Degrees of Membership > 1 and < 0 of the Elements With Respect to a Neutrosophic OffSet

Prof. Florentin Smarandache, Ph D
University of New Mexico
Mathematics & Science Department
705 Gurley Ave., Gallup, NM 87301, USA

E-mail: smarand@unm.edu

Abstract. We have defined the *Neutrosophic Over-/Under-/Off-Set* and *-Logic* for the first time in 1995 and published in 2007. During 1995-2016 we presented them to various national and international conferences and seminars ([13]-[34]) and did more publishing during 2007-2016 ([1]-[9]). These new notions are totally different from other sets/logics/probabilities.

We extended the neutrosophic set respectively to *Neutrosophic Overset* {when some neutrosophic component is > 1}, to *Neutrosophic Underset* {when some neutrosophic component is < 0}, and to *Neutrosophic Offset* {when some neutrosophic components are off the interval [0, 1], i.e. some neutrosophic component > 1 and other neutrosophic component < 0}.

This is no surprise since our real-world has numerous examples and applications of over-/under-/off-neutrosophic components.

Keywords: neutrosophic overset, neutrosophic underset, neutrosophic offset, neutrosophic overlogic, neutrosophic underlogic, neutrosophic offlogic, neutrosophic overprobability, neutrosophic underprobability, neutrosophic offprobability, overmembership (membership degree > 1), undermembership (membership degree < 0), offmembership (membership degree off the interval [0, 1]).

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1. Introduction.

In the classical set and logic theories, in the fuzzy set and logic, and in intuitionistic fuzzy set and logic, the degree of membership and degree of nonmembership have to belong to, or be included in, the interval [0, 1]. Similarly, in the classical probability and in imprecise probability the probability of an event has to belong to, or respectively be included in, the interval [0, 1].

Yet, we have observed and presented to many conferences and seminars around the globe {see [13]-[34]} and published {see [1]-[9]} that in our real world there are many cases when the degree of membership is greater than 1. The set, which has elements whose membership is over 1, we called it *Overset*.

Even worst, we observed elements whose membership with respect to a set is under 0, and we called it *Underset*.

In general, a set that has elements whose membership is above 1 and elements whose membership is below 0, we called it *Offset* (i.e. there are elements whose memberships are off (over and under) the interval [0, 1]).

"Neutrosophic" means based on three components *T (truth-membership)*, *I (indeterminacy)*, and *F (falsehood-nonmembership)*. And "over" means above 1, "under" means below 0, while "offset" means behind/beside the set on both sides of the interval [0, 1], over and under, more and less, supra and below, out of, off the set. Similarly, for "offlogic", "offmeasure", "offprobability", "offstatistics" etc.

It is like a pot with boiling liquid, on a gas stove, when the liquid swells up and leaks out of pot. The pot (the interval [0, 1]) can no longer contain all liquid (i.e., all neutrosophic truth / indeterminate / falsehood values), and therefore some of them fall out of the pot (i.e., one gets neutrosophic truth / indeterminate / falsehood values which are > 1), or the pot cracks on the bottom and the liquid pours down (i.e., one gets neutrosophic truth / indeterminate / falsehood values which are < 0).

Mathematically, they mean getting values off the interval [0, 1].

The American aphorism "think outside the box" has a perfect resonance to the neutrosophic offset, where the box is the interval [0, 1], yet values outside of this interval are permitted.

2. Example of Overmembership and Undermembership.

In a given company a full-time employer works 40 hours per week. Let's consider the last week period.

Helen worked part-time, only 30 hours, and the other 10 hours she was absent without payment; hence, her membership degree was 30/40 = 0.75 < 1.

John worked full-time, 40 hours, so he had the membership degree 40/40 = 1, with respect to this company.

But George worked overtime 5 hours, so his membership degree was (40+5)/40 = 45/40 = 1.125 > 1. Thus, we need to make distinction between employees who work overtime, and those who work full-time or part-time. That's why we need to associate a degree of membership strictly greater than 1 to the overtime workers.

Now, another employee, Jane, was absent without pay for the whole week, so her degree of membership was 0/40 = 0.

