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F-Root Square Mean Labeling of Graphs Obtained From Paths

S. Arockiaraj

(Department of Mathematics, Kamarajar Government Arts College, Surandai - 627859, Tamilnadu, India)

A. Durai Baskar and A. Rajesh Kannan

(Department of Mathematics, Mepco Schlenk Engineering College, Sivakasi- 626 005, Tamilnadu, India)

E-mail: psarockiaraj@gmail.com, a.duraibaskar@gmail.com, rajmaths@gmail.com

Abstract: A function f is called a F-root square mean labeling of a graph $G(V,E)$ with p vertices and q edges if $f : V(G) \to \{1,2,3,\ldots,q+1\}$ is injective and the induced function f^* defined as

$$f^*(uv) = \left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$$

for all $uv \in E(G)$, is bijective. A graph that admits a F-root square mean labeling is called a F-root square mean graph. In this paper, we study the F-root square meanness of the path P_n, the graph $P_n \circ S_m$, the graph $P_n \circ K_2$, the graph $TW(P_n)$, the graph $[P_n; S_m]$, the graph $S(P_n \circ K_1)$, the graph $M(P_n)$, the graph $T(P_n)$, the graph P_n^2, the ladder graph L_n and the slanting ladder graph SL_n.

Key Words: Labeling, Smarandache power root mean labeling, F-root square mean labeling, F-root square mean graph.

AMS(2010): 05C12.

§1. Introduction

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let $G(V,E)$ be a graph with p vertices and q edges. For notations and terminology, we follow [3]. For a detailed survey on graph labeling, we refer [2].

Path on n vertices is denoted by P_n. A star graph S_n is the complete bipartite graph $K_{1,n}$. The graph $G \circ S_m$ is obtained from G by attaching m pendant vertices to each vertex of G. A Twig $TW(P_n), n \geq 4$ is a graph obtained from a path by attaching exactly two pendant vertices to each internal vertices of the path P_n. If $v_1^{(i)}, v_2^{(i)}, v_3^{(i)}, \cdots, v_{m+1}^{(i)}$ and $u_1, u_2, u_3, \cdots, u_n$ be the vertices of ith copy of the star graph S_m and the path P_n respectively, then the graph $[P_n; S_m]$ is obtained from n copies of S_m and the path P_n by joining u_i with the central vertex $v_1^{(i)}$ of the ith copy of S_m by means of an edge, for $1 \leq i \leq n$.

A subdivision of a graph G, denoted by $S(G)$, is a graph obtained by subdividing edge of

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G by a vertex.

The middle graph $M(G)$ of a graph G is the graph whose vertex set is $\{v : v \in V(G)\} \cup \{e : e \in E(G)\}$ and the edge set is $\{e_1e_2 : e_1, e_2 \in E(G)$ and e_1 and e_2 are adjacent edges of $G\} \cup \{ve : v \in V(G), e \in E(G)$ and e is incident with $v\}$. The total graph $T(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent if and only if either they are adjacent vertices of G or adjacent edges of G or one is a vertex of G and the other one is an edge incident on it. Square of a graph G, denoted by G^2, has the vertex set as in G and two vertices are adjacent in G^2 if they are at a distance either 1 or 2 apart in G. Let G_1 and G_2 be any two graphs with p_1 and p_2 vertices respectively. Then the *Cartesian product* $G_1 \times G_2$ has p_1p_2 vertices which are $\{(u, v) : u \in G_1, v \in G_2\}$ and the edges are obtained as follows: (u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \times G_2$ if either $u_1 = u_2$ and v_1 and v_2 are adjacent in G_2 or u_1 and u_2 are adjacent in G_1 and $v_1 = v_2$. A ladder graph L_n is the graph $P_2 \times P_n$.

The slanting ladder SL_n is a graph obtained from two paths $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ by joining each v_i, with $u_{i+1}, 1 \leq i \leq n-1$.

The concept of root square mean labeling was introduced and studied by S.S. Sandhya et al. in [4,5]. Motivated by the works of so many authors in the area of graph labeling, we introduce a new type of labeling called F-root square mean labeling.

A labeling f on a graph $G(V, E)$ with p vertices and q edges is called a *Smarandache power root mean labeling* for an integer $m \geq 1$ if $f : V(G) \to \{1, 2, 3, \cdots, q+1\}$ is injective and the induced function f^* defined by

$$f^*(uv) = \left\lfloor \sqrt[m]{\frac{f(u)^m + f(v)^m}{2}} \right\rfloor$$

is bijective for all $uv \in E(G)$. Particularly, if $m = 1$, such a Smarandache power root mean labeling is nothing else but the mean labeling and $m = 2$ is called the *F-root square mean labeling* on graph $G(V, E)$ with an injective $f : V(G) \to \{1, 2, 3, \cdots, q+1\}$ and an induced bijective

$$f^*(uv) = \left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$$

for all $uv \in E(G)$. A graph that admits a F-root square mean labeling is called a F-root square mean graph.

