International Journal of Intelligent Computing and Cybernetics

Data fusion and sensor selection from imperfect sources with regards to the operating environment

Edwin El-Mahassni,

Article information:

IJICC
6,4

Received 28 September 2011
Revised 16 August 2012
Accepted 20 August 2012

Data fusion and sensor selection from imperfect sources with regards to the operating environment

Edwin El-Mahassni
ISRD 200 Labs, Edinburgh, Australia

Abstract

Purpose – The purpose of this paper is to extend the work of fusing sensors with a Bayesian method to incorporate the sensor's reliability with regard to their operating environment. The results are then to be used with the expected decision formula, conditional entropy and mutual information for suboptimally selecting which types of sensors should be fused where there are operational constraints.

Design/methodology/approach – The approach is an extension of previous work incorporating an environment parameter. The expected decision formula then forms the basis for sensor selection.

Findings – The author found that the performance of the sensors is correlated to the environment of operation, given that the likelihood of error will be higher in a difficult terrain than would otherwise be the case. However, the author also shows the sensors for fusion will vary if the author knows specifically which terrain the sensors will be operating in.

Research limitations/implications – The author notes that in order for this technique to be effective, a proper understanding of the limitations of the sensors, possible terrain types and targets have to be assumed.

Practical implications – The practical implication of this work is the ability to assess the performance of fused sensors according to the environment or terrain they might be operating under, thus providing a greater level of sensitivity than would otherwise be the case.

Originality/value – The author has extended previous ideas on sensor fusion from imprecise and uncertain sources using a Bayesian technique, as well as developed techniques regarding which sensors should be chosen for fusion given payload or other constraints.

Keywords Decision theory, Information fusion

Paper type Research paper

1. Introduction

Data fusion can be defined as combining information from difference sources in order to obtain a better picture of an environment than would otherwise be the case from single disparate sources.

One of the first definitions of data fusion came from the North American Joint Directors of Laboratories (JDL) (White, 1990) who defined data fusion as a "multilevel, multifaceted process dealing with the automatic detection, association, correlation, estimation and combination of data from single and multiple sources".

In addition, although an advantage of fusing information is that it has the ability to mitigate or minimise the impact of erroneous advice from a single source

The author is grateful to all reviewers who provided constructive feedback and which have led to an improvement in the quality of the paper.

International Journal of Intelligent Computing and Cybernetics
Vol. 6 No. 4, 2013
pp. 358-369

1756-378X
DOI 10.1108/IJICC-09-2011-0040

(Hall and Llinas, 1997), the fusion of conflicting and/or imprecise information nonetheless does occur and a way to rigorously capture errors has proved problematic. Several techniques have been developed to overcome some of the issues. These include: fuzzy logic (Zadeh, 1975) and Dempster-Shafer theory (DST) and variants thereof (Dempster, 1968; Shafer, 1976; Smets and Kennes, 1994; Dezert and Smarandache, 2005), such as Dezert-Smaradanche theory (DSmT). The works using DST and other related variants have attracted strong criticism from Bayesian staticians, namely because on occassions, it can lead to counterintuitive results (Gelman, 2006; Zadeh, 1975). Further, since we assume in this paper that we have prior information about the performance of the means by which the information gathered is then fused then Bayesian statistics are optimal and are much more well-established and accepted in the statistical community than these other techniques. Thus, if we can incorporate uncertainty within a Bayesian environment, then one can make the case that DST and these other methods become redundant. Maskell (2008), developed a new approach using Bayesian statistics. This technique is novel in that it allows the modelling of uncertain or imprecise information within a Bayesian environment. Here, we extend this method of accommodating the probability of erroneous advice so that the advice is not only dependent on its accuracy, but also within the environment it is operating. By types of environments we might mean terrain type, weather conditions or time of day. That is, difficult environments will be more prone to error than others. For example, we would expect that the probability of the advice being correct from a radar under sunny, cloudless conditions will be greater than on a rainy, foggy day. Moreover, it is reasonable to assume that for surveillance applications, a foe is likely to have different plans and courses of actions, according to the environment or situation.

