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Further Results on 4-Total Mean Cordial Graphs

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Abstract: Let G be a graph. Let $f : V(G) \to \{0, 1, 2, \ldots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label

$$f(uv) = \left\lceil \frac{f(u) + f(v)}{2} \right\rceil$$

and f is called a k-total mean cordial labeling of G if $|t_{mf}(i) - t_{mf}(j)| \leq 1$, for all $i, j \in \{0, 1, 2, \ldots, k-1\}$, where $t_{mf}(x)$ denotes the total number of vertices and edges labelled with x, $x \in \{0, 1, 2, \ldots, k-1\}$. A graph with admit a k-total mean cordial labeling is called k-total mean cordial graph. In this paper we investigate the 4-Total mean cordial labeling behavior of some graphs like $C_4 \times P_n$, middle graph of P_n, total graph of P_n, middle graph of C_n, total graph of C_n and kayak paddale graph.

Key Words: Total mean cordial labelling, Smarandachely total mean cordial labeling, middle graph, total graph.

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§1. Introduction

In this paper we consider simple, finite and undirected graphs only. Cordial labeling was introduced by Cahit [1]. The notion of k-total mean cordial labeling has been introduced in [5]. The 4-total mean cordial labeling behaviour of several graphs like cycle, complete graph, star, bistar, comb and crown have been studied in [5, 6, 7, 8, 9, 10, 11, 12, 13]. Edge-Odd gracefulness of middle graphs and total graphs of certain graphs was studied in [4]. In this paper we investigate the 4- total mean cordial labeling of middle graph of the path P_n, total graph of the path P_n, middle graph of the cycle C_n, total graph of the cycle C_n, $C_4 \times P_n$ and kayak paddale graph. Let x be any real number. Then $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. Terms are not defined here follow from Harary [3] and Gallian [2].

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§2. k-Total Mean Cordial Graph

Definition 2.1 *Let G be a graph. Let $f : V(G) \rightarrow \{0, 1, 2, \ldots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $f(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called a k-total mean cordial labeling of G if $|t_{mf}(i) - t_{mf}(j)| \leq 1$, for all $i, j \in \{0, 1, 2, \ldots, k-1\}$, where $t_{mf}(x)$ denotes the total number of vertices and edges labelled with x, $x \in \{0, 1, 2, \ldots, k-1\}$. A graph with admit a k-total mean cordial labeling is called k-total mean cordial graph.*

Such a labeling f is called a Smarandachely k-total mean cordial labeling of G if there are integers $i, j \in \{0, 1, 2, \cdots, k-1\}$ hold with $|t_{mf}(i) - t_{mf}(j)| \geq 2$ and G is called a Smarandachely k-total mean cordial graph.

§3. Preliminaries

Definition 3.1([3]) *A middle graph $M(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it.*

Definition 3.2([3]) *A total graph $T(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent whenever they are either adjacent or incident in G.*

Definition 3.3([3]) *A Kayak Paddale $KP(m, n, l)$ is the graph obtained by joining the cycles C_m and C_n with the path P_{l+1} of length l. Let C_m be the cycle $u_1\ u_2\ \ldots\ u_n\ u_1$ and C_n be the cycle $v_1\ v_2\ \ldots\ v_n\ v_1$. Let P_{l+1} be the path $w_1\ w_2\ \ldots\ w_n$. Identify u_1 with w_1 and w_n with v_1.*

§4. Main Results

Theorem 4.1 *A graph $C_4 \times P_n$ is a 4-total mean cordial for all $n \geq 2$.*