In [4], S.S. Sandhya et al. defined the root square mean labeling as follows:

A graph $G(V, E)$ with p vertices and q edges is said to be Root Square Mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1, 2, 3, \cdots, q+1$ in such a way that when each edge $e = uv$ is labeled with

$$\left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil \quad \text{or} \quad \left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor,$$

then the edge labels are distinct. In this case f is called a root square mean labeling of G.

In the above definition, the readers will get some confusion in finding the edge labels which

edge is assigned by flooring function and which edge is assigned by ceiling function. To avoid the confusion of assigning the edge labels in their definition, we just consider the flooring function

$$\left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$$

for our discussion. Based on our definition, a F-root square mean labeling of the graph $K_5 - e$ is given in Figure 1.

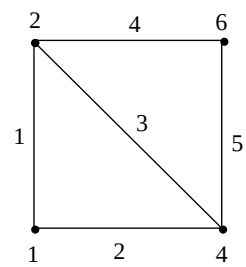

Figure 1. A F-root square mean labeling of $K_4 - e$.

In this paper, we study the F-root square meanness of the path P_n, the graph $P_n \circ S_m$, the graph $P_n \circ K_2$, the twig graph $TW(P_n)$, the graph $[P_n; S_m]$, the graph $S(P_n \circ K_1)$, the middle graph $M(P_n)$, the total graph $T(P_n)$, the square graph P_n^2, the ladder graph L_n and the slanting ladder graph SL_n.

§2. Main Results

Theorem 2.1 *Every path P_n is a F-root square mean graph.*

Proof Let $v_1, v_2, v_3, \cdots, v_n$ be the vertices of the path P_n. Define $f : V(G) \to \{1, 2, 3, \cdots, n\}$ as $f(v_i) = i$, for $1 \leq i \leq n$. Then the induced edge labeling is $f^*(v_i v_{i+1}) = i$, for $1 \leq i \leq n-1$. Hence f is a F-Root Square Mean labeling of path P_n. Thus the path P_n is a F-root square mean graph. □

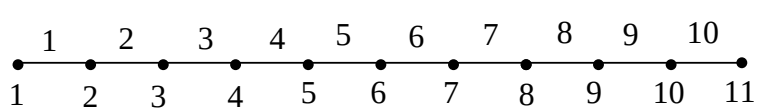

Figure 2. A F-root square mean labeling of P_{11}.

Theorem 2.2 *The graph $P_n \circ S_m$ is a F-root square mean graph for $n \geq 1$ and $m \leq 4$.*

Proof Let $v_1, v_2, v_3, \cdots, v_n$ be the vertices of the path P_n and $u_1^{(i)}, u_2^{(i)}, u_3^{(i)}, \cdots, u_m^{(i)}$ be the pendant vertices at each v_i, for $1 \leq i \leq n$.

Case 1. $m = 1$.

Define $f : V(P_n \circ S_1) \to \{1, 2, 3, \cdots, 2n\}$ as follows

$$f(v_i) = 2i - 1, \text{ for } 1 \leq i \leq n,$$
$$f\left(u_1^{(i)}\right) = 2i, \text{ for } 1 \leq i \leq n.$$

Then the induced edge labeling is obtained as follows

$$f^*(v_i v_{i+1}) = 2i, \text{ for } 1 \leq i \leq n-1,$$
$$f^*\left(v_i u_1^{(i)}\right) = 2i - 1, \text{ for } 1 \leq i \leq n.$$

Case 2. $m = 2$.

Define $f : V(P_n \circ S_2) \to \{1, 2, 3, \cdots, 3n\}$ as follows

$$f(v_i) = 3i - 1, \text{ for } 1 \leq i \leq n,$$
$$f\left(u_1^{(i)}\right) = 3i - 2, \text{ for } 1 \leq i \leq n,$$
$$f\left(u_2^{(i)}\right) = 3i, \text{ for } 1 \leq i \leq n.$$

Then the induced edge labeling is obtained as follows

$$f^*(v_i v_{i+1}) = 3i, \text{ for } 1 \leq i \leq n-1,$$
$$f^*\left(v_i u_1^{(i)}\right) = 3i - 2, \text{ for } 1 \leq i \leq n,$$
$$f^*\left(v_i u_2^{(i)}\right) = 3i - 1, \text{ for } 1 \leq i \leq n.$$

Case 3. $m = 3$.

Define $f : V(P_n \circ S_3) \to \{1, 2, 3, \cdots, 4n\}$ as follows

$$f(v_i) = 4i - 2, \text{ for } 1 \leq i \leq n,$$
$$f\left(u_1^{(i)}\right) = 4i - 3, \text{ for } 1 \leq i \leq n,$$
$$f\left(u_2^{(i)}\right) = 4i - 1, \text{ for } 1 \leq i \leq n,$$
$$f\left(u_3^{(i)}\right) = 4i, \text{ for } 1 \leq i \leq n.$$

Then the induced edge labeling is obtained as follows

$$f^*(v_i v_{i+1}) = 4i, \text{ for } 1 \leq i \leq n-1,$$
$$f^*\left(v_i u_1^{(i)}\right) = 4i - 3, \text{ for } 1 \leq i \leq n,$$
$$f^*\left(v_i u_2^{(i)}\right) = 4i - 2, \text{ for } 1 \leq i \leq n,$$
$$f^*\left(v_i u_3^{(i)}\right) = 4i - 1, \text{ for } 1 \leq i \leq n.$$

Case 4. $m = 4$.

