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On the M-polynomial and Degree-Based Topological Indices of Dandelion Graph

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Abstract: For a graph G, the M-polynomial is defined to be

$$M(G;x,y)=\sum_{\delta\le\alpha\le\beta\le\Delta} m_{\alpha\beta}(G)x^{\alpha}y^{\beta},$$

where $m_{\alpha\beta}(\alpha,\beta\ge 1)$ is the number of edges ab of G such that $deg_G(a)=\alpha$ and $deg_G(b)=\beta$; and δ is the minimum degree and Δ is the maximum degree of G. The physicochemical properties of chemical graphs are found by topological indices, in particular, the degree-based topological indices, which can be determined from an algebraic formula called M-polynomial. In this paper, we first compute the M-polynomial of the Dandelion graph and the line graph of Dandelion graph. Further, we derive some degree-based topological indices of these graphs from their respective M-polynomial.

Key Words: M-polynomial, Smarandachely M-polynomial, degree-based topological indices, Dandelion graph, line graph.

AMS(2010): 05C07,05C31.

§1. Introduction

Throughout this paper, by a graph $G=(V,E)$, we mean a simple, undirected, and finite graph of order n and size m. Let $V(G)$ and $E(G)$ denote the vertex set and edge set of G, respectively. A *chemical graph* is a labeled graph where the atoms correspond to the vertices and the chemical bonds of the compound corresponds to the edges. A numerical quantity which is used to analyse both the physical and chemical properties of compounds is termed as a *topological index*. A topological index is also called a *graph invariant*. In general, the physicochemical properties and boiling activities of a chemical graph are investigated using topological indices.

The number of vertices of G adjacent to a given vertex v is the *degree* of the vertex v and is denoted by $deg_G(v)$. In a chemical graph, the degree of any vertex is at most 4. For all terms and definitions, not defined specifically in this paper, we refer to [5].

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The study of topological indices was first initiated by H. Wiener [16] in the year 1947. He introduced Weiner index in order to understand the correlation of the measured properties of molecules in a compound with their structural properties. In the year 1972, the Weiner index was interpreted by Hosoya [6] using distances between vertices in a graph. Over the last decade, various topological indices were introduced and studied by different authors [1,3,4].

Historically, various topological indices have been computed based on their mathematical definitions. Efforts have been made to explore a more streamlined approach capable of recovering multiple topological indices within a specific category. In this pursuit, the concept of a general polynomial was introduced, designed to yield the values of necessary topological indices through its derivatives and integrals. For instance, the Hosoya polynomial [7] is employed for calculating distance-based topological indices, while the NM-polynomial [15] is used to derive neighborhood degree sum-based indices. In 2015, Deutsch and Klažar [2] introduced the concept of M-polynomial to address the computation of degree-based topological indices. For more details on degree-based topological indices using the M-polynomial, we refer the readers to the references [11,12,13,14].

Definition 1.1 *For a graph G, the M-polynomial is defined to be*

$$M(G;x,y)=\sum_{\delta\leq\alpha\leq\beta\leq\Delta} m_{\alpha\beta}(G)x^{\alpha}y^{\beta},$$

where $m_{\alpha\beta}(\alpha,\beta\geq 1)$ is the number of edges ab of G such that $deg_G(a)=\alpha$ and $deg_G(b)=\beta$.

Particularly, if $deg_G(a)\neq deg_G(b)$ for all edges $ab\in E(G)$ such a M-polynomial is called a *Smarandachely M-polynomial* because it posses the character of Smarandachely denied axiom. For example, a star S_{n-1}. The authors in [8] introduced the concept of Dandelion graph while studying the Wiener inverse interval problem.

Definition 1.2 *A star graph, written S_{n-1}, is a graph on n vertices, consisting of some vertex, connected to $n-1$ leaves.*

Definition 1.3 *A Dandelion graph, written $D(n,l)$, is a graph on n vertices, consisted of a copy of the star S_{n-1} and copy of a path p_l on vertices $p_0,p_1,p_2,\ldots,p_{l-1}$, where p_0 is identified with a star center,*

Figure 1 shows an example of Dandelion graph $D(17,8)$.

