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A New Characterization of Ruled Surfaces According to q-Frame Vectors in Euclidean 3-Space

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Abstract: In this work, we study new families of ruled surfaces generated by q- frame vectors called quasi vectors in 3-dimensional Euclidean space. First, the characterizations of these ruled surfaces such as first and second fundamental forms, Gaussian and mean curvatures are given. After we work on the ruled surfaces generated by the general vector field and give the same caharacterizations for these surfaces whose director is general vector field, we investigate some geometric properties such as developability, minimality, striction curve, and distribution parameter. Lastly, we visualize the surfaces whose directors are tangent, q-normal, q-binormal and general vector field by taking two different curves.

Key Words: Gaussian curvature, mean curvature, quasi frame, ruled surface.

AMS(2010): 53A05, 53A10, 57R25.

§1. Introduction

In order to understand what's going on around us, we need to work on the surfaces. Therefore, it is important to have an idea about how to construct the surfaces. Considering the structural advantage of ruled surfaces and the ease of constructing their geometries, ruled surfaces are one of the most attractive surfaces to work on. The ruled surface is a special type of surface which is generated by the motion of a straight line (ruling) along a curve.

After these surfaces were found and investigated by Gaspard Mongea, Ravani and Ku studied ruled surface and examined some properties of them in 1991. Some of the studies have been done by Aydemir and Kasap in 2005, Sarioglugil and Tutar in 2007, Ali et. al. in 2013, Senturk and Yuce in 2015, Unluturk et. al., in 2016, Dede et al. in 2017, Kaymanli in 2020 and Gozutok et al., in 2020 in Euclidean space [1], [4], [5], [7], [9], [12]-[14], [17] while Turgut and Hacisalihoglu in 1998, Kimm and Yoon in 2004, Tosun and Gungor in 2005, Orbay and Aydemir in 2019 and in 2010, Kaymanli et. al., in 2020 in Minkowski space [2], [8], [10], [11], [15], [16].

In this work, we study new families of ruled surfaces generated by q- frame vectors called quasi vectors in 3-dimensional Euclidean space. First, the characterizations of these ruled surfaces such as first and second fundamental forms, Gaussian and mean curvatures are given.

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After we work on the ruled surfaces generated by the general vector field and give the same caharacterizations for these surfaces whose director is general vector field, we investigate some geometric properties such as developability, minimality, striction curve, and distribution parameter. Lastly, we visualize the surfaces whose directors are tangent, q-normal, q-binormal and general vector field by taking two different curves.

§2. Preliminaries

In this section, we give some background information about Frenet frame and how to construct q-frame. Let $\alpha(s)$ be a space curve with a non-vanishing second derivative. The Frenet frame is written as

$$\mathbf{t} = \frac{\alpha'}{\|\alpha'\|},\ \mathbf{b} = \frac{\alpha' \wedge \alpha''}{\|\alpha' \wedge \alpha''\|},\ \mathbf{n} = \mathbf{b} \wedge \mathbf{t}.$$

The curvature κ and the torsion τ are given by

$$\kappa = \frac{\|\alpha' \wedge \alpha''\|}{\|\alpha'\|^3}, \tau = \frac{\det(\alpha', \alpha'', \alpha''')}{\|\alpha' \wedge \alpha''\|^2}.$$

The well-known Frenet formulas are given by

$$\begin{bmatrix} \mathbf{t}' \\ \mathbf{n}' \\ \mathbf{b}' \end{bmatrix} = v \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix} \tag{1}$$

where $v = \|\alpha'(s)\|$.

Besides Frenet frame, we use another frame called q-frame consists of unit tangent vector $\mathbf{t}$, the q-normal $\mathbf{n}_q$ and the q-binormal vector $\mathbf{b}_q$ along a space curve $\alpha(t)$.

