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ORIGINAL ARTICLE

Generalized fuzzy cognitive maps: a new extension of fuzzy cognitive maps

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Abstract A fuzzy cognitive maps (FCM) is a cognitive map within the relations between the elements. FCM has been widely used in many applications such as experts system and knowledge engineering. However, classical FCM is inherently short of sufficient capability of representing and aggregating uncertain information. In this paper, generalized FCM (GFCM) is proposed based on genetic algorithm and interval numbers. An application frame of GFCM is detailed. At last, a numerical example about socio-economic system is used to illustrate the effectiveness of the proposed methodology.

Keywords Generalized fuzzy cognitive maps · Interval number · Genetic algorithm · Decision making

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1 Introduction

FCM has received special attentions from the scientific community and done many achievements since it can provide a powerful tool to manipulate knowledge imitating human reasoning and thinking. FCM has used to solve many problems like fuzzy control (Stylios and Groumpos 1999), approximate reasoning (Khan and Quaddus 2004), strategic planning (Konar and Chakraborty 2005), data mining analysis (Yang and Peng 2009), virtual worlds and network models (Dickerson and Kosko 1993), and so on (Gupta and Gandhi 2013, 2014; Kandasamy and Indra 2000; Jorge et al. 2011; Yesil et al. 2013; Ganguli 2014; Papageorgiou and Iakovidis 2013; Salmeron and Papageorgiou 2014; Glykas 2013; Nápoles et al. 2013; Gray et al. 2014; Stylios and Groumpos 2000). It is noted in the real application that Papageorgiou (2011) presents a novel framework for the construction of augmented FCMs based on fuzzy rule-extraction methods for decisions in medical informatics. The study extracted the available knowledge from data in the form of fuzzy rules and inserted them into the FCM, contributing to the development of a dynamic decision support system. FCM has also been investigated for risk analysis of pulmonary infections during patient admission into the hospital (Papageorgiou et al. 2011; Parsopoulos et al. 2004).

Although FCM has achieved success in many fields, there are some limitations inherent in FCM, such as lack of adequate capability to handle uncertain information and lack of enough ability to aggregate the information from different sources. Recently, the uncertain information processing has been heatedly researched in the field of risk analysis (Deng et al. 2011b; Shafiqul Islam et al. 2012) and decision making (Sengupta and Pal 2000; Deng and Chan

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2011; Deng et al. 2011a; Liu et al. 2012; Siraj et al. 2001), environment assessment (Deng et al. 2014), social science (Stakias et al. 2013; Carvalho 2013) and and other fields (Papageorgiou 2013; Kang et al. 2012; Zhang et al. 2013a, b; Chen et al. 2013; Stach et al. 2005; Du et al. 2014; Malik 2013). Some attention has been paid to the first issue by some researchers. For example, Salmeron (2010) proposes an innovative and flexible model based on Grey Systems Theory, called fuzzy grey cognitive maps (FGCM), which can be adapted to a wide range of problems, especially in multiple meaning-based environments. Iakovidis and Papageorgiou (2011) propose an approach based on cognitive maps and intuitionistic fuzzy logic, which is called intuitionistic fuzzy cognitive map (IFCM) to extend the existing FCM by considering the experts hesitancy in the determination of the causal relations between the concepts of a domain. Similarly, after the introduction of neutrosophic logic (similar to intuitionistic fuzzy sets) by Samarandache (2002), indeterminacy has been introduced into causal relationships between some of concepts of FCMs. This is a generalization of FCMs and the structure is called neutrosophic cognitive maps (NCMs) (Kandasamy and Smarandache 2003). However, how to extend the ability of FCM to aggregate the information from different sources under uncertain environment is a significant question in the application of FCM and is still an open issue.

Hence, a method of aggregating the information from different source base on genetic algorithm and interval number is proposed to enhance the ability of classical FCM to handle the fuzzy information from different sources. The combination of genetic algorithm and FCM base on interval number is shown to be a valuable approach through illustrations.

This paper is organized as follows. In Sect. 2, classical FCMs, genetic algorithms and basic operation of interval numbers are introduced. The generalized FCM (GFCM) is proposed in Sect. 3. In Sect. 4, The application frame of GFCMs is proposed. A numerical example about socio-economic is used to illustrate the effectiveness of the proposed methodology in Sect. 5. At last, a conclusion is made in Sect. 6.

2 Preliminaries

2.1 Classical FCMs

Classical fuzzy cognitive map (FCM), an extension of cognitive map, is an illustrative causative representation of the description and models the behavior of any system (Kosko 1986, 1996). Figure 1 (left) illustrates a simple FCM consisting of six concepts C_i (i $= 1, \ldots, 6$). The value C_i is denoted by A_i'. Weight $\omega_{ij} \in [-1, 1]$ represents the causal relationship between concept i and concept j, where a negative sign represents inverse causation. This scheme may give rise to the following three types of interactions:

$\omega_{ij} > 0$, a positive causality, where an increase in the value of the ith concept causes an increase in the value of the jth concept;

$\omega_{ij} < 0$, a negative causality, where an increase in the value of the ith concept causes an decrease in the value of the jth concept;

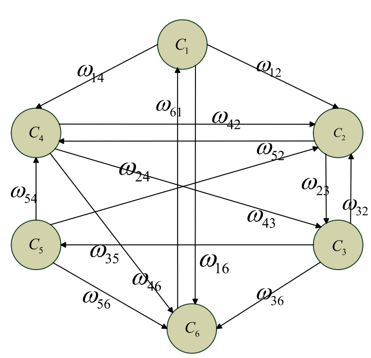
$\omega_{ij} = 0$, no causal relationship between the ith concept and the jth concept.

The edge matrix of fuzzy cognitive map is denoted as the matrix in Fig. 1 (right) correspondingly.

