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Mathematical Combinatorics

– My Philosophy Promoted on Science Internationally

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Abstract: Mathematical science is the human recognition on the evolution laws of things that we can understand with the principle of logical consistency by mathematics, i.e., mathematical reality. So, *is the mathematical reality equal to the reality of thing?* The answer is not because there always exists contradiction between things in the eyes of human, which is only a local or conditional conclusion. Such a situation enables us to extend the mathematics further by combinatorics for the reality of thing from the local reality and then, to get a combinatorial reality of thing. This is the *combinatorial conjecture for mathematical science*, i.e., CC conjecture that I put forward in my postdoctoral report for *Chinese Academy of Sciences* in 2005, namely *any mathematical science can be reconstructed from or made by combinatorialization*. After discovering its relation with Smarandache multi-spaces, it is then be applied to generalize mathematics over 1-dimensional topological graphs, namely the *mathematical combinatorics* that I promoted on science internationally for more than 20 years. This paper surveys how I proposed this conjecture from combinatorial topology, how to use it for characterizing the non-uniform groups or contradictory systems and furthermore, why I introduce the continuity flow G^L as a mathematical element, i.e., vectors in Banach space over topological graphs with operations and then, how to apply it to generalize a few of important conclusions in functional analysis for providing the human recognition on the reality of things, including the subdivision of substance into elementary particles or quarks in theoretical physics with a mathematical supporting.

Key Words: Science's limitation, CC conjecture, Smarandachely denied axiom, Smarandache mutispace, non-harmonious group, non-solvable equation system, continuity flow, combinatorial notion, neutrosophic set, recognitive philosophy.

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§1. Introduction

Science is the recognition of human on the law of things in the universe under conditions by the "*six sense organs*" of human, i.e., the eyes, ears, nose, tongue, body and mind, including their extension using by the scientific instruments or facilities, and mathematics is the formal

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system of symbols in accordance with the principle of logical consistency in order to describe the evolution of thing by abstract symbols, i.e, the mathematical reality $T_{\mathcal{M}}$ holds on thing T, which is a recognitive process from non-being to being of human and greatly depends on the human recognition from the known to the unknown, including the characteristics, indicators and methods of recognition on nature of thing. For example, let the observable characteristics of thing T be $\chi_1, \chi_2, \cdots$ such as the spatial location, geometry, color, state, odor, rate and direction of change, melting point, boiling point, hardness, density, structure, acidity, alkalinity, oxidation, reducibility, thermal stability, metabolism, growth, reproduction and development, heredity and variation, etc. Then, T is recognized on its characteristics of $\{\chi_1, \chi_2, \cdots\}$ by humans. Notice that the recognition on characteristics of $\chi_1, \chi_2, \cdots$ of thing T is a gradual process in general. That is why it needs to constantly improve, modify or extend our theory on thing T so as to approach to the reality of thing T infinitely. For example, the six blind men touched the elephant's teeth, trunk, ears, stomach, legs and the tail in fable of the blind men with an elephant and then, an elephant was characterized respectively by the blind men as a big radish, pipe, a leaf fan, a wall, a pillar or a rope, such as those shown in Figure 1.

(a) (b)

Figure 1 An elephant's shape recognized by blind men

In this case, *why did the blind men argue for the shape of an elephant?* The answer is because each of them touched different parts of the elephant's body, which results in the recognitions on the elephant different. Similarly, the human recognition on a thing T by its characteristics of $\chi_1, \chi_2, \cdots$ is similar to that of the case of a blind man. It is also in local recognition on T one by one characterizes of $\chi_1, \chi_2, \cdots$. However, *can such a recognition really equal to the reality of thing T and realize $T_{\mathcal{M}} = T$?* The answer is not because it is the human in recognizing things T and it mainly depends on the sense and reason of human, which has been asserted $T_{\mathcal{M}} \neq T$ in the discussion of sages. For example, "*Tao told is not the eternal Tao; Name named is not the eternal Name*" in Chapter 1 of Lao Zi's *Tao Te Ching*, "*Color is not different from the Empty, Empty is not different from the Color and the Color is the Empty, the Empty is the Color*" in *Heart Sutra* and also Kant's *Critique of Pure Reason* or "*what can I know?* " and so on. All of their discussions show that the human recognition is relative or conditional reality $T_{\mathcal{M}}$, not equivalent to the reality of thing T but only a gradual process, i.e., $T_{\mathcal{M}} \to T$. Furthermore, *can the mathematical reality of things be realized $T_{\mathcal{M}} \to T$ by human under the principle of logical consistency?* The answer is also not because the mathematical system follows the logical consistency but a contradiction exists everywhere in human recognition. It is impossible to completely describe the evolution of thing T with a logical consistency system of symbols. In this case, it is necessary to recognize that such a contradiction is caused by human's describing on the evolution, not the truth colour of thing T with mathematics. Thus, there are 3 questions need further to discuss at least in recognition of thing T by characteristics of $\chi_1, \chi_2, \cdots$ and then, we realize the thing T, including the blind men with the radish, pipe, fan, wall, pillar, rope and others for characterizing the shape of an elephant, respectively.

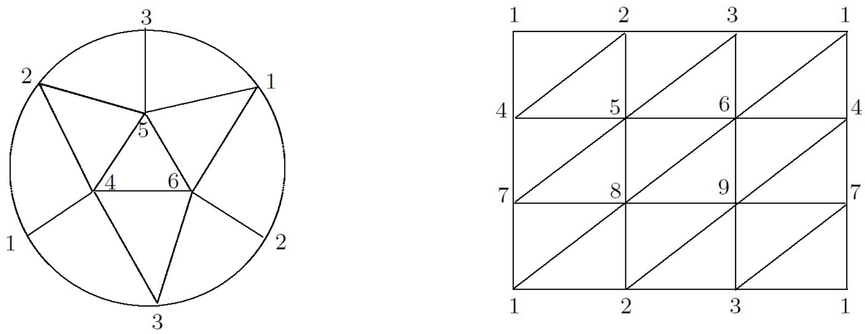
(Q1) *For an integer $i \geq 1$, is it complete for understanding thing T only by the characteristic χ_i?* The answer is certainly not because χ_i is only one characteristic of thing T, not the whole. In the fable of blind men with an elephant, although the sophist told the blind men that "*you are all right about the elephant*", he also said that "*the reason why you think the elephant's shape different is because each of you touches the different part of the elephant's body. In fact, an elephant has those all characteristics that you are talking about*", namely the sophist pointed out that each recognition of them is local also. Similarly, knowing thing T in terms of characteristic χ_i is necessarily incomplete but it is the normal case of human recognition. And so, all human activities led by the incomplete scientific recognitions are bound to be constrained by their application field, scope and achieving conditions.

(Q2) *How to recognize the characteristics of $\chi_1, \chi_2, \cdots$ of thing T?* In fact, there are many methods for the recognition on characteristics of $\chi_1, \chi_2, \cdots$ of thing T in science, including mathematical, physical, chemical and biological methods such as a radish can be eaten, a pillar can support others and a rope is soft but can tie others, etc. But as long as its characteristics are quantitatively described by data χ_i, $i \geq 1$, it must be assumed that the characteristic χ_i follows a mathematical system S, namely the characteristic χ_i is described in accordance with the principle of logical consistency in mathematics. Now, the question is *whether it is correct in assuming that the characteristic χ_i follows the rules of mathematical system S, and whether the change of characteristic χ_i of thing T can be fully described?*

(Q3) *Are any combination of characteristics of $\chi_1, \chi_2, \cdots$ necessarily the thing T?* For example, in fable of the blind men with an elephant, *is any combination of* 2 *big radishes,* 1 *pipe,* 2 *leaf fans,* 1 *wall,* 4 *pillars and* 1 *rope be the shape of an elephant recognized by the blind men?* The answer is not because these six known objects can be combined to create a variety of geometrical objects, they do not necessarily be the shape of an elephant. In other words, the shape of an elephant made from 2 big radishes, 1 pipe, 2 leaf fans, 1 wall, 4 pillars and 1 rope is combined on a 1-dimensional topology or topological graph G^L, and this 1-dimensional topology G^L is accompanied by human recognition of things T, which is inevitable.

Different understandings on the previous 3 questions will inevitably lead to different developing ways of science. Most researchers are at the first level, namely acknowledging tacitly that a local characteristic of thing T is equal to thing T and so, the thing T is subdivided into microscopic particles, including cells and genes in biology to reduce the *effect* of thing T to *cause* of the behavior of microscopic particles, which is to recognize the whole in a partial one. Unlike the ordinary scholars, Prof.Smarandache introduced the neutrosophic set for describing thing T with characteristics of $\chi_1, \chi_2, \cdots$, lead a lot of mathematicians researching it deeply and obtained many academic achievements. So, *what is a neutrosophic set?* A neutrosophic set is such a set that associates each element $x \in \chi_i$ of a recognitive set with a ternary array $(\mathcal{T}, \mathcal{I}, \mathcal{F})$, where $\mathcal{T}, \mathcal{I}, \mathcal{F} \subseteq [0,1]$, are respectively the confident set $\mathcal{T}$, indefinite set $\mathcal{I}$ and fail set $\mathcal{F}$, see [36] for details. However, I believe personally that the human recognition of thing T should follow the rule of extending the known to the unknown, from the local to the whole because humans are bound to be unable to give definite recognition for an uncertain or unrecognizable thing that appear in recognition. Thus, it is undoubtedly a useable or feasible way that extends mathematics over the topological structure G^L inherited in human recognition by reduction on thing T and then, apply it to the recognition of unknown things.

I was working on compact 2-dimensional manifolds without boundary, namely the partition of a closed surface into regular polygons and counted the non-isomorphic ways of partition during my doctoral and postdoctoral periods. For this work, there is a classical conclusion in algebraic topology, namely ([33]) *there exists a finite triangulation* $\{T_1, T_2, \cdots, T_n\}$ *on a closed surface* S, where for any integer $1 \leq i \leq n$, T_i is homeomorphic to a triangle $\triangle$, i.e., an open disk $\{(x,y)|x^2+y^2<1\}$ on Euclideian plane $\mathbb{R}^2$, called a 2-*cell.* That is, a closed surface can be obtained by adhering triangles. For example, the partitions (a) and (b) in Figure 2 are triangulation of the projective plane and the torus, respectively.



(a) Projective plane **(b) Torus**

Figure 2 Triangulation of surface

Notice that the 1-dimensional skeleton in a 2-cell partition of closed surface corresponds to a topological graph G^L. Conversely, a 2-cell embedded of graph G^L on surface S is nothing else but a 2-cell partition of surface S ([7]), called also a *combinatorial map* ([10],[11]), which is adhering a closed surface with regular polygons. Similarly, the assembling objects of space by tetrahedrons, hexahedrons, octahedrons, dodecahedrons and icosahedrons, and the algebraic systems such as those of rings, fields consisted of commutative groups is also such a combinatorial one. Working on combinatorial topology years motivated me realized suddenly that the essence of this way is a combinatorial notion which can be applied for generalizing mathematical science in general. After thinking for a long time, I proposed the notion that of applying combinatorics for generalizing mathematical sciences, namely the words of "*a good idea is pullback measures on combinatorial objects again, ignored by the classical combinatorics and reconstructed or make combinatorial generalization for the classical mathematics such as the algebra, differential geometry, Riemann geometry,* $\cdots$ *and the mechanics, theoretical physics,* $\cdots$" in Introduction of Chapter 5 of my post-doctoral report "*On the Automorphisms of Maps*

& Klein Surface" ([11] 1st edition) for *Chinese Academy of Sciences* in 2005. And then, I formally proposed the *combinatorial conjecture for mathematical science* in my report "*Combinatorial speculations and combinatorial conjecture for mathematics*" at the *2nd Conference on Combinatorics and Graph Theory of China* (August 16-19, 2006, Tianjin), namely

Combinatorial Conjecture for Mathematics([14]) *Any mathematical science can be reconstructed from or made by combinatorialization.*

The combinatorial conjecture for mathematics, abbreviated to CC conjecture is not so much as a mathematical conjecture but a generalization of mathematical science for extending the local recognition of human on thing by a combinatorial approach, which implies that one can select a limited number of combinatorial rules and axioms to reconstruct or generalize mathematics so that classical mathematics is its special or partial. And meanwhile, different branches of mathematics can be combined into a union one and then, applied to generalize other mathematics and sciences, which is the *mathematical combination*. Even so, *how to generalize mathematical science by mathematical combinations?* For this objective, an effective way is to establish the *Smarandache multi-space* or *continuity flow theory* ([31]) by vectors in a Banach space over 1-dimensional topological structures G^L for extending mathematics, including the contradiction avoided in mathematics for the recognition of reality of thing T. This is nothing else but the recognitive way explained by the sophist to the blind men in fable of the blind men with an elephant. In this way, the human recognition of reality of thing T should be a combined one or combinatorial reality. Essentially, the complex network obtained by *reduction* in the human recognition of thing T happens to be such a 1-dimensional topological structure G^L but we are short of a mathematical theory that regards it as an element, which is also the reason in the previous assertion that mathematical reality can not induce $T_{\mathcal{M}} \to T$.