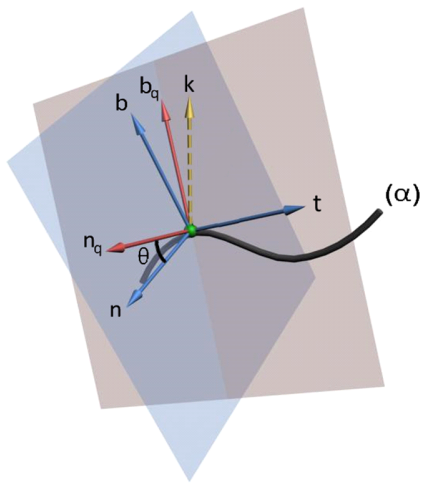

Figure 1 The q-frame and Frenet frame

The q-frame $\{\mathbf{t}, \mathbf{n}_q, \mathbf{b}_q, \mathbf{k}\}$ is defined by

$$\mathbf{t} = \frac{\alpha'}{\|\alpha'\|}, \mathbf{n}_q = \frac{\mathbf{t} \wedge \mathbf{k}}{\|\mathbf{t} \wedge \mathbf{k}\|}, \mathbf{b}_q = \mathbf{t} \wedge \mathbf{n}_q \tag{2}$$

shown in Figure 1, where $\mathbf{k}$ is the projection vector [3].

Without loss of generality, we chose the projection vector $\mathbf{k} = (0, 0, 1)$ in this study. However, the q-frame is singular in all cases where $\mathbf{t}$ and $\mathbf{k}$ are parallel. Thus, in those cases where $\mathbf{t}$ and $\mathbf{k}$ are parallel the projection vector $\mathbf{k}$ can be chosen as $\mathbf{k} = (0, 1, 0)$ or $\mathbf{k} = (1, 0, 0)$.

In order to define a relation between q-frame and Frenet frame, we pick Euclidean angle θ between the principal normal $\mathbf{n}$ and q-normal $\mathbf{n}_q$ vectors. Then the relation matrix may be expressed as

$$\begin{bmatrix} \mathbf{t} \\ \mathbf{n}_q \\ \mathbf{b}_q \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix} \tag{3}$$

or

$$\begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \mathbf{t} \\ \mathbf{n}_q \\ \mathbf{b}_q \end{bmatrix}. \tag{4}$$

Let $\alpha(s)$ be a curve that is parameterized by arc length s. Differentiating (3) with respect to s, then substituting (4) into the results gives the variation equations of the q-frame in the following form

$$\begin{bmatrix} \mathbf{t}' \\ \mathbf{n}_q' \\ \mathbf{b}_q' \end{bmatrix} = \begin{bmatrix} 0 & k_1 & k_2 \\ -k_1 & 0 & k_3 \\ -k_2 & -k_3 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{t} \\ \mathbf{n}_q \\ \mathbf{b}_q \end{bmatrix}, \tag{5}$$

where the q-curvatures are

$$\begin{aligned} k_1 &= \langle \mathbf{t}', \mathbf{n}_q \rangle \\ k_2 &= \langle \mathbf{t}', \mathbf{b}_q \rangle \\ k_3 &= \left\langle \mathbf{n}_q', \mathbf{b}_q \right\rangle. \end{aligned} \tag{6}$$

The parametric equation of ruled surface $\varphi(s, v)$ is given as

$$\varphi(s, v) = \alpha(s) + vX(s), \tag{7}$$

where $\alpha(s)$ is a curve and $X(s)$ is a generator vector. The distribution parameter of the ruled surface is identified by (see [6], [14])

$$P_X = \frac{\det(\alpha_s, X, X_s)}{\langle X_s, X_s \rangle}. \tag{8}$$

The striction point on the ruled surface is the foot of the common perpendicular line

successive rulings on the main ruling. It is given as

$$\beta_X(s) = \alpha(s) - \frac{\langle \alpha_s, X_s \rangle}{\langle X_s, X_s \rangle} X(s) \tag{9}$$

Let M be a regular surface given with the parameterization $\varphi(s,v)$ in E^3. The tangent space of M at an arbitrary point is spanned by the vectors φ_s and φ_v. The coefficients of the first fundamental form of M are defined as

$$E = \langle \varphi_s, \varphi_s \rangle, F = \langle \varphi_s, \varphi_v \rangle, G = \langle \varphi_v, \varphi_v \rangle, \tag{10}$$

where $\langle , \rangle$ is the Euclidean inner product. Then the unit normal vector field of M is defined as

$$N = \frac{\varphi_s \wedge \varphi_v}{||\varphi_s \wedge \varphi_v||}. \tag{11}$$

The coefficients of the second fundamental form of M are defined as

$$e = \langle \varphi_{ss}, N \rangle, f = \langle \varphi_{sv}, N \rangle, g = \langle \varphi_{vv}, N \rangle. \tag{12}$$

The Gaussian curvature and the mean curvature of M are given by

$$K = \frac{eg - f^2}{EG - F^2} \tag{13}$$

and

$$H = \frac{Eg + Ge - 2Ff}{2(EG - F^2)}, \tag{14}$$

respectively.

