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Decision and Reasoning in Incompleteness or Uncertainty conditions

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ABSTRACT When we try to solve new or known problems to which we want to give new solutions, we create new knowledge and realize new discoveries. To date, the scientific methods have used the probability in order to analyze problems, make inference and build forecasts. However, everyone agreed that most problems do not follow standard probabilistic rules. In this study we will build an uncertainty logic by using the concept of probability, with those of plausibility, credibility and possibility. We will provide several models which treats uncertainty information and allow to perform more reliable forecasts. After that, we will prove the models reliability through a final simulation on the Biometrics and Sport fields using one of the models; these simulation are fully replicabile for each field and for each of the provided models.

INDEX TERMS Biometrics, Credibility, Incompleteness, Plausibility, Possibility, Probability, Sports odds, Uncertainty.

I. INTRODUCTION

THE plausibility is a concept used in many inferential contexts and processes. A plausible inference theory goes through a preliminary definition of what is "uncertain" and how it affects the inferential process. A plausible interference can be invalidated, corrected or simply updated at the entry of new data or information. In this sense, plausibility is not only non-monotonous, but operates in an open conceptual space. The data, the available information and inferential rules are of a temporary nature and reviewable so, they are constantly changing.

Polya (1954) defines the plausible reasoning as opposed to the demonstrative reasoning. The plausible inference helps to acquire new knowledge, using the inductions and the analogy which are not certain and do not transmit the truth. The plausible inference has a fluid, temporary and risky behavior. Rescher (1976), conceives the plausible reasoning as a form of deduction starting from uncertain premises, aimed at the treatment of "cognitive dissonances". In this theory the premises and conclusions are provisional, controversial and fluid.

For Dezert (2002), the plausible reasoning is an extension of the evidence theory of Dempster-Shafer, and therefore of the Bayesian probability. His theory allows to deal with information not only uncertain, but also paradoxical[1]. This reasoning has been fully studied: Cuzzolin in [1] defines the properties of plausibility. Friedman, Halpern and Barnett calculated and measured the plausibility [2] [3]. This concept has been used in real world applications like [4] as well. While Shved, Wanga and Wen provide a way to measure uncertainty [5] [6] [7]. In this paper, we are going to define a new perspective on the plausible reasoning.

II. CONCEPTUAL OVERVIEW

This section in inspired by the work of author in [8]. The formulation of a theory of plausibility which has been known in antiquity is that of Sextus Empiricus. Truthly, he attributes this theory to Carneade of Cyrene, in his composition "Against the Logicians". Plausible here means "to persuade". The criteria are:

- something is plausible if it seems to be true,
- is even more plausible if it seems to be true and is compatible with other things that seem to be true (i.e. it is stable),
- is stable and is tested.

[1]admitting contradictory information thanks to the "hyper" set -power"

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So it is not necessary that something be true or believed, to be plausible. It simply has to satisfy some requirements, so as to allow something to be assessed as in accordance or consistent with related facts.
Uncertainty can be of two types:

- Aleatory (also known as stochastic), when it concerns the randomness of the object of knowledge system;
- Epistemic (also known as subjective), i.e. when it refers to the state of knowledge of the subject over the system.

The plausibility uncertainty belongs to the second type; it is an expression of a subjective relationship.
According to Walton [9], it is a controversial question if the plausibility is different from the probability, and it is difficult to rule out that plausibility can be resolved in some special case of probability.
In order to model the epistemic uncertainty with the probability, the Cox's theorem is used. It establishes the existence of a precise relationship between the notion of plausibility, **pl** and the theory of probability calculation, **pr**. This theorem proves that any system of plausible inferences, which is not isomorphic to probability theory, must violate at least one of the following conditions:

- pl $(A|X)$ = 0 $\leftrightarrow$ A false in X
- pl $(A|X) = 1 \leftrightarrow$ A true in X
- $0 \leq pl(A|X) \leq 1$
- pl (A$\wedge$B$|X$) = pl $(A|X)pl(B|X)$
- pl $(\neg A|X) = 1 - pl(A|X)$,

where pl $(A|X)$ indicates the plausibility of A over X. If it does not violate any of the constraints it is possible to replace **pr** a **pl** and use the probability calculation without losing information.

A. POLYA

According to Polya, the prototypes of plausible interference are the analogy and the induction; in fact "the analogy and the particular cases are the most abundant sources of plausible arguments" [10] [11]. In particular "the inference by analogy seems the most common and essential form of inference. It produces more or less plausible conjectures which may or may not be confirmed by experience or by more rigorous reasoning". The plausible argument, as defined by Polya, is not definitive: it produces provisional conclusions. The plausibilistic reasoning is not subjective: the models of reasonings are impersonal, while it is the force of the conclusions which is of a subjective nature and therefore not representable by means of quantity. As for the fundamental inductive model, they have a correspondence in the demonstrative logic. In [10], the impersonal rules which establish some kind of evidence are described. While it is personal and subjective to establish "whether a test has a sufficient weight or not". For Polya the probability is the core in the plausibility notion. The Bayes theorem is the foundation of Polya's concept of plausibility. The credibility of B is the confidence of A if B is true. In fact, credibility can be a conditional probability $Pr\{E/H\}$ and therefore "have a numerical value, equal to the probability that an event of the type predicted by E will happen, computed on the basis of the statistical hypothesis H" although "credibility and probability are differently defined" [10]. When we deal with the plausibility of A, we are dealing with the reliability of A and the strength of evidence in favor of A, in other words, our trust in A. Credibility is expressed in probabilistic terms, readjusting the Bayes' theorem with the substitution of probability (Pr) with credibility (Cr). In this view, plausibility and probability are indistinguishable, as the former can be reduced to the second; however it is not always possible to compare numerically the plausibility of two different events. The probabilistic conception of Polya is affected by all the limitations of the Bayesian conception.

B. DEMPSTER-SHAFER

To overcome the limits of the bayesian conception and the relative vision of plausible inference, the evidence theory of Dempster-Shafer has been defined, hereafter denoted as DS. Also for this theory the bayesian conception is the basis for the plausibility notion. In particular, it "includes the bayesain theory as a special case and therefore preserves at least some of the attractions of that theory" [12] [13]. The DS theory renounces the one-dimensional representation of belief: in other words, it renounces to assign the remaining portion of belief to the negation of the proposition. The DS theory starts from what is called a discernment or universe of discourse u, which is a series of mutually exclusive alternatives. The DS theory assigns a basic probability, a belief function and a plausibility function to the u subsets which are called propositions. We define the following assignment to a *basic probability*:

$$\text{m: } 2^u \rightarrow [0,1] \text{ such that m } () = 0, \sum_{A \subseteq u} m(A) = 1$$

which represents and expresses numerically the force of evidence, the exact belief that an agent has in A.
The function m: $2^u \rightarrow [0,1]$ is called *belief function Bel*, when it satisfies the conditions:

- $Bel() = 0$,
- $Bel(u) = 1$.