Yet, Richard, who was also hired as a full-time, not only didn't come to work last week at all (0 worked hours), but he produced, by accidentally starting a devastating fire, much damage to the company, which was estimated at a value half of his salary (i.e. as he would have gotten for working 20 hours that week). Therefore, his membership degree has to be less that Jane's (since Jane produced no damage). Whence, Richard's degree of membership, with respect to this company, was - 20/40 = - 0.50 < 0.

Consequently, we need to make distinction between employees who produce damage, and those who produce profit, or produce neither damage no profit to the company.

Therefore, the membership degrees > 1 and < 0 are real in our world, so we have to take them into consideration.

Then, similarly, the Neutrosophic Logic/Measure/Probability/Statistics etc. were extended to respectively *Neutrosophic Over-/Unde-r/Off-Logic, -Measure, -Probability, -Statistics* etc. [Smarandache, 2007].

3. Another Example of Membership Above 1 and Membership Below 0.

Let's consider a spy agency $S = \{S_1, S_2, \ldots, S_{1000}\}$ of a country Atara against its enemy country Batara. Each agent S_j, $j \in \{1, 2, \ldots, 1000\}$, was required last week to accomplish 5 missions, which represent the full-time contribution/membership.

Last week agent S_{27} has successfully accomplished his 5 missions, so his membership was $T(A_{27}) = 5/5 = 1 = 100\%$ (full-time membership).

Agent S_{32} has accomplished only 3 missions, so his membership is $T(S_{32}) = 3/5 = 0.6 = 60\%$ (part-time membership).

Agent S_{41} was absent, without pay, due to his health problems; thus $T(S_{41}) = 0/5 = 0 = 0\%$ (null-membership).

Agent S_{53} has successfully accomplished his 5 required missions, plus an extra mission of another agent that was absent due to sickness, therefore $T(S_{53}) = (5+1)/5 = 6/5 = 1.2 > 1$ (therefore, he has membership above 1, called over-membership).

Yet, agent S_{75} is a double-agent, and he leaks highly confidential information about country Atara to the enemy country Batara, while simultaneously providing misleading information to the country Atara about the enemy country Batara. Therefore S_{75} is a negative agent with respect to his country Atara, since he produces damage to Atara, he was estimated to having intentionally done wrongly all his 5 missions, in addition of compromising a mission of another agent of country Atara, thus his membership $T(S_{75}) = -(5+1)/5 = -6/5 = -1.2 < 0$ (therefore, he has a membership below 0, called under-membership).

4. Definition of Single-Valued Neutrosophic Overset.

Let U be a universe of discourse and the neutrosophic set $A_1 \subset U$.

Let T(x), I(x), F(x) be the functions that describe the degrees of membership, indeterminate-membership, and nonmembership respectively, of a generic element $x \in U$, with respect to the neutrosophic set A_1:

$T(x), I(x), F(x) : U \rightarrow [0, \Omega]$

where $0 < 1 < \Omega$, and Ω is called overlimit,

$T(x), I(x), F(x) \in [0, \Omega]$.

A Single-Valued Neutrosophic Overset A_1 is defined as:

$A_1 = \{(x, <T(x), I(x), F(x)>), x \in U\}$,

such that there exists at least one element in A_1 that has at least one neutrosophic component that is > 1, and no element has neutrosophic components that are < 0.

For **example**: $A_1 = \{(x_1, <1.3, 0.5, 0.1>), (x_2, <0.2, 1.1, 0.2>)\}$, since $T(x_1) = 1.3 > 1$, $I(x_2) = 1.1 > 1$, and no neutrosophic component is < 0.

Also $O_2 = \{(a, <0.3, -0.1, 1.1>)\}$, since $I(a) = -0.1 < 0$ and $F(a) = 1.1 > 1$.

5. Definition of Single-Valued Neutrosophic Underset.

Let U be a universe of discourse and the neutrosophic set $A_2 \subset U$.

Let T(x), I(x), F(x) be the functions that describe the degrees of membership, indeterminate-membership, and nonmembership respectively, of a generic element $x \in U$, with respect to the neutrosophic set A_2:

$T(x), I(x), F(x) : U \rightarrow [\Psi,1]$

where $\Psi < 0 < 1$, and Ψ is called underlimit,

$T(x), I(x), F(x) \in [\Psi,1]$.

