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## An Atlas of Roman Domination Polynomials of Graphs of Order at Most Six

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**Abstract:** The Roman domination polynomial of a graph G of order p is defined as  $R(G,x) = \sum_{\substack{j=\gamma_R(G)\\ G}}^{2n} r(G,j)x^j$ , where r(G,j) is the number of Roman dominating functions of G of weight j [5]. The roots of a Roman domination polynomial of a graph are called the Roman domination roots of that graph. In this article, the Roman domination polynomials of all the connected graphs of order less than or equal to six are obtained and their roots are computed. Furthermore, all these graphs and their Roman domination polynomials and roots are illustrated in a table.

**Key Words**: Atlas, Roman domination polynomial, Roman domination roots, graphs of order at most 6.

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## §1. Introduction

Throughout this paper all the considered graph are finite simple graphs i.e. all the graphs here are finite, undirected and have no self-loops or multiple edges. Let G = (V, E) be a graph. The order and the size of G are denoted respectively by |V(G)| = n and |E(G)| = m.

The Roman domination number of a graph G = (V, E),  $\gamma_R(G)$ , has been defined in [7] as the smallest weight, W(f(V)), of a function  $f : V(G) \to \{0, 1, 2\}$  satisfying the condition that every vertex u for which f(u) = 0 is adjacent to at least one vertex v for which f(v) = 2, where  $W(f(V)) = \sum_{u \in V(G)} f(u)$ . A function  $f : V(G) \to \{0, 1, 2\}$  with this condition is called a Roman

dominating function of the graph G = (V, E) or in brief an RDF of G. For more details about Roman domination and its properties, the reader is referred to [6].

In [5], Deepak *et al.* introduced the Roman domination polynomial of a graph G as

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 $R(G,x) = \sum_{j=\gamma_R(G)}^{2n} r(G,j)x^j$ , where r(G,j) is the number of Roman dominating functions of G of weight j and studied some of its properties. The roots of the Roman domination polynomial of a graph G are called the Roman domination roots of G. In addition, the Roman domination polynomials of paths and cycles are studied in details in [4] and [3], respectively.

As with all the types of graph polynomials, the analysis of the Roman domination polynomial of graphs can give us some informations about graphs. Similar to the domination polynomial of graphs [2, 1], an atlas for the Roman domination polynomials of graphs of order at most six is presented in this article. Moreover, the Roman domination polynomials of all the connected graphs of order less than or equal to six and their roots are illustrated in a table. Furthermore, for computing the Roman domination polynomials of the disconnected graphs of order less than or equal to six the following lemma can be used.

**Lemma** 1.1([5]) If a graph G consists of m components  $G_1, \dots, G_m$ , then  $R(G, x) = R(G_1, x) \times R(G_2, x) \times \dots \times R(G_m, x)$ .

Some coefficients of the polynomials are computed by using the following theorem.

**Theorem 1.2**([5]) Let G be a graph on n vertices with i isolated vertices, t vertices of degree one and l vertices of degree two. Suppose  $R(G, x) = \sum_{j=\gamma_R(G)}^{2n} r(G, j) x^j$  is the Roman domination polynomial of G. Then the following hold:

- (i) r(G, 2n 1) = n;(ii)  $i = \frac{n(n+1)}{2} - r(G, 2n - 2);$ (iii)  $r(G, 2n - 3) = 2\binom{n}{2} + \binom{n}{3} - i(n - 1) - t;$
- (iv) If G has s  $K_2$ -components, then

$$r(G, 2n-4) = \binom{n}{2} + 3\binom{n}{3} + \binom{n}{4} - i(n-1) + \binom{i}{2} - t(n-1) + s - l.$$

(v) If  $G \neq K_2$ , then  $r(G, 2) = |\{v \in V(G) : |deg(v) = n - 1\}|$ 

and the other coefficients are computed by determining all the possible functions  $f: V(G) \rightarrow \{0, 1, 2\}$  of some size and reduce the cases when  $f: V(G) \rightarrow \{0, 1, 2\}$  is not an RDF function of the graph G. For instance, all the possible functions of size 2n - 5, 2n - 6, 2n - 7 and 2n - 8 are given as:

(i) For size 
$$2n - 5$$
 there is  $3\binom{n}{3} + 4\binom{n}{4} + \binom{n}{5}$  possible function;  
(ii) For size  $2n - 6$  there is  $\binom{n}{3} + 6\binom{n}{4} + 5\binom{n}{5} + \binom{n}{6}$  possible function;  
(iii) For size  $2n - 7$  there is  $4\binom{n}{4} + 10\binom{n}{5} + 6\binom{n}{6} + \binom{n}{7}$  possible function;  
(iv) For size  $2n - 8$  there is  $\binom{n}{4} + 10\binom{n}{5} + 15\binom{n}{6} + 7\binom{n}{7} + \binom{n}{8}$  possible function.

All the roots of the polynomials are found by using the Matlab program. On the other hand, the repetition of any root is expressed as an exponent on that root. For example, the three times repetition of the zero root is expressed as  $(0)^3$ .

## §2. Roman Domination Polynomials of All Connected Graphs of order $\leq 6$

In the following, a table illustrates all the connected graphs of order less than of equal to six with their Roman domination polynomials and roots.

Graph	Roman Domination Polynomial	Roman Domination Roots
•	$x^2 + x$	0, -1
••	$x^4 + 2x^3 + 3x^2$	$(0)^2, \ -1\pm\sqrt{2}i$
••••	$x^6 + 3x^5 + 6x^4 + 5x^3 + x^2$	$(0)^2, -1, -0.2848, -0.8576 \pm 1.6662i$
$\bigtriangleup$	$x^6 + 3x^5 + 6x^4 + 7x^3 + 3x^2$	$(0)^2, \ (-1)^2, \ \frac{-1 \pm \sqrt{11}i}{2}$
	$x^{8} + 4x^{7} + 10x^{6} + 13x^{5} + 10x^{4} + 3x^{3} + x^{2}$	$(0)^2, -0.1062 \pm 0.3824i, -0.9827 \pm 0.8465i, -0.9111 \pm 1.7159i$
	$x^8 + 4x^7 + 10x^6 + 14x^5 + 11x^4 + 2x^3$	$(0)^3, -0.2471, -1.2146 \pm 0.8713i, -0.6618 \pm 1.7846i$
	$x^8 + 4x^7 + 10x^6 + 15x^5 + 16x^4 + 5x^3$	$(0)^3, -0.4599, -0.2992 \pm 1.7264i, -1.4708 \pm 1.174i$
	$x^8 + 4x^7 + 10x^6 + 16x^5 + 15x^4 + 4x^3$	$(0)^3, -0.402, -1.4178 \pm 0.8204i, -0.3812 \pm 1.8877i$
	$x^{8} + 4x^{7} + 10x^{6} + 16x^{5} + 17x^{4} + 8x^{3} + 2x^{2}$	$(0)^2, -0.3296 \pm 0.3569i, -1.352 \pm 0.9293i, -0.3184 \pm 1.7455i$