Theorem 2.1([12]) *The ruled surface is developable if and only if* $P_X = 0$.

Theorem 2.2 *The ruled surface is minimal if and only if* $H = 0$.

§3. Ruled Surfaces Generated by q-Frame Vectors

The ruled surfaces generated by q-frame vectors t, n_q, b_q are given as

$$\begin{aligned} \phi^t(s,v) &= \alpha(s) + vt(s), \\ \phi^{n_q}(s,u) &= \alpha(s) + un_q(s), \\ \phi^{b_q}(s,z) &= \alpha(s) + zb_q(s), \end{aligned}$$

respectively. The ruled surface generated by general vector filed X is written as

$$\phi^X(s,w) = \alpha(s) + wX(s) \tag{15}$$

where $X(s) = x_1(s)t + x_2(s)n_q + x_3(s)b_q$.

Theorem 3.1 *The distribution parameters of surfaces ϕ^t, ϕ^{n_q} and ϕ^{b_q} are*

$$P_t = 0, \quad P_{n_q} = \frac{k_3}{k_1^2 + k_3^2} \quad \text{and} \quad P_{b_q} = \frac{k_3}{k_2^2 + k_3^2},$$

respectively.

Theorem 3.2 *The striction curves on the ruled surfaces ϕ^t, ϕ^{n_q} and ϕ^{b_q} are given by*

$$\beta_t(s) = \alpha(s), \quad \beta_{n_q}(s) = \alpha(s) + \frac{k_1}{k_1^2 + k_3^2} n_q \quad \text{and} \quad \beta_{b_q}(s) = \alpha(s) + \frac{k_2}{k_2^2 + k_3^2} b_q,$$

respectively.

Theorem 3.3 *The distribution parameter and striction curve of ϕ^X are calculated as*

$$\begin{aligned} P_X &= \frac{-x_3(k_1x_1 + x_2' - k_3x_3) + x_2(k_1x_1 + k_3x_2 + x_3')}{(x_1' - k_1x_2 - k_2x_3)^2 + (k_1x_1 + x_2' - k_3x_3)^2 + (k_2x_1 + k_3x_2 + x_3')^2}, \\ \beta_X(s) &= \alpha(s) - \frac{(x_1' - k_1x_2 - k_2x_3)(x_1t + x_2n_q + x_3b_q)}{(x_1' - k_1x_2 - k_2x_3)^2 + (k_1x_1 + x_2' - k_3x_3)^2 + (k_2x_1 + k_3x_2 + x_3')^2} \end{aligned}$$

respectively.

Proof Taking derivative of $\alpha(s)$ and $X(s)$ with respect to s, we can easily find $\alpha'(s) = \mathbf{t}$ and

$$X'(s) = (x_1' - x_2k_1 - x_3k_2)\,\mathbf{t} + (x_1k_1 + x_2' - x_3k_3)\,\mathbf{n}_q + (x_1k_2 + x_2k_3 + x_3')\,\mathbf{b}_q,$$

respectively. With the help of the obtained equation and equations (8) and (9), an algebraic calculus gives us desired results. □

Theorem 3.4 *The Gaussian curvatures of surfaces ϕ^t, ϕ^{n_q} and ϕ^{b_q} are*

$$K_t = 0, \quad K_{n_q} = 0 \quad \text{and} \quad K_{b_q} = \frac{-k_3^2}{(k_3^2z^2 + (1 - k_2z)^2)^2},$$

respectively.

Theorem 3.5 *The mean curvatures of surfaces ϕ^t, ϕ^{n_q} and ϕ^{b_q} are*

$$H_t = \frac{-k_3}{2v\sqrt{k_1^2 + k_2^2}}, \quad H_{n_q} = \frac{k_2(1 - 2k_1u + (k_1^2 + k_3^2)u^2)}{2(k_3^2u^2 + (1 - k_1u)^2)^{3/2}}$$

and

$$H_{b_q} = \frac{-k_1}{2(k_3^2z^2 + (k_2z - 1)^2)},$$

respectively.

