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On Bitopological Supra B-Open Sets

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Abstract: In this paper, we introduce and investigate a new class of sets and maps between bitopological spaces called supra(1,2) b-open, and supra (1,2) b-continuous maps, respectively. Furthermore, we introduce the concepts of supra(1,2) locally-closed, supra(1,2) locally b-closed sets. We also introduce supra(1,2) extremely disconnected. Finally, additional properties of these sets are investigated.

Key Words: Supra(1,2) b-open set, supra(1,2) locally closed, supra(1,2) b-closed, supra(1,2) extremely disconnected.

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§1. Introduction

In 1983 A.S.Mashhour et al [5] introduced supra topological spaces and studied s-continuous maps and s^*-continuous maps. Andrijevic [1] introduced a class of generalized open sets in a topological space,the called b-open sets in 1996. In 1963, J.C.Kelly [3] introduced the concept of bitopological spaces. The purpose of this present paper is to define some properties by using supra(1,,2) b-open sets, supra(1,2) locally-closed, supra(1,2) locally b-closed in supra bitopological spaces and investigate the relationship between them.

§2. Preliminaries

Throughout this paper by (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2). (or simply X, Y and Z) represent bitopological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, A^c denote the complement of A. A subcollection μ is called a supra topology [5] on X if $X \in \mu$, where μ is closed under arbitrary union. (X, μ) is called a supra topological space. The elements of μ are said to be supra open in (X, μ) and the complement of a supra open set is called a supra closed set. The supra topology μ is associated with the topol-

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ogy τ if $\tau \subset \mu$. A subset A of X is $\tau_1\tau_2$-open [4] if A$\in \tau_1 \cup \tau_2$ and $\tau_1\tau_2$-closed if its complement is $\tau_1\tau_2$-open in X. The $\tau_1\tau_2$-closure of A is denoted by $\tau_1\tau_2cl(A)$ and $\tau_1\tau_2cl(A) = \cap\{\ F : A \subset F$ and F^c is $\tau_1\tau_2$-open$\}$. Let (X,μ_1, μ_2) be a supra bitopological space. A set A is $\mu_1\mu_2$-open if $A \in \mu_1 \cup \mu_2$ and $\mu_1\mu_2$-closed if its complement is $\mu_1\mu_2$-open in (X, μ_1, μ_2). The $\mu_1\mu_2$-closure of A is denoted by $\mu_1\mu_2cl(A)$ and $\mu_1\mu_2cl(A) = \cap\{\ F : A \subset F$ and F^c is $\mu_1\mu_2 -$ open$\}$.

Definition 2.1 *Let (X, μ) be a supra topological space. A set A is called*

(1) *supra α-open set [2] if $A \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(A)))$;*

(2) *supra semi-open set [2] if $A \subseteq cl^{\mu}(int^{\mu}(A))$;*

(3) *supra b-open set [6] if $A \subseteq cl^{\mu}(int^{\mu}(A)) \cup int^{\mu}(cl^{\mu}(A))$.*

Definition 2.2([4]) *Let (X, τ_1, τ_2) be a bitopological space. A subset A of (X, τ_1, τ_2) is called*

(1) *(1,2)semi-open set if $A \subseteq \tau_1\tau_2cl(\tau_1int(A))$;*

(2) *(1,2)pre-open set if $A \subseteq \tau_1int(\tau_1\tau_2cl(A))$;*

(3) *(1,2)α-open-set if $A \subseteq \tau_1int(\tau_1\tau_2cl(\tau_1int(A)))$;*

(4) *(1,2)b-open-set $A \subseteq \tau_1\tau_2cl(\tau_1int(A)) \cup \tau_1int(\tau_1\tau_2cl(A))$.*

§3. Comparison

In this section we introduce a new class of generalized open sets called supra(1,2) b-open sets and investigate the relationship between some other sets.

Definition 3.1 *Let (X, τ_1, τ_2) be a supra bitopological space. A set A is called a supra(1,2) b-open set if $A \subseteq \mu_1\mu_2cl(\mu_1int(A)) \cup \mu_1int(\mu_1\mu_2cl(A))$.The compliment of a supra(1,2) b-open is called a supra(1,2) b-closed set.*

Definition 3.2 *Let X be a supra bitopological space. A set A is called*

(1) *supra (1,2) semi-open set if $A \subseteq \mu_1\mu_2cl(\mu_1int(A))$;*

(2) *supra (1,2) pre-open set if $A \subseteq \mu_1int(\mu_1\mu_2cl(A))$;*

(3) *supra (1,2) α-open-set if $A \subseteq \mu_1int(\mu_1\mu_2cl(\mu_1int(A)))$.*