Kosko (1986, 1996) proposed a rule to calculate the value of each concept based on the influence of the interconnected concepts, where the content of the following function is normalized in the interval $[-1, 1]$:

$$A_j^t = f\left(k_1^i \sum_{\substack{i=1 \\ i \neq j}}^{n} A_i^{t-1} \omega_{ij} + k_2^j A_j^{t-1} \right) \quad 0 \le k_1^i \le 1 \quad 0 \le k_2^i \le 1 \tag{1}$$

where A_j^t is the normalized ($A_j^t \in [0, 1]$) value (a.k.a activation level) of concept C_j at time step t, and $f(x)$ is a threshold function. Generally, a sigmoid function $f(x) = \frac{1}{1+e^{-\lambda x}}$ is used to constrain the value of $f(x)$ in the interval [0,1], where $\lambda > 0$ determines the steepness of $f(x)$. The coefficient k_1^i express the influence of interconnected concepts in the configuration of the new value of concept A_i. For example, in Fig. 1 the concept C_6 receives inputs from concepts C_1, C_3,



$$W = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{pmatrix} 0 & \omega_{12} & 0 & \omega_{14} & 0 & \omega_{16} \\ 0 & 0 & \omega_{23} & \omega_{24} & 0 & 0 \\ 0 & \omega_{32} & 0 & 0 & \omega_{35} & \omega_{36} \\ 0 & \omega_{42} & \omega_{43} & 0 & 0 & \omega_{46} \\ 0 & \omega_{52} & 0 & \omega_{54} & 0 & \omega_{56} \\ \omega_{61} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Fig. 1 Structural diagram of fuzzy cognitive map

C_4 and C_5. If experts perceive that C_4 and C_5 interact in such a way that both are fully participating in impacting C_6 the the k_1^i associated with them will be closer to 1. Similarly, k_2^j accounts for the importance of C_6 being at its activation level in the previous time step. The selection of coefficients k_1^i and k_2^j depends on the nature and type of each concept, and may naturally differ from concept to concept.

2.2 Genetic algorithms

The simple genetic algorithm over populations defined as multi-sets $P(t) = (a_0(t), a_1(t), \ldots, a_{r-1}(t))$ consisting of r individual binary $l-$tuples $a_k(t) = (a_{k,0}(t), a_{k,1}(t), \ldots, a_{k,l-1}(t)) \in \Omega$ with fitness values $f(a_k(t))$. For the creation of offspring individual in each generation t random genetic operators like crossover χ_Ω and mutation μ_Ω are applied to parental individuals which are selected according to their fitness values as follows. The population $P(0)$ is initialised appropriately, e.g. by randomly choosing individuals in Ω (Holland 1975).

```
%procedure : simple genetic algorithm
=======================================
t := 0;
while end of adaptation ≠ ture do
    for k = 0 to r − 1 do
        select parental l − tuples b(t) and c(t)
        apply crossover χΩ and mutation μΩ
            a_k(t + 1) := μΩ(χΩ(b(t), c(t)));
        evaluate fitness f(a_k(t + 1));
    end
    t := t + 1;
end
=======================================
```

2.3 Interval number

The distance between two interval numbers $A(a_1, a_2)$ and $B(b_1, b_2)$ is defined as (Tran and Duckstein 2002):

$$D^2(A,B) = \int_{-1/2}^{1/2} \left\{ \left[\left(\frac{a_1 + a_2}{2} \right) + x(a_2 - a_1) \right] - \left[\left(\frac{b_1 + b_2}{2} \right) + x(b_2 - b_1) \right] \right\}^2 dx \quad (2)$$

Let $\tilde{X}$, $\tilde{Y}$ be two interval numbers, $\tilde{X} = [X_L, X_U]$, $\tilde{Y} = [Y_L, Y_U]$, the operation addition and multiplication of two interval numbers is denoted as

$$\tilde{Z} = \tilde{X} \oplus \tilde{Y} = [X_L + Y_L, X_U + Y_U]; \ \tilde{Z} = \tilde{X} \otimes \tilde{Y} = [z_L, z_U] \quad (3)$$

such that

$$z_L = \min\{X_L Y_L, X_L Y_U, X_U Y_L, X_U Y_U\}; \quad z_U = \max\{X_L Y_L, X_L Y_U, X_U Y_L, X_U Y_U\} \quad (4)$$

3 Proposed generalized fuzzy cognitive maps based on genetic algorithm and interval number

3.1 Generalized fuzzy cognitive maps based on interval number

Generalized fuzzy cognitive map (GFCM), an extension of classical fuzzy cognitive map, is also a directed graph with feedback, consisting of nodes and weighted arcs. Nodes of the graph stands for the concepts that used to described the behavior of the system and they are connected by weighted arcs representing the causal relationships that exist between the concepts. Each concepts $\tilde{C}_i$ is characterized by an interval $\tilde{A}_i$ that represents its value and it results from the transformation of the fuzzy value of the system's variable. In this way, the representation of the concept is more flexible that the representation of the concept in classical FCM using crisp number. According to the essence of classical cognitive map, experts' opinions are reflected on the estimated of the degree of the cause that is between nodes in the referred concept set, namely weight estimate. Generally, due to the complexity of the relation of concepts and limitation of knowledge and experience of experts, the value of weight estimate should not be simply represented by a crisp number. Hence, the interval number is applied to soften the opinions of experts.

Figure 2 illustrates a simple GFCM consisting of six concepts $\tilde{C}_i$ (i = 1, ..., 6). Weight $\tilde{\omega}_{ij} = [a, b]$ represents the causal relationship between concept i and concept j, where $a \times b > 0, -1 < a, b < 1$. This scheme may give rise to the following three types of interactions:

(1) $\tilde{\omega}_{ij} = [a, b] > 0$, a positive causality from concept $\tilde{C}_i$ to concept $\tilde{C}_j$, where $0 \le a < b \le 1$;

(2) $\tilde{\omega}_{ij} = [a, b] < 0$, a negative causality from concept $\tilde{C}_i$ to concept $\tilde{C}_j$, where $-1 \le a < b \le 0$;

(3) $\tilde{\omega}_{ij} = [a, b] = 0$, no causal relation from concept $\tilde{C}_i$ to concept $\tilde{C}_j$, where $a = b = 0$.

