorial numbers then adding or subtracting 1

**Abstract.** In this paper I state the following two conjectures: (I) There exist an infinity of primes obtained concatenating two consecutive primorial numbers and adding 1 to the resulted number; example: concatenating the tenth and eleventh primorials then adding 1 is obtained the prime 6469693230200560490131; (II) There exist an infinity of primes obtained concatenating two consecutive primorial numbers and subtracting 1 from the resulted number; example: concatenating the ninth and tenth primorials then subtracting 1 is obtained the prime 2230928706469693229.

**The sequence of primorial numbers:**  
(A002110 in OEIS)

: 1, 2, 6, 30, 210, 2310, 30030, 510510, 9699690, 223092870, 6469693230, 200560490130, 7420738134810, 304250263527210, 13082761331670030, 614889782588491410, 32589158477190044730, 1922760350154212639070 (...)

### Conjecture I:

There exist an infinity of primes  $p$  obtained concatenating two consecutive primorial numbers and adding 1 to the resulted number; example: concatenating the tenth and eleventh primorials then adding 1 is obtained the prime 6469693230200560490131.

**The sequence of primes  $p$ :**

: 13, 631, 30211, 2102311, 231030031, 9699690223092871, 6469693230200560490131, 7420738134810304250263527211 (...)

### Conjecture 2:

There exist an infinity of primes  $p$  obtained concatenating two consecutive primorial numbers and subtracting 1 from the resulted number; example: concatenating the ninth and tenth primorials then subtracting 1 is obtained the prime 2230928706469693229.

**The sequence of primes  $p$ :**

: 11, 5105109699689, 2230928706469693229 (...)

### Observation:

The numbers obtained this way are products of very few prime factors; for instance, the numbers  $7858321551080267055879090557940830126698960967415390 \pm 1$  obtained concatenating the nineteenth and twentieth primorials then adding/subtracting 1 are products of two, respectively three prime factors.

## 51. Primes obtained concatenating $30 \cdot p$ with $30 \cdot q$ then adding or subtracting 1, where $p$ and $q = p + 6$ primes

**Abstract.** In this paper I state the following three conjectures: let  $[p, q]$  be a pair of sexy primes ( $q = p + 6$ ); then: (I) there exist an infinity of primes obtained concatenating  $30 \cdot p$  with  $30 \cdot q$  and adding 1 to the resulted number; example: for  $[p, q] = [23, 29]$ , the number 690871 is prime; (II) there exist an infinity of primes obtained concatenating  $30 \cdot p$  with  $30 \cdot q$  and subtracting 1 from the resulted number; example: for  $[p, q] = [23, 29]$ , the number 690869 is prime; (III) there exist an infinity of pairs of twin primes obtained concatenating  $30 \cdot p$  with  $30 \cdot q$  and adding/subtracting 1 from the resulted number; example: for  $[p, q] = [101, 107]$ , the numbers 30303209 and 30303211 are primes.

**The sequence of primes  $p$  such that  $q = p + 6$  is also prime:**  
(A023201 in OEIS)

: 5, 7, 11, 13, 17, 23, 31, 37, 41, 47, 53, 61, 67, 73, 83, 97, 101, 103, 107, 131, 151, 157, 167, 173, 191, 193, 223, 227, 233, 251, 257, 263, 271, 277, 307, 311, 331, 347, 353, 367, 373, 383, 433, 443, 457, 461, 503, 541, 557, 563, 571, 587, 593, 601, 607, 613, 641, 647 (...)

### Conjecture I:

There exist an infinity of primes  $r$  obtained concatenating  $30 \cdot p$  with  $30 \cdot q$  and adding 1 to the resulted number, where  $[p, q]$  is a pair of sexy primes; example: for  $[p, q] = [23, 29]$ , the number 690871 is prime.