Theorem 3.3 *In a supra bitopological space (X, μ_1, μ_2), any supra open set in (X, μ_1) is supra(1,2) b-open set and any supra open set in (X, μ_2) is supra (2,1) b-open set.*

Proof Let A be any supra open in (X, μ_1). Then $A = \mu_1int(A)$. Now $A \subseteq \mu_1\mu_2cl(A) = \mu_1\mu_2cl(\mu_1int(A)) \subseteq \mu_1\mu_2cl(\mu_1int(A) \cup \mu_1int(\mu_1\mu_2cl(A))$. Hence A is supra(1,2) b-open set. Similarly, any supra open in (X, μ_2) is supra(2,1) b-open set. □

Remark 3.4 The converse of the above theorem need not be true as shown by the following example.

Example 3.5 Let $X = \{a, b, c, d\}$, $\mu_1 = \{\phi, X, \{a, b\}, \{a, c, d\}\}$, $\mu_2 = \{\phi, \{a\}, \{a, b\}, \{b, c, d\}, X\}$; $\mu_1\mu_2$-open $= \{\phi, \{a\}, \{a, b\}, \{a, c, d\}, \{b, c, d\}, X\}$, $\mu_1\mu_2$-closed $= \{\phi, \{a\}, \{b\}, \{c, d\}, \{b, c, d\}, X\}$, supra(1,2) $bO(X) = \{\phi, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, X\}$. It is obvious that {a,d}$\in$ supra(1,2) b-open but $\{a,d\} \notin \mu_1$-open. Also, supra(2,1) $bO(X) = (\phi, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,c,d\}, \{a,b,d\}, \{b,c,d\}, X\}$. Here $\{a,c\} \in$ supra(2,1) b-open set but $\{a,c\} \notin \mu_2$-open.

Theorem 3.6 *In a supra bitopological space (X, μ_1, μ_2), any supra open set in (X, μ_1) is supra(1,2) α-open set and any supra open set in (X, μ_2) is supra (2,1) α-open set.*

Proof Let A be any supra open in (X, μ_1). Then $A = \mu_1 int(A)$. Now $A \subseteq \mu_1\mu_2 cl(A)$. Then $\mu_1 int(A) \subseteq \mu_1 int(\mu_1\mu_2 cl(A))$. Since $A = \mu_1 int(A)$, $A \subseteq \mu_1 int(\mu_1\mu_2 cl(\mu_1 int(A)))$. Hence A is supra(1,2) α-open set. Similarly, any supra open in (X, μ_2) is supra(2,1) α-open set. □

Remark 3.7 The converse of the above theorem need not be true as shown in the following example.

Example 3.8 Let $X = \{a,b,c,d\}$, $\mu_1 = \{\phi, \{a,c\}, \{a,b,c\}, \{a,b,d\}, X\}$, $\mu_2 = \{\phi, \{c,d\}, \{a,b,d\}, \{b,c,d\}, X\}$, $\mu_1\mu_2$-open $= \{\phi, \{a,c\}, \{c,d\}, \{a,b,c\}, \{a,b,d, \{b,c,d\}, X\}$, $\mu_1\mu_2$-closed $= \{\phi, \{a\}, \{c\}, \{d\}, \{a,b\}, \{b,d\}, X\}$. supra(1,2) $\alpha O(X) = \{\phi, \{a,c\}, \{a,b,c\}, \{a,c,d\}, \{a,b,d\}\}, X\}$. Here $\{a,c,d\} \in supra(1,2)$ α-open but $\{a,c,d\} \notin \mu_1$-open. Also, supra(2,1) $\alpha O(X) = (\phi, \{c,d\}, \{a,c,d\}, \{a,b,d\}, \{b,c,d\}, X\}$. Here $\{a,c,d\} \in supra(2,1)\alpha$-open but $\{a,c,d\} \notin \mu_2$-open.

Theorem 3.9 *Every supra(1,2) α-open is supra(1,2) semi-open.*

Proof Let A be a supra (1,2) α-open set in X. Then $A \subseteq \mu_1 int(\mu_1\mu_2 cl(\mu_1 int(A))) \subseteq \mu_1\mu_2 cl(\mu_1 int(A))$. Therefore, $A \subseteq \mu_1\mu_2 cl(\mu_1 int(A))$. Hence A is supra(1,2) semi-open set. □

Remark 3.10 The converse of the above theorem need not be true as shown below.

