# Correlation coefficients of simplified neutrosophic sets and their multiple attribute decision-making method

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**Abstract**

The paper presents two new correlation coefficients of simplified neutrosophic sets (SNSs) as the further extension of the correlation coefficient of single valued neutrosophic sets (SVNSs) and investigates their properties. Then a multiple attribute decision-making method is proposed based on the weighted correlation coefficients of SNSs, in which the evaluation information for alternatives with respect to attributes is represented by the form of simplified neutrosophic values under simplified neutrosophic environment. We utilize the weighted correlation coefficients between each alternative and the ideal alternative to rank the alternatives and to determine the best one(s). Finally, an illustrative example demonstrates the application and effectiveness of the proposed decision-making method.

**Keywords**: Neutrosophic set; Simplified neutrosophic set; Correlation coefficient; Multiple attribute decision making

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## 1. Introduction

Neutrosophic set [1], which was proposed by Smarandache in 1999, is a powerful general formal framework which generalizes the concept of the classic set, fuzzy set, intuitionistic fuzzy set, interval valued fuzzy set, interval valued intuitionistic fuzzy set, paraconsistent set, dialetheist set, paradoxist set, tautological set [1]. Then, it can handle not only incomplete information but also the indeterminate information and inconsistent information which exist commonly in real situations. In a neutrosophic set $A$ in $X$, a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$ can be expressed independently. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ in the neutrosophic set $A$ are real standard or nonstandard subsets of $]^-0, 1^+[$, i.e., $T_A(x) \subseteq\ ]^-0, 1^+[$, $I_A(x) \subseteq\ ]^-0, 1^+[$, and $F_A(x) \subseteq\ ]^-0, 1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, i.e. $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$. However, the neutrosophic set generalizes the above mentioned sets from philosophical point of view. From scientific or engineering point of view, it is difficult to apply in real scientific and engineering areas. Therefore, Wang et al. [2, 3] proposed an interval neutrosophic set (INS) and a single valued neutrosophic set (SVNS), respectively, which are an instance of neutrosophic set, and provided the set-theoretic operators and various properties of SVNSs and INSs. SVNSs and INSs can be used for the scientific and engineering applications because the SVNS theory and the INS theory are valuable in handling uncertain, imprecision and inconsistent information and easily reflect the ambiguous nature of subjective judgments. After that, Ye [4] presented the correlation coefficient of SVNSs based on the extension of the correlation of intuitionistic fuzzy sets and proved that the cosine similarity measure of SVNSs is a special case of the correlation coefficient of SVNSs, and then applied it to decision-making problems with single valued neutrosophic information. Ye [5] proposed a cross-entropy measure for SVNSs and applied it to decision-making problems under single valued neutrosophic environment. On the other hand, Ye [6] also introduced the Hamming and Euclidean distances between INSs and their similarity measures, and then applied them to decision-making problems in interval neutrosophic setting. Furthermore, Ye [7] presented a concept of a simplified neutrosophic set (SNS), which is a subclass of the neutrosophic set and encompasses that of a SVNS and an INS as special cases of a SNS, and defined some operations of SNSs, and then developed a simplified neutrosophic weighted averaging (SNWA) operator, a simplified neutrosophic weighted geometric (SNWG) operator, and a multicriteria decision-making method based on the SNWA and SNWG operators and the cosine measure of SNSs under simplified neutrosophic environment.

As mentioned above, SNSs are the extension of SVNSs and INSs and suitable for capturing imprecise, uncertain, and inconsistent information in multiple attribute decision making. Then, correlation coefficients are one of important tools in many scientific and engineering applications. Therefore, motivated by [4], the purposes of this paper are to propose two correlation coefficients of SNSs as a further generalization of the correlation coefficient of SVNSs proposed by Ye [4] and to develop a multiple attribute decision making method using the proposed correlation coefficients of SNSs under simplified neutrosophic environment. An illustrative example demonstrates the application and effectiveness of the proposed decision-making method.

The rest of the paper is organized as follows. Section 2 briefly describes some concepts of

SNSs and the correlation coefficient of SVNSs. Section 3 proposes two correlation coefficients for SNSs and investigates their properties. Section 4 establishes a decision-making approach based on the proposed correlation coefficients of SNSs. An illustrative example validating our approach and the comparative analysis are given in Section 5. Section 6 contains a conclusion and future research.

## 2. Preliminaries

### *2.1. Simplified neutrosophic set*

Smarandache [1] presented the neutrosophic set from philosophical point of view and gave the following definition of a neutrosophic set.

**Definition 1** [1]. Let $X$ be a space of points (objects), with a generic element in $X$ denoted by $x$. A neutrosophic set $A$ in $X$ is characterized by a truth-membership function $T_A(x)$, a indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$, i.e., $T_A(x)$: $X \to ]^-0, 1^+[$, $I_A(x)$: $X \to ]^-0, 1^+[$, and $F_A(x)$: $X \to ]^-0, 1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

Obviously, it is difficult to apply the neutrosophic set to practical problems. Therefore, Ye [7] introduced the concept of a SNS, which is a subclass of the neutrosophic set.

