International J.Math. Combin. Vol.1(2021), 62-78

A Study on
Neighbourly Pseudo Irregular Neutrosophic Bipolar Fuzzy Graph

N.R.Santhi Maheswari and D.Subburaj

PG and Research Department of Mathematics, G.Venkataswamy Naidu College,
Manomaniam Sundarnar University, Kovilpatti-628 502, Tamil Nadu, India

E-mail: nrsmaths@yahoo.com, subburajphd2019@gmail.com, subburajdharmaraj@gmail.com

Abstract: In this paper, the concepts of neighbourly pseudo irregular neutrosophic bipolar fuzzy graphs, neighbourly pseudo totally irregular neutrosophic bipolar fuzzy graphs are introduced. Some basic theorems related to the stated graphs have also been presented.

Key Words: Neutrosophic bipolar fuzzy graphs, pseudo degree in neutrosophic bipolar fuzzy graphs, total pseudo degree in neutrosophic bipolar fuzzy graphs, neighbourly pseudo irregular neutrosophic bipolar fuzzy graphs, neighbourly pseudo totally irregular neutrosophic bipolar fuzzy graphs.

AMS(2010): 05C12, 03E72, 05C72.

§1. Introduction

F.Smarandache [4] introduced notion of neutrosophic set which is useful for dealing real life problems having imprecise, indeterminacy and inconsistent data. They are generalization of the theory of fuzzy sets, intuitionistics fuzzy set, interval valued fuzzy set and interval valued intuitionistic fuzzy sets. N. Shah and Hussain[2, 6] introduced the notion of soft neutrosophic graphs. N. Shah introduces the notion of neutrosophic graphs and different operations like union, intersection and complement in his work. A neutrosophic set is characterized by a truth membership degree (t), an indeterminacy membership degree(i), falsity membership degree(f) independently, which are with in the real standard or non standard unit interval $]^{-}0, 1^{+}[$. See [4] for details.

Divya and Dr. J. Malarvizhi[7] introduced the notion of neutrosophic fuzzy graph and few fundamental operation on neutrosophic fuzzy graph. Zhang initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. Bipolar fuzzy sets whose range of membership degree is [-1,1]. In bipolar fuzzy sets, membership degree of an element means that the element is irrelevant to the corresponding property, the membership degree within (0, 1] of an element indicates that the element somewhat satisfies the property, and the membership degree within [-1,0) of an element indicates the element somewhat satisfies the implicit counter property.

[1]Received November 2, 2020, Accepted March 8, 2021.

It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible [9]. M.Akram and Wieslaw A.Dudek introduced regular and totally regular bipolar fuzzy graphs. Also, they introduced the notion of bipolar fuzzy line graphs and presents some of their properties [9].

In 2011, Akram introduced the concept of bipolar fuzzy graphs and defined different operations on it. Bipolar fuzzy graph theory is now growing and expanding its applications. The theoretical developments in this area is discussed. In 2012,Sovan Samanta and Madhumangal Pal introduced the concept of irregular bipolar fuzzy graph and defined different operations on it. N.R.Santhi Maheswari and C.Sekar introduced Pseudo regular bipolar fuzzy graph and pseudo irregular bipolar fuzzy graph and discussed its properties [16].

N.R.Santhi Maheswari and V.Jeyapratha introduced Neighbourly Pseudo irregular fuzzy graph and discussed its properties [15]. N.R.Santhi Maheswari and C.Sekar introduced Neighbourly pseudo and Strongly Pseudo irregular bipolar fuzzy graph and discussed its properties [14]. These idea motivates us to introduce Neighbourly pseudo irregular neutrosophic bipolar fuzzy graphs.

§2. Preliminaries

We present some known definition and results for ready references to go through the work presented in the paper.

Definition 2.1([4]) *Let X be a space of points with generic elements in X denoted by x. A neutrosophic set A (NSA) is an object having the form*

$$A = < x : T_A(x), I_A(x), F_A(x) >, x \in X,$$

where the functions $T, I, F \to]0^-, 1^+[$ define respectively a truth membership function, an indeterminacy membership function and a falsity membership function of the element $x \in X$ to the set A with the condition

$$0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+.$$

The functions $T_A(x), I_A(x), F_A(x)$ are real standard or non standard subsets of $]0^-, 1^+[$.

Definition 2.2([5]) *Let V be a non empty finite set and $\sigma : V \to [0,1]$. Again, let $\mu : V \times V \to [0,1]$ such that $\mu(x,y) \leq \sigma(x) \wedge \sigma(y)$ for all $(x,y) \in V \times V$. Then the pair $G : (\sigma, \mu)$ is called a fuzzy graph over the set V. Here σ and μ are respectively called fuzzy vertex set and fuzzy edge set of the fuzzy graph $G : (\sigma, \mu)$.*

Definition 2.3([7]) *Let X be a space of points with generic elements in X denoted by x. A neutrosophic fuzzy set A(NFSA) is characterized by truth membership function $T_A(x)$, an indeterminacy membership fuctions $I_A(x)$ and a falsity membership fuction $F_A(x)$. For each point $x \in X$, $T_A(x), I_A(x), F_A(x) \in [0,1]$. A NFS A can be written as*

$$A = < x : T_A(x), I_A(x), F_A(x) >, \quad x \in X.$$

Definition 2.4([7]) *Let* $A = (T_A(x), I_A(x), F_A(x))$ *and* $B = (T_A(x), I_A(x), F_A(x))$ *be neutrosophic fuzzy sets on a set X. If* $A = (T_A(x), I_A(x), F_A(x))$ *is a neutrosophic fuzzy relation on a set X, then* $A = (T_A(x), I_A(x), F_A(x))$ *is called a neutrosophic fuzzy relation on* $B = (T_A(x), I_A(x), F_A(x))$ *if*

$$T_B(x, y) \leq T_A(x).T_A(y), \ \ I_B(x, y) \leq I_A(x).I_A(y) \text{ and } F_B(x, y) \leq F_A(x).F_A(y)$$

for all $x, y \in X$*, Where the notation "." means the ordinary multiplication.*

Definition 2.5([7]) *A neutrosophic fuzzy graph (NF graph) with underlying set V is defined to be a pair* $N_G = (A, B)$, where

(*i*) *The functions* $(T_A, I_A, F_A) : V \to [0, 1]$ *denote the degree of truth membership,degree of indeterminacy membership and the degree of falsity membership of the element* $v_i \in V$ *respectively and* $0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$*;*