Thus, the aim of our paper is to extend the ideas presented in Maskell (2008) to show that it can be also used to model the performance of sensors in different environments. However, a follow-on problem is what to do if we are forced to select a subset of all the possible sensors available to us? How should we go about such a task? That is, in situations where there are payload constraints such as with unmanned aerial vehicles (UAV), it might not always be possible to use all the sensors at our disposal and instead we are forced to choose a subset of these for fusion. This would allows us to select the best sensors, in a suboptimal sense, from an available list being mindful of potential errors within the environment where they will be operating. Then, based on the results using this extension from previous work (Maskell, 2008), we can employ tools such as conditional entropy, mutual information and the expected value of a decision formula (Green and Swets, 1988) for sensor selection. In particular, with the expected decision value, we will also be establishing a link between sensor selection and decision making. The implications of this work are that we would be able to gauge the performance of fused sensors with regard to the environment they might be operating under, which could be then be used as the basis for sensor selection. The techniques can also be employed as a means of selecting the most appropriate sensors to fuse where we are ignorant of the operating environment. We illustrate this approach and further discuss these issues with an example that fuses sensor outputs. We note that although here we restrict ourselves to sensors in a defence scenario, the techniques here can be applied to humans giving advice and other areas, such as medicine or economics. The rest of the paper will be organised as follows. Section 2 will give some background on fusion of imperfect sources, the decision value function, conditional entropy and mutual information.

In Section 3, we show how we adapt a previous technique to include the operating environment. In Section 4, we provide a couple of examples using our techniques along with some discussion and in Section 5 we provide some concluding remarks.

2. Background

2.1 Fusion from imperfect sources using a Bayesian method

Recently (Maskell, 2008), a technique is used where more than one hypothesis can be accommodated. There, it was employed to model two cases: one when a human gives correct advice and the other when this advice is incorrect. For the latter, a uniform distribution was assumed. This paper considered Zadeh's paradox (Zadeh, 1975), which is summarised as Table I. The diagnosis of two experts regarding a patient where three possible classes are listed as M for meningitis, C for concussion and T for brain tumor.

Using Bayes' rule or the Dempter-Shafer approach, the counter-intuitive result of having 100 percent belief that a patient was having a concussion was obtained. Maskell (2008), addressed this problem by adding a parameter, e_i, which accounts for the possibility that an expert has made an error ($e_i = 1$) or not ($e_i = 0$), where $i = 1, \ldots, N$ is the number of separate pieces of evidence fused. Hence, the values in Zadeh's example are for when:

$$p(x|q, e_i = 0) = 1, x \in \{M, T, C\}, i \in \{0, 1\},$$

with the actual paradox summarised in the following table.

However, Maskell (2008) then revised this example by letting:

$$p(x|q_i, e_i = 1) = 1/3, x \in \{M, T, C\}, i \in \{0, 1\}, \tag{1}$$

and the results then become much more plausible so that the larger the error probability, the more likely that the probability of the correct diagnosis being meningitis or concussion was 0.5 for either case.

Further, the probability of a particular outcome given human advice can now be given:

$$p(x|q_1, q_2) = \sum_{e_1}\sum_{e_2} p(x, e_1, e_2|q_1, q_2) \tag{2}$$

where:

$$\begin{aligned} p(x, e_1, e_2|q_1, q_2) &= \frac{p(q_1|x, e_1, e_2, q_2)p(x, e_1, e_2|q_2)}{p(q_1|q_2)} \\ &= \frac{p(q_1|x, e_1)p(q_2|x, e_1, e_2)p(x, e_1, e_2)}{p(q_1, q_2)} \\ &\propto p(q_1|x, e_1)p(q_2|x, e_2)p(x)p(e_1)p(e_2) \end{aligned} \tag{3}$$

Table I. Zadeh's paradox

Human	M	C	T
q_1	0.99	0.00	0.01
q_2	0.00	0.99	0.01

with:

$$p(q_i|x, e_i) = \frac{p(x|q_i, e_i)p(q_i|e_i)}{p(x|e_i)} \propto p(x|q_i, e_i)p(e_i|q_i)p(q_i)$$

Hence:

$$p(x, e_1, e_2|q_1, q_2) \propto p(x)\left(\prod_{i=1,2} p(x|q_i, e_i)p(e_i|q_i)p(e_i)\right),$$

noting that $p(q_i) = 1, i = 1, 2$. This method also allows us to obtain the probability of erroneous or correct advice. This is given by:

$$p(e_1, e_2|q_1, q_2) = \sum_x p(x, e_1, e_2|q_1, q_2). \quad (4)$$

Hence, for the example above, if $p(e_1) = p(e_2) = 0.01$, the probability of at least one expert giving incorrect advice is around 0.9462, whilst the probability that the condition is actually concussion is 0.0282 and for either meningitis or a brain tumor their probabilities are both 0.4859.