The main purpose of this paper is to summarize the contribution of CC conjecture to the generalization of classical algebraic systems, topology and geometry, analyze its relationship with Smarandachely denied axiom, multi-spaces and the philosophy of mathematical combinational G^L for recognizing the combinatorial reality of thing, show its contribution to scientific recognition. All terminologies and notations not defined in this paper are standard such as those of the algebra, topology, complex systems, functional analysis and topological graph are respectively referred to [4]-[7], and terminologies in Smarandache geometry and multi-space are referred to [11]-[12],[15] and [36]-[38].

§2. Smarandachely Denied Axiom

Generally, it is believed that the application of mathematics to describe the reality of thing T depends on the closed algebraic operation system such as groups, rings and fields for describing the evolving rule of thing T, regular geometrical bodies approaching the appearance of thing T and the combinatorial relations between elements of thing T in mathematics. In the early of 2005, I completed my post-doctoral report "*On the Automorphisms of Maps & Klein Surface*" ([11] 1st edition). Also in the year earlier, I received an email from Dr.Preze Mihn, the editor of American Research Press. He told me that they would fund me to publish a book in USA if it contains Smarandache geometry. I personally appreciate the notion that recognizes things by combinatorics proposed in my post-doctoral report and thought I should let more ones know this notion for developing science. So, I reorganized my post-doctoral report and emailed it to this publishing company in USA, told them that a lots of classical mathematical problems such as Riemannian surfaces, Riemannian geometry and algebraic curves were discussed by combinatorics in my book. Dr.Preze Mihn emailed me a few of documents after reading it and told me that Smarandache geometry is more extensive than Riemannian geometry. He suggested me to increase the content of Smarandache geometry in my book, which motivated me to turn my research on Smarandachely denied axiom and Smarandache geometry.

2.1.Smarandache Geometry So, *what is Smarandachely denied axiom, what is Smarandache geometry, and what things that can be described by them?* Surprisingly, a Smarandache geometry no longer complies with the principle of logical consistency but includes contradictions.

Definition 2.1([37],[38]) *An axiom is said Smarandachely denied if the axiom behaves in at least two different ways within the same space, i.e., validated and invalided, or only invalided but in multiple distinct ways. A Smarandache geometry is a geometry which has at least one Smarandachely denied axiom.*

What really interests me on Smarandache geometry is that it is different from the classical one, namely it includes contradictions, and I always believe that this is the most important question in human recognition of thing. For example, the Euclidean geometry is a geometry without contradiction, based on five axioms but the fifth axiom, i.e., "*given a line and a point exterior this line, there is one line parallel to this line*" is always felt to be less obvious than the other axioms. And then, the Lobachevshy-Bolyai-Gauss geometry replaces it by "*there are infinitely many line parallels to a given line passing through an exterior point*" and the Riemannian geometry replaces it by "*there is no parallel to a given line passing through an exterior point*". All of them are complied with the principle of logical consistency, but a Smarandache geometry can be partly Euclidean geometry or Riemannian geometry and partly Lobachevshy-Bolyai-Gauss geometry ([9]), which is probably the natural state of thing because it is always evolving non-harmoniously.

Usually, the research object in Riemannian geometry is the n-dimensional Riemannian manifolds for integers $n \geq 2$, which is endowed on n-dimensional topological manifold with smooth nature, establish further its vector field, tensors and connections with geometrical behaviors. So, *what is an n-dimensional topological manifold?* By definition, an n-dimensional topological manifold is a Hausdoff space M holds with the separation axiom, namely for two distinct points $p_1, p_2 \in M$ there are neighborhoods $U(p_1), U(p_2) \in M$ of p_1, p_2 such that $U(p_1) \bigcap U(p_2) = \emptyset$ and for any point $p \in M$ there exists a neighborhood $U(p)$ homeomorphic to n-dimensional Euclidean space $\mathbb{R}^n$, called also the *locally Euclidean space M*. After my post-doctoral report ([11] 1st edition) published in USA, I further studied Iseri's book [8] on Smarandache manifolds and found an easier way for constructing 2-dimensional Smarandache manifolds than that of [8], i.e., by the 2-cell embeddings M of graphs on surfaces, namely endowed with a real number $\mu(u)$, $\mu(u)\rho_M(u)$ (mod2π) on any vertex $u \in V(M)$, $\rho_M(u) \geq 3$ of 2-cell embedding M to get a 2-dimensional Smarandache manifold (M, μ), is said to be *map* *geometry* and points of (M,μ) are classified into *elliptic*, *Euclidean* or *hyperbolic* if $\rho(u)\mu(u) < 2\pi$, $\rho(u)\mu(u) = 2\pi$ or $\rho(u)\mu(u) > 2\pi$, where $\rho_M(u)$ is the valency of vertex u in M and $\mu : u \in V(M) \to (0,\pi)$ is said to be an *angle factor*, see [12] for details.

Notice that the generalization of this way can be applied to Euclidean space $\mathbb{R}^n$ for constructing n-dimensional pseudo-Euclidean space and then, to get n-dimensional Smarandache manifolds ([15]). Generally, let $\mathbb{R}^n$ be an n-dimensional Euclidean space with normal basis $\boldsymbol{\epsilon}_1 = (1,0,\cdots,0)$, $\boldsymbol{\epsilon}_2 = (0,1,\cdots,0)$, $\cdots$, $\boldsymbol{\epsilon}_n = (0,0,\cdots,1)$. Then, an *$n$-dimensional pseudo-Euclidean space* is defined to be a 2-tuple $(\mathbf{R^n}, \omega|_{\overrightarrow{O}})$, where $\omega|_{\overrightarrow{O}} : \mathbf{R}^n \to \mathscr{O}$ is such a continuous function that a straight line of orientation $\overrightarrow{O}$ passing through point $\mathbf{u} \in \mathbb{R}^n$ will turn its orientation to $\overrightarrow{O} + \omega|_{\overrightarrow{O}}(\mathbf{u})$. Certainly, an n-dimensional pseudo-Euclidean space $(\mathbb{R}^n, \omega|_{\overrightarrow{O}}) = \mathbb{R}^n$ if and only if $\omega|_{\overrightarrow{O}}(\mathbf{u}) = \mathbf{0}$ for any point $\mathbf{u} \in \mathbb{R}^n$, i.e., a *flat space*. And then, an n-dimensional Smarandache manifold is defined to be a local n-dimensional pseudo-Euclidean space $(M^n, \mathcal{A}^\omega)$, namely for any point $p \in M^n$ there exists a neighborhood U_p homeomorphic to n-dimensional pseudo-Euclidean space $(\mathbf{R}^n, \omega|_{\overrightarrow{O}})$, where (U_p, φ_p^ω) is a chart at point p with a homeomorphism $\varphi_p^\omega : U_p \to (\mathbf{R}^n, \omega|_{\overrightarrow{O}})$ and $\mathcal{A} = \{(U_p, \varphi_p^\omega) | p \in M^n\}$ is an atlas on manifold M^n.

Generally, the six sense organs of human can not feel the distortion of space but thinks that the space is flat priorly. This is why the Euclidean spaces $\mathbb{R}^n$ of $n \geq 3$ are often used to be the reference of thing, but it is not necessarily the nature of thing in universe. For example, one of Einstein's contributions to the gravitational field is to show that the substance field of universe is not flat but a curved one under gravitation, not even with the light, namely the nature of substance field of universe is a 3-dimensional Smarandache manifold rather than a Euclidean space $\mathbb{R}^3$, which should be the proper contribution of rational thinking of human.

2.2.Smarandache Multi-Space. Unlike the classical geometry, an axiom of Smarandache geometry behaves simultaneously validated and invalided, or only invalided but in multiple ways, which is not easy to find by the geometrical intuition of human but it is easy to construct Smarandache systems on algebra such as the combination of two non-isomorphic groups or rings defined on a set and then, the resulting system must be Smarandachely denied, which is essentially the application of CC conjecture to that of algebraic systems. I emailed this thing to Dr.Preze Mihn and told him that I was going to generalize algebra systems along this way. He wrote back that this way had been proposed by Smarandache a few years ago, i.e., the *Smarandache multi-space* and encouraged me to follow this thinking on mathematics.

Definition 2.2([12],[36]) *For an integer $n \geq 1$, let $(\mathcal{S}_1, \mathcal{O}_1), (\mathcal{S}_2, \mathcal{O}_2), \cdots, (\mathcal{S}_n, \mathcal{O}_n)$ be n distinct mathematical systems or spaces, namely for any integer $1 \leq i \neq j \leq n$, $\mathcal{O}_i \subset \mathcal{S}_i \times \mathcal{S}_i$ and $\mathcal{S}_i \neq \mathcal{S}_j$ or $\mathcal{S}_i = \mathcal{S}_j$ but $\mathcal{O}_i \neq \mathcal{O}_j$, $1 \leq i, j \leq n$. Then, a Smarandache multi-space is defined to be*

$$(\mathscr{S};\mathscr{O}) = \bigcup_{i=1}^{n} (\mathcal{S}_i, \mathcal{O}_i), \tag{2.1}$$

i.e., $\mathscr{S} = \bigcup\limits_{i=1}^{n} \mathcal{S}_i$ and $\mathscr{O} = \bigcup\limits_{i=1}^{n} \mathcal{O}_i$, where $\mathcal{S}_i$ is a set and $\mathcal{O}_i$ is operations on $\mathcal{S}_i$ for integers $1 \leq i \leq n$.

Now, *how to understand each characteristic of $\chi_1, \chi_2, \cdots$ in the human recognition of* *thing T?* Usually, these characteristics of $\chi_1, \chi_2, \cdots$ in human recognition of thing T are not only numerical values or data but a family of sets $\mathcal{S}_1, \mathcal{S}_2, \cdots$ with respective characteristics of $\chi_1, \chi_2, \cdots$, where $\chi(T)$ is a kind of nature of thing T, i.e.,

$$T_{\mathcal{M}} = \bigcup_{i=1}^{\infty} (\mathcal{S}_i; \mathcal{O}_i), \tag{2.2}$$

where $\mathcal{O}_i$ is the evolving rule of elements in $\mathcal{S}_i$ and $T_{\mathcal{M}}$ is nothing else but a Smarandache multi-space (2.1). For example, the sophist told the blind men that the shape of elephant is

$$\begin{aligned} \text{An Elephant} = \{\ 2 \text{ Big Radish}\} \bigcup \{1 \text{ Pipe}\} \bigcup \{2 \text{ Leaf Fans}\} \\ \bigcup \{1 \text{ Wall}\} \bigcup \{4 \text{ Pillars}\} \bigcup \{1 \text{ Rope}\}. \end{aligned}$$

in fable of the blind men with an elephant, which is a Smarandache multi-space. Notice that different characteristics χ_i corresponds to different evolving systems $\mathcal{S}_i$ if $n \geq 2$, namely a Smarandache multi-space $(\mathscr{S}; \mathscr{O})$ holds with Smarandachely denied axiom. Conversely, all elements in a Smarandachely denied system $\mathscr{S}$ can be classified into systems by each of Smarandachely denied axiom $\mathcal{A}$ validated or invalided, or each invalided case to get a mathematical system or space $(\mathcal{S}_i; \mathcal{A})$, namely a Smarandachely denied system $\mathscr{S}$ is nothing else but a Smarandache multi-space $(\mathscr{S}; \mathscr{O})$. Whence, a Smarandachely denied system is equivalent to a Smarandache multi-space. Then, *how to determine evolving rules in set $\mathcal{S}_i$ for an integer $i \geq 1$?* Usually, we assume that all elements with characteristic χ_i comply with the mathematical operations in $\mathcal{O}_i$ and then, describe the evolution of things in $\mathcal{S}_i$ by mathematics.

