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Gallai and Anti-Gallai Symmetric n-Sigraphs

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Abstract: An *n-tuple* $(a_1, a_2, \cdots, a_n)$ is *symmetric*, if $a_k = a_{n-k+1}, 1 \leq k \leq n$. Let $H_n = \{(a_1, a_2, \cdots, a_n) : a_k \in \{+, -\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$ be the set of all symmetric n-tuples. A *symmetric n-sigraph (symmetric n-marked graph)* is an ordered pair $S_n = (G, \sigma)$ $(S_n = (G, \mu))$, where $G = (V, E)$ is a graph called the *underlying graph* of S_n and $\sigma : E \to H_n$ $(\mu : V \to H_n)$ is a function. In this paper, we introduced a new notions Gallai and anti-Gallai symmetric n-sigraph of a symmetric n-sigraph and its properties are obtained. Also we give the relation between Gallai symmetric n-sigraphs and anti-Gallai symmetric n-sigraphs. Further, we discuss structural characterizations of these notions.

Key Words: Symmetric n-sigraph, Smarandachely symmetric n-sigraph, symmetric n-marked graph, Smarandachely symmetric n-marked graph, balance, switching, Gallai symmetric n-sigraphs, Smarandachely Gallai symmetric n-sigraph, anti-Gallai symmetric n-sigraph, Smarandachely anti-Gallai n-sigraph, complementation.

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§1. Introduction

Unless mentioned or defined otherwise, for all terminology and notion in graph theory the reader is refer to [3]. We consider only finite, simple graphs free from self-loops.

Let $n \geq 1$ be an integer. An *n-tuple* $(a_1, a_2, \cdots, a_n)$ is *symmetric*, if $a_k = a_{n-k+1}, 1 \leq k \leq n$. Let $H_n = \{(a_1, a_2, \cdots, a_n) : a_k \in \{+, -\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$ be the set of all symmetric n-tuples. Note that H_n is a group under coordinate wise multiplication, and the order of H_n is 2^m, where $m = \lceil \frac{n}{2} \rceil$.

A *symmetric n-sigraph (symmetric n-marked graph)* is an ordered pair $S_n = (G, \sigma)$ $(S_n = (G, \mu))$, where $G = (V, E)$ is a graph called the *underlying graph* of S_n and $\sigma : E \to H_n$ $(\mu : V \to H_n)$ is a function. Generally, a *Smarandachely symmetric n-sigraph* (*Smarandachely*

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symmetric n-marked graph) for a subgraph $H \prec G$ is such a graph that $G - E(H)$ is *symmetric n-sigraph (symmetric n-marked graph)*. For example, let H be a path $P_2 \succ G$ or a claw $K_{1,3} \prec G$. Certainly, if $H = \emptyset$, a Smarandachely symmetric n-sigraph (or Smarandachely symmetric n-sigraph) is nothing else but a symmetric n-sigraph (or symmetric n-marked graph).

In this paper by an *n-tuple/n-sigraph/n-marked graph* we always mean a symmetric n-tuple/symmetric n-sigraph/symmetric n-marked graph.

An n-tuple $(a_1, a_2, \cdots, a_n)$ is the *identity n-tuple*, if $a_k = +$, for $1 \leq k \leq n$, otherwise it is a *non-identity n-tuple*. In an n-sigraph $S_n = (G, \sigma)$ an edge labelled with the identity n-tuple is called an *identity edge*, otherwise it is a *non-identity edge*.

Further, in an n-sigraph $S_n = (G, \sigma)$, for any $A \subseteq E(G)$ the *n-tuple* $\sigma(A)$ is the product of the n-tuples on the edges of A.

In [11], the authors defined two notions of balance in n-sigraph $S_n = (G, \sigma)$ as follows (See also R. Rangarajan and P.S.K.Reddy [7]):

Definition 1.1 *Let $S_n = (G, \sigma)$ be an n-sigraph. Then,*

(*i*) *S_n is* identity balanced *(or* i-balanced*), if product of n-tuples on each cycle of S_n is the identity n-tuple, and*

(*ii*) *S_n is* balanced*, if every cycle in S_n contains an even number of non-identity edges.*

Notice that an i-balanced n-sigraph need not be balanced and conversely. The following characterization of i-balanced n-sigraphs is obtained in [11].