Graph	Roman Domination Polynomial	Roman Domination Roots
	$x^{8} + 4x^{7} + 10x^{6} + 16x^{5} + 19x^{4} + 12x^{3} + 4x^{2}$	$(0)^2, -0.5274 \pm 0.5087i, -1.3019 \pm 1.0899i, -0.1708 \pm 1.5986i$
$\times$	$x^{10} + 5x^9 + 15x^8 + 26x^7 + 29x^6 + 21x^5 + 10x^4 + 4x^3 + x^2$	$(0)^2, -1, -0.575, 0.0259 \pm 0.5197i, -0.69 \pm 0.9483i, -1.0485 \pm 1.89i$
·	$x^{10} + 5x^9 + 15x^8 + 27x^7 + 32x^6 + 21x^5 + 6x^4 + x^3$	$(0)^3, -1, -0.1803 \pm 0.2468i,$ $-0.6458 \pm 1.7634i, -1.1739 \pm 1.2873i$
•••••	$x^{10} + 5x^9 + 15x^8 + 28x^7 + 34x^6 + 23x^5 + 6x^4$	$(0)^4, -1, -0.5973, -1.1535 \pm 1.1497i, -0.5479 \pm 1.8674i$
$\times$	$x^{10} + 5x^9 + 15x^8 + 28x^7 + 35x^6 + 27x^5 + 12x^4 + 4x^3 + x^2$	$(0)^2, -1, -0.6222, -0.0191 \pm 0.4241i, -1.0795 \pm 1.1832i, -0.5904 \pm 1.7686i$
$\prec$	$x^{10} + 5x^9 + 15x^8 + 28x^7 + 36x^6 + 27x^5 + 10x^4 + 2x^3$	$(0)^3, -1, -0.2632 \pm 0.3289i,$ -1.2977 ± 1.2965 <i>i</i> , -0.4391 ± 1.7768 <i>i</i>
$\succ$	$x^{10} + 5x^9 + 15x^8 + 29x^7 + 38x^6 + 31x^5 + 12x^4 + x^3$	$(0)^3, \ (-1)^2, \ -0.1113, \ -0.3914 \pm 1.7864i, -1.053 \pm 1.2558i$
	$x^{10} + 5x^9 + 15x^8 + 29x^7 + 38x^6 + 29x^5 + 10x^4 + x^3$	$(0)^3, -1, -0.1613, -0.574, -1.2552 \pm 1.1673i, -0.3771 \pm 1.8796i$
$\dot{\Sigma}$	$x^{10} + 5x^9 + 15x^8 + 30x^7 + 40x^6 + 31x^5 + 10x^4$	$(0)^4, (-1)^2, -0.3193 \pm 1.9689i, -1.1807 \pm 1.058i$
	$x^{10} + 5x^9 + 15x^8 + 29x^7 + 40x^6 + 35x^5 + 16x^4 + 3x^3$	$(0)^3, -1, -0.4934 \pm 0.2123i,$ $-1.2496 \pm 1.3342i, -0.2571 \pm 1.7452i$
\$~·	$x^{10} + 5x^9 + 15x^8 + 29x^7 + 39x^6 + 33x^5 + 16x^4 + 5x^3 + x^2$	$(0)^2, -1, -0.6085, -0.11 \pm 0.3958i,$ $-1.2109 \pm 1.2087i, -0.3749 \pm 1.785i$

Graph	Roman Domination Polynomial	Roman Domination Roots
X	$x^{10} + 5x^9 + 15x^8 + 30x^7 + 41x^6 + 37x^5 + 18x^4 + 4x^3 + x^2$	$(0)^2, -1, -1.3489, -0.0581 \pm 0.2926i, -0.9508 \pm 1.2585i, -0.3167 \pm 1.8022i$
$\hat{\Box}$	$x^{10} + 5x^9 + 15x^8 + 30x^7 + 42x^6 + 37x^5 + 16x^4 + 2x^3$	$(0)^3, (-1)^2, -0.1977, -1.1768 \pm 1.2281i, -0.2244 \pm 1.8564i$
	$x^{10} + 5x^9 + 15x^8 + 30x^7 + 42x^6 + 37x^5 + 16x^4 + 2x^3$	$(0)^3, \ (-1)^2, \ -0.1977, \ -1.1768 \pm 1.2281i, -0.2244 \pm 1.8564i$
À	$x^{10} + 5x^9 + 15x^8 + 29x^7 + 41x^6 + 39x^5 + 22x^4 + 7x^3 + x^2$	$(0)^2, -1, -0.4598, -0.3386 \pm 0.3367i, -0.2004 \pm 1.6457i, -1.2311 \pm 1.398i$
$\Leftrightarrow$	$x^{10} + 5x^9 + 15x^8 + 30x^7 + 42x^6 + 39x^5 + 22x^4 + 8x^3 + 2x^2$	$(0)^2, (-1)^2, -0.1147 \pm 0.4777i, -1.0835 \pm 1.1503i, -0.3018 \pm 1.7963i$
$\bigtriangleup$	$x^{10} + 5x^9 + 15x^8 + 30x^7 + 43x^6 + 41x^5 + 22x^4 + 6x^3 + x^2$	$(0)^2, \ (-1)^2, \ -0.1639 \pm 0.28i, \ -1.1513 \pm 1.291i, \ -0.1848 \pm 1.7724i$
$\Rightarrow$	$x^{10} + 5x^9 + 15x^8 + 30x^7 + 44x^6 + 43x^5 + 22x^4 + 4x^3$	$(0)^3, \ (-1)^2, \ -0.3665, \ -1.2308 \pm 1.3839i, -0.0859 \pm 1.7817i$
\$	$x^{10} + 5x^9 + 15x^8 + 30x^7 + 44x^6 + 45x^5 + 28x^4 + 10x^3 + 2x^2$	$(0)^2, (-1)^2, -0.2436 \pm 0.4032i, -1.1373 \pm 1.3645i, -0.1191 \pm 1.686i$
$\boxtimes$	$x^{10} + 5x^9 + 15x^8 + 30x^7 + 45x^6 + 47x^5 + 28x^4 + 8x^3 + x^2$	$(0)^2, (-1)^2, -0.2484 \pm 0.1789i, -1.2315 \pm 1.4418i, -0.0201 \pm 1.7228i$
à	$x^{10} + 5x^9 + 15x^8 + 30x^7 + 45x^6 + 49x^5 + 34x^4 + 14x^3 + 3x^2$	$(0)^2, (-1)^2, -0.3355 \pm 0.477i, -0.0235 \pm 1.6156i, -1.141 \pm 1.4413i$
	$x^{10} + 5x^9 + 15x^8 + 30x^7 + 45x^6 + 51x^5 + 40x^4 + 20x^3 + 5x^2$	$(0)^2, \ (-1)^2, \ -0.5 \pm 0.6887i, \ 0.0198 \pm 1.469i, \ -1.0198 \pm 1.469i$