Thus, if we do not consider the combinatorial structure G^L inherited in recognitive sets $\mathcal{S}_1, \mathcal{S}_2, \cdots$, we can apply CC conjecture to generalize algebraic multi-systems, including group, ring and field, geometrical multi-spaces, compact n-dimensional manifolds in topology, etc., which already appears for a few of simple cases in classical mathematics. For example, the continuous groups, Lie groups are both Smarandache multi-space of $n = 2$ in (2.1). By the Smarandache multi-space, Kandasamy, Smarandache and others extensively generalized algebraic systems such as those groups, rings and algebraic properties ([39]-[41]). For example, let $(G_1; \circ), (G_2; \bullet)$ be 2 different groups. Then, $(G_1 \bigcup G_2; \{\circ, \bullet\})$ is said to be a *bigroup*. Particularly, if $(G_1 \bigcup G_2; \circ)$ is an Abelian group with unit 0, $(G_1 \bigcup G_2 \setminus \{0\}; \bullet)$ is a group and for any elements $x, y, z \in G_1 \bigcup G_2$, there is

$$x \bullet (y \circ z) = (x \bullet y) \circ (x \bullet z), \quad (y \circ z) \bullet x = (y \bullet x) \circ (z \bullet x), \tag{2.3}$$

then the bigroup $(G_1 \bigcup G_2; \{\circ, \bullet\})$ is a skew field. Furthermore, if $(G_1 \bigcup G_2 \setminus \{0\}; \bullet)$ is also an Abelian group, then the bigroup $(G_1 \bigcup G_2; \{\circ, \bullet\})$ is nothing else but a field. Generally, for an integer $n \geq 1$, let $(\mathcal{S}_1, \mathcal{O}_1), (\mathcal{S}_2, \mathcal{O}_2), \cdots, (\mathcal{S}_n, \mathcal{O}_n)$ be n groups, rings or modules. Then, a Smarandache multi-space defined by (2.1) on $(\mathcal{S}_1, \mathcal{O}_1), (\mathcal{S}_2, \mathcal{O}_2), \cdots, (\mathcal{S}_n, \mathcal{O}_n)$ is said to be respectively the *n-group, n-ring* or *n-module*, and we can determine their multi-subgroups, multi-subrings, multi-subideals with a homomorphic theorem on associative systems following, and the decomposition structure of n-module, see [15] for details.

Theorem 2.3([15]) *Let ω be an onto homomorphism from an associative multi-system $(\mathscr{H}_1; \widetilde{O}_1)$*

to $(\mathscr{H}_2;\widetilde{O}_2)$ *and let* $(\mathcal{I}(\widetilde{O}_2);\widetilde{O}_2)$ *be a multi-system with unit* $1_{\circ^-}$ *for* $\forall\circ^-\in\widetilde{O}_2$ *and inverse* x^{-1} *for* $\forall x\in\mathcal{I}(\widetilde{O}_2)$ *in* $((\mathcal{I}(\widetilde{O}_2);\circ^-)$. *Then there are representation pairs* $(R_1,\widetilde{P}_1)$ *and* $(R_2,\widetilde{P}_2)$ *with* $\widetilde{P}_1\subset\widetilde{O},\widetilde{P}_2\subset\widetilde{O}_2$ *such that*

$$\left.\frac{(\mathscr{H}_1;\widetilde{O}_1)}{(\widetilde{\mathrm{Ker}\omega};\widetilde{O}_1)}\right|_{(R_1,\widetilde{P}_1)}\cong\left.\frac{(\mathscr{H}_2;\widetilde{O}_2)}{(\mathcal{I}(\widetilde{O}_2);\widetilde{O}_2)}\right|_{(R_2,\widetilde{P}_2)}\tag{2.4}$$

if each element of $\widetilde{\mathrm{Ker}\omega}$ *has an inverse in* $(\mathscr{H}_1;\circ)$ *for* $\circ\in\widetilde{O}_1$, *where* $\mathcal{I}(\widetilde{O}_2)$ *denotes the set consisting of all units* $1_\circ,\circ\in\widetilde{O}_2$ *in multi-system* $(\mathscr{H}_2;\widetilde{O}_2)$, $\widetilde{\mathrm{Ker}\omega}=\{a|\omega(a)=1_\circ,a\in\mathscr{H}_1,\circ\in\widetilde{O}_1\}$, *a multi-system* $(\mathscr{H};\widetilde{O})$ *is associative if for* $\forall a,b,c\in\mathscr{H}$, $\forall\circ_1,\circ_2\in\widetilde{O}$, $(a\circ_1 b)\circ_2 c=a\circ_1(b\circ_2 c)$ *and* $(R_1,\widetilde{P}_1)$, $(R_2,\widetilde{P}_2)$ *denotes the pairs of* $\mathscr{H}_1=\bigcup\limits_{a\in R_1,\circ\in\widetilde{P}_1}a\circ\widetilde{\mathrm{Ker}\omega}$, $\mathscr{H}_2=\bigcup\limits_{a\in R_2,\circ\in\widetilde{P}_2}a\circ\widetilde{\mathrm{Ker}\omega}$.

Particularly, let $(\mathscr{H}_1;\widetilde{O}_1)$ and $(\mathscr{H}_2;\widetilde{O}_2)$ be n-groups. Then,

Corollary 2.4([12]) *If homomorphism* $\omega:(\mathscr{H}_1;\widetilde{O}_1)\to(\mathscr{H}_2;\widetilde{O}_2)$ *is onto then* $\mathscr{H}_1/\mathrm{Ker}\omega\simeq\mathrm{Im}\omega$.

Now, if $(\mathcal{S}_1,\rho_1),(\mathcal{S}_2,\rho_2),\cdots,(\mathcal{S}_n,\rho_n)$ are n metric spaces, then a Smarandache multi-space determined by (2.1) is said to be a *n-metric space* and we can introduce also the Cauchy sequence, complete space and the contraction mapping on such a space to generalize Banach fixed-point theorem.

Theorem 2.5([12]) *Let* $\widetilde{M}=\bigcup\limits_{i=1}^{m}M_i$ *be a completed multi-metric space. For an* ϵ*-disk sequence* $\{B(\epsilon_n,x_n)\}$ *with* $\epsilon_n>0$ *for* $n=1,2,3,\cdots$, *if* $B(\epsilon_1,x_1)\supset B(\epsilon_2,x_2)\supset\cdots\supset B(\epsilon_n,x_n)\supset\cdots$ *and* $\lim\limits_{n\to+\infty}\epsilon_n=0$, *then* $\bigcap\limits_{n=1}^{+\infty}B(\epsilon_n,x_n)$ *has only one point.*

Theorem 2.6([12]) *If* $\widetilde{M}=\bigcup\limits_{i=1}^{m}M_i$ *is a completed multi-metric space and* T *a contraction on* $\widetilde{M}$ *then* $1\leq|T_{\mathrm{fix}}|\leq m$, *where* $|T_{\mathrm{fix}}|$ *is the cardinality of fixed point set of* T.

Notice that a Smarandache multi-space defined by (2.1) with $\mathscr{S}=\bigcup\limits_{i=1}^{n}\mathcal{S}_i$ and $\mathscr{O}=\bigcup\limits_{i=1}^{n}\mathcal{O}_i$ is the union of elements in $\mathcal{S}_i$ with operations in $\mathcal{O}_i,1\leq i\leq n$. However, any thing does not exist in isolation. We can determine the combinatorial structure G^L inherited in systems or spaces $(\mathcal{S}_1,\mathcal{R}_1),1\leq i\leq n$ by CC conjecture further.

Definition 2.7 *For an integer* $n\geq1$, *let* $(\mathscr{S};\mathscr{O})=(\bigcup\limits_{i=1}^{n}\mathcal{S}_i;\bigcup\limits_{i=1}^{n}\mathcal{O}_i)$ *be a Smarandache multi-space with an inherited combinatorial structure or vertex-edge labeled graph* $G^L[\mathscr{S},\mathscr{O}]$ *determined by*

$$\begin{aligned}V(G^L[\mathscr{S},\mathscr{O}])&=\{\mathcal{S}_1,\mathcal{S}_2,\cdots,\mathcal{S}_n\},\\E(G^L[\mathscr{S},\mathscr{O}])&=\{(\mathcal{S}_i,\mathcal{S}_j)|\mathcal{S}_i\bigcap\mathcal{S}_j\neq\emptyset,\ 1\leq i\neq j\leq n\}\end{aligned}$$

and label mapping

$$L:\ \mathcal{S}_i\to L(\mathcal{S}_i)=\mathcal{S}_i,\quad(\mathcal{S}_i,\mathcal{S}_j)\to L(\mathcal{S}_i,\mathcal{S}_j)=\mathcal{S}_i\bigcap\mathcal{S}_j,\quad 1\leq i\neq j\leq n.$$

Thus, there is a bijection

$$(\mathscr{S};\mathscr{O})\ \stackrel{1-1}{\longleftrightarrow}\ G^L[\mathscr{S},\mathscr{O}]\tag{2.5}$$

between Smarandache multi-space $(\mathscr{S};\mathscr{O})$ and the labeled graph $G^L[\mathscr{S},\mathscr{O}]$. For example, let $\mathscr{G}_1=\langle\alpha,\beta\rangle$, $\mathscr{G}_2=\langle\alpha,\gamma,\theta\rangle$, $\mathscr{G}_3=\langle\beta,\gamma\rangle$, $\mathscr{G}_4=\langle\beta,\delta,\theta\rangle$ be freely Abelian groups generated by elements $\alpha,\beta,\gamma,\delta$ and θ with $\alpha\neq\beta\neq\gamma\neq\delta\neq\theta$. Calculation shows that $\mathscr{G}_1\bigcap\mathscr{G}_2=\langle\alpha\rangle$, $\mathscr{G}_2\bigcap\mathscr{G}_3=\langle\gamma\rangle$, $\mathscr{G}_3\bigcap\mathscr{G}_4=\langle\delta\rangle$, $\mathscr{G}_1\bigcap\mathscr{G}_4=\langle\beta\rangle$ and $\mathscr{G}_2\bigcap\mathscr{G}_4=\langle\theta\rangle$. So, the vertex-edge labeled graph $G^L[\mathscr{S},\mathscr{O}]$ determined by Smarandache multi-group $(\mathscr{S};\mathscr{O})$ is shown in Figure 3.

2.3.Combinatorial Manifold. A generalization of manifolds in geometry by combinatorics is the combinatorial manifolds. By definition, a *combinatorial manifold* $\widetilde{M}$ is the combination of finite manifolds $M_1,M_2,\cdots,M_m$ over a topological graph G^L, namely the space $(\mathcal{S}_i,\mathcal{O}_i)$ is a manifold M_i for any integer $1\leq i\leq m$ in Definition 2.7 ([14],[15]). Such a Smarandache multi-space $(\mathscr{S};\mathscr{O})$ is denoted by $\widetilde{M}(n_1,n_2,\cdots,n_m)$. It should be noted that a topological graph G^L is inherited in the combinatorial manifold $\widetilde{M}$, namely

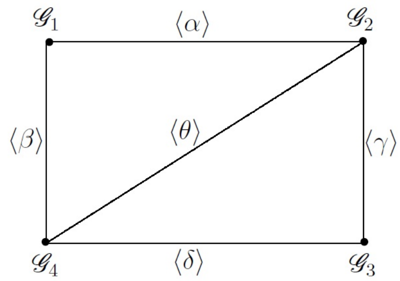


Figure 3. A labeled graph

$$\begin{aligned} V\left(G^L\right) &= \{M_1,M_2,\cdots,M_m\}, \\ E\left(G^L\right) &= \left\{(M_i,M_j)|M_i\bigcap M_j\neq\emptyset, 1\leq i,j\leq m\right\} \end{aligned}$$

and

$$L:\ M_i\rightarrow L(\mathcal{S}_i)=M_i,\quad (M_i,M_j)\rightarrow L(M_i,M_j)=M_i\bigcap M_j,\quad 1\leq i\neq j\leq m,$$

such as those shown in Figure 4, where M^3 is a 3-dimensional manifold, B^1 and T^2 are respectively a bouquet and a torus.