Define $f : V(P_n \circ S_4) \to \{1, 2, 3, \cdots, 5n\}$ as follows

$$f(v_1) = 2, \quad f(v_i) = 5i - 2, \text{ for } 1 \le i \le n, \quad f\left(u_1^{(1)}\right) = 1,$$
$$f\left(u_1^{(i)}\right) = 5i - 5, \text{ for } 2 \le i \le n, \quad f\left(u_2^{(1)}\right) = 3,$$
$$f\left(u_2^{(i)}\right) = 5i - 3, \text{ for } 2 \le i \le n,$$
$$f\left(u_3^{(i)}\right) = 5i - 1, \text{ for } 1 \le i \le n,$$
$$f\left(u_4^{(i)}\right) = 5i + 1, \text{ for } 1 \le i \le n - 1, \quad f\left(u_4^{(n)}\right) = 5n.$$

Then the induced edge labeling is obtained as follows

$$f^*(v_i v_{i+1}) = 5i, \text{ for } 1 \le i \le n - 1, \quad f^*\left(v_i u_1^{(i)}\right) = 5i - 4, \text{ for } 1 \le i \le n,$$
$$f^*\left(v_i u_2^{(i)}\right) = 5i - 3, \text{ for } 1 \le i \le n, \quad f^*\left(v_i u_3^{(i)}\right) = 5i - 2, \text{ for } 1 \le i \le n,$$
$$f^*\left(v_i u_4^{(i)}\right) = 5i - 1, \text{ for } 1 \le i \le n.$$

Hence, f is a F-root square mean labeling of the graph $P_n \circ S_m$. Thus the graph $P_n \circ S_m$ is a F-root square mean graph for $n \ge 1$ and $m \le 4$. □

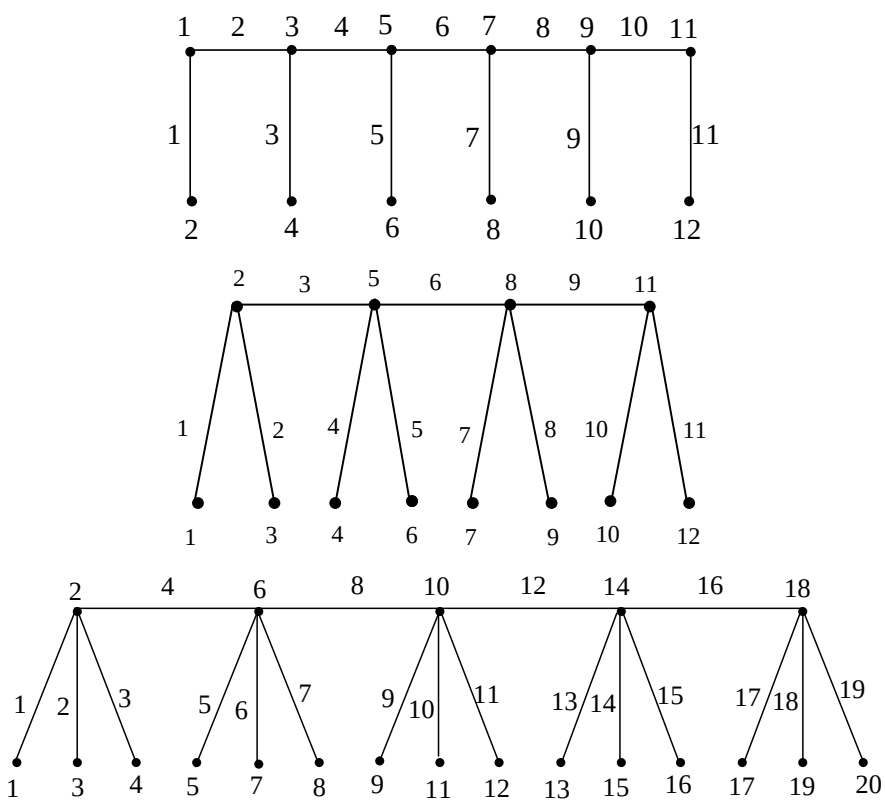

Figure 3. A F-root square mean labeling of $P_6 \circ S_1$, $P_4 \circ S_2$ and $P_5 \circ S_3$.