**Definition 2** [7]. Let $X$ be a space of points (objects), with a generic element in $X$ denoted by $x$. A neutrosophic set $A$ in $X$ is characterized by a truth-membership function $T_A(x)$, a indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. If the functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are singleton subintervals/subsets in the real standard [0, 1], that is $T_A(x)$: $X \to [0, 1]$, $I_A(x)$: $X \to [0, 1]$, and $F_A(x)$: $X \to [0, 1]$. Then, a simplified neutrosophic set $A$ is defined by

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}.$$

It is a subclass of neutrosophic sets and includes the concepts of INSs and SVNSs.

When we use the SNS whose $T_A(x)$, $I_A(x)$ and $F_A(x)$ values are single points in the real standard [0, 1] instead of subintervals/subsets in the real standard [0, 1], the SNS reduce to the SVNS which was proposed by Wang et al. [3]. Thus, each SNS can be described by three real numbers in the real unit interval [0, 1]. Therefore, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x) \in [0, 1]$ satisfies the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$. In this case, we introduce the following definitions [3, 7].

**Definition 3**. A SNS $A$ is contained in the other SNS $B$, $A \subseteq B$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, and $F_A(x) \ge F_B(x)$ for every $x$ in $X$.

**Definition 4**. The complement of a SNS $A$ is denoted by $A^c$ and is defined as $T_A{}^c(x) = F_A(x)$, $I_A{}^c(x) = 1 - I_A(x)$, $F_A{}^c(x) = T_A(x)$ for any $x$ in $X$.

**Definition 5**. Two SNSs $A$ and $B$ are equal, written as $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$.

When we only consider three functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ in the SNS as subunitary intervals in the real standard [0, 1], the SNS reduce to the INS which was proposed by Wang et al. [2]. Thus, a SNS $A$ can be described by three interval numbers in the real unit interval [0, 1]. Therefore, for each point $x$ in $X$, there are the three interval pairs $T_A(x) = [\inf T_A(x), \sup T_A(x)] \subseteq [0, 1]$, $I_A(x) = [\inf I_A(x), \sup I_A(x)] \subseteq [0, 1]$ and $F_A(x) = [\inf F_A(x), \sup F_A(x)] \subseteq [0, 1]$ and their sum satisfies the condition $0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3$ for any $x$ in $X$. In this case, we introduce the following definitions [2, 7].

**Definition 6**. The complement of a SNS $A$ is denoted by $A^c$ and is defined as $T_A{}^c(x) = F_A(x) = [\inf F_A(x), \sup F_A(x)]$, $I_A{}^c(x) = [1 - \sup I_A(x), 1 - \inf I_A(x)]$, $F_A{}^c(x) = T_A(x) = [\inf T_A(x), \sup T_A(x)]$ for any $x$ in $X$.

**Definition 7**. A SNS $A$ is contained in the other SNS $B$, $A \subseteq B$, if and only if $\inf T_A(x) \le \inf T_B(x)$, $\sup T_A(x) \le \sup T_B(x)$, $\inf I_A(x) \ge \inf I_B(x)$, $\sup I_A(x) \ge \sup I_B(x)$, $\inf F_A(x) \ge \inf F_B(x)$, and $\sup F_A(x) \ge \sup F_B(x)$ for any $x$ in $X$.

**Definition 8**. Two SNSs $A$ and $B$ are equal, written as $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$.

If for an SNS $A$ the lower and super end points of the three interval pairs $T_A(x) = [\inf T_A(x), \sup T_A(x)]$, $I_A(x) = [\inf I_A(x), \sup I_A(x)]$and $F_A(x) = [\inf F_A(x), \sup F_A(x)]$ for any $x$ in $X$ are identical, the SNS $A$ reduce to the SVNS $A$.

However, the INS $A$ and the SVNS $A$ belong to the SNS $A$. Then, this paper only considers the SNS whose $T(x)$, $I(x)$ and $F(x)$ values are interval numbers.

*2.2. Correlation coefficient of SVNSs*

Based on the extension of the correlation of intuitionistic fuzzy sets, Ye [4] defined the informational energy of a SVNS $A$, the correlation of two SVNSs $A$ and $B$, and the correlation coefficient of two SVNSs $A$ and $B$.

For a SVNS $A$ in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$, the informational energy of the SVNS $A$ is defined as

$$T(A) = \sum_{i=1}^{n} \left[ T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i) \right]. \quad (1)$$

For two SVNSs $A$ and $B$ in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$, then the correlation of the SVNSs $A$ and $B$ is defined as

$$C(A,B) = \sum_{i=1}^{n} \left[ T_A(x_i) \cdot T_B(x_i) + I_A(x_i) \cdot I_B(x_i) + F_A(x_i) \cdot F_B(x_i) \right]. \quad (2)$$

Therefore, the correlation coefficient of the SVNSs $A$ and $B$ is defined by the following formula:

$$K(A,B)=\frac{C(A,B)}{\left[T(A)\cdot T(B)\right]^{1/2}}=\frac{\sum_{i=1}^{n}\left[T_A(x_i)\cdot T_B(x_i)+I_A(x_i)\cdot I_B(x_i)+F_A(x_i)\cdot F_B(x_i)\right]}{\sqrt{\sum_{i=1}^{n}\left[T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i)\right]}\cdot\sqrt{\sum_{i=1}^{n}\left[T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i)\right]}}. \quad (3)$$

The correlation coefficient $K(A, B)$ satisfies the following properties [4]:

(1) $K(A, B) = K(B, A)$;

(2) $0 \le K(A,B) \le 1$;

(3) $K(A, B) = 1$, if $A = B$.