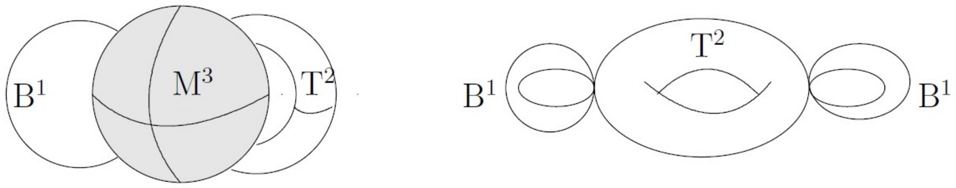


Figure 4. Examples of combinatorial manifolds

Locally, for any integer sequence $0<n_1<n_2<\cdots<n_m$, a combinatorial manifold can be geometrically defined also to be a Hausdoff space $\widetilde{M}$ holds with the separation axiom and there always is a neighborhood U_p with a homeomorphism $\varphi_p:U_p\rightarrow\widetilde{\mathbb{R}}(n_1(p),n_2(p),\cdots,n_{s(p)})$ for point $p\in\widetilde{M}$, where $\widetilde{\mathbb{R}}(n_1(p),n_2(p),\cdots,n_{s(p)})$ is a combinatorial Euclidean space by $s(p)$ Euclidean spaces $\mathbb{R}^{n_1},\mathbb{R}^{n_2},\cdots,\mathbb{R}^{n_{s(p)}}$, is said to be a *combinatorial Euclidean fan-space*, i.e., for integers $1\leq i\neq j\leq s(p)$, $\mathbb{R}^{n_i}\bigcap\mathbb{R}^{n_j}=\bigcap\limits_{k=1}^{s(p)}\mathbb{R}^{n_k}$ and

$$\bigcup_{p\in\widetilde{M}}\left\{n_1(p),n_2(p),\cdots,n_{s(p)}(p)\right\}=\{n_1,n_2,\cdots,n_m\}. \tag{2.6}$$

Now, *is a combinatorial manifold a topological manifold or Smarandache manifold?* By the definition in topology, the intersection $M_i \bigcap M_j$ of two n-dimensional manifolds M_i and M_j is also an n-dimensional manifold, which is consistent with the visual perception of human. Whence, if the intersection of manifolds $M_1, M_2, \cdots, M_m$ in same dimension complies with the intersection rule of topology in a combinatorial manifold $\widetilde{M}$, $\widetilde{M}$ is a manifold also. Otherwise, if the dimensions $\dim(M_1) = n_1, \dim(M_2) = n_2, \cdots, \dim(M_n) = n_m$ of manifolds $M_1, M_2, \cdots, M_m$ in combinatorial manifold $\widetilde{M}$ are not all the same, or not complies with the intersection rule of topology, i.e., for integers $1 \leq i \neq j \leq m$, the intersection $M_i \bigcap M_j$ satisfies

$$\dim(M_i \bigcap M_j) < \min\{\dim(M_i), \dim(M_j)\} \tag{2.7}$$

then, the combinatorial manifold $\widetilde{M}$ is no longer a topological manifold but a Smarandache manifold. In this way, the combinatorial manifold includes both the case of topological manifold and Smarandache manifold.

A typical nature of combinatorial manifold is that it simultaneously displays both the nature of manifold and topological graph, and so it can be characterized by the natures of manifolds and topological graphs. For example, let the combinatorial manifolds $\widetilde{M}_1$ and $\widetilde{M}_2$ be respectively consisted of manifolds $M_i^1, 1 \leq i \leq m$ and $M_k^2, 1 \leq k \leq s$. If there is such an isomorphism $\varphi : V(G^L[\widetilde{M}_2]) \to V(G^L[\widetilde{M}_2])$ between labeled graphs $G^L[\widetilde{M}_1]$ and $G^L[\widetilde{M}_2]$ that for any integers $1 \leq i, j \leq m$, if $\varphi : M_i^1 \to M_j^2$ then there is a homeomorphism $h_{M_i^1} : \varphi\left(M_i^1\right) \to M_j^2$ such that $\varphi(M_i^1)$ homeomorphic to M_j^2, then $h \circ \varphi$ is a homeomorphism between combinatorial manifold $\widetilde{M}_1$ and $\widetilde{M}_2$. We can therefore characterize the connectivity, d-dimensional fundamental group, homology group of combinatorial manifold $\widetilde{M}$ by topological graph $G^L[\widetilde{M}]$, and also the main objects in Riemannian geometry such as those of vector field, tensor field with local coordinates, Riemannian tenor with connection ([14]). Among them, what can reflect the most of CC conjecture is to establish the $|\Gamma|$-multiple covering space of combinatorial manifold $\widetilde{M}$ by voltage graph ([6]), where Γ is the finite group in voltage graph $G^L[\widetilde{M}]$.

(1)Regular covering space. Let G^L be a topological graph and let $(\Gamma; \circ)$ be a finite group. So, *what is a voltage graph and what is the lifting of a voltage graph?* Firstly, let $e = (v, u) \in E(G^L)$ be an edge of G^L. Its plus and minus orientations e^+, e^- on e are defined to be $v \to u$ and $u \to v$, respectively. And then, a voltage assignment $\alpha : E(G^L) \to \Gamma$ is a mapping from the plus-edges e^+ to Γ for $e \in E(G^L)$, i.e., $\alpha(e)$ is an element in group $(\Gamma; \circ)$ holding with $\alpha(e^-) = \alpha^{-1}(e)$. A topological graph G^L with voltage $\alpha(e)$ for any edge $e \in E(G^L)$ is said to be a *voltage graph* on group $(\Gamma; \circ)$, denoted by $(G^L; \alpha)$.

Notice that the voltage graph $(G^L; \alpha)$ is only a labeled graph with edge labels in a finite group $(\Gamma, \circ)$. Certainly, it is also a Smarandache multi-space of a topological graph G^L with a finite group $(\Gamma, \circ)$. However, the most interesting of voltage graph $(G^L; \alpha)$ is its *lifting* G^{L^α} defined by (see [7] for details)

$$\begin{aligned} V\left(G^{L^\alpha}\right) &= \left\{(v, a) = v_a \in V\left(G^L\right) \times \Gamma\right\}, \\ E\left(G^{L^\alpha}\right) &= \left\{(v_a, u_{a \circ b}) | e^+ = (v, u) \in E\left(G^L\right), \alpha\left(e^+\right) = b\right\} \end{aligned}$$

for regular covering of G^L. For example, let $G^L = K_3^L$, $\Gamma = \mathbb{Z}_2$ be respectively a topological graph and a finite group. Then, (a) is a voltage graph $(K_3^L;\alpha)$ and (b) is the lifting $K_3^{L^\alpha}$ of voltage graph $(K_3^L;\alpha)$ of (a) in Figure 5.

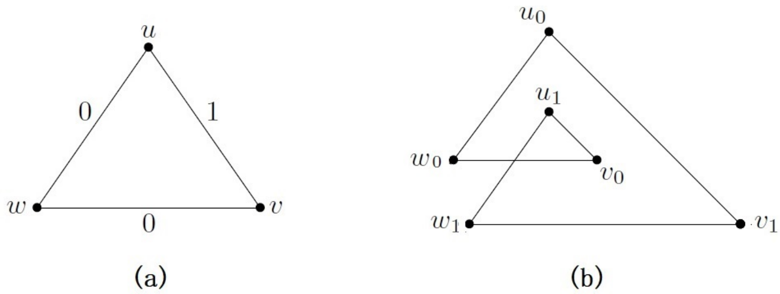


Figure 5. A voltage graph with lifting

There is a natural projection $\pi : G^{L^\alpha} \to G^L$ from the lifting G^{L^α} of voltage graph $(G^L;\alpha)$ on the topological graph G^L, namely for any vertices $v,u,\in V(G^L)$ and edge $(v,u)\in E(G^L)$ with $\alpha(v,u) = h$, define $\pi(v_g) = v$, $\pi(u_g) = u$ and $\pi(v_g, u_{g\circ h}) = (v,u)$. So, all vertices in the lifting G^{L^α} projected on $v \in V(G^L)$ consists of vertices $v_g, g\in\Gamma$ and edges that projected on edge (v,u) consists of edges $(v_g, u_{g\circ h})$, denoted by $\pi^{-1}(v)$ and $\pi^{-1}(v,u)$, respectively.

Notice that the lifting G^{L^α} of voltage graph $(G^L;\alpha)$ is a regular covering of topological graph G^L ([33]), i.e., a $|\Gamma|$-multiple covering on topological space G^L of dimension 1, and this method can be generalized for constructing regular covering space of combinatorial manifold $\widetilde{M}$ by the inherited topological structure $G^L[\widetilde{M}]$ of $\widetilde{M}$. That is, to assign voltages $\alpha : e\in E(G^L[\widetilde{M}])$ on edges of $G^L[\widetilde{M}]$ to get a voltage graph $(G^L;\alpha)$ with its lifting combinatorial manifold $\widetilde{M}^*$, where for any vertex $v_g \in V(G^L[\widetilde{M}^*])$, the labeling mapping $L^\alpha(v_g) = M^*$ is a covering space of M at vertex $v_M \in V(G^L[\widetilde{M}])$. In this case, for any manifold $M \in V(G^L[\widetilde{M}])$, let h_M be a covering mapping $h_M : M^* \to M$, $\varsigma_M : x \in M \to M$ and define $\pi^* = h_M \circ (\varsigma_M^{-1}\circ\pi\circ\varsigma_M)$. Then, the mapping $\pi^* : \widetilde{M}^* \to \widetilde{M}$ is a $|\Gamma|$-multiple covering mapping, which shows that the combinatorial manifold $\widetilde{M}^*$ is a regular covering space of combinatorial manifold $\widetilde{M}$.

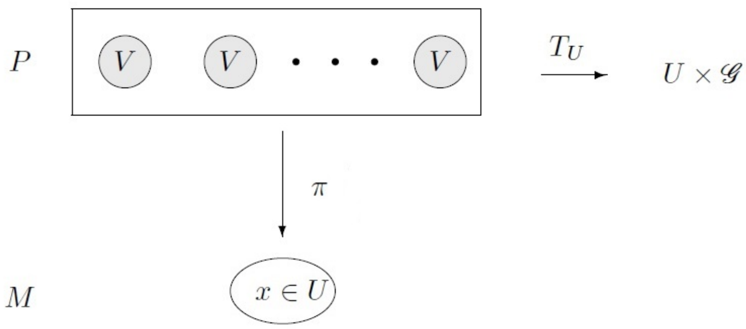


Figure 6. Principal fiber bundle on manifold

(2)Principal fiber bundle. Now, we turn our attention to differentiable manifolds. Firstly, a Lie group $(G;\circ)$ is a Smarandache multi-space that all elements in G both have algebraic and geometrical nature, namely for any $x,y\in G$, $x\circ y$ and x^{-1} both are C^∞-mapping, where G is a differentiable manifold in geometry and each point is an element in group

$(G;\circ)$ also. So, *what is a principal fiber bundle on a differentiable manifold M?* A principal fiber bundle is essentially a covering space on a differentiable manifold M. By definition, the *principal fiber bundle* on a differentiable manifold M is a 3-tuple $\{P, M;\mathscr{G}\}$ with covering space P, manifold M, a projection π and a mapping T_u such as those shown in Figure 6, where $\pi : P \to M$ is a projection, $\mathscr{G}$ is a Lie group acting on P such that $(g,h) \to g\circ h$ is C^∞ for any $g,h \in \mathscr{G}$ and holding with 3 conditions following:

(C_1) *The right action of Lie group $\mathscr{G}$ acting on P is free, namely for any $g \in \mathscr{G}$ there is a diffeomorphism $R_g : P \to P$ such that $R_g(p) = pg$ for any $p \in P$, $p(g_1g_2) = (pg_1)g_2$ for any $g_1, g_2 \in \mathscr{G}$, and $pe = p$ for any $e \in \mathscr{G}$ if and only if e is the identity of Lie group $\mathscr{G}$;*

(C_2) *The mapping $\pi : P \to M$ is onto, and $\pi^{-1}(\pi(p)) = \{pg|g \in \mathscr{G}\}$;*

(C_3) *For any point $x \in M$ there is an opened set U such that $x \in U$ and there is a diffeomorphism $T_U : \pi^{-1}(U) \to U \times \mathscr{G}$, i.e., $T_U(p) = (\pi(p), s_U(p))$, where $s_U : \pi^{-1}(U) \to \mathscr{G}$ holds with $s_U(pg) = s_U(p)g$ for any $g \in \mathscr{G}, p \in \pi^{-1}(U)$.*

Now, if a combinatorial manifold $\widetilde{M}$ consisted of differentiable manifolds $M_1, M_2, \cdots, M_m$ is differentiable and 3-tuples $\{P_1, M_1;\mathscr{G}_1\}$, $\{P_2, M_2;\mathscr{G}_2\}$, $\cdots$, $\{P_m, M_m;\mathscr{G}_m\}$ are respectively the principal fiber bundles on manifolds $M_1, M_2, \cdots, M_m$ with projections $\pi_i : P_i \to M_i$ for integers $1 \le i \le m$, then

$$\widetilde{P} = \bigcup_{i=1}^{m} P_i, \qquad \left(\widetilde{\mathscr{G}};\mathscr{O}\right) = \bigcup_{i=1}^{m} (\mathscr{G}_i;\circ_i) \tag{2.8}$$

are respectively the covering spaces and Lie multi-groups on combinatorial manifold $\widetilde{M}$, namely $\widetilde{\mathscr{G}}$ is a Lie multi-group in algebra and a differentially combinatorial manifold in geometry. Then, $\{\widetilde{P}, \widetilde{M};\widetilde{\mathscr{G}}\}$ is a principal fiber bundle on $\widetilde{M}$.