Example 3.11 Let $X = \{a,b,c,d\}$, $\mu_1 = \{\phi, \{b\}, \{a,d\}, \{a,b,c\}, X\}$, $\mu_2 = \{\phi, \{b,c\}, \{a,b,d\}, X\}$, $\mu_1\mu_2$-open $=\{\phi, \{b\}, \{a,d\}, \{b,c\}, \{a,b,c, \{a,b,d\}, X\}$, $\mu_1\mu_2$-closed $= \{\phi, \{c\}, \{d\}, \{a,d\}, \{b,c\}, \{a,c,d\}, X\}$. supra(1,2) $\alpha O(X) = \{\phi, \{b\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}\}, X\}$, supra(1,2) $SO(X) = \{\phi, \{b\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}\}, X\}$. Here $\{b,c\}$ is a supra(1,2) α-open but not supra(1,2) semi-open.

Theorem 3.12 *Every supra(1,2) semi-open set is supra(1,2) b-open.*

Proof Let A be a supra(1,2) semi-open set X. Then $A \subseteq \mu_1\mu_2 cl(\mu_1 int(A))$. Hence $A \subseteq \mu_1\mu_2 cl(\mu_1 int(A)) \cup \mu_1 int(\mu_1\mu_2 cl(A))$. Thus A is supra(1,2) b-open set. □

Remark 3.13 The converse of the above theorem need not be true as shown in the following example.

Example 3.14 Let $X = \{a,b,c,d\}$, $\mu_1 = \{\phi, \{a\}, \{a,b\}, \{b,c,d\}, X\}$, $\mu_2 = \{\phi, \{b\}, \{a,d\}, \{a,b,d\}, \{b,c,d\}, X\}$, $\mu_1\mu_2$-open $= \{\phi, \{a\}, \{b\}, \{a,b\}, \{a,d\}, \{a,b,d\}, \{b,c,d\}, X\}$, $\mu_1\mu_2$-closed $= \{\phi, \{a\}, \{c\}, \{b,c\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}, X\}$, supra(1,2) $bO(X) = \{\phi, \{a\}, \{a,b\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}, X\}$, supra(1,2) $SO(X) = \{\phi, \{a\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}\}, X\}$. Here $\{b,d\} \in supra(1,2)b$-open set but $\{b,d\} \notin$ supra(1,2) semi-open.

Theorem 3.15 *Every supra(1,2) α-open is supra(1,2) b-open.*

Proof Let A be an supra(1,2) α-open in X. Then $A \subseteq \mu_1 int(\mu_1\mu_2 cl(\mu_1 int(A)))$. It is obvious that $\mu_1 int(\mu_1\mu_2 cl(\mu_1 int(A))) \subseteq \mu_1\mu_2 cl(\mu_1 int(A)) \subseteq \mu_1\mu_2 cl(\mu_1 int(A)) \cup \mu_1 int(\mu_1\mu_2 cl(A))$. Hence $A \subseteq \mu_1\mu_2 cl(\mu_1 int(A)) \cup \mu_1 int(\mu_1\mu_2 cl(A))$. Thus A is supra(1,2) b-open set. □

Remark 3.16 The reverse claim in Theorem 3.15 is not usually true.

Example 3.17 Let $X = \{a, b, c, d\}$,$\mu_1 = \{\phi, \{a, c\}, \{a, b, c\}, \{a, b, d\}, X\}$,$\mu_2 = \{\phi, \{c, d\}, \{b, c, d\}, \{a, b, d\}X\}$,$\mu_1\mu_2$-open$= \{\phi, \{a, c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\{b, c, d\}, X\}$ $\mu_1\mu_2$-closed$= \{\phi, \{a\}, \{c\}, \{d\}, \{a, b\}, \{b, d\}, X\}$, supra(1,2) $\alpha O(X) = \{\phi, \{a, c\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}\}, X\}$, supra(1,2) $bO(X) = \{\phi, \{a, c\}, \{a, d\}, \{b, c\}, \{c, d\}\{a, b, c\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}, X\}$. Here $\{a, d\} \in$ supra(1,2) b-open but $\{a, d\} \notin$ supra(1,2) α-open.

Theorem 3.18 *In a supra bitopological space (X, μ_1, μ_2), any supra open set in (X, μ_1) is supra(1,2) semi-open set and any supra open set in (X, μ_2) is supra (2,1) semi-open set.*

Proof This follows immediately from Theorems 3.6 and 3.9. □

Remark 3.19 The converse of the above theorem need not be true as shown in the Example 3.8, $\{a, c, d\}$ is both supra(1,2) semi-open and supra(2,1) semi-open but it is not supra μ_1-open and also is not μ_2-open.

Remark 3.20 From the above discussions we have the following diagram. $A \rightarrow$B represents A implies B, $A \nrightarrow B$ represents A does not implies B.