(*ii*) $E \subseteq V \times V$ *where the functions* $(T_B, I_B, F_B) : V \times V \to [0, 1]$ *are defined by*

$$T_B(v_i, v_j).T_A(v_i) \leq T_A(v_j), I_B(v_i, v_j).I_A(v_i) \leq I_A(v_j)$$

and

$$F_B(v_i, v_j).F_A(v_i) \leq F_A(v_j)$$

for all $v_i, v_j \in V$*, where the notation "." means ordinary multiplication denotes the degrees of truth membership, indeterminacy membership and falsity membership of the edge* $v_i, v_j \in E$ *respectively, where*

$$0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3$$

for all $v_i, v_j \in E$ $(j = 1, 2, \cdots, n)$*.*

Definition 2.6([17]) *A bipolar neutrosophic set A in X is defined as an object of the form*

$$A = \left\{< x, T_A^+(x), I_A^+(x), F_A^+(x), T_A^-(x), I_A^-(x), F_A^-(x) >: x \in X\right\},$$

where $(T_A^+, I_A^+, F_A^+) : X \to [0, 1]$ *and* $(T_A^-, I_A^-, F_A^-) : X \to [-1, 0]$*. The positive membership degree* $T_A^+(x), I_A^+(x), F_A^+(x)$ *denotes the truth membership, indeterminate membership and false membership of an element* $\in X$ *corresponding to a bipolar neutrosophic set A and the negative membership degree* $T_A^-(x), I_A^-(x), F_A^-(x)$ *denotes the truth membership, indeterminate membership and false membership of an element* $\in X$ *to some implicit counter-property corresponding to a bipolar neutrosophic set A.*

Definition 2.7 *A bipolar neutrosophic set B in V is defined as an object of the form*

$$B = \left\{< v, T_B^+(v), I_B^+(v), F_B^+(v), T_B^-(v), I_B^-(v), F_B^-(v) >: v \in V\right\},$$

where $(T_B^+, I_B^+, F_B^+) : V \to [0, 1]$ *and* $(T_B^-, I_B^-, F_B^-) : V \to [-1, 0]$*. The positive membership* *degree* $T_B^+(v), I_B^+(v), F_B^+(v)$ *denotes the truth membership, indeterminate membership and false membership of an element* $\in V$ *corresponding to a bipolar neutrosophic set A and the negative membership degree* $T_B^-(v), I_B^-(v), F_B^-(v)$ *denotes the truth membership, indeterminate membership and false membership of an element* $\in V$ *to some implicit counter-property corresponding to a bipolar neutrosophic set B.*

Definition 2.8 *A bipolar fuzzy graph with an underlying set V is defined to be a pair (A,B), where* $A = (m_1^+, m_1^-)$ *is a bipolar fuzzy set on V and* $B = (m_2^+, m_2^-)$ *is a bipolar fuzzy set on E such that*

$$m_2^+(x,y) \leq min\left\{m_1^+(x), m_1^+(y)\right\} \text{ and } m_2^+(x,y) \geq max\left\{m_1^-(x), m_1^-(y)\right\}$$

for all (x,y) *in E. Here, A is called bipolar fuzzy vertex set on V and B is called bipolar fuzzy edge set on E.*

Definition 2.9 *Let* $G = (A,B)$ *be a bipolar fuzzy graph on* $G^* = (V,E)$*. The positive degree and the negative degree of a vertex u in G is defined respectively as*

$$d^+(u) = \sum m_2^+(u,v)$$

and

$$d^-(u) = \sum m_2^-(u,v)$$

for uv in E, and the degree of a vertex u is defined as

$$d(u) = (d^+(u), d^-(u)).$$

Definition 2.10 *Let* $G = (A,B)$ *be a bipolar fuzzy graph on* $G^* = (V,E)$*. The positive total degree and the negative total degree of a vertex u in G is defined respectively as*

$$td^+(u) = \sum m_2^+(u,v) + m_1^+(u)$$

and

$$td^-(u) = \sum m_2^-(u,v) + m_1^-(u)$$

for uv in E.

§3. Neutrosophic Bipolar Fuzzy Graph

Definition 3.1 *A neutrosophic fuzzy graph (NF graph) with underlying set V is defined to be a pair* $N_G = (A,B)$ *, where,*

(i) The functions $(T_A^+, I_A^+, F_A^+) : V \to [0,1]$ *denote the degree of truth membership,degree of indeterminacy membership and the degree of falsity membership of the element* $v_i \in V$ *re-*

spectively and

$$0 \leq T_A^+(v_i) + I_A^+(v_i) + F_A^+(v_i) \leq 3;$$

(ii) $E \subseteq V \times V$ *where the functions* $(T_B^+, I_B^+, F_B^+) : V \times V \to [0,1]$ *are defined by*

$$T_B^+(v_i, v_j).T_A^+(v_i) \leq T_A^+(v_j), I_B^+(v_i, v_j).I_A^+(v_i) \leq I_A^+(v_j)$$

and

$$F_B^+(v_i, v_j).F_A^+(v_i) \leq F_A^+(v_j)$$

for all $v_i, v_j \in V$*, where, the notation "." means ordinary multiplication denotes the degrees of truth membership, indeterminacy membership and falsity membership of the edge* $v_i, v_j \in E$ *respectively, where*

$$0 \leq T_B^+(v_i, v_j) + I_B^+(v_i, v_j) + F_B^+(v_i, v_j) \leq 3$$

for all $v_i, v_j \in E$ $(j = 1, 2, \cdots, n)$*.*

Definition 3.2 *A neutrosophic fuzzy graph (NF graph) with underlying set* V *is defined to be a pair* $N_G = (A, B)$ *, where,*