With this representing method, the uncertainty of the causal relation between two concepts is more clearly described and handled comparing with the frame of classical FCM. The value $\tilde{A}_i$ of concept $\tilde{C}_i$ expresses the degree which corresponds to physical value. At each simulation step, the value $\tilde{A}_i$ of a concept $\tilde{C}_i$ is calculated by computing the influence of the interconnected concepts $\tilde{C}_j's$ on the specific concept $\tilde{C}_i$ following the calculation rule:

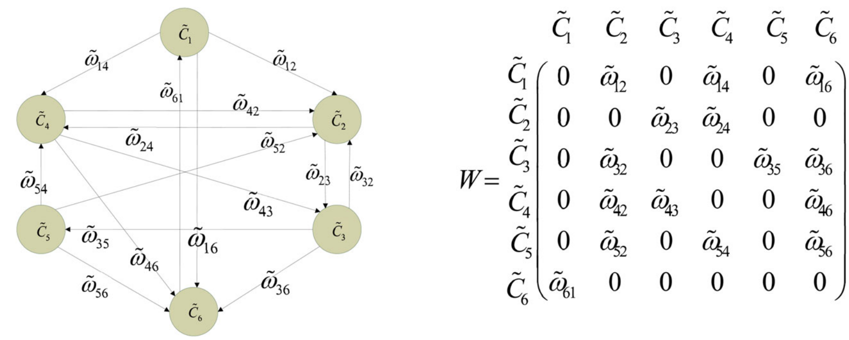


Fig. 2 The structural diagram of generalized fuzzy cognitive map

$$\tilde{A}_j^t = f\left(k_1^i \bigoplus_{\substack{i=1\\ i\neq j}}^{n} \left(\tilde{A}_i^{t-1} \otimes \tilde{\omega}_{ij}\right) \oplus k_2^j \tilde{A}_j^{t-1}\right) \quad 0 \le k_1^i \le 1 \quad 0 \le k_2^i \le 1 \tag{5}$$

where $\tilde{A}_j^t$ is the value of concept $\tilde{C}_j$ at simulation step t, $\tilde{A}_i^{t-1}$ is the value of concept $\tilde{C}_i$ at simulation step $k-1$. The meaning of k_1^i and k_2^i here is the same as meaning of k_1^i and k_2^i in FCM. $\tilde{\omega}_{ij}$ is the weight of the interconnection from concept $\tilde{C}_i$ to concept $\tilde{C}_j$ and f is the nonlinear mapping function which derives from function $f(x) = \frac{1}{1+e^{-\lambda x}}$.

$$f(\tilde{X}) = \frac{1}{1+e^{-\lambda\tilde{X}}} \tag{6}$$

where $\tilde{X}$ is an interval numbers and λ is a parameter determining its steepness. The output of f is also an interval number. It can approximately handle the uncertain information from concepts and connection weights. The meaning of the f can be illustrated with Fig. 3 ($x > 0$).

3.2 Aggregation of GFCM using genetic algorithm

This paper integrates the fuzzy numbers by applying the distance based on interval numbers to find a special interval number ($[x_1, x_2], a \le x_1 \le x_2 \le b$) whose distance is the optimal value from all other normalized interval numbers with the searching method of Genetic Algorithms. The objective function $f(x_1, x_2)$ is constructed with the summation of all the distance between $[x_1, x_2]$ and each normalized interval number. Hence, the problem is mapped into find the optimal solution of $f(x_1, x_2)$ under the constraint $a \le x_1 < x_2 \le b$ with the method of Genetic Algorithms. This process can be mathematically described by Eq. (7).

$$\min SDF(x_1, x_2) = \sum_{i=1}^{n} D^2(O, O_i)$$
$$s.t. \begin{cases} O = [x_1, x_2] \\ a \le x_1 \le b \\ a \le x_2 \le b \\ x_1 < x_2 \end{cases} \tag{7}$$

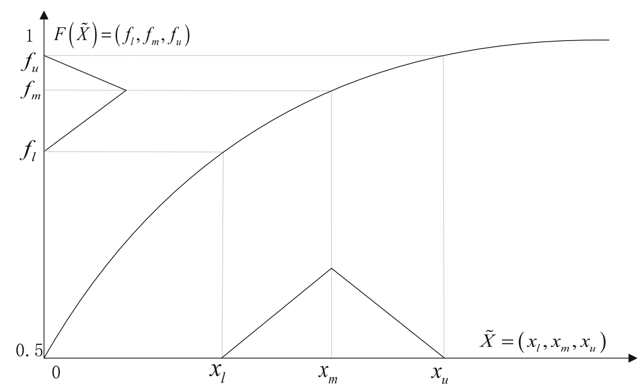


Fig. 3 The nonlinear mapping function

where O_i is the interval value of opinion of the ith expert. $D^2(O, O_i)$ is the interval distance between O and O_i. a and b is the boundary range of x_1 and x_2.

Now the procedure of aggregating interval numbers (objective factors) is made by GA in detail as follows:

(1) Determination of the encoding method

The first step is to determine the encoding strategy, in other words, how to represent the data between a and b with the computer language. In this paper, the binary encoding strategy is adopted to represent the chromosome. The length of chromosome depends on the accuracy of encoding. Assume the domain of variable x_1 is [a, b], and the accuracy of encoding *prec* is the digit after decimal point. The length of a binary string variable L can be calculated as follows:

$$L = \max\_\mathrm{int}\left(\log_2^{\left[\frac{b-a}{\mathrm{prec}}+1\right]}\right) \tag{8}$$

where *prec* is to represent the precision of the result, max_int(x) means the maximum integer which is not more than x.

(2) Determination of the decoding method

The decoding of chromosome is to translate the chromosome from binary data to a decimal data in the interval number accordingly. Assume the binary encoding of a chromosome is represented with $(b_{L-1}\ldots, b_0)$. Firstly, the binary string $(b_L, b_{L-1}\ldots, b_0)$ should be translate to decimal number x', which can be denoted as:

$$(b_L, b_{L-1}, \ldots, b_0)_2 = \left(\sum_{i=0}^{L-1} b_i \times 2^i\right)_{10} = x' \tag{9}$$

Then, the final decoding data in the interval number [a, b] can be calculated as

$$x = a + x' \cdot \frac{b-a}{2^L - 1} \tag{10}$$

where L is the encoding length of the chromosome.

(3) Construction of the initial population

According to Eq. (8), the encoding length of chromosome for each variable can be obtained. Hence, the total length of chromosome for variable weight can be accumulated to be composed of the length of a single long chromosome. For the point position of each chromosome, the method of generating population can be denoted as:

$$b_i = \begin{cases} 1, & \xi_i > 0.5 \\ 0, & \xi_i \le 0.5 \end{cases} \quad where\ \xi_i \in U(0,1) \tag{11}$$

(4) Determination of the adaptive function and adaptive value

Generally, the adaptive function is designed according to the objective function $f^*(x_1, x_2)$, and the adaptive function is denoted as $F(x_1, x_2)$. In order to lay the foundation for calculating the selected probability of each individual behind, the optimalizing direction of adaptive function $f^*(x_1, x_2)$ should adapt to the incremental direction of adaptive value. Due to the value of objective function $f^*(x_1, x_2) > 0$, the adaptive function $F(x_1, x_2)$ is defined as:

$$F(x_1, x_2) = e^{-f^*(x_1, x_2)} \tag{12}$$

In this paper, the objective function $f^*(x_1, x_2)$ is determined by *minSD* according to Eq. (7).