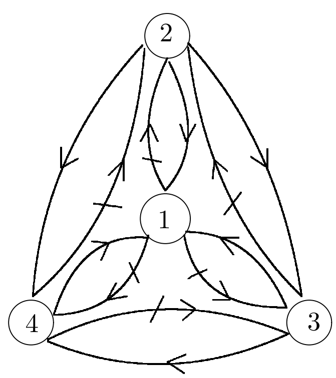

Fig. 1 1=supra (1,2) b-open, 2=μ_1-open,
3=supra (1,2) α-open, 4=supra (1,2) semi-open

§4. Properties of Supra(1,2) b-Open Sets

Theorem 4.1 *A finite union of supra(1,2) b-open sets is always supra(1,2) b-open.*

Proof Let A and B be two supra(1,2) b-open sets. Then $A \subseteq \mu_1\mu_2 cl(\mu_1 int(A)) \cup$

$\mu_1 int(\mu_1\mu_2 cl(A))$ and $B \subseteq \mu_1\mu_2 cl(\mu_1 int(B)) \cup \mu_1 int(\mu_1\mu_2 cl(B))$. Now, $A \cup B \subseteq \mu_1\mu_2 cl(\mu_1 int(A \cup B)) \cup \mu_1 int(\mu_1\mu_2 cl(A \cup B))$. Hence $A \cup B$ is supra(1,2) b-open set. □

Remark 4.2 Finite intersection of supra(1,2) b-open sets may fail to be supra(1,2) b-open since, in Example 3.14, both $\{a, b\}$ and $\{b, d\}$ are supra(1,2) b-open sets, but their intersection $\{c\}$ is not supra(1,2) b-open.

Definition 4.3 *The supra(1,2) b-closure of a set A is denoted by $supra(1,2)bcl(A)$ and defined as $supra(1,2)bcl(A) = \cap\{B :$ B is a supra(1,2) b-closed set and $A \subset B\}$. The supra(1,2) interior of a set A is denoted by supra(1,2)bint(A), and defined as supra(1,2)b int(A)$= \cup\{B :$ B is a supra(1,2) b-open set and $A \supseteq B\}$.*

Remark 4.4 It is clear that $supra(1,2)bint(A)$ is a supra(1,2) b-open and $supra(1,2)bcl(A)$ is supra(1,2) b-closed set.

Definition 4.5 *A subset A of supra bitopological space X is called*

(1) *supra(1,2)locally-closed if $A = U \cap V$, where $U \in \mu_1$ and V is supra$\mu_1\mu_2$ closed;*
(2) *supra(1,2) locally b-closed if $A = U \cap V$, where $U \in \mu_1$ and V is supra(1,2) b-closed;*
(3) *supra (1,2)D(c,b) set if $\mu_1 int(A) = supra(1,2)bint(A)$.*

Theorem 4.6 *The intersection of a supra open in (X, μ_1) and a supra(1,2) b-open set is a supra(1,2) b-open set.*

Proof Let A be supra open in (X, μ_1). Then A is supra(1,2) b-open and $A = \mu_1 int(A) \subseteq supra(1,2)bint(A)$. Let B be supra(1,2) b-open then $B = supra(1,2)bint(B)$. Now $A \cap B \subseteq supra(1,2)bint(A) \cap supra(1,2)bint(B) = supra(1,2)bint(A \cap B)$. Hence the intersection of supra open set in (X, μ_1) and a supra(1,2) b-open set is a supra(1,2) b-open set. □

Theorem 4.7 *For a subset A of X, the following are equivalent:*

(1) *A is supra-open in (X, μ_1);*
(2) *A is supra (1,2) b-open and supra(1,2) D(c,b)-set.*

Proof (1)⇒(2) If A is supra-open in (X, μ_1), then A is supra (1,2) b-open and $A = \mu_1 int(A)$, $A = supra(1,2)bint(A)$. Hence $\mu_1 int(A) = supra(1,2)bint(A)$. Therefore, A is $supra(1,2)D(c,b)$-set.

(2)⇒(1) Let A be supra (1,2) b-open and supra $(1,2)D(c,b)$-set. Then $A = supra(1,2)bint(A)$ and $\mu_1 int(A) = supra(1,2)bint(A)$. Hence $A = \mu_1 int(A)$. This implies that A is supra-open in (X, μ_1). □

Definition 4.8 *A space X is called an supra(1,2) extremely disconnected space (briefly supra(1,2) E.D) if supra$\mu_1\mu_2$ closure of each supra-open in (X, μ_1) is supra open set in (X, μ_1). Similarly supra$\mu_1\mu_2$ closure of each supra-open in (X, μ_2) is supra open set in (X, μ_2).*

Example 4.9 Let $X = \{a, b, c\}$, $\mu_1 = \{\phi, \{b\}, \{a, b\}, \{a, c\}, X\}$, $\mu_2 = \{\phi, \{a\}, \{b, c\}, \{a, c\}\}$, $\mu_1\mu_2$open $= \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$, $\mu_1\mu_2$ closed $= \{\phi, \{a\}, \{b\}, \{c\}, \{a, c\}\{b, c\}, X\}$.