Theorem 3.6 *The Gaussian and mean curvatures of* ϕ^X *are*

$$\begin{aligned} H_X &= \frac{1}{2W\sqrt{(Bx_3-Cx_2)^2+(Cx_1-Ax_3)^2+(Ax_2-Bx_1)^2}}\left[\left((Bx_3-Cx_2)(A'-Bk_1-Ck_2)\right.\right.\\ &\quad +(Cx_1-Ax_3)(Ak_1+B'-Ck_3)+(Ax_2-Bx_1)(Ak_2-Bk_3+C'))\\ &\quad -2\left(AB^2x_1x_3(A/B)^{\cdot}+A^3x_1x_2(C/A)^{\cdot}+AC^2x_1^2(B/C)^{\cdot}+B^3x_2x_3(A/B)^{\cdot}\right)\\ &\quad +2\left(BA^2x_2^2(C/A)^{\cdot}+BC^2x_1x_2(B/C)^{\cdot}+CB^2x_3^2(A/B)^{\cdot}+CA^2x_2x_3(C/A)\right.\\ &\quad \left.\left.+C^3x_1x_3(B/C)^{\cdot}\right)\right]\\ K_X &= -\frac{1}{W}\left(\frac{x_3B^2(A/B)^{\cdot}+x_2A^2(C/A)^{\cdot}+x_1C^2(B/C)^{\cdot}}{\sqrt{(Bx_3-Cx_2)^2+(Cx_1-Ax_3)^2+(Ax_2-Bx_1)^2}}\right)^2 \end{aligned}$$

where $A=w(x_1'-k_1x_2-k_2x_3)+1,\quad B=w(k_1x_1+x_2'-k_3x_3),\ C=w(k_2x_1+k_3x_2+x_3')$, $W=A^2+B^2+C^2-A^2x_1^2+B^2x_2^2+2ABx_1x_2+C^2x_3^2+2ACx_1x_3+2BCx_2x_3,\ Y^{\cdot}=\dfrac{dY}{dw}$ *and* $Y'=\dfrac{dY}{ds}$.

Proof First and second partial derivatives of the surface given in (15) with respect to s and w are expressed as

$$\begin{aligned} \phi_s^X &= A\mathbf{t}+B\mathbf{n}_q+C\mathbf{b}_q\\ \phi_w^X &= x_1\mathbf{t}+x_2\mathbf{n}_q+x_3\mathbf{b}_q \end{aligned}$$

and

$$\begin{aligned} \phi_{ss}^X &= (A'-Bk_1-Ck_2)\mathbf{t}+(Ak_1+B'-Ck_3)\mathbf{n}_q+(Ak_2-Bk_3+C')\mathbf{b}_q\\ \phi_{sw}^X &= A^{\cdot}\mathbf{t}+B^{\cdot}\mathbf{n}_q+C^{\cdot}\mathbf{b}_q\\ \phi_{ww}^X &= 0, \end{aligned}$$

respectively. The coefficients of the first and second fundamental forms are calculated by $E=A^2+B^2+C^2,\ F=Ax_1+Bx_2+Cx_3,\ G=1$ and

$$\begin{aligned} e &= \frac{1}{\sqrt{(Bx_3-Cx_2)^2+(Cx_1-Ax_3)^2+(Ax_2-Bx_1)^2}}\left[(Bx_3-Cx_2)(A'-Bk_1-Ck_2)\right.\\ &\quad \left.+(Cx_1-Ax_3)(Ak_1+B'-Ck_3)+(Ax_2-Bx_1)(Ak_2-Bk_3+C')\right] \end{aligned}$$

$$\begin{aligned} f &= \frac{x_3B^2\left(\frac{A}{B}\right)^{\cdot}+x_2A^2\left(\frac{C}{A}\right)^{\cdot}+x_1C^2\left(\frac{B}{C}\right)^{\cdot}}{\sqrt{(Bx_3-Cx_2)^2+(Cx_1-Ax_3)^2+(Ax_2-Bx_1)^2}}\\ g &= 0 \end{aligned}$$

respectively. Using equations (13) and (14), the Gaussian and mean curvatures of ϕ^X are presented easily. This completes the proof. □

Corollary 3.7 *The ruled surface ϕ^t is developable and the ruled surfaces ϕ^{n_q} and ϕ^{b_q} are developable if and only if $k_3 = 0$.*