Plausibility can now be defined. We observe that "a proposition is plausible in the light of evidence to the point where the evidence does not support its opposite" [12]. Then we introduce the dubious function, *Dub*, placing it as equal to $Cr(\neg A)$ where $\neg A$ is the complement of A in u and we define the function of plausibility, Pl, as Pl (A) = 1-Dub (A) = 1 $(Cr\neg A)$. The plausibility function expresses what should be believed in proposition A if all the not known facts at present state support A; it also expresses the maximum probabilistic value that can be allocated to a proposition A: in particular "it measures the total mass of belief that can be moved to A" [14]. The lower limit corresponds to the belief function, and therefore the relation Cr (A) $\leq$ Pl (A) is valid. The difference between belief and plausibility is also reflected at the functional level: indeed the belief

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function "is often zero for all atomic propositions in complex domains, unless a large number of trials are available". The other big difference between bayesian theory and DS theory is the evidence combination, that is the belief updating rule when there is a new evidence. In fact, in the DS' theory the combination rule substitutes the bayesian rule. The combination rule is also called *orthogonal sum*. The DS combination rule suffers from some well-known structural limitations: in fairly simple situations it could generate unexpected and non-intuitive results, which strongly limit effectiveness [15].

C. DEZERT-SMARANDACHE

The *Dezert-Smarandache*'s theory [16], hereafter denoted as DSm, has born as an explicit attempt to overcome some difficulties of the DS theory. Plausibility is an upper limit of the probabilistic value which a proposition can assume. Ds' plausibility theory conceives plausible reasoning as a conceptual model for guiding decision-making under uncertainty conditions, which is broadly defined respecting the DS' theory. In particular, the uncertainty referred by Dezert extends the one conceived in probability theory and DS' theory. A situation or an information state is "rational" when the basic assignment m is a sum one and the closure of the operators $\cap$ and $\cup$ on the universe elements is 0; it is strictly "uncertain" when it is a sum one, and the closing of the operator $\cup$ can be different from zero; it is "paradoxical" when it is sum one and the closing of the operator $\cap$ can be different from zero; it is "uncertain and paradoxical" when the closure of both operators $\cap$ and $\cup$ can be different from zero. The assumptions' modification produces effects on the developed conceptual model, but does not allow to exceed its limits. Although Dezert tends to a more extensive discussion of the plausible reasoning, his approach contributes to leaving it anchored to a strongly reductionist view.

D. THE RESCHER'S DEDUCTIVIST CONCEPTION

Among the non-probabilistic approaches to plausibility theory, *Rescher's* deductivist approach stands out. According to Nicholas Rescher, the "plausibility theory aims to provide a rational instrument to treat cognitive dissonances" [17]. Rescher provides a preliminary distinction between plausibility and probability: the first "classifies propositions according to the status of the evidentiary sources or the validating principles that guarantee them", while "the probability weights various alternatives and evaluates them". Instead of adopting the probability theory as a basis for modeling the plausibility notion, he bases his approach on the traditional Theophrastus' principle, pointing to a return to the historical roots of the plausibility theory. It never develops calculations which merge quantities into new results, but proceeds only with comparisons between different degrees of data. A further aspect of the deductivistic approach to the plausibility notion is the link with the entymatic argument where the common knowledge makes it possible to integrate the missing steps, that is the non-explicit assumptions within an inferences sequence. The plausibility theory allows us to estabilish if the propositions are "candidates" to the truth or not. An argument plausibility can therefore be defined as the maximum value among the minimum plausibilistic values of the entemematic integrations which allows a deductive conclusion derivated from the premises.

E. NOT MONOTONICITY AND PLAUSIBILITY

Another non-probabilistic approach aimed at treating plausible inference is non-monotonous logics' one. This approach is characterized by the violation of one of the strongest conditions of classical logic, known as "monotonicity". If the conclusion φ is a consequence of a premises set Γ, then it remains a consequence of any premises set Δ that contains Γ as its subset. According to this condition, a conclusion can not be invalidated by the addition of new information, it remains true once and for all, regardless of the premises that we can add to the premises set Γ. The valid propositions number increases monotonously to the premises added to Γ increase. If the propositions can be invalidated by the addition of new premises, the valid assertions number may not only increase, but may also decrease. The non-monotonous logic is based on a reduced vision of uncertainty and it simply treats inferences with exceptions. Inference forms modeled by non-monotonic logics are uncertain as they produce provisional conclusions which can be invalidated or reviewed in the light of new information available, and this is a big limitation. Furthermore, the non-monotonous approach conceives the plausible reasoning as essentially addressed to the inferences treatment proper to everyday experience. Thus, it neglects the whole part of the plausible inference directly connected to the scientific disciplines, including Mathematics. As theorized by Polya and Cellucci [18], the plausible inference is full of experimental reasoning that the classical logic does not allow to handle.

F. THE COLLINS-MICHALSKI'S COGNITIVE APPROACH

The cognitive approach to plausibility aims to show how factors of a subjective and psychological nature can play a decisive role in the process that leads to a choice. The cognitive approach to plausibility is modeled in the Collins-Michalscki's plausibility theory hereafter denoted as C-M. It intends to formalize the plausible inferences which frequently recur in people's responses to questions for which they do not have a ready answer [19].

Information is defined as common knowledge to a group of people if: all the group members know it; they know that others know it; they know that others know that they know it; and so on. It is different from the simple mutual information, "which implies only that specific possession of information and not even the awareness of the others' knowledge". That mutual knowledge may become a common knowledge, in presence of an "independent arbitrariness" to the various agents who make inferences and share information. These reasoning forms create plausible inference models which, although dependent on cognitive and psychological factors, are not included or traceable to the theory of Collins-Michalsky.

This theory leaves out some facts which the plausible reasoning explicitly intended to treat and model.

G. OPEN SYSTEMS AND NETWORKS

Thanks to its recent developments, the networks' theory can be viewed as an analysis source and plausible inference investigation. The knowledge representation in graph form is capable of detecting essential aspects about the plausible inference like nature and dynamics. We can consider an hypothesis as 'plausible' when it is connected to existing knowledge and has a capability of self-adjustment and robustness (resistance to error): the greater the degree of connection, the more likely the hypotesis to be plausible. Connectivity will measure its integration and compatibility with existing knowledge degree and robustness. The success of this approach depends essentially on an adequate knowledge representation in a reticular form, and in particular on the chosen granularity. Plausible inferences can have very complex forms, composed of many steps resulting from different rules application (deductions, inductions, analogies), which create equally complex graphs. It is possible to define plausibility in terms of local or global properties. In particular, a node can be plausible if it has numerous incident edges or if there are different paths leading to it.

III. DECISION IN UNCERTAINTY DOMAINS

A. SCENARIO

In the knowledge and collective intelligence era, like ever, we are called to make decisions. Generally, we use problem solving to achive goals (decision making). It is obvious that a target could be reached by joining "elementary" decisions. This highlights an important issue: in the modern decision-making support systems, it is important to focus both on the objectives and on the decisionaly path. The objective is reached by joining sub-objectives through an optimal path.
If we were in Physics, with a material system that has to transit from A to B in n finite steps and through m decision-making paths, it would be easy to estimate the problem complexity and reduce it with principles of minimization, i.e. the problems of transaction energy, the transaction disorder or time minimization.
In the case of a logistic transport problem, in which the objective is moving merchandise or people from A to B, the optimal solution may be achieved by maximizing merchandise transported in a given time interval.
When there are more solutions or a target solution reachable through different competitive strategies, these examples and their generalizations give us the opportunity to formalize the problem in (fig. 1):

- initial conditions definition,
- constraints fixing,
- target individuation,
- solution strategy definition,
- solution optimization.