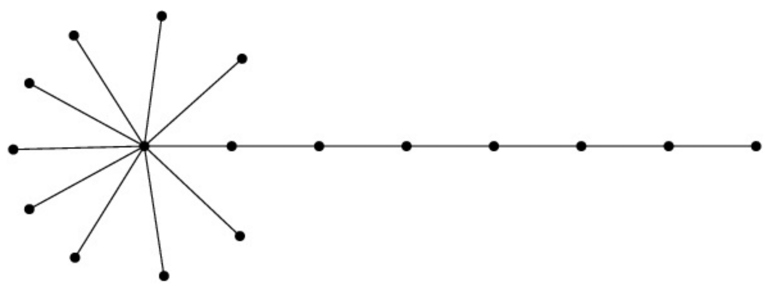

Figure 1. Dandelion graph $D(17,8)$

Siros Ghobadi et al. [9] computed the F-polynomial of Dandelion graphs. Again in [10], Siros Ghobadi et al. computed the first Zagreb index, F-index, and F-coindex of the line graph of Dandelion graph using subdivision concept. Motivated by this, we aim to calculate several algebraic polynomials and degree-based topological indices of Dandelion graph and the line graph of Dandelion graph.

§2. Methodology

We first divide the edge set of Dandelion graph and the line graph of Dandelion graph into different classes based on the degree of end vertices. With the help of this edge division, we compute the M-polynomial of Dandelion graph and the line graph of Dandelion graph. Further, by using M-polynomial, we compute the degree-based topological indices as listed in Table 1.

Table 1. Operations to derive degree-based topological indices form M-polynomial

Notation	Topological Index	Derivation from $M(G;x,y)$
$M_1(G)$	First Zagreb index	$(D_x+D_y)(M(G;x,y))\|_{x=y=1}$
$M_2(G)$	Second Zagreb index	$(D_xD_y)(M(G;x,y))\|_{x=y=1}$
${}^mM_2(G)$	Second modified Zagreb index	$(S_xS_y)(M(G;x,y))\|_{x=y=1}$
$R_\alpha(G)$	Randić index	$(D_x^\alpha D_y^\alpha)(M(G;x,y))\|_{x=y=1}$
$RR_\alpha(G)$	Inverse Randić index	$(S_x^\alpha S_y^\alpha)(M(G;x,y))\|_{x=y=1}$
$SSD(G)$	Symmetric division index	$(D_xS_y+D_yS_x)(M(G;x,y))\|_{x=y=1}$
$H(G)$	Harmonic index	$2S_xJ(M(G;x,y))\|_{x=1}$

where

$$\begin{aligned} M(G;x,y) &= f(x,y),\quad D_x = x\frac{\partial f(x,y)}{\partial x},\quad D_y = y\frac{\partial f(x,y)}{\partial y}, \\ S_x &= \int_o^x \frac{f(t,y)}{t}dt,\quad S_y = \int_o^y \frac{f(x,t)}{t}dt,\quad J(f(x,y)) = f(x,x) \end{aligned}$$

are the operators. As discussed in [2], each of these topological indices can be found using M-polynomial as given in Table 1.

§3. M-polynomial of Dandelion Graph

In this section we find the M-polynomial of Dandelion graph.

Theorem 3.1 *Let $G = D(n,l)$ be the Dandelion graph. Then the M-polynomial of G is*

$$M(G;x,y) = xy^2 + (l-3)x^2y^2 + x^2y^{n-l+1} + (n-l)xy^{n-l+1}.$$

Proof Let $G = D(n,l)$ be the Dandelion graph. From Figure 1, $|V(G)| = n$ and $|E(G)| =$

$n-1$. Since each of the vertices of G is of degree either 1 or 2 or $n-l+1$, the vertex set of G has three partitions with respect to degree

$V_1(G) = \{u \in V(G) : deg_G(u) = 1\}$;

$V_2(G) = \{u \in V(G) : deg_G(u) = 2\}$;

$V_3(G) = \{u \in V(G) : deg_G(u) = n-l+1\}$ such that $|V_1(G)| = n-l+1$, $|V_2(G)| = l-2$, $|V_3(G)| = 1$. Further, the edge set of G has four partitions based on the degree of the end vertices.

