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Open Neighborhood Coloring of a Generalized Antiprism Graph

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Abstract: An open neighborhood coloring of a graph is a coloring in which vertices adjacent with a common vertex are colored differently. The minimum number of colors used in an open neighborhood coloring of a graph G is called the open neighborhood chromatic number of G. We determine this parameter for a generalization of the antiprism graph in this paper.

Key Words: Coloring, chromatic number, open neighborhood coloring, Smarandachely open neighborhood coloring, antiprism.

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§1. Introduction

A *vertex coloring* or simply, a *coloring* of a graph $G = (V, E)$ is an assignment of colors to the vertices of G. A k-coloring of G is a surjection $c : V \to \{1, 2, \cdots, k\}$. A proper coloring of G is an assignment of colors to the vertices of G so that adjacent vertices are colored differently. A *proper k-coloring* of G is a surjection $c : V \to \{1, 2, \cdots, k\}$ such that $c(u) \neq c(v)$ if u and v are adjacent in G. The minimum k for which there is a proper k-coloring of G is called the *chromatic number* of G denoted $\chi(G)$.

An *open neighborhood coloring* [5] of a graph is a coloring in which vertices adjacent with a common vertex are colored differently. In other words, an open neighborhood coloring of a graph $G(V, E)$ is a coloring $c : V \to Z^+$ such that for each $w \in V$ and every $u, v \in N(w)$, $c(u) \neq c(v)$ and generally, for a subgraph Γ such as $P_2, K_{1,3}$ of G if there is an open neighborhood coloring c on graph $G - \Gamma$, G is said to be a *Smarandachely open neighborhood coloring* on Γ. Clearly, if $\Gamma = \emptyset$, a Smarandachely open neighborhood coloring of G is nothing else but an open neighborhood coloring of G. The minimum number of colors used in an open neighborhood coloring of a graph G is called the *open neighborhood chromatic number* of G, denoted $\chi_{onc}(G)$.

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The concept of open neighborhood coloring was introduced in the year 2013 by Geetha *et* al. [5]. Further, in [6], this parameter has been obtained for the Prism graph which is a particular case of the generalised Petersen graph $GP(n, k)$. In [8, 9], the open neighborhood chromatic number has been obtained for the class of antiprism graphs and some path related graphs such as line graph, total graph and transformation graphs of a path.

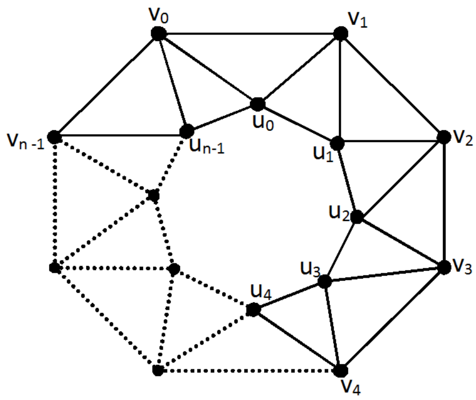

Figure 1. n-Antiprism graph

The graph obtained by replacing the faces of a polyhedron with its edges and vertices is called the *skeleton* [3] of the polyhedron. An *n-antiprism* [2], $n \geq 3$, is a semiregular polyhedron constructed with $2n$-gons and $2n$ triangles. It is made up of two n-gons on top and bottom, separated by a ribbon of $2n$ triangles, with the two n-gons being offset by one ribbon segment. The graph corresponding to the skeleton of an n-antiprism is called the $n-$*antiprism graph*, denoted by Q_n, $n \geq 3$ as shown in Figure 1. As seen from this figure, Q_n has $2n$ vertices and $4n$ edges, and is isomorphic to the circulant graph $\mathrm{Ci}_{2n}(1, 2)$.