**The sequence of primes  $r$ :**

: 210391, 510691, 690871, 11101291, 15901771, 30303211, 47104891, 57905971, 66906871, 93309511, 993010111, 1383014011, 1509015271, 1803018211, 1923019411 (...),  
obtained for  $[p, q] = [7, 13], [17, 23], [23, 29], [37, 43], [53, 59], [101, 107], [157, 163], [193, 199], [223, 229], [311, 317], [331, 337], [461, 467], [503, 509], [601, 607], [641, 647]...$

### Conjecture II:

There exist an infinity of primes  $r$  obtained concatenating  $30 \cdot p$  with  $30 \cdot q$  and subtracting 1 from the resulted number, where  $[p, q]$  is a pair of sexy primes; example: for  $[p, q] = [23, 29]$ , the number 690869 is prime.

**The sequence of primes  $r$ :**

: 150329, 330509, 690869, 12301409, 30303209, 32103389, 50105189, 66906869, 68106989, 69907169, 75307709, 78908069, 81308309, 1671016889, 1761017789, 1803018209, 1821018389, 1923019409 (...),



obtained for  $[p, q] = [5, 11], [11, 17], [23, 29], [41, 47], [101, 107], [107, 113], [167, 173], [223, 229], [227, 233], [233, 239], [251, 257], [263, 269], [271, 277], [557, 563], [587, 593], [601, 607], [607, 613], [641, 647]...$

### **Conjecture III:**

There exist an infinity of pairs of twin primes obtained concatenating  $30 \cdot p$  with  $30 \cdot q$  and adding/subtracting 1 from the resulted number, where  $[p, q]$  is a pair of sexy primes; example: for  $[p, q] = [101, 107]$ , the numbers 30303211 and 30303209 are primes.

### **The sequence of such pairs of twin primes:**

:  $[690869, 690871], [30303209, 30303211], [66906869, 66906869], [1803018209, 1803018211], [1923019409, 1923019411]...$   
obtained for  $[p, q] = [23, 29], [101, 107], [223, 229], [601, 607], [641, 647]...$

## ANNEX A

### Summary of the methods we have used in order to obtain Smarandache-Coman sequences

**Note:** I shall use the notation  $a(n)$  for a term of a Smarandache concatenated sequence and  $b(n)$  for a term of a Smarandache-Coman sequence.

(1)

SC(n) is defined as follows:  $b(n) = a(n+1) - a(n) - 2$  if the last digit of the term  $a(n+1)$  is even and  $b(n) = a(n+1) - a(n) + 2$  if the last digit of the term  $a(n+1)$  is odd.

I applied this method to the terms of *Consecutive numbers sequence* obtaining the following terms of SC(n) (primes by definition): 113, 1109, 11113, 111109, 11111113, 12222222119, 122222221210099 (...)

(2)

SC(n) is defined as follows:  $b(n) = a(n)1$ , i.e. the terms of the Smarandache sequence concatenated to the right with the number 1.

I applied this method to the terms of *Consecutive numbers sequence* obtaining the following terms of SC(n) (primes by definition): 11, 1231, 1234567891, 12345678910111, 123456789101112131415161, 12345678910111213141516171819202122232425261 (...)

I also applied this method to the terms of *Reverse sequence* obtaining the following terms of SC(n) (primes by definition): 11, 211, 876543211, 9876543211, 222120191817161514131211109876543211 (...)

I also applied this method to the terms of *Consecutive odd sequence* obtaining the following terms of SC(n) (primes by definition): 11, 131, 13579111315171, 1357911131517191, 13579111315171921231, 13579111315171921232527291 (...)

I also applied this method to the terms of *Concatenated  $n2*n$  sequence* obtaining the following terms of SC(n) (primes by definition): 241, 5101, 6121, 8161, 9181, 12241, 14281, 17341, 19381 (...)

I also applied this method to the terms of "*Concatenated  $nn^2$* " sequence obtaining the following terms of SC(n) (primes by definition): 241, 6361, 8641, 9181, 111211, 121441, 298411 (...)