### 3. Correlation coefficients of SNSs

SNSs are a subclass of a neutrosophic set and a generalization of fuzzy sets and intuitionistic fuzzy sets, interval valued intuitionistic fuzzy sets, SVNSs, and INSs. To extend the correlation coefficient of SVNSs [4] to SNSs, we define the informational energy of a SNS, the correlation of two SNSs, and the correlation coefficient of two SNSs, which can be used in real scientific and engineering applications, in the following.

**Definition 9.** Let any SNS be $A=\{\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle \mid x_i \in X\}$ in the universe of discourse $X = \{x_1, x_2,\dots, x_n\}$, where $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i) \subseteq [0, 1]$ for every $x_i \in X$. Then, the informational energy of the SNS $A$ is defined as

$$E(A)=\sum_{i=1}^{n}\left[\inf T_A^2(x_i)+\inf I_A^2(x_i)+\inf F_A^2(x_i)+\sup T_A^2(x_i)+\sup I_A^2(x_i)+\sup F_A^2(x_i)\right]. \quad (4)$$

**Definition 10**. For two SNSs $A$ and $B$ in the universe of discourse $X = \{x_1, x_2,\dots, x_n\}$, the correlation of the SNSs $A$ and $B$ is defined as

$$N_1(A,B)=\sum_{i=1}^{n}\big[\inf T_A(x_i)\cdot\inf T_B(x_i)+\inf I_A(x_i)\cdot\inf I_B(x_i)+\inf F_A(x_i)\cdot\inf F_B(x_i)$$
$$+\sup T_A(x_i)\cdot\sup T_B(x_i)+\sup I_A(x_i)\cdot\sup I_B(x_i)+\sup F_A(x_i)\cdot\sup F_B(x_i)\big]. \quad (5)$$

It is obvious that the correlation of the SNSs *A* and *B* satisfies the following properties:

(1) $N_1(A, A) = E(A)$,

(2) $N_1(A, B) = N_1(B, A)$.

According to Definitions 9 and 10, we can derive the correlation coefficient for SNSs.

**Definition 11**. For two SNSs *A* and *B* in the universe of discourse $X = \{x_1, x_2,\ldots, x_n\}$, the correlation coefficient between two SNSs *A* and *B* is given by

$$M_1(A,B)=\frac{N_1(A,B)}{\left[N_1(A,A)\cdot N_1(B,B)\right]^{1/2}}$$
$$=\frac{\sum_{i=1}^{n}\begin{bmatrix}\inf T_A(x_i)\cdot\inf T_B(x_i)+\inf I_A(x_i)\cdot\inf I_B(x_i)+\inf F_A(x_i)\cdot\inf F_B(x_i)\\+\sup T_A(x_i)\cdot\sup T_B(x_i)+\sup I_A(x_i)\cdot\sup I_B(x_i)+\sup F_A(x_i)\cdot\sup F_B(x_i)\end{bmatrix}}{\left(\begin{array}{l}\sum_{i=1}^{n}\left[\inf T_A^2(x_i)+\inf I_A^2(x_i)+\inf F_A^2(x_i)+\sup T_A^2(x_i)+\sup I_A^2(x_i)+\sup F_A^2(x_i)\right]\\ \cdot\sum_{i=1}^{n}\left[\inf T_B^2(x_i)+\inf I_B^2(x_i)+\inf F_B^2(x_i)+\sup T_B^2(x_i)+\sup I_B^2(x_i)+\sup F_B^2(x_i)\right]\end{array}\right)^{1/2}}.$$

(6)

Thus, we can derive the following Theorem 1 from the correlation coefficient between two SNSs *A* and *B*.

**Theorem 1**. For two SVNSs *A* and *B* in the universe of discourse $X = \{x_1, x_2,\ldots, x_n\}$, the correlation coefficient $M_1(A, B)$ satisfies the following properties:

(4) $M_1(A, B) = M_1(B, A)$;

(5) $0 \le M_1(A,B) \le 1$;

(6) $M_1(A, B) = 1$, if $A = B$.

**Proof 1**:

(1) It is straightforward.