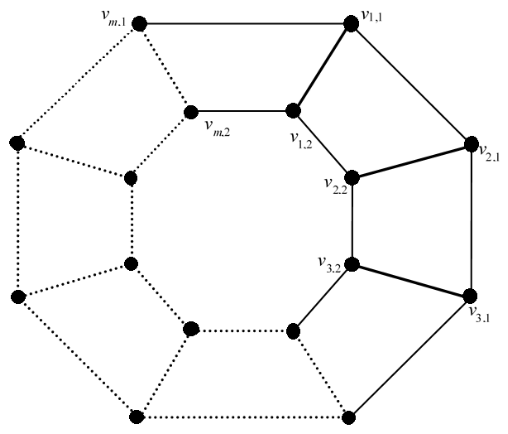

Figure 2. Prism graph

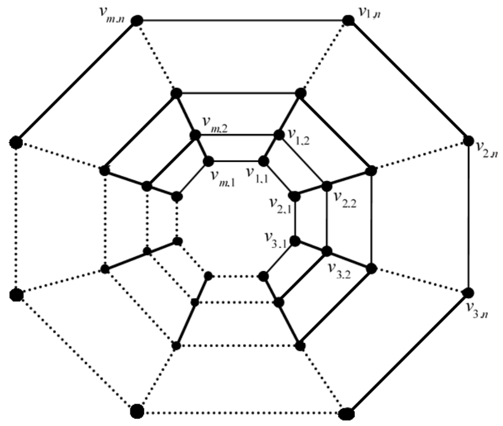

Figure 3. Generalized prism graph

A *prism graph* [7] Y_n is a graph corresponding to the skeleton of an n-prism and has $2n$ vertices and $3n$ edges as shown in Figure 2. A *generalized prism* graph [4], denoted $Y_{m,n} = C_m \times P_n$, is the graph having mn vertices and $m(2n - 1)$ edges as shown in Figure 3. Its vertex set is given by $V(Y_{m,n}) = V(C_m \times P_n) = \{v_{i,j} : 1 \leqslant i \leqslant m, 1 \leqslant j \leqslant n\}$ and its edge set $E(Y_{m,n}) = E(C_m \times P_n) = \{v_{i,j}v_{i+1,j} : 1 \leqslant i \leqslant m-1, 1 \leqslant j \leqslant n\} \bigcup \{v_{m,j}v_{1,j} : 1 \leqslant j \leqslant n\} \bigcup \{v_{i,j}v_{i,j+1} : 1 \leqslant i \leqslant m, 1 \leqslant j \leqslant n-1\}$.

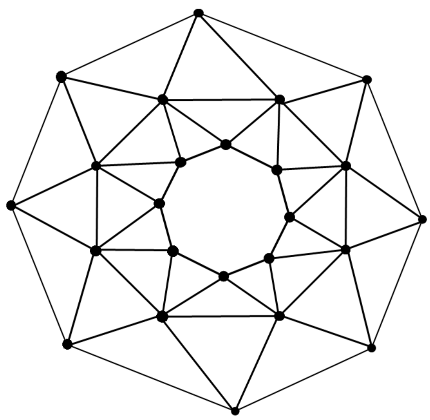

Figure 4. Generalized antiprism graph A_8^3

As introduced in [1], the *generalized antiprism* A_m^n can be obtained by completing the generalized prism $C_m \times P_n$ by edges $\{v_{i,j+1}v_{i+1,j} : 1 \leqslant i \leqslant m-1, 1 \leqslant j \leqslant n-1\} \bigcup \{v_{m,j+1}v_{1,j} : 1 \leqslant j \leqslant n-1\}$ where $V(A_m^n) = V(C_m \times P_n) = \{v_{i,j} : 1 \leqslant i \leqslant m, 1 \leqslant j \leqslant n\}$ is the vertex set of A_m^n. Thus, $E(A_m^n) = E(C_m \times P_n) \bigcup \{v_{i,j+1}v_{i+1,j} : 1 \leqslant i \leqslant m, 1 \leqslant j \leqslant n-1\}$ is the edge set of A_m^n, where i is taken modulo m. In particular, if $n = 2$, we obtain the antiprism graph Q_m. For example, the graph A_8^3 is as shown in Figure 4.

§2. Open Neighborhood Coloring of A_m^3

In this section, we obtain the open neighborhood chromatic number of the generalized antiprism graph A_m^n for $m \geqslant 3$ and $n = 3$. Using this result, we determine the open neighborhood chromatic number of the generalized antiprism graph A_m^n for any $m \geqslant 3, n \geqslant 2$ in the following section.

To begin with, we recall some of the important definitions and results obtained by various authors for immediate reference.