2.2 The decision value function

A function that we will also be using is the expected value function. We note that originally it was framed in terms of signal detection theory (Green and Swets, 1988) and whether a target was present or not. That is, first, we let q_0 and q_1 denote the hypotheses indicating the absence or presence of a target, respectively. We also let Q_0 and Q_1 be the decision maker accepting the q_0 or q_1 hypothesis, respectively. We also define the following:

- Let V_{00} be the reward value associated with a correct choice of Q_0, which occurs with probability $p(Q_0|q_0)$. This is known as a correct rejection.
- Let V_{01} be the cost value associated with an incorrect choice of Q_1 (when, in fact, Q_0 is the correct alternative); that is, the person loses V_{01} when this type of incorrect choice is made which occurs with probability $p(Q_1|q_0)$. This is also known as a false alarm.
- Let V_{11} be the reward value associated with a correct choice of Q_1, which occurs with probability $p(Q_1|q_1)$. This is also known as a hit.
- Let V_{10} be the cost value associated with an incorrect choice of Q_0 (when, in fact, Q_1 is the correct alternative); that is, the person loses V_{10} when this type of incorrect choice is made, which occurs with probability $p(Q_0|q_1)$. This is also known as a miss.

Thus, the expected value of the decision strategy is given by:

$$\begin{aligned} E = {} & V_{00}P(q_0)P(Q_0|q_0) + V_{11}P(q_1)P(Q_1|q_1) \\ & - V_{10}P(q_1)P(Q_0|q_1) - V_{01}P(q_0)P(Q_1|q_0) \end{aligned} \quad (5)$$

noting that, of course, $\sum_{i=0,1} P(Q_i|q_j) = 1$ for $j = 0, 1$.

For the multinomial case, this can be reformulated in the following manner:

$$E = \sum_{i}^{N} V_i P(q_i) P(Q_i|q_i) - \sum_{i}^{N} V_{\tilde{i}} P(q_i) P(\tilde{Q}_i|q_i) \tag{6}$$

where V_i and $V_{\tilde{i}}$ are the rewards and costs of making the right and wrong decision, respectively, given q_i; and $P(\tilde{Q}_i|q_i)$ is the probability of selecting a hypothesis other than q_i.

2.3 Conditional entropy and mutual information

Entropy is a measure of uncertainty associated with a random variable. The concept was introduced by Shannon (1948) and can be thought of as the average information content when the value of the random variable is not known. A related concept to entropy is conditional entropy or equivocation. It quantifies the remaining entropy of a random variable given that another random variable is known. If the random variable X is known with support $\mathcal{X}$ and we wish to find the remaining entropy of the random variable Y with support $\mathcal{Y}$ then the conditional entropy $H(Y|X)$ is given:

$$H(Y|X) = \sum_{x\in\mathcal{X}}\sum_{y\in\mathcal{Y}} p(x,y) ln(p(y|x)). \tag{7}$$

Further, once we have the conditional entropy, the amount of mutual information which measures the mutual dependence of two variables can be given. If we define this as $I(Y;X)$, then this is intrinsically related to conditional entropy by the following formula:

$$I(Y;X) = H(Y) - H(Y|X).$$

For the purposes of this paper, we attempt to discover the correlation between the accuracy of the sensors' reports and their operating environment (this will be shown for one of the two example we demonstrate in Section 4). That is, if we know the operating environment of the sensors, how certain can we be regarding their accuracy?