(i) *The functions* $(T_A^-, I_A^-, F_A^-) : V \to [-1, 0]$ *denote the degree of truth membership,degree of indeterminacy membership and the degree of falsity membership of the element* $v_i \in V$ *respectively and*

$$0 \geq T_A^-(v_i) + I_A^-(v_i) + F_A^-(v_i) \geq -3;$$

(ii) $E \subseteq V \times V$ *where the functions* $(T_B^-, I_B^-, F_B^-) : V \times V \to [-1, 0]$ *are defined by*

$$T_B^-(v_i, v_j).T_A^-(v_i) \geq T_A^-(v_j), I_B^-(v_i, v_j).I_A^-(v_i) \geq I_A^-(v_j)$$

and

$$F_B^-(v_i, v_j).F_A^-(v_i) \geq F_A^-(v_j)$$

for all $v_i, v_j \in V$ *, where "." means ordinary multiplication denotes the degrees of truth membership, indeterminacy membership and falsity membership of the edge* $v_i, v_j \in E$ *respectively, where*

$$0 \geq T_B^-(v_i, v_j) + I_B^-(v_i, v_j) + F_B^-(v_i, v_j) \geq -3$$

for all $v_i, v_j \in E$ $(j = 1, 2, \cdots, n)$*.*

Definition 3.3 *Let* $BN_G = (A, B)$*, where* $A = (m_1^+, m_1^-)$ *and* $A = (m_2^+, m_2^-)$ *be a neutrosophic bipolar fuzzy graph. The neighborhood positive degree of a vertex* x *in* BN_G *defined by*

$$deg(x)^+ = (deg_T(x)^+, deg_I(x)^+,\ deg_F(x)^+),$$

where,

$$deg_T(x)^+ = \sum_{xy \in E} T_B(xy)^+,\ deg_I(x)^+ = \sum_{xy \in E} I_B(xy)^+,\ deg_F(x)^+ = \sum_{xy \in E} F_B(xy)^+.$$

The neighborhood negative degree of a vertex x in BN_G defined by

$$deg(x)^- = (deg_T(x)^-, deg_I(x)^-, deg_F(x)^-),$$

where

$$deg_T(x)^- = \sum_{xy \in E} T_B(xy)^-,\ deg_I(x)^- = \sum_{xy \in E} I_B(xy)^-,\ deg_F(x)^- = \sum_{xy \in E} F_B(xy)^-.$$

Therefore, the degree of a vertex x in BN_G is $(deg(x)^+, deg(x)^-)$.

Example 3.4 Let BN_G be the neutrosophic bipolar fuzzy graph shown in Figure 1.

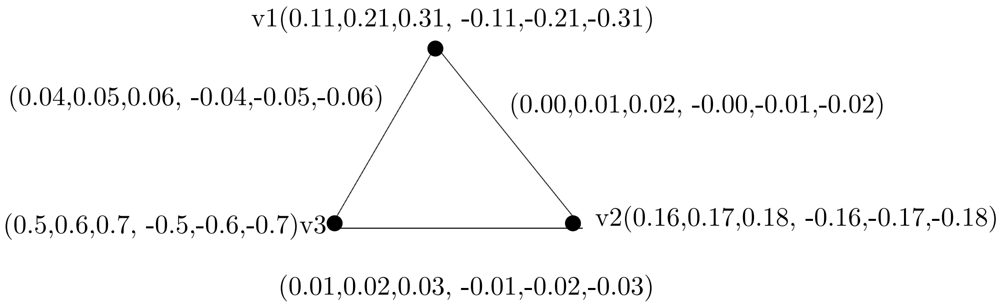

Figure 1

Then,

$$\begin{aligned} d(v_1)^+ &= (0.04, 0.06, 0.08),\ d(v_1)^- = (-0.04, -0.06, -0.08), \\ d(v_2)^+ &= (0.01, 0.03, 0.05),\ d(v_2)^- = (-0.01, -0.03, -0.05), \\ d(v_3)^+ &= (0.05, 0.07, 0.09),\ d(v_3)^- = (-0.05, -0.07, -0.09). \end{aligned}$$

Definition 3.5 *Let $BN_G = (A, B)$, where $A = (m_1^+, m_1^-)$ and $B = (m_2^+, m_2^-)$ be a neutrosophic bipolar fuzzy graph. The closed positive neighborhood degree of a vertex x in BN_G defined by*

$$deg[x]^+ = (deg_T[x]^+, deg_I[x]^+, deg_F[x]^+)$$

where,

$$\begin{aligned} deg_T(x)^+ &= \sum_{xy \in E} T_B(xy)^+ + T_A(x)^+, \\ deg_I(x)^+ &= \sum_{xy \in E} I_B(xy)^+ + I_A(x)^+, \\ deg_F(x)^+ &= \sum_{xy \in E} F_B(xy)^+ + F_A(x)^+. \end{aligned}$$

The closed negative neighborhood degree of a vertex BN_G defined by

$$deg[x]^- = (deg_T[x]^-, deg_I[x]^-, deg_F[x]^-)$$

where,

$$\begin{aligned} deg_T(x)^- &= \sum_{xy\in E} T_B(xy)^- + T_A(x)^-, \\ deg_I(x)^- &= \sum_{xy\in E} I_B(xy)^- + I_A(x)^-, \\ deg_F(x)^- &= \sum_{xy\in E} F_B(xy)^- + F_A(x)^-. \end{aligned}$$

Therefore, the closed degree of a vertex x in BN_G is $(deg[x]^+, deg[x]^-)$.

Example 3.6 Let BN_G be a neutrosophic bipolar fuzzy graph shown in Figure 1. We calculate closed degree of a vertices in the above neutrosophic bipolar fuzzy graphs as follows:

$$\begin{aligned} d[v_1]^+ &= (0.15, 0.27, 0.39), d[v_1]^- = (-0.15, -0.27, -0.39), \\ d[v_2]^+ &= (0.17, 0.20, 0.23), d[v_2]^- = (-0.17, -0.20, -0.23), \\ d[v_3]^+ &= (0.55, 0.67, 0.79), d[v_3]^- = (-0.55, -0.67, -0.79). \end{aligned}$$

§4. Pseudo Degree and Total Pseudo Degree in Neutrosophic Bipolar Fuzzy Graph

Definition 4.1 *Let BN_G be a neutrosophic bipolar fuzzy graph on $G^*(V,E)$. The 2-degree of a vertex v in BN_G is defined as the sum of the degrees of the vertices adjacent to v and is denoted by*

$$td_{BN_G}(v) = (td^+_{BN_G}(v), td^-_{BN_G}(v)).$$

That is, the positive 2-degree of v is

$$td^+_{BN_G}(v) = \sum d^+_{BN_G}(u),$$

where $d^+_{BN_G}(u)$ is the positive degree of the vertex u which is adjacent with the vertex v and the negative 2-degree of v is

$$td^-_{BN_G}(v) = \sum d^-_{BN_G}(u),$$

where $d^-_{BN_G}(u)$ is the negative degree of the vertex u which is adjacent with the vertex v.