Corollary 3.8 *The ruled surface ϕ^t is minimal if and only if $k_3 = 0$, the ruled surface ϕ^{b_q} is minimal if and only if $k_1 = 0$ and the ruled surface ϕ^{n_q} is minimal if and only if either $k_2 = 0$ or $u = \frac{k_1(1\pm\sqrt{1-k_3^2})}{k_1^2+k_3^2}$.*

Corollary 3.9 *There is a relation between K_{b_q}, H_{b_q} and P_{b_q} as follows*

$$\frac{K_{b_q}}{H_{b_q}} = \frac{2k_3^2}{k_1(k_3^2z^2 + (1 - k_2z)}$$

and

$$\frac{K_{b_q}}{P_{b_q}} = \frac{-k_3(k_2^2 + k_3^2)}{(k_3^2z^2 + (1 - k_2z)^2)^2}.$$

§4. Examples

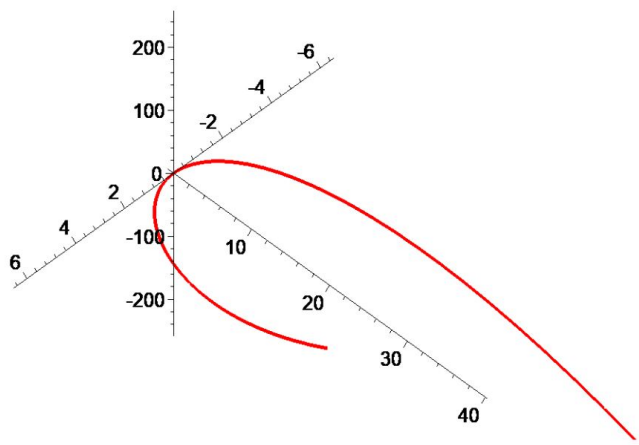

Figure 2 The curve $\alpha(s) = (s, s^2, s^3)$

Example 4.1 Consider the curve, shown in Figure 2,

$$\alpha(s) = (s, s^2, s^3)$$

with q-vectors and curvatures

$$\begin{aligned} t &= \frac{1}{\sqrt{1+4s^2+9s^4}}\left(1, 2s, 3s^2\right) \\ n_q &= \frac{1}{\sqrt{1+4s^2}}\left(2s, -1, 0\right) \\ b_q &= \frac{1}{\sqrt{1+4s^2+9s^4}\sqrt{1+4s^2}}\left(3s^2, 6s^3, -\sqrt{1+4s^2}\right) \end{aligned}$$

and

$$\begin{aligned} k_1 &= -\frac{2}{(1+4s^2+9s^4)\sqrt{1+4s^2}} \\ k_2 &= -\frac{6s(1+2s^2)}{(1+4s^2+9s^4)^{3/2}\sqrt{1+4s^2}} \\ k_3 &= \frac{6s^2}{(1+4s^2+9s^4)(1+4s^2)} \end{aligned}$$

respectively.