Furthermore, let $(\Gamma;\circ)$ be a finite group, $\alpha : E(G^L[\widetilde{M}]) \to \Gamma$ is a voltage assignment on topological graph $G^L[\widetilde{M}]$. Then, by the lifting of voltage graph $(G^L[\widetilde{M}];\alpha)$, we can obtain the principal fiber bundles $\{\widetilde{P}^\alpha, \widetilde{M};\widetilde{\mathscr{G}}\}$ on differentiable manifold $\widetilde{M}$ in general, namely let $L^\alpha(v_g) = P$ for any vertex $v_g \in V(G^{L^\alpha}[\widetilde{M}])$ for a projection $\pi : G^L[\widetilde{P}] \to G^L[\widetilde{M}]$, i.e., for any label $M \in V(G^L[\widetilde{M}])$ if $\pi(P_M) = M$ then $\pi^{-1}(M) = \{P_M^1, P_M^2, \cdots, P_M^m\}$, where P_M^i is differentially homeomorphic to P_M for integers $1 \le i \le m$ such as those shown in Figure 7.

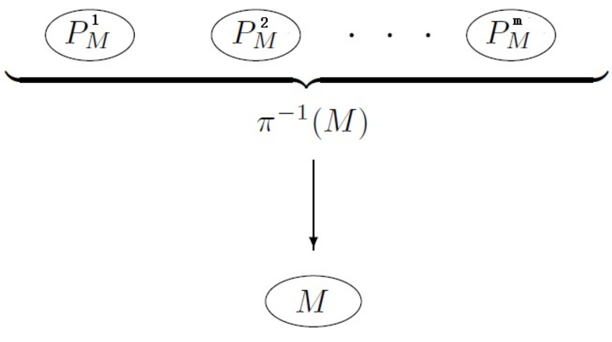


Figure 7. A principal fiber bundle on combinatorial manifold

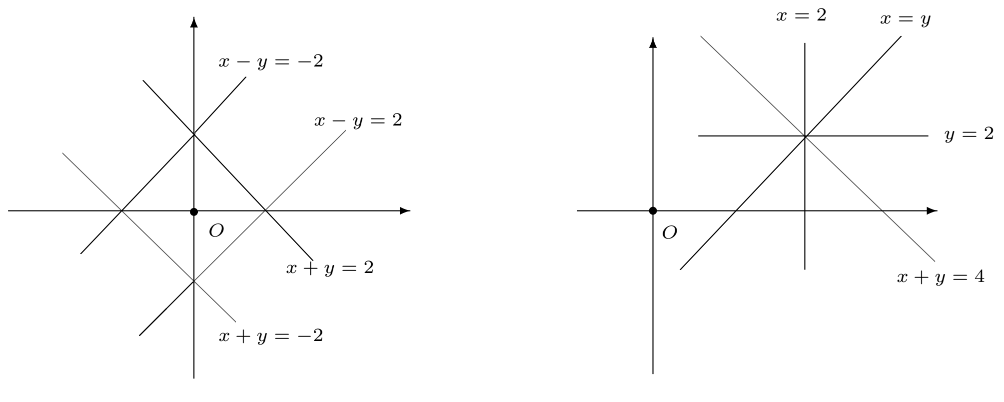
Now, let $\{\widetilde{P}^\alpha, \widetilde{M};\widetilde{\mathscr{G}}\}$ be a principal fiber bundles constructed in this way. Then, we can introduce the connection on differentially combinatorial manifold $\widetilde{M}$ and get the general form of curvature for characterizing differentially combinatorial manifolds. For example, a local and global connection on a principal fiber bundle $\{\widetilde{P}^{\alpha}, \widetilde{M}; \mathscr{G}\}$ are respectively a local linear mapping ${}^i\Gamma_u : T_x(\widetilde{M}) \to T_u(\widetilde{P}), u \in \Pi_i^{-1}(x) = {}^iF_x, x \in M_i$ for an integer $1 \le i \le l$ and a global linear mapping $\Gamma_u : T_x(\widetilde{M}) \to T_u(\widetilde{P})$ for $u \in \Pi^{-1}(x) = F_x$, $x \in \widetilde{M}$ holding with (i) $(d\Pi_i){}^i\Gamma_u$ =identity and $(d\Pi)\Gamma_u$ = identity mapping on $T_x(\widetilde{M})$; (ii) ${}^i\Gamma_{{}^iR_g \circ_i u} = d^iR_g \circ_i {}^i\Gamma_u$ and $\Gamma_{R_g \circ u} = dR_g \circ \Gamma_u$ for $\forall g \in \mathscr{L}_G$, $\forall \circ \in \mathscr{O}(\mathscr{L}_G)$, where ${}^iR_g$, R_g are the right translation respectively on P_{M_i} and $\widetilde{P}$; (iii) the mappings $u \to {}^i\Gamma_u$ and $u \to \Gamma_u$ both are C^∞, and a *curvature form* of a local or global connection is a $\mathfrak{Y}(\mathscr{H}_{\circ_i}, \circ_i)$ or $\mathfrak{Y}(\mathscr{L}_G)$-valued 2-form ${}^i\Omega = (d\ {}^i\omega)h$ or $\Omega = (d\omega)h$, where $(d^i\omega)h(X,Y) = d^i\omega(hX, hY)$ and $(d\omega)h(X,Y) = d\omega(hX, hY)$ for $X, Y \in \mathscr{X}(P_{M_i})$ or $X, Y \in \mathscr{X}(\widetilde{P})$. Then, the generalizations of Cartan's theorem and Bianchi identity on differentially combinatorial manifolds are obtained in the following.

Theorem 2.8([15]) *Let ${}^i\omega$, $1 \le i \le l$ and ω be respectively a local or global connection forms on a principal fiber bundle $\{\widetilde{P}^{\alpha}, \widetilde{M}; \mathscr{G}\}$. Then $(d^i\omega)(X,Y) = -[{}^i\omega(X), {}^i\omega(Y)] + {}^i\Omega(X,Y)$ and $d\omega(X,Y) = -[\omega(X), \omega(Y)] + \Omega(X,Y)$ for vector fields $X, Y \in \mathscr{X}(P_{M_i})$ and $\mathscr{X}(\widetilde{P})$, respectively.*

Theorem 2.9([15]) *Let ${}^i\omega$, $1 \le i \le l$ and ω be respectively a local or global connection forms on a principal fiber bundle $\{\widetilde{P}^{\alpha}, \widetilde{M}; \mathscr{G}\}$. Then, $(d^i\Omega)h = 0$ and $(d\Omega)h = 0$.*

§3. Non-Harmonious Groups

There is an implicit assumption in human recognition by the reduction when subdividing a thing T into microscopic particles, cells or genes, namely the evolving behavior of microscopic particles, cells or genes are all match in step and a solvable equation can be applied to describe its behavior, predict its evolution further. However, this assumption is not true in general unless the evolution of all microscopic particles, cells or genes that constitute thing T is synchronous. Otherwise, it is wrong even in the macroscopic world. For example, let $A = \{H_1, H_2, H_3, H_4\}$ and $B = \{H_1', H_2', H_3', H_4'\}$ be 2 families consisting of 4 horses that run respectively along 4 lines (LES^N) or (LES^S) on Euclidean plane $\mathbb{R}^2$ in Figure 8.



(LES_4^N) (LES_4^S)

Figure 8. Horses running on 4 straight lines

So, we are easily know the systems of linear equations

$$(LES_4^N)\begin{cases} x+y &= 2\\ x+y &= -2\\ x-y &= -2\\ x-y &= 2\end{cases}\qquad\qquad (LES_4^S)\begin{cases} x=y\\ x+y=4\\ x=2\\ y=2\end{cases}$$

that each horse in families A or B runs along 4 lines (LES^N) or (LES^S) such as those shown in Figure 8.

Now, *how to characterize the running behavior of horses in families A or B?* Generally, we use to the solution of equation systems (LES_4^N) or (LES_4^S) for characterizing the horse behaviors in families A or B. However, the equation system (LES_4^N) is non-solvable and the solution of equation system (LES_4^S) is $x=2, y=2$, which is only a point $(2,2)$ on Euclidean plan $\mathbb{R}^2$, and both of them can not be used to characterize the horse behavior in families A or B, even their running orbits. Here, a central question is the horses in families A or B are all conscious, not necessarily in synchronization or consistence in their respective running.

Similarly, there are also the biological populations, communities and the self-organizing or self-regulating system of cells, genes etc., whose evolution can not be characterized by a solvable equation. Whence, it is impossible to describe the evolution of groups in nature such as the self-organizing or self-regulating systems without an extending of mathematical elements, including the unified field by a solvable equation that of Einstein, which is the essence for discussing the non-harmonious groups.

The original thinking on non-harmonious groups came from my characterizing Smarandachely denied axiom on equations for reality of thing. In the first half of year 2012, I finished paper [16] on systems of linear equations. It is so happen that I went on a business trip to Guangzhou during the time that Prof.Smarandache's visiting Guangdong University of Technology in 2012. I visited him and introduced my thinking on Smarandachely denied axiom with the combinatorial characterizing of non-solvable systems of linear equations [16] that I just finished to him, which shows the necessity for the suitable form of Smarandachely denied axiom by non-solvable equations because a physical law is always described by differential equations on the evolution of thing. I got his greatly approval for this thinking.

Definition 3.1([29],[31]) *A non-harmonious group is such a group $\mathcal{T}$ consisting of elements P_i, $1 \leq i \leq p, p \geq 2$ with internal relations that P_i is constrained on an equation $\mathscr{F}_i(\mathbf{x},\mathbf{y}) = 0$ on time t in space, namely its system state equation in n-dimensional Euclidean space $\mathbb{R}^n$ is*

$$\mathcal{T} \triangleq \begin{cases} \mathscr{F}_1(\mathbf{x},\mathbf{y}) = 0\\ \mathscr{F}_2(\mathbf{x},\mathbf{y}) = 0\\ \cdots\cdots\cdots\cdots\\ \mathscr{F}_m(\mathbf{x},\mathbf{y}) = 0\end{cases} \tag{3.1}$$

where $\mathscr{F}_i(\mathbf{x}^0,\mathbf{y}^0) = 0$ and $\mathscr{F}_i$ holds at a neighborhood U of point $(\mathbf{x}^0,\mathbf{y}^0)$ with the condition of *implicit function theorem, i.e., each equation* $\mathscr{F}_i(\mathbf{x},\mathbf{y})=0$ *is solvable for integers* $1\leq i\leq m$.

So, *how to characterize the evolution of a non-harmonious group* $\mathcal{T}$*?* Notice that each equation $\mathscr{F}_i(\mathbf{x},\mathbf{y})=0$ holds with the condition of implicit function theorem for any integer $1\leq i\leq m$. It must exist a solution manifold $S_{\mathscr{F}_i}\subset\mathbb{R}^n$ with $\mathscr{F}_i: S_{\mathscr{F}_i}\to 0$ for any integer $1\leq i\leq m$. Then, the condition for system (3.1) of equations having no solution or having a solution geometrically is

$$\bigcap_{i=1}^{p} S_{\mathscr{F}_i}=\emptyset \quad \text{or} \quad \bigcap_{i=1}^{p} S_{\mathscr{F}_i}\neq\emptyset. \tag{3.2}$$

In this case, *how to explain that system* (3.1) *has or has no solution?* Notice that the solution of system (3.1) represents only the overlap state of elements $P_1,P_2,\cdots,P_m$ at time t, not the state of elements $P_1,P_2,\cdots,P_m$ because the state of element P_i is the solution manifold $S_{\mathscr{F}_i}, 1\leq i\leq m$. Correspondingly, the non-solvable case of system (3.1) indicates only that there are no overlap state in system elements, not that the they do not exist because the states of elements $P_1,P_2,\cdots,P_p$ are described by the solution manifold $S_{\mathscr{F}_i}$ for integers $1\leq i\leq m$. And so, the system state of a non-harmonious group $\mathcal{T}$ should be described by Smarandache multi-space $\bigcup\limits_{i=1}^{m} S_{\mathscr{F}_i}$, not $\bigcap\limits_{i=1}^{m} S_{\mathscr{F}_i}$ as the usual, namely the system solution of equation (3.1) of a non-harmonious group $\mathcal{T}$ should be characterized by a combinatorial manifold $G^L[\widetilde{S}]$.