A first problem classification is to establish whether it is deterministic or stochastic. The variables on which the problem

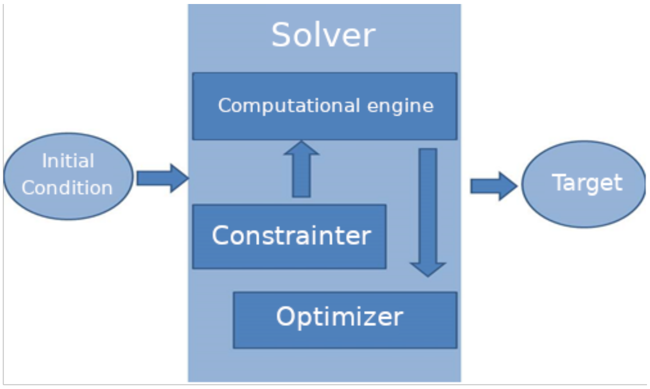

FIGURE 1: How a problem is defined

depends could be hence described either by dynamic defined laws or by some dynamics which are out of the analyst's control.

This indetermination could concern both the single variable and the way it affects the output. In the first case we have a direct indeterminacy (cause indeterminacy) that concerns a problem variable. In the second case we have a derivative or effect indeterminacy.
Imagine an individual that has to move from A to B by bus. The first indetermination concerne when the bus will arrive (temporal indeterminacy), so on the input variable "time". It is obvious that reaching the B position could depend on the traffic which represents an effect indeterminacy given by the constraint "traffic" or road filling coefficient. Having knowledge of multiple paths and their road status ("traffic" or road filling coefficient) it is an obvious advantage to minimize the B reaching time, also paying a route cost, corresponding to a longer journey in terms of space.
This example allows to understand the simplicity of dealing with non-deterministic problems when they can be reduced to deterministic ones with a series of choices (constraints).
If we think about how to choose which path to take, the answer leads to another dualism: infocompletness and infoincompletness. With the previous example it is simple to understand how the knowledge of a free path allows the driver to easily choose which road to follow to reach point B (goal of the trip) in the shortest time. With this method we can transform a non deterministic problem in a deterministic one thanks to a choice based on the scenario knowledge.
In this way, we have introduced complex systems and decision theory characterization: the info completeness-incompleteness.
From the following three characterizations the stochasticity is defined [2]:

1) variables determinism, variables indeterminism;
2) functional link between variables and their impact on the target determinism, target indeterminism;

[2]That is the non-determinism of many problems in most of the cognitive areas where it is necessary to make decisions

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3) completeness and incompleteness of the starting scenario (input varables), the arrival scenario (output variables), and the constraints (functional models which link inputs to outputs).

We give an example in the financial context where the target is the reaching of a price target:

1) first type indetermination: tick instant;
2) second type indetermination: ticks succession;
3) third type indetermination, info incompleteness: exchanged volumes and price variation.

It is important to specify that the word stochastic could not mean full random, because it is possible to build models which manipulate stochasticity and make predictions. In other words, taking the previous example, we will be able to give an estimation based on known constraints or analyst's needs, so we will be able to make predictions on the price in a time frame. Stochastic dynamic systems solve many problems even if they do not provide a perfectly deterministic answer.

It is important to define the right position of the problem. This problem concerns the certainty of the target. About the previous examples, we must ask if the target price is reachable, or if the considered bus line stops at point A etc. So the question is if the target can be possibly achieved or it is impossible.

Therefore, a set of linguistic nuances emerged between the certain event and the impossible one.

In fact, other terms have emerged linguistically: possible, plausible and credible. These terms need to be formally defined. Shortly, our study highlights that decision-making problems in complexity contexts are generated by dualisms:

- determinism, indeterminism;
- completeness, incompleteness;

which generate another dualism: *certainty, uncertainty* of the target which have two extremes of solution. This target is between *certain event, impossible event and uncertain event,* which is reality and it is in the middle.

Uncertain dualism is faced by quantification of:

- plausible, implausible;
- credible, incredible;
- probable, improbable;
- possible, impossible;

The goal of our work is to understand how to use, quantify and generalize these dichotomies in order to build decision-making support systems which are not only probability-based.

Someone could be confused and wonder why to use so many different concepts and not rely on probability theory or Bayesian processes.

There is a simple reason, some phenomena which consider complexity or complex systems are not Gaussian (or other known distributions) distributed, but they have heavy tails which are not Gaussian.

When someone uses the term heavy tails, he is putting a patch on a problem, because he does not have suitable methodologies to solve and to describe the uncertainty in a rigorous framework. An example is the price distribution of a financial instrument around a sample price in which a not-normal distribution is observed, with heavy tails (larger than Gaussian tails).

This means that some improbable events can still happen, therefore it is important to estimate whether the event is plausible, credible or possible and estimate reliability.

Info-incompleteness or the input indecision generate emotions and beliefs that make decision-making process subjective.

So we understand why the rational operator becomes emotional-affected and takes sentiment and subjective choices.

The purpose of decision-making support systems must be to build simulated scenarios, like those of game theory. In game theory, different solutions/targets cooperate or compete to the best choice, where the best word is based on different parameters of decision-maker interest (target constraints).

Thanks to this methodology, related to technological help and the support of simulated scenarios, an emotional-affected operator will be brought to rationally (induced rational operator) decide before the result achievement (decision forward in time).

Information incompleteness and the variables/information, or functional bonds indetermination, generate emotions in the decision maker which induce beliefs.

Thanks to this work we bring back the operator to be rational on a larger cognitive basis (expanded knowledge); we use other approaches like sentiment analysis or emotional analysis to control this beliefs.

The new realization moment will lead to the hyperconsciousness status where the operator is conscious of the augmented scenario called hyperscenario.

B. CONTINUOUS OCCURENCE OF UNLIKELY POSSIBLE

How many times have we witnessed the occurrence of highly improbable events?

How many times was a football match conclusion plausible and things have gone differently?

Moreover, how many times the price dynamics of a financial instrument have made us to believe that we have understood the dominant direction, but as soon as we entered the market it had an opposite behavior?

These are some examples of how complex it is to make previsions under uncertainty conditions (input, models and results uncertainty) or information incompleteness.

Overcome determinism and entered the stochastic or uncertainty phenomena context we find the terms: probable, plausible, credible, possible.

In literature there are different research lines and thousands works which describe, distinguish and model these terms sometimes in a conflictual way.

In this study, we present a graded organic vision of the terms, starting from the best modeled concept, rather probability,

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using that to model probable event, but that extend it to describe other concepts where possible.
In other words, while György Pólya[3] or Arthur Dempster - Glenn Shafer[4] have tried to extract,in probabilistic terms the belief function from the describable component. Here we accept the difference among the terms; we hierarchize them in strenght terms and we describe them with a method that is more general and not probability exclusive.
In our vision the concepts:

probable(P_r),

plausible(P_l),

credible(C_r),

possible(P_o),

have a decreasing strength, even if they all describe uncertain events. So a:

- possible event is not necessarily credible;
- credible event is surely possible;
- credible event is not necessarily plausible, but it is surely possible;
- plausible event is not necessarily probable, but it is surely credible and possible;
- probable event is surely plausible, credible and possible.

From now on, it will be necessary to indicate a characteristic multiple function for a process or phenomenon that is a combination of more functions (i.e. distribution):

F1 probability

F2 plausibility

F3 credibility

F4 possibility

where F_i are multiscale single-mode functions to 1 sum[5]. We conduct an analysis that starts from a different perspective which naturally emerges from artificial intelligence studies when it is desidered to extend decision-making capability of an artificial agent which operates in uncertain conditions, with increasing cognitive capacities which try to tend to human ones.