Proposition 1.1 (E. Sampathkumar et al. [11]) *An n-sigraph $S_n = (G, \sigma)$ is i-balanced if, and only if, it is possible to assign n-tuples to its vertices such that the n-tuple of each edge uv is equal to the product of the n-tuples of u and v.*

Let $S_n = (G, \sigma)$ be an n-sigraph. Consider the n-marking μ on vertices of S_n defined as follows: each vertex $v \in V$, $\mu(v)$ is the n-tuple which is the product of the n-tuples on the edges incident with v. *Complement* of S_n is an n-sigraph $\overline{S_n} = (\overline{G}, \sigma^c)$, where for any edge $e = uv \in \overline{G}$, $\sigma^c(uv) = \mu(u)\mu(v)$. Clearly, $\overline{S_n}$ as defined here is an i-balanced n-sigraph due to Proposition 1 in [11].

In [11], the authors also have defined switching and cycle isomorphism of an n-sigraph $S_n = (G, \sigma)$ as follows: (See also [5, 8 - 10, 13 - 23]).

Let $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$, be two n-sigraphs. Then S_n and S'_n are said to be *isomorphic*, if there exists an isomorphism $\phi : G \to G'$ such that if uv is an edge in S_n with label $(a_1, a_2, \cdots, a_n)$ then $\phi(u)\phi(v)$ is an edge in S'_n with label $(a_1, a_2, \cdots, a_n)$.

Given an n-marking μ of an n-sigraph $S_n = (G, \sigma)$, *switching* S_n with respect to μ is the operation of changing the n-tuple of every edge uv of S_n by $\mu(u)\sigma(uv)\mu(v)$. The n-sigraph obtained in this way is denoted by $\mathcal{S}_\mu(S_n)$ and is called the *μ-switched n-sigraph* or just *switched n-sigraph*.

Further, an n-sigraph S_n *switches* to n-sigraph S'_n (or that they are *switching equivalent* to each other), written as $S_n \sim S'_n$, whenever there exists an n-marking of S_n such that $\mathcal{S}_\mu(S_n) \cong S'_n$.

Two n-sigraphs $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$ are said to be *cycle isomorphic*, if there exists an isomorphism $\phi : G \to G'$ such that the n-tuple $\sigma(C)$ of every cycle C in S_n equals to the n-tuple $\sigma(\phi(C))$ in S'_n.

We make use of the following known result (see [11]).

Proposition 1.2 (E. Sampathkumar et al. [11]) *Given a graph G, any two n-sigraphs with G as underlying graph are switching equivalent if, and only if, they are cycle isomorphic.*

§2. Gallai n-Sigraphs

The Gallai graph $\mathcal{GL}(G)$ of a graph $G = (V, E)$ is the graph whose vertex-set $V(\mathcal{GL}(G)) = E(G)$ and two distinct vertices e_1 and e_2 are adjacent in $\mathcal{GL}(G)$ if e_1 and e_2 are incident in G, but do not span a triangle in G (see [4]). In fact, this concept was introduced by Gallai [2] in his examination of comparability graphs and this notation was suggested by Sun [24]. The author Sun wasted Gallai graphs $\mathcal{GL}(G)$ to characterize a nice class of perfect graphs. Gallai graphs are also wasted in the polynomial time algorithm to determinate complete bipartite $K_{1,3}$-free perfect graphs by the authors Chvátal and Sbihi [1].

Motivated by the existing definition of complement of an n-sigraph, we extend the notion of Gallai graphs to n-sigraphs as follows:

The *Gallai n-sigraph* $\mathcal{GL}(S_n)$ of an n-sigraph $S_n = (G, \sigma)$ is an n-sigraph whose underlying graph is $\mathcal{GL}(G)$ and the n-tuple of any edge uv in $\mathcal{GL}(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical n-marking of S_n and similarly, the *Smarandachely Gallai symmetric n-sigraph* on s subgraph $H \prec G$ is the Gallai Smarandachely symmetric n-sigrpah on H. Further, an n-sigraph $S_n = (G, \sigma)$ is called Gallai n-sigraph if $S_n \cong \mathcal{GL}(S'_n)$ for some n-sigraph S'_n. The following result indicates the limitations of the notion $\mathcal{GL}(S_n)$ as introduced above, since the entire class of i-unbalanced n-sigraphs is forbidden to be Gallai n-sigraphs.