Graph	Roman Domination Polynomial	Roman Domination Roots
*	$x^{12} + 6x^{11} + 21x^{10} + 45x^9 + 65x^8 + 66x^7 + 51x^6 + 30x^5 + 15x^4 + 5x^3 + x^2$	$\begin{array}{l} (0)^2, \ 0.1235 \pm 0.63i, \ -0.3468 \pm 0.3391i, \\ -0.5 \pm 0.866i, \ -1.1904 \pm 0.6991i, \\ -1.0862 \pm 2.0572i \end{array}$
· <u>+</u> ··	$x^{12} + 6x^{11} + 21x^{10} + 46x^9 + 69x^8 + 69x^7 + 45x^6 + 18x^5 + 6x^4 + x^3$	$\begin{array}{c} (0)^3, \ -0.3121, \ -0.0323 \pm 0.4384i, \\ -1.0811 \pm 0.6413i, \ -0.8774 \pm 1.3707i, \\ -0.8532 \pm 1.7983i \end{array}$
$\succ$	$x^{12} + 6x^{11} + 21x^{10} + 46x^9 + 70x^8 + 70x^7 + 43x^6 + 14x^5 + 3x^4$	$(0)^4, -0.1845 \pm 0.3661i, -1.0193 \pm 0.5608i, -0.569 \pm 1.7459i, -1.2272 \pm 1.5512i$
	$x^{12} + 6x^{11} + 21x^{10} + 47x^9 + 73x^8 + 75x^7 + 46x^6 + 12x^5 + x^4$	$(0)^4$ , -0.1569, -0.2961, -1.1689 $\pm$ 0.632 <i>i</i> , -0.4916 $\pm$ 1.8189 <i>i</i> , -1.1129 $\pm$ 1.4817 <i>i</i>
1	$x^{12} + 6x^{11} + 21x^{10} + 47x^9 + 73x^8 + 75x^7 + 48x^6 + 16x^5 + 3x^4$	$(0)^4, \ -0.2402 \pm 0.3241i, \ -1.0707 \pm 0.6568i, -0.5512 \pm 1.855i, \ -1.1379 \pm 1.351i$
•••••	$x^{12} + 6x^{11} + 21x^{10} + 48x^9 + 76x^8 + 80x^7 + 51x^6 + 14x^5 + x^4$	$(0)^4, -0.1065, -0.4101, -1.289 \pm 0.6654i, -0.466 \pm 1.9189i, -0.9867 \pm 1.348i$
$\times$	$x^{12} + 6x^{11} + 21x^{10} + 47x^9 + 73x^8 + 78x^7 + 59x^6 + 32x^5 + 15x^4 + 5x^3 + x^2$	$\begin{array}{l} (0)^2, \ 0.1005 \pm 0.5357i, \ -0.3634 \pm 0.3076i, \\ -0.9047 \pm 1.0954i, \ -1.0993 \pm 0.7781i, \\ -0.733 \pm 1.8761i \end{array}$
×	$x^{12} + 6x^{11} + 21x^{10} + 47x^9 + 74x^8 + 77x^7 + 55x^6 + 23x^5 + 7x^4 + x^3$	$(0)^3, -0.2808, -0.1229 \pm 0.4157i, -0.6917 \pm 0.9552i, -0.5851 \pm 1.8882i, -1.46 \pm 1.1647i$
Å	$x^{12} + 6x^{11} + 21x^{10} + 47x^9 + 75x^8 + 81x^7 + 54x^6 + 30x^5 + 3x^4$	$(0)^4, -0.1226, -1.7056, -0.1608 \pm 0.8745i, -0.4263 \pm 1.8985i, -1.4987 \pm 1.5959i$
<b>.</b> ⊽	$x^{12} + 6x^{11} + 21x^{10} + 48x^9 + 77x^8 + 85x^7 + 61x^6 + 24x^5 + 6x^4 + x^3$	$\begin{array}{c} (0)^3, \ -0.4477, \ -0.0846 \pm 0.3091i, \\ -1.2938 \pm 0.6559i, \ -0.4577 \pm 1.7908i, \\ -0.9401 \pm 1.4635i \end{array}$

Graph	Roman Domination Polynomial	Roman Domination Roots
Ľ,	$x^{12} + 6x^{11} + 21x^{10} + 48x^9 + 77x^8 + 84x^7 + 60x^6 + 25x^5 + 7x^4 + x^3$	$(0)^3, -0.333, -0.135 \pm 0.3698i,$ -1.1933 $\pm 0.7199i, -0.4782 \pm 1.8756i,$ -1.027 $\pm 1.2682i$
<u>.</u>	$x^{12} + 6x^{11} + 21x^{10} + 48x^9 + 78x^8 + 87x^7 + 62x^6 + 22x^5 + 4x^4$	$(0)^4, -0.2759 \pm 0.2872i, -1.302 \pm 0.7156i, -0.3364 \pm 1.8063i, -1.0857 \pm 1.4852i$
	$x^{12} + 6x^{11} + 21x^{10} + 48x^9 + 78x^8 + 86x^7 + 59x^6 + 20x^5 + 3x^4$	$(0)^4, \ -0.2987 \pm 0.2091i, \ -1.1757 \pm 0.6497i, -0.3324 \pm 1.8639i, \ -1.1932 \pm 1.4373i$
	$x^{12} + 6x^{11} + 21x^{10} + 48x^9 + 78x^8 + 86x^7 + 61x^6 + 24x^5 + 5x^4$	$(0)^4, -0.3289 \pm 0.392i, -1.0706 \pm 0.6814i, -1.2184 \pm 1.3314i, -0.382 \pm 1.8693i$
$\Xi$	$x^{12} + 6x^{11} + 21x^{10} + 48x^9 + 78x^8 + 88x^7 + 65x^6 + 26x^5 + 5x^4$	$(0)^4, \ -0.3725 \pm 0.3183i, \ -1.2292 \pm 0.6886i, -0.3643 \pm 1.7517i, \ -1.034 \pm 1.4858i$
₩.:	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 81x^8 + 93x^7 + 70x^6 + 28x^5 + 5x^4$	$(0)^4, -0.3903 \pm 0.2555i, -0.8845 \pm 1.4059i, -1.3704 \pm 0.678i, -0.3548 \pm 1.8539i$
	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 81x^8 + 92x^7 + 65x^6 + 20x^5 + 2x^4$	$(0)^4$ , -0.193, -0.3455, -1.444 ± 0.647 <i>i</i> , -0.2776 ± 1.8986 <i>i</i> , -1.0092 ± 1.4952 <i>i</i>
$\dot{\Delta}$	$ \begin{aligned} x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 81x^8 + \\ 91x^7 + 64x^6 + 22x^5 + 3x^4 \end{aligned} $	$(0)^4, -0.3236 \pm 0.1375i, -1.3071 \pm 0.6816i, -1.0564 \pm 1.3279i, -0.3129 \pm 1.9443i$
$\bigcirc$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 84x^8 + 96x^7 + 69x^6 + 24x^5 + 3x^4$	$(0)^4, -0.2931, -0.3912, -0.9289 \pm 1.2843i, -1.4442 \pm 0.6431i, -0.2848 \pm 2.0214i$
Ľ.≺́	$x^{12} + 6x^{11} + 21x^{10} + 48x^9 + 78x^8 + 88x^7 + 69x^6 + 37x^5 + 16x^4 + 5x^3 + x^2$	$ \begin{array}{c} (0)^2, \ 0.0483 \pm 0.4866i, \ -0.3807 \pm 0.322i, \\ -1.0655 \pm 0.7386i, \ -0.4789 \pm 1.8345i, \\ -1.1231 \pm 1.2337i \end{array} $