Definition 4.2 *Let BN_G be a neutrosophic bipolar fuzzy graph on $G^*(V,E)$. A positive pseudo (average) degree of a vertex v in BN_G is denoted by $pd^+_{BN_G}(v)$ and is defined by*

$$pd^+_{BN_G}(v) = \frac{td^+_{BN_G}(v)}{d^*_{BN_G}(v)},$$

where $d^*_{BN_G}(v)$ is the number of edges incident at v. The negative pseudo (average) degree of a vertex v in BN_G is denoted by $pd^-_{BN_G}(v)$ and is defined by

$$pd^-_{BN_G}(v) = \frac{td^-_{BN_G}(v)}{d^*_{BN_G}(v)},$$

where $d^*_{BN_G}(v)$ is the number of edges incident at v.

The pseudo degree of a vertex v in neutrosophic bipolar fuzzy graph BN_G is defined as $pd_{BN_G}(v) = (pd^+_{BN_G}(v), pd^-_{BN_G}(v))$.

Example 4.3 Let BN_G be the neutrosophic bipolar fuzzy graph shown in Figure 2.

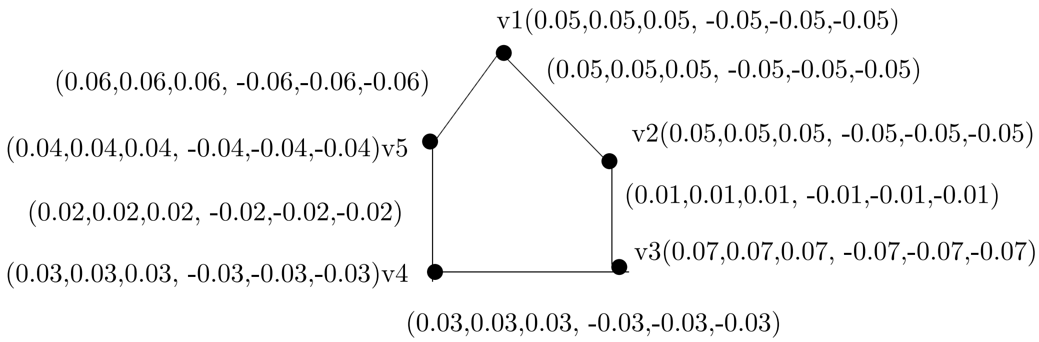

Figure 2

We calculate pseudo degree of vertex of the neutrosophic bipolar fuzzy graph in Figure 2 as follows:

$$\begin{aligned}
d(v_1)^+ &= (0.11, 0.11, 0.11), d(v_1)^- = (-0.11, -0.11, -0.11),\\
d(v_2)^+ &= (0.06, 0.06, 0.06),\ d(v_2)^- = (-0.06, -0.06, -0.06),\\
d(v_3)^+ &= (0.04, 0.04, 0.04),\ d(v_3)^- = (-0.04, -0.04, -0.04),\\
d(v_4)^+ &= (0.05, 0.05, 0.05),\ d(v_4)^- = (-0.05, -0.05, -0.05),\\
d(v_5)^+ &= (0.08, 0.08, 0.08),\ d(v_5)^- = (-0.08, -0.08, -0.08),
\end{aligned}$$

$$pd_{BN_G}(v_1)^+ = \frac{td^+_{BN_G}(v_1)}{d^*_{BN_G}(v_1)} = (-0.07, -0.07, -0.07),$$

$$pd_{BN_G}(v_1)^- = \frac{td^-_{BN_G}(v_1)}{d^*_{BN_G}(v_1)} = (-0.07, -0.07, -0.07),$$

$$\begin{aligned}
pd_{BN_G}(v_2)^+ &= (0.075, 0.075, 0.075),\\
pd_{BN_G}(v_2)^- &= (-0.075, -0.075, -0.075),\\
pd_{BN_G}(v_3)^+ &= (0.055, 0.055, 0.055),\\
pd_{BN_G}(v_3)^- &= (-0.055, -0.055, -0.055),
\end{aligned}$$

$$\begin{aligned}
pd_{BN_G}(v_4)^+ &= (0.06, 0.06, 0.06),\\
pd_{BN_G}(v_4)^- &= (-0.06, -0.06, -0.06),\\
pd_{BN_G}(v_5)^+ &= (0.08, 0.08, 0.08),\\
pd_{BN_G}(v_5)^- &= (-0.08, -0.08, -0.08).
\end{aligned}$$

Definition 4.4 *Let BN_G be a neutrosophic bipolar fuzzy graph on $G^*(V, E)$. A positive total pseudo degree of a vertex v in BN_G is denoted by $tpd^+_{BN_G}(v)$ and is defined by*

$$tpd^+_{BN_G}(v) = pd^+_{BN_G}(v) + (T_A, I_A, F_A)^+(v).$$

The negative total pseudo degree of a vertex v in BN_G is denoted by $tpd^-_{BN_G}(v)$ and is defined by

$$tpd^-_{BN_G}(v) = pd^-_{BN_G}(v) + (T_A, I_A, F_A)^-(v).$$

The total pseudo degree of a vertex v in BN_G is denoted by

$$tpd_{BN_G}(v) = (tpd^+_{BN_G}(v), tpd^-_{BN_G}(v))$$

for all $v \in V$.

Example 4.5 Let BN_G be the a neutrosophic bipolar fuzzy graph in Figure 2. We calculate the total pseudo degree of vertex of this neutrosophic bipolar fuzzy graphs as follows:

$$\begin{aligned}
tpd_{BN_G}(v_1)^+ &= pd^+_{BN_G}(v_1) + (T_A, I_A, F_A)^+(v_1) = (0.12, 0.12, 0.12),\\
tpd_{BN_G}(v_1)^- &= pd^-_{BN_G}(v_1) + (T_A, I_A, F_A)^-(v_1) = (-0.12, -0.12, -0.12),\\
tpd_{BN_G}(v_2)^+ &= (0.125, 0.125, 0.125),\\
tpd_{BN_G}(v_2)^- &= (-0.125, -0.125, -0.125),\\
tpd_{BN_G}(v_3)^+ &= (0.125, 0.125, 0.125),\\
tpd_{BN_G}(v_3)^- &= (-0.125, -0.125, -0.125),\\
tpd_{BN_G}(v_4)^+ &= (0.09, 0.09, 0.09),\\
tpd_{BN_G}(v_4)^- & \quad = (-0.09, -0.09, -0.09),\\
tpd_{BN_G}(v_5)^+ &= (00.12, 0.12, 0.12),\\
tpd_{BN_G}(v_5)^- &= (-0.12, -0.12, -0.12).
\end{aligned}$$

§5. Neighbourly Pseudo and Pseudo Totally Irregular Neutrosophic Bipolar Fuzzy Graphs

Definition 5.1 *Let $BN_G = (A, B)$ be a neutrosophic bipolar fuzzy graph. Then BN_G is said to be neighbourly pseudo irregular neutrosophic bipolar fuzzy graph if every pair of adjacent vertices have distinct pseudo degree.*

Definition 5.2 *Let $BN_G = (A, B)$ be a neutrosophic bipolar fuzzy graph. Then BN_G is said to be neighbourly pseudo totally irregular neutrosophic bipolar fuzzy graph if every pair of adjacent vertices have distinct total pseudo degree.*

Remark 5.3 A graph which is both neighbourly pseudo irregular neutrosophic bipolar fuzzy graph and neighbourly pseudo totally irregular neutrosophic bipolar fuzzy graph.