(5) Determination of the selection criteria

In this paper, the proportional selection strategy of adaptive value is introduced, and the the proportion of every individual is defined as selected probability P_i. Assume the population whose scale is n as $pop = \{a_1, a_2, a_3, \ldots, a_n\}$, and the adaptive of a_i as F_i, then the selected probability P_i is denoted as:

$$P_i = \frac{F_i}{\sum_{i=1}^{n} F_i}, \quad i = 1, 2, 3, \ldots, n \tag{13}$$

then the accumulative probability Q_i of every chromosome is denoted as follows:

$$Q_i = \sum_{j=1}^{i} P_j, \quad j = 1, 2, \ldots \tag{14}$$

After a random data $r, r \in [0, 1]$ is generated, The selection of chromosome U_i for a new population can be selected if $Q_{i-1} \le r \le Q_i$.

(6) Determination of the genetic operators

Assume that we have a population *pop*(1) including four individuals described as follows:

pop(1) = {
$<1101011101001100011110>$, %%U_1
$<1000011001010001000010>$, %%U_2
$<0001100111010110000000>$,% %U_3
$<0110101001101110010101>$ }%%U_4

After several Roulette Wheel testing, assume chromosome U_2 occupy the most area of the whole circle and chromosome U_3 occupy the least area of the whole circle. According to the selecting criteria, chromosome U_2 is selected to make a reproduction, while chromosome U_3 is fell into disuse.

newpop(1) = {
$<1101011101001100011110>$, %%U_1
$<1000011001010001000010>$, %%U_2
$<1000011001010001000010>$, %%U_2
$<0110101001101110010101>$ }%%U_4

The crossover operator in this paper adopts the strategy of a single cutting crossover. This method considers the two flanks of the cutting into two substrings, then the right substring should be exchanged with each other to get two new individuals. If the crossover probability $P_c = 25\,\%$, it means that 25 % of the chromosomes on average exchange each other.

<110101110 **1001100011110** $>$
crossover :
$<$**100001100** $1010001000010>$

new : $<1101011101010001000010>$
new : $<1000011001001100011110>$

Mutation operator is to change some gene of chromosome with a tiny possibility. If the mutation probability $P_m = 0.01$, it means that 1 % of all the genes are expected to mutate.

(7) Determine the process parameter and terminal condition

Through the prior test, the crossover probability *Pcro* is between 0.4 and 0.99, the mutation probability *Pmut* is 0.0001 and 0.01, the scale of population is between 20 and 100, the terminated condition may be determined by the precise *N* iterated generation or determined by the minimum bias δ, which satisfies

$$|fitness_{\max} - fitness^*| \leq \delta \tag{15}$$

where $fitness_{\max}$ is the maximum fitness value, and $fitness^*$ is the objective fitness value. If the judgement condition is not satisfied with the terminal condition, then goto step(4).

A numerical example is used to illustrate the procedure of our method. Three experts, E_1, E_2 and E_3, give their linguistic estimation of concept $\tilde{C}_5$ and concept $\tilde{C}_6$ as *medium*, *strong*, *very strong* and their values using interval number are as follows:

$$O_1 = [0.35, 0.65]; \quad O_1 = [0.5, 0.8]; \quad O_1 = [0.65, 0.9]$$

Assume the importance of each expert is same. First, we suppose that the integrated opinion of the three experts is represented with an interval numbers $O = (x_1, x_2)$, $0 \leq x_1 < x_2 \leq 1$. So the distance between O and each expert $E_i (i = 1, 2, 3)$ can be showed as follows according the distance of the interval numbers.

$$\begin{aligned} f(x_1, x_2) &= \sum_{i=1}^{3} D^2(O, O_i) = D^2(O, O_1) + D^2(O, O_2) \\ &\quad + D^2(O, O_3) \\ &= x_1^2 + x_1 x_2 + x_2^2 - \frac{31}{15} x_1 - \frac{107}{60} x_2 + \frac{97}{75} \end{aligned} \tag{16}$$

where $0 \leq x_1 < x_2 \leq 1$. Hence, the integration of three interval numbers is mapped into solve the minimum of the formula $f(x_1, x_2)$ under the constraint of $0 \leq x_1 < x_2 \leq 1$.

Assuming the method of encoding is binary, the size of population pop_size is 20, the max generation of iteration is 100, the crossover probability *pcro* is 0.4, the mutation probability *pmut* is 0.1. The adaptive function is $F(x_1, x_2) = e^{-f(x_1, x_2)} = e^{-(x_1^2 + x_1 x_2 + x_2^2 - \frac{31}{15} x_1 - \frac{107}{60} x_2 + \frac{97}{75})}$. After 100 times repeat, we get the convergent result is [0.5,0.7833], the minimal value the objective function is 0.0381. The trend of objective function can be shown by Fig. 4.