(3)

SC(n) is defined as follows:  $b(n) = a(n+1) + a(n) - S(a(n+1)) - S(a(n)) + 2$ , where  $S(a(n))$  is the sum of the digits of the term  $a(n)$ .

I applied this method to the terms of *Consecutive odd sequence* obtaining the following terms of SC(n) (primes by definition): 11, 137, 14897, 1371431, 13714902317 (...)

(4)

SC(n) is defined as follows:  $b(n) = a(n+1) + a(n) - S(a(n+1)) - S(a(n)) + 1$ , where  $S(a(n))$  is the sum of the consecutive even numbers which form the term  $a(n)$ .

I applied this method to the terms of *Consecutive even sequence* obtaining the following terms of SC(n) (primes by definition): 1 19, 2683, 249229, 2492782129 (....)

(5)

SC(n) is defined as follows:  $b(n) = 3a(n)$ , i.e. the terms of the Smarandache sequence concatenated to the left with the number 3.

I applied this method to the terms of *Concatenated prime sequence* obtaining the following terms of SC(n) (primes by definition): 3235711, 323571113171923, 32357111317192329313741 (....)

(6)

SC(n) is defined as follows:  $b(n) = 2*a(n) - 1$ .

I applied this method to the terms of *Back concatenated odd sequence* obtaining the following terms of SC(n) (primes by definition): 61, 1061, 15061, 262395061 (....)

I also applied this method to the terms of *Generalized  $nk*n$  sequence* obtaining the following terms of SC(n) (primes by definition): 47, 962431, 1020304049, 14284256708497, 1632486480971327 (....)

(7)

SC(n) is defined as follows:  $b(n) = a(n) - 1$ .

I applied this method to the terms of *Back concatenated even sequence* obtaining the following terms of SC(n) (primes by definition): 41, 641, 8641, 18161412108641 (....)

I also applied this method to the terms of "*n concatenated n times*" sequence obtaining the following terms of SC(n) (primes by definition): 43, 8887, 111109, 1333331 (....)

(8)

SC(n) is defined as follows:  $b(n) = 2*a(n) + 1$ .

I applied this method to the terms of *Concatenated odd square sequence* obtaining the following terms of SC(n) (primes by definition): 3851, 38509963, 38509962242338451 (....)

(9)

SC(n) is defined as follows:  $b(n) = a(n) + 5$ .

I applied this method to the terms of *Concatenated even square sequence* obtaining the following terms of SC(n) (primes by definition): 421, 41641, 4163669 (....)

(10)

SC(n) is defined as follows:  $b(n) = a(n)(a(n+1) + 1)$ , i.e. b(n) is obtained concatenating the term a(n) with the value of the term a(n+1) added to 1.

I applied this method to the terms of *Concatenated even square sequence* obtaining the following terms of SC(n) (primes by definition): 41641637, 41636641001444163664100144197 (...)

(11)

SC(n) is defined as follows:  $b(n) = a(n)9$ , i.e. the terms of the Smarandache sequence concatenated to the right with 9.

I applied this method to the terms of *Concatenated triangular numbers sequence* obtaining the following terms of SC(n) (primes by definition): 19, 139, 13610159 (...)

(12)

SC(n) is defined as follows:  $b(n) = a(n+1) - a(n)$ .

I applied this method to the terms of "*n concatenated n times*" sequence obtaining the following terms of SC(n) (primes by definition): 311, 4111, 611111 (...)

(13)

SC(n) is defined as follows:  $b(n) = m(n)/6 + 1$ , where m(n) is the number obtained concatenating a(n) with a(n+1) then with a(n+2).

I applied this method to the terms of "*Concatenated n2\*n*" sequence obtaining the following terms of SC(n) (primes by definition): 20407, 40609, 102119137 (...)

(14)

SC(n) is defined as follows:  $b(n) = m - 1$ , where m(n) is the number obtained concatenating a(n) with a(n+1).