Example 5.4 Let BN_G be the neutrosophic bipolar fuzzy graph shown in Figure 3.

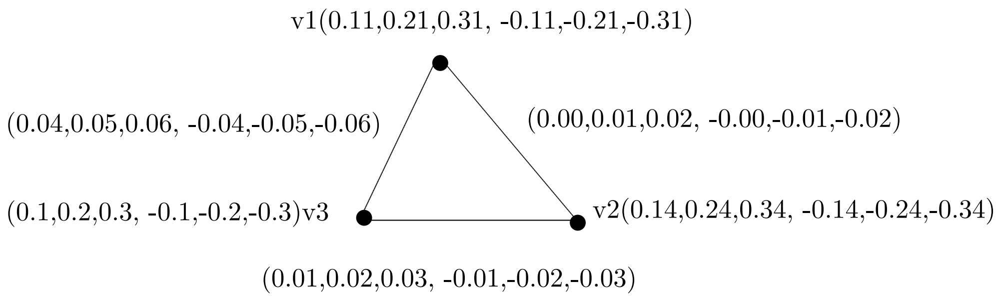

Figure 3

Then,

$$\begin{aligned}
d(v_1)^+ &= (0.04, 0.06, 0.08),\ d(v_1)^- = (-0.04, -0.06, -0.08),\\
d(v_2)^+ &= (0.01, 0.03, 0.05),\ d(v_2)^- = (-0.01, -0.03, -0.05),\\
d(v_3)^+ &= (0.05, 0.07, 0.09),\ d(v_3)^- = (-0.05, -0.07, -0.09)
\end{aligned}$$

and

$$\begin{aligned}
pd_{BN_G}(v_1)^+ &= (0.03, 0.05, 0.07),\\
pd_{BN_G}(v_1)^- &= (-00.03, -0.05, -0.07),\\
pd_{BN_G}(v_2)^+ &= (0.045, 0.065, 0.085),\\
pd_{BN_G}(v_2)^- &= (-0.045, -0.065, -0.085),\\
pd_{BN_G}(v_3)^+ &= (0.025, 0.045, 0.065),\\
pd_{BN_G}(v_3)^- &= (-0.025, -0.045, -0.065),\\
tpd_{BN_G}(v_1)^+ &= (0.14, 0.26, 0.38),\\
tpd_{BN_G}(v_1)^- &= (-0.14, -0.26, -0.38),\\
tpd_{BN_G}(v_2)^+ &= (0.185, 0.305, 0.435),\\
tpd_{BN_G}(v_2)^- &= (-0.185, -0.305, -0.435),\\
tpd_{BN_G}(v_3)^+ &= (0.135, 0.245, 0.365),\\
tpd_{BN_G}(v_3)^- &= (-0.135, -0.245, -0.365).
\end{aligned}$$

Here every pair of adjacent vertices have distinct pseudo degree and every pair of adjacent vertices have distinct total pseudo degree. Hence the graph given in Figure 3, is a neighbourly pseudo irregular neutrospphic bipolar fuzzy graph and neighbourly pseudo totally irregular neutrosophic bipolar fuzzy graph.

Remark 5.5 Every neighbourly pseudo irregular neutrosophic bipolar fuzzy graph need not be a neighbourly pseudo totally irregular neutrosophic bipolar fuzzy graph.

Example 5.6 Let BN_G be the neutrosophic bipolar fuzzy graph shown in Figure 4.

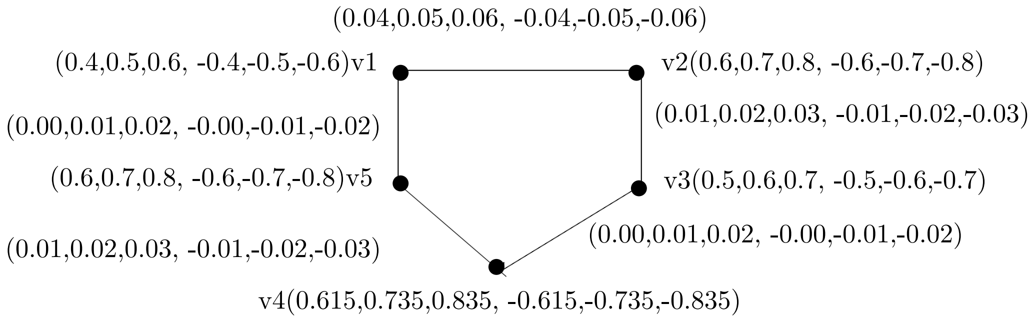

Figure 4

Then,

$$\begin{aligned}
d(v_1)^+ &= (0.04, 0.06, 0.08),\ d(v_1)^- = (-0.04, -0.06, -0.08),\\
d(v_2)^+ &= (0.05, 0.07, 0.09),\ d(v_2)^- = (-0.05, -0.07, -0.09),\\
d(v_3)^+ &= (0.01, 0.03, 0.05),\ d(v_3)^- = (-0.01, -0.03, -0.05),\\
d(v_4)^+ &= (0.01, 0.03, 0.05),\ d(v_4)^- = (-0.01, -0.03, -0.05),\\
d(v_5)^+ &= (0.01, 0.03, 0.05),\ d(v_5)^- = (-0.01, -0.03, -0.05)
\end{aligned}$$