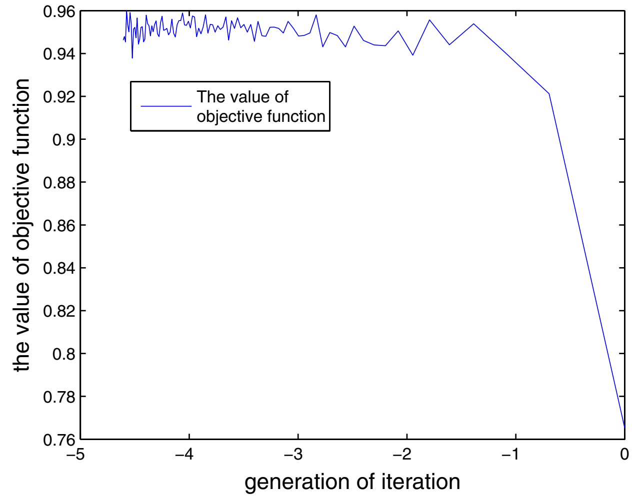


Fig. 4 The trend of objective function value in knowledge aggregation

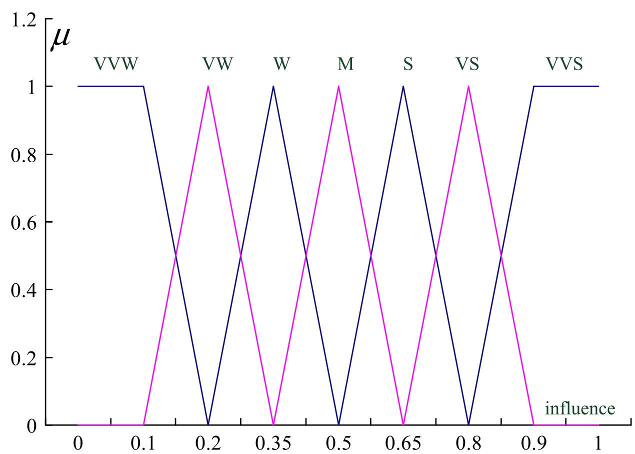


Fig. 5 Membership function of the linguistic variable influence

4 Application frame of GFCMs

4.1 Knowledge acquisition

GFCM can be constructed by a procedure which mainly includes the following three steps: (1) acquisition of the whole concepts related to this issue; (2) refining the concepts; (3) definition of the relation between the concepts (causal or influential relation). Suppose we have m experts, and they are divided into n groups according to their knowledge and background. For each group, the relationship (or the edge weight) between the concepts (or the nodes) is decided by experts exclusively and independently.

The linguistic weight is used so that experts can make a decision more flexible. The values of the linguistic weight are usually very very weak (*vvw*), very weak (*vw*), weak (*w*), medium (*m*), strong (*s*), very strong (*vs*), very very strong (*vvs*), which is described by Fig. 5. For example in the socio-economic system, three experts are selected to evaluate the influence from population to economic condition, the first expert think that the more scale of the population, the little worse of the economic condition, and the opinion can be shown as $-vw$. The second expert's opinion is the same as the first one, but more worse, whose opinion is described as $-w$. The third expert insists that the

Table 1 Experts' knowledge in group 1 (values of aggregated knowledge are shown in bold)

Item	$\widetilde{C_1}$	$\widetilde{C_3}$	$\widetilde{C_5}$
$\widetilde{C_1}$	–	E1: $-vw$ (−[0.1,0.35]) E2: $-w$ (−[0.2,0.5]) E3: vw ([0.1,0.35]) **E123:(−[0.15,0.425])**	E1: m ([0.35,0.65]) E2: w ([0.2,0.5]) E3: s ([0.5,0.8]) **E123:([0.275,0.575])**
$\widetilde{C_3}$	–	–	E1: $-vs$ (−[0.65,0.9]) E2: $-s$ (−[0.5,0.8]) E3: $-vvs$ (−[0.8,1]) **E123:(−[0.575,0.85])**
$\widetilde{C_5}$	–	–	–

large scale of the population may improve the economic condition for the labors provided for the economic development are more dominate, the relationship from population to economic is positive, hence, the third opinion can be shown as *vw*.

4.2 Knowledge aggregation

This step allows the aggregation of knowledge acquired from various sources to develop a comprehensive GFCM, which will represent the understanding of the experts about the special issue. The comprehensive GFCM combines partial GFCMs from inner groups and outer groups. The aggregation of knowledge from inner groups is for the opinions of the experts of each group and the aggregation of knowledge from outer groups is for the edges of the partial GFCMs.

In this part, the main problem is how to aggregate the relationship of the each single GFCM and the framework of all the GFCMs. For the relationship between criteria, assume that the weight of all domain experts (whose opinions are described by linguistic language, like Fig. 5) is equal, we aggregate opinions from experts (interval number) by trying to find a general interval number whose total distance from other opinions is minimum. Hence the question of fusion is converted into an optimization problem. GA is good at solving optimization problem than others such as "ant colony optimization" for its global search capability. The detailed solution is show in the second part of the proposed methodology.

4.3 Training and interpreting GFCM

Let $A^0 = [\tilde{A}_1^0, \tilde{A}_2^0, \ldots, \tilde{A}_n^0]$ $(\tilde{A}_i^0 = [x_1, x_2], 0 <= x_1 < x_2 <= 1, i = 1, \ldots, n)$ be an initial vector state in the GFCM and let $k_1^i = 1$, $k_2^j = 1$ and the sigmoid function with $\lambda = 1$ be used as a threshed function. After several times iteration, it can be seen that the FCM reaches an equilibrium state approximately. GFCM is a dynamic system, whether the dynamic is reaches equilibrium, the paper (Boutalis et al. 2009) has provided some inspiration for the interpretation of the condition.

Once the GFCM reaches equilibrium, the activation values provide the 'triggering or firing' strength of those concepts for a given scenario. Generally, when the GFCM reaches equilibrium, the activation levels are transformed back to the corresponding values. These activation levels may be interpreted 'quantitatively' or 'qualitatively'. The

Table 2 Experts' knowledge in group 2 (values of aggregated knowledge are shown in bold)

Item	$\widetilde{C_1}$	$\widetilde{C_2}$	$\widetilde{C_3}$	$\widetilde{C_4}$
$\widetilde{C_1}$	–	–	E1: $-m$ (−[0.35,0.65]) E2: $-w$ (−[0.2,0.5]) E3: vvw ([0, 0.2]) **E123: (−[0.275,0.575])**	–
$\widetilde{C_2}$	–	–		E1: vs ([0.65,0.9]) E2: vss ([0.8,1]) E3: vss ([0.8,1]) **E123: ([0.725,0.95])**
$\widetilde{C_3}$	–	E1: $-m$ (−[0.35,0.65]) E2: $-s$ (−[0.5,0.8]) E3: $-vs$ (−[0.65,0.9]) **E123:(−[0.425,0.725])**	–	–
$\widetilde{C_4}$	E1: $-m$ (−[0.35,0.65]) E2: $-w$ (−[0.2,0.5]) E3: $-s$ (−[0.5,0.8]) **E123:(−[0.275,0.575])**,	E1: s ([0.5,0.8]) E2: vs ([0.65.0.9]) E3: vss ([0.8,1]) **E123:(0.575,0.85)**	–	–

Table 3 Experts' knowledge in group 3 (values of aggregated knowledge are shown in bold)