Hence every $\mu_1\mu_2$ closure of supra-open is (X,μ_1) and also every supra$\mu_1\mu_2$ closure of supra-open in (X,μ_2).

Theorem 4.10 *Let A be a subset of supra bitopological space (X,μ_1,μ_2) if A is supra(1,2) locally b-closed, then*

(1) *$supra(1,2)bcl(A)-A$ is supra(1,2) b-closed set:*
(2) *$[A\cup(X-supra(1,2)bcl(A))]$ is supra(1,2) b-open;*
(3) *$A\subseteq supra(1,2)bint(A\cup(X-supra(1,2)bcl(A))$.*

Proof (1) If A is an supra(1,2) locally b-closed, there exist an U is supra-open in (X,μ_1) such that $A=U\cap supra(1,2)bcl(A)$. Now, $supra(1,2)bcl(A)-A=supra(1,2)bcl(A)-[U\cap supra(1,2)bcl(A)]=supra(1,2)bcl(A)\cap[X-(U\cap supra(1,2)bcl(A))]=supra(1,2)bcl(A)\cap[(X-U)\cup(X-supra(1,2)bcl(A))]=supra(1,2)bcl(A)\cap(X-U)$, which is supra(1,2) b-closed by Theorem 4.5.

(2) Since $supra(1,2)bcl(A)-A$ is supra(1,2) b closed, then $[X-(supra(1,2)bcl(A)-A)]$ is supra(1,2) b-open and $[X-(supra(1,2)bcl(A)-A)]=(X-supra(1,2)bcl(A))\cup(X\cap A)=A\cup[X-supra(1,2)bcl(A)]$. Hence $[A\cup(X-supra(1,2)bcl(A))]$ is supra(1,2) b-open.

(3) It is clear that

$$A\subseteq[A\cup(X-supra(1,2)bcl(A)]=supra(1,2)bint[A\cup(X-supra(1,2)bcl(A))].\qquad\square$$

§5. Supra (1,2) b-Continuous Functions

In this section, We introduce a new class of continuous maps called a supra (1,2) b-continuous maps and obtain some of their properties.

Definition 5.1 *Let (X,τ_1,τ_2) and (Y,σ_1,σ_2) be two bitopological spaces and μ_1,μ_2 be an associated supra bitopology with τ_1,τ_2. A map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is called a supra (1,2) b-continuous map [resp. supra (1,2) α-continuous, supra (1,2) semi-continuous] if the inverse image of each $\sigma_1\sigma_2$-open set in Y is supra (1,2) b-open set [resp. supra (1,2) α-open, supra (1,2) semi-open] in X.*

Definition 5.2 *Let (X,τ_1,τ_2) and (Y,σ_1,σ_2) be two bitopological spaces and μ_1,μ_2 be an associated supra bitopology with τ_1,τ_2. A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is called supra (1,2) continuous if $f^{-1}(V)$ is μ_1-open in X for each $\sigma_1\sigma_2$-open set V of Y.*

Theorem 5.3 *Every (1,2) continuous is supra (1,2) b-continuous.*

Proof Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be an (1,2)-continuous map and let A be an $\sigma_1\sigma_2$-open set in (Y,σ_1,σ_2). Then $f^{-1}(A)$ is an τ_1-open set in (X,τ_1,τ_2). Since μ_1 and μ_2 are associated with τ_1 and τ_2, then $\tau_1\subseteq\mu_1$. This implies that $f^{-1}(A)$ is μ_1-open in X and it is supra (1,2) b-open in X. Hence f is supra (1,2)b-continuous. $\square$

Theorem 5.4 *Every supra (1,2)-continuous is supra (1,2) b-continuous function.*

Proof Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be an supra (1,2)-continuous and let A be an $\sigma_1\sigma_2$ open set in Y. Since f is supra (1,2)-continuous and μ_1, μ_2 associated with τ_1, τ_2, $f^{-1}(A)$ is μ_1-open in X and it is supra (1,2) b-open in X. Hence f is supra (1,2) b-continuous. □

Remark 5.5 The converse of Theorems 5.3 and 5.4 need not be true. We can shown this by the following example.