Theorem 2.1 ([5]) *If f is an open neighborhood k-coloring of $G(V,E)$ with $\chi_{onc}(G) = k$, then $f(u) \neq f(v)$ holds where u, v are the end vertices of a path of length 2 in G.*

Theorem 2.2 ([5]) *For any graph $G(V,E)$, $\chi_{onc}(G) \geq \Delta(G)$.*

Theorem 2.3 ([5]) *If H is a connected subgraph of G, then $\chi_{onc}(H) \leq \chi_{onc}(G)$.*

Theorem 2.4 ([5]) *Let $G(V,E)$ be a connected graph on $n \geq 3$ vertices. Then $\chi_{onc}(G) = n$ if and only if $N(u) \bigcap N(v) \neq \emptyset$ holds for every pair of vertices $u, v \in V(G)$.*

Theorem 2.5 ([8]) *For an antiprism graph $Q_n, n \geq 3$,*

$$\chi_{onc}(Q_n) = \begin{cases} 5, & \text{if } n \equiv 0 (mod\ 5), \\ 7, & \text{if } n = 7, \\ 8, & \text{if } n = 4, \\ 6, & \text{otherwise.} \end{cases}$$

Observation 2.6 For $m \geqslant 3$, $Q_m \subseteq A_m^3$ so that $\chi_{onc}(A_m^3) \geqslant \chi_{onc}(Q_m)$.

Definition 2.7 ([6]) *In a graph G, a subset V_1 of $V(G)$ such that no two vertices of V_1 are end vertices of a path of length two in G is called a P_3-independent set of G.*

Lemma 2.8 *For the generalized antiprism graph $A_m^3, m \geq 3$, $\chi_{onc}(A_m^3) \geq 7$.*

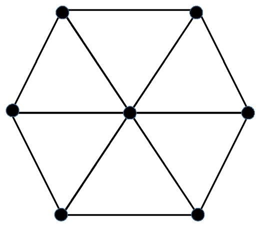

Figure 5. Subgraph H of A_m^3

Proof For each $m \geq 3$, it is easy to observe that A_m^3 contains the subgraph H in Figure 5. Further, in H, there is a path of length two between every pair of vertices so that $\chi_{onc}(H) = 7$. Hence, by Theorem 2.3, $\chi_{onc}(A_m^3) \geq 7$. □

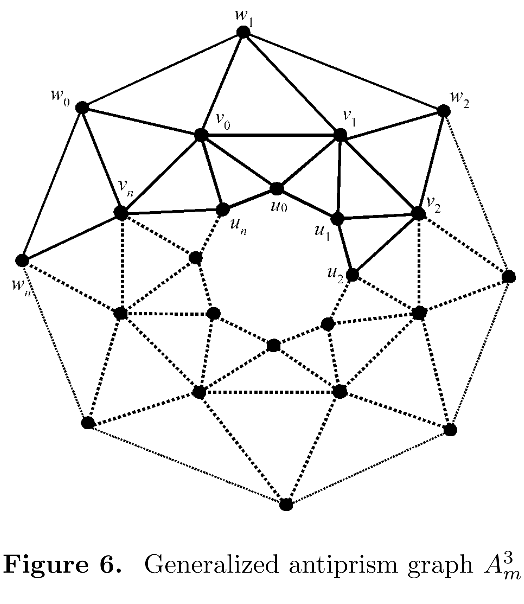

Figure 6. Generalized antiprism graph A_m^3

Observation 2.9 In the generalized antiprism graph A^3_m as shown in Figure 6,

(i) The only vertices that are connected to a vertex $u_i, 0 \leq i \leq m-1$ by a path of length two are $u_{i\pm1}, u_{i\pm2}, v_i, v_{i\pm1}, v_{i+2}, w_i, w_{i+1}, w_{i+2}$ where the suffix is under modulo m;

(ii) The only vertices that are connected to a vertex $v_i, 0 \leq i \leq m-1$ by a path of length two are $u_i, u_{i\pm1}, u_{i-2}, v_{i\pm1}, v_{i\pm2}, w_{i\pm1}, w_i, w_{i+2}$ where the suffix is under modulo m;

(iii) The only vertices that are connected to a vertex $w_i, 0 \leq i \leq m-1$ by a path of length two are $u_i, u_{i-1}, u_{i-2}, v_{i\pm1}, v_i, v_{i-2}, w_{i\pm1}, w_{i\pm2}$ where the suffix is under modulo m.

Lemma 2.10 *For the generalized antiprism graph A^3_m,*

$$\chi_{onc}(A^3_m) = \begin{cases} 9, & \textit{if } m = 3, 6, \\ 8, & \textit{if } m = 4, 5. \end{cases}$$

Proof We prove the result by considering the following cases.