3. Fusion of imperfect sensors and/or advice with sensitivity to the operating environment

In this section, we show how to extend the main idea by Maskell (2008) so that the sensor's advice can be encapsulated according to the operating environment and the likelihood of a target given said environment. First we note that we can think of the variable e (Maskell, 2008), not only as a possible error measurement, but as a variable parameter in general. Noting that originally (Maskell, 2008), $e \in \{0, 1\}$ represented correct or incorrect advice, respectively, this is now extended to $e^{(n)} \in \{0, 1\}$ where $n = \{1, \ldots, N\}$ are the number of distinct environments or terrains the sensors might be operating under. For instance, we could have three terrain environments (e.g. plains, hills, mountains) and within those environments the advice might be correct or incorrect. Hence, if $N = 3$ so that for instance $e^{(1)} = 0$ or $e^{(1)} = 1$ is when the sensor is operating in the plains and giving correct or incorrect advice, respectively. Similarly, when $e^{(2)} = 0$ and $e^{(2)} = 1$, this indicates the performance of the sensor in a hilly terrain is providing correct or incorrect advice, respectively, and so on for $e^{(3)}$. In order to fuse the sensors under this paradigm, we still use equations (2) and (3) but with a few slight changes. We interestingly note that if both sensors are operating in the same environment at the same time then $p(x, e_1^{(n_1)}, e_2^{(n_2)}|s_1, s_2) = 0$, if $n_1 \neq n_2$.

Thus, for the fusion of two sensors, we are able to estimate the following:

- As before (Maskell, 2008), we are still able to derive the same probabilities of one or both sensors making errors, as well as the probability of a particular target being present are $p(x, e_1^{(n_1)}, e_2^{(n_2)}|q_1, q_2)$ and $p(x|q_1, q_2)$.
- The probability of being provided correct advice about a particular target given the operating environment n is:

$$p(x, e_1^{(n)} = 0, e_2^{(n)} = 0|q_1, q_2) \quad = \frac{p(x, e_1^{(n)} = 0, e_2^{(n)} = 0|q_1, q_2)}{\sum_{e_1^{(n)}=0}^{1}\sum_{e_1^{(n)}=0}^{1} p(x, e_1^{(n)} = 0, e_2^{(n)} = 0|q_1, q_2)}.$$

- The probability that the fused sensors have or have not made an error (for any target) in a particular given environment n. For instance, the probability of both sensors providing correct advice in given environment $n_1 = n_2 = n$, is given by:

$$p\left(e_1^{(n)} = 0, e_2^{(n)} = 0|q_1, q_2\right) = \frac{\sum_x p\left(x, e_1^{(n)} = 0, e_2^{(n)} = 0|q_1, q_2\right)}{\sum_x \sum_{e_1^{(n)}=0}^{1}\sum_{e_2^{(n)}=0}^{1} p\left(x, e_1^{(n)}, e_2^{(n)}|q_1, q_2\right)}.$$

- The probability of a correct detection when environment is unknown. Here, we make the assumption that we will follow the advice of a set of fused sensors when a majority of those fused sensors provide correct advice. For instance, for the two-sensor case, we have the probability that making the right decision for the presence of a target type x is given by:

$$P(H_x|h_x) = \sum_{n=1}^{N} p\left(x, e_1^{(n)} = 0, e_2^{(n)} = 0|q_1, q_2\right),$$

- which is the sum of all probabilities when both sensors give correct advice for target x. The more general case of S sensors is similarly formulated so that we sum all the probabilities where at least $\lfloor S/2 \rfloor + 1$ sensors are providing correct advice.

4. Examples

Consider the following examples of sensor performance under different weather conditions. Suppose also we are given Table II and that for the purposes of this paper, we assume that a fusion centre is trying to distinguish between three different types of targets – say, a bomber (B), a fighter plane (F) and a reconnaissance plane (R),

Table II. Performance of sensors in different weather conditions

Sensor	Sunny			Cloudy			Rainy		
Advice	B	F	R	B	F	R	B	F	R
s_1	0.9	0.05	0.05	0.7	0.15	0.15	0.6	0.3	0.1
s_2	0.85	0.05	0.1	0.75	0.15	0.1	0.55	0.2	0.25
s_3	0.8	0.1	0.1	0.7	0.05	0.25	0.65	0.25	0.1

each with prior probabilities of 1/3 and in three possible distinct weather conditions, also with uniformly distributed priors.

We also let the probability of any sensor making an error in sunny conditions be 0.05, the probability of an error in a cloudy day be 0.15, whilst in rainy conditions the probability of incorrect advice is 0.25.

4.1 Known different environments

In this case, suppose that we know the environments where our sensors will be operating. However, here we assume they will differ. This situation might occur if we are wishing to fuse the tracking of an object between two sensors with a significant time delay. For instance, we might wish to fuse two sensors which will be operating at the same time on two consecutive days, but for one day, it will be sunny, whereas the forecast for the next day will be cloudy; and because there is a considerable time delay for a sensor to arrive back and resend it, on each occasion, it must be a different sensor. Here are the results.