Proof Let $V(C_4 \times P_n) = \{a_i, b_i, c_i, d_i : 1 \leq i \leq n\}$ and $E(C_4 \times P_n) = \{a_i a_{i+1}, b_i b_{i+1}, c_i c_{i+1}, d_i d_{i+1} : 1 \leq i \leq n-1\} \bigcup \{a_i b_i, b_i c_i, c_i d_i, d_i a_i : 1 \leq i \leq n\}$. Obviously,

$$|V(C_4 \times P_n)| + |E(C_4 \times P_n)| = 12n - 4.$$

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r \in \mathbb{N}$. Assign the label 0 to the $4r - 1$ vertices a_1, a_2, $\cdots$, a_{4r-1}. Now we assign the label 2 to the vertex a_{4r}. Next we assign the label 3 to the $4r$ vertices b_1, b_2, $\cdots$, b_{4r}. We now assign the label 0 to the $2r$ vertices c_1, c_2, $\ldots$, c_{2r}. Now we assign the label 1 to the $r-1$ vertices c_{2r+1}, c_{2r+2}, $\cdots$, c_{3r-1}. Next we assign the label 3 to the r vertices c_{3r}, c_{3r+1}, $\cdots$, c_{4r-1}. Now we assign the label 0 to the vertex c_{4r}. We now assign the label 1 to the $2r$ vertices d_1, d_2, $\cdots$, d_{2r}. Now we assign the label 2 to the $2r-1$ vertices d_{2r+1}, d_{2r+2}, $\cdots$, d_{4r-1}. Finally we assign the label 0 to the vertex d_{4r}.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r + 1$, $r \in \mathbb{N}$. Assign the label 0 to the $4r + 1$ vertices $a_1, a_2, \cdots, a_{4r+1}$. Next we assign the label 3 to the $4r + 1$ vertices $b_1, b_2, \cdots, b_{4r+1}$. Now we assign the label 0 to the $2r + 1$ vertices $c_1, c_2, \cdots, c_{2r+1}$. We now assign the label 1 to the r vertices $c_{2r+2}, c_{2r+3}, \cdots, c_{3r+1}$. Next we assign the label 3 to the r vertices $c_{3r+2}, c_{3r+3}, \cdots, c_{4r+1}$. We now assign the label 1 to the $2r+1$ vertices $d_1, d_2, \cdots, d_{2r+1}$. Now we assign the label 2 to the $2r - 1$ vertices $d_{2r+2}, d_{2r+3}, \cdots, d_{4r}$. Finally we assign the label 3 to the vertex d_{4r+1}.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r + 2$, $r \geq 0$. Label the vertices a_i, b_i, c_i, d_i $(1 \leq i \leq 4r + 1)$ as in Case 1. Next we assign the labels 2, 3, 0, 0 to the vertices $a_{4r+2}, b_{4r+2}, c_{4r+2}, d_{4r+2}$.

Case 4. $n \equiv 3 \pmod 2$.

Let $n = 4r + 3$, $r \geq 0$. Assign the label 0 to the $4r + 3$ vertices $a_1, a_2, \cdots, a_{4r+3}$. Now we assign the label 3 to the $4r + 3$ vertices $b_1, b_2, \cdots, b_{4r+3}$. Next we assign the label 0 to the $2r + 2$ vertices $c_1, c_2, \cdots, c_{2r+2}$. Now we assign the label 1 to the r vertices $c_{2r+3}, c_{2r+4}, \cdots, c_{3r+2}$. Next we assign the label 3 to the $r + 1$ vertices $c_{3r+3}, c_{3r+4}, \ldots, c_{4r+3}$. We now assign the label 1 to the $2r + 2$ vertices $d_1, d_2, \cdots, d_{2r+2}$. Now we assign the label 2 to the $2r + 1$ vertices $d_{2r+3}, d_{2r+4}, \cdots, d_{4r+3}$.

Thus, this vertex labeling f is a 4-total mean cordial labeling follows from the Table 1.