Case 1. $m = 3$.

It is easy to observe that, in A^3_3, every vertex is connected to every other vertex by a path of length two. Thus, in any open neighborhood coloring of A^3_3, every vertex has to be given a different color so that $\chi_{onc}(A^3_3) = 9$.

Case 2. $m = 4$.

By Observation 2.6 and Theorem 2.5, we have $\chi_{onc}(A^3_4) \geqslant \chi_{onc}(Q_4) = 8$. The reverse inequality can be established from Figure 7. Hence, $\chi_{onc}(A^3_4) = 8$.

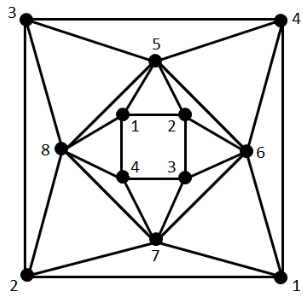

Figure 7. An open neighborhood coloring of the graph A^3_4

Case 3. $m = 5, 6$.

By observation, it is seen that a color can be assigned to not more than two vertices in any open neighborhood coloring of A^3_5 and A^3_6 so that $\chi_{onc}(A^3_5) \geqslant 8$ and $\chi_{onc}(A^3_6) \geqslant 9$. Further, the open neighborhood coloring of A^3_5 and A^3_6 as shown in Figure 8 ensure that $\chi_{onc}(A^3_5) \leqslant 8$ and $\chi_{onc}(A^3_6) \leqslant 9$.

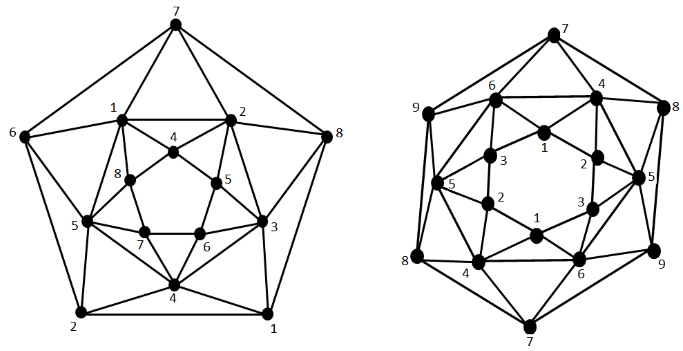

Figure 8. An open neighborhood coloring of the graphs A_5^3 and A_6^3

This completes the proof. □

Lemma 2.11 *In the generalized antiprism graph* $A_m^3, m \geqslant 7$, *for each* l, $0 \leq l \leq 6$, *the set* $S_l = \{u_i, v_j, w_k \mid i \equiv l(mod\ 7), j \equiv l+3(mod\ 7) and\ k \equiv l-1(mod\ 7)\}$ *is a* P_3- *independent set if and only if* $m \equiv 0(mod\ 7)$.

Proof Let $m \equiv 0(mod\ 7)$. It is given that $S_l = \{u_i, v_j, w_k \mid i \equiv l(mod\ 7), j \equiv l + 3(mod\ 7)\ and\ k \equiv l - 1(mod\ 7)\}$. It is easy to observe that each $S_i, 0 \leqslant i \leqslant 6$, is a P_3-independent set of A_m^3.

We prove the converse by the method of contraposition. Suppose that $m \not\equiv 0(mod\ 7)$. Then, we have the following cases.

Case 1. $m \equiv 1(mod\ 7)$. Then, $u_0, u_{m-1} \in S_0$. In such a case, S_0 is not a P_3- independent set as u_0 and u_{m-1} are end vertices of a path of length two.

Case 2. $m \equiv 2(mod\ 7)$. Then, $u_0, u_{m-2} \in S_0$. But u_0 and u_{m-2} are end vertices of a path of length two so that S_0 is not a P_3- independent set.

The other cases follow similarly. □

Theorem 2.12 *For the generalized antiprism graph* $A_m^3, m \geqslant 3$, $\chi_{onc}(A_m^3) = 7$ *if and only if* $m \equiv 0(mod\ 7)$.

Proof Suppose $m \equiv 0(mod\ 7)$. From Lemma 2.8, we have $\chi_{onc}(A_m^3) \geqslant 7$. Further, by Lemma 2.11, each $S_l = \{u_i, v_j, w_k \mid i \equiv l(mod\ 7), j \equiv l + 3(mod\ 7) and\ k \equiv l - 1(mod\ 7)\}, 0 \leqslant l \leqslant 6$ is a P_3-independent set of A_m^3.