A Single-Valued Neutrosophic Underset A_2 is defined as:

$A_2 = \{(x, <T(x), I(x), F(x)>), x \in U\}$,

such that there exists at least one element in A_2 that has at least one neutrosophic component that is < 0, and no element has neutrosophic components that are > 1.

For **example**: $A_2 = \{(x_1, <-0.4, 0.5, 0.3>), (x_2, <0.2, 0.5, -0.2>)\}$, since $T(x_1) = -0.4 < 0$, $F(x_2) = -0.2 < 0$, and no neutrosophic component is > 1.

6. Definition of Single-Valued Neutrosophic Offset.

Let U be a universe of discourse and the neutrosophic set $A_3 \subset U$.

Let T(x), I(x), F(x) be the functions that describe the degrees of membership, indeterminate-membership, and nonmembership respectively, of a generic element $x \in U$, with respect to the set A_3:

$T(x), I(x), F(x) : U \rightarrow [\Psi,\Omega]$

where $\Psi < 0 < 1 < \Omega$, and Ψ is called underlimit, while Ω is called overlimit,

$T(x), I(x), F(x) \in [\Psi,\Omega]$.

A Single-Valued Neutrosophic Offset A_3 is defined as:

$A_3 = \{(x, <T(x), I(x), F(x)>), x \in U\}$,

such that there exist some elements in A_3 that have at least one neutrosophic component that is > 1, and at least another neutrosophic component that is < 0.

For **examples**: $A_3 = \{(x_1, <1.2, 0.4, 0.1>), (x_2, <0.2, 0.3, -0.7>)\}$, since $T(x_1) = 1.2 > 1$ and $F(x_2) = -0.7 < 0$.

Also $B_3 = \{(a, <0.3, -0.1, 1.1>)\}$, since $I(a) = -0.1 < 0$ and $F(a) = 1.1 > 1$.

7. Single Valued Neutrosophic Overset / Underset / Offset Operators.

Let U be a universe of discourse and $A = \{(x, <T_A(x), I_A(x), F_A(x)>), x \in U\}$ and

and $B = \{(x, <T_B(x), I_B(x), F_B(x)>), x \in U\}$ be two single-valued neutrosophic oversets / undersets / offsets.

$T_A(x), I_A(x), F_A(x), T_B(x), I_B(x), F_B(x): U \to [\Psi, \Omega]$

where $\Psi \le 0 < 1 \le \Omega$, and Ψ is called underlimit, while Ω is called overlimit,

$T_A(x), I_A(x), F_A(x), T_B(x), I_B(x), F_B(x) \in [\Psi, \Omega]$.

We take the inequality sign ≤ instead of < on both extremes above, in order to comprise all three cases: overset {when $\Psi = 0$, and $1 < \Omega$}, underset {when $\Psi < 0$, and $1 = \Omega$}, and offset {when $\Psi < 0$, and $1 < \Omega$}.

7.1. Single Valued Neutrosophic Overset / Underset / Offset Union.

Then $A \cup B = \{(x, <\max\{T_A(x), T_B(x)\}, \min\{I_A(x), I_B(x)\}, \min\{F_A(x), F_B(x)\}>), x \in U\}$

7.2. Single Valued Neutrosophic Overset / Underset / Offset Intersection.

Then $A \cap B = \{(x, <\min\{T_A(x), T_B(x)\}, \max\{I_A(x), I_B(x)\}, \max\{F_A(x), F_B(x)\}>), x \in U\}$

7.3. Single Valued Neutrosophic Overset / Underset / Offset Complement.

The neutrosophic complement of the neutrosophic set A is

$C(A) = \{(x, <F_A(x), \Psi + \Omega - I_A(x), T_A(x)>), x \in U\}$.

8. Definition of Interval-Valued Neutrosophic Overset.

Let U be a universe of discourse and the neutrosophic set $A_1 \subset U$.

Let T(x), I(x), F(x) be the functions that describe the degrees of membership, indeterminate-membership, and nonmembership respectively, of a generic element $x \in U$, with respect to the neutrosophic set A_1:

$T(x), I(x), F(x) : U \to P([0, \Omega])$,

where $0 < 1 < \Omega$, and Ω is called overlimit,

$T(x), I(x), F(x) \subseteq [0, \Omega]$, and $P([0, \Omega])$ is the set of all subsets of $[0, \Omega]$.