Graph	Roman Domination Polynomial	Roman Domination Roots
÷	$x^{12} + 6x^{11} + 21x^{10} + 48x^9 + 78x^8 + 88x^7 + 67x^6 + 32x^5 + 11x^4 + 2x^3$	$\begin{array}{l} (0)^3, \ -0.4201, \ -0.1104 \pm 0.4858i, \\ -1.1744 \pm 0.6852i, \ -0.4308 \pm 1.7973i, \\ -1.0744 \pm 1.3725i \end{array}$
1	$x^{12} + 6x^{11} + 21x^{10} + 48x^9 + 79x^8 + 91x^7 + 69x^6 + 30x^5 + 8x^4 + x^3$	$(0)^3, -0.3093, -0.1986 \pm 0.3323i,$ $-1.1854 \pm 0.6472i, -0.2844 \pm 1.7695i,$ $-1.177 \pm 1.516i$
Z<	$x^{12} + 6x^{11} + 21x^{10} + 48x^9 + 79x^8 + 92x^7 + 72x^6 + 33x^5 + 9x^4 + x^3$	$(0)^3, -0.2376, -0.2756 \pm 0.3559i,$ $-1.2094 \pm 0.6657i, -0.2836 \pm 1.7083i,$ $-1.1126 \pm 1.5483i$
	$x^{12} + 6x^{11} + 21x^{10} + 48x^9 + 80x^8 + 94x^7 + 73x^6 + 32x^5 + 7x^4$	$(0)^4, -0.418 \pm 0.4055i, -1.117 \pm 0.6553i, -0.2117 \pm 1.7592i, -1.2534 \pm 1.5328i$
	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 81x^8 + 94x^7 + 75x^6 + 38x^5 + 15x^4 + 5x^3 + x^2$	$\begin{array}{l} (0)^2, \ 0.0702 \pm 0.4345i, \ -0.4437 \pm 0.2135i, \\ -0.8269 \pm 1.3436i, \ -1.3747 \pm 0.6844i, \\ -0.425 \pm 1.8568i \end{array}$
$\dot{\Delta}$	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 82x^8 + 96x^7 + 74x^6 + 32x^5 + 8x^4 + x^3$	$(0)^3, -0.3786, -0.1866 \pm 0.2785i,$ $-1.3112 \pm 0.6703i, -0.2771 \pm 1.8538i,$ $-1.0359 \pm 1.4185i$
	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 82x^8 + 97x^7 + 77x^6 + 35x^5 + 9x^4 + x^3$	$\begin{array}{l} (0)^3, \ -0.2893, \ -0.2676 \pm 0.2811i, \\ -1.3411 \pm 0.6708i, \ -0.2853 \pm 1.7984i, \\ -0.9614 \pm 1.4678i \end{array}$
⊠	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 82x^8 + 97x^7 + 77x^6 + 35x^5 + 9x^4 + x^3$	$\begin{array}{c} (0)^{3}, \ -0.2893, \ -0.2676 \pm 0.2811i, \\ -1.3411 \pm 0.6708i, \ -0.2853 \pm 1.7984i, \\ -0.9614 \pm 1.4678i \end{array}$
$\Rightarrow$	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 82x^8 + 96x^7 + 76x^6 + 36x^5 + 10x^4 + x^3$	$\begin{array}{l} (0)^3, \ -0.184, \ -0.3372 \pm 0.4009i, \\ -1.2633 \pm 0.7621i, \ -0.332 \pm 1.8505i, \\ -0.9755 \pm 1.2738 \end{array}$

Graph	Roman Domination Polynomial	Roman Domination Roots
<⊥.	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 83x^8 + 99x^7 + 78x^6 + 34x^5 + 7x^4$	$(0)^4, -0.4344 \pm 0.3425, -1.2443 \pm 0.7076i, \\ -1.1133 \pm 1.4237i, -0.208 \pm 1.8371i$
☑	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 83x^8 + 100x^7 + 81x^6 + 37x^5 + 8x^4$	$(0)^4, -0.4721 \pm 0.3624i, -1.2814 \pm 0.7156i, -0.2047 \pm 1.7874i, -1.0418 \pm 1.4677i$
	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 83x^8 + 99x^7 + 76x^6 + 30x^5 + 5x^4$	$\begin{array}{l} (0)^4, \ -0.3847 \pm 0.2078i, \ -1.2966 \pm 0.6407i, \\ -0.1699 \pm 1.856i, \ -1.1487 \pm 1.5097i \end{array}$
÷	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 83x^8 + 99x^7 + 76x^6 + 30x^5 + 5x^4$	$\begin{array}{l} (0)^4, \ -0.3847 \pm 0.2078i, \ -1.2966 \pm 0.6407i, \\ -0.1699 \pm 1.856i, \ -1.1487 \pm 1.5097i \end{array}$
$\hat{\mathbf{X}}$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 85x^8 + 102x^7 + 81x^6 + 37x^5 + 9x^4 + x^3$	$(0)^3, -0.4793, -0.2359 \pm 0.2063i,$ $-1.34 \pm 0.5136i, -0.2629 \pm 1.9138i,$ $-0.9216 \pm 1.3839i$
¢	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 86x^8 + 104x^7 + 83x^6 + 37x^5 + 7x^4$	$\begin{array}{l} (0)^4, \ -0.5422 \pm 0.2074i, \ -1.2695 \pm 0.6782i, \\ -0.983 \pm 1.3109i, \ -0.2053 \pm 1.9216i \end{array}$
$\Leftrightarrow$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 86x^8 + 104x^7 + 81x^6 + 32x^5 + 5x^4$	$(0)^4, -0.3956 \pm 0.1422i, -1.4133 \pm 0.6206i, -1.0285 \pm 1.4517i, -0.1626 \pm 1.9301i$
$\bigtriangleup$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 86x^8 + 104x^7 + 81x^6 + 32x^5 + 5x^4$	$(0)^4, \ -0.3956 \pm 0.1422i, \ -1.4133 \pm 0.6206i, \\ -1.0285 \pm 1.4517i, \ -0.1626 \pm 1.9301i$
X	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 86x^8 + 106x^7 + 91x^6 + 46x^5 + 11x^4$	$(0)^4, -0.5786 \pm 0.39i, -0.7173 \pm 1.4495i, -1.4467 \pm 0.734i, -0.2575 \pm 1.7935i$
X	$x^{12} + 6x^{11} + 21x^{10} + 48x^9 + 80x^8 + 96x^7 + 81x^6 + 45x^5 + 18x^4 + 5x^3 + x^2$	$(0)^2, -0.0184 \pm 0.4127i, -0.4318 \pm 0.3685i, -1.129 \pm 0.6906i, -0.2618 \pm 1.6586i, -1.159 \pm 1.5295i$