$E_1(G) = \{e = uv \in E(G) : deg_G(u) = 1, deg_G(v) = 2\}$,

$E_2(G) = \{e = uv \in E(G) : deg_G(u) = 2, deg_G(v) = 2\}$,

$E_3(G) = \{e = uv \in E(G) : deg_G(u) = 2, deg_G(v) = n-l+1\}$,

$E_4(G) = \{e = uv \in E(G) : deg_G(u) = 1, deg_G(v) = n-l+1\}$ such that $|E_1(G)| = 1; |E_2(G)| = l-3$, $|E_3(G)| = 1$, $|E_4(G)| = n-l$.

Now, from the definition of the M-polynomial,

$$\begin{aligned} M(G;x,y) &= \sum_{\alpha\le\beta} m_{\alpha\beta}(G)x^\alpha y^\beta \\ &= m_{12}(G)xy^2 + m_{22}(G)x^2y^2 + m_{2(n-l+1)}(G)x^2y^{n-l+1} + m_{1(n-l+1)}(G)xy^{n-l+1} \\ &= xy^2 + (l-3)x^2y^2 + x^2y^{n-l+1} + (n-l)xy^{n-l+1} \end{aligned}$$

This completes the proof. □

Now, we compute some degree-based topological indices of the Dandelion graph from this M-polynomial.

Theorem 3.2 *Let $G = D(n,l)$ be the Dandelion graph. Then*

$$\begin{aligned} M_1(G) &= n^2 + (3-2l)n + l^2 + l - 6, \\ M_2(G) &= n^2 + (3-2l)n + l^2 + l - 8, \\ {}^mM_2(G) &= \frac{(l+3)n - l^2 - 2l + 1}{4n - 4l + 4}, \\ R_\alpha(G) &= 2^\alpha(n-l+1) + (n-l)(n-l+1) + 4^\alpha(l-3) + 2^\alpha, \\ RR_\alpha(G) &= \frac{2^\alpha + 4^\alpha(n-l) + ((l-3)n - l^2 + 4l - 3) + 2^\alpha(n-l+1)}{4^\alpha(n-l+1)}, \\ SSD(G) &= \frac{2n^3 + (5-6l)n^2 + (6l^2 - 6l - 1)n - 2l^3 + l^2 + 5l - 2}{2n - 2l + 2}, \\ H(G) &= \frac{(3l+7)n^2 + (-6l^2 + l + 23)n + 3l^3 - 8l^2 - 5l - 6}{6n^2 + (30 - 12l)n + 6l^2 - 30l + 36}. \end{aligned}$$

Proof From Theorem 3.1, we have

$$M(G;x,y) = f(x,y) = xy^2 + (l-3)x^2y^2 + x^2y^{n-l+1} + (n-l)xy^{n-l+1}.$$

Then, we get the following:

$$
\begin{aligned}
D_x f(x,y) &= 2x^2y^{n-l+1} + (n-l)xy^{n-l+1} + 2(l-3)x^2y^2 + xy^2, \\
D_y f(x,y) &= (n-l+1)x^2y^n + (n-l)(n-l+1)xy^n + 2(l-3)x^2y^2 + 2xy^2, \\
(D_yD_x)(f(x,y)) &= 2(n-l+1)x^2y^n + (n-l)(n-l+1)xy^n + 4(l-3)x^2y^2 + 2xy^2, \\
S_x(f(x,y)) &= \frac{1}{2}x^2y^{n-l+1} + (n-l)xy^{n-l+1} + \frac{1}{2}(l-3)x^2y^2 + xy^2, \\
S_y(f(x,y)) &= \frac{1}{n-l+1}x^2y^{n-l+1} + \frac{n-l}{n-l+1}xy^{n-l+1} \\
&\quad + \frac{\left((l-3)n - l^2 + 4l - 3\right)x^2y^2}{2(n-l+1)} + \frac{1}{2}xy^2, \\
S_xS_y(f(x,y)) &= \frac{1}{2(n-l+1)}x^2y^{n-l+1} + \frac{n-l}{n-l+1}xy^{n-l+1} \\
&\quad + \frac{\left((l-3)n - l^2 + 4l - 3\right)x^2y^2}{4(n-l+1)} + \frac{1}{2}xy^2, \\
S_yD_x(f(x,y)) &= \frac{2x+n-l}{n-l+1}xy^{n-l+1} + \frac{\left((2l-6)n - 2l^2 + 8l - 6\right)}{2(n-l+1)}x^2y^2 + \frac{1}{2}xy^2, \\
S_xD_y(f(x,y)) &= \frac{n-l+1}{2}x^2y^{n-l+1} + \frac{\left(2n^2 + (2-4l)n + 2l^2 - 2l\right)}{2}xy^{n-l+1} \\
&\quad +(l-3)x^2y^2 + 2xy^2, \\
2S_xJ(f(x,y)) &= \frac{\left((3l-9)n^2 + (-6l^2 + 33l - 45)n + 3l^3 - 24l^2 + 63l - 54\right)x^4}{6n^2 + (30-12l)n + 6l^2 - 30l + 36} \\
&\quad + \frac{\left(4n^2 + (20-8l)n + 4l^2 - 20l + 24\right)x^3}{6n^2 + (30-12l)n + 6l^2 - 30l + 36} \\
&\quad + \frac{\left((12n - 12l + 24)x^3 + (12n^2 + (36-24l)n + 12l^2 - 36l)x^2\right)x^{n+l}}{6n^2 + (30-12l)n + 6l^2 - 30l + 36}
\end{aligned}
$$

Now, we have the following from Table 1:

(1) The first Zagreb index

$$M_1(G) = (D_x + D_y)(f(x,y))|_{x=y=1} = n^2 + (3-2l)n + l^2 + l - 6.$$

(2) The second Zagreb index

$$M_2(G) = (D_xD_y)(f(x,y))|_{x=y=1} = n^2 + (3-2l)n + l^2 + l - 8.$$

(3) The second modified Zagreb index

$${}^mM_2(G) = (S_xS_y)(f(x,y))|_{x=y=1} = \frac{(l+3)n - l^2 - 2l + 1}{4n - 4l + 4}.$$

(4) The Randić index

$$R_\alpha(G) = (D_x^\alpha D_y^\alpha)(f(x,y))|_{x=y=1} = 2^\alpha(n-l+1) + (n-l)(n-l+1) + 4^\alpha(l-3) + 2^\alpha.$$

(5) The inverse Randić index

$$RR_\alpha(G) = (S_x^\alpha S_y^\alpha)(f(x,y))|_{x=y=1} = \frac{2^\alpha + 4^\alpha(n-l) + ((l-3)n - l^2 + 4l - 3) + 2^\alpha(n-l+1)}{4^\alpha(n-l+1)}.$$

(6) The symmetric division index

$$SSD(G) = (D_xS_y + D_yS_x)(f(x,y))|_{x=y=1} = \frac{2n^3 + (5-6l)n^2 + (6l^2 - 6l - 1)n - 2l^3 + l^2 + 5l - 2}{2n - 2l + 2}.$$

(7) The harmonic index

$$H(G) = 2S_xJ(f(x,y))|_{x=1} = \frac{(3l+7)n^2 + (-6l^2 + l + 23)n + 3l^3 - 8l^2 - 5l - 6}{6n^2 + (30-12l)n + 6l^2 - 30l + 36}.$$