and

$$\begin{aligned}
pd_{BN_G}(v_1)^+ &= (0.03, 0.05, 0.07),\\
pd_{BN_G}(v_1)^- &= (-0.03, -0.05, -0.07),\\
pd_{BN_G}(v_2)^+ &= (0.025, 0.065, 0.085),\\
pd_{BN_G}(v_2)^- &= (-0.025, -0.065, -0.085),\\
pd_{BN_G}(v_3)^+ &= (0.03, 0.05, 0.07),\\
pd_{BN_G}(v_3)^- &= (-0.03, -0.05, -0.07),\\
pd_{BN_G}(v_4)^+ &= (0.01, 0.03, 0.05),\\
pd_{BN_G}(v_4)^- &= (-0.01, -0.03, -0.05),\\
pd_{BN_G}(v_5)^+ &= (0.025, 0.065, 0.085),\\
pd_{BN_G}(v_5)^- &= (-0.025, -0.065, -0.085),
\end{aligned}$$

$$\begin{aligned}
tpd_{BN_G}(v_1)^+ &= (0.43, 0.55, 0.67),\\
tpd_{BN_G}(v_1)^- &= (-0.43, -0.55, -0.67),\\
tpd_{BN_G}(v_2)^+ &= (0.625, 0.765, 0.885),\\
tpd_{BN_G}(v_2)^- &= (-0.625, -0.765, -0.885),\\
tpd_{BN_G}(v_3)^+ &= (0.53, 0.65, 0.77),\\
tpd_{BN_G}(v_3)^- &= (-0.53, -0.65, -0.77),\\
tpd_{BN_G}(v_4)^+ &= (0.625, 0.765, 0.885),\\
tpd_{BN_G}(v_4)^- &= (-0.625, -0.765, -0.885),\\
tpd_{BN_G}(v_5)^+ &= (0.625, 0.765, 0.885),\\
tpd_{BN_G}(v_5)^- &= (-0.625, -0.765, -0.885).
\end{aligned}$$

Here every pair of adjacent vertices have distinct pseudo degree. But the pair of adjacent vertices v_4 and v_5 have same total pseudo degree. Hence the graph is neighbourly pseudo irregular neutrosophic bipolar fuzzy graph. But not neighbourly pseudo totally irregular neutrosophic bipolar fuzzy graph.

Remark 5.7 Every neighbourly pseudo totally irregular neutrosophic bipolar fuzzy graph need not be a neighbourly pseudo irregular neutrosophic bipolar fuzzy graph.

Example 5.8 Let BN_G be the neutrosophic bipolar fuzzy graph shown in Figure 5.

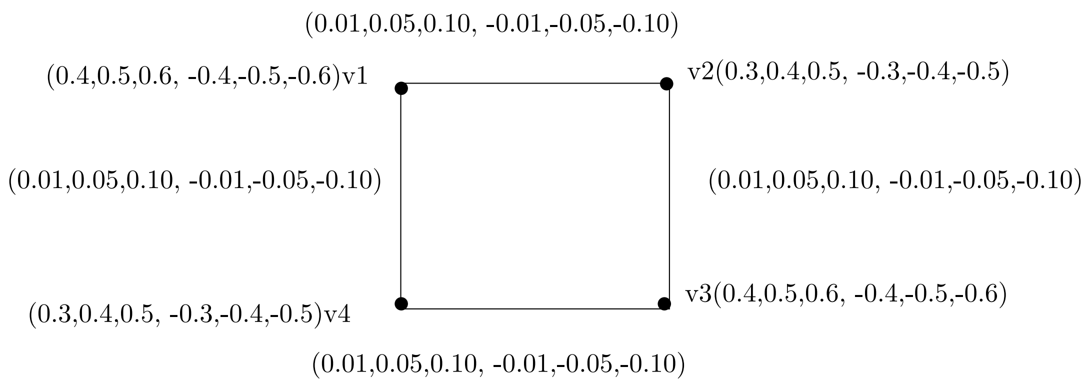

Figure 5

$$\begin{aligned}
d(v_1)^+ &= (0.02, 0.10, 0.20),\ d(v_1)^- = (-0.02, -0.10, -0.20),\\
d(v_2)^+ &= (0.02, 0.10, 0.20),\ d(v_2)^- = (-0.02, -0.10, -0.20),\\
d(v_3)^+ &= (0.02, 0.10, 0.20),\ d(v_3)^- = (-0.02, -0.10, -0.20),\\
d(v_4)^+ &= (0.02, 0.10, 0.20),\ d(v_4)^- = (-0.02, -0.10, -0.20)
\end{aligned}$$

and

$$\begin{aligned}
pd_{BN_G}(v_1)^+ &= (0.02, 0.10, 0.20),\\
pd_{BN_G}(v_1)^- &= (-0.02, -0.10, -0.20),\\
pd_{BN_G}(v_2)^+ &= (0.02, 0.10, 0.20),\\
pd_{BN_G}(v_2)^- &= (-0.02, -0.10, -0.20),\\
pd_{BN_G}(v_3)^+ &= (0.02, 0.10, 0.20),\\
pd_{BN_G}(v_3)^- &= (-0.02, -0.10, -0.20),\\
pd_{BN_G}(v_4)^+ &= (0.02, 0.10, 0.20),\\
pd_{BN_G}(v_4)^- &= (-0.02, -0.10, -0.20),\\
tpd_{BN_G}(v_1)^+ &= (0.42, 0.60, 0.80),\\
tpd_{BN_G}(v_1)^- &= (-0.42, -0.60, -0.80),\\
tpd_{BN_G}(v_2)^+ &= (0.32, 0.50, 0.70),\\
tpd_{BN_G}(v_2)^- &= (-0.32, -0.50, -0.70),\\
tpd_{BN_G}(v_3)^+ &= (0.42, 0.60, 0.80),\\
tpd_{BN_G}(v_3)^- &= (-0.42, -0.60, -0.80),\\
tpd_{BN_G}(v_4)^+ &= (0.32, 0.50, 0.70),\\
tpd_{BN_G}(v_4)^- &= (-0.32, -0.50, -0.70).
\end{aligned}$$

Here every pair of adjacent vertices have same pseudo degree. But every pair of adjacent vertices have distinct total pseudo degree. Hence the graph is neighbourly pseudo totally irregular neutrosophic bipolar fuzzy graph. But not neighbourly pseudo irregular neutrosophic bipolar fuzzy graph.