(2) The inequality $M_1(A, B) \geq 0$ is obvious. Below let us prove $M_1(A, B) \leq 1$:

$$\begin{aligned} N_1(A,B) = &[\inf T_A(x_1)\cdot \inf T_B(x_1) + \inf I_A(x_1)\cdot \inf I_B(x_1) + \inf F_A(x_1)\cdot \inf F_B(x_1) \\ &+ \sup T_A(x_1)\cdot \sup T_B(x_1) + \sup I_A(x_1)\cdot \sup I_B(x_1) + \sup F_A(x_1)\cdot \sup F_B(x_1)] \\ &+ [\inf T_A(x_2)\cdot \inf T_B(x_2) + \inf I_A(x_2)\cdot \inf I_B(x_2) + \inf F_A(x_2)\cdot \inf F_B(x_2) \\ &+ \sup T_A(x_2)\cdot \sup T_B(x_2) + \sup I_A(x_2)\cdot \inf I_B(x_2) + \inf F_A(x_2)\cdot \inf F_B(x_2)] \\ &+ \cdots + [\inf T_A(x_n)\cdot \inf T_B(x_n) + \inf I_A(x_n)\cdot \inf I_B(x_n) + \inf F_A(x_n)\cdot \inf F_B(x_n) \\ &+ \sup T_A(x_n)\cdot \sup T_B(x_n) + \sup I_A(x_n)\cdot \inf I_B(x_n) + \inf F_A(x_n)\cdot \inf F_B(x_n)] \end{aligned}.$$

Using the Cauchy-Schwarz inequality:

$$(x_1y_1 + x_2y_2 + \cdots + x_ny_n)^2 \leq (x_1^2 + x_2^2 + \cdots + x_n^2)\cdot(y_1^2 + y_2^2 + \cdots + y_n^2),$$

where $(x_1, x_2,\ldots, x_n) \in R^n$ and $(y_1, y_2,\ldots, y_n) \in R^n$, we obtain

$$\begin{aligned} [N_1(A,B)]^2 \leq &[\inf T_A^2(x_1) + \inf I_A^2(x_1) + \inf F_A^2(x_1) + \sup T_A^2(x_1) + \sup I_A^2(x_1) + \sup F_A^2(x_1)] \\ &+ [\inf T_A^2(x_2) + \inf I_A^2(x_2) + \inf F_A^2(x_2) + \sup T_A^2(x_2) + \sup I_A^2(x_2) + \sup F_A^2(x_2)] \\ &+ \cdots + [\inf T_A^2(x_n) + \inf I_A^2(x_n) + \inf F_A^2(x_n) + \sup T_A^2(x_n) + \sup I_A^2(x_n) + \sup F_A^2(x_n)] \\ &+ [\inf T_B^2(x_1) + \inf I_B^2(x_1) + \inf F_B^2(x_1) + \sup T_B^2(x_1) + \sup I_B^2(x_1) + \sup F_B^2(x_1)] \\ &+ [\inf T_B^2(x_2) + \inf I_B^2(x_2) + \inf F_B^2(x_2) + \sup T_B^2(x_2) + \sup I_B^2(x_2) + \sup F_B^2(x_2)]. \\ &+ \cdots + [\inf T_B^2(x_n) + \inf I_B^2(x_n) + \inf F_B^2(x_n) + \sup T_B^2(x_n) + \sup I_B^2(x_n) + \sup F_B^2(x_n)] \\ &= \sum_{i=1}^{n}\left[\inf T_A^2(x_i) + \inf I_A^2(x_i) + \inf F_A^2(x_i) + \sup T_A^2(x_i) + \sup I_A^2(x_i) + \sup F_A^2(x_i)\right] \\ &\cdot \sum_{i=1}^{n}\left[\inf T_B^2(x_i) + \inf I_B^2(x_i) + \inf F_B^2(x_i) + \sup T_B^2(x_i) + \sup I_B^2(x_i) + \sup F_B^2(x_i)\right] \\ &= N_1(A,A)\cdot N_1(B,B) \end{aligned}$$

Therefore

$$N_1(A,B) \leq [N_1(A,A)]^{1/2}\cdot[N_1(B,B)]^{1/2}.$$

Thus, $0 \leq M_1(A,B) \leq 1$.

(3) $A = B \Rightarrow \inf T_A(x_i) = \inf T_B(x_i)$, $\sup T_A(x_i) = \sup T_B(x_i)$, $\inf I_A(x_i) = \inf I_B(x_i)$ , $\sup I_A(x_i) = \sup I_B(x)$, $\inf F_A(x_i) = \inf F_B(x_i)$, and $\sup F_A(x_i) = \sup F_B(x_i)$ for any $x_i \in X \Rightarrow M_1(A, B) = 1$. □

Especially, when both the lower and super end points of the interval numbers of $T_A(x_i)$, $I_A(x_i)$ and $F_A(x_i)$ in the SNS $A$ and the lower and super end points of the interval numbers of $T_B(x_i)$, $I_B(x_i)$ and $F_B(x_i)$ in the SNS $B$ are identical for any $x_i$ in $X$, there are the three real numbers of $T_A(x_i)$, $I_A(x_i)$,

$F_A(x_i) \in [0, 1]$ in $A$ and the three real numbers of $T_B(x_i)$, $I_A(x_i)$, $F_B(x_i) \in [0, 1]$ in $B$. Thus, Eq. (6) reduces to Eq. (3). Therefore, the correlation coefficient of SVNSs is a special case of the correlation coefficient of SNSs.

As a generalization of the correlation coefficient used in interval intuitionistic fuzzy sets [8], we give another formula of the correlation coefficient of SNSs.