Thus, the probability that sensor 1, sent on a sunny day, fused with sensor 2, sent on a cloudy day, both detect a bomber is 0.8801. Hence, in this case, we should send sensors 1 and 2 (for sunny and cloudy days, respectively), since the probability that they both give correct advice is 0.8964 (0.8801 + 0.0098 + 0.0065) and is the largest value compared to all other possible sensor combinations.

4.2 Same environment

Here, we would like to assess the overall performance of fusing any two of those sensors when they operate in the same, but unknown environment. Such a constraint might be imposed because of payload constraints (such as UAV) or simply because we have a limited budget and we can only purchase a finite number of sensors. We are now able to give some performance measures for fusing sensors 1 and 2, 1 and 3, and 2 and 3.

The results, presented in Tables III-V, outline the probabilities of each target being present with none, one or both sensors giving correct advice. (Note that due to rounding off the probabilities, they may not add up to exactly 1.) For instance, the probability that a bomber plane is correctly detected by both sensors 1 and 2 on a cloudy day is 0.2224.

Given that this example is more complex than the first, we explain how different measure such as conditional entropy, mutual entropy and the expected decision formula can be used to determine which pair of fused sensors should be employed.

Further, another even more complex example can be derived where we combine both examples above. That is, if we have to fuse two sensors which may not even be in the same environment and we have complete ignorance as to which environments they may be. In this case, we would have 36 probability values for any given sensor pair.

4.3 Probability results

These tables also allows us to derive other interesting statistics. For instance, if we assume that incorrect advice occurs when at least one of the sensors is incorrect, then we notice that the probability of correct advice given its a sunny day for fused sensors 1 and 2 will be:

Fused sensors	Sensor report		Bomber	Fighter	Recon.
(1,2)	Correct	Correct	0.8801	0.0098	0.0065
(1,2)	Correct	Incorrect	0.069	0.0038	0.0038
(1,2)	Incorrect	Correct	0.0172	0.0034	0.0023
(1,2)	Incorrect	Incorrect	0.0013	0.0013	0.0013
(2,1)	Correct	Correct	0.8537	0.0108	0.0215
(2,1)	Correct	Incorrect	0.0717	0.0042	0.0084
(2,1)	Incorrect	Correct	0.0176	0.0038	0.0038
(2,1)	Incorrect	Incorrect	0.0015	0.0015	0.0015
(1,3)	Correct	Correct	0.8696	0.0035	0.0173
(1,3)	Correct	Incorrect	0.0731	0.0041	0.0041
(1,3)	Incorrect	Correct	0.017	0.0012	0.0061
(1,3)	Incorrect	Incorrect	0.0014	0.0014	0.0014
(3,1)	Correct	Correct	0.8365	0.0224	0.0224
(3,1)	Correct	Incorrect	0.0703	0.0088	0.0088
(3,1)	Incorrect	Correct	0.0183	0.0039	0.0039
(3,1)	Incorrect	Incorrect	0.0015	0.0015	0.0015
(3,2)	Correct	Correct	0.8517	0.0213	0.0142
(3,2)	Correct	Incorrect	0.0668	0.0084	0.0084
(3,2)	Incorrect	Correct	0.0187	0.0037	0.0025
(3,2)	Incorrect	Incorrect	0.0015	0.0015	0.0015
(2,3)	Correct	Correct	0.8476	0.0036	0.0356
(2,3)	Correct	Incorrect	0.0712	0.0042	0.0084
(2,3)	Incorrect	Correct	0.0175	0.0012	0.0062
(2,3)	Incorrect	Incorrect	0.0015	0.0015	0.0015

Table III.
Target probabilities when fusing sensors sent on a sunny and then cloudy day, respectively

Environment	Sensor report		Bomber	Fused sensors (1,2) Fighter	Recon.
Sunny	Correct	Correct	0.5056	0.0017	0.0033
Sunny	Correct	Incorrect	0.0104	0.0006	0.0006
Sunny	Incorrect	Correct	0.0099	0.0006	0.0012
Sunny	Incorrect	Incorrect	0.0002	0.0002	0.0002
Cloudy	Correct	Correct	0.2224	0.0095	0.0064
Cloudy	Correct	Incorrect	0.0174	0.0037	0.0037
Cloudy	Incorrect	Correct	0.0187	0.0037	0.0025
Cloudy	Incorrect	Incorrect	0.0015	0.0015	0.0015
Rainy	Correct	Correct	0.0847	0.0154	0.0064
Rainy	Correct	Incorrect	0.0171	0.0086	0.0029
Rainy	Incorrect	Correct	0.0157	0.0057	0.0071
Rainy	Incorrect	Incorrect	0.0032	0.0032	0.0032