[3]Pólya's theory

[4]Dempster-Shafer's theory manages the distinction between uncertainty and ignorance and calculates a proposition evidence, in their work it is calculated the belief function Bel(X), that is the probability that a proposition is supported by evidence.

[5]When an E event is highly improbable i.e. P(E) $\leq$ 1%, P(E) $\leq$ 0,1%, P(E) $\leq$ 0,01% we are moving in the plausibility of improbable events. Then it is not important how the event is improbable, but how much the improbable event is plausible. Is similar for credibility and possibility.

With this assumption it is obvious that in uncertainty or incompleteness information conditions, the improbable events occurrence opens the space to emotions which significantly demarcate the passage from strict uncertainty methods (probability) to the uncertain events treatment under emotional effects action.
In this scenario the closer concept to probability is plausibility, even if plausibility and credibility both concern an estimation theory under emotional coditioning. For what concerns affectivity it tends to provide a subjective answer that aims to being objective. So we assign a probability to an event based on the several people opinion, in the belief case this comparison is not required, there is no subjective evaluation objectification. Belief brings out the personal and subjective evaluation to be shared, surely possible, but not necessarily sharable.
In other words, for the belief it is sufficient that the proposition to which a subjective probability is to be assigned is possible.
From these considerations it emerges that as the concept of probability is more stringent than plausibility one, in the first we assume that even if there is uncertainty, there is no emotionally driven assessment; instead in the plausibility definition, we associate an emotional component to uncertainty, but it represents the most people opinion. In the belief, emotionality make brings the ego out, that is a personal and subjective vision that does not require comparison with the community, but only needs the possibility.
We summarize this in the conceptual provocation:
"when the improbable is highly possible we should not be surprised if it is realized".
Formulated as the same way as a Edward Murphy[6] law it should be a scientific truth to all.
In other words the current price p(t) is a certain form of uniformity, while the trend p(t-1)-p(t) is a more or less reliable, but not sure source.
Using the Dempster-Shafer's (denoted as DS) theory rather than bayesian model allows to exceed the multidimensionality limit to sum one.
In other words, price growth possibility with a subjective probability estimated, does not imply that the decrescence has the complement to 1, so a decreasing trend probability is equal to one minus uptrend probability.
Thanks to the DS theory it is possible to give a probability to an E event and an E negation ($\neg E$), that could be another probability different from 1-P(E) with P(E)+P($\neg$E)$\leq$1.
In DS theory, C_r(E)$\leq$$P_l$(E) means that credible is less strong than plausible, because the plausibility is the probabilistic maximum value that can be allocated to E proposition, while belief is the probabilistic minimum assignable value to E, above all because for plausible we mean something believable for more people.

[6]Murphy law is a pseudo-scientific paradoxes set with an ironic character realised by Arthur Bloch with didactic phrases in statistical-mathematical form

Another difference element between our theory and DS theory is: while the DS theory provides a method to change previous opinions because of new beliefs, equally treating the new and previous evidence, our dynamic systems experience leads us to give more weight to the new evidence than old, to refining the cognitive process and a gradual approach to the solution.
In financial markets context, this means: if we do a price average for assuming a future price, then an average that weighs recent prices more than older ones will be more appropriate than an average that equally weighs new prices and older ones.

Nicholas Rescher's non-probabilistic theory of plausibility can be useful when there are equal reliability conflicting information (news or indicators).
As an example, think that a set of news or indicators tells us that the price of a financial instrument is destined to rise in a time frame and to fall in another one.
Thanks to the Rescher's theory, which is not sum 1, we can associate a value of bullish belief, for example 0.9, and a bearer of the same value, 0.9 too, since we have not bounded it by sum one total like it happens to sum one total theories as the probability one.
Consequently, the result is that the price will not certainly be stationary, so a financial instrument with a lateral behavior is to be excluded.

C. INFERENCE METHODS FOR FINANCIAL INSTRUMENTS DOMINATED BY UNCERTAINTY: INTERACTION BETWEEN FINANCIAL OPERATOR AND MARKET

When a financial operator joins the market, he performs an analysis (or someone does it for him) in order to decide whether to enter on a financial instrument, and what kind of position to take, in terms of size and of upward or downward directions.
The financial operator does not remain an observer, interacts with the financial market and disturbs it.
In other words, the analysis becomes an interaction with the market, once the operator decides to enter.
This reasoning recalls that of Richard Feynman[7], where he explains the quantum systems measurement process and the consequent Werner Karl Heisenberg's uncertainty principle[8].
About this Feynman explains that the fluctuations or uncertainties that introduce a description in terms of expectation and probability values to the state of a dynamic system, proposes the following example. Suppose that the dynamic system under study is an electron, or at least a particle of the microscopic world described through Quantum Mechanics. We suppose that on this dynamic system we want to make a position and speed (or momentum) measurement. Make a measurement means to send one or more photons (i.e. a beam) in order to know in each instant where the electron is and the its speed.
Feynman's observation is that by making this measure we are changing the state of motion of the electron system; we are perturbing it, because, to provide information about the electron speed and the electron position, the photon, bumping it will change its speed and its motion.
So, the effect of the measure represents a perturbation. The more we are sure of the position, having sent more photons to measure the position, the less we can say about the speed because doing the different measures we have perturbed (if not distorted) the electron motion and vice versa. The Heisenberg principle says that in the combined measure of position and momentum it is impossible to be more precise than a certain threshold value given by the reduced Planck constant[9] .
In the macroscopic scale reality, we understand that what explained by Quantum Mechanics does not respond to our common sense. Let the motion of a car, and we consider the measurement process link to a beam of photons that collide with the vehicle to know the position and the speed. It is evident that the photons can not disturb the vehicle motion state.
In this case the observer or measurer is unrelated to the vehicle motion and any uncertainty is generated.
For this reason the macroscopic reality is described in Classical Mechanics, starting from the Isaac Newton's principles of dynamics (inertia principle, action and reaction principle).
Many macroscopic dynamical systems have a similar behavior to quantum systems.
Considering the price dynamics of a financial instrument, it is clear that what Feynman explained is in complete analogy. If we replace the electron position with the price level of a financial instrument, we formulate an uncertainty financial markets principle similar to Heisenberg's principle. In this way it is impossible to certainty predict the price and volatility of a financial instrument in a future moment.
This because of the decision taken by the financial operator and therefore the entry to the market automatically could imply the market transition from one state to another one. A state could be different from the previous one depending on the operator size, (his liquidity introduced to the market) giving a direction to it.
Think of the effects caused by market mover (such as the effects of the decisions carried out by FED, ECB, BOE etc ...).
What has been described explains the need to place the financial markets in the complexity theory and to describe them with increasingly accurate tools which can look at

[7]Feynman's reasoning says that only a quantum system, where there is uncertainty or even worse ambiguity, can simulate the behavior of another quantum system.

[8]The Heisenberg principle establishes the limits in the knowledge of values that physical quantities associated with operators who do not interact with each other take on a physical system at the same time.

[9]Planck's constant, also called action quantum and denoted by h, is a physical constant that represents the elementary action quantity, determining that fundamental physical quantities do not evolve continuously, but are quantized, that is, they can only assume values multiples of this constant.