Theorem 5.9 *Let BN_G be a neutrosophic bipolar fuzzy graph and let*

$$(T_A, I_A, F_A)(u) = ((T_A, I_A, F_A)^+)(u), (T_A, I_A, F_A)^-)(u))$$

for all $u \in V$ be a constant function, then the following are equivalent:

(1) *BN_G is a neighbourly pseudo irregular neutrosophic bipolar fuzzy graph;*

(2) *BN_G is a neighbourly pseudo totally irregular neutrosophic bipolar fuzzy graph.*

Proof Assume that

$$(T_A, I_A, F_A)(u) = ((T_A, I_A, F_A)^+(u),\ (T_A, I_A, F_A)^-(u)) = ((c_T^+, c_I^+, c_F^+), (c_T^-, c_I^-, c_F^-))$$

for all $u \in V$ is a constant function. Suppose BN_G is neighbourly pseudo irregular neutrosophic bipolar fuzzy graph. Then, every pair of adjacent vertices having distinct pseudo degree.

Let v_i and v_j be two adjacent vertices having distinct pseudo degree (x_i, y_i, z_i) and (x_j, y_j, z_j)

respectively. Then $(x_i, y_i, z_i) \neq (x_j, y_j, z_j)$i.e,

$$\begin{aligned}
&pd_{BN_G}(v_i) = (pd_{BN_G}(v_i)^+, pd_{BN_G}(v_i)^-),\ pd_{BN_G}(v_j) = (pd_{BN_G}(v_j)^+, pd_{BN_G}(v_j)^-) \text{ and} \\
&pd_{BN_G}(v_i)^+ \neq pd_{BN_G}(v_j)^+ \\
&\qquad \Rightarrow (x_i, y_i, z_i)^+ \neq (x_j, y_j, z_j)^+ \\
&\qquad \Rightarrow (x_i, y_i, z_i)^+ + (c_T^+, c_I^+, c_F^+) \neq (x_j, y_j, z_j)^+ + (c_T^+, c_I^+, c_F^+) \\
&\qquad \Rightarrow (x_i, y_i, z_i)^+ + (T_A, I_A, F_A)^+(v_i) \neq (x_j, y_j, z_j)^+ + (T_A, I_A, F_A)^+(v_j) \\
&\qquad \Rightarrow tpd_{BN_G}(v_i)^+ \neq tpd_{BN_G}(v_j)^+.
\end{aligned}$$

Similarly, we prove that

$$\begin{aligned}
&pd_{BN_G}(v_i)^- \neq pd_{BN_G}(v_j)^- \\
&\qquad \Rightarrow tpd_{BN_G}(v_i)^- \neq tpd_{BN_G}(v_j)^-.
\end{aligned}$$

In general

$$pd_{BN_G}(v_i) \neq pd_{BN_G}(v_j) \Rightarrow tpd_{BN_G}(v_i) \neq tpd_{BN_G}(v_j).$$

Therefore, every pair of adjacent vertices having distinct total pseudo degree, i.e, (1) $\Rightarrow$ (2) is proved.

Now, suppose BN_G is neighbourly pseudo totally irregular neutrosophic bipolar fuzzy graph. Then every pair of adjacent vertices have distinct total pseudo degree. Let v_i and v_j be two adjacent vertices having distinct total pseudo degree (tx_i, ty_i, tz_i) and (tx_j, ty_j, tz_j) respectively. Then

$$(tx_i, ty_i, tz_i) \neq (tx_j, ty_j, tz_j),$$

i.e,

$$\begin{aligned}
tpd_{BN_G}(v_i) &= (tpd_{BN_G}(v_i)^+, tpd_{BN_G}(v_i)^-),\ tpd_{BN_G}(v_i)^+ \neq tpd_{BN_G}(v_j)^+ \text{ and} \\
tpd_{BN_G}(v_j) &= (tpd_{BN_G}(v_j)^+, tpd_{BN_G}(v_j)^-) \\
&\Rightarrow (tx_i, ty_i, tz_i)^+ \neq (tx_j, ty_j, tz_j)^+ \\
&\Rightarrow (x_i, y_i, z_i)^+ + (T_A, I_A, F_A)^+(v_i) \neq (x_j, y_j, z_j)^+ + (T_A, I_A, F_A)^+(v_j) \\
&\Rightarrow (x_i, y_i, z_i)^+ + (c_T^+, c_I^+, c_F^+) \neq (x_j, y_j, z_j)^+ + (c_T^+, c_I^+, c_F^+) \\
&\Rightarrow (x_i, y_i, z_i)^+ \neq (x_j, y_j, z_j)^+ \\
&\Rightarrow pd_{BN_G}(v_i)^+ \neq pd_{BN_G}(v_j)^+.
\end{aligned}$$

Similarly, we prove that

$$tpd_{BN_G}(v_i)^- \neq tpd_{BN_G}(v_j)^- \Rightarrow pd_{BN_G}(v_i)^- \neq pd_{BN_G}(v_j)^-.$$

In general

$$tpd_{BN_G}(v_i) \neq tpd_{BN_G}(v_j) \Rightarrow pd_{BN_G}(v_i) \neq pd_{BN_G}(v_j).$$

Therefore, every pair of adjacent vertices having distinct pseudo degree and (2) $\Rightarrow$ (1) is proved.

Hence (1) and (2) are equivalent. □

Remark 5.10 The converse of Theorem 5.9 needs not be true.

Example 5.11 Let BN_G be the neutrosophic bipolar fuzzy graph shown in Figure 6.

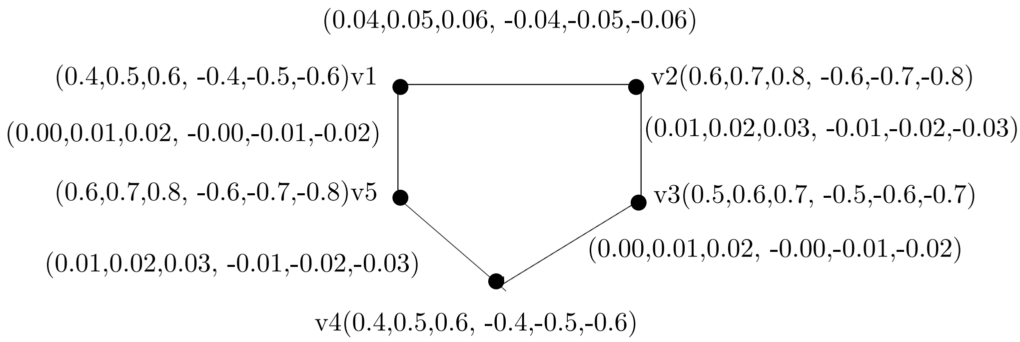

Figure 6

Then,

$$\begin{aligned}
d(v_1)^+ &= (0.04, 0.06, 0.08),\ d(v_1)^- = (-0.04, -0.06, -0.08),\\
d(v_2)^+ &= (0.05, 0.07, 0.09),\ d(v_2)^- = (-0.05, -0.07, -0.09),\\
d(v_3)^+ &= (0.01, 0.03, 0.05),\ d(v_3)^- = (-0.01, -0.03, -0.05),\\
d(v_4)^+ &= (0.01, 0.03, 0.05),\ d(v_4)^- = (-0.01, -0.03, -0.05),\\
d(v_5)^+ &= (0.01, 0.03, 0.05),\ d(v_5)^- = (-0.01, -0.03, -0.05)
\end{aligned}$$