Order of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 4r$	$12r - 1$	$12r - 1$	$12r - 1$	$12r - 1$
$n = 4r + 1$	$12r + 2$	$12r + 2$	$12r + 2$	$12r + 2$
$n = 4r + 2$	$12r + 5$	$12r + 5$	$12r + 5$	$12r + 5$
$n = 4r + 3$	$12r + 8$	$12r + 8$	$12r + 8$	$12r + 8$

Table 1

This completes the proof. □

Theorem 4.2 *A middle graph of the path P_n, $M(P_n)$ is a 4-total mean cordial for all values of $n \geq 2$.*

Proof Let $u_1, u_2, \cdots, u_n$ be the vertices of path P_n and let $v_1, v_2, \cdots, v_{n-1}$ be the added vertices corresponding to the edges $e_1, e_2, \cdots, e_n$ of P_n to obtain $M(P_n)$. Let $V(M(P_n)) = \{u_i : 1 \leq i \leq n\} \bigcup \{v_i : 1 \leq i \leq n-1\}$, $E(M(P_n)) = \{u_i v_i, v_i u_{i+1} : 1 \leq i \leq n-1\} \bigcup \{v_i v_{i+1} : 1 \leq i \leq n-2\}$. Clearly, $|V(M(P_n))| + |E(M(P_n))| = 5n - 5$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r \geq 1$. Assign the label 0 to the r vertices $u_1, u_2, \cdots, u_r$. Now we assign the label 1 to the r vertices $u_{r+1}, u_{r+2}, \cdots, u_{2r}$. Next we assign the label 2 to the $r - 1$ vertices $u_{2r+1}, u_{2r+2}, \cdots, u_{3r-1}$. We now assign the label 3 to the r vertices $u_{3r}, u_{3r+1}, \cdots, u_{4r-1}$. Next we assign the label 0 to the vertex u_{4r}.

Now we assign the label 0 to the r vertices $v_1, v_2, \cdots, v_r$. Next we assign the label 1 to the $r-1$ vertices $v_{r+1}, v_{r+2}, \cdots, v_{2r-1}$. We now assign the label 2 to the r vertices $v_{2r}, v_{2r+1}, \cdots, v_{3r-1}$. Next we assign the label 3 to the $r-1$ vertices $v_{3r}, v_{3r+1}, \cdots, v_{4r-2}$. Finally, we assign the label 2 to the vertex v_{4r-1}.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r+1$, $r \geq 1$. Assign the label 0 to the $r+1$ vertices $u_1, u_2, \cdots, u_{r+1}$. Next we assign the label 1 to the r vertices $u_{r+2}, u_{r+3}, \cdots, u_{2r+1}$. Now we assign the label 2 to the r vertices $u_{2r+2}, u_{2r+3}, \cdots, u_{3r+1}$. We now assign the label 3 to the r vertices $u_{3r+2}, u_{3r+3}, \cdots, u_{4r+1}$.

Next we assign the label 0 to the r vertices $v_1, v_2, \cdots, v_r$. Now we assign the label 1 to the r vertices $v_{r+1}, v_{r+2}, \cdots, v_{2r}$. We now assign the label 2 to the r vertices $v_{2r+1}, v_{2r+2}, \cdots, v_{3r}$. Next we assign the label 3 to the r vertices $v_{3r+1}, v_{3r+2}, \cdots, v_{4r}$.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r+2$, $r \geq 1$. Label the vertices u_i $(1 \leq i \leq 4r+1)$, v_i $(1 \leq i \leq 4r)$ as in Case 2. Now we assign the labels 0, 2 to the vertex u_{4r+2},v_{4r+1}.

Case 4. $n \equiv 3 \pmod 2$.

Let $n = 4r+3$, $r \geq 1$. Assign the label 0 to the $r+1$ vertices $u_1, u_2, \cdots, u_{r+1}$. Now we assign the label 1 to the $r+1$ vertices $u_{r+2}, u_{r+3}, \cdots, u_{2r+2}$. We now assign the label 2 to the r vertices $u_{2r+3}, u_{2r+4}, \cdots, u_{3r+2}$. Next we assign the label 3 to the $r+1$ vertices $u_{3r+3}, u_{3r+4}, \cdots, u_{4r+3}$.