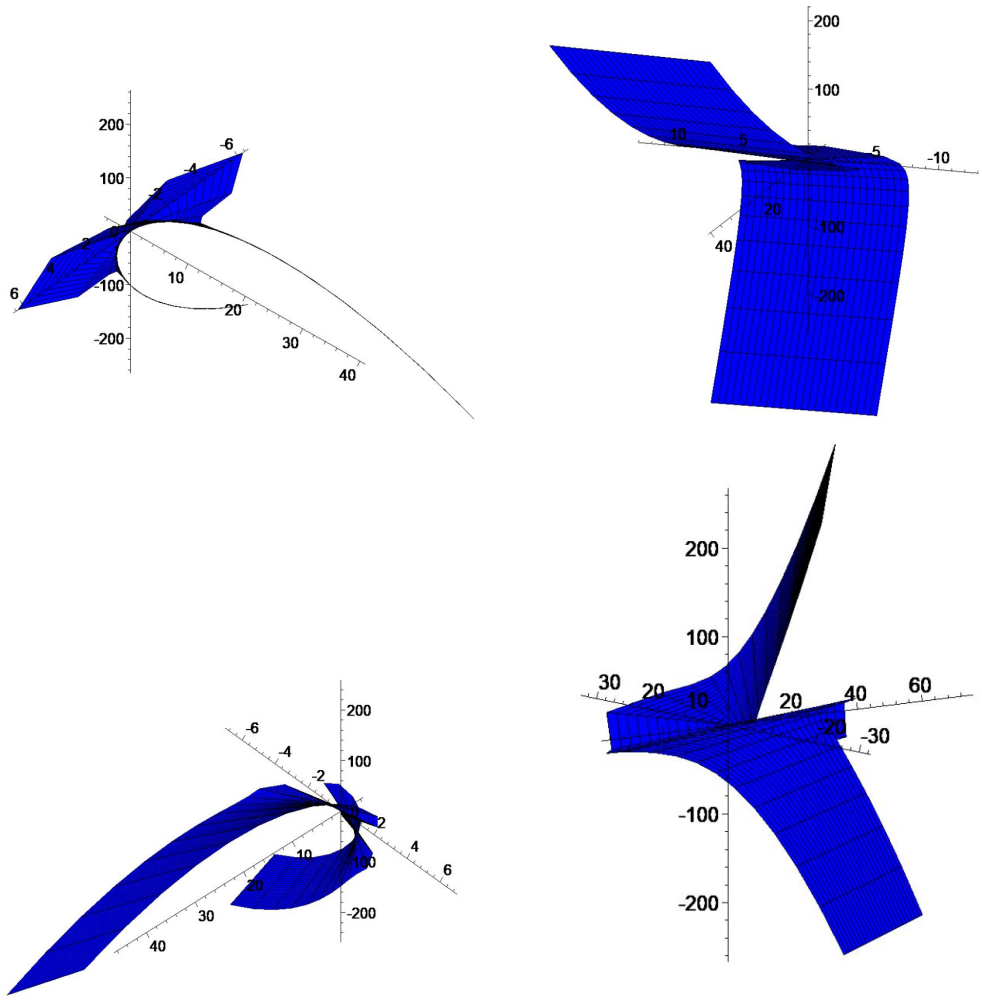

Figure 3 $\phi_1^t(s,v)$ (left top), $\phi_1^{n_q}(s,u)$ (right top), $\phi_1^{b_q}(s,z)$ (left bottom) and $\phi_1^X(s,w)$ for $x_1(s)=2,\ x_2(s)=3,\ x_3(s)=5$ (right bottom)

The ruled surfaces generated by q-frame vectors t, n_q, b_q and X shown in Figure 3, are given as

$$\begin{aligned} \phi_1^t(s,v) &= (s,s^2,s^3) + v\frac{1}{\sqrt{1+4s^2+9s^4}}\left(1,2s,3s^2\right), \\ \phi_1^{n_q}(s,u) &= (s,s^2,s^3) + u\frac{1}{\sqrt{1+4s^2}}\left(2s,-1,0\right), \end{aligned}$$

$$\phi_1^{b_q}(s,z) = (s,s^2,s^3) + z\frac{1}{\sqrt{1+4s^2+9s^4}\sqrt{1+4s^2}}\left(3s^2,6s^3,-\sqrt{1+4s^2}\right),$$

and

$$\begin{aligned}\phi_1^{X}(s,w) &= (s,s^2,s^3) + w\Bigg(x_1(s)(\frac{1}{\sqrt{1+4s^2+9s^4}}\left(1,2s,3s^2\right)) \\ &+x_2(s)(\frac{1}{\sqrt{1+4s^2}}\left(2s,-1,0\right)) \\ &+x_3(s)(\frac{1}{\sqrt{1+4s^2+9s^4}\sqrt{1+4s^2}}\left(3s^2,6s^3,-\sqrt{1+4s^2}\right))\Bigg),\end{aligned}$$

respectively.

Example 4.2 Consider the curve, shown in the Figure 4,

$$\alpha(s) = (\frac{5}{13}\sin\frac{s}{13}, -\frac{5}{13}\cos\frac{s}{13}, \frac{s}{169})$$

with q-vectors and curvatures

$$\begin{aligned} t &= \left(\frac{5}{\sqrt{26}}\cos\tfrac{s}{13}, \frac{5}{\sqrt{26}}\sin\tfrac{s}{13}, \frac{1}{\sqrt{26}}\right) \\ n_q &= \left(\sin\tfrac{s}{13}, -\cos\tfrac{s}{13}, 0\right) \\ b_q &= \left(\frac{1}{\sqrt{26}}\cos\tfrac{s}{13}, \frac{1}{\sqrt{26}}\sin\tfrac{s}{13}, -\frac{5}{\sqrt{26}}\right)\end{aligned}$$

and

$$\begin{aligned} k_1 &= -\frac{5}{2} \\ k_2 &= 0 \\ k_3 &= \frac{1}{2}\end{aligned}$$

respectively.