and

$$\begin{aligned}
pd_{BN_G}(v_1)^+ &= (0.03, 0.05, 0.07),\\
pd_{BN_G}(v_1)^- &= (-0.03, -0.05, -0.07),\\
pd_{BN_G}(v_2)^+ &= (0.025, 0.065, 0.085),\\
pd_{BN_G}(v_2)^- &= (-0.025, -0.065, -0.085),\\
pd_{BN_G}(v_3)^+ &= (0.03, 0.05, 0.07),\\
pd_{BN_G}(v_3)^- &= (-0.03, -0.05, -0.07),\\
pd_{BN_G}(v_4)^+ &= (0.01, 0.03, 0.05),\\
pd_{BN_G}(v_4)^- &= (-0.01, -0.03, -0.05),\\
pd_{BN_G}(v_5)^+ &= (0.025, 0.065, 0.085),\\
pd_{BN_G}(v_5)^- &= (-0.025, -0.065, -0.085),
\end{aligned}$$

$$\begin{aligned}
tpd_{BN_G}(v_1)^+ &= (0.43, 0.55, 0.67),\\
tpd_{BN_G}(v_1)^- &= (-0.43, -0.55, -0.67),\\
tpd_{BN_G}(v_2)^+ &= (0.625, 0.765, 0.885),\\
tpd_{BN_G}(v_2)^- &= (-0.625, -0.765, -0.885),\\
tpd_{BN_G}(v_3)^+ &= (0.53, 0.65, 0.77),\\
tpd_{BN_G}(v_3)^- &= (-0.53, -0.65, -0.77),\\
tpd_{BN_G}(v_4)^+ &= (0.41, 0.53, 0.65),\\
tpd_{BN_G}(v_4)^- &= (-0.41, -0.53, -0.65),\\
tpd_{BN_G}(v_5)^+ &= (0.625, 0.765, 0.885),\\
tpd_{BN_G}(v_5)^- &= (-0.625, -0.765, -0.885).
\end{aligned}$$

Here the graph is both neighbourly pseudo irregular neutrosophic bipolar fuzzy graph and neighbourly pseudo totally irregular neutrosophic bipolar fuzzy graph. But here

$$(T_A, I_A, F_A)(u) = ((T_A, I_A, F_A)^+(u), (T_A, I_A, F_A)^-(u))$$

for all u in V is not constant.

References

[1] A. Rosenfeld,*Fuzzy Graphs, Fuzzy Sets And Their Applications To Cognitive and Decision Process*, M.Eds. Academic Press, New York,1975,77-95.

[2] N. Shah, A. Hussain, Neutrosophic soft graphs, *Neutrosophic Sets and Systems*, Vol.11, 2016, 31- 44.

[3] N.Shah, Some studies in neutrosophic graphs, *Neutrosophic Sets and Systems*, Vol.12(2016), 54- 64.

[4] F. Smarandache, Neutrosophic set, a generalisation of the intuitionistic fuzzy sets, *Inter. J.Pure Appl. Math.*, 24 (2005), 287-297.

[5] A. Nagoor Gani and S.R Latha, On irregular fuzzy graphs, *Applied Mathematical Sciences*, Vol.6, No.11(2012), 517-523.

[6] Shah Nasir and Said Broumi,Irregular neutrosophic graphs, *Neutrosophic Sets and Systems*, Vol. 13(2016), 46-55.

[7] G. Divya and J. Malarvizhi, Some operations on neutrosophic fuzzy graph, *International Journal of Mathematical Archieve*, 8(9), 2017, 120-125.

[8] Sovan Samanta and Madhumangal Pal, Irregular bipolar fuzzy graph, *Inernational Journal of Applications of Fuzzy Sets*, Vol. 2 (2012), 91-102.

[9] M.Akram and A.Dudek, Regular bipolar fuzzy graphs,*Neural Comput. And App;ic.*, 21 (Suppl 1) (2012), 197-205.

[10] H.B. Walikar, S.B. Halkarni, H.S. Ramane, M. Tavakoli and A.R. Ashrafi, On nNeighbourly irregular graphs, *Kragujevac Journal of Mathematics* , Vol.39, 1(2015), 31-39.

[11] N.R.Santhi Maheswari and M.Sudha, Pseudo Irregular fuzzy graph and Highly Pseudo

Irregular fuzzy graph, *International Journal of Mathematical Archive* , 7(4), 2016, 99-106.

[12] N.R.Santhi Maheswari and C.Sekar, On pseudo regular fuzzy graph, *Annals of Pure and Applied Mathematics*, Vol. 11, No. 1, 2016, 105-113.

[13] N.R.Santhi Maheswari and M.Rajeswari, Strongly pseudo irregular fuzzy graph, *International Journal of Mathematical Archive*, 7(6), 2016, 145-151.

[14] N.R.Santhi Maheswari and C.Sekar, On neighbourly pseudo and strongly pseudo irregular bipolar fuzzy graph, *International Journal of Innoviative Research in Science, Engineering and Technology*, Vol 5, Issue 8, August 2016, 15795-15800.

[15] N.R.Santhi Maheswari and V.Jeyapratha, Neighbourly pseudo irregular fuzzy graph, *International J.Math. Combin.*, Vol.4(2018), 45-52.

[16] N.R.Santhi Maheswari and C.Sekar, On pseudo regular and irregular bipolar fuzzy graph, *Annals of Pure Applied Mathematics*, Vol.11(2), 2016, 123-131.

[17] S.Satham Huissain, R.jahir Hussain, Young Bae Jun and Florentin Smarandache, Neutrosophic bipolar vague set and its application to neutrosophic bipolar vague graphs, *Neutrosophic Set and Systems*, Vol.28, 2019, 69-86.