Now we assign the label 0 to the $r+1$ vertices $v_1, v_2, \cdots, v_{r+1}$. Next we assign the label 1 to the r vertices $v_{r+2}, v_{r+3}, \cdots, v_{2r+1}$. We now assign the label 2 to the $r+1$ vertices $v_{2r+2}, v_{2r+3}, \cdots, v_{3r+2}$. Finally, we assign the label 3 to the r vertices $v_{3r+3}, v_{3r+4}, \ldots, v_{4r+2}$.

Thus, this vertex labeling f is a 4-total mean cordial labeling follows from the Table 2.

Order of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 4r$	$5r-1$	$5r-2$	$5r-1$	$5r-1$
$n = 4r+1$	$5r$	$5r$	$5r$	$5r$
$n = 4r+2$	$5r+1$	$5r+1$	$5r+1$	$5r+2$
$n = 4r+3$	$5r+3$	$5r+2$	$5r+3$	$5r+2$

Table 2

Case 5. $n = 2$ or 3.

A 4-total mean cordial labeling of $M(P_n)$ is given in Tabel 3.

Value of n	u_1	u_2	u_3	v_1	v_2
2	0	3		2	
3	0	1	3	0	3

Table 3

This completes the proof. □

Theorem 4.3 *A total graph of the path P_n, $T(P_n)$ is a 4-total mean cordial for all values of $n \geq 2$.*

Proof Clearly, the vertex labeling of Theorem 4.2 is also a 4-total mean cordial labeling of $T(P_n)$. □

Theorem 4.4 *A middle graph of the cycle C_n, $M(C_n)$ is a 4-total mean cordial for all values of $n \geq 3$.*

Proof Let $u_1, u_2, \cdots, u_n$ be the vertices of cycle C_n and let $v_1, v_2, \cdots, v_n$ be the added vertices corresponding to the edges $e_1, e_2, \cdots, e_n$ of C_n to obtain $M(C_n)$. Let $V(M(C_n)) = \{u_i, v_i : 1 \leq i \leq n\}$ and let $E(M(C_n)) = \{u_i v_i : 1 \leq i \leq n\} \bigcup \{v_i v_{i+1}, v_i u_{i+1} : 1 \leq i \leq n-1\} \bigcup \{v_1 v_n, v_n u_1\}$. Notice that $|V(M(C_n))| + |E(M(C_n))| = 5n$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r \geq 1$. Assign the label 0 to the r vertices $v_1, v_2, \cdots, v_r$. Next we assign the label 1 to the r vertices $v_{r+1}, v_{r+2}, \ldots, v_{2r}$. We now assign the label 2 to the r vertices $v_{2r+1}, v_{2r+2}, \cdots, v_{3r}$. Now we assign the label 3 to the r vertices $v_{3r+1}, v_{3r+2}, \cdots, v_{4r}$.

Next we assign the label 0 to the r vertices $u_1, u_2, \cdots, u_r$. Now we assign the label 1 to the r vertices $u_{r+1}, u_{r+2}, \cdots, u_{2r}$. We now assign the label 2 to the $r-1$ vertices $u_{2r+1}, u_{2r+2}, \cdots, u_{3r-1}$. Next we assign the label 3 to the r vertices $u_{3r}, u_{3r+1}, \cdots, u_{4r-1}$. Finally we assign the label 0 to the vertex u_{4r}.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r+1$, $r \geq 1$. Assign the label 0 to the $r+1$ vertices $v_1, v_2, \cdots, v_{r+1}$. Now we assign the label 1 to the r vertices $v_{r+2}, v_{r+3}, \ldots, v_{2r+1}$. Next we assign the label 2 to the r vertices $v_{2r+2}, v_{2r+3}, \cdots, v_{3r+1}$. We now assign the label 3 to the r vertices $v_{3r+2}, v_{3r+3}, \cdots, v_{4r+1}$.