Proposition 2.1 *For any n-sigraph $S_n = (G, \sigma)$, its Gallai n-sigraph $\mathcal{GL}(S_n)$ is i-balanced.*

Proof Since the n-tuple of any edge uv in $\mathcal{GL}(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical n-marking of S_n, by Proposition 1.1, $\mathcal{GL}(S_n)$ is i-balanced. □

For any positive integer k, the k^{th} iterated Gallai n-sigraph $\mathcal{GL}(S_n)$ of S_n is defined as

$$(\mathcal{GL})^0(S_n) = S_n, \quad (\mathcal{GL})^k(S_n) = \mathcal{GL}((\mathcal{GL})^{k-1}(S_n)).$$

Corollary 2.1 *For any n-sigraph $S_n = (G, \sigma)$ and any positive integer k, $(\mathcal{GL})^k(S_n)$ is i-balanced.*

In [4], the author characterize the graphs for which $\mathcal{GL}(G) \cong G$.

Theorem 2.1 *Let $G = (V, E)$ be any graph, Gallai graph $\mathcal{GL}(G)$ is isomorphic to G if, and only if, $G \cong C_n$, where $n \geq 4$.*

In view of the above result, we now characterize the n-sigraphs for which Gallai n-sigraph

$\mathcal{GL}(S_n)$ and S_n are switching equivalent.

Theorem 2.2 *For any n-sigraph $S_n = (G, \sigma)$, the Gallai n-sigraph $\mathcal{GL}(S_n)$ and S_n are switching equivalent if, and only if, S_n is i-balanced n-sigraph and G is isomorphic to C_n, where $n \geq 4$.*

Proof Suppose $S_n \sim \mathcal{GL}(S_n)$. This implies, $G \cong \mathcal{GL}(G)$ and hence G is isomorphic to C_n, where $n \geq 4$. Now, if S_n is any n-sigraph with underlying graph as cycle C_n, where $n \geq 4$, Proposition 2.1 implies that $\mathcal{GL}(S_n)$ is i-balanced and hence if S_n is i-unbalanced and its $\mathcal{GL}(S_n)$ being i-balanced can not be switching equivalent to S_n in accordance with Proposition 1.2. Therefore, S_n must be i-balanced.

Conversely, suppose that S_n is an i-balanced n-sigraph and G is isomorphic to C_n, where $n \geq 4$. Then, since $\mathcal{GL}(S_n)$ is i-balanced as per Proposition 2.1 and since $G \cong \mathcal{GL}(G)$, the result follows from Proposition 1.2 again. □

Proposition 2.2 *For any two S_n and S'_n with the same underlying graph, their Gallai n-sigraphs are switching equivalent.*

Now, we characterize Gallai n-sigraphs. The following result characterize n-sigraphs which are Gallai n-sigraphs.

Theorem 2.3 *An n-sigraph $S_n = (G, \sigma)$ is a Gallai n-sigraph if, and only if, S_n is i-balanced n-sigraph and its underlying graph G is a Gallai graph.*

Proof Suppose that S_n is i-balanced and G is a $\mathcal{GL}(G)$. Then there exists a graph H such that $\mathcal{GL}(H) \cong G$. Since S_n is i-balanced, by Proposition 1.1, there exists an n-marking μ of G such that each edge uv in S_n satisfies $\sigma(uv) = \mu(u)\mu(v)$. Now consider the n-sigraph $S'_n = (H, \sigma')$, where for any edge e in H, $\sigma'(e)$ is the n-marking of the corresponding vertex in G. Then clearly, $\mathcal{GL}(S'_n) \cong S_n$. Hence S_n is a Gallai n-sigraph.

Conversely, suppose that $S_n = (G, \sigma)$ is a Gallai n-sigraph. Then there exists an n-sigraph $S'_n = (H, \sigma')$ such that $\mathcal{GL}(S'_n) \cong S_n$. Hence G is the $\mathcal{GL}(G)$ of H and by Proposition 2.1, S_n is i-balanced. □

§3. Anti-Gallai n-Sigraph of a n-Sigraph

The anti-Gallai graph $\mathcal{AGL}(G)$ of a graph $G = (V, E)$ is the graph whose vertex-set $V(\mathcal{AGL}(G)) = E(G)$; two distinct vertices e_1 and e_2 are adjacent in $\mathcal{AGL}(G)$ if e_1 and e_2 are incident in G and lie on a triangle in G (see [4]). Equivalently, the anti-Gallai graph $\mathcal{AGL}(G)$ is the complement of Gallai graph $\mathcal{GL}(G)$ in the line graph $L(G)$. We can easily observe that the Gallai graphs $\mathcal{GL}(G)$ and anti-Gallai graphs $\mathcal{AGL}(G)$ are the spanning subgraphs of the line graph $L(G)$ (See [4] for details).