Example 5.6 Let $X = \{a, b, c, d\}$, $Y = \{p, q, r, s\}$, $\tau_1 = \{\phi, \{a\}, \{a, b\}, \{a, d\}, X\}$ and $\tau_2 = \{\phi, \{a\}, \{a, b\}, X\}$ are topologies on (X, τ_1, τ_2), $\sigma_1 = \{\phi, \{p\}, \{r\}, \{p, r\}, Y\}$, $\sigma_2 = \{\phi, \{p, r\}, Y\}$, $\sigma_1\sigma_2$-open $= \{\phi, \{p\}, \{r\}, \{p, r\}, Y\}$. The supra topologies μ_1, μ_2 are defined as follows:

$\mu_1 = \{\phi, \{a\}, \{a, b\}, \{a, d\}, \{b, c\}, X\}$, $\mu_2 = \{\phi, \{a\}, \{a, b\}, \{b, c\}, X\}$, $\mu_1\mu_2$ open $= \{\phi, \{a\}, \{a, b\}, \{a, d\}, \{b, c\}, X\}$, $\mu_1\mu_2$ closed $= \{\phi, \{a, d\}, \{b, c\}, \{b, d\}, \{b, c, d\}, X\}$, supra (1,2) b-open $= \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, X\}$. Define a map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by $f(a) = p$, $f(b) = q$, $f(c) = r$, $f(d) = s$. Clearly f is supra (1,2) b-continuous. But $f^{-1}(\{p, r\}) = \{a, c\}$ is not μ_1-open set in X where $\{p, r\}$ is $\sigma_1\sigma_2$-open in Y. So f is not supra (1,2) continuous. And also f is not (1,2)-continuous functions because $f^{-1}(\{p, r\}) = \{a, c\}$ is not τ_1-open in X where where $\{p, r\}$ is $\sigma_1\sigma_2$-open in Y.

Theorem 5.7 *Every supra (1,2) α-continuous map is supra (1,2)b-continuous.*

Proof It is obvious that every supra (1,2) α-open is (1,2) b-open. □

Remark 5.8 The converse of the above theorem need not be true as shown in the following example.

Example 5.9 Let $X = \{a, b, c, d\}$, $Y = \{p, q, r, s\}$, $\tau_1 = \{\phi, \{a, b, c\}, X\}$ and $\tau_2 = \{\phi, \{a, b, d\}, X\}$ are topologies on (X, τ_1, τ_2), $\sigma_1 = \{\phi, \{p, r\}, Y\}$, $\sigma_2 = \{\phi, \{p, q\}, \{p, q, s\}, Y\}$, $\sigma_1\sigma_2$-open $= \{\phi, \{p, q\}, \{p, r\}, \{p, q, s\}, Y\}$. The supra topologies μ_1, μ_2 are defined as follows:

$\mu_1 = \{\phi, \{a, c\}, \{a, b, c\}, \{a, b, d\}, X\}$, $\mu_2 = \{\phi, \{c, d\}, \{b, c, d\}, \{a, b, d\}, X\}$. Define a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by $f(a) = q$, $f(b) = r$, $f(c) = p$, $f(d) = s$. Then f is supra (1,2) b-continuous but not (1,2) α-continuous because $f^{-1}(\{p, r\}) = \{b, c\}$ is not supra (1,2) α-open where $\{p, r\}$ is $\sigma_1\sigma_2$-open in Y.

Theorem 5.10 *Let (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2) be three bitopological spaces. If a map $f : (X, \tau_1, \tau_2 \to (Y, \sigma_1, \sigma_2)$ is supra(1,2) b-continuous and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ is a (1,2)-continuous map, then $g \circ f : (X, \tau_1, \tau_2) \to (Z, \eta_1, \eta_2)$ is a supra(1,2) b-continuous.*

Proof Let A be a $\eta_1\eta_2$-open set in Z. Since g is (1,2)-continuous, then $g^{-1}(A)$ is σ_1-open in Y. Every σ_1-open is $\sigma_1\sigma_2$-open. Thus $g^{-1}(A)$ is $\sigma_1\sigma_2$-open in Y. Since f is supra (1,2) b-continuous, then $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is supra (1,2) b-open set in X. Therefore $g \circ f$ is supra(1,2) b-continuous. □

Theorem 5.11 *Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be bitopological spaces. Let μ_1, μ_2 and v_1, v_2 be the associated supra bitopologies with τ_1, τ_2 and σ_1, σ_2, respectively. Then $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a supra(1,2) b-continuous map if one of the following holds:*

(1) *$f^{-1}(supra(1,2)bint(A)) \subseteq \tau_1 int(f^{-1}(A))$ for every set A in Y;*

(2) *$\tau_1\tau_2 cl(f^{-1}(A)) \subseteq f^{-1}(supra(1,2)bcl(A))$ for every set A in Y;*

(3) *$f(\tau_1\tau_2 cl(B)) \subseteq supra(1,2)bcl(f(B))$ for every set B in X.*

Proof Let A be any $\sigma_1\sigma_2$-open set of Y. If condition (1) is satisfied, then

$$f^{-1}(supra(1,2)bint(A)) \subseteq \tau_1 int(f^{-1}(A)).$$

We get, $f^{-1}(A) \subseteq \tau_1 int(f^{-1}(A))$. Therefore $f^{-1}(A)$ is supra open set in (X, μ_1). Every supra open set in (X, μ_1) is supra(1,2) b-open set. Hence f is supra (1,2) b-continuous function.