**Definition 12**. For two SNSs $A$ and $B$ in the universe of discourse $X = \{x_1, x_2,\ldots, x_n\}$, the correlation coefficient between the two SNSs $A$ and $B$ is defined by

$$M_2(A,B) = \frac{N_1(A,B)}{\max[N_1(A,A), N_1(B,B)]}$$
$$= \frac{\sum_{i=1}^{n}\begin{bmatrix} \inf T_A(x_i)\cdot\inf T_B(x_i) + \inf I_A(x_i)\cdot\inf I_B(x_i) + \inf F_A(x_i)\cdot\inf F_B(x_i) \\ + \sup T_A(x_i)\cdot\sup T_B(x_i) + \sup I_A(x_i)\cdot\sup I_B(x_i) + \sup F_A(x_i)\cdot\sup F_B(x_i) \end{bmatrix}}{\max\begin{pmatrix} \sum_{i=1}^{n}\left[\inf T_A^2(x_i) + \inf I_A^2(x_i) + \inf F_A^2(x_i) + \sup T_A^2(x_i) + \sup I_A^2(x_i) + \sup F_A^2(x_i)\right], \\ \sum_{i=1}^{n}\left[\inf T_B^2(x_i) + \inf I_B^2(x_i) + \inf F_B^2(x_i) + \sup T_B^2(x_i) + \sup I_B^2(x_i) + \sup F_B^2(x_i)\right] \end{pmatrix}} \quad (7)$$

**Theorem 2**. The correlation coefficient $M_2(A, B)$ follows the same properties listed in Theorem 1 as follows:

(1) $M_2(A, B) = M_2(B, A)$;

(2) $0 \le M_2(A,B) \le 1$;

(3) $M_2(A, B) = 1$, if $A = B$.

**Proof 2:**

The process to prove the properties (1) and (3) is analogous to that in Theorem 1 (omitted).

(2) The inequality $M_2(A, B) \ge 0$ is obvious. Now, we only prove $M_2(A, B) \le 1$. Based on the proof process of Theorem 1, we have

$$N_1(A,B) \le [N_1(A,A)]^{1/2}\cdot[N_1(B,B)]^{1/2},$$

and then

$$N_1(A,B) \le \max\left[N_1(A,A), N_1(B,B)\right].$$

Thus, $0 \le M_2(A,B) \le 1$. □

Especially, when both the lower and super end points of the interval numbers of $T_A(x_i)$, $I_A(x_i)$ and $F_A(x_i)$ in the SNS $A$ and the lower and super end points of the interval numbers of $T_B(x_i)$, $I_B(x_i)$ and $F_B(x_i)$ in the SNS $B$ are identical for any $x_i$ in $X$, there are the three real numbers of $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i) \in [0, 1]$ in $A$ and the three real numbers of $T_B(x_i)$, $I_A(x_i)$, $F_B(x_i) \in [0, 1]$ in $B$. Thus, Eq. (7) reduces to the following formula:

$$M_3(A,B) = \frac{C(A,B)}{\max\left[T(A),T(B)\right]} = \frac{\sum_{i=1}^{n}\left[T_A(x_i)\cdot T_B(x_i) + I_A(x_i)\cdot I_B(x_i) + F_A(x_i)\cdot F_B(x_i)\right]}{\max\left\{\sum_{i=1}^{n}\left[T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)\right], \sum_{i=1}^{n}\left[T_B^2(x_i) + I_B^2(x_i) + F_B^2(x_i)\right]\right\}}. \quad (8)$$

Obviously, it is another formula of the correlation coefficient between the SVNSs $A$ and $B$, which is a special case of the correlation coefficient between the SNSs $A$ and $B$.

However, the differences of importance are considered in the elements in the universe. Therefore, we need to take the weights of the elements $x_i$ ($i$ = 1, 2,…, $n$) into account. In the following, we develop two weighted correlation coefficients between SNSs.

Let $w_i$ be the weight for each element $x_i$ ($i$ = 1, 2,…, $n$), $w_i \in [0, 1]$, and $\sum_{i=1}^{n} w_i = 1$, then we have the following two weighted correlation coefficients between the SNSs $A$ and $B$, respectively, as follows:

$$M_4(A,B) = \frac{N_2(A,B)}{[N_2(A,A)\cdot N_2(B,B)]^{1/2}}$$
$$= \frac{\sum_{i=1}^{n} w_i \begin{bmatrix} \inf T_A(x_i)\cdot \inf T_B(x_i) + \inf I_A(x_i)\cdot \inf I_B(x_i) + \inf F_A(x_i)\cdot \inf F_B(x_i) \\ + \sup T_A(x_i)\cdot \sup T_B(x_i) + \sup I_A(x_i)\cdot \sup I_B(x_i) + \sup F_A(x_i)\cdot \sup F_B(x_i) \end{bmatrix}}{\left( \begin{array}{l} \sum_{i=1}^{n} w_i [\inf T_A^2(x_i) + \inf I_A^2(x_i) + \inf F_A^2(x_i) + \sup T_A^2(x_i) + \sup I_A^2(x_i) + \sup F_A^2(x_i)] \\ \cdot \sum_{i=1}^{n} w_i [\inf T_B^2(x_i) + \inf I_B^2(x_i) + \inf F_B^2(x_i) + \sup T_B^2(x_i) + \sup I_B^2(x_i) + \sup F_B^2(x_i)] \end{array} \right)^{1/2}} \tag{9}$$