I applied this method to the terms of "*Concatenated n2\*n*" sequence obtaining the following terms of SC(n) (primes by definition): 1223, 510611, 612713, 9181019, 14281529 (...)

(15)

SC(n) is defined as follows: b(n) are obtained concatenating both to the left and to the right the terms a(n) with 1.

I applied this method to the terms of "*Concatenated n2\*n*" sequence obtaining the following terms of SC(n) (primes by definition): 1361, 1481, 15101, 19181, 112241, 114281, 115301, 118361 (...)

I also applied this method to the terms of “Concatenated  $nn^2$ ” sequence obtaining the following terms of SC(n) (primes by definition): 16361, 17491, 111211, 1183241, 1266761, 1287841 (...)

(16)

SC(n) is defined as follows:  $b(n) = a(n) + n + 1$ .

I applied this method to the terms of “Concatenated  $nn^2$ ” sequence obtaining the following terms of SC(n) (primes by definition): 13, 43, 421, 643, 757, 991, 10111, 12157, 15241, 13183 (...)

(17)

SC(n) is defined as follows: b(n) are obtained concatenating to the right the terms a(n) with 11.

I applied this method to the terms of “Concatenated  $nn^2$ ” sequence obtaining the following terms of SC(n) (primes by definition): 2411, 3911, 41611, 52511, 63611, 1419611, 1522511, 1728911 (...)

(18)

SC(n) is defined as follows:  $b(n) = a(n) + a(n+1) - 1$ .

I applied this method to the terms of *Smarandache-Wellin sequence* obtaining the following terms of SC(n) (primes by definition): 257, 2591, 23806823, 23806824303642526067, 23806824303642526068877 (...)

I applied this method to the terms of *Concatenated odd sequence* obtaining the following terms of SC(n) (primes by definition): 37, 36067, 360682429, 360682430364251, 36068243036425260687883, 360682430364252606878849099, 3606824303642526068788491011321293943 (...)

(19)

SC(n) is defined as follows:  $b(n) = 1a(n)$ , i.e. the terms of the Smarandache sequence concatenated to the left with the number 1.

I applied this method to the terms of *Smarandache reverse sequence* obtaining the following terms of SC(n) (primes by definition): 11, 1321, 14321, 154321, 113121110987654321, 11413121110987654321 (...)

I also applied this method to the terms of *Smarandache back concatenated odd sequence* obtaining the following terms of SC(n) (primes by definition): 131, 1531, 151494745434139373533312927252321191715131197531 (...)

I also applied this method to the terms of *Smarandache back concatenated square sequence* obtaining the following terms of SC(n) (primes by definition): 11, 12516941, 16449362516941, 1100816449362516941 (...)

## ANNEX B

### List of few Smarandache concatenated sequences liable to lead to Smarandache-Coman sequences

**Note:** I will not list the Smarandache sequences which have been already used in this book in order to obtain Smarandache-Coman sequences.

(1)

#### The back concatenated prime sequence

$S(n)$  is defined as the sequence obtained through the concatenation of the first  $n$  primes, in reverse order.

The first ten terms of the sequence (A038394 in OEIS):

2, 32, 532, 7532, 117532, 13117532, 1713117532, 191713117532, 23191713117532, 2923191713117532.

(2)

#### The concatenated square sequence

$S(n)$  is defined as the sequence obtained through the concatenation of the first  $n$  squares.

The first ten terms of the sequence (A019521 in OEIS):

1, 14, 149, 14916, 1491625, 149162536, 14916253649, 1491625364964, 149162536496481, 149162536496481100.

(3)

#### The concatenated cubic sequence

$S(n)$  is defined as the sequence obtained through the concatenation of the first  $n$  cubes.

The first ten terms of the sequence (A019521 in OEIS):

1, 18, 1827, 182764, 182764125, 182764125216, 182764125216343, 182764125216343512, 182764125216343512729, 1827641252163435127291000.