Graph	Roman Domination Polynomial	Roman Domination Roots
A	$x^{12} + 6x^{11} + 21x^{10} + 48x^9 + 80x^8 + 96x^7 + 81x^6 + 41x^5 + 13x^4 + 2x^3$	$(0)^3, -0.3915, -0.2307 \pm 0.4047i,$ $-0.1986 \pm 1.6219i, -1.3076 \pm 0.8313i,$ $-1.0673 \pm 1.5915i$
\$-	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 82x^8 + 97x^7 + 81x^6 + 46x^5 + 20x^4 + 6x^3 + x^2$	$\begin{array}{l} (0)^2, \ -0.0193 \pm 0.5142i, \ -0.3809 \pm 0.2499i, \\ -0.9361 \pm 1.1908i, \ -1.2662 \pm 0.7793i, \\ -0.3976 \pm 1.8522i \end{array}$
$\dot{\Delta}$	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 83x^8 + 100x^7 + 83x^6 + 44x^5 + 17x^4 + 5x^3 + x^2$	$\begin{array}{l} (0)^2, \ 0.0212 \pm 0.4112i, \ -0.4442 \pm 0.2853i, \\ -1.2421 \pm 0.7185i, \ -0.2605 \pm 1.8109i, \\ -1.0744 \pm 1.3845i \end{array}$
Ŷ	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 83x^8 + 101x^7 + 84x^6 + 42x^5 + 13x^4 + 2x^3$	$\begin{array}{l} (0)^3, \ -0.4247, \ -0.2232 \pm 0.3935i, \\ -1.3145 \pm 0.6601i, \ -0.2104 \pm 1.7613i, \\ -1.0396 \pm 1.5166i \end{array}$
Ś	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 83x^8 + 101x^7 + 84x^6 + 42x^5 + 13x^4 + 2x^3$	$\begin{array}{l} (0)^3, \ -0.4247, \ -0.2232 \pm 0.3935i, \\ -1.3145 \pm 0.6601i, \ -0.2104 \pm 1.7613i, \\ -1.0396 \pm 1.5166i \end{array}$
<b>☆</b> -•	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 84x^8 + 104x^7 + 88x^6 + 44x^5 + 12x^4 + x^3$	$\begin{array}{l} (0)^3, \ -0.1341, \ -0.4273 \pm 0.3811i, \\ -1.2532 \pm 0.6966i, \ -0.1351 \pm 1.7632i, \\ -1.1174 \pm 1.5131i \end{array}$
Ś	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 84x^8 + 104x^7 + 86x^6 + 40x^5 + 10x^4 + x^3$	$\begin{array}{l} (0)^3, \ -0.2416, \ -0.3148 \pm 0.2456i, \\ -1.3003 \pm 0.6331i, \ -0.1038 \pm 1.7952i, \\ -1.1603 \pm 1.5787i \end{array}$
$\Leftrightarrow$	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 82x^8 + 96x^7 + 72x^6 + 32x^5 + 8x^4 + x^3$	$(0)^3, (-1)^2, -0.6702, -0.2146 \pm 0.2651i, -1.1889 \pm 1.4508i, -0.2615 \pm 1.8914i$
Å	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 84x^8 + 105x^7 + 93x^6 + 51x^5 + 15x^4 + x^3$	$\begin{array}{c} (0)^3, \ -0.0902, \ -0.584 \pm 0.4736i, \\ -1.2491 \pm 0.8035i, \ -0.1482 \pm 1.6641i, \\ -0.9736 \pm 1.4954i \end{array}$

Graph	Roman Domination Polynomial	Roman Domination Roots
$\Rightarrow$	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 85x^8 + 107x^7 + 92x^6 + 46x^5 + 11x^4$	$(0)^4, -0.5542 \pm 0.4259i, -1.1816 \pm 0.7297i, \\ -0.0686 \pm 1.7718i, -1.1956 \pm 1.5114i$
$\overline{\mathbf{A}}$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 86x^8 + 106x^7 + 91x^6 + 49x^5 + 18x^4 + 5x^3 + x^2$	$\begin{array}{l} (0)^2, \ 0.0065 \pm 0.377 i, \ -0.4814 \pm 0.2683 i, \\ -0.8424 \pm 1.3964 i, \ -0.2552 \pm 1.8481 i, \\ -1.4275 \pm 0.6819 i \end{array}$
会	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 86x^8 + 106x^7 + 89x^6 + 44x^5 + 13x^4 + 2x^3$	$(0)^3, -0.4773, -0.2069 \pm 0.35i, -1.4357 \pm 0.626i, -0.2084 \pm 1.8498i, -0.9105 \pm 1.4676i$
$\Leftrightarrow$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 86x^8 + 106x^7 + 93x^6 + 52x^5 + 17x^4 + 2x^3$	$(0)^3, -0.2189, -0.5046 \pm 0.4894i, -0.6196 \pm 1.2791i, -0.3357 \pm 1.8302i, -1.4307 \pm 0.7725i$
$\bigcirc$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 87x^8 + 109x^7 + 93x^6 + 46x^5 + 12x^4 + x^3$	$\begin{array}{c} (0)^3, \ -0.1411, \ -0.4254 \pm 0.3143i, \\ -1.3917 \pm 0.6763i, \ -0.1365 \pm 1.8433i, \\ -0.9758 \pm 1.4644i \end{array}$
$\Leftrightarrow$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 87x^8 + 109x^7 + 93x^6 + 46x^5 + 12x^4 + x^3$	$\begin{array}{l} (0)^3, \ -0.1411, \ -0.4254 \pm 0.3143i, \\ -1.3917 \pm 0.6763i, \ -0.1365 \pm 1.8433i, \\ -0.9758 \pm 1.4644i \end{array}$
ا	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 87x^8 + 109x^7 + 93x^6 + 46x^5 + 12x^4 + x^3$	$\begin{array}{l} (0)^3, \ -0.1411, \ -0.4254 \pm 0.3143i, \\ -1.3917 \pm 0.6763i, \ -0.1365 \pm 1.8433i, \\ -0.9758 \pm 1.4644i \end{array}$
$\bigtriangleup$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 88x^8 + \\112x^7 + 95x^6 + 44x^5 + 9x^4$	$(0)^4, -0.485 \pm 0.2888i, -1.3653 \pm 0.6551i, -1.1019 \pm 1.5178i, -0.0479 \pm 1.8706i$
Ş	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 87x^8 + 110x^7 + 98x^6 + 53x^5 + 15x^4 + x^3$	$\begin{array}{c} (0)^3, \ -0.092, \ -0.5575 \pm 0.4075i, \\ -0.8202 \pm 1.4852i, \ -0.1641 \pm 1.7577i, \\ -1.4123 \pm 0.7401i \end{array}$