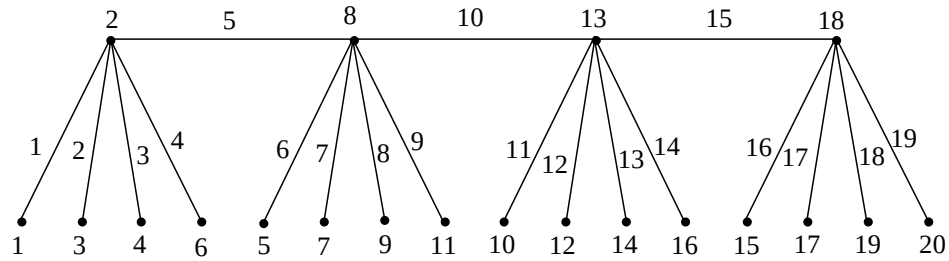

Figure 4. A F-root square mean labeling of $P_4 \circ S_4$.

Theorem 2.3 *The graph $P_n \circ K_2$ is a F-root square mean graph for $n \geq 1$.*

Proof Let $v_1, v_2, v_3, \cdots, v_n$ be the vertices of the path P_n and $u_i^{(1)}, u_i^{(2)}$ be the vertices of ith copy of K_2 attached with v_i, for $1 \leq i \leq n$.

Define $f : V(P_n \circ K_2) \to \{1, 2, 3, \cdots, 4n\}$ as follows

$$f(v_i) = 4i - 2, \text{ for } 1 \leq i \leq n,$$
$$f\left(u_i^{(1)}\right) = 4i - 3, \text{ for } 1 \leq i \leq n,$$
$$f\left(u_i^{(2)}\right) = 4i, \text{ for } 1 \leq i \leq n.$$

Then the induced edge labeling is obtained as follows

$$f^*(v_i v_{i+1}) = 4i, \text{ for } 1 \leq i \leq n-1,$$
$$f^*\left(u_i^{(1)} u_i^{(2)}\right) = 4i - 2, \text{ for } 1 \leq i \leq n,$$
$$f^*\left(u_i^{(1)} v_1\right) = 4i - 3, \text{ for } 1 \leq i \leq n,$$
$$f^*\left(u_i^{(2)} v_i\right) = 4i - 1, \text{ for } 1 \leq i \leq n.$$

Hence f is a F-root square mean labeling of the graph $P_n \circ K_2$. Thus the graph $P_n \circ K_2$ is a F-root square mean graph for $n \geq 1$. □

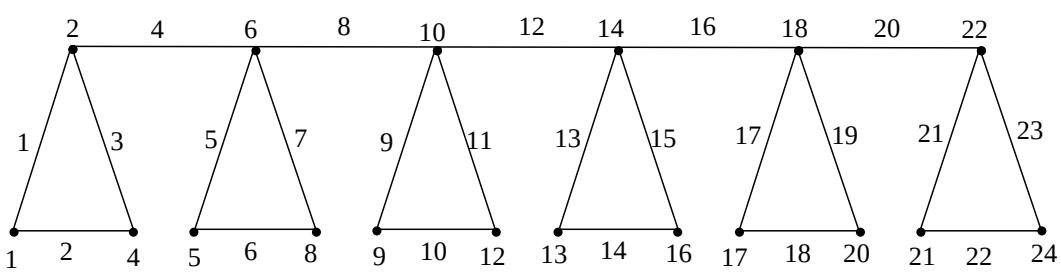

Figure 5. A F-root square mean labeling of $P_6 \circ K_2$.

Theorem 2.4 *The twig graph $TW(P_n)$ of the path P_n is a F-root square mean graph for $n \geq 4$.*

Proof Let $v_1, v_2, v_3, \cdots, v_n$ be the vertices of the path P_n and $u_1^{(i)}, u_2^{(i)}$ be the pendant vertices at each vertex v_i, for $2 \leq i \leq n-1$. Define $f : V(TW(P_n)) \to \{1, 2, 3, \cdots, 3n-4\}$ as follows

$$\begin{aligned} f(v_1) = 1, \quad f(v_2) = 2, \quad f(v_i) = 3i-3, \text{ for } 3 \leq i \leq n-1, \\ f(v_n) = 3n-4, \quad f\left(u_1^{(2)}\right) = 3, \quad f\left(u_1^{(i)}\right) = 3i-4, \text{ for } 3 \leq i \leq n-1, \\ f\left(u_2^{(i)}\right) = 3i-2, \text{ for } 2 \leq i \leq n-1. \end{aligned}$$

Then the induced edge labeling is obtained as follows

$$\begin{aligned} f^*(v_i v_{i+1}) = 3i-2, \text{ for } 1 \leq i \leq n-1, \\ f^*\left(v_i u_1^{(i)}\right) = 3i-4, \text{ for } 2 \leq i \leq n-1 \\ f^*\left(v_i u_2^{(i)}\right) = 3i-3, \text{ for } 2 \leq i \leq n-1. \end{aligned}$$

Hence f is a F-root square mean labeling of the graph $TW(P_n)$. Thus the graph $TW(P_n)$ is a F-root square mean graph for $n \geq 1$. □

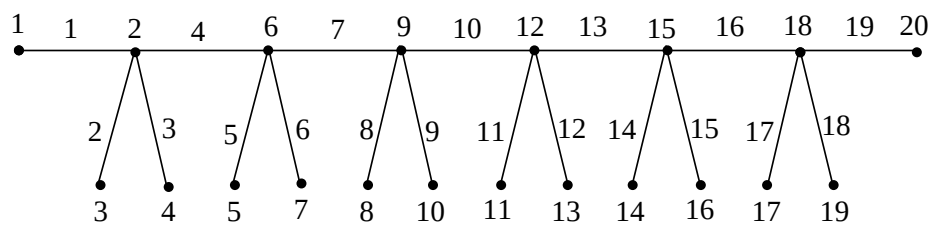

Figure 6. A F-root square mean labeling of $TW(P_8)$.