This completes the proof. □

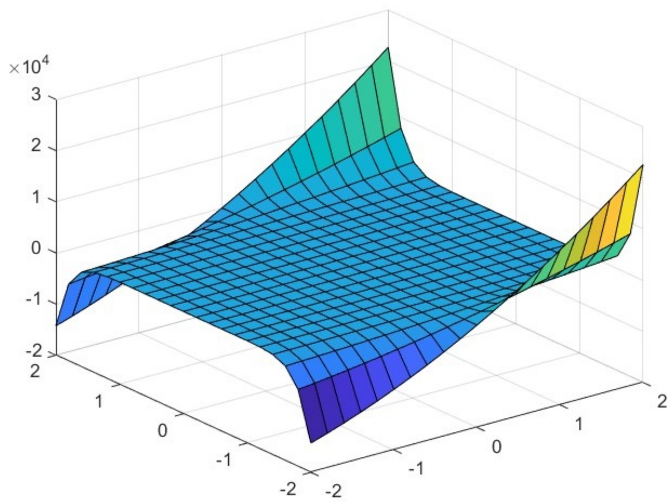

Figure 2 Plot of M-polynomial of Dandelion graph $D(17, 8)$

§4. M-polynomial of the Line Graph of Dandelion Graph

There are many graph operators (or graph valued functions) with which one can construct a new graph from a given graph, such as the line graphs, line cut-vertex graphs; total graphs; middle graphs; and their generalizations. A *line graph* of a graph G, written $L(G)$, is the graph whose vertices are the edges of G, with two vertices of $L(G)$ adjacent whenever the corresponding edges of G have a vertex in common.

In the next theorem, we find the M-polynomial of the line graph of Dandelion graph.

Theorem 4.1 *Let $G = D(n,l), l > 4$, be a Dandelion graph. Then, the M-polynomial of $L(G)$ is*

$$M(L(G); x, y) = xy^2 + (l-4)x^2y^2 + x^2y^{n-l+1} + (n-l)x^{n-l}y^{n-l+1} + \frac{n^2+l^2-2nl-n+l}{2}x^{n-l}y^{n-l}.$$

Proof Let $G = D(n,l), l > 4$, be the Dandelion graph. Then $|V(L(G))| = n-1$. Since each of the vertices of $L(G)$ is of degree either 1 or 2 or $n-l$ or $n-l+1$, the vertex set of $L(G)$ has four partitions with respect to degree:

$V_1(L(G)) = \{u \in V(L(G)) : deg_{L(G)}(u) = 1\}$,

$V_2(L(G)) = \{u \in V(L(G)) : deg_{L(G)}(u) = 2\}$,

$V_3(L(G)) = \{u \in V(L(G)) : deg_{L(G)}(u) = n-l\}$ and

$V_4(L(G)) = \{u \in V(L(G)) : deg_{L(G)}(u) = n-l+1\}$ such that

$$|V_1(L(G))| = 1, \quad |V_2(L(G))| = l-3, \quad |V_3(L(G))| = n-l, \quad |V_4(L(G))| = 1.$$

Further, the edge set of G has five partitions based on the degree of the end vertices.

$E_1(L(G)) = \{e = uv \in E(L(G)) : deg_{L(G)}(u) = 1, deg_{L(G)}(v) = 2\}$,

$E_2(L(G)) = \{e = uv \in E(L(G)) : deg_{L(G)}(u) = 2, deg_{L(G)}(v) = 2\}$,

$E_3(L(G)) = \{e = uv \in E(L(G)) : deg_{L(G)}(u) = 2, deg_{L(G)}(v) = n-l+1\}$,

$E_4(L(G)) = \{e = uv \in E(L(G)) : deg_{L(G)}(u) = n-l, deg_{L(G)}(v) = n-l+1\}$,

$E_5(L(G)) = \{e = uv \in E(L(G)) : deg_{L(G)}(u) = n-l, deg_{L(G)}(v) = n-l\}$ such that

$$|E_1(L(G))| = 1, \quad |E_2(L(G))| = l-4, \quad |E_3(L(G))| = 1, \quad |E_4(L(G))| = n-l,$$

and
$$|E_5(L(G))| = \frac{n^2+l^2-2nl-n+l}{2}.$$

Now, from the definition of the M-polynomial,

$$M(L(G); x, y) = xy^2 + (l-4)x^2y^2 + x^2y^{n-l+1} + (n-l)x^{n-l}y^{n-l+1} + \frac{n^2+l^2-2nl-n+l}{2}x^{n-l}y^{n-l}.$$

This completes the proof. □

Now, we compute some degree-based topological indices of the line graph of Dandelion graph from this M-polynomial.