Theorem 3.2([29][31]) *For any integer* $m\geq 1$*, the combinatorial solution or G-solution to system* (3.1) *of a non-harmonious group* $\mathcal{T}$ *is a combinatorial manifold* $\widetilde{S}$ *inherited a topological graph* $G^L[\widetilde{S}]$ *with*

$$\begin{aligned} V\left(G^L[\widetilde{S}]\right) &= \{S_{\mathscr{F}_i},\ 1\leq i\leq m\};\\ E\left(G^L[\widetilde{S}]\right) &= \left\{(S_{\mathscr{F}_i},S_{\mathscr{F}_j})|S_{\mathscr{F}_i}\bigcap S_{\mathscr{F}_j}\neq\emptyset, 1\leq i,j\leq m\right\}\end{aligned}$$

with a labelling

$$L:\ S_{\mathscr{F}_i}\to S_{\mathscr{F}_i},\quad (S_{\mathscr{F}_i},S_{\mathscr{F}_j})\to S_{\mathscr{F}_i}\bigcap S_{\mathscr{F}_j},\quad 1\leq i,j\leq m.$$

For example, let the orbit of a horse in families A or B running on a straight line $ax+by=c$ be a point set $L_{a,b,c}=\{(x,y)|ax+by=c, ab\neq 0\}$. Then, the system states of horse families A and B can be characterized respectively by the combinatorial solutions $C_4^L[LES_4^N]$ and $K_4^L[LES_4^S]$ of equation systems (LES_4^N) and (LES_4^S), such as those shown in Figure 9,

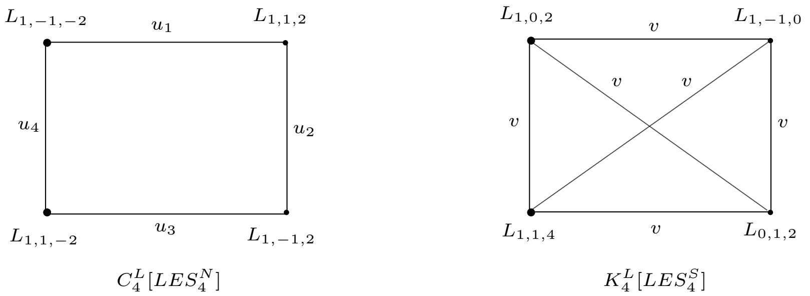


Figure 9. Combinatorial solutions of systems (LES_4^N) and (LES_4^S)

where

$$\begin{aligned} u_1 &= L_{1,-1,-1}\bigcap L_{1,1,2}, \quad u_2 = L_{1,1,2}\bigcap L_{1,-1,2}, \\ u_3 &= L_{1,-1,2}\bigcap L_{1,1,-2}, \quad u_4 = L_{1,1,-2}\bigcap L_{1,-1,-2}. \end{aligned}$$

Generally, we can apply Theorem 3.2, i.e., a combinatorial approach to discuss the non-solvable systems of algebraic equation, ordinary differential equations or partial differential equation for describing the system states of non-harmonious groups with the stability of systems, including biological systems that they correspond, respectively. Furthermore, the non-solvable systems of homogeneous algebraic equations in 3-variables can be used to determine the genus $g(\widetilde{S})$ of combinatorial surfaces $\widetilde{S}$ and normalization of complex non-singular curves, etc., see [17]-[18],[21],[23],[25] for details. Let us take the systems $(LDES_m^1)$ and (LDE_m^n) of non-solvable ordinary differential equations

$$(LDES_m^1)\begin{cases} \dot{X} = A_1X \\ \dot{X} = A_2X \\ \cdots\cdots\cdots \\ \dot{X} = A_mX \end{cases}, \qquad (LDE_m^n)\begin{cases} x^{(n)} + a_{11}^{[0]}x^{(n-1)} + \cdots + a_{1n}^{[0]}x = 0 \\ x^{(n)} + a_{21}^{[0]}x^{(n-1)} + \cdots + a_{2n}^{[0]}x = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ x^{(n)} + a_{m1}^{[0]}x^{(n-1)} + \cdots + a_{mn}^{[0]}x = 0 \end{cases}$$

as examples, which are a system of 1-order of linear ordinary differential equations and an n-order of linear differential equations with constant coefficients respectively, where $a_{ij}^{[k]}$ is a real number for integers $0 \leq k \leq m$, $1 \leq i \leq n$, $1 \leq j \leq s$, $A_k = \left(a_{ij}^k\right)_{n\times s}$ is a matrix and $X = (x_1, x_2, \cdots, x_n)^T$. Notice that the solution manifolds of $(LDES_m^1)$ and (LDE_m^n) are both linear spaces spanned by their basic solutions. We can therefore replace labels $\mathcal{S}_{\mathscr{F}_i}$, $\mathcal{S}_{\mathscr{F}_j}, 1 \leq i, j \leq m$ in Theorem 3.2 by basic solutions and then, obtain their uniquely basic graphs $G^L[LDES_m^1]$ and $G^L[LDE_m^n]$ of systems $(LDES_m^1)$ and (LDE_m^n), respectively. For example, let $m = 6$ with a system of linear ordinary differential equations

$$(LDES_6^1)\begin{cases} \ddot{x} - 3\dot{x} + 2x = 0 & (1) \\ \ddot{x} - 5\dot{x} + 6x = 0 & (2) \\ \ddot{x} - 7\dot{x} + 12x = 0 & (3) \\ \ddot{x} - 9\dot{x} + 20x = 0 & (4) \\ \ddot{x} - 11\dot{x} + 30x = 0 & (5) \\ \ddot{x} - 7\dot{x} + 6x = 0 & (6) \end{cases}$$

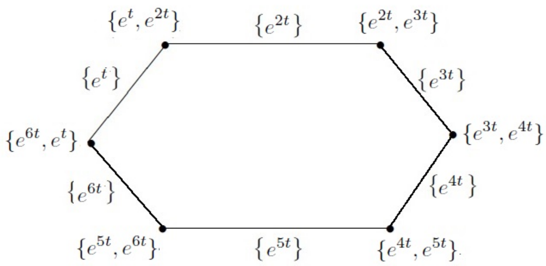


Figure 10. A basic graph

Then, the basic solutions of differential equations $(1) - (6)$ are respectively

$$\{e^t, e^{2t}\}, \{e^{2t}, e^{3t}\}, \{e^{3t}, e^{4t}\}, \{e^{4t}, e^{5t}\}, \{e^{5t}, e^{6t}\}, \{e^{6t}, e^t\},$$

with a combinatorial solution or basic graph $G^L[LDES_6^1]$ shown in Figure 10.

Notice that there always exists a combinatorial solution $G^L[\widetilde{S}]$ of a non-harmonious group

(3.1) by Theorem 3.2, which enables us to introduce the system stability of non-harmonious group. For example, a combinatorial solution $G[LDES_m^1]$ or $G^L[LDE_m^n]$ is said to be *prod-stable* or *asymptotically prod-stable* if

$$\left\|\prod_{v\in V(H)} Y_v(t) - \prod_{v\in V(H)} X_v(t)\right\| < \varepsilon \quad \text{or} \quad \lim_{t\to 0}\left\|\prod_{v\in V(H)} Y_v(t) - \prod_{v\in V(H)} X_v(t)\right\| = 0. \tag{3.3}$$

holds with all solutions $Y_v(t)$, $v\in V(G^L)$ of $G[LDES_m^1]$ or $G^L[LDE_m^n]$ with $\|Y_v(0)-X_v(0)\| < \delta_v$ exists for all $t\geq 0$. In this case, we have

Theorem 3.3([18]) *A combinatorial zero-solution, i.e., all labels on basic graphs $G^L[LDES_m^1]$ and $G^L[LDE_m^n]$ are 0 of systems $(LDES_m^1)$ and $(LDES_m^n)$ of linear homogeneous differential equations is asymptotically prod-stable if and only if*

$$\sum_{v\in V(G^L[LDES_m^1])} \mathrm{Re}\alpha_v \ < \ 0 \quad \textit{or} \quad \sum_{v\in V(G^L[LDE_m^n])} \mathrm{Re}\lambda_v \ < \ 0 \tag{3.4}$$

for any basic solutions $\overline{\beta}_v(t)e^{\alpha_v t}\in\mathscr{B}_v$ of $(LDES_m^1)$ or $t^{l_v}e^{\lambda_v t}\in\mathscr{C}_v$ of (LDE_m^n).

In classical meaning, all systems correspondent to non-harmonious groups (3.1) are non-mathematical systems in general, namely they do not comply with the principle of logical consistency. So, *how to transform a non-mathematical system into a mathematical system and characterize the evolution of thing?* The answer is to transform a system of non-mathematics to mathematics by combinatorial approach discussed profitably in [19], including those of non-groups, non-rings, non-fields, non-solvable equations in algebra, non-solvable differential equations in calculus, non-spaces, non-manifolds, non-differentiable manifolds in geometry and others, which are all decomposing into mathematical systems over topological graphs G^L. This is exactly the application of my CC conjecture, i.e., mathematical combinatorics on non-mathematical groups.

§4. Continuity Flows

In the human recognition, the essence of subdividing a substance T into microscopic particles such as those of elementary particles, cells or genes by reduction is holding on the reality of thing T by the combinatorial solution $G^L[\widetilde{S}]$ on the behavior of microscopic particles. Notice that an edge $(S_{\mathscr{F}_i}, S_{\mathscr{F}_i})\in V(G^L[\widetilde{S}])$ if and only if $S_{\mathscr{F}_i}\bigcap S_{\mathscr{F}_i}\neq\emptyset$ by definition, namely the action $S_{\mathscr{F}_i}$ and $S_{\mathscr{F}_i}$ in system $\widetilde{S}$ is symmetric. However, the interaction between particles or in general, the energy, information transmission are mostly not symmetric but a unidirectional one. For example, the transformation of energy with conservation. According to the uncertainty principle of microscopic particle, the random evolution of particle is introduced and then, the microscopic particle is described by complex networks ([5]). For example, Barabaśi and Albert described the growth of nodes of complex network by randomness in [2],[3]. But, *is the mechanism of natural evolution really random?* If so, *how can it be possible to describe the evolution of thing by random models established by humans?* Among them, a necessary way should be to understand the non-harmonious groups $\widetilde{S}$ by the recognizability of thing rather than infinitely subdividing a thing

T into microscopic particles or recognizing a thing T by randomness because the randomness is a by-product of the limitation of human recognition for lacking of information, may not the truth colour of thing T.

I reflected deeply the previous questions on the combinatorial solutions $G^L[\widetilde{S}]$ of non-harmonious groups in 2014. After finishing the paper [19], I began to think about how a combinational notion should correctly describe the evolution of thing T by reduction in human recognition and then, vectors $\mathbf{v}$ in a Banach space $\mathscr{B}$ were used for labelling the edge of topological graph or network G^L following with the conservation law at each vertex of G^L ([20]), i.e., the $\overrightarrow{G}$*-flow*, a generalization of network with operations. Subsequently, I proposed the *action flow* $\overrightarrow{G}^L$ in my plenary report at the "*National Conference on Emerging Trends in Mathematics and Mathematical Sciences*" (December 17-19, 2015, Kolkata) invited by the *Calcutta Mathematical Society* as an honorary guest, which is an extension from the conservation of vertices in $\overrightarrow{G}$-flow to the conservation of action flows by edge operators ([22]) and then, the *continuity flow* $\overrightarrow{G}^L$([24]) was put forward in my J.C.& K.L.Saha memorial lecture at the "*International Conference on Geometry and Mathematical Models in Complex Phenomena*" (December 5-7, 2017, Kolkata), which can be viewed also as a mathematical element with algebraic, differential and integral operations. In this way, the Banach and Hilbert flow spaces are established for providing theories of human recognition, especially for the reduction step by step.

So, *what is a continuity flow?* A continuity flow $(\overrightarrow{G}; L, \mathscr{A})$ is essentially a Banach space over a topological graph G^L, a generalization of mathematics by applying my CC conjecture.

Definition 4.1([24]) *A continuity flow* $\left(\overrightarrow{G}; L, \mathscr{A}\right)$ *is an oriented topological graph* $\overrightarrow{G}^L$ *in space* $\mathscr{S}$ *associated with a mapping* $L: v \to L(v),\ (v,u) \to L(v,u)$, 2 *end-operators* $A^+_{vu}: L(v,u) \to L^{A^+_{vu}}(v,u)$ *and* $A^+_{uv}: L(u,v) \to L^{A^+_{uv}}(u,v)$ *on a Banach space* $\mathscr{B}$ *over a field* $\mathscr{F}$ *such as those shown in Figure* 11 *with* $L(v,u) = -L(u,v)$, $A^+_{vu}(-L(v,u)) = -L^{A^+_{vu}}(v,u)$ *for* $\forall (v,u) \in E\left(G^L\right)$ *and meanwhile, holding with the continuity equation*

$$\sum_{u \in N^-_G(v)} L^{A^+_{uv}}(u,v) - \sum_{u \in N^+_G(v)} L^{A^+_{uv}}(u,v) \ = \ L(v) \qquad (4.1)$$

at any vertex $v \in V(G^L)$ *of topological graph* G^L, *where* $N^-_G(v), N^+_G(v)$ *are respectively the in-neighborhood and out-neighborhood of vertex* $v \in V(G^L)$, *namely all vertices in* $N^-_G(v) \subset N_G(v)$ *or* $N^+_G(v) \subset N_G(v)$ *flow into or out of the vertex* v *and* $N^-_G(v) \cup N^+_G(v) = N_G(v)$.