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uncertainty from different perspectives and not only from the probabilistic, plausibilistic, credibilistic or possibility.
Because all these visions in an instant, in different quantities, contribute to the formation of the price.
We intend to provide a further enrichment level for the MRQF theory [20] as a complementary work. For this aim, we work on the modeling of uncertainty, and on how the various probable, plausible, credible, possible models contribute to the construction of a market state function which absorbs the individual probability, plausibility, credibility, possibility functions. After that harmonizes them to allow the analyst to make inferences about future market conditions and to help him to make decisions about entering the market, in what dimensions, and whether to bull or bear.

D. UNCERTAINTY TREATMENT BY INFORMATION FUSION AND EXPECTATION FUNCTION MODELING

We define the expectation function a: $2^A \rightarrow [0,1]$ obtained from an appropriate functional composition in terms of P_r, P_l, C_r, P_o, such that a $(\emptyset) = 0$ and $\sum_{i=1}^{2} a(A_i) = 1$.

1) First model: Average model

We indicate with P1 = Pr, P2 = Pl, P3 = Cr, P4 = Po. The expectation function simplest model we can build is the following average model

$$a_1 = a_1(P_1, P_2, P_3, P_4) = \frac{1}{4}\sum_{i=1}^{4} P_i.$$

Evidently this model assigns the same importance to the different distributions of the probability, plausibility, credibility and possibility, so it is both an advantage and a disadvantage depending on how much expert the analyst is and how much we want to make expert the system. This model, as in the probability theory is used for calculate more alternative events expectation. Let us make a weather forecast example:

- P_1=place A forecasts of the military air force;
- P_2=place B forecasts by the sentiment;
- P_3=experts' forecast for place C;
- P_4=users' forecasts for place D;

This model can be used for calculate the expectation function for the event "it will rain in the place A or in the place B or in the place C or in the place D".

2) Second model: Product model

Another way of estimating the event expectation is through the Pi product, formally

$$a_2 = a_1(P_1, P_2, P_3, P_4) = \prod_{i=1}^{4} P_i$$

where $P_i : 2^A \rightarrow [0,1]$.
In general, we observe that

$$\prod_{i=1}^{4} P_i \leq \sum_{i=1}^{4} P_i.$$

To be able to say that $a_2 \leq a_1$, it is required a more detailed study that includes the P_i functional models, considering that the function a_1 has the normalization factor equal to $\frac{1}{4}$. In this model, as in the previous one, all the Pi have the same importance. Let us make another weather forecast example:

- P_1=military air force rain forecasts;
- P_2=sentiment for the wind;
- P_3=experts' forecast for the temperatures;
- P_4=users' forecasts for the humidity .

This model can be used for calculate the expectation function for the event "it will rain, the wind will blow at 31km/h, there will be 18 C° and the humidity will be 80%". It is important to give a different importance to the Probability, Plausibility, Credibility and Possibility; so, we define weights for each of these concepts.

3) Third model: Weighted average method

To extend the previous models we assume that not all P_i have the same importance in the expectation function construction. So, we build the following weighted average type model,

$$a_3 = \frac{(\sum_{i=1}^{4} \alpha_i P_i)}{\sum_{i=1}^{4} \alpha_i},$$

where α_i are weights relative to the different P_i.
An example could be to give a 50% of weight to P_1 that is: $\alpha_1 = 0,5$; 25% to P_2, so $\alpha_2 = 0,25$; 15% to P_3, so $\alpha_3 = 0,15$, and 10% to P_4 , $\alpha_4 = 0,1$;
in this way the expectation function is

$$a_3 = \frac{\sum_{i=1}^{4} \alpha_i P_i}{\sum_{i=1}^{4} \alpha_i} = 0,5P_1 + 0,25P_2 + 0,15P_3 + 0,1P_4.$$

4) Fourth model: Weighted product model

In analogy, to extend the product model for the expectation function, we consider the following weigthed product model

$$a_4 = \frac{\prod_{i=1}^{4} \alpha_i P_i}{\sum_{i=1}^{4} \alpha_i}.$$

Also in this way, we will have the possibility to weigh the different contribution of the probability, plausibility, credibility and possibility to the expectation function. In both the weighted models we have the constraint of

$$\sum_{i=1}^{4} a_i = 1.$$

This is because when we use an expectation function model, we are trying to rebuild a probability which had uncertain inputs.

5) Overlap model with shift based on probability

First and third models represent overlap models and weighted overlap of the different P_i contributions to the expectation function.
A further possible generalization could include a P_i hierarchical use.
Let P_r be the probability of the event, In detail we can see probability, plausibility, credibility and possibility as concepts to be used in descending order of importance.
For example we can consider

$$a_5 = \begin{cases} P_1 & \text{if } 1\% < P_r \leq 100\% \\ P_2 & \text{if } 0,1\% < P_r \leq 1\% \\ P_3 & \text{if } 0,01\% < P_r \leq 0,1\% \\ P_4 & \text{if } P_r \leq 0,01\%. \end{cases}$$

This would mean that until 1% of the probability distribution we will use only the classical probability(P1); beyond that probability we will define the improbable event, with a plausibility (P2) coefficient between 0% and 100% in the interval in which 1% $\leq P_r \leq$ 0,1%; similarly, however, 0.1% $< P_r \leq$ 0.01% we define the improbable and implausible event, with credibility (P3) between 0% and 100%; finally, for even lower values of probability we will have unlikely and implausible events, with a coefficient of possibility (P4) between 0% and 100%.
At the ends of these evaluations there is the certain event $P_1 = 100\%$ and the impossible one $P_1 = 0\%$:

$$P_1 : 2^A \rightarrow [0, 1].$$

6) Overlap model with shift based on Pi hierarchical

As an alternative to the previous model, without making to the probability play a pivotal role, but generalizing the Pi and evaluating, with own importance, also the emotional and cognitive elements, leads to the formulation of an expectation function which can be defined as

$$a_6 = \begin{cases} P_1 & \text{if } \overline{P}_{P_1} - 3\sigma_{P_1} \leq p_t \leq \overline{P}_{P_1} + 3\sigma_{P_1} \\ P_2 & \text{if } \overline{P}_{P_2} - 3\sigma_{P_2} \leq p_t \leq \overline{P}_{P_2} + 3\sigma_{P_2} \\ & \text{ and } p_t > \overline{P}_{P_1} + 3\sigma_{P_1} \\ P_3 & \text{if } \overline{P}_{P_3} - 3\sigma_{P_3} \leq p_t \leq \overline{P}_{P_3} + 3\sigma_{P_3} \\ & \text{ and } p_t > \overline{P}_{P_2} + 3\sigma_{P_2} \\ P_4 & \text{if } \overline{P}_{P_4} - 3\sigma_{P_4} \leq p_t \leq \overline{P}_{P_4} + 3\sigma_{P_4} \\ & \text{ and } p_t > \overline{P}_{P_3} + 3\sigma_{P_3}. \end{cases}$$

Where $\overline{P}$ is the average price on the considered set of data and σ is the standard deviation; the subscript P_i indicates with which distribution $\overline{P}$ and σ are calculated.
From the provided models, we understand how general the issue of decisions in uncertainty conditions is and how easy it is to build models using the expectation function which describe the inference processes in info-incompleteness and input uncertainty terms.
The proposed reasoning theory is a meta model where defining expectation function models makes it possible to construct new and more opportune operative models contextualised to a cognitive and informative domain.