Now we assign the label 0 to the r vertices $u_1, u_2, \cdots, u_r$. Next we assign the label 1 to the r vertices $u_{r+1}, u_{r+2}, \cdots, u_{2r}$. We now assign the label 2 to the r vertices $u_{2r+1}, u_{2r+2}, \cdots, u_{3r}$. Now we assign the label 3 to the r vertices $u_{3r+1}, u_{3r+2}, \cdots, u_{4r}$. Finally we assign the label 2 to the vertex u_{4r+1}.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r+2$, $r \geq 1$. Assign the label 0 to the $r+1$ vertices $v_1, v_2, \cdots, v_{r+1}$. Next we assign the label 1 to the $r+1$ vertices $v_{r+2}, v_{r+3}, \cdots, v_{2r+2}$. We now assign the label 2 to the r vertices $v_{2r+3}, v_{2r+4}, \cdots, v_{3r+2}$. Now we assign the label 3 to the r vertices $v_{3r+3}, v_{3r+4}, \cdots, v_{4r+2}$.

Next we assign the label 0 to the $r+1$ vertices $u_1, u_2, \cdots, u_{r+1}$. Now we assign the label 1 to the r vertices $u_{r+2}, u_{r+3}, \cdots, u_{2r+1}$. We now assign the label 2 to the r vertices $u_{2r+2}, u_{2r+3}, \cdots, u_{3r+2}$. Finally we assign the label 3 to the $r+1$ vertices $u_{3r+3}, u_{3r+4}, \cdots, u_{4r+2}$.

Case 4. $n \equiv 3 \pmod 2$.

Let $n = 4r+3$, $r \geq 1$. Assign the label 0 to the $r+1$ vertices $v_1, v_2, \cdots, v_{r+1}$. Now we assign the label 1 to the $r+1$ vertices v_{r+2}, v_{r+3}, $\cdots$, v_{2r+2}. We now assign the label 2 to the r vertices v_{2r+3}, v_{2r+4}, $\cdots$, v_{3r+2}. Now we assign the label 3 to the $r+1$ vertices v_{3r+3}, v_{3r+4}, $\cdots$, v_{4r+3}.

Next we assign the label 0 to the $r+1$ vertices u_1, u_2, $\cdots$, u_{r+1}. Now we assign the label 1 to the r vertices u_{r+2}, u_{r+3}, $\cdots$, u_{2r+1}. We now assign the label 2 to the $r+1$ vertices u_{2r+2}, u_{2r+3}, $\cdots$, u_{3r+2}. Next we assign the label 3 to the r vertices u_{3r+3}, u_{3r+4}, $\cdots$, u_{4r+2}. Finally we assign the label 2 to the vertex u_{4r+3}.

Thus this vertex labeling f is a 4-total mean cordial labeling follows from the Table 4.

n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n=4r$	$5r$	$5r$	$5r$	$5r$
$n=4r+1$	$5r+1$	$5r+1$	$5r+2$	$5r+1$
$n=4r+2$	$5r+3$	$5r+3$	$5r+2$	$5r+2$
$n=4r+3$	$5r+3$	$5r+4$	$5r+4$	$5r+4$

Table 4

This completes the proof. □

Theorem 4.5 *A total graph of the cycle C_n, $T(C_n)$ is a 4-total mean cordial if $n \equiv 0, 2 \pmod 4$.*

Proof Obviously, the vertex labeling of Theorem **??** is also a 4-total mean cordial labeling of $T(C_n)$. □

Theorem 4.6 *A Kayak Paddale $KP(n,n,n)$ is a 4-total mean cordial for all values of $n \geq 3$.*