An Interval-Valued Neutrosophic Overset A_1 is defined as:

$A_1 = \{(x, <T(x), I(x), F(x)>), x \in U\}$,

such that there exists at least one element in A_1 that has at least one neutrosophic component that is partially or totally above 1, and no element has neutrosophic components that is partially or totally below 0.

For **example**: $A_1 = \{(x_1, <(1, 1.4], 0.1, 0.2>), (x_2, <0.2, [0.9, 1.1], 0.2>)\}$, since $T(x_1) = (1, 1.4]$ is totally above 1, $I(x_2) = [0.9, 1.1]$ is partially above 1, and no neutrosophic component is partially or totally below 0.

9. Definition of Interval-Valued Neutrosophic Underset.

Let U be a universe of discourse and the neutrosophic set $A_2 \subset U$.

Let T(x), I(x), F(x) be the functions that describe the degrees of membership, indeterminate-membership, and nonmembership respectively, of a generic element $x \in U$, with respect to the neutrosophic set A_2:

$T(x), I(x), F(x) : U \rightarrow [\Psi,1]$,

where $\Psi < 0 < 1$, and Ψ is called underlimit,

$T(x), I(x), F(x) \subseteq [\Psi,1]$, and $P([\Psi,1])$ is the set of all subsets of $[\Psi,1]$.

An Interval-Valued Neutrosophic Underset A_2 is defined as:

$A_2 = \{(x, <T(x), I(x), F(x)>), x \in U\}$,

such that there exists at least one element in A_2 that has at least one neutrosophic component that is partially or totally below 0, and no element has neutrosophic components that are partially or totally above 1.

For **example**: $A_2 = \{(x_1, <(-0.5,-0.4), 0.6, 0.3>), (x_2, <0.2, 0.5, [-0.2, 0.2]>)\}$, since $T(x_1) = (-0.5, -0.4)$ is totally below 0, $F(x_2) = [-0.2, 0.2]$ is partially below 0, and no neutrosophic component is partially or totally above 1.

10. Definition of Interval-Valued Neutrosophic Offset.

Let U be a universe of discourse and the neutrosophic set $A_3 \subset U$.

Let T(x), I(x), F(x) be the functions that describe the degrees of membership, indeterminate-membership, and nonmembership respectively, of a generic element $x \in U$, with respect to the set A_3:

$T(x), I(x), F(x) : U \rightarrow P([\Psi,\Omega])$,

where $\Psi < 0 < 1 < \Omega$, and Ψ is called underlimit, while Ω is called overlimit,

$T(x), I(x), F(x) \subseteq [\Psi,\Omega]$, and $P([\Psi,\Omega])$ is the set of all subsets of $[\Psi,\Omega]$.

An Interval-Valued Neutrosophic Offset A_3 is defined as:

$A_3 = \{(x, <T(x), I(x), F(x)>), x \in U\}$,

such that there exist some elements in A_3 that have at least one neutrosophic component that is partially or totally above 1, and at least another neutrosophic component that is partially or totally below 0.

For **examples**: $A_3 = \{(x_1, <[1.1, 1.2], 0.4, 0.1>), (x_2, <0.2, 0.3, (-0.7, -0.3)>)\}$, since $T(x_1) = [1.1, 1.2]$ that is totally above 1, and $F(x_2) = (-0.7, -0.3)$ that is totally below 0.

Also $B_3 = \{(a, <0.3, [-0.1, 0.1], [1.05, 1.10]>)\}$, since $I(a) = [-0.1, 0.1]$ that is partially below 0, and $F(a) = [1.05, 1.10]$ that is totally above 1.

11. Interval-Valued Neutrosophic Overset / Underset / Offset Operators.

Let U be a universe of discourse and $A = \{(x, <T_A(x), I_A(x), F_A(x)>), x \in U\}$

and $B = \{(x, <T_B(x), I_B(x), F_B(x)>), x \in U\}$ be two interval-valued neutrosophic oversets / undersets / offsets.