Item	$\widetilde{C_1}$	$\widetilde{C_4}$	$\widetilde{C_5}$
$\widetilde{C_1}$	–	–	E1: *m* ([0.35,0.65]) E2: *w* ([0.2,0.5]) E3: −*vvw* (−[0, 0.2]) **E123:([0.275,0.575])**
$\widetilde{C_4}$	E1: *m* ([0.35,0.65]) E2: *s* ([0.5,0.8]) E3: *vw* ([0.1,0.35]) **E123:([0.425,0.725])**	–	–
$\widetilde{C_5}$	–	E1: *s* ([0.5,0.8]) E2: *vs* ([0.65,0.9]) E3: *vvs* ([0.8,1]) **E123:([0.575,0.85])**	–

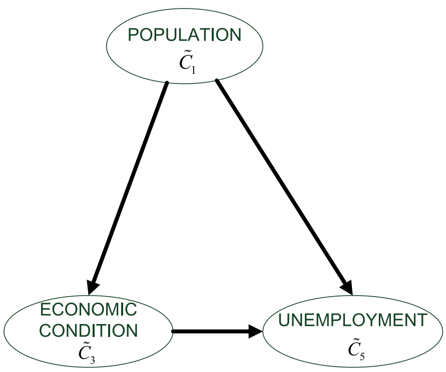


Fig. 6 Result of GFCM simulations

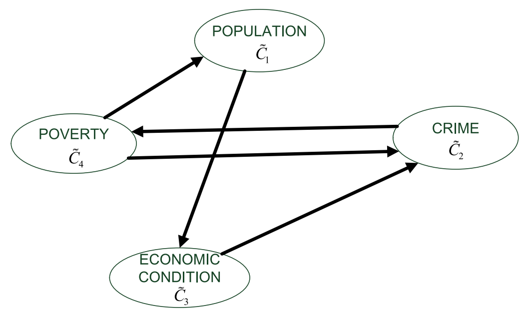


Fig. 7 Result of GFCM simulations

interpretation of these concepts will determine the judgement for a given scenario.

5 A numerical example

This section illustrates the application of the proposed method to a socio-economic model. It is constructed with Population, Crime, Economic condition, Poverty, and Unemployment as nodes or concepts. Our purpose is to evaluate the trend of factors changing with any one factor using GFCM.

First, the structure of GFCM should be established using several sources of partial knowledge. All the available experts are divided into three groups (group1, group2, and grou3). and the opinions are provided in Tables 1, 2 and 3. They can be described as Figs. 6, 7 and 8 accordingly.

Next, the opinions from different experts and the partial GFCMs are combined together based on genetic algorithm

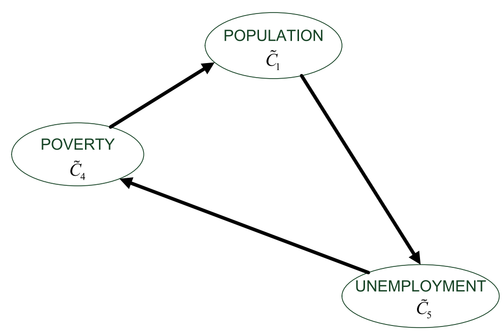


Fig. 8 Result of GFCM simulations

(see Table 4; Fig. 9), then the final combined interval adjacency matric (denoted as Table 5) of the GFCM for socio-economic scenario is established.

For considering the influence of node *Population* to the socio-economic system, the node of *Population* is made triggered, and others are non-triggered. Let $A^0 = [\tilde{A}^0_1, \tilde{A}^0_2, \tilde{A}^0_3, \tilde{A}^0_4, \tilde{A}^0_5] = [vvs, 0, 0, 0, 0] = [[0.8, 1], 0, 0, 0, 0]$ be

Table 4 Aggregation of experts' knowledge in groups (values of aggregated knowledge are shown in bold)

Item	$\widetilde{C}_1$	$\widetilde{C}_2$	$\widetilde{C}_3$	$\widetilde{C}_4$	$\widetilde{C}_5$
$\widetilde{C}_1$	–	–	E1: −[0.15,0.425] E2: −[0.275,0.575] **E12: −[0.212,0.5]**		E1: [0.275,0.575] E2: [0.275,0.575] **E12: [0.275,0.575]**
$\widetilde{C}_2$	–	–	–	E1: [0.725,0.95]	–
$\widetilde{C}_3$	–	E1: −[0.425,0.725],	–	–	E1: −[0.575,0.85]
$\widetilde{C}_4$	E1: −[0.275,0.575] E2: [0.425,0.725] **E12: [−0.075,0.225]**	E1: [0.575,0.85],	–	–	–
$\widetilde{C}_5$	–	–	–	E1: [0.575,0.85]	–

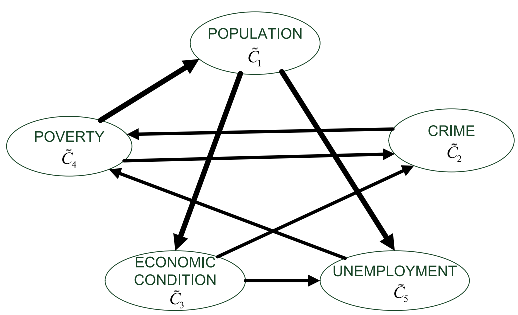


Fig. 9 Result of GFCM simulations

an initial vector state in the GFCM and let $k_1^i = 1$, $k_2^j = 1$ and the sigmoid function with $\lambda = 1$ be used as a threshed function. After six times iteration, it can be seen that the FCM reaches an equilibrium state approximately.

The process of the convergence in this simulation can be shown as Figs. 10, 11 and 12. The final mean state of the nodes "*Population*", "*Crime*", "*Economic condition*", "*Poverty*" and "*Unemployment*" is [0.68, 0.73, 0.58, 0.87, 0.61]. It can be concluded the population is initially triggering, the rate of crime is increasing, the poverty is more serious, and the economic condition (volume of economic) may be improved; the population is decreasing gradually at the same time, and all the nodes reach a new equilibrium state.