Theorem 2.5 *The graph $[P_n; S_m]$ is a F-root square mean graph for $m \leq 2$ and $n \geq 1$.*

Proof Let $u_1, u_2, u_3, \ldots, u_n$ be the vertices of the path P_n and $v_1^{(i)}, v_2^{(i)}, v_3^{(i)}, \ldots, v_{m+1}^{(i)}$ be the vertices of the star graph S_m such that $v_1^{(i)}$ is the central vertex of the star graph $S_m, 1 \leq i \leq n$.

Case 1. $m = 1$

Define $f : V([P_n; S_1]) \to \{1, 2, 3, \ldots, 3n\}$ as follows

$$f(u_i) = \begin{cases} 3i, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 3i-2, & 1 \leq i \leq n \text{ and } i \text{ is even,} \end{cases}$$

$$f\left(v_1^{(i)}\right) = 3i-1, \text{for } 1 \leq i \leq n,$$

$$f\left(v_2^{(i)}\right) = \begin{cases} 3i-2, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 3i, & 1 \leq i \leq n \text{ and } i \text{ is even.} \end{cases}$$

Then the induced edge labeling is obtained as follows

$$f^*(u_iu_{i+1}) = 3i, \text{ for } 1 \le i \le n-1,$$

$$f^*\left(u_iv_1^{(i)}\right) = \begin{cases} 3i-1, & 1 \le i \le n \text{ and } i \text{ is odd} \\ 3i-2, & 1 \le i \le n \text{ and } i \text{ is even} \end{cases}$$

$$f^*\left(v_1^{(i)}v_2^{(i)}\right) = \begin{cases} 3i-2, & 1 \le i \le n \text{ and } i \text{ is odd} \\ 3i-1, & 1 \le i \le n \text{ and } i \text{ is even.} \end{cases}$$

Case 2. $m = 2$

Define $f : V([P_n; S_2]) \to \{1, 2, 3, \ldots, 4n\}$ as follows

$$f(u_i) = \begin{cases} 4i-1, & 1 \le i \le n \text{ and } i \text{ is odd} \\ 4i-3, & 1 \le i \le n \text{ and } i \text{ is even,} \end{cases}$$

$$f\left(v_1^{(i)}\right) = 4i-2, \text{for } 1 \le i \le n,$$

$$f\left(v_2^{(i)}\right) = \begin{cases} 4i-3, & 1 \le i \le n \text{ and } i \text{ is odd} \\ 4i-1, & 1 \le i \le n \text{ and } i \text{ is even} \end{cases}$$

$$f\left(v_3^{(i)}\right) = 4i, \text{ for } 1 \le i \le n.$$

Then the induced edge labeling is obtained as follows

$$f^*(u_iu_{i+1}) = 4i, \text{ for } 1 \le i \le n-1,$$

$$f^*\left(u_iv_1^{(i)}\right) = \begin{cases} 4i-2, & 1 \le i \le n \text{ and } i \text{ is odd} \\ 4i-3, & 1 \le i \le n \text{ and } i \text{ is even,} \end{cases}$$

$$f^*\left(v_1^{(i)}v_2^{(i)}\right) = \begin{cases} 4i-3, & 1 \le i \le n \text{ and } i \text{ is odd} \\ 4i-2, & 1 \le i \le n \text{ and } i \text{ is even} \end{cases}$$

$$f^*\left(v_1^{(i)}v_3^{(i)}\right) = 4i-1, \text{ for } 1 \le i \le n.$$

Hence f is a F-root square mean labeling of the graph $[P_n; S_m]$. Thus the graph $[P_n; S_m]$ is a F-root square mean graph for $n \ge 1$. □

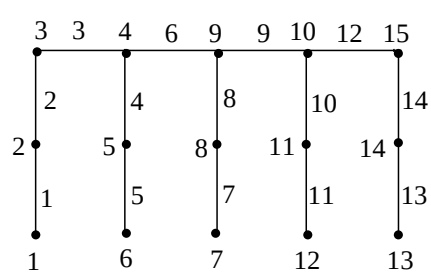

Figure 7. A F-root square mean labeling of $[P_5; S_1]$.