$$M_5(A,B) = \frac{N_2(A,B)}{\max[N_2(A,A), N_2(B,B)]}$$
$$= \frac{\sum_{i=1}^{n} w_i \begin{bmatrix} \inf T_A(x_i)\cdot \inf T_B(x_i) + \inf I_A(x_i)\cdot \inf I_B(x_i) + \inf F_A(x_i)\cdot \inf F_B(x_i) \\ + \sup T_A(x_i)\cdot \sup T_B(x_i) + \sup I_A(x_i)\cdot \sup I_B(x_i) + \sup F_A(x_i)\cdot \sup F_B(x_i) \end{bmatrix}}{\max \left( \begin{array}{l} \sum_{i=1}^{n} w_i [\inf T_A^2(x_i) + \inf I_A^2(x_i) + \inf F_A^2(x_i) + \sup T_A^2(x_i) + \sup I_A^2(x_i) + \sup F_A^2(x_i)], \\ \sum_{i=1}^{n} w_i [\inf T_B^2(x_i) + \inf I_B^2(x_i) + \inf F_B^2(x_i) + \sup T_B^2(x_i) + \sup I_B^2(x_i) + \sup F_B^2(x_i)] \end{array} \right)} \tag{10}$$

If $w = (1/n, 1/n, \ldots, 1/n)^T$, then Eqs. (9) and (10) reduce to Eqs. (6) and (7), respectively. Note that both $M_4(A, B)$ and $M_5(A, B)$ also satisfy the three properties of Theorem 1.

**Theorem 3**. Let $w_i$ be the weight for each element $x_i$ ($i$ = 1, 2,…, $n$), $w_i \in [0, 1]$, and $\sum_{i=1}^{n} w_i = 1$, then the weighted correlation coefficient $M_4(A, B)$ defined in Eq. (9) satisfies the following properties:

(1) $M_4(A, B) = M_4(B, A)$;

(2) $0 \le M_4(A,B) \le 1$;

(3) $M_4(A, B) = 1$, if $A = B$.

Since the process to prove these properties is similar to that in Theorem 1, we do not repeat it here.

**Theorem 4**. Let $w_i$ be the weight for each element $x_i$ ($i$ = 1, 2,…, $n$), $w_i \in [0, 1]$, and $\sum_{i=1}^{n} w_i = 1$, then the weighted correlation coefficient $M_5(A, B)$ defined in Eq. (10) satisfies the following properties:

(1) $M_5(A, B) = M_4(B, A)$;

(2) $0 \le M_5(A, B) \le 1$;

(3) $M_5(A, B) = 1$, if $A = B$.

Since the process to prove these properties is similar to that in Theorem 2, we do not repeat it here.

**4. Decision-making method based on correlation coefficients**

In this section, we propose a multiple attribute decision-making method based on two correlation coefficients between SNSs under simplified neutrosophic environment.

Let $A = \{A_1, A_2,\ldots, A_m\}$ be a set of alternatives and $C = \{C_1, C_2,\ldots, C_n\}$ be a set of attributes. Assume that the weight of an attribute $C_j$ ($j = 1, 2,\ldots, n$), entered by the decision-maker, is $w_j$, $w_j \in [0, 1]$ and $\sum_{j=1}^{n} x_j = 1$. In this case, the characteristic of an alternative $A_i$ ($i = 1, 2,\ldots, m$) on an attribute $C_j$ ($j = 1, 2,\ldots, n$) is represented by the following SNS:

$$A_i = \{\langle C_j, T_{A_i}(C_j), I_{A_i}(C_j), F_{A_i}(C_j)\rangle \mid C_j \in C\}.$$

Here, we only consider that the three interval pairs $T_{A_i}(C_j)$ = [$\inf T_{A_i}(C_j)$, $\sup T_{A_i}(C_j)$], $I_{A_i}(C_j)$ = [$\inf I_{A_i}(C_j)$, $\sup I_{A_i}(C_j)$], $F_{A_i}(C_j)$ = [$\inf F_{A_i}(C_j)$, $\sup F_{A_i}(C_j)$] $\subseteq$ [0, 1] are given in a SNS $A_i$, where $0 \le \sup T_{A_i}(C_j) + \sup I_{A_i}(C_j) + \sup F_{A_i}(C_j) \le 3$ for $C_j \in C$, $j = 1, 2, \ldots, n$, and $i = 1, 2, \ldots, m$, because a SNS $A_i$ is reduced to a SVNS $A_i$ when $F_{A_i}(C_j) = \inf F_{A_i}(C_j) = \sup F_{A_i}(C_j)$, $I_{A_i}(C_j) = \inf I_{A_i}(C_j) = \sup I_{A_i}(C_j)$, and $F_{A_i}(C_j) = \inf F_{A_i}(C_j) = \sup F_{A_i}(C_j)$ are three real numbers in the real unit interval [0, 1].