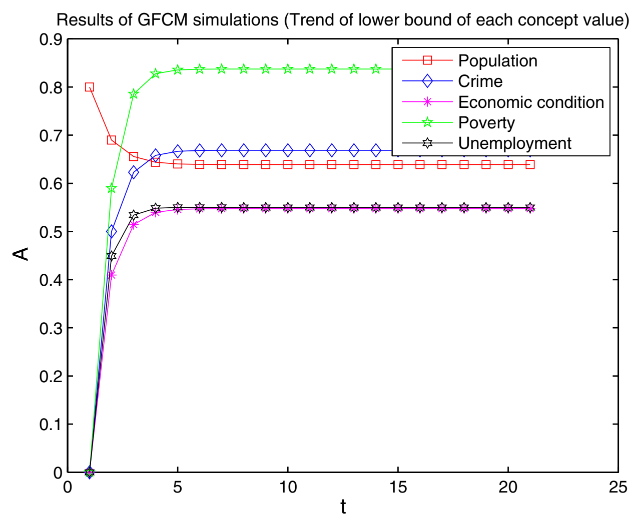


Fig. 10 Results of GFCM simulations (trend of *lower bound* of each concept value)

Table 5 Aggregation of experts' knowledge in 3 groups

Item	$\widetilde{C}_1$	$\widetilde{C}_2$	$\widetilde{C}_3$	$\widetilde{C}_4$	$\widetilde{C}_5$
$\widetilde{C}_1$	0	0	**−[0.212,0.5]**	0	**[0.275,0.575]**
$\widetilde{C}_2$	0	0	0	**[0.725,0.95]**,	0
$\widetilde{C}_3$	0	**−[0.425,0.725]**,	0	0	**−[0.725,0.85]**
$\widetilde{C}_4$	**[−0.075,0.225]**	**[0.575,0.85]**	0	0	0
$\widetilde{C}_5$	0	0	0	**[0.575,0.85]**	0

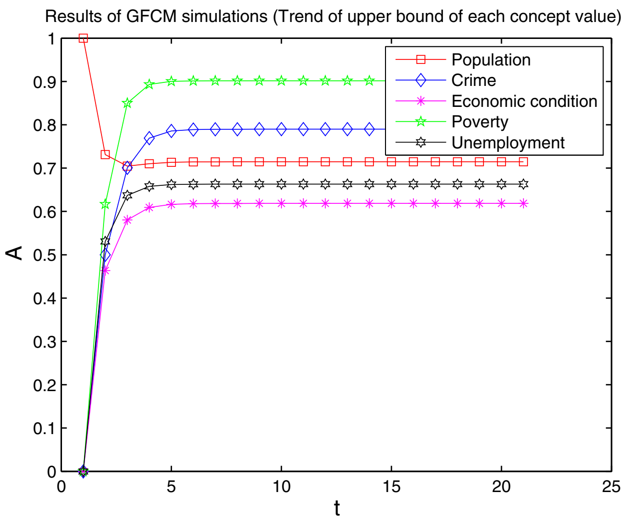


Fig. 11 Results of GFCM simulations (trend of *upper bound* of each concept value)

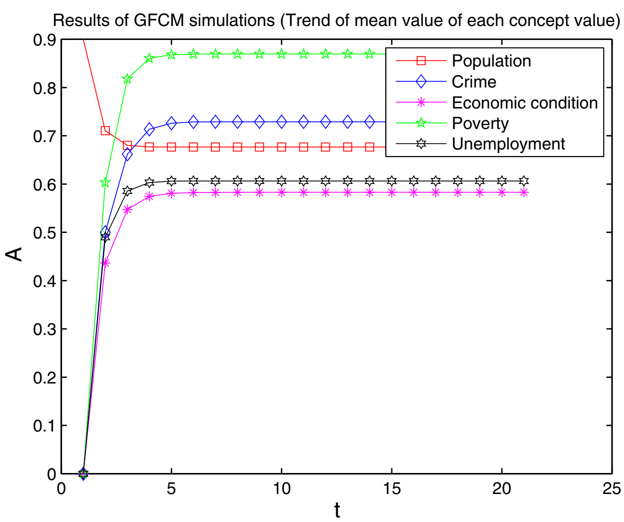


Fig. 12 Results of GFCM simulations (trend of mean value of *lower* and *upper* bound of each concept value)

6 Conclusion

GFCMs are uncertain-graph structures for representing causal reasoning. GFCM can be considered as the development of FCMs with considering aggregating information from different sources under uncertain environment. It can be widely used in many applications such as decision making and uncertain reasoning. The frame of GFCMs is detailed and a simulation about socio-economic system is also shown to introduce the application of GFCMs.

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References

Boutalis Y, Kottas TL, Christodoulou M (2009) Adaptive estimation of fuzzy cognitive maps with proven stability and parameter convergence. IEEE Trans Fuzzy Syst 17(4):874–889

Carvalho JP (2013) On the semantics and the use of fuzzy cognitive maps and dynamic cognitive maps in social sciences. Fuzzy Sets Syst 214:6–19

Chen S, Deng Y, Jiyi W (2013) Fuzzy sensor fusion based on evidence theory and its application. Appl Artif Intell 27(3):235–248

Deng Y, Chan FTS (2011) A new fuzzy dempster MCDM method and its application in supplier selection. Expert Syst Appl 38(8):9854–9861

Deng Y, Jiang W, Sadiq R (2011a) Modeling contaminant intrusion in water distribution networks: a new similarity-based dst method. Expert Syst Appl 38(1):571–578

Deng Y, Sadiq R, Jiang W, Tesfamariam S (2011b) Risk analysis in a linguistic environment: a fuzzy evidential reasoning-based approach. Expert Syst Appl 38(12):15438–15446

Deng X, Yong H, Deng Y, Mahadevan S (2014) Environmental impact assessment based on d numbers. Expert Syst Appl 41(2):635–643

Dickerson JA, Kosko B (1993) Virtual worlds as fuzzy cognitive maps. In: Virtual reality annual international symposium, IEEE, pp 471–477

Du Y , Mo H, Deng X, Sadiq R, Deng Y (2014) A new method in failure mode and effects analysis based on evidential reasoning. Int J Syst Assur Eng Manag 5(1):1–10

Ganguli R (2014) Fuzzy cognitive maps for structural damage detection. In: Papageorgiou IE (ed) Fuzzy cognitive maps for applied sciences and engineering. Springer Berlin, Heidelberg, pp 267–290

Glykas M (2013) Fuzzy cognitive strategic maps in business process performance measurement. Expert Syst Appl 40(1):1–14