Proof Let $V(KP(n,n,n)) = \{u_i, v_i : 1 \leq i \leq n\} \bigcup \{u_1 = w_{1, v_1 = w_{n+1}}\} \bigcup \{w_i : 2 \leq i \leq n\}$ and let $E(KP(n,n,n)) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \leq i \leq n\} \bigcup \{u_1u_n, v_1v_n\} \bigcup \{w_{1-1}w_i : 2 \leq i \leq n\}$. Notice that $|V(KP(n,n,n))| + |E(KP(n,n,n))| = 6n - 1$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r \geq 1$. Assign the label 0 to the $r+1$ vertices u_1, u_2, $\cdots$, u_{r+1}. Next we assign the label 1 to the $3r-1$ vertices u_{r+2}, u_{r+3}, $\cdots$, u_{4r}.
Now we assign the label 3 to the r vertices v_1, v_2, $\cdots$, v_r. Now we assign the label 2 to the $3r-1$ vertices v_{r+1}, v_{r+2}, $\cdots$, v_{4r-1}. Then we assign the label 0 to the vertex v_{4r}.

Next we assign the label 0 to the $2r-1$ vertices w_2, w_2, $\cdots$, w_{2r}. Finally we assign the label 3 to the $2r$ vertices w_{2r+1}, w_{2r+2}, $\cdots$, w_{4r}.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r+1$, $r \geq 1$. Now we assign the label 0 to the $r+1$ vertices u_1, u_2, $\cdots$, u_{r+1}. Next we assign the label 1 to the $3r$ vertices u_{r+2}, u_{r+3}, $\cdots$, u_{4r+1}.

Next we assign the label 2 to the $r+1$ vertices v_1, v_2, $\cdots$, v_{r+1}. We now assign the label 3 to the $3r$ vertices v_{r+2}, v_{r+3}, $\cdots$, v_{4r+1}.

Now we assign the label 0 to the $2r$ vertices $w_2, w_3, \cdots, w_{2r+1}$. Next we assign the label 2 to the $2r$ vertices $w_{2r+2}, w_{2r+3}, \cdots, w_{4r+1}$.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r + 2$, $r \geq 1$. We now assign the label 0 to the $r + 2$ vertices $u_1, u_2, \cdots, u_{r+2}$. Next we assign the label 1 to the $3r$ vertices $u_{r+3}, u_{r+4}, \cdots, u_{4r+2}$.

Now we assign the label 2 to the $r + 1$ vertices $v_1, v_2, \cdots, v_{r+1}$. Next we assign the label 3 to the $3r + 1$ vertices $v_{r+2}, v_{r+3}, \cdots, v_{4r+2}$.

We now assign the label 0 to the $2r$ vertices $w_2, w_3, \cdots, w_{2r+1}$. Finally we assign the label 2 to the $2r + 1$ vertices $w_{2r+2}, w_{2r+3}, \cdots, w_{4r+2}$.

Case 4. $n \equiv 3 \pmod 2$.

Let $n = 4r + 3$, $r \geq 1$. Assign the label 0 to the $r + 2$ vertices $u_1, u_2, \cdots, u_{r+2}$. Now we assign the label 1 to the $3r + 1$ vertices $u_{r+3}, u_{r+4}, \cdots, u_{4r+3}$.
We now assign the label 3 to the $r + 1$ vertices $v_1, v_2, \cdots, v_{r+1}$. Next we assign the label 2 to the $3r + 1$ vertices $v_{r+2}, v_{r+3}, \cdots, v_{4r+2}$. Now we assign the label 1 to the vertex v_{4r+3}.

Next we assign the label 0 to the $2r + 1$ vertices $w_2, w_3, \cdots, w_{2r+2}$. Finally we assign the label 3 to the $2r + 1$ vertices $w_{2r+3}, w_{2r+4}, \cdots, w_{4r+3}$.

Thus, this vertex labeling f is a 4-total mean cordial labeling follows from the Table 5.

n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 4r$	$6r$	$6r$	$6r - 1$	$6r$
$n = 4r + 1$	$6r + 1$	$6r + 2$	$6r + 1$	$6r + 1$
$n = 4r + 2$	$6r + 3$	$6r + 2$	$6r + 3$	$6r + 3$
$n = 4r + 3$	$6r + 5$	$6r + 4$	$6r + 4$	$6r + 4$

Table 5

This completes the proof. □

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