For convenience, the interval pairs $T_{A_i}(C_j)$ = [inf $T_{A_i}(C_j)$, sup $T_{A_i}(C_j)$ ], $I_{A_i}(C_j)$ = [inf $I_{A_i}(C_j)$, sup $I_{A_i}(C_j)$], $F_{A_i}(C_j)$ = [inf $F_{A_i}(C_j)$, sup $F_{A_i}(C_j)$ ] $\subseteq$ [0, 1] are denoted by a simplified neutrosophic value (SNV) $\alpha_{ij} = \langle[a_{ij}, b_{ij}], [c_{ij}, d_{ij}], [e_{ij}, f_{ij}]\rangle$ ($i$ = 1, 2, …, $m$; $j$ = 1, 2,…, $n$), which is usually derived from the evaluation of an alternative $A_i$ with respect to a criterion $C_j$ by the expert or decision maker. Thus, we can elicit a simplified neutrosophic decision matrix $D = (\alpha_{ij})_{m\times n}$.

In multiple attribute decision making problems, the concept of ideal point has been used to help identify the best alternative in the decision set. Although the ideal alternative does not exist in real world, it does provide a useful theoretical construct against which to evaluate alternatives [6].

Generally, the evaluation attributes can be categorized into two kinds: benefit attributes and cost attributes. Let $H$ be a collection of benefit attributes and $L$ be a collection of cost attributes. In the decision-making method, an ideal alternative can be identified by using a maximum operator for the benefit attributes and a minimum operator for the cost attributes to determine the best value of each attribute among all alternatives. Therefore, we define an ideal SNV for a benefit attribute in the ideal alternative $A^*$ as

$$\begin{aligned}\alpha_j^* &= \left\langle [a_j^*, b_j^*], [c_j^*, d_j^*], [e_j^*, f_j^*] \right\rangle \\ &= \left\langle \left[\max_i(a_{ij}), \max_i(b_{ij})\right], \left[\min_i(c_{ij}), \min_i(d_{ij})\right], \left[\min_i(e_{ij}), \min_i(f_{ij})\right] \right\rangle\end{aligned} \quad \text{for } j \in H;$$

while for a cost attributes, we define an ideal SNV in the ideal alternative $A^*$ by

$$\begin{aligned}\alpha_j^* &= \left\langle [a_j^*, b_j^*], [c_j^*, d_j^*], [e_j^*, f_j^*] \right\rangle \\ &= \left\langle \left[\min_i(a_{ij}), \min_i(b_{ij})\right], \left[\max_i(c_{ij}), \max_i(d_{ij})\right], \left[\max_i(e_{ij}), \max_i(f_{ij})\right] \right\rangle\end{aligned} \quad \text{for } j \in L.$$

Hence, by applying Eq. (9) the weighted correlation coefficient between an alternative $A_i$ ($i$ = 1, 2, …, $m$) and the ideal alternative $A^*$ is given by

$$M_4(A_i, A^*) = \frac{\sum_{j=1}^{n} w_j \left(a_{ij}a_j^* + b_{ij}b_j^* + c_{ij}c_j^* + d_{ij}d_j^* + e_{ij}e_j^* + f_{ij}f_j^*\right)}{\sqrt{\sum_{j=1}^{n} w_j \left(a_{ij}^2 + b_{ij}^2 + c_{ij}^2 + d_{ij}^2 + e_{ij}^2 + f_{ij}^2\right)} \cdot \sqrt{\sum_{j=1}^{n} w_j \left[\left(a_j^*\right)^2 + \left(b_j^*\right)^2 + \left(c_j^*\right)^2 + \left(d_j^*\right)^2 + \left(e_j^*\right)^2 + \left(f_j^*\right)^2\right]}}. \tag{11}$$

Or by applying Eq. (10), the weighted correlation coefficient between an alternative $A_i$ ($i$ = 1, 2, …, $m$) and the ideal alternative $A^*$ is given by

$$M_5(A_i, A^*) = \frac{\sum_{j=1}^{n} w_j \left(a_{ij}a_j^* + b_{ij}b_j^* + c_{ij}c_j^* + d_{ij}d_j^* + e_{ij}e_j^* + f_{ij}f_j^*\right)}{\max\left\{\sum_{j=1}^{n} w_j \left(a_{ij}^2 + b_{ij}^2 + c_{ij}^2 + d_{ij}^2 + e_{ij}^2 + f_{ij}^2\right), \sum_{j=1}^{n} w_j \left[\left(a_j^*\right)^2 + \left(b_j^*\right)^2 + \left(c_j^*\right)^2 + \left(d_j^*\right)^2 + \left(e_j^*\right)^2 + \left(f_j^*\right)^2\right]\right\}} \tag{12}$$

Through the correlation coefficient $M_k(A_i, A^*)$ ($k$ = 4 or 5; $i$ = 1, 2, …, $m$), we can obtain the ranking order of all alternatives and the best one(s).

## 5. Illustrative example and comparative analysis

### *5.1 Illustrative example*

In this subsection, an illustrative example for the multiple attribute decision-making problem of investment alternatives is given to demonstrate the application and effectiveness of the proposed decision-making method.