Gray SA, Zanre Erin, Gray SRJ (2014) Fuzzy cognitive maps as representations of mental models and group beliefs. In: Papageorgiou IE (ed) Fuzzy cognitive maps for applied sciences and engineering. Springer Berlin, Heidelberg, pp 29–48

Gupta P, Gandhi OP (2013) Ontological modeling of spatial shaft-position knowledge for steam turbine rotor. Int J Syst Assur Eng Manag 4(3):284–292

Gupta P, Gandhi OP (2014) Equipment redesign feasibility through maintenance-work-order records using fuzzy cognitive maps. Int J Syst Assur Eng Manag 5(1):21–31

Holland JH (1975) Adaptation in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence. Univ. of Michigan Press, Ann Arbor

Iakovidis DK, Papageorgiou E (2011) Intuitionistic fuzzy cognitive maps for medical decision making. IEEE Trans Inf Technol Biomed 15(1):100–107

Kandasamy W, Indra V (2000) Applications of fuzzy cognitive maps to determine the maximum utility of a route. J Fuzzy Math 8:65–77

Kandasamy WBV, Smarandache F (2003) Fuzzy cognitive maps and neutrosophic cognitive maps. American Research Press, Rehoboth

Kang B, Deng Y, Sadiq R, Mahadevan S (2012) Evidential cognitive maps. Knowl-Based Syst 35:77–86

Khan MS, Quaddus M (2004) Group decision support using fuzzy cognitive maps for causal reasoning. Group Dec Negot 13(5):463–480

Konar A, Chakraborty UK (2005) Reasoning and unsupervised learning in a fuzzy cognitive map. Inf Sci 170(2):419–441
Kosko B (1986) Fuzzy cognitive maps. Int J Man-Mach Stud 24(1):65–75
Kosko B (1996) Fuzzy Engineering. Prentice-Hall, Inc., Englewood Cliffs
Liu J, Chan FTS, Li Y, Zhang Y, Deng Y (2012) A new optimal consensus method with minimum cost in fuzzy group decision. Knowl-Based Syst 35:357–360
Malik SC (2013) Reliability modeling of a computer system with preventive maintenance and priority subject to maximum operation and repair times. Int J Syst Assur Eng Manag 4(1):94–100
Nápoles G, Grau I, León M, Grau R (2013) Modelling, aggregation and simulation of a dynamic biological system through fuzzy cognitive maps. In: Batyrshin I, Mendoza MG (eds) Advances in computational intelligence. Springer Berlin, Heidelberg, pp 188–199
Papageorgiou EI (2011) A new methodology for decisions in medical informatics using fuzzy cognitive maps based on fuzzy rule-extraction techniques. Appl Soft Comput 11(1):500–513
Papageorgiou EI (2013) Review study on fuzzy cognitive maps and their applications during the last decade. In: Glykas M (ed) Business process management. Springer Berlin, Heidelberg, pp 281–298
Papageorgiou EI, Iakovidis DK (2013) Intuitionistic fuzzy cognitive maps. Fuzzy Syst IEEE Trans 21(2):342–354
Parsopoulos KE, Papageorgiou EI, Groumpos PP, Vrahatis MN (2004) Evolutionary computation techniques for optimizing fuzzy cognitive maps in radiation therapy systems. Presence 3102:402–413
Papageorgiou EI, Papandrianos N, Karagianni G, Kyriazopoulos G, Sfyras D (2011) A fuzzy inference map approach to cope with uncertainty in modeling medical knowledge and making decisions. Intell Decis Technol 5(3):219–235
Salmeron JL (2010) Modelling grey uncertainty with fuzzy grey cognitive maps. Expert Syst Appl 37(12):7581–7588
Sengupta A, Pal TK (2000) On comparing interval numbers. Eur J Oper Res 127(1):28–43
Salmeron JL, Papageorgiou EI (2014) Using fuzzy grey cognitive maps for industrial processes control. In: Papageorgiou IE (ed) Fuzzy cognitive maps for applied sciences and engineering. Springer Berlin, Heidelberg, pp 237–252
Shafiqul Islam M, Zargar A, Dyck R, Mohapatra A, Sadiq R (2012) Data fusion-based risk assessment framework: an example of benzene. Intl J Syst Assur Eng Manag 3(4):267–283
Simões JM, Gomes CF, Yasin MM (2011) A literature review of maintenance performance measurement: a conceptual framework and directions for future research. J Qual Maint Eng 17(2):116–137
Siraj A, Bridges SM, Vaughn RB (2001) Fuzzy cognitive maps for decision support in an intelligent intrusion detection system. In: IFSA world congress and 20th NAFIPS international conference, 2001. Joint 9th, vol 4, IEEE, pp 2165–2170
Smarandache F (2002) Definitions derived from neutrosophics. Mult Valued Log Int J 8(1):591–603
Stach W, Kurgan L, Pedrycz W, Reformat M (2005) Genetic learning of fuzzy cognitive maps. Fuzzy Sets Syst 153(3):371–401
Stakias G, Psoras M, Glykas M (2013) Fuzzy cognitive maps in social and business network analysis. Bus Process Manag 444:241–279
Stylios CD, Groumpos PP (1999) Fuzzy cognitive maps: a model for intelligent supervisory control systems. Comput Ind 39(3):229–238
Stylios CD, Groumpos PP (2000) Fuzzy cognitive maps in modeling supervisory control systems. J Intell Fuzzy Syst 8(1):83–98
Tran L, Duckstein L (2002) Comparison of fuzzy numbers using a fuzzy distance measure. Fuzzy Sets Syst 130(3):331–341
Yang B, Peng Z (2009) Fuzzy cognitive map and a mining methodology based on multi-relational data resources. Fuzzy Inf Eng 1(4):357–366
Yesil E, D MF, Sakalli A, Ozturk C, Guzay C (2013) Self-tuning pi controllers via fuzzy cognitive maps. In: Joe Turner A, Seneca SC (eds) Artificial intelligence applications and innovations. Springer Berlin, Heidelberg, pp 567–576
Zhang X, Deng Y, Chan FTS, Xu P, Mahadevan S, Hu Y (2013) IFSJSP: a novel methodology for the job-shop scheduling problem based on intuitionistic fuzzy sets. Int J Prod Res 51(17):5100–5119
Zhang Y, Zhang Z, Deng Y, Mahadevan S (2013b) A biologically inspired solution for fuzzy shortest path problems. Appl Soft Comput 13(5):2356–2363