Theorem 4.2 *Let $G = D(n,l), l > 4$, be the Dandelion graph. Then,*

$$M_1(L(G)) = 2n^2 + (-4l + 2k + 2)n + 2l^2 + (2-2k)l - 10,$$

$$
\begin{aligned}
M_2(L(G)) &= n^3+(-3l+k+1)n^2+(3l^2+(-2k-2)l+2)n-l^3+(k+1)l^2+2l-12,\\
{}^mM_2(L(G)) &= \frac{(l-2)n^3+(-3l^2+7l+4)n^2+(3l^3-8l^2-8l+4k)n-l^4+3l^3+4l^2-4kl+4k}{4n^3+(4-12l)n^2+(12l^2-8l)n-4l^3+4l^2},\\
R_\alpha(L(G)) &= 2^\alpha(n-l+1)+(n-l)(n-l+1)+4^\alpha(l-3)+2^\alpha,\\
RR_\alpha(L(G)) &= \frac{2^\alpha+4^\alpha(n-l)+((l-3)n-l^2+4l-3)+2^\alpha(n-l+1)}{4^\alpha(n-l+1)},\\
SSD(L(G)) &= \frac{5n^2+(-6l+4k-5)n+l^2+(9-4k)l+4k-4}{2n-2l+2},\\
H(L(G)) &= \frac{A}{B}.
\end{aligned}
$$

where

$$
\begin{aligned}
A &= (6l-4)n^3+(-18l^2+33l+12k+4)n^2+(18l^3-54l^2\\
&\quad +(1-24k)l+42k-12)n-6l^4+25l^3+(12k-5)l^2+(12-42k)l+18k,\\
B &= 12n^3+(42-36l)n^2+(36l^2-84l+18)n-12l^3+42l^2-18l.
\end{aligned}
$$

Proof From Theorem 4.1, we have

$$M(L(G);x,y)=xy^2+(l-4)x^2y^2+x^2y^{n-l+1}+(n-l)x^{n-l}y^{n-l+1}+kx^{n-l}y^{n-l},$$

where

$$k=\frac{n^2+l^2-2nl-n+l}{2}.$$

Then, we have the following

$$
\begin{aligned}
D_xf(x,y) &= (n-l)^2x^{n-l}y^{n-l+1}+2x^2y^{n-l+1}+k(n-l)x^{n-l}y^{n-l}+2(l-4)x^2y^2+xy^2,\\
D_yf(x,y) &= (n-l)(n-l+1)x^{n-l}y^{n-l+1}+(n-l+1)x^2y^{n-l+1}+k(n-l)x^{n-l}y^{n-l}\\
&\quad +2(l-4)x^2y^2+2xy^2,\\
(D_yD_x)(f(x,y)) &= (n-l)^2(n-l+1)x^{n-l}y^{n-l+1}+2(n-l+1)x^2y^{n-l+1}+k(n-l)^2x^{n-l}y^{n-l}\\
&\quad +4(l-4)x^2y^2+2xy^2.
\end{aligned}
$$

$$
\begin{aligned}
S_x(f(x,y)) &= \frac{((l-4)n-l^2+4l)x^{l+2}y^{l+2}+(2n-2l)x^{l+1}y^{l+2}+(2n-2l)x^ny^{n+1}+(n-l)x^{l+2}y^{n+1}}{(2n-2l)x^ly^l}\\
&\quad +\frac{2kx^ny^n}{(2n-2l)x^ly^l}
\end{aligned}
$$

$$
\begin{aligned}
S_y(f(x,y)) &= \frac{((l-4)n^2+(-2l^2+9l-4)n+l^3-5l^2+4l)x^{l+2}+(n^2+(1-2l)n+l^2-l)x^{l+2}y^{l+2}}{(2n^2+(2-4l)n+2l^2-2l)x^ly^l}\\
&\quad +\frac{(2n^2-4nl+2l^2)x^ny^{n+1}+(2n-2l)x^{l+2}y^{n+1}+(2kn-2kl+2k)x^ny^n}{(2n^2+(2-4l)n+2l^2-2l)x^ly^l}
\end{aligned}
$$