7) Model based on Dempster's composition rules

Another significant way to construct the expectation function is to use the rule that Dempster has defined for the plausibility, extending it to the application of Pi case calculating the mass (m) function, the belief(bel) function and the Dempster's plausibility(we call it Dpl) for each P:

- $m(P_1)$, relative $bel(P_1)$ and $Dpl(P_1)$;
- $m(P_2)$, relative $bel(P_2)$ and $Dpl(P_2)$;
- $m(P_3)$, relative $bel(P_3)$ and $Dpl(P_3)$;
- $m(P_4)$, relative $bel(P_4)$ and $Dpl(P_4)$;

And finally compose it with the Dempster's composition rules.

E. RELIABILITY

In classical reasoning, the method, the element or the system reliability, is the probability of not breaking for a certain period t, under defined operating and environmental conditions; i.e. reliability definition is in frequentistic terms:

$$R = \frac{nf}{n},$$

with nf number of times that the method, element or system has worked, and n tests number and samples used for the test. By using our approach, reliability is obtained by changing the probability P with the expectation function a. Similarly we can define unreliability as Fault

F=1-R.

So we are in a 1-dimensional one-sum logic and therefore R and F are time functions.
The reliability is linked to the number of successes or failures obtained after a t time.
Formally the reliability R is the following time function:

$$R : T \rightarrow [0, 1].$$

We also define average operating time of the method, element, system, or mean time to failure (mtf) the following quantity

$$mtF = 1/\lambda,$$

with λ number of faults / failures at t time.
In our study we considered R = R (T); in general R = R (T, C, A) where t is the time, C is the method for judging successes and failures and A are environmental conditions.
This reliability concept can be applied as an estimator to the probability, plausibility, credibility and possibility functions, in which backtesting can be seen how the different inferences worked.
When we do not refer to the method but to the result, we need to introduce the trustworthiness concept.

F. TRUSTWORTHINESS

In some contexts, simple reliability and trustworthiness, as well as probability, plausibility, credibility and possibility, can be considered synonyms, but this is not the case of domains and events where complexity theory is required.
After setting the measurement conditions, the trustworthiness indicates the costancy of a set of results.
With this definition we understand how in an uncertainty context, in terms of probabilities and distributions which are

dependent on emotion (such as plausibility and credibility), the reliability can precisely express how the same dynamics are differently considered.
For example, a trader with the same trend at different times may feel the need to act / react differently.
At this point the result trustworthiness acquires a different, but equally important role from the previous concepts.
Trustworthiness is not reliability, accuracy or validity of a data synonymous; but we will say that an estimate is trustworthy if the results remain constant over time.
If this does not happen, we will say that the estimation or the result is untrustworthiness.
There are several methods to estimate trustworthiness; we will use the Pearson[10] correlation index which is the basis of the various reliability estimates.
The Pearson correlation index between two variables x and y is

$r = \frac{\sigma x y}{\sigma x \sigma y}$ with -1≤r< 1

where σ x y denote the covariance and σ x σ y the standard deviations of x and y respectively.
We distinguish the following cases:
$r < 0$: related variables
$r = 0$: non related variables
$r > 0$: anticorrelated variables
$0 < r < 0.3$: weakly correlated variables
$0.3 < r < 0.7$: moderately correlated variables
$r > 0.7$: highly correlated variables
and similarly for anti-correlation cases.
Generally in the case of more than two variables, the Pearson index will become

$$r = r_{ij}.$$

We have a correlation matrix of r rows and r symmetric column $r_{i_j} = r_{j_i}$ with coefficients on the diagonal equal to 1, as $r_{i_i} = \sigma_{i_i} / \sigma_i^2$.
For example, in the financial case we can estimate the reliability of two or more periods of results.

IV. SIMULATION

In this section we will describe a simulation based on the described theory. This simulation has been developed in two fields:

- Biometric fusion choiches,
- Sport odds.

For each field, we suppose to have different features to be applied at probability, plausibility, credibility and possibility. Our aim is to calculate how much these features weight in the formula:

$$a_3 = \frac{\sum_{i=1}^{4} \alpha_i P_i}{\sum_{i=1}^{4} \alpha_i}.$$

In particular in the biometrics field we have:

[10]The Pearson correlation index between two statistical variables is defined as their covariance divided by the standard deviations product of the two variables and expresses a possible linearity relationship between them.

	Fingerprint	Face	Iris	Retina
Accuracy	90	76	93	94
	Vascul.	Ear	Voice	Signature
Accuracy	96	74	66	67
	Hand	Palm	BodyMot.	DNA
Accuracy	67	88	78	98

TABLE 1: Biometrics accuracy

	Fingerprint	Iris	Face
Accuracy	90	93	76

TABLE 2: Selected biometrics

- probability: the biometrics statistical reliability that is in the [0,1] interval,
- plausibility: the biometrics sentiment that is in the [0,1] interval,
- credibility: the awareness and reliability of the recognition algorithm that is in the interval [0,1],
- possibility: the system fault statistic.

In Sport odds, all the features are in the [0,1] interval:

- probability: the bookmaker quotes average,
- plausibility: the experts opinions,
- credibility: the sentiment analysis,
- possibility: the users forecasts.

The simulation has been developed with r studio; we created the two simulated datasets[11] and they have been disturbed in order to simulate uncertainty.

A. BIOMETRICS SIMULATION

When we fuse biometrics in a multibiometric system, we are dealing with a decision-making problem. The decisions can be joined by several methods: voting method, where, each classifier votes for a class, then the pattern will be assigned to the most voted class, we focused on this method; Gupta, Singh Walia and Sharma [22] propose a score based fusion method; Divyakant et al. [21] describe other fusion methods. We initialize each biometric with the related accuracy:

Delac et al. [23] and Tripathi [24] calculated the biometrics reliability; we have extracted the biometrics accuracies from these works (table 1). Suppose to build a multibiometrics system that use the most plausible biometrics in terms of accuracy and user-acceptability (user acceptability is described by Tripathi [24] and Rui [25]). We select:

- Fingerprint,
- Iris,
- Face.

In the table 2, we can see the selected biometrics accuracy.

After that, we initialized three algorithms for each biometrics and relative success rate: random high, medium and lower rate. The success rate can be found in table 3.

[11]one field, one dataset

	Fingerprint	Iris	Face
A1	85	93	73
A2	76	88	71
A3	90	85	70

TABLE 3: Supposed algorithm success rate

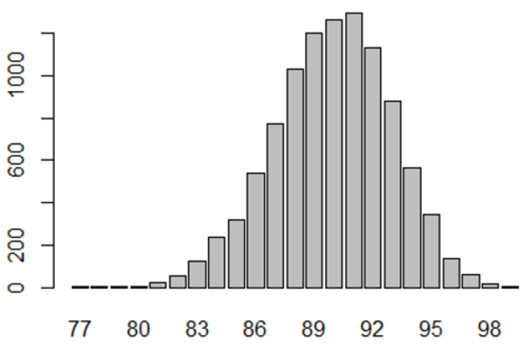

FIGURE 2: Multibiometric system distribution

We simulated this multibiometric system 10000 times with 100 hypothetical positive subjects and we obtained the distribution in fig. 2.