Motivated by the existing definition of complement of an n-sigraph, we extend the notion of anti-Gallai graphs to n-sigraphs as follows:

The *anti-Gallai n-sigraph* $\mathcal{AGL}(S_n)$ of an n-sigraph $S_n = (G, \sigma)$ is an n-sigraph whose underlying graph is $\mathcal{AGL}(G)$ and the n-tuple of any edge uv is $\mathcal{AGL}(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical n-marking of S_n. Similarly, the *Smarandachely anti-Gallai n-sigraph* of a Smarandachely n-sigraph $S_n = (G, \sigma)$ on $H \prec G$ is the anti-Gallai n-sigraph of the Smarandachely n-sigraph on H. Further, an n-sigraph $S_n = (G, \sigma)$ is called anti-Gallai n-sigraph, if $S_n \cong \mathcal{AGL}(S'_n)$ for some n-sigraph S'_n. The following result indicates the limitations of the notion $\mathcal{AGL}(S_n)$ as introduced above, since the entire class of i-unbalanced n-sigraphs is forbidden to be anti-Gallai n-sigraphs.

Proposition 3.1 *For any n-sigraph $S_n = (G, \sigma)$, its anti-Gallai n-sigraph $\mathcal{AGL}(S_n)$ is i-balanced.*

Proof Since the n-tuple of any edge uv in $\mathcal{AGL}(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical n-marking of S_n, by Proposition 1.1, $\mathcal{AGL}(S_n)$ is i-balanced. □

For any positive integer k, the k^{th} iterated anti-Gallai n-sigraph $\mathcal{AGL}(S_n)$ of S_n is defined to be

$$(\mathcal{AGL})^0(S_n) = S_n, \quad (\mathcal{AGL})^k(S_n) = \mathcal{AGL}((\mathcal{AGL})^{k-1}(S_n)).$$

Corollary 3.1 *For any n-sigraph $S_n = (G, \sigma)$ and any positive integer k, $(\mathcal{AGL})^k(S_n)$ is i-balanced.*

In [4], the author characterize the graphs for which $\mathcal{AGL}(G) \cong G$.

Theorem 3.1 *Let $G = (V, E)$ be any graph, anti-Gallai graph $\mathcal{AGL}(G)$ is isomorphic to G if, and only if $G \cong K_3$.*

In view of the above result, we now characterize the n-sigraphs for which anti-Gallai n-sigraph $\mathcal{AGL}(S)$ and S are switching equivalent.

Theorem 3.2 *For any n-sigraph $S_n = (G, \sigma)$, the anti-Gallai signed graph $\mathcal{AGL}(S_n)$ and S are switching equivalent if, and only if, S_n is i-balanced and G is isomorphic to K_3.*

Proof Suppose $S_n \sim \mathcal{AGL}(S_n)$. This implies, $G \cong \mathcal{AGL}(G)$ and hence G is isomorphic to K_3. Now, if S_n is any n-sigraph with underlying graph as C_3, Proposition 2.1 implies that $\mathcal{AGL}(S_n)$ is i-balanced and hence if S_n is i-unbalanced and its $\mathcal{AGL}(S_n)$ being i-balanced can not be switching equivalent to S_n in accordance with Proposition 1.2. Therefore, S_n must be i-balanced.

Conversely, suppose that S_n is an i-balanced n-sigraph and G is isomorphic to C_3. Then, since $\mathcal{AGL}(S_n)$ is i-balanced as per Proposition 3 and since $G \cong \mathcal{AGL}(G)$, the result follows from Proposition 1.2 again. □

Proposition 3.2 *For any two S_n and S'_n with the same underlying graph, their anti-Gallai n-sigraphs are switching equivalent.*

Now, we characterize Gallai n-sigraphs. The following result characterize n-sigraphs which are Gallai n-sigraphs.

Theorem 3.3 *An n-sigraph $S_n = (G, \sigma)$ is an anti-Gallai n-sigraph if, and only if, S_n is i-balanced n-sigraph and its underlying graph G is an anti-Gallai graph.*

Proof Suppose that S_n is i-balanced and G is a $\mathcal{AGL}(G)$. Then there exists a graph H such that $\mathcal{AGL}(H) \cong G$. Since S_n is i-balanced, by Proposition 1.1, there exists an n-marking μ of G such that each edge uv in S_n satisfies $\sigma(uv) = \mu(u)\mu(v)$. Now consider the n-sigraph $S'_n = (H, \sigma')$, where for any edge e in H, $\sigma'(e)$ is the n-marking of the corresponding vertex in G. Then clearly, $\mathcal{AGL}(S'_n) \cong S_n$. Hence S_n is an anti-Gallai n-sigraph.