Define a function $c : V(A_m^3) \to \{1, 2, \cdots, 7\}$ as $c(v) = l + 1$ such that $v \in S_l, 0 \leqslant l \leqslant 6$. Clearly, c is an open neighborhood coloring of A_m^3 so that $\chi_{onc}(A_m^3) \leqslant 7$. Thus, $\chi_{onc}(A_m^3) = 7$ if $m \equiv 0(mod\ 7)$.

Conversely, let $\chi_{onc}(A_m^3) = 7$. In view of Theorem 2.5 and Lemma 2.10, we have $m \geqslant 7$. By observation, in the graph A_m^3, none of the vertices $v_{i-1}, v_i, v_{i+1}, u_{i-1}, u_i, w_i, w_{i+1}$, the suffix taken under modulo 7, can be given the same color in an open neighborhood coloring. Thus, if $m \not\equiv 0(mod\ 7)$, then seven colors are not sufficient to have an open neighborhood coloring of A_m^3 so that $m \equiv 0(mod\ 7)$. □

Observation 2.13 Every integer $m \geqslant 11$, with $m \not\equiv 0(mod\ 7), m \neq 13, 17$ can be written as $m = 4k + 7l$ for integers $k \geqslant 1, l \geqslant 0$.

Theorem 2.14 *For $m \geqslant 3$,*

$$\chi_{onc}(A^3_m) = \begin{cases} 9, & \textit{if } m = 3, 6, \\ 7, & \textit{if } m \equiv 0 (mod\ 7), \\ 8, & \textit{otherwise.} \end{cases}$$

Proof The result holds for $m \leqslant 6$ and $m \equiv 0(mod\ 7)$ from Lemma 2.10 and Theorem 2.12. We now consider the case when $m \geqslant 3$ is an integer such that $m \neq 3, 6$ and $m \not\equiv 0(mod\ 7)$. In view of Theorem 2.12, $\chi_{once}(A^3_m) \geqslant 8$. Hence, it suffices to show that, in this case, $\chi_{once}(A^3_m) \leqslant 8$ for which we take various cases as follows.

Case 1. $m = 8$. Consider the coloring $c : V(A^3_8) \to \{1, 2, \cdots, 8\}$ defined by $c(v) = i + 1$ for $v = u_i, v = v_{i+6}$ or $v = w_{i+3}$ for $0 \leqslant i \leqslant 7$, the suffix taken under modulo 8. It is easy to verify that c is an open neighborhood coloring of A^3_8 using eight colors so that $\chi_{once}(A^3_8) \leqslant 8$.

Case 2. $m = 9$. Observing that the coloring of A^3_9 in Figure 9 is an open neighborhood coloring, $\chi_{once}(A^3_9) \leqslant 8$.

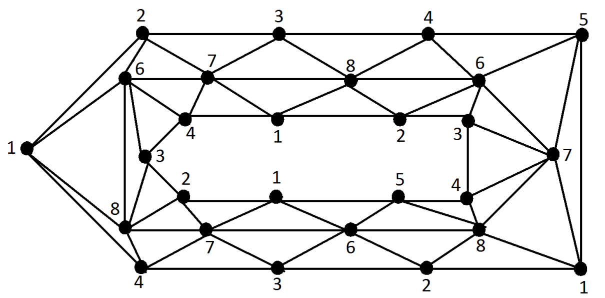

Figure 9. An open neighborhood coloring of A^3_9

Case 3. As seen from Figure 10, A^3_{10} can be colored with eight colors in an open neighborhood coloring so that $\chi_{once}(A^3_{10}) \leqslant 8$.

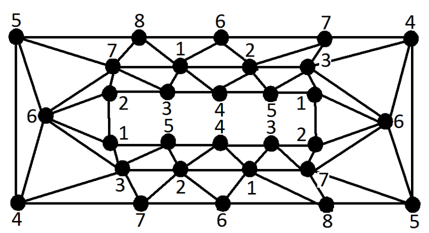

Figure 10. An open neighborhood coloring of A^3_{10}

Case 4. For $m = 13$ and $m = 17$, the coloring patterns are similar to Case 2.