The skewness of this distribution is 0.6456361, which represents an asimmetric distribution. The kurtosis is -1.261064 which represents a platykurtic distribution. These results follows because of randomic selection of positive or negative biometrics system's choice.

B. ADDING UNCERTAINTY

Now we add uncertainty to the multibiometric system's probability. We estimate it having a combination of:

- biometrics reliability estimation (10% of uncertainty factor), in the figure 3 we can found the distribution of this feature,
- sentiment analysis on biometrics (30% of uncertainty factor[12]), in the figure 4 we can found the distribution of this feature,
- Awareness and reliability of the recognition algorithm (no uncertainty, we have a well defined statistic when we develop a biometric algorithm), in the figure 5 we can found the distribution of this feature,
- system fault statistic (in which there is not uncertainty, but there are few experiments), in the figure 6 we can found the distribution of this feature.

So, we simulated the system 300 times for 100 hypothetical recognized subjects. The simulated result (Fig. 7) have a skewness of 0.09512331 and a kurtosis of -1.369442 that means we are in presence of heavy tails in this distribution. In the following paragraph we used these simulated results as dataset in order to calculate the weights of Pr,Pl,Cr and Po.

[12]Default users are not expert and we have few data on this argument

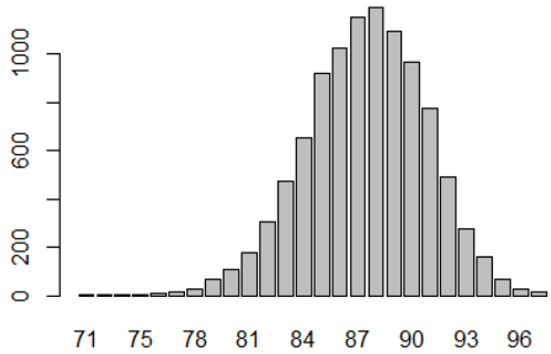

FIGURE 3: Multibiometric system distribution based only on reliability, the skewness is 0.69 and the kurtosis is -1.20

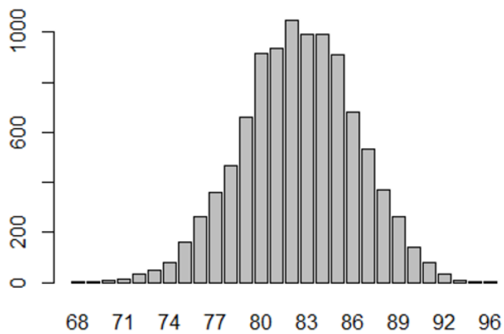

FIGURE 4: Multibiometric system distribution based only on sentiment analysis, the skewness is 0.64 and the kurtosis is -1.24

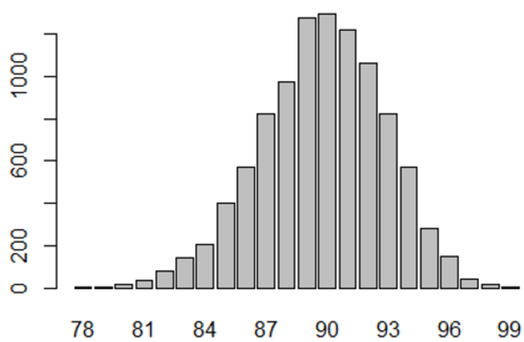

FIGURE 5: Multibiometric system distribution based only on the best three algorithms, the skewness is 0.58 and the kurtosis is -1.04

C. WEIGHT CALCULATION

To calculate the weights, we based the method on the mean square deviations by backtesting the uncertainty data with the real probability data. The values which weigh most are those that have the average difference closest to the real probability, after that, we calculated the scraps and the distribution of the weights. Let P1=Pr, P2=Pl, P3=Cr, P4=Po and a_i the related

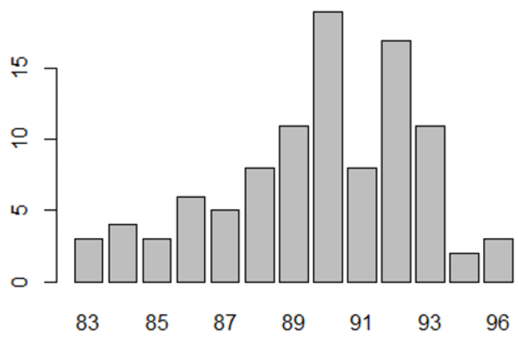

FIGURE 6: Multibiometric system distribution based only on system faults, the skewness is 0.83 and the kurtosis is -0.69

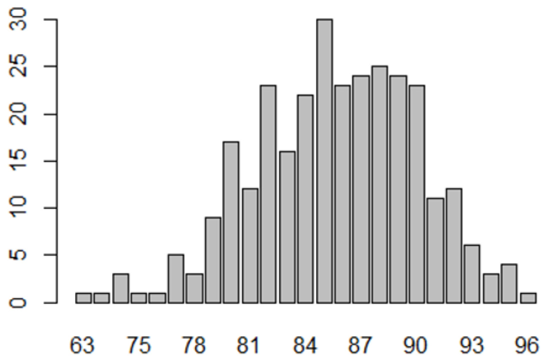

FIGURE 7: Multibiometric system distribution with all P_r, P_l, C_r, P_o; there are few experiments because of the bound with the system faults feature.

weight, let x_j the uncertain value and p the real probability, then:

$$\sigma_i = \sqrt{\frac{\sum_{j=1}^{N}(x_j - p)^2}{N}}$$

. Then we normalize the results:

$$a_i = \frac{\sigma_i}{\sum_{j=1}^{4} \sigma j}$$

The results are showed in the table 4.

So, the best feature for the biometrics area is the biometric reliability estimation. After this, we are going to calculate the standard deviations between the real probability and the estimated expectation in 300 simulations. Thanks to this simulation we have recovered an approximation of the real probability (expectation) of the multibiometrics system, we remember that the real probability is 88%, the expectation calculated using the retrivied weights with the a_3 formula

	Pr	Pl	Cr	Po
Weight	0.58%	0.13%	0.15%	0.14%

TABLE 4: Weights in the Biometrics simulation

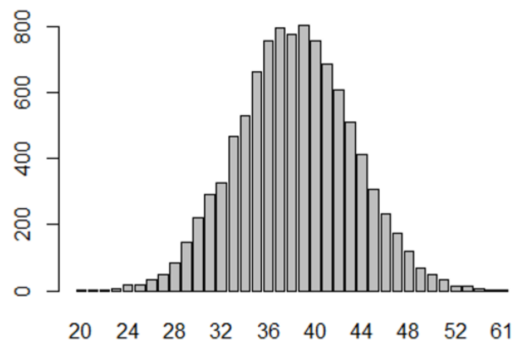

FIGURE 8: Victory of the first team distribution

$$a_3 = \frac{\sum_{i=1}^{4} \alpha_i P_i}{\sum_{i=1}^{4} \alpha_i},$$

for the first individual is 91.33%. For 300 individuals, the standard deviation between real probability and estimated expectation is 2.52%.

D. SPORTS ODDS SIMULATION

Suppose we want to estimate the probability of a result in a sport event as a soccer match. In our simulation we figured the strenght of the italian teams basing on the number of achieved points in the Serie A season 2017/2018.

We created a function for quotas calculation. In this function, factors as home-away factor, difference of points and team form status are used in order to calculate quotas.