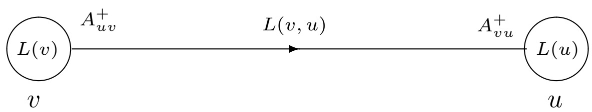


Figure 11. Flow with end-operators on an edge

Now, *why is the continuity flow important to human?* The answer is that the continuity flow provides us with a mathematical support for reduction on thing T, also answer the 3 questions in Section 1 because the result of human recognition by reduction to a thing T is such a continuity flow G^L. However, all existing sciences, including the mathematics can be only used for describing the evolution of a particle or particles of a system evolving all in synchronization, namely there are few evolutionary theory that regards a thing T as a self-organized or self-adjusted system by mathematics.

4.1.Continuity Flow Space. All operations on continuity flows are the composition of the union of topological graphs with composition of mappings. Generally, let $G^L, G'^{L'}$ be continuity flows on Banach space $\mathscr{B}$ over field $\mathscr{F}$, $\lambda \in \mathscr{F}$. Then, the addition, multiplication and scalar multiplication on continuity flows are defined by

$$\begin{aligned} G^L + G'^{L'} &= (G \setminus G')^L \bigcup \left(G \bigcap G'\right)^{L+L'} \bigcup (G' \setminus G)^{L'}, \quad (4.2) \\ G^L \cdot G'^{L'} &= (G \setminus G')^L \bigcup \left(G \bigcap G'\right)^{L \cdot L'} \bigcup (G' \setminus G)^{L'}, \quad (4.3) \\ \lambda \cdot G^L &= G^{\lambda \cdot L} \quad (4.4) \end{aligned}$$

where $L(v), L'(v), L(v,u), L'(v,u) \in \mathscr{B}$ for any vertex $v \in V(G)$ and edge $(v,u) \in E(G)$ with

$$\begin{aligned} L + L' &: v \to L(v) + L'(v),\ (v,u) \to L(v,u) + L'(v,u), \\ L \cdot L' &: v \to L(v) \cdot L'(v),\ (v,u) \to L(v,u) \cdot L'(v,u), \\ \lambda \cdot L &: v \to \lambda \cdot L(v),\ (v,u) \to \lambda \cdot L(v,u), \end{aligned}$$

and $L(v) \cdot L'(v)$, $L(v,u) \cdot L'(v,u)$ denote the operation of Hadamard product on vectors in Banach space $\mathscr{B}$, namely

$$(x_1, x_2, \cdots, x_n) \cdot (y_1, y_2, \cdots, y_n) = (x_1y_1, x_2y_2, \cdots, x_ny_n). \quad (4.5)$$

Generally, let $\mathscr{G} = \{G_1, G_2, \cdots, G_m\}$ be a closed family under the union operation of topological graphs and let $\mathscr{B}$ be a Banach space. All continuity flows with vectors in $\mathscr{B}$ over topological graph $G^L \in \mathscr{G}$ are denoted by $\mathscr{G}_{\mathscr{B}} = \left\{G^L | G \in \mathscr{G}, L : v \to L(v) \in \mathscr{B}, (v,u) \to L(v,u) \in \mathscr{B},\right.$ $\left. v \in V(G), (v,u) \in E(G)\right\}$. Then, *$(\mathscr{G}_{\mathscr{B}}; +, \cdot)$ is a bigroup under operations of addition "+" and Hadamard product "·"*. Furthermore, *if $(\mathscr{B}; +, \cdot)$ is a field and all end-operators are $1_{\mathscr{B}}$ on any continuity flow $G^L \in \mathscr{G}_{\mathscr{B}}$, then the bigroup $(\mathscr{G}_{\mathscr{B}}; +, \cdot)$ is also a field*. In addition, *$(\mathscr{G}_{\mathscr{B}}; \mathscr{F})$ is always a linear space under operations of addition and scalar multiplication.*

In this case, if each end-operator on a continuity flow $G^L \in \mathscr{G}_{\mathscr{B}}$ is linearly continuous on $\mathscr{B}$, then the norm of continuity flow G^L is defined by

$$\left\|G^L\right\| = \sum_{(v,u) \in E(G)} \left\| L^{A^+_{vu}}(v,u) \right\|, \quad (4.6)$$

where $\|\cdot\|$ denotes the norm on Banach space $\mathscr{B}$. And then, *the continuity flow space $(\mathscr{G}_{\mathscr{B}}; \mathscr{F})$ is a normed space*. Furthermore, for any $G^L, G'^{L'} \in \mathscr{G}_{\mathscr{B}}$ define the metric of continuity flows $G^L, G'^{L'}$ to be

$$\rho\left(G^L, G'^{L'}\right) = \left\|G^L - G'^{L'}\right\|. \quad (4.7)$$

Then, *each Cauchy sequence in continuity flow space $(\mathscr{G}_{\mathscr{B}}; \mathscr{F})$ is complete*, and *if the linear space $(\mathscr{B}; \mathscr{F})$ is a Banach or Hilbert space, then $(\mathscr{G}_{\mathscr{B}}; \mathscr{F})$ is a Banach or Hilbert space* ([24]).

4.2.G-Isomorphic Operator. By functional analysis, an operator on Banach space $\mathscr{B}$ maps one vector to another one. As a Banach space, an operator on continuity flow space $(\mathscr{G}_{\mathscr{B}};\mathscr{F})$ can be also defined to be a mapping that maps one continuity flow G^L to another $G'^{L'}$. However, such a definition does not reflect the topological nature in continuous flow G^L. So, it is of little significance. Whence, it is necessary to define a typical operator that leaves the topological structure of continuity flow G^L unchanged, which is nothing else but the G-isomorphic operator.

Definition 4.2([29],[31]) *Let $G_1^{L_1}, G_2^{L_2} \in \mathscr{G}_{\mathscr{B}}$ be continuity flows. A mapping $f : G_1^{L_1} \to G_2^{L_2}$ is said to be a G-isomorphic operator between continuity flows $G_1^{L_1}$, $G_2^{L_2}$ and the continuity flow $G_1^{L_1}$ is said to be G-isomorphic to $G_2^{L_2}$ if*

(1) *G_1, G_2 are isomorphic in graphs, i.e., there is an isomorphism $\varphi : G_1 \to G_2$ of graph;*

(2) *$L_2 = f \circ \varphi \circ L_1$ for $\forall (v,u) \in E(G_1)$.*

Notice that Definition 4.2 can be applicable only if $G_1^{L_1}, G_2L_2$ are isomorphic in labeled graphs, which should be extended to the general case. Usually, it is conventionalized that $\widehat{G}^{\widehat{L}} = G^L$ for a topological graph $\widehat{G} \supset G$ if $\widehat{L}(x) = L(x)$ for $x \in V(G) \cup E(G)$ and $\widehat{L}(x) = \mathbf{0}$ for $x \notin V(G) \cup E(G)$, which reflects the essence of continuity flow. And by this convention, a $\widehat{G}$-isomorphism between continuity flows $G_1^{L_1}, G_2^{L_2}$ can be generally defined even if $G_1^{L_1}, G_2^{L_2}$ are non-isomorphic but with a supergraph $\widehat{G}$ as $\widehat{G} \supseteq G_1 \bigcup G_2$.

Definition 4.3([29],[31]) *A mapping $f : G_1^{L_1} \to G_2^{L_2}$ is said to be a G-isomorphic operator between continuity flows $G_1^{L_1}$ and $G_2^{L_2}$ if*

(1) *there is an isomorphism $\varphi : \widehat{G} \to \widehat{G}$ with $\widehat{G} \supset G_1, G_2$ in graph;*

(2) *for $\forall (v,u) \in E(G_1)$ there is $L_2 = f \circ \varphi \circ L_1$ but for $\forall (v,u) \in E(G_2 \setminus G_1)$, $f : \mathbf{0} \to L_2(v,u)$ and for $\forall (v,u) \in E(G_1 \setminus G_2)$ and $\forall (v,u) \in E(\widehat{G} \setminus (G_1 \bigcup G_2))$, $f : L(v,u) \to \mathbf{0}$.*

Particularly, let $\varphi = \mathrm{id}_G$, i.e., the identity mapping on topological graph G. Then, a G-isomorphic operator f is determined by the equation

$$L_2(v,u) \ = \ f \circ L_1(v,u), \quad \forall (v,u) \in E(G) \tag{4.8}$$

and the linearity of G-isomorphic operator with its nature such as the continuous, bounded, image and others can be introduced similar to the usual Banach space, and generalize a few of well-known results such as those of *a G-isomorphic linear operator $f : \mathscr{G}_{\mathscr{B}} \to \mathscr{G}_{\mathscr{B}}$ is continuous if and only if it is bounded* and *if $f : \mathscr{G}_{\mathscr{B}} \to \mathscr{G}_{\mathscr{B}}$ is closed then f is continuous*, etc. Furthermore, if the Banach space $\mathscr{B}$ is a function field on variable $\mathbf{x}$ then the G-isomorphic equation (4.8) is equivalent to

$$f\left(G^L[\mathbf{x}]\right) \ = \ G^{f(L[\mathbf{x}])}. \tag{4.9}$$

For example, let $\mathscr{B}$ be a real number field. We can construct the power $G^{aL}[\mathbf{x}]$ and exponent $a^{G^L[\mathbf{x}]}$ of a continuity flow $G^L[\mathbf{x}]$, define the limitation of continuity flow sequence, differential and integral operations, i.e., the theory of calculus on continuity flows $G^L[\mathbf{x}]$ and obtain the fundamental theorem ([29])

$$\int_a^b f\frac{d}{dt}\left(\overrightarrow{G}^L[t]\right)dt = f\left(\overrightarrow{G}^L[t]\right)\Big|_{t=b} - f\left(\overrightarrow{G}^L[t]\right)\Big|_{t=a} \tag{4.10}$$

similar to that of calculus.

In this case, assume the mapping $\mathscr{L} : (v,u) \in E(G) \to \mathscr{L}[\mathcal{L}(t,\mathbf{x}(t),\dot{\mathbf{x}}(t))(v,u)]$ is differentiable and commutative with all end-operators A_{vu}^{+}, then the action $J\left[G^{\mathscr{L}}[t]\right]$ and variation $\delta J\left[G^{\mathscr{L}}[t]\right]$ on a continuity flow $G^{\mathscr{L}}[t]$ are respectively defined by

$$J\left[G^{\mathscr{L}}[t]\right] = \left|\int_{t_1}^{t_2} G^{\mathscr{L}[\mathcal{L}(t,\mathbf{x}(t),\dot{\mathbf{x}}(t))]}dt\right|, \quad \delta J\left[G^{\mathscr{L}}[t]\right] = \left|\delta\int_{t_1}^{t_2} G^{\mathscr{L}[\mathcal{L}(t,\mathbf{q}(t),\dot{\mathbf{q}}(t))]}dt\right|, \tag{4.10}$$

where the variation $\delta : \mathscr{G}_{\mathscr{B}} \to \mathscr{G}_{\mathscr{B}}$ is a G-isomorphic operator. Then, by the least action principle $\delta J\left[G^{\mathscr{L}}[t]\right](v,u) = 0$ for $\forall(v,u) \in E\left(G^{\mathscr{L}}[t]\right)$ and the norm property in Banach flow space $\mathscr{G}_{\mathscr{B}}$, we can induce Euler-Lagrange equations on continuity flow $G^{\mathscr{L}}[t]$ to be

$$\frac{\partial G^{\mathscr{L}}}{\partial q_i} - \frac{d}{dt}\frac{\partial G^{\mathscr{L}}}{\partial \dot{q}_i} \;=\; \mathbf{O}, \quad 1 \le i \le n. \tag{4.11}$$

and the 3 interesting exponential identities ([29]) following

$$e^x = 1 + \frac{x}{1!} + \frac{x^{2]}}{2!} + \cdots + \frac{x^n}{n!} + \cdots, \tag{4.12}$$

$$e^{tA} = \mathbf{I} + \frac{tA}{1!} + \frac{t^2A^2}{2!} + \cdots + \frac{t^nA^n}{n!} + \cdots, \tag{4.13}$$

$$e^{G^L[\mathbf{x}]} = \mathbf{I} + \frac{G^L[\mathbf{x}]}{1!} + \frac{G^{2L}[\mathbf{x}]}{2!} + \cdots + \frac{G^{nL}[\mathbf{x}]}{n!} + \cdots, \tag{4.14}$$

where for an integer $n \ge 1$, A is an $n \times n$ matrix, $G^L[\mathbf{x}]$ is a continuity flow that continuous in variable $\mathbf{x}$, namely the formula (4.12) is the exponential identity in calculus, (4.13) is the exponential identity on matrix A which is a generalization of the exponential identity (4.12) and (4.14) is the exponential identity on continuity flow $G^L[\mathbf{x}]$, which is a generalization of the exponential identity (4.13) on matrix A.