Table IV.
Target probabilities when fusing sensors 1 and 2

$$p\left(e_1^{(1)}, e_2^{(1)} | q_1, q_2\right) = \frac{\sum_x p\left(x, e_1^{(1)} = 0, e_2^{(1)} = 0 | q_1, q_2\right)}{\sum_x \sum_{i,j \in \{0,1\}} p\left(x, e_1^{(1)} = i, e_2^{(1)} = j | q_1, q_2\right)} = 0.5106/0.5345$$

$$= 0.955,$$

Table V.
Target probabilities when fusing sensors 1 and 3

Environment	Sensor report		Fused sensors (1,3) Bomber	Fighter	Recon.
Sunny	Correct	Correct	0.4877	0.0034	0.0034
Sunny	Correct	Incorrect	0.0107	0.0006	0.0006
Sunny	Incorrect	Correct	0.0095	0.0012	0.0012
Sunny	Incorrect	Incorrect	0.0002	0.0002	0.0002
Cloudy	Correct	Correct	0.2127	0.0033	0.0163
Cloudy	Correct	Incorrect	0.0179	0.0038	0.0038
Cloudy	Incorrect	Correct	0.0179	0.0013	0.0064
Cloudy	Incorrect	Incorrect	0.0015	0.0015	0.0015
Rainy	Correct	Correct	0.1026	0.0197	0.0026
Rainy	Correct	Incorrect	0.0175	0.0088	0.0029
Rainy	Incorrect	Correct	0.019	0.0073	0.0029
Rainy	Incorrect	Incorrect	0.0032	0.0032	0.0032

whereas if the sensors fused were numbers 2 and 3, this figure is 0.951. However, on a rainy day, we notice that fusing sensors 2 and 3 is more beneficial than fusing sensors 1 and 2, where the probabilities of correct advice are 0.625 and 0.615, respectively. This implies that our choice of sensors can be influenced if we have knowledge of the type of environment they will be operating in. If we are unsure, then we marginalise the probabilities of all errors over $i = 1, 2, 3$ for $e^{(i)}$. For fused sensors 1 and 2, the probability of correct advice given by both sensors is 0.8554; for sensors 1 and 3, it is 0.8288; and for sensors 2 and 3, it is 0.8478.

4.4 Conditional entropy and mutual information

For our example, we let Y be the error variable and X be the environment. We recall that under our definition of conditional entropy we are calculating the average entropy of the accuracy of a report (determined by both sensors giving correct reports) averaged over all possible environments. We recall that when both sensors give correct advice we deem that to be correct and anything else is labelled as incorrect. Conditional entropy, in our case, is the amount of uncertainty remaining about the accuracy of a report when the weather is known. Thus, it would be desirable to have our value $H(Y|X)$ as small as possible. For instance, we see that when $y =$ Correct we have the unnormalised value of:

$$\mathcal{P}(\text{Correct}|\text{Sunny}) = 0.5056 + 0.0017 + 0.0033 = 0.5106.$$

Similarly, we can then notice the unnormalised value of:

$$\mathcal{P}(\text{Incorrect}|\text{Sunny}) = 0.5345 - 0.5106 = 0.0239.$$

Note that the value of 0.5345 is calculated by adding all the values under the sunny environment. Thus, the respective conditioned probabilities become:

$$p(\text{Correct}|\text{Sunny}) = 0.5106/0.5345 = 0.955 \quad \text{and}$$
$$p(\text{Incorrect}|\text{Sunny}) = 0.0239/0.5345 = 0.045.$$

Hence, repeating this process for all weather conditions and recalling equation (7), we have that:

$$
\begin{aligned}
H^{(1,2)}(Y|X) &= -0.5106ln(0.955) + 0.0239ln(0.045) + 0.2383ln(0.815) \\
&\quad + 0.0542ln(0.185) + 0.1065ln(0.615) + 0.0667ln(0.385) \\
&= 0.353.
\end{aligned}
$$

$H^{(1,3)}(Y|X) = 0.365$ and $H^{(2,3)}(Y|X) = 0.370$.