Graph	Roman Domination Polynomial	Roman Domination Roots
$\bigcirc$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 88x^8 + 112x^7 + 99x^6 + 52x^5 + 13x^4$	$(0)^4, -0.6273 \pm 0.4267i, -0.9408 \pm 1.3728i, -1.3306 \pm 0.8333i, -0.1013 \pm 1.8161i$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 88x^8 + 112x^7 + 97x^6 + 48x^5 + 11x^4$	$(0)^4, \ -0.5453 \pm 0.3684i, \ -1.34 \pm 0.7294i, \\ -1.0427 \pm 1.4531i, \ -0.072 \pm 1.8455i$
$\otimes$	$ \begin{aligned} x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 88x^8 + \\ & 112x^7 + 95x^6 + 44x^5 + 9x^4 \end{aligned} $	$(0)^4, -0.485 \pm 0.2888i, -1.3653 \pm 0.6551i, -1.1019 \pm 1.5178i, -0.0479 \pm 1.8706i$
\$.	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 84x^8 + 105x^7 + 93x^6 + 54x^5 + 22x^4 + 6x^3 + x^2$	$(0)^2, -0.0786 \pm 0.4225i, -0.4153 \pm 0.2981i, -1.252 \pm 0.7062i, -0.182 \pm 1.7154i, -1.0722 \pm 1.4899i$
	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 84x^8 + 106x^7 + 94x^6 + 52x^5 + 18x^4 + 3x^3$	$(0)^3, -0.4495, -0.2757 \pm 0.4629i,$ -1.3173 ± 0.6507 <i>i</i> , -0.1142 ± 1.6899 <i>i</i> , -1.0681 ± 1.6039 <i>i</i>
	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 85x^8 + 108x^7 + 95x^6 + 52x^5 + 19x^4 + 5x^3$	$(0)^3, -0.8284, -0.0878 \pm 0.4909i,$ -1.2316 ± 0.7055 <i>i</i> , -0.0743 ± 1.763 <i>i</i> , -1.1922 ± 1.5644 <i>i</i>
$\Rightarrow$	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 85x^8 + 109x^7 + 98x^6 + 54x^5 + 17x^4 + 2x^3$	$(0)^3, -0.2333, -0.4386 \pm 0.4226i,$ -1.2605 ± 0.6864 <i>i</i> , -0.0447 ± 1.7103 <i>i</i> , -1.1397 ± 1.592 <i>i</i>
÷	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 85x^8 + 109x^7 + 100x^6 + 58x^5 + 19x^4 + 2x^3$	$(0)^3, -0.1758, -0.5658 \pm 0.5017i,$ -1.2057 $\pm 0.7781i, -0.064 \pm 1.6618i,$ -1.0767 $\pm 1.5277i$
÷	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 86x^8 + 106x^7 + 95x^6 + 60x^5 + 29x^4 + 10x^3 + 2x^2$	$\begin{array}{l} (0)^2, \ -0.0012 \pm 0.581 i, \ -0.4883 \pm 0.315 i, \\ -0.7487 \pm 1.114 i, \ -0.3524 \pm 1.9001 i, \\ -1.4095 \pm 0.7885 i \end{array}$

Graph	Roman Domination Polynomial	Roman Domination Roots
À	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 87x^8 + 110x^7 + 98x^6 + 56x^5 + 22x^4 + 6x^3 + x^2$	$(0)^2, -0.0566 \pm 0.4029i, -0.4434 \pm 0.2502i, -1.3964 \pm 0.682i, -0.1827 \pm 1.8053i, -0.9209 \pm 1.4429i$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 87x^8 + 111x^7 + 99x^6 + 54x^5 + 18x^4 + 3x^3$	$\begin{array}{c} (0)^3, \ -0.493, \ -0.2562 \pm 0.4156i, \\ -1.4321 \pm 0.6182i, \ -0.1255 \pm 1.7715i, \\ -0.9397 \pm 1.5633i \end{array}$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 88x^8 + 113x^7 + 102x^6 + 58x^5 + 21x^4 + 5x^3 + x^2$	$\begin{array}{l} (0)^2, \ -0.027 \pm 0.3248i, \ -0.5297 \pm 0.3493i, \\ -1.345 \pm 0.7385i, \ -0.1065 \pm 1.8048i, \\ -0.9918 \pm 1.4336i \end{array}$
<u>×</u>	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 88x^8 + $ 114x <sup>7</sup> + 107x <sup>6</sup> + 65x <sup>5</sup> + 24x <sup>4</sup> + 5x <sup>3</sup> + x <sup>2</sup>	$\begin{array}{l} (0)^2, \ -0.0431 \pm 0.2804i, \ -0.6378 \pm 0.4173i, \\ -0.8158 \pm 1.4693i, \ -0.1205 \pm 1.7096i, \\ -1.3828 \pm 0.8164i \end{array}$
Ŕ	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 88x^8 + 114x^7 + 103x^6 + 56x^5 + 17x^4 + 2x^3$	$\begin{array}{l} (0)^3, \ -0.2542, \ -0.422 \pm 0.3556i, \\ -1.3894 \pm 0.6657i, \ -0.0554 \pm 1.7847i, \\ -1.006 \pm 1.5497i \end{array}$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 88x^8 + 114x^7 + 105x^6 + 60x^5 + 19x^4 + 2x^3$	$\begin{array}{l} (0)^3, \ -0.1831, \ -0.5358 \pm 0.4304i, \\ -0.9159 \pm 1.5054i, \ -0.0806 \pm 1.7439i, \\ -1.3761 \pm 0.7416i \end{array}$
$\bigotimes$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 88x^8 + 114x^7 + 103x^6 + 56x^5 + 17x^4 + 2x^3$	$\begin{array}{l} (0)^3, \ -0.2542, \ -0.422 \pm 0.3556i, \\ -1.3894 \pm 0.6657i, \ -0.0554 \pm 1.7847i, \\ -1.006 \pm 1.5497i \end{array}$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 88x^8 + 114x^7 + 105x^6 + 60x^5 + 19x^4 + 2x^3$	$(0)^{3}, -0.1831, -0.5358 \pm 0.4304i, -0.9159 \pm 1.5054i, -0.0806 \pm 1.7439i, -1.3761 \pm 0.7416i$
$\Leftrightarrow$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 88x^8 + 114x^7 + 105x^6 + 60x^5 + 19x^4 + 2x^3$	$(0)^3, -0.1831, -0.5358 \pm 0.4304i,$ $-0.9159 \pm 1.5054i, -0.0806 \pm 1.7439i,$ $-1.3761 \pm 0.7416i$