$$
\begin{aligned}
S_xS_y(f(x,y)) &= \frac{((l-4)n^3+(-3l^2+13l-4)n^2+(3l^3-14l^2+8l)n-l^4-5l^3-4l^2)x^{l+2}y^{l+2}}{(2n-2l)(2n^2+(2-4l)n+2l^2-2l)x^ly^l} \\
&+\frac{(2n^3+(2-6l)n^2+(6l^2-4l)n-2l^3+2l^2)x^{l+2}y^{l+2}}{(2n-2l)(2n^2+(2-4l)n+2l^2-2l)x^ly^l} \\
&+\frac{(4n^2-8ln+4l^2)x^ny^{n+1}+(2n^2-4nl+2l^2)x^{l+2}y^{n+1}+(4kn-4kl+4k)x^ny^n}{(2n-2l)(2n^2+(2-4l)n+2l^2-2l)x^ly^l}, \\
S_yD_x(f(x,y)) &= \frac{\left((2l-8)n-2l^2+10l-8)x^2+(n-l+1)x\right)x^ly^{l+2}}{(2n-2l+2)x^ly^l} \\
&+\frac{\left((2n^2-4ln+2l^2)x^n+4x^{l+2}\right)y^{n+1}+(2kn-2kl+2k)x^ny^n}{(2n-2l+2)x^ly^l}, \\
S_yD_x(f(x,y)) &= \frac{((2l-8)x^2+4x)x^ly^{l+2}+(2n-2l+2)x^ny^{n+1}+(n-l+1)x^{l+2}y^{n+1}+2kx^ny^n}{2x^ly^l}, \\
2S_xJ(f(x,y)) &= \frac{(24n^2+(12-48l)n+24l^2-12l)x^{n+l+3}}{(12n^3+(42-36l)n^2+(36l^2-84l+18)n-12l^3+42l^2-18l)\,x^{2l}} \\
&+\frac{\left((6l-24)n^3+(-18l^2+93l-84)n^2+(18l^3-114l^2+177l-36)n\right)x^4}{(12n^3+(42-36l)n^2+(36l^2-84l+18)n-12l^3+42l^2-18l)} \\
&+\frac{\left(-6l^4+45l^3-93l^2+36l)\right)x^4}{(12n^3+(42-36l)n^2+(36l^2-84l+18)n-12l^3+42l^2-18l)} \\
&+\frac{\left((8n^3+(28-24l)n^2+(24l^2-56l+12)n-8l^3+28l^2-12l)\right)x^3}{(12n^3+(42-36l)n^2+(36l^2-84l+18)n-12l^3+42l^2-18l)} \\
&+\frac{\left(12n^3+(36-36l)n^2+(36l^2-72l)n-12l^3+36l^2\right)x^{2n+1}}{(12n^3+(42-36l)n^2+(36l^2-84l+18)n-12l^3+42l^2-18l)\,x^{2l}} \\
&+\frac{\left(12kn^2+(42k-24kl)n+12kl^2-42kl+18k\right)x^{2n}}{(12n^3+(42-36l)n^2+(36l^2-84l+18)n-12l^3+42l^2-18l)\,x^{2l}}
\end{aligned}
$$

We finally get the results by applying the appropriate operations given in Table 1. □