We assigned the expectation features in this way, all the following features are in the [0,1] interval:

- probability: the bookmaker forecasts,
- plausibility: the expert opinions,
- credibility: the sentiment analysis,
- possibility: the users forecasts.

We simulated the match between the leader of the league team and second one 10000 for 100 times. We took as reference the victory of the first classificated team (fig. 8).

The skewness is 0.6705781, which represents an right asimmetric distribution. The kurtosis is -1.160554, which represents a platykurtic distribution. These results follows because of randomic, but weighted with team strenght, selection of the match result.

E. ADDING UNCERTAINTY

Now we add uncertainty to the victory probability of the leader team, we estimate it having a combination of:

- the bookmaker forecasts (10% of uncertainty factor on the team form and -7% of bookmaker work fee), in the figure 9 we can found the distribution of this feature,
- expert opinions (20% of uncertainty factor on the team form), in the figure 12 we can found the distribution of this feature,
- sentiment analysis: combination between bookmaker forecasts and 10% of uncertainty on the team form be-

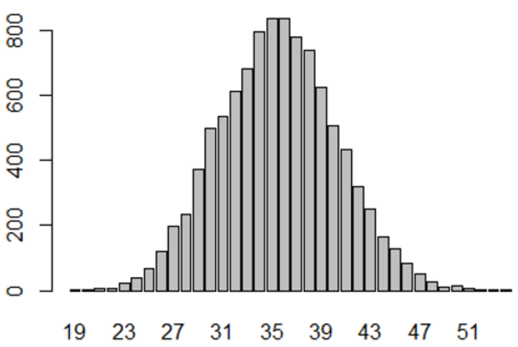

FIGURE 9: Victory of the first team distribution related to bookmakers forecasts, the skewness is 0.65 and the kurtosis is -1.17

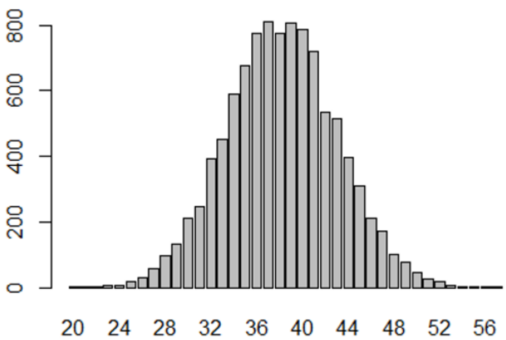

FIGURE 10: Victory of the first team distribution related to sentiment, the skewness is 0.74 and the kurtosis is -1.07

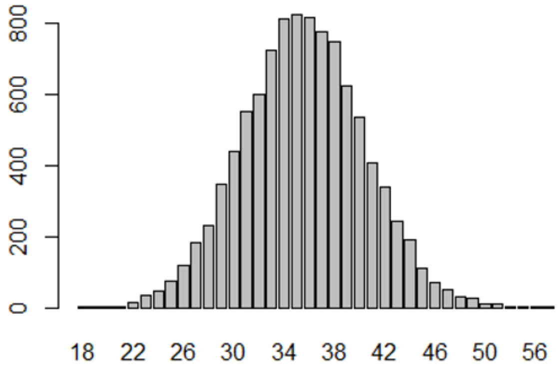

FIGURE 11: Victory of the first team distribution related to users forecasts, the skewness is 0.74 and the kurtosis is -1.05

cause the collective opinion is influenced by bookmakers quotas, in the figure 10 we can found the distribution of this feature,

- users forecasts (random uncertainty factor of team forms combined with bookmaker quotas). The random factor is in the interval 0-50%, in the figure 11 we can found the distribution of this feature.

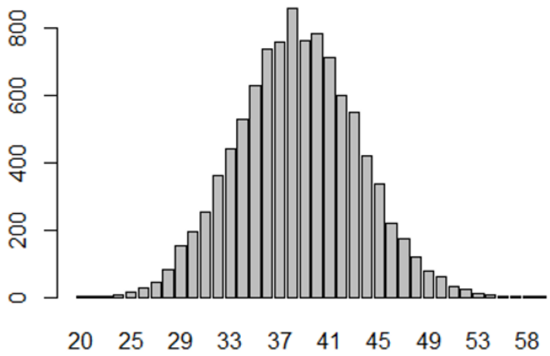

FIGURE 12: Victory of the first team distribution, related to experts forecasts, the skewness is 0.72 and the kurtosis is -1.07

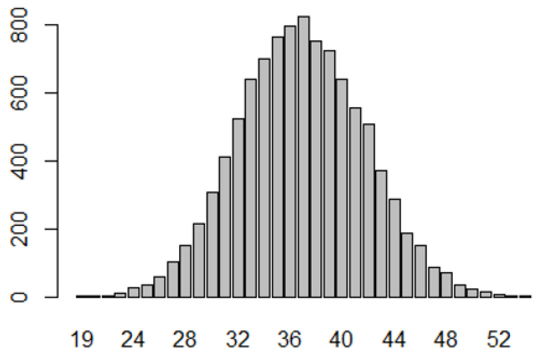

FIGURE 13: Victory of the first team distribution with uncertainty

So, we simulated the system 10000 times adding uncertainty (fig. 13).

The simulated result have a skewness of 0.5768954 and a kurtosis of -1.315472 so, we are in presence of heavy tails in this distribution.

F. WEIGHT CALCULATION

We calculated the weights using means of standard deviations method, doing backtesting on simulated data. We assign a weight for each feature based on standard deviation, the more the feature is far from the real probability, the less it weights in the formula. The results are shown in the table 4.

So, the best feature for the sports odds field is the sentiment estimation. After this, we are going to calculate the standard deviations between the real probability and the estimated expectation in 10000 simulations. Thanks to this simulation

	Pr	Pl	Cr	Po
Weight	0.06%	0.45%	0.46%	0.03%

TABLE 5: Weights in the sport simulation

we have recovered an approximation of the real probability (expectation) of the leader team victory, we remember that the real probability of this event is 38,83%, the expectation calculated using the retrivied weights with the a_3 formula

$$a_3 = \frac{\sum_{i=1}^{4} \alpha_i P_i}{\sum_{i=1}^{4} \alpha_i},$$

for the first simulation is 38.61%. For 10000 simulations, the standard deviation between real probability and estimated expectation is 0.25%.

V. CONCLUSIONS AND FUTURE WORK

In this work, we extended the Dempster Shafer's theory introducing the novel concept of Credibility and Possibility in addition to the already existing Probability and Plausibility ones and we combined them in several models which can be applied to real problems. Thanks to the information fusion, we obtained a versatile environment where the weights of the objective function a can be modelled. We proposed seven models which extends and includes the classical probability by giving a more suitable tool for describing and forecasting events which happens more frequently than their probability and in which there are uncertainty conditions. These models are generalizable, so they could be applied to all the fields in which there is uncertainty on data input using an historical dataset as test for weight calculation. For example, in the financial field, we can assign to Pr = fundamental analysis; Pl = technical analysis; Cr = experts' opinions; Po = sentiment analysis. Select the data and calculate the weights for the expectation function. Another example could be the forecast of success of a product; in this case, we could assign: Pr = success rate of products of the same category; Pl = sentiment analysis; Cr = experts' opinions; Po = customers opinions. In the future we will apply this models in the financial field with historical data. Again, in the financial area, we are developing a Token Evaluation System which permit to evaluate digital assets like tokens on the Blockchain and which is based on the proposed theories.

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