Thus, under this measure fusing sensors 1 and 2 should be preferred.

Now, suppose we consider mutual information. In this case, the value $I(Y;X)$ represents the mutual dependence between the environment and the accuracy of the advice from the fused sensors. To calculate $H(Y)$ we merely have to determine the overall probabilities of a correct report (again determined by both sensors indicating correct reports) and an incorrect report (all other cases). For instance, for fused sensors 1 and 2, we have that the overall probability of a correct report is given by:

$$
\begin{aligned}
p(\text{Correct}) &= 0.5056 + 0.0017 + 0.0033 \\
&\quad 0.2224 + 0.0095 + 0.0064 \\
&\quad 0.0847 + 0.0154 + 0.0064 = 0.8554,
\end{aligned}
$$

thus $H^{(1,2)}(Y) = -(0.8554ln(0.8554) + 0.1446ln(0.1446)) = 0.413$. Combining this with $H^{(1,2)}(Y|X) = 0.353$, we then see that $I^{(1,2)}(Y;X) = 0.06$. Similarly, since $H^{(1,3)} = 0.419$ and $H^{(2,3)} = 0.426$ we have that $I^{(1,3)}(Y;X) = 0.054$ and $I^{(2,3)}(Y;X) = 0.056$. This means that there is the least correlation between the environment and the scenario where both reports are correct, when we fuse sensors 1 and 3. However, there was not much difference between all the fused sensors' cases.

4.5 Lower bound on expected value of a decision

Let $i = 1, 2$ and 3 denote a bomber (B), fighter (F) and reconnaissance plane (R), respectively. Using equation (5) for this example we then let $V_i = V_{\bar{i}} = 1, i = \{1\ldots3\}$. We can now obtain a lower bound for the expected value of a decision. As stated, we assume that when both sensors report correctly, then the correct decision will be made and that an incorrect decision will be taken otherwise. Now, given that probability of all three targets is 1/3, we can see, for example, that for fused sensors 1 and 2 we have:

$$
\begin{aligned}
P(H_B|h_B) &= 0.5056 + 0.2224 + 0.0847 = 0.8127 \\
P(H_F|h_F) &= 0.0017 + 0.0095 + 0.0154 = 0.0266 \\
P(H_R|h_R) &= 0.0033 + 0.0064 + 0.0064 = 0.0161.
\end{aligned}
$$

For calculating the probability of making the wrong decision we simply add all the other entries in a target's column. For instance:

$$
\begin{aligned}
P(H_{\bar{B}}|h_B) &= 0.0104 + 0.0099 + 0.0002 + 0.0174 \\
&\quad + 0.0187 + 0.0015 + 0.0171 + 0.0157 + 0.0032 \\
&= 0.0941
\end{aligned}
$$

$$E^{(1,2)} \geq 1/3((0.8127 + 0.0266 + 0.0161) - (0.0941 + 0.0278 + 0.0229)) = 0.2369$$

$$E^{(1,3)} \geq 1/3((0.798 + 0.0264 + 0.0223) - (0.0974 + 0.0279 + 0.0227)) = 0.2329$$

$$E^{(2,3)} \geq 1/3((0.8028 + 0.0203 + 0.0247) - (0.0991 + 0.0255 + 0.027)) = 0.2321$$

Hence, using this approach, and with cost values given above, we should fuse and use sensors 1 and 2. If, however, the reward and cost values change, our preferred fused sensors might also change. For example, if $V_1 = V_2 = 1$, $V_3 = 10$ and $V_{\tilde{1}} = V_{tilde2} = V_{\tilde{3}} = 1$ then sensors 2 and 3 should be employed in the fusion process. We note also that if we know in which type of weather the sensors will be operating under, then we can still use the expected decision formula by simply marginalising the probabilities belonging to that environment as listed in Table VI.

5. Concluding remarks

The purpose of this paper was to further extend the ideas shown by Maskell (2008) and use this as a basis for sensor selection. We have done this by allowing opinions of sensors, humans, etc. to differ within different operating environments. That is, since the accuracy of sensors will vary according to their operating environment, it is also reasonable that these different levels of accuracy should also be modelled. We have been able to show that the choice of fused sensors will depend not only on the operating environment, but also on the perceived importance of potential targets when we use the expected decision formula. We note that although in our example we used the weather as a variable, there is no reason why the operating environment variable might not be the type of terrain, time of day, etc. Further, another novelty of this paper is that we were able to incorporate the expected decision formula, originally formulated by Green and Swets (1988) to derive a lower bound for the expected value of a decision as a method for selecting which sensors to fuse. Other alternatives such as conditional entropy and mutual information were also shown as a means for sensor selection.