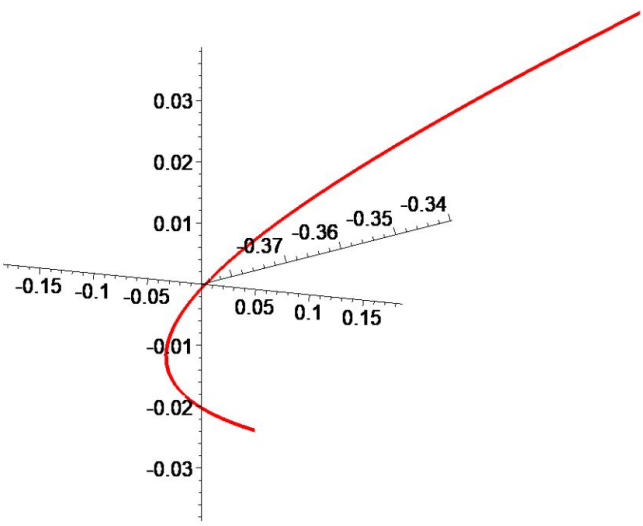

Figure 4 The curve $\alpha(s) = (\frac{5}{13}\sin\frac{s}{13}, -\frac{5}{13}\cos\frac{s}{13}, \frac{s}{169})$

The ruled surfaces generated by q-frame vectors t, n_q, b_q and X shown in Figure 5