Notice that a linear functional $f : \mathscr{B} \to \mathbb{R}$ or $\mathbb{C}$ is a linear operator on a Banach space by definition. So, *how to extend a linear functional on a Banach space to the Banach flow space $\mathscr{G}_{\mathscr{B}}$?* If there really exists such a G-isomorphic linear operator $f : \mathscr{G}_{\mathscr{B}} \to \mathbb{R}$ or $\mathbb{C}$, f is referred to a functional on $\mathscr{G}_{\mathscr{B}}$. And so, a fundamental question is to determine *whether there exists a continuously linear functional on the Banach flow space $\mathscr{G}_{\mathscr{B}}$?* The answer is certainly yes because we can generalize the functional extension theorem, i.e., Hahn-Banach theorem on Banach space to the Banach flow space $\mathscr{G}_{\mathscr{B}}$.

Theorem 4.4([26]) *Let $\mathscr{H}_{\mathscr{B}}$ be a subspace of Banach flow space $\mathscr{G}_{\mathscr{B}}$, $F : \mathscr{H}_{\mathscr{B}} \to \mathbb{C}$ is a continuously linear G-isomorphic functional on $\mathscr{H}_{\mathscr{B}}$. Then, there is a continuously linear G-isomorphic functional $\widetilde{F} : \mathscr{G}_{\mathscr{B}} \to \mathbb{C}$ holding with (i) if $G^L \in \mathscr{H}_{\mathscr{B}}$ then $\widetilde{F}\left(G^L\right) = F\left(G^L\right)$ and (ii) $\|\widetilde{F}\| = \|F\|$.*

Particularly, if $\mathbf{O} \ne G_0^{L_0} \in \mathscr{G}_{\mathscr{B}}$, there is a continuously linear G-isomorphic functional F such that $\|F\| = 1$ and $\|F(G_0^{L_0})\| = \|G_0^{L_0}\|$, where $G_0^{L_0}$ is the continuity flow $\mathbf{O}$, i.e., $L : v \to \mathbf{0}$ and $(v,u) \to \mathbf{0}$ for $\forall \in V(G_0^{L_0})$ and $\forall(v,u) \in E(G_0^{L_0})$.

Furthermore, by Theorem 4.4 of the functional extension theorem on $\mathscr{G}_{\mathscr{B}}$ we have

Corollary 4.5 *For a continuity flow $G^L \in \mathscr{G}_{\mathscr{B}}$ if $F\left(G^L\right) = 0$ holds with all linear functionals F, there must be $\overrightarrow{G}^L = \mathbf{O}$.*

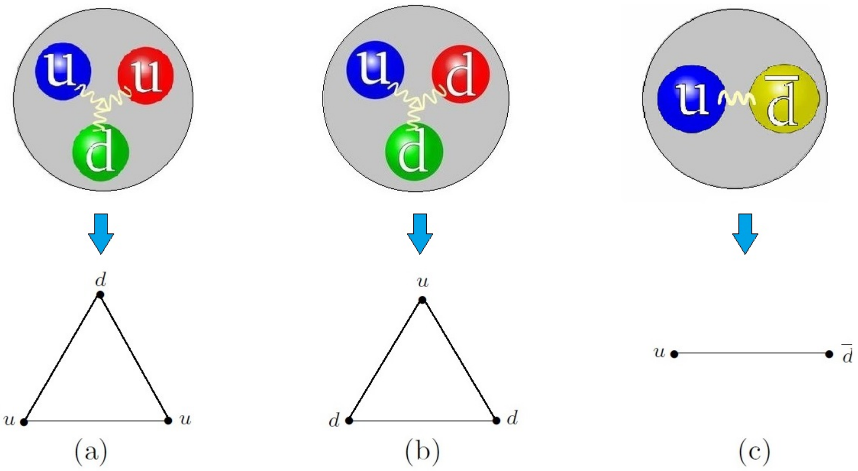


Figure 12. Hadron's quark model with continuity flow

Notice that there are 3 assumptions in quantum mechanics with a Hilbert space as the model of quantum states ([35]), i.e., (i) A pure state of quantum can be characterized in terms of a normalized vector $|\psi\rangle$ in Hilbert space $\mathcal{H}$ with $\langle\psi|\psi\rangle = 1$; (ii) For a physical quantity a of quantum, an observation on a in state $|\psi\rangle$ is the eigenvalue λ_j of an Hermitian operator A acting on $\mathcal{H}$ that exists, i.e., $A\,|\lambda_j\rangle = \lambda_j\,|\lambda_j\rangle$; (iii) The evolution of quantum state is governed by Schrödinger equation $i\hbar d\,|\psi\rangle/dt = H\,|\psi\rangle$, where $\hbar$ is the Planck's constant and H denotes a Hermitian operator corresponding to the energy of system. However, if we describe a microscopic particle such as a hadron, i.e., a proton, neutron or a meson by quarks, its model is no longer a particle but a continuity flow G^L ([31],[35]) shown in Figure 12.

In this case, *are there any reason to conclude there is a Hermitian operator A holding with the* 3 *assumptions in quantum mechanics?* Certainly, there are no such an affirmatively answer unless we priorly assume that all quarks $u, d, \overline{d}$ are evolving in synchronization. However, this assumption is incorrect on a self-organizing or self-regulating system such as a biological system consisting of cells. But its correctness can be verified by Theorem 4.4, i.e., let $f : \mathscr{G}_{\mathscr{B}} \to \mathbb{C}$ be a continuously linear G-isomorphism on continuity flows G^L. Then, there exists a uniquely continuity flow $\widehat{G}^{\widehat{L}} \in \mathscr{G}_{\mathscr{B}}$ holding with $f\left(G^L\right) = \left\langle G^L, \widehat{G}^{\widehat{L}}\right\rangle$ for a continuity flow $G^L \in \mathscr{G}_{\mathscr{B}}$ by Theorem 4.4, i.e., no matter how we subdivide a particle into a continuity flow G^L, there always exists a Hermitian operator A which holds with the 3 assumptions in quantum mechanics.

4.4.Example. A typical example of continuity flow G^L is the twelve meridians on human body ([31]), which consist of the lung meridian of Hand-Taiyin (LU) belongs to the lung and connects with the large intestine, the heart meridian of Hand-Shaoyin (HT) belongs to the heart and connects with the small intestine, the pericardium meridian of Hand-Jueyin (PC) is belongs to the pericardium and connects with the Sanjiao, the spleen meridian of Foot-Taiyin (SP) belongs to the spleen and connects with the stomach, the kidney meridian of Foot-Shaoyin (KI) belongs to the kidney and connects with the bladder, the liver meridian of Foot-Jueyin (LR) belongs to the liver and connects with gallbladder; the large intestine meridian of Hand-Yangming

(LI) belongs to the large intestine and connects with the lung, the small intestine meridian of Hand-Taiyang (SI) belongs to the small intestine and connects with the heart, the Sanjiao meridian of Hand-Shaoyang (SJ) belongs to the Sanjiao and connects with the pericardium, the stomach meridian of Foot-Yangming (ST) belongs to the stomach and connects with the spleen, the bladder meridian of Foot-Taiyang (BL) belongs to the bladder and connects with the kidney, and the gallbladder meridian of Foot-Shaoyang (GB) belongs to the gallbladder and connects with the liver of human body. All of the meridians on the way are shown in Figure 13 with 310 acupoints in the national standard of China, i.e., *Body Model for Both Meridian and Extraordinary Points of China* (GB 12346-90) for details.

Figure 13. Twelve meridians on human body

Now, *how to construct a continuity flow G^L on* 12 *meridians of human body?* Certainly, we are easily to construct a continuity flow G^L of human body, namely let all vertices v of continuity flow G^L be all acupoints on the twelve meridians of human body and let all edges (v, u) be paths between two successive acupoints v, u on one of the twelve meridians with a labelling $L : V \to$ *internal organs* of human body and $L : (v, u) \to$ *vital energy flow* $L(v, u)$ on (v, u) of human meridians. Notice that the flow of vital energy between two successive acupoints varies at different times of the day and depends on the state of human body. And so, the traditional Chinese medicine classifies $L(v, u)$ into two parts, i.e., *Yin* $\mathbf{Y}^-$ and *Yang* $\mathbf{Y}^+$, which can be regarded as a pair of interacting vectors and believes that the essence of normal operating of human body lies in the balance of Yin and Yang following the natural law, i.e., for $\forall v \in V(G^L)$ and any direction $\overrightarrow{O}$ on v in space, there must be

$$\mathbf{Y}^-(v) + \mathbf{Y}^+(v) \ = \ \mathbf{C}\left(v, \overrightarrow{O}\right) \tag{4.13}$$

which is a general ruler for determining whether the human body is operating normally, where $\mathbf{C}(v, \overrightarrow{O})$ is a constant vector at the point v in direction $\overrightarrow{O}$. Thus, an illness of human body is abstractly equivalent to the imbalance of Yin $\mathbf{Y}^-$ and Yang $\mathbf{Y}^+$ on some meridians of human body, and it is necessary to adjust the flows of vital energy on human meridians by "*reducing the excess with supply the insufficient*" to recover the balance of Yin $\mathbf{Y}^-$ and Yang $\mathbf{Y}^+$, i.e., the natural law of vital energy operating on human body.

§5. Conclusion

Clearly, the original motivation of CC conjecture is to apply the combinatorial approach for extending mathematical sciences in order to improve human's ability for recognizing the nature. But since it is a conjecture, it is necessary to give a proof on its correctness. So, *how to prove its correctness of CC conjecture?* In fact, the CC conjecture is not so much as a mathematical conjecture but a recognitive thought. Its correctness lies in the fact that humans extend their local recognitions of thing T to the whole, i.e., by characteristics of $\chi_1, \chi_2, \cdots$ of thing T showing up in front of human. Certainly, the recognitive conclusion must be a combination of local recognitive outcomes on characteristics of $\chi_1, \chi_2, \cdots$ over an inherited 1-dimensional topological structure G^L of thing T, namely the human recognition on reality of thing T can only be a combinatorial one ([31]), including the science and technology ([1],[32]). This is exactly what the sophist told the blind men in fable of the blind men with an elephant. Surely, the mathematics follows the principle of logical consistency in human recognition, which inevitably leads to the limitation of effect in human recognition by mathematics, namely the mathematical reality $T_{\mathcal{M}}$ is only a local recognitive or conditional conclusion. My CC conjecture is only consistent with the extension of human recognition from the local to the whole. That is, the combinatorial approach on reality $T_{\mathcal{M}}$ of thing T is prior to human recognition, including science and mathematics, which is a philosophy of human recognition, no proof further is required, namely the combinatorial notion implied by the human recognition of reduction is on the first and the recognition or science is followed, only on the second, i.e., the essence of mathematical combinatorics following.

Mathematical Combinatorics. *All mathematical sciences should be generalized or reconstructed over topological graph G^L which is consistent with the 1-dimensional topological graph G^L inherited in human recognition of thing by reduction.*

Notice that the "*reduction*" here is subdivided thing T into the recognizable elements that humans understand the reality of thing T, not an infinitely subdividing of thing T and there is essentially no need to subdivide any thing $T \to$ molecule $\to$ atom $\to$ elementary particle or any living $T \to$ biological macromolecule $\to$ cell $\to$ gene. Otherwise, it will artificially cause the complexity in the recognition with no benefits for recognizing the truth colour of thing. For example, the number of cells of an adult is about $4 \times 10^{14} - 6 \times 10^{14}$. So, *does an analysis on the behavior of an adult need to subdivide it into cells?* The answer is certainly not because the essence of reducing of thing T into elementary particles, cells and genes is to treat thing T as a complex network, which simultaneously increases the complexity in recognition. In contrast, the Chinese science established on the interaction of Yin $\mathbf{Y}^-$ and Yang $\mathbf{Y}^+$, the promoting and restraining potentials of five elements, i.e., the *metal, wood, water, fire, earth* is more suitable for leading the developing of humans, which is essentially a system science for the harmonious coexistence of humans with nature on the human recognition in today's terms.

In 2003, Prof.Tagmark of *Massachusetts Institute of Technology* proposed a *mathematical universe hypothesis* ([42]) which claims that *the natural reality outside of human is a mathematical structure, namely the universe can not only be characterized by mathematics but the universe itself is a mathematical structure.* Of course, this is an encouraging hypothesis that excites most researchers because it allows mathematics to describe and model the evolution of everything in the universe. However, *can the mathematics that follows the principle of logical consistency be applied already to describe the reality of everything in the universe?* The answer is certainly not because the mathematical reality is still a local or conditional one. It is essentially not equivalent to the reality of thing. And meanwhile, this hypothesis can not be verified because humans have not yet been able to arrive at each corner in the universe. In this case, it is necessary to extend mathematics including contradictions, i.e., Smarandache multi-spaces over the 1-dimensional topological graphs G^L inherited in things in order to understand the nature of things. This is my philosophy of mathematical combinatorics, which includes also the mathematical universe hypothesis of Prof.Tagmark as a deduction.

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