Case 5. m is any other integer. Then, by Observation 2.13, $m = 4k + 7l$ for some integers $k \geqslant 1, l \geqslant 0$. Define a coloring $c : V(A_m^3) \to \{1, 2, 3, 4, 5, 6, 7, 8\}$ as

$$c(u_i) = \begin{cases} 1, & \text{if } i \equiv 0 (\text{mod } 4) \text{ and } 0 \leq i \leq 4k-1 \text{ or } i - 4k \equiv 0(\text{mod } 7) \text{ and } 4k \leq i \leq m-1 \\ 2, & \text{if } i \equiv 1 (\text{mod } 4) \text{ and } 0 \leq i \leq 4k-1 \text{ or } i - 4k \equiv 1(\text{mod } 7) \text{ and } 4k \leq i \leq m-1 \\ 3, & \text{if } i \equiv 2 (\text{mod } 4) \text{ and } 0 \leq i \leq 4k-1 \text{ or } i - 4k \equiv 2(\text{mod } 7) \text{ and } 4k \leq i \leq m-1 \\ 4, & \text{if } i \equiv 3 (\text{mod } 4) \text{ and } 0 \leq i \leq 4k-1 \text{ or } i - 4k \equiv 3(\text{mod } 7) \text{ and } 4k \leq i \leq m-1 \\ 5, & i - 4k \equiv 4(\text{mod } 7) \text{ and } 4k \leq i \leq m-1 \\ 6, & i - 4k \equiv 5(\text{mod } 7) \text{ and } 4k \leq i \leq m-1 \\ 7, & \text{otherwise} \end{cases}$$

$$c(v_i) = \begin{cases} 1, & i - 4k - 2 \equiv 3(\text{mod } 7) \text{ and } 4k+2 \leq i \leq m-1 \\ 2, & i - 4k - 2 \equiv 4(\text{mod } 7) \text{ and } 4k+2 \leq i \leq m-1 \\ 3, & i - 4k - 2 \equiv 5(\text{mod } 7) \text{ and } 4k+2 \leq i \leq m-1 \text{ or } i = 0 \\ 4, & i - 4k - 2 \equiv 6(\text{mod } 7) \text{ and } 4k+2 \leq i \leq m-1 \text{ or } i = 1 \\ 5, & \text{if } i - 2 \equiv 0(\text{mod } 4) \text{ and } 2 \leq i \leq 4k+1 \\ & \text{or } i - 4k - 2 \equiv 0(\text{mod } 7) \text{ and } 4k+2 \leq i \leq m-1 \\ 6, & \text{if } i - 2 \equiv 1(\text{mod } 4) \text{ and } 2 \leq i \leq 4k+1 \\ & \text{or } i - 4k - 2 \equiv 1(\text{mod } 7) \text{ and } 4k+2 \leq i \leq m-1 \\ 7, & \text{if } i - 2 \equiv 2(\text{mod } 4) \text{ and } 2 \leq i \leq 4k+1 \\ & \text{or } i - 4k - 2 \equiv 2(\text{mod } 7) \text{ and } 4k+2 \leq i \leq m-1 \\ 8, & \text{otherwise} \end{cases}$$

and

$$c(w_i) = \begin{cases} 1, & \text{if } i \equiv 3(\text{mod } 4) \text{ and } 3 \leq i \leq 4k+2 \\ & \text{or } i - 4k \equiv 3(\text{mod } 7) \text{ and } 4k+3 \leq i \leq m-1 \\ 2, & \text{if } i \equiv 0(\text{mod } 4) \text{ and } 3 \leq i \leq 4k+2 \\ & \text{or } i - 4k \equiv 4(\text{mod } 7) \text{ and } 4k+3 \leq i \leq m-1 \\ 3, & \text{if } i \equiv 1(\text{mod } 4) \text{ and } 3 \leq i \leq 4k+2 \\ & \text{or } i - 4k \equiv 5(\text{mod } 7) \text{ and } 4k+3 \leq i \leq m-1 \\ 4, & \text{if } i \equiv 2(\text{mod } 4) \text{ and } 3 \leq i \leq 4k+2 \\ & \text{or } i - 4k \equiv 6(\text{mod } 7) \text{ and } 4k+3 \leq i \leq m-1 \\ 5, & i - 4k \equiv 0(\text{mod } 7) \text{ and } 4k+3 \leq i \leq m-1 \text{ or } i = 0 \\ 6, & i - 4k \equiv 1(\text{mod } 7) \text{ and } 4k+3 \leq i \leq m-1 \text{ or } i = 1 \\ 7, & \text{otherwise.} \end{cases}$$