(4)

#### The concatenated triangular numbers sequence

$S(n)$  is defined as the sequence obtained through the concatenation of the first  $n$  triangular numbers:  $1(2^3)(3^3)\dots(n^3)$ .

The first ten terms of the sequence (A078795 in OEIS):

1, 13, 136, 13610, 1361015, 136101521, 13610152128, 1361015212836, 136101521283645, 13610152128364555.

(5)

### The permutation sequence

$S(n)$  is defined as the sequence of numbers obtained through concatenation and permutation in the following way:  $13\dots(2n-3)(2n-1)(2n)(2n-2)(2n-4)\dots42$ .

The first seven terms of the sequence (A007943 in OEIS):

12, 1342, 135642, 13578642, 13579108642, 135791112108642, 1357911131412108642, 13579111315161412108642.

(6)

### The pierced chain sequence

The sequence obtained in the following way: the first term is 101 and every next term is obtained through concatenation of the previous term with the group of digits 0101.

The first six terms of the sequence (A031982 in OEIS):

101, 1010101, 10101010101, 101010101010101, 1010101010101010101, 10101010101010101010101.

(7)

### The concatenated Fibonacci sequence

The sequence obtained through concatenation of the terms of Fibonacci sequence.

The first ten terms of the sequence (A019523 in OEIS):

1, 11, 112, 1123, 11235, 112358, 11235813, 1123581321, 112358132134, 11235813213455.

(8)

### The symmetric numbers sequence

$S(n)$  is defined as the sequence obtained through the concatenation in the following way: if  $n$  is odd, the  $n$ -th term of the sequence is obtained through concatenation  $123\dots(m-1)m(m-1)\dots321$ , where  $m = (n+1)/2$ ; if  $n$  is even, the  $n$ -th term of the sequence is obtained through concatenation  $123\dots(m-1)mm(m-1)\dots321$ , unde  $m = n/2$ .

The first nine terms of the sequence (A007907 in OEIS):

1, 11, 121, 1221, 12321, 123321, 1234321, 12344321, 123454321, 1234554321.

(9)

### The antisymmetric numbers sequence

$S(n)$  is defined as the sequence obtained through the concatenation in the following way:  $12\dots(n)12\dots(n)$ .

The first ten terms of the sequence (A019524 in OEIS):

11, 1212, 123123, 12341234, 1234512345, 123456123456, 12345671234567,  
1234567812345678, 123456789123456789.

**(10)**

### **The mirror sequence**

$S(n)$  is defined as the sequence obtained through the concatenation in the following way:  $n(n - 1) \dots 32123 \dots (n - 1)n$ .

The first ten terms of the sequence (A007942 in OEIS):

1, 212, 32123, 4321234, 543212345, 65432123456, 7654321234567, 876543212345678,  
98765432123456789, 109876543212345678910.

### **Open problem**

Which from the sequences from above, combined with the methods summarized in the Annex A, are liable to lead to Smarandache-Coman sequences?



The purpose of this book is to show that the method of concatenation can be a powerful tool in number theory and, in particular, in obtaining possible infinite sequences of primes. Part One of this book, “Primes in Smarandache concatenated sequences and Smarandache-Coman sequences of primes” , contains 12 papers on various sequences of primes that are distinguished among the terms of the well known Smarandache concatenated sequences. The sequences presented in this part are related to concatenation in three different ways: the sequence is obtained by the method of concatenation but the operation applied on its terms is some other arithmetical operation; the sequence is not obtained by concatenation but the operation applied on its terms is concatenation or both the sequence and the operation applied on its terms (in order to find sequences of primes) are using the method of concatenation. Part Two of this book, “Sequences of primes obtained by the method of concatenation” brings together 51 articles which aim, using the mentioned method, to highlight sequences of numbers which are rich in primes or are liable to lead to large primes. The method of concatenation is applied to different classes of numbers, e.g. Poulet numbers, twin primes, reversible primes, triangular numbers, repdigits, factorial numbers, fibonorial numbers, primorial numbers in order to obtain sequences of primes.

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