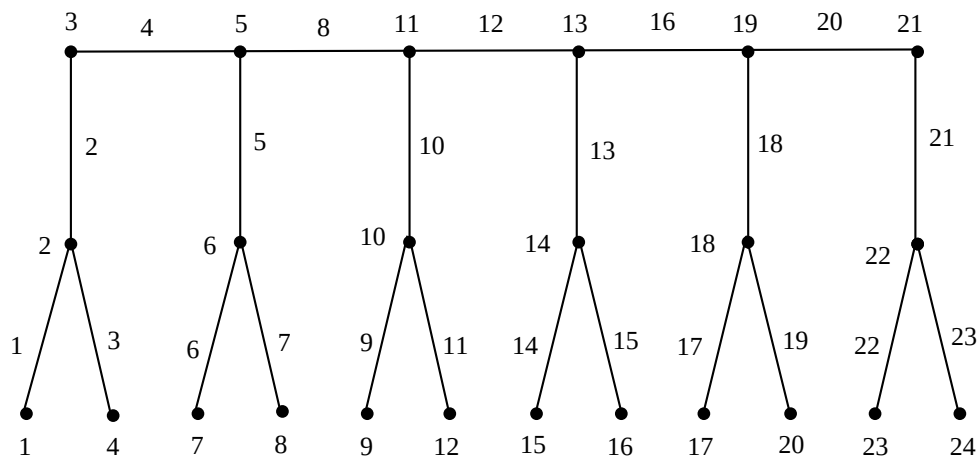

Figure 8. A F-root square mean labeling of $[P_6; S_2]$.

Theorem 2.6 *The graph $S(P_n \circ K_1)$ is a F-root square mean graph for $n \geq 1$.*

Proof In $P_n \circ K_1$, let $u_i, 1 \leq i \leq n$, be the vertices on the path P_n and v_i be the vertex attached at each vertex $u_i, 1 \leq i \leq n$.

Let x_i be the vertex which divides the edge $u_i v_i$, for $1 \leq i \leq n$ and y_i be the vertex which divides the edge $u_i u_{i+1}$, for $1 \leq i \leq n-1$. Then,

$$V(S(P_n \circ K_1)) = \{u_i, v_i, x_i, y_j; 1 \leq i \leq n \text{ and } 1 \leq j \leq n-1\},$$
$$E(S(P_n \circ K_1)) = \{u_i x_i, v_i y_i; 1 \leq i \leq n\} \bigcup \{u_i y_i, y_i u_{i+1}; 1 \leq i \leq n-1\}.$$

Define $f : V(S(P_n \circ K_1)) \to \{1, 2, 3, \ldots, 4n-1\}$ as follows

$$\begin{aligned} f(u_i) &= 4i-3, \text{ for } 1 \leq i \leq n, \\ f(y_i) &= 4i-1, \text{ for } 1 \leq i \leq n-1, \\ f(x_i) &= 4i-2, \text{ for } 1 \leq i \leq n, \\ f(v_i) &= 4i, \text{ for } 1 \leq i \leq n-1, \\ f(v_n) &= 4n-1. \end{aligned}$$

Then the induced edge labeling is obtained as follows

$$\begin{aligned} f^*(u_i y_i) &= 4i-2, \text{ for } 1 \leq i \leq n-1, \\ f^*(y_i u_{i+1}) &= 4i, \text{ for } 1 \leq i \leq n-1, \\ f^*(u_i x_i) &= 4i-3, \text{ for } 1 \leq i \leq n, \\ f^*(x_i v_i) &= 4i-1, \text{ for } 1 \leq i \leq n-1, \\ f^*(x_n v_n) &= 4n-2. \end{aligned}$$

Hence f is a F-root square mean labeling of $S(P_n \circ K_1)$. Thus the graph $S(P_n \circ K_1)$ is a

F-root square mean graph for $n \geq 1$. □

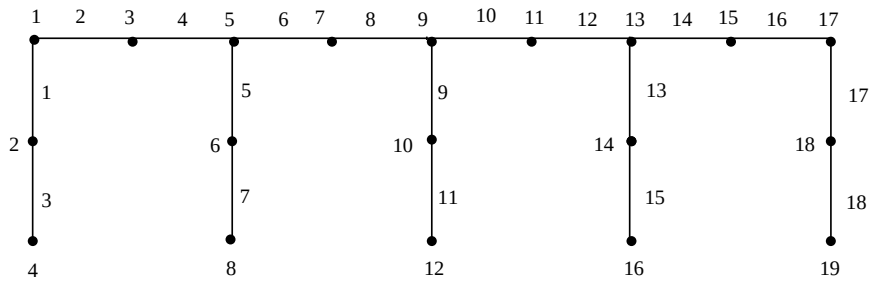

Figure 9. A F-root square mean labeling of $S(P_5 \circ K_1)$.