It is easy to verify that c is an open neighborhood coloring of A_m^3 so that $\chi_{onc}(A_m^3) \leqslant 8$. □

§3. Open Neighborhood Coloring of A_m^n

We recall that the *generalized antiprism* graph A_m^n is obtained by completing the generalized prism $C_m \times P_n$ by edges $\{v_{i,j+1}v_{i+1,j} : 1 \leqslant i \leqslant m-1, 1 \leqslant j \leqslant n-1\} \bigcup \{v_{m,j+1}v_{1,j} : 1 \leqslant j \leqslant n-1\}$ where $V(A_m^n) = V(C_m \times P_n) = \{v_{i,j} : 1 \leqslant i \leqslant m, 1 \leqslant j \leqslant n\}$ is the vertex set of A_m^n. Thus, $E(A_m^n) = E(C_m \times P_n) \bigcup \{v_{i,j+1}v_{i+1,j} : 1 \leqslant i \leqslant m, 1 \leqslant j \leqslant n-1\}$ is the edge set of A_m^n, where i is taken modulo m.

Observation 3.1 For $n_2 \geqslant n_1 \geqslant 2$ and $m \geqslant 3$, $Q_m = A_m^2 \subseteq A_m^{n_1} \subseteq A_m^{n_2}$ so that $\chi_{onc}(Q_m) \leqslant \chi_{onc}(A_m^{n_1}) \leqslant \chi_{onc}(A_m^{n_2})$.

Lemma 3.2 *For the antiprism graph A_3^n and A_6^n, $n \geqslant 3$, $\chi_{onc}(A_3^n) = \chi_{onc}(A_6^n) = 9$.*

Proof By Lemma 2.10, we have $\chi_{onc}(A_3^3) = \chi_{onc}(A_6^3) = 9$. Thus, By Observation 3.1, for $n \geqslant 3, \chi_{onc}(A_3^n) = \chi_{onc}(A_6^n) \geqslant 9$.

To establish the reverse inequality, for $m = 3$ or 6, define a function $c : V(A_m^n) \to \{1, 2, \cdots, 9\}$ as $c(_{i,j}) = l$ with $i \equiv h(mod\ 3)$ & $j \equiv k(mod\ 3)$ and l corresponding to the respective hk^{th} entry in the following table.

$h \setminus k$	0	1	2
0	9	3	6
1	7	1	4
2	8	2	5

Table 1.

It is easy to verify that the above coloring is an open neighborhood coloring of A_m^n. Hence, $\chi_{onc}(A_m^n) \leqslant 9$.

To conclude, $\chi_{onc}(A_3^n) = \chi_{onc}(A_6^n) = 9$ for $n \geqslant 3$. □

Theorem 3.3 *For any integers $m \geqslant 3, n \geqslant 2$,*

$$\chi_{onc}(A_m^n) = \begin{cases} 7, & \text{if } m \equiv 0 (mod\ 7), \\ 5, & \text{if } n = 2 \text{ and } m \equiv 0 (mod\ 5), \\ 6, & \text{if } n = 2 \text{ and } m \neq 4, \\ 9, & \text{if } n \geqslant 3 \text{ and } m = 3, 6, \\ 8, & \text{otherwise.} \end{cases}$$

Proof In view of Theorems 2.5 and 2.14, the result holds for $n = 2$ and $n = 3$. For $n \geqslant 4$, we consider the following cases.

Case 1. For $m = 3, 6$, the result follows from Lemma 3.2.

Case 2. For $m = 5, 9, 10, 13, 17$, following the respective coloring patterns as in Lemma 2.10 and Theorem 2.14 yields an open neighborhood coloring with eight colors.