Graph	Roman Domination Polynomial	Roman Domination Roots
$\bigotimes$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 89x^8 + 117x^7 + 107x^6 + 58x^5 + 16x^4 + x^3$	$\begin{array}{l} (0)^3, \ -0.0849, \ -0.5511 \pm 0.3827i, \\ -1.3403 \pm 0.715i, \ 0.0048 \pm 1.8007i, \\ -1.071 \pm 1.5332i \end{array}$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 89x^8 + 118x^7 + 109x^6 + 62x^5 + 18x^4 + x^3$	$(0)^3, -0.0709, -0.6957 \pm 0.5181i,$ -1.1718 $\pm 0.5179i, 0.0102 \pm 1.7809i,$ -1.1072 $\pm 1.5419i$
$\Leftrightarrow$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 89x^8 + 117x^7 + 107x^6 + 58x^5 + 16x^4 + x^3$	$(0)^3, -0.0849, -0.5511 \pm 0.3827i,$ -1.3403 ± 0.715 <i>i</i> , 0.0048 ± 1.8007 <i>i</i> , -1.071 ± 1.5332 <i>i</i>
	$\begin{aligned} x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 90x^8 + \\ 120x^7 + 111x^6 + 60x^5 + 15x^4 \end{aligned}$	$(0)^4$ , $-0.6364 \pm 0.4i$ , $-1.283 \pm 0.7667i$ , $-1.1381 \pm 1.5159i$ , $0.0575 \pm 1.8177i$
$\bigotimes$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 90x^8 + 120x^7 + 111x^6 + 60x^5 + 15x^4$	$(0)^4, -0.6364 \pm 0.4i, -1.283 \pm 0.7667i, -1.1381 \pm 1.5159i, 0.0575 \pm 1.8177i$
À	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 85x^8 + 110x^7 + 103x^6 + 64x^5 + 27x^4 + 7x^3 + x^2$	$(0)^2, -0.2107 \pm 0.4016i, -0.3587 \pm 0.3219i, -1.2598 \pm 0.695i, -0.0725 \pm 1.6492i, -1.0983 \pm 1.5824i$
à	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 85x^8 + 111x^7 + 106x^6 + 66x^5 + 25x^4 + 4x^3$	$(0)^3, -0.3743, -0.4699 \pm 0.5431i, -1.2836 \pm 0.712i, -0.014 \pm 1.596i, -1.0454 \pm 1.6378i$
$\Rightarrow$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 88x^8 + 114x^7 + 107x^6 + 68x^5 + 31x^4 + 10x^3 + 2x^2$	$\begin{array}{l} (0)^2, \ -0.0443 \pm 0.4978i, \ -0.5229 \pm 0.3243i, \\ -0.9295 \pm 1.4096i, \ -0.1523 \pm 1.7575i, \\ -1.3511 \pm 0.7477i \end{array}$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 88x^8 + 115x^7 + 108x^6 + 66x^5 + 27x^4 + 7x^3 + x^2$	$\begin{array}{c} (0)^2, \ -0.1532 \pm 0.3941i, \ -0.4109 \pm 0.2588i, \\ -1.3937 \pm 0.6711i, \ -0.0866 \pm 1.7314i, \\ -0.9557 \pm 1.5443i \end{array}$

Graph	Roman Domination Polynomial	Roman Domination Roots
Ş	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 88x^8 + 116x^7 + 111x^6 + 68x^5 + 25x^4 + 4x^3$	$\begin{array}{l} (0)^3, \ -0.4065, \ -0.4318 \pm 0.4781 i, \\ -1.4156 \pm 0.6723 i, \ -0.0405 \pm 1.6731 i, \\ -0.9089 \pm 1.6189 i \end{array}$
$\Rightarrow$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 89x^8 + 118x^7 + 112x^6 + 68x^5 + 26x^4 + 6x^3 + x^2$	$\begin{array}{l} (0)^2, \ -0.0835 \pm 0.3041i, \ -0.5301 \pm 0.3671i, \\ -1.3446 \pm 0.7231i, \ -0.0164 \pm 1.7521i, \\ -1.0255 \pm 1.5257i \end{array}$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 89x^8 + 118x^7 + 114x^6 + 72x^5 + 28x^4 + 6x^3 + x^2$	$\begin{array}{l} (0)^2, \ -0.0847 \pm 0.2693i, \ -0.6226 \pm 0.4356i, \\ -0.9221 \pm 1.4781i, \ -0.0345 \pm 1.7066i, \\ -1.3362 \pm 0.8198i \end{array}$
Ŕ	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 89x^8 + 119x^7 + 115x^6 + 70x^5 + 24x^4 + 3x^3$	$(0)^3, -0.243, -0.5545 \pm 0.4425i,$ -1.3725 $\pm 0.7272i, 0.0242 \pm 1.7089i,$ -0.9757 $\pm 1.5901i$
À	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 89x^8 + 119x^7 + 115x^6 + 70x^5 + 24x^4 + 3x^3$	$(0)^3, -0.243, -0.5545 \pm 0.4425i,$ -1.3725 $\pm 0.7272i, 0.0242 \pm 1.7089i,$ -0.9757 $\pm 1.5901i$
$\Rightarrow$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 89x^8 + 119x^7 + 115x^6 + 70x^5 + 24x^4 + 3x^3$	$\begin{array}{l} (0)^3, \ -0.243, \ -0.5545 \pm 0.4425 i, \\ -1.3725 \pm 0.7272 i, \ 0.0242 \pm 1.7089 i, \\ -0.9757 \pm 1.5901 i \end{array}$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 90x^8 + 121x^7 + 116x^6 + 70x^5 + 25x^4 + 5x^3 + x^2$	$\begin{array}{l} (0)^2, \ -0.0363 \pm 0.2693i, \ -0.6238 \pm 0.3994i, \\ -1.2869 \pm 0.779i, \ 0.043 \pm 1.7737i, \\ -1.096 \pm 1.5049i \end{array}$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 90x^8 + 122x^7 + 119x^6 + 72x^5 + 23x^4 + 2x^3$	$(0)^{3}, -0.1297, -0.6492 \pm 0.4324i, -1.324 \pm 0.7875i, 0.0788 \pm 1.74i, -1.0408 \pm 1.5613i$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 90x^8 + 122x^7 + 119x^6 + 72x^5 + 23x^4 + 2x^3$	$\begin{array}{l} (0)^3, \ -0.1297, \ -0.6492 \pm 0.4324i, \\ -1.324 \pm 0.7875i, \ 0.0788 \pm 1.74i, \\ -1.0408 \pm 1.5613i \end{array}$