$T_A(x), I_A(x), F_A(x), T_B(x), I_B(x), F_B(x): U \rightarrow P([\Psi, \Omega])$,

where $P([\Psi, \Omega])$ means the set of all subsets of $[\Psi, \Omega]$,

and $T_A(x), I_A(x), F_A(x), T_B(x), I_B(x), F_B(x) \subseteq [\Psi, \Omega]$,

with $\Psi \leq 0 < 1 \leq \Omega$, and Ψ is called underlimit, while Ω is called overlimit.

We take the inequality sign $\leq$ instead of $<$ on both extremes above, in order to comprise all three cases: overset {when $\Psi = 0$, and $1 < \Omega$}, underset {when $\Psi < 0$, and $1 = \Omega$}, and offset {when $\Psi < 0$, and $1 < \Omega$}.

11.1. Interval-Valued Neutrosophic Overset / Underset / Offset Union.

Then $A \cup B =$

$\{(x, <[\max\{\inf(T_A(x)), \inf(T_B(x))\}, \max\{\sup(T_A(x)), \sup(T_B(x))\}],$

$[\min\{\inf(I_A(x)), \inf(I_B(x))\}, \min\{\sup(I_A(x)), \sup(I_B(x))\}],$

$[\min\{\inf(F_A(x)), \inf(F_B(x))\}, \min\{\sup(F_A(x)), \sup(F_B(x))\}]>, x \in U\}.$

11.2. Interval-Valued Neutrosophic Overset / Underset / Offset Intersection.

Then $A \cap B =$

$\{(x, <[\min\{\inf(T_A(x)), \inf(T_B(x))\}, \min\{\sup(T_A(x)), \sup(T_B(x))\}],$

$[\max\{\inf(I_A(x)), \inf(I_B(x))\}, \max\{\sup(I_A(x)), \sup(I_B(x))\}],$

$[\max\{\inf(F_A(x)), \inf(F_B(x))\}, \max\{\sup(F_A(x)), \sup(F_B(x))\}]>, x \in U\}.$

11.3. Interval-Valued Neutrosophic Overset / Underset / Offset Complement.

The complement of the neutrosophic set A is

$C(A) = \{(x, <F_A(x), [\Psi + \Omega - \sup\{I_A(x)\}, \Psi + \Omega - \inf\{I_A(x)\}], T_A(x)>), x \in U\}$.

14. Conclusion.

The membership degrees over 1 (overmembership), or below 0 (undermembership) are part of our real world, so they deserve more study in the future.

The neutrosophic overset / underset / offset together with neutrosophic overlogic / underlogic / offlogic and especially neutrosophic overprobability / underprobability / and offprobability have many applications in technology, social science, economics and so on that the readers may be interested in exploring.

After designing the neutrosophic operators for single-valued neutrosophic overset/underset/offset, we extended them to interval-valued neutrosophic overset/underset/offset operators. We also presented another example of membership above 1 and membership below 0.

Of course, in many real world problems the neutrosophic union, neutrosophic intersection, and neutrosophic complement for interval-valued neutrosophic overset/underset/offset can be used. Future research will be focused on practical applications.

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The author has presented the
- *neutrosophic overset, neutrosophic underset, neutrosophic offset;*
- *neutrosophic overlogic, neutrosophic underlogic, neutrosophic offlogic;*
- *neutrosophic overmeasure, neutrosophic undermeasure, neutrosophic offmeasure;*
- *neutrosophic overprobability, neutrosophic underprobability, neutrosophic offprobability;*
- *neutrosophic overstatistics, neutrosophic understatistics, neutrosophic offstatistics; as follows:*

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31. *An Introduction to Neutrosophic Logic and Set*, Invited Speaker at and sponsored by University Kristen Satya Wacana, Salatiga, Indonesia, May 24, 2006.
32. *Introduction to Neutrosophics and their Applications*, Invited speaker at Pushchino Institute of Theoretical and Experimental Biophysics, Pushchino (Moscow region), Russia, August 9, 2005.

33. *Neutrosophic Probability, Set, and Logic*, Second Conference of the Romanian Academy of Scientists, American Branch, New York City, February 2, 1999.
34. *Paradoxist Mathematics*, Department of Mathematics and Computer Sciences, Bloomsburg University, PA, USA, November 13, 1995, 11:00 a.m. - 12:30 p.m.