Theorem 2.7 *The middle graph $M(P_n)$ of a path P_n is a F-root square mean graph.*

Proof Let $V(P_n) = \{v_1, v_2, v_3, \cdots, v_n\}$ and $E(P_n) = \{e_i = v_i v_{i+1}; 1 \leq i \leq n-1\}$ be the vertex set and edge set of the path P_n. Then,

$$V(M(P_n)) = \{v_1, v_2, v_3, \cdots, v_n, e_1, e_2, e_3, \cdots, e_{n-1}\},$$
$$E(M(P_n)) = \{v_i e_i, e_i v_{i+1}; 1 \leq i \leq n-1\} \bigcup \{e_i e_{i+1}; 1 \leq i \leq n-2\}.$$

Define $f : V(M(P_n)) \to \{1, 2, 3, \ldots, 3n-3\}$ as follows

$$f(v_i) = \begin{cases} 1, & i = 1 \\ 3i-3, & 2 \leq i \leq n, \end{cases}$$
$$f(e_i) = 3i - 1, \text{ for } 1 \leq i \leq n-1.$$

Then the induced edge labeling is obtained as follows

$$f^*(v_i e_i) = 3i - 2, \text{ for } 1 \leq i \leq n-1,$$
$$f^*(e_i v_{i+1}) = 3i - 1, \text{ for } 1 \leq i \leq n-1,$$
$$f^*(e_i e_{i+1}) = 3i, \text{ for } 1 \leq i \leq n-2.$$

Hence f is a F-root square mean labeling of $M(P_n)$. Thus the graph $M(P_n)$ is a F-root square mean graph. □

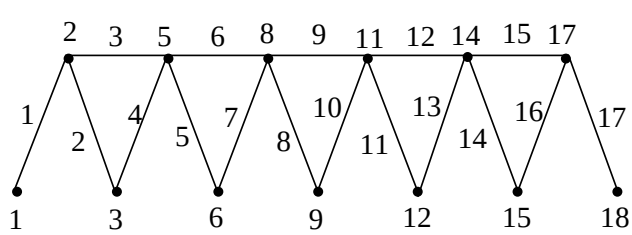

Figure 10. A F-root square mean labeling of $M(P_7)$.

Theorem 2.8 *The total graph $T(P_n)$ of a path P_n is a F-root square mean graph for $n \geq 1$.*

Proof Let $V(P_n) = \{v_1, v_2, v_3, \cdots, v_n\}$ and $E(P_n) = \{e_i = v_i v_{i+1}; 1 \leq i \leq n-1\}$ be the vertex set and edge set of the path P_n. Then,

$$V(T(P_n)) = \{v_1, v_2, v_3, \cdots, v_n, e_1, e_2, e_3, \cdots, e_{n-1}\}$$
$$E(T(P_n)) = \{v_i v_{i+1}, e_i v_i, e_i v_{i+1}; 1 \leq i \leq n-1\} \cup \{e_i e_{i+1}; 1 \leq i \leq n-2\}.$$

Define $f : V(T(P_n)) \to \{1, 2, 3, \ldots, 4(n-1)\}$ as follows

$$f(v_1) = 1, \quad f(v_i) = 4i - 4, \text{ for } 2 \leq i \leq n$$
$$f(e_i) = 4i - 2, \text{ for } 1 \leq i \leq n-1.$$

Then the induced edge labeling is obtained as follows

$$f^*(v_i v_{i+1}) = 4i - 2, \text{ for } 1 \leq i \leq n-1, \quad f^*(e_i e_{i+1}) = 4i, \text{ for } 1 \leq i \leq n-2,$$
$$f^*(v_i e_i) = 4i - 3, \text{ for } 1 \leq i \leq n-1$$
$$f^*(e_i v_{i+1}) = 4i - 1, \text{ for } 1 \leq i \leq n-1.$$

Hence f is a F-root square mean labeling of $T(P_n)$. Thus the graph $T(P_n)$ is a F-root square mean graph. □

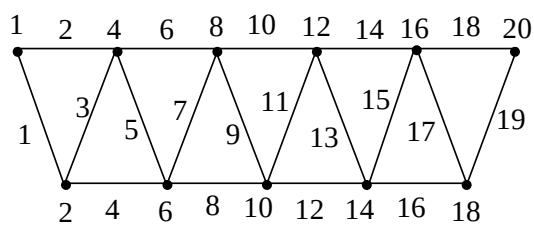

Figure 11. A F-root square mean labeling of $T(P_6)$.

Theorem 2.9 *The square graph P_n^2 of the path P_n is a F-root square mean graph for $n \geq 1$.*

Proof Let $v_1, v_2, v_3, \cdots, v_n$ be the vertices of the path P_n. Define $f : V\left(P_n^2\right) \to \{1, 2, 3, \cdots, 2(n-1)\}$ as $f(v_1) = 1$ and $f(v_i) = 2i - 2$ for $2 \leq i \leq n$. Then the induced edge labeling is obtained as follows

$$f^*(v_i v_{i+1}) = 2i - 1, \text{ for } 1 \leq i \leq n-1,$$
$$f^*(v_i v_{i+2}) = 2i, \text{ for } 1 \leq i \leq n-2.$$