Let us consider the decision-making problem adapted from [6]. There is an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible alternatives to invest the money: (1) $A_1$ is a car company; (2) $A_2$ is a food company; (3) $A_3$ is a computer company; (4) $A_4$ is an arms company. The investment company must take a decision according to the three attributes: (1) $C_1$ is the risk; (2) $C_2$ is the growth; (3) $C_3$ is the environmental impact, where $C_1$ and $C_2$ are benefit attributes, and $C_3$ is a cost attribute. The weight vector of the attributes is given by $w$ = $(0.35, 0.25, 0.4)^T$ [6]. The four possible alternatives are to be evaluated under the above three attributes by the form of SNVs, as shown in the following simplified neutrosophic decision matrix $D$:

$$D=\begin{bmatrix} \langle[0.4,0.5],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.4,0.6],[0.1,0.3],[0.2,0.4]\rangle & \langle[0.7,0.9],[0.2,0.3],[0.4,0.5]\rangle \\ \langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.3,0.6],[0.3,0.5],[0.8,0.9]\rangle \\ \langle[0.3,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.5,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.4,0.5],[0.2,0.4],[0.7,0.9]\rangle \\ \langle[0.7,0.8],[0.0,0.1],[0.1,0.2]\rangle & \langle[0.6,0.7],[0.1,0.2],[0.1,0.3]\rangle & \langle[0.6,0.7],[0.3,0.4],[0.8,0.9]\rangle \end{bmatrix}.$$

Then, we utilize the developed approach to obtain the most desirable alternative(s).

From the simplified neutrosophic decision matrix $D$ we can obtain the following ideal alternative:

$$A^* = \{\langle[0.7,0.8],[0.0,0.1],[0.1,0.2]\rangle, \langle[0.6,0.7],[0.1,0.2],[0.1,0.3]\rangle, \langle[0.3,0.5],[0.3,0.5],[0.8,0.9]\rangle\}.$$

Then by using Eq. (11), we can obtain the values of the correlation coefficient $M_4(A_i, A^*)$ ($i$ =1, 2, 3, 4):

$M_4(A_1, A^*)$ = 0.8535, $M_4(A_2, A^*)$ = 0.9909, $M_4(A_3, A^*)$ = 0.9445, and $M_4(A_4, A^*)$ = 0.9839.

Thus, the ranking order of the four alternatives is $A_2 \succ A_4 \succ A_3 \succ A_1$. Therefore, the alternative $A_2$ is the best choice among the four alternatives.

Or by using Eq. (12), we can also obtain the values of the correlation coefficient $M_5(A_i, A^*)$ ($i$ =1, 2, 3, 4):

$M_5(A_1, A^*)$ = 0.7642, $M_5(A_2, A^*)$ = 0.9895, $M_5(A_3, A^*)$ = 0.8745, and $M_5(A_4, A^*)$ = 0.9336.

Therefore, the ranking order of the four alternatives is $A_2 \succ A_4 \succ A_3 \succ A_1$. Obviously, the alternative $A_2$ is also the best choice among the four alternatives.

From the above results we can see that the same ranking order of the four alternatives and the same best choice are obtained by use of different correlation coefficients, which are in agreement with the results of Ye's methods [6]. The above example clearly indicates that the proposed decision-making method is applicable and effective under simplified neutrosophic environment.

*5.2 Comparisons to relative methods*

As mentioned above, the SNS include the SVNS and the INS, which are special cases of the SNS. Therefore, the two correlation coefficients of SNSs proposed in this paper are the further extension of the correlation coefficient of SVNSs proposed in [4]. On the one hand, compared with the decision making methods in [4-6], the decision-making method in this paper uses the simplified neutrosophic information, while the decision making methods in [4-6] uses the single valued neutrosophic information in [4, 5] and the interval neutrosophic information in [6]. Furthermore, the simplified neutrosophic decision making method proposed in this paper is a further generalization of the single valued neutrosophic decision-making method proposed by Ye [4]. The later is a special case of the former. Therefore, the decision-making method proposed in this paper can deal with not only single valued neutrosophic decision making problems but also interval neutrosophic decision-making problems. To some extent, the proposed simplified neutrosophic decision-making method is more general and more practical than existing decision-making methods [4-6]. On the other hand, compared with the decision making method in [7], although the decision making methods in this paper and [7] all use simplified neutrosophic information, the decision-making method proposed in this paper is more simple and more convenient than the decision-making method in [7] since in the decision-making process the former uses relatively simple calculations and steps, and then the later uses relatively complex calculations and steps.

## 6. Conclusion

This paper has developed two correlation coefficients between SNSs as a generalization of the single neutrosophic correlation coefficient. Then a multicriteria decision-making method has been established based the proposed two correlation coefficients of SNSs under simplified neutrosophic environment. Through the correlation coefficients between each alternative and the ideal alternative, we can obtain the ranking order of all alternatives and the best alternative. Finally, an illustrative example demonstrated the application and effectiveness of the developed decision-making approach. The proposed decision-making method is suitable for decision making problems with the incomplete, indeterminate, and inconsistent information which exist commonly in real situations. Furthermore, the techniques proposed in this paper extend existing decision-making methods in [4-6] and can provide a useful and simple method for decision-makers. In the future, we shall continue working in the application of the correlation coefficients between SNSs to other domains, such as pattern recognitions and medical diagnoses.

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