Graph	Roman Domination Polynomial	Roman Domination Roots
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 90x^8 + 122x^7 + 121x^6 + 76x^5 + 25x^4 + 2x^3$	$\begin{array}{l} (0)^3, \ -0.1121, \ -0.736 \pm 0.4347i, \\ 0.0763 \pm 1.7018i, \ -1.3382 \pm 0.8915i, \\ -0.946 \pm 1.5361i \end{array}$
	$x^{12} + 6x^{11} + 21x^{10} + 49x^9 + 85x^8 + 112x^7 + 111x^6 + 76x^5 + 35x^4 + 9x^3 + x^2$	$\begin{array}{l} (0)^2, \ -0.2865 \pm 0.1324i, \ -0.4153 \pm 0.6139i, \\ -0.0178 \pm 1.5072i, \ -1.2854 \pm 0.7218i, \\ -0.995 \pm 1.6462i \end{array}$
$\Rightarrow$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 89x^8 + 119x^7 + 117x^6 + 78x^5 + 36x^4 + 11x^3 + 2x^2$	$\begin{array}{l} (0)^2, \ -0.1224 \pm 0.4731 i, \ -0.5155 \pm 0.3399 i, \\ -1.3496 \pm 0.7314 i, \ -0.0412 \pm 1.6919 i, \\ -0.9713 \pm 1.5201 i \end{array}$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 87x^8 + 110x^7 + 98x^6 + 56x^5 + 23x^4 + 6x^3 + x^2$	$\begin{array}{l} (0)^2, \ -0.095 \pm 0.4168i, \ -0.3314 \pm 0.3237i, \\ -1.448 \pm 0.67i, \ -0.1814 \pm 1.8122i, \\ -0.9443 \pm 1.459i \end{array}$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 89x^8 + 121x^7 + 123x^6 + 82x^5 + 32x^4 + 5x^3$	$\begin{array}{l} (0)^3, \ -0.3499, \ -0.5934 \pm 0.4954i, \\ 0.0784 \pm 1.6146i, \ -1.4059 \pm 0.7324i, \\ -0.9041 \pm 1.6807i \end{array}$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 90x^8 + 123x^7 + 124x^6 + 82x^5 + 33x^4 + 7x^3 + x^2$	$(0)^2, -0.1152 \pm 0.2405i, -0.6359 \pm 0.439i, -1.3318 \pm 0.7978i, 0.074 \pm 1.685i, -0.9911 \pm 1.5662i$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 90x^8 + 122x^7 + 123x^6 + 84x^5 + 37x^4 + 10x^3 + 2x^2$	$(0)^2, -0.0913 \pm 0.3631i, -0.7025 \pm 0.4349i, -0.9207 \pm 1.4437i, 0.0185 \pm 1.6731i, -1.3039 \pm 0.9196i$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 90x^8 + 123x^7 + 124x^6 + 82x^5 + 33x^4 + 7x^3 + x^2$	$\begin{array}{c} (0)^2, \ -0.1152 \pm 0.2405i, \ -0.6359 \pm 0.439i, \\ -1.3318 \pm 0.7978i, \ 0.074 \pm 1.685i, \\ -0.9911 \pm 1.5662i \end{array}$

Graph	Roman Domination Polynomial	Roman Domination Roots
$\Leftrightarrow$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 90x^8 + 124x^7 + 127x^6 + 84x^5 + 31x^4 + 4x^3$	$(0)^3, -0.2388, -0.6756 \pm 0.4587i,$ -1.3675 $\pm 0.7958i, 0.1242 \pm 1.6615i,$ -0.9617 $\pm 1.6404i$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 90x^8 + 124x^7 + 127x^6 + 84x^5 + 31x^4 + 4x^3$	$(0)^3, -0.2388, -0.6756 \pm 0.4587i, -1.3675 \pm 0.7958i, 0.1242 \pm 1.6615i, -0.9617 \pm 1.6404i$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 89x^8 + $ 121x <sup>7</sup> + 125x <sup>6</sup> + 90x <sup>5</sup> + 44x <sup>4</sup> + 13x <sup>3</sup> + 2x <sup>2</sup>	$\begin{array}{l} (0)^2, \ -0.3369 \pm 0.3633i, \ -0.4276 \pm 0.4667i, \\ 0.0164 \pm 1.5594i, \ -1.3856 \pm 0.7407i, \\ -0.8664 \pm 1.6238i \end{array}$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 90x^8 + 123x^7 + 126x^6 + 90x^5 + 45x^4 + 15x^3 + 3x^2$	$\begin{array}{l} (0)^2, \ -0.1346 \pm 0.524i, \ -0.5926 \pm 0.389i, \\ 0.0122 \pm 1.655i, \ -1.299 \pm 0.8058i, \\ -0.986 \pm 1.4882i \end{array}$
$\langle \langle \rangle$	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 90x^8 + 124x^7 + 129x^6 + 92x^5 + 43x^4 + 12x^3 + 2x^2$	$\begin{array}{l} (0)^2, \ -0.1808 \pm 0.3545i, \ -0.618 \pm 0.4469i, \\ 0.0759 \pm 1.6192i, \ -1.341 \pm 0.8078i, \\ -0.936 \pm 1.5796i \end{array}$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 90x^8 + 125x^7 + 132x^6 + 94x^5 + 41x^4 + 9x^3 + x^2$	$\begin{array}{c} (0)^2, \ -0.1737 \pm 0.1812i, \ -0.6682 \pm 0.4706i, \\ 0.1377 \pm 1.6032i, \ -1.377 \pm 0.8028i, \\ -0.9187 \pm 1.6633i \end{array}$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 90x^8 + 125x^7 + 134x^6 + 102x^5 + 53x^4 + 17x^3 + 3x^2$	$\begin{array}{c} (0)^2, \ -0.2721 \pm 0.4424i, \ -0.5909 \pm 0.4568i, \\ 0.0924 \pm 1.542i, \ -1.3515 \pm 0.8172i, \\ -0.8779 \pm 1.606i \end{array}$
A	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 90x^8 + 126x^7 + 135x^6 + 96x^5 + 39x^4 + 6x^3$	$(0)^{3}, -0.3189, -0.7075 \pm 0.4716i, 0.1916 \pm 1.5988i, -1.4076 \pm 0.7946i, -0.917 \pm 1.7322i$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 90x^8 + 126x^7 + 139x^6 + 112x^5 + 63x^4 + 22x^3 + 4x^2$	$ \begin{array}{c} (0)^2, \ -0.4209 \pm 0.5088i, \ -0.527 \pm 0.4644i, \\ 0.1355 \pm 1.4616i, \ -1.3631 \pm 0.8255i, \\ -0.8245 \pm 1.6489i \end{array} $

Graph	Roman Domination Polynomial	Roman Domination Roots
A	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 90x^8 + $ 126x <sup>7</sup> + 137x <sup>6</sup> + 104x <sup>5</sup> + 51x <sup>4</sup> + 14x <sup>3</sup> + 2x <sup>2</sup>	$\begin{array}{l} (0)^2, \ -0.2378 \pm 0.2739 i, \ -0.6605 \pm 0.4862 i, \\ 0.164 \pm 1.5421 i, \ -1.3872 \pm 0.8092 i, \\ -0.8785 \pm 1.6945 i \end{array}$
	$x^{12} + 6x^{11} + 21x^{10} + 50x^9 + 90x^8 + 126x^7 + 141x^6 + 120x^5 + 75x^4 + 30x^3 + 6x^2$	$\begin{array}{l} (0)^2, \ -0.5662 \pm 0.3387i, \ -0.5 \pm 0.866i, \\ 0.1321 \pm 1.3332i, \ -0.7327 \pm 1.5959i, \\ -1.3332 \pm 0.8446i \end{array}$

Now, all connected graphs of order  $\leq 6$  with their Roman domination polynomials and roots are listed in the table.

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