Conversely, suppose that $S_n = (G, \sigma)$ is an anti-Gallai n-sigraph. Then there exists an n-sigraph $S'_n = (H, \sigma')$ such that $\mathcal{AGL}(S'_n) \cong S_n$. Hence G is the $\mathcal{AGL}(G)$ of H and by Proposition 2.1, S_n is i-balanced. □

We now characterize n-sigraphs whose Gallai n-sigraphs and anti-Gallai n-sigraphs are switching equivalent. In case of graphs the following result is due to Palathingal and Aparna Lakshmanan [6].

Theorem 3.4 *For any graph $G = (V, E)$, the graphs $\mathcal{GL}(G)$ and $\mathcal{AGL}(G)$ are isomorphic if, and only if, G is $nK_3 \cup nK_{1,3}$.*

Theorem 3.5 *For any n-sigraph $S_n = (G, \sigma)$, $\mathcal{GL}(S_n) \sim \mathcal{AGL}(S_n)$ if, and only if, G is $nK_3 \cup nK_{1,3}$.*

Proof Suppose $\mathcal{GL}(S_n) \sim \mathcal{AGL}(S_n)$. This implies, $\mathcal{GL}(G) \cong \mathcal{AGL}(G)$ and hence by Theorem 3.4, we see that the graph G must be isomorphic to $nK_3 \cup nK_{1,3}$.

Conversely, suppose that G is isomorphic to $nK_3 \cup nK_{1,3}$. Then $\mathcal{GL}(G) \cong \mathcal{AGL}(G)$ by Theorem 3.4. Now, if S_n is an n-sigraph with underlying graph as $nK_3 \cup nK_{1,3}$, by Propositions 2.1 and 3.1, $\mathcal{GL}(S_n)$ and $\mathcal{GL}(S_n)$ are i-balanced. The result follows from Proposition 1.2. □

§4. Complementation

In this section, we investigate the notion of complementation of a graph whose edges have signs (a *sigraph*) in the more general context of graphs with multiple signs on their edges. We look at two kinds of complementation: complementing some or all of the signs, and reversing the order of the signs on each edge.

For any $m \in H_n$, the *m-complement* of $a = (a_1, a_2, \cdots, a_n)$ is: $a^m = am$. For any $M \subseteq H_n$, and $m \in H_n$, the *m-complement* of M is $M^m = \{a^m : a \in M\}$.

For any $m \in H_n$, the *m-complement* of an n-sigraph $S_n = (G, \sigma)$, written (S_n^m), is the same graph but with each edge label $a = (a_1, a_2, \cdots, a_n)$ replaced by a^m.

For an n-sigraph $S_n = (G, \sigma)$, the $\mathcal{GL}(S_n)$ ($\mathcal{AGL}(S_n)$) is i-balanced. We now examine, the condition under which m-complement of $\mathcal{GL}(S_n)$ is i-balanced, where for any $m \in H_n$.

Proposition 4.1 *Let $S_n = (G, \sigma)$ be an n-sigraph. Then, for any $m \in H_n$, if $\mathcal{GL}(G)$ ($\mathcal{AGL}(G)$) is bipartite then $(\mathcal{GL}(S_n))^m$ ($(\mathcal{AGL}(S_n))^m$) is i-balanced.*

Proof Since, by Proposition 2.1 (Proposition 3.1), $\mathcal{GL}(S_n)$ ($\mathcal{AGL}(S_n)$) is i-balanced, for each k,

$1 \leq k \leq n$, the number of n-tuples on any cycle C in $\mathcal{GL}(S_n)$ ($\mathcal{AGL}(S_n)$) whose k^{th} co-ordinate are $-$ is even. Also, since $\mathcal{GL}(G)$ ($\mathcal{AGL}(G)$) is bipartite, all cycles have even length; thus, for each k, $1 \leq k \leq n$, the number of n-tuples on any cycle C in $\mathcal{GL}(S_n)$ ($\mathcal{AGL}(S_n)$) whose k^{th} co-ordinate are $+$ is also even. This implies that the same thing is true in any m-complement, where for any $m, \in H_n$. Hence $(\mathcal{GL}(S_n))^t$ $((\mathcal{AGL}(S_n))^t)$ is i-balanced. $\square$

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