Hence f is a F-root square mean labeling of P_n^2. Thus the graph P_n^2 is a F-root square mean graph. □

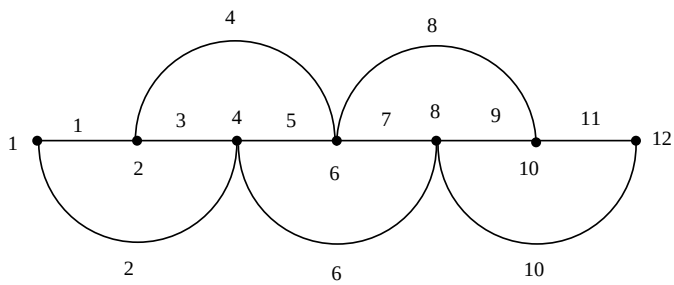

Figure 12. A F-root square mean labeling of P_7^2.

Theorem 2.10 *The ladder graph L_n is a F-root square mean graph for $n \geq 1$.*

Proof Let $G = P_2 \times P_n$ be the ladder graph for any positive integer n, having $2n$ vertices and $3n-2$ edges. Let $u_1, u_2, \cdots, u_n$ and $v_1, v_2, \cdots, v_n$ be the vertices of G. Then the edge set of G is $\{u_iu_{i+1}, v_iv_{i+1} : 1 \leq i \leq n-1\} \bigcup \{u_iv_i : 1 \leq i \leq n\}$.

Define $f : V(G) \to \{1, 2, 3, \ldots, 3n-1\}$ as follows

$$f(u_i) = 3i - 1, \text{ for } 1 \leq i \leq n,$$
$$f(v_i) = 3i - 2, \text{ for } 1 \leq i \leq n.$$

Then the induced edge labeling is obtained as follows

$$f^*(u_iu_{i+1}) = 3i, \text{ for } 1 \leq i \leq n-1,$$
$$f^*(v_iv_{i+1}) = 3i - 1, \text{ for } 1 \leq i \leq n-1$$
$$f^*(u_iv_i) = 3i - 2, \text{ for } 1 \leq i \leq n.$$

Hence f is a F-root square mean labeling of L_n. Thus the graph L_n is a F-root square mean graph. □

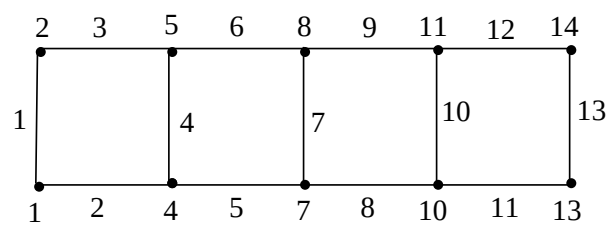

Figure 13. A F-root square mean labeling of L_5.

Theorem 2.11 *The slanting ladder graph SL_n is a F-root square mean graph for $n \geq 2$.*

Proof Let the vertex set of SL_n be $\{u_1, u_2, u_3, \cdots, u_n, v_1, v_2, v_3, \cdots, v_n\}$ and the edge set of SL_n be $\{u_iu_{i+1}; 1 \leq i \leq n-1\} \bigcup \{v_iv_{i+1}; 1 \leq i \leq n-1\} \bigcup \{v_iu_{i+1}; 1 \leq i \leq n-1\}$. Then SL_n has $2n$ vertices and $3n-3$ edges.

Define $f : V(SL_n) \to \{1, 2, 3, \cdots, 3n-2\}$ as follows

$$f(u_1) = 1, \quad f(u_i) = 3i - 4, \text{ for } 2 \le i \le n,$$
$$f(v_i) = 3i, \text{ for } 1 \le i \le n-1 \quad \text{and } f(v_n) = 3n - 2.$$

Then the induced edge labeling is obtained as follows

$$f^*(u_i u_{i+1}) = \begin{cases} 1, & i = 1 \\ 3i - 3, & 2 \le i \le n-1, \end{cases}$$
$$f^*(v_i v_{i+1}) = 3i + 1, \text{ for } 1 \le i \le n-2,$$
$$f^*(v_{n-1} v_n) = 3n - 3$$
$$f^*(v_i u_{i+1}) = 3i - 1, \text{ for } 1 \le i \le n-1.$$

Hence f is a F-root square mean labeling of SL_n. Thus the graph SL_n is a F-root square mean graph. □

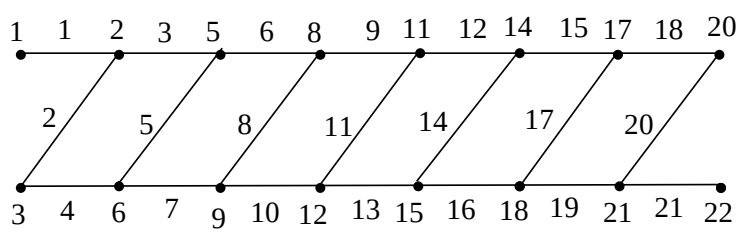

Figure 14. A F-root square mean labeling of SL_8.

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