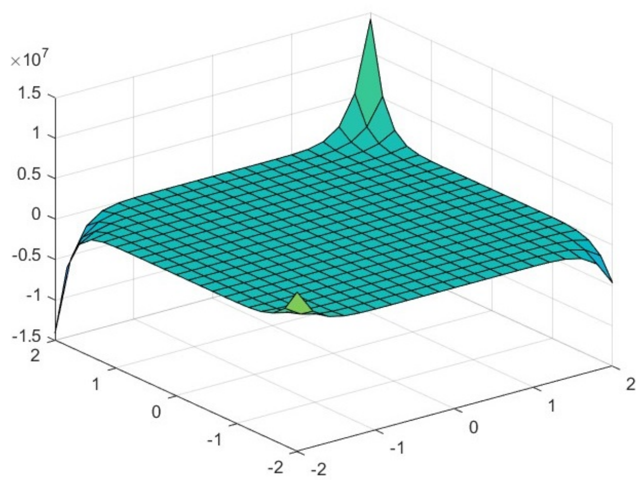

Figure 3 Plot of M-polynomial of the line graph of Dandelion graph $D(17,8)$

§5. Conclusion

Topological indices play an important role in understanding many physical and chemical properties of a chemical compound. Some of the degree-based topological indices can be found by means of the M-polynomial of the corresponding chemical graph. In this paper, we have determined some of these topological indices using the closed form of the M-polynomial of the Dandelion graph and the line graph of Dandelion graph. The M-polynomial can be determined for graph operations, graph products, and graph powers also. The study on M-polynomials with respect to different types of graph operators also seem to be much promising.

References

[1] F. M. Bruckler, T. Došlić, A. Graovac, I. Gutman,On a class of distance - based molecular structure descriptors, *Chemical Physics Letters* 503, (4-6), 336-338, 2011.

[2] E. Deutsch, S. Klavźar,M-polynomial and degree-based topological indices, *Iranian Journal of Mathematical Chemistry*, 6(2), 93-102, 2015.

[3] K. C. Das, N. Trinajstić,Comparison between first geometric - arithmetic index and atom-bond connectivity index, *Chem. Phys. Lett.* 497, 149-151, 2010.

[4] G. Fath-Tabar, B. Furtula, I. Gutman,A new geometric-arithmetic index, *J. Math. Chem.* 47, 477-486, 2010.

[5] F. Harary, *Graph Theory*, Addison Wesley, Reading, 1969.

[6] H. Hosoya,Topological index, a newly proposed quantity characterizing the topological nature of structure isomers of saturated hydrocarbons, *Bull. Chem. Soc. Jpn*, 44, 2332-2339, 1971.

[7] H. Hosoya,On some counting polynomials in chemistry, *Discrete Appl. Math.*, 19, 239-257, 1988.

[8] Matjaž Krnca, Riste Škrekovski, On Wiener inverse interval problem, *MATCH Commun. Math. Comput. Chem.*, 75, 71-80, 2016.

[9] Sirous Ghobadi, Mobina Ghorbaninejad, On F-polynomial, multiple and hyper F-index of some molecular graphs, *Bulletin of Mathematical Sciences and Applications*, 20, 36-43, 2018.

[10] Sirous Ghobadi, Mobina Ghorbaninejad, First Zagreb index, F-index and F-coindex of the line subdivision graphs, *Turkish Journal of Analysis and Number Theory*, 5 (1), 23-26, 2017.

[11] H. C. Shilpa, K. Gayathri, H. M. Nagesh, N. Narahari, On the M-polynomials and degree-based topological indices of an important class of graphs, *International J.Math. Combin.*, 2, 38-50, 2023.

[12] H. C. Shilpa, K. Gayathri, H. M. Nagesh, N. Narahari,On the M-polynomials and degree-based topological indices of Mycielskian of paths and cycles, *Communications in Mathematics and Applications* (in Press).

[13] H. C. Shilpa, K. Gayathri, H. M. Nagesh, M-polynomial and neighborhood M-polynomial of some concise drug structures: A-315675, Oseltamivir carboxylate, Zanamivir, and BCX-

1812, *Letters in Applied NanoBioScience* (in Press).

[14] N. N. Swamy, C. K. Gangappa, P. Poojary, B. Sooryanarayana, H. M. Nagesh, Topological indices of the subdivision graphs of the nanostructure $TUC_4C_8(R)$ using M-polynomials, *Journal of Discrete Mathematical Sciences and Cryptography*, 25(1), 265-282, 2022.

[15] A. Verma, S. Mondal, N. De, A. Pal,Topological properties of bismuth tri-iodide using neighborhood M-polynomial, *Int. J. Math. Trends Technol.*, 67, 83-90, 2019.

[16] H. Wiener,Structural determination of the paraffin boiling points, *J. Am. Chem. Soc.*, 69, 17-20, 1947.