If condition (2) is satisfied, then we can easily prove that f is supra(1,2) b-continuous function.

Now if the condition (3) is satisfied and A be any $\sigma_1\sigma_2$-open set of Y. Then $f^{-1}(A)$ is a set in X and $f(\tau_1\tau_2 cl(f^{-1}(A))) \subseteq supra(1,2)bcl(f(f^{-1}(A)))$. This implies $f(\tau_1\tau_2 cl(f^{-1}(A))) \subseteq supra(1,2)bcl(A)$. It is nothing but just the condition (2). Hence f is a supra(1,2) b-continuous map. □

§6. Applications

Now we introduce a new class of space called a supra(1,2)-extremely disconnected space.

Definition 6.1 *A space X is called an supra(1,2)-extremely disconnected space (briefly supra (1,2)-E.D) if $\mu_1\mu_2$ closure of each supra-open in (X, μ_1) is supra-open set in (X, μ_1). Similarly $\mu_1\mu_2$-closure of each supra-open in (X, μ_2) is supra-open set in (X, μ_2).*

Theorem 6.2 *For a subset A of a supra(1,2) extremely disconnected space X, the following are equivalent:*

(1) *A is supra-open in (X, μ_1);*

(2) *A is supra(1,2) b-open and supra(1,2) locally closed.*

Proof (1)⇒(2) It is obvious.

(2)⇒(1) Let A be supra(1,2) b-open and supra(1,2) locally closed. Then

$$A \subseteq \mu_1\mu_2 cl(\mu_1 int(A)) \cup \mu_1 int(\mu_1\mu_2 cl(A)) \quad \text{and} \quad A = U \cap \mu_1\mu_2 cl(A),$$

where U is supra-open in (X, μ_1). So $A \subseteq U \cap (\mu_1 int(\mu_1\mu_2 cl(A)) \cup \mu_1\mu_2 cl(\mu_1 int(A)) \subseteq [\mu_1 int(U \cap \mu_1\mu_2 cl(A))] \cup [U \cap \mu_1\mu_2 cl(\mu_1 int(A))] \subseteq [\mu_1 int(U \cap \mu_1\mu_2 cl(A))] \cup [U \cap \mu_1 int(\mu_1\mu_2 cl(A))]$ (since X is supra(1,2) E.D)$\subseteq [\mu_1 int(U \cap \mu_1\mu_2 cl(A))] \cup [\mu_1 int(U \cap \mu_1\mu_2 cl(A))] = \mu_1 int(A) \cup \mu_1 int(A) = \mu_1 int(A)$. Hence $A \subseteq \mu_1 int(A)$. Therefore A is supra-open in (X, μ_1). □

Theorem 6.3 *Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be two bitopological spaces and μ_1, μ_2 be associated supra topologies with τ_1, τ_2. Let $f: X \to Y$ be a map. Then the following are equivalent.*

(1) *f is supra (1,2) b-continuous map;*

(2) *The inverse image of a $\sigma_1\sigma_2$-closed set in Y is a supra (1,2) b-closed set in X;*

(3) *Supra* $(1,2)bcl(f^{-1}(A) \subseteq f^{-1}(\sigma_1\sigma_2 cl(A))$ *for every set* A *in* Y;

(4) $f(supra(1,2)bcl(A)) \subseteq \sigma_1\sigma_2 cl(f(A))$ *for every set* $A \in X$;

(5) $f^{-1}(\sigma_1(B)) \subseteq supra(1,2)int(f^{-1}(B))$ *for every* B *in* Y.

Proof (1)⇒(2) Let A be a $\sigma_1\sigma_2$ closed set in Y, then $Y - A$ is $\sigma_1\sigma_2$ open set in Y. Since f is supra (1,2) b-continuous, $f^{-1}(Y - A) = X - f^{-1}(A)$ is a supra (1,2) b-open set in X. This implies that $f^{-1}(A)$ is a supra (1,2) b-closed subset of X.