Case 3. For any other $m \geqslant 3$, following Observation 3.1 and Theorem 2.14, we see that $\chi_{onc}(A_m^n) \geqslant 8$. To prove the reverse inequality, define the coloring $c : V(A_m^n) \to \{1, 2, \cdots, 8\}$ as

$$(i)\quad c(v_{i,1}) = \begin{cases} 1, & \text{if } i \equiv 1 (\text{mod } 4) \text{ and } 0 \leq i \leq 4k \text{ or } i - 4k \equiv 1 (\text{mod } 7) \text{ and } 4k+1 \leq i \leq m \\ 2, & \text{if } i \equiv 2 (\text{mod } 4) \text{ and } 0 \leq i \leq 4k \text{ or } i - 4k \equiv 2 (\text{mod } 7) \text{ and } 4k+1 \leq i \leq m \\ 3, & \text{if } i \equiv 3 (\text{mod } 4) \text{ and } 0 \leq i \leq 4k \text{ or } i - 4k \equiv 3 (\text{mod } 7) \text{ and } 4k+1 \leq i \leq m \\ 4, & \text{if } i \equiv 0 (\text{mod } 4) \text{ and } 0 \leq i \leq 4k \text{ or } i - 4k \equiv 4 (\text{mod } 7) \text{ and } 4k+1 \leq i \leq m \\ 5, & i - 4k \equiv 5 (\text{mod } 7) \text{ and } 4k+1 \leq i \leq m \\ 6, & i - 4k \equiv 6 (\text{mod } 7) \text{ and } 4k+1 \leq i \leq m \\ 7, & \text{otherwise} \end{cases}$$

$$(ii)\quad c(v_{i,2}) = \begin{cases} 1, & i - 4k - 2 \equiv 4 (\text{mod } 7) \text{ and } 4k+3 \leq i \leq m \\ 2, & i - 4k - 2 \equiv 5 (\text{mod } 7) \text{ and } 4k+3 \leq i \leq m \\ 3, & i - 4k - 2 \equiv 6 (\text{mod } 7) \text{ and } 4k+3 \leq i \leq m \text{ or } i = 1 \\ 4, & i - 4k - 2 \equiv 0 (\text{mod } 7) \text{ and } 4k+3 \leq i \leq m \text{ or } i = 2 \\ 5, & \text{if } i \equiv 3 (\text{mod } 4) \text{ and } 3 \leq i \leq 4k+2 \text{ or } i - 4k \equiv 3 (\text{mod } 7) \text{ and } 4k+3 \leq i \leq m \\ 6, & \text{if } i \equiv 0 (\text{mod } 4) \text{ and } 3 \leq i \leq 4k+2 \text{ or } i - 4k \equiv 4 (\text{mod } 7) \text{ and } 4k+3 \leq i \leq m \\ 7, & \text{if } i \equiv 1 (\text{mod } 4) \text{ and } 3 \leq i \leq 4k+2 \text{ or } i - 4k \equiv 5 (\text{mod } 7) \text{ and } 4k+3 \leq i \leq m \\ 8, & \text{otherwise} \end{cases}$$

and (iii) $c(v_{i,j}) = c(v_{i-3,j-2})$ for $j \geqslant 3$ where i is taken under modulo m.

An illustration of the coloring c for the graph A_{29}^5 is given in Figure 11.

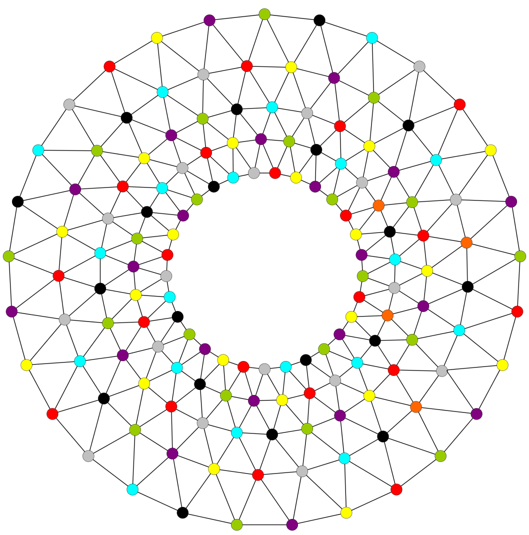

Figure 11. An open neighborhood coloring of A_{29}^5

It is easy to verify that the above coloring is an open neighborhood coloring of A_m^n. Hence, $\chi_{onc}(A_m^n) \leqslant 8$. □

§4. Conclusion

The open neighborhood chromatic number of an antiprism graph Q_n has been determined in [8]. We have obtained this parameter for the generalized antiprism graph A_m^3 in this paper, by means of which, we have solved the problem of finding the open neighborhood chromatic number of A_m^n, $m \geqslant 3, n \geqslant 2$.

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