Table VI.
Target probabilities when fusing sensors 2 and 3

Environment	Sensor report		Fused sensors (2,3) Bomber	Fighter	Recon.
Sunny	Correct	Correct	0.4725	0.0035	0.0069
Sunny	Correct	Incorrect	0.0104	0.0006	0.0012
Sunny	Incorrect	Correct	0.0098	0.0012	0.0012
Sunny	Incorrect	Incorrect	0.0002	0.0002	0.0002
Cloudy	Correct	Correct	0.2338	0.0033	0.0111
Cloudy	Correct	Incorrect	0.0196	0.0039	0.0026
Cloudy	Incorrect	Correct	0.0183	0.0013	0.0065
Cloudy	Incorrect	Incorrect	0.0015	0.0015	0.0015
Rainy	Correct	Correct	0.0965	0.0135	0.0067
Rainy	Correct	Incorrect	0.0165	0.006	0.0075
Rainy	Incorrect	Correct	0.0195	0.0075	0.003
Rainy	Incorrect	Incorrect	0.0033	0.0033	0.0033

References

Dempster, A.P. (1968), "A generalization of Bayesian inference", *Journal of the Royal Statistical Society Series B*, Vol. 30, pp. 205-247.

Dezert, J. and Smarandache, F. (2005), "A new paradigm shift for information fusion", *Proceedings of Cogis 06, Paris*, pp. 15-17.

Gelman, A. (2006), "The boxer, the wresler, and the coin flip: a paradox of robust Bayesian inference and belief functions", *The American Statistician*, Vol. 60 No. 2, pp. 146-150.

Green, D.G. and Swets, J.A. (1988), *Signal Detection Theory and Psychophysics*, Peninsula Publishing, Newport Beach, CA.

Hall, D.L. and Llinas, J. (1997), "An introduction to multisensor data fusion", *IEEE Proceedings*, Vol. 85, pp. 6-23.

Maskell, S. (2008), "A Bayesian approach to fusing uncertain, imprecise and conflicting information", *Information Fusion*, Vol. 9, pp. 259-277.

Shafer, G.A. (1976), *Mathematical Theory of Evidence*, Princeton University Press, Princeton, NJ.

Shannon, C.E. (1948), "A mathematical theory of communication", *Bell System Technical Journal*, Vol. 27, pp. 379-423 and 623-656.

Smets, P.h. and Kennes, R. (1994), "The transferable belief model", *Artificial Intelligence*, Vol. 66 No. 2, pp. 191-234.

White, F.E. Jr (1990), "Joint directors of laboratories data fusion subpanel report", *Proceedings of the Joint Service Data Fusion Symposium, DFS-90*, pp. 476-484.

Zadeh, L. (1975), "Fuzzy logic and approximate reasoning", *Synthese*, Vol. 30, pp. 407-428.

Further reading

Steinberg, A.N., Bowman, C.L. and White, F.E. (1999), "Revisions to the JDL data fusion model", *Proceedings of SPIE*, Vol. 3719, pp. 430-441.

About the author

Edwin El-Mahassni received his BA Arts (Hons) (mathematics), MSc (theoretical computer science) and PhD (mathematics) from Macquarie University, Australia, in 1999, 2001 and 2008, respectively. He has published in the area of pseudorandom number generators, and decision-making in the presence of uncertainty which is where his current research interest lies. For the last ten years, he has been employed by the Defence Science and Technology Organisation (DSTO), Australia. Edwin El-Mahassni can be contacted at: Edwin.El-Mahassni@dsto.defence.gov.au

This article has been cited by:

1. Aang SoenandiIwan, Iwan Aang Soenandi, DjatnaTaufik, Taufik Djatna, SuryaniAni, Ani Suryani, Irzaman, Irzaman. 2017. Real-time optimization using gradient adaptive selection and classification from infrared sensors measurement for esterification oleic acid with glycerol. *International Journal of Intelligent Computing and Cybernetics* **10**:2, 130-144. [Abstract] [Full Text] [PDF]