(2)⇒(3) Let A be any subset of Y. Since $\sigma_1\sigma_2 cl(A)$ is $\sigma_1\sigma_2$ closed in Y, then $f^{-1}(\sigma_1\sigma_2 cl(A))$ is supra (1,2) b-closed in X. Hence supra $(1,2)bcl(f^{-1}(A) \subseteq supra(1,2)bcl(f^{-1}(\sigma_1\sigma_2 cl(A))) = f^{-1}(\sigma_1\sigma_2 cl(A))$.

(3)⇒(4) Let A be any subset of X. By (3), we obtain

$$f^{-1}(\sigma_1\sigma_2 cl(f(A))) \supseteq supra(1,2)bclf^{-1}(f(A)) \supseteq supra(1,2)bcl(A).$$

Hence $f(supra(1,2)cl(A)) \subseteq \sigma_1\sigma_2 cl(f(A))$.

(4)⇒(5) Let B be any subset of Y. By (5), $f(supra(1,2)bcl(X - f^{-1}(B))) \subset \sigma_1\sigma_2 cl(f(X - f^{-1}(B)))$ and $f(X - supra(1,2)bint(f^{-1}(B))) \subseteq \sigma_1\sigma_2 cl(Y - B) = Y - \sigma_1 int(B)$. Therefore we have

$$X - supra(1,2)bint(f^{-1}(B)) \subset f^{-1}(Y - \sigma_1 int(B))$$

and

$$f^{-1}(\sigma_1 int(B)) \subset supra(1,2)bint(f^{-1}(B)).$$

(5)⇒(1) Let B be a σ_1-open set in Y. Then by (4), $f^{-1}(\sigma_1 int(B)) \subseteq supra(1,2)int(f^{-1}(B))$. Therefore $f^{-1}(B) \subseteq supra(1,2)int(f^{-1}(B))$. But $supra(1,2)bint(f^{-1}(B)) \subseteq f^{-1}(B)$. Hence $f^{-1}(B) = supra(1,2)bint(f^{-1}(B))$. Therefore $f^{-1}(B)$ is supra (1,2) b-open in X. Thus f is supra (1,2) b-continuous map. □

We introduce the following definition.

Definition 6.4 *Let* (X, τ_1, τ_2) *and* (Y, σ_1, σ_2) *be two bitopological spaces and* μ_1, μ_2 *be associated supra bitopologies with* τ_1, τ_2. *A map* $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ *is called a supra (1,2) locally closed continuous [resp. supra (1,2) D(c,b) continuous, supra (1,2) locally b-closed continuous] if* $f^{-1}(B)$ *is supra (1,2) locally closed [resp. supra (1,2) D(c,b) set, supra (1,2) locally b-closed] in* X *for each* $\sigma_1\sigma_2$ *open set* V *of* Y.

Theorem 6.5 *Let* X *be supra (1,2) extremely disconnected space, the function* $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ *is supra (1,2)-continuous iff* f *is supra (1,2) b-continuous and supra (1,2) locally closed continuous.*

Proof Let V be a $\sigma_1\sigma_2$-open set in Y. Since f is supra (1,2)-continuous, $f^{-1}(V)$ is μ_1-open in X. Then by Theorem 3.3, $f^{-1}(V)$ is supra (1,2) b-open and supra (1,2) locally closed in X. Hence f is supra (1,2) b-continuous and supra (1,2) locally closed continuous. Conversely, let U be a $\sigma_1\sigma_2$-open set in Y. Since f is supra (1,2) b-continuous and supra (1,2) locally closed continuous, $f^{-1}(U)$ is supra (1,2) b-open and supra (1,2) locally-closed in X. Since X is supra (1,2) extremely disconnected, by Theorem 6.1, $f^{-1}(U)$ is μ_1-open in X. Hence f is supra (1,2)-continuous. □

Theorem 6.6 *The function $f:\ (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is supra (1,2) continuous iff f is supra (1,2) b-continuous and supra (1,2) D(c,b)-continuous.*

Proof Let V be a $\sigma_1\sigma_2$ open set in Y. Since f is supra (1,2) continuous, $f^{-1}(V)$ is μ_1-open in X. By Theorem 4.7, $f^{-1}(V)$ is supra (1,2) b-open and supra (1,2) D(c,b)set. Then f is supra (1,2) b-continuous and supra (1,2) D(c,b)continuous. Conversely, let U be a $\sigma_1\sigma_2$ open in Y. Since f is supra (1,2) b-continuous and supra (1,2) D(c,b)-continuous, $f^{-1}(U)$ is supra (1,2) b-open and supra (1,2) D(c,b)set. By Theorem 4.7, $f^{-1}(U)$ is supra open in (X,μ_1). Hence f is supra (1,2) continuous. □

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