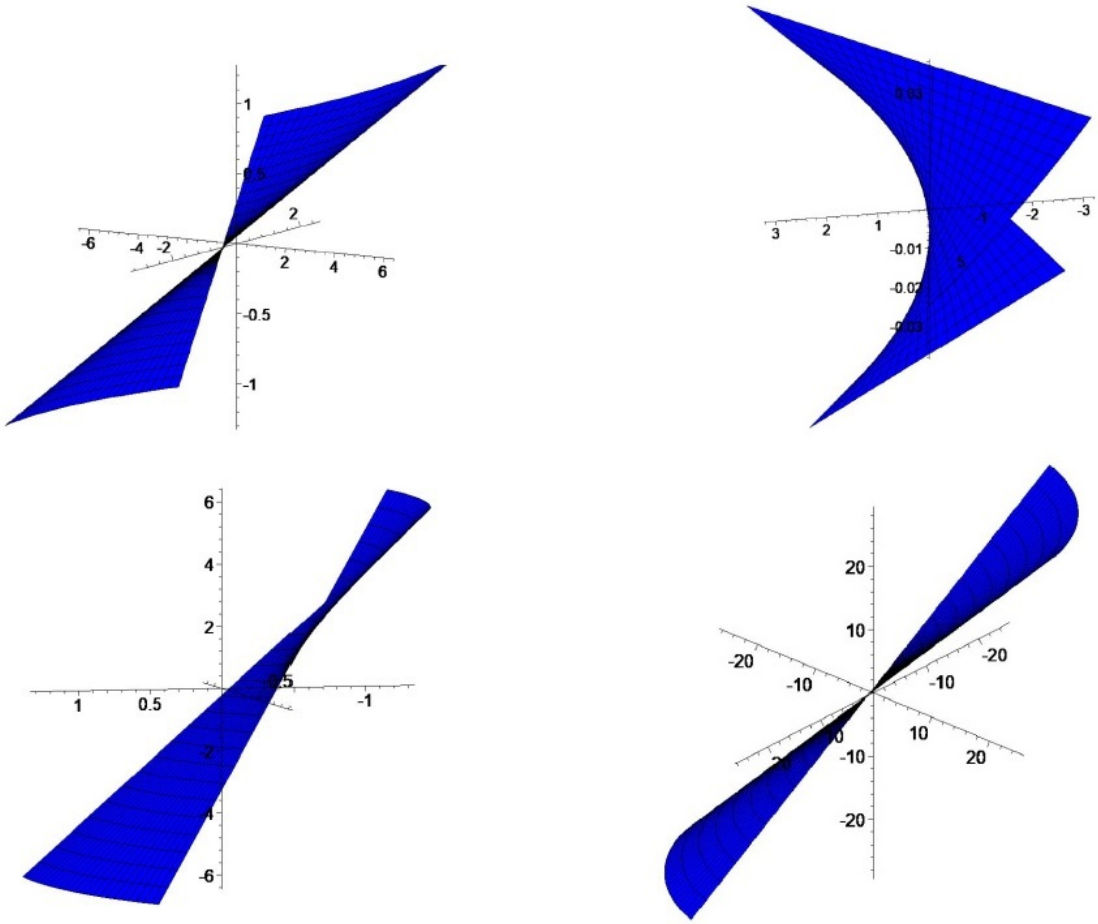

Figure 5 $\phi_2^t(s,v)$ (left top), $\phi_2^{n_q}(s,u)$ (right top), $\phi_2^{b_q}(s,z)$ (left bottom) and $\phi_2^X(s,w)$ for $x_1(s)=2,\ x_2(s)=3,\ x_3(s)=5$ (right bottom)

are given respectively by

$$\begin{aligned}
\phi_2^t(s,v) &= \left(\tfrac{5}{13}\sin\tfrac{s}{13}, -\tfrac{5}{13}\cos\tfrac{s}{13}, \tfrac{s}{169}\right) + v\left(\frac{5}{\sqrt{26}}\cos\tfrac{s}{13}, \frac{5}{\sqrt{26}}\sin\tfrac{s}{13}, \frac{1}{\sqrt{26}}\right),\\
\phi_2^{n_q}(s,u) &= \left(\tfrac{5}{13}\sin\tfrac{s}{13}, -\tfrac{5}{13}\cos\tfrac{s}{13}, \tfrac{s}{169}\right) + u\left(\sin\tfrac{s}{13}, -\cos\tfrac{s}{13}, 0\right),\\
\phi_2^{b_q}(s,z) &= \left(\tfrac{5}{13}\sin\tfrac{s}{13}, -\tfrac{5}{13}\cos\tfrac{s}{13}, \tfrac{s}{169}\right) + z\left(\frac{1}{\sqrt{26}}\cos\tfrac{s}{13}, \frac{1}{\sqrt{26}}\sin\tfrac{s}{13}, -\frac{5}{\sqrt{26}}\right),
\end{aligned}$$

and

$$\begin{aligned}
\phi_2^X(s,w) &= \left(\tfrac{5}{13}\sin\tfrac{s}{13}, -\tfrac{5}{13}\cos\tfrac{s}{13}, \tfrac{s}{169}\right) + w\left(x_1(s)\left(\frac{5}{\sqrt{26}}\cos\tfrac{s}{13}, \frac{5}{\sqrt{26}}\sin\tfrac{s}{13}, \frac{1}{\sqrt{26}}\right)\right.\\
&\quad \left. + x_2(s)\left(\sin\tfrac{s}{13}, -\cos\tfrac{s}{13}, 0\right) + x_3(s)\left(\frac{1}{\sqrt{26}}\cos\tfrac{s}{13}, \frac{1}{\sqrt{26}}\sin\tfrac{s}{13}, -\frac{5}{\sqrt{26}}\right)\right).
\end{aligned}$$

For $x_1(s) = 2,\ x_2(s) = 3,\ x_3(s) = 5$, the Gaussian curvature, mean curvature, distribution parameter and striction curve of the surface $\phi_2^X(s,w)$, generated by X are calculated as

$$\begin{aligned}
K_X &= -\frac{7056}{(68+1140w+8721w^2)^2},\\
H_X &= \frac{\sqrt{1228}(1228+26220w+200583w^2)}{4\sqrt{884+14820w+113373w^2}(68+1140w+8721w^2)},\\
P_X &= -\frac{56}{153}\\
\beta_X(s) &= \left(\frac{125}{663}\sin\frac{s}{13}-\frac{25\sqrt{26}}{663}\cos\frac{s}{13}, -\frac{125}{663}\cos\frac{s}{13}-\frac{25\sqrt{26}}{663}\sin\frac{s}{13}, \frac{s}{169}+\frac{115\sqrt{26}}{1989}\right),
\end{aligned}$$

respectively.

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