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# PD-Divisor Labeling of Graphs

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**Abstract**: Let $G = (V(G), E(G))$ be a simple, finite and undirected graph of order $n$. Given a bijection $f : V(G) \to \{1, 2, \cdots, |V(G)|\}$, we associate two integers $P = f(u)f(v)$ and $D = |f(u) - f(v)|$ with every edge $uv$ in $E(G)$. The labeling $f$ induces on edge labeling $f' : E(G) \to \{0, 1\}$ such that for any edge $uv$ in $E(G)$, $f'(uv) = 1$ if $D \mid P$ and $f'(uv) = 0$ if $D \nmid P$. Let $e_{f'}(i)$ be the number of edges labeled with $i \in \{0, 1\}$. We say $f$ is an PD-divisor labeling if $f'(uv) = 1$ for all $uv \in E(G)$. Moreover, $G$ is PD-divisor if it admits an PD-divisor labeling. We say $f$ is an PD-divisor cordial labeling if $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Moreover, $G$ is PD-divisor cordial if it admits an PD-divisor cordial labeling. In this paper, we define PD-divisibility and PD-divisor pair of numbers and establish some of its properties. Also, we are dealing in PD-divisor labeling of some standard graphs.

**Key Words**: Divisor cordial labeling; PD-divisor labeling; PD-divisor graph, Smarandachely SD-divisor cordial labeling, Smarandachely SD-divisor cordial graph.

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## §1. Introduction

Let $G = (V(G), E(G))$ (or $G = (V, E)$) be a simple, finite and undirected graph of order $|V(G)| = n$ and size $|E(G)| = m$. All notations not defined in this paper can be found in [4].

**Definition** 1.1 ([2]) *Let $a$ and $b$ be two integers. If $a$ divides $b$ means that there is a positive integer $k$ such that $b = ka$. It is denoted by $a \mid b$. If $a$ does not divide $b$ , then we denote $a \nmid b$.*

**Definition** 1.2 ([1]) *Let $G = (V, E)$ be a graph. A mapping $f : V(G) \to \{0, 1\}$ is called binary vertex labeling of $G$ and $f(v)$ is called the label of the vertex $v$ of $G$ under $f$. For an edge $e = uv$, the induced edge labeling $f' : E(G) \to \{0, 1\}$ is given by $f'(e) = |f(u) - f(v)|$. Let $v_f(0), v_f(1)$ be the number of vertices of $G$ having labels 0 and 1 respectively under $f$ and $e_{f'}(0), e_{f'}(1)$ be the number of edges having labels 0 and 1 respectively under $f'$. This labeling is called cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_{f'}(0) - e_{f'}(1)| \leq 1$. A graph $G$ is cordial if it admits cordial labeling.*

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**Definition** 1.3 ([9]) *A bijection $f : V \to \{1, 2, \cdots, n\}$ induces an edge labeling $f' : E \to \{0, 1\}$ such that for any edge $uv$ in $G$, $f'(uv) = 1$ if $gcd(f(u), f(v)) = 1$, and $f'(uv) = 0$ otherwise. We say that $f$ is a prime cordial labeling if $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Moreover, $G$ is prime cordial if it admits a prime cordial labeling.*

**Definition** 1.4 ([10]) *Let $G = (V, E)$ be a simple graph and $f : V \to \{1, 2, \cdots, n\}$ be a bijection. For each edge $uv$, assign the label 1 if either $f(u) \mid f(v)$ or $f(v) \mid f(u)$ and the label 0 otherwise. We say that $f$ is a divisor cordial labeling if $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Moreover, $G$ is divisor cordial if it admits a divisor cordial labeling.*

Given a bijection $f : V \to \{1, 2, \cdots, n\}$, we associate two integers $S = f(u) + f(v)$ and $D = |f(u) - f(v)|$ with every edge $uv$ in $E$.

**Definition** 1.5 ([7]) *A bijection $f : V \to \{1, 2, \cdots, n\}$ induces an edge labeling $f' : E \to \{0, 1\}$ such that for any edge $uv$ in $G$, $f'(uv) = 1$ if $gcd(S, D) = 1$, and $f'(uv) = 0$ otherwise. We say $f$ is an SD-prime labeling if $f'(uv) = 1$ for all $uv \in E$. Moreover, $G$ is SD-prime if it admits an SD-prime labeling.*

**Definition** 1.6 ([6]) *A bijection $f : V \to \{1, 2, \cdots, n\}$ induces an edge labeling $f' : E \to \{0, 1\}$ such that for any edge $uv$ in $G$, $f'(uv) = 1$ if $gcd(S, D) = 1$, and $f'(uv) = 0$ otherwise. The labeling $f$ is called an SD-prime cordial labeling if $|e_{f'}(0) - e_{f'}(1)| \leq 1$. We say that $G$ is SD-prime cordial if it admits an SD-prime cordial labeling.*

**Definition** 1.7 ([5]) *Let $G = (V(G), E(G))$ be a simple graph and a bijection $f : V(G) \to \{1, 2, 3, \cdots, |V(G)|\}$ induces an edge labeling $f' : E(G) \to \{0, 1\}$ such that for any edge $uv$ in $E(G)$, $f'(uv) = 1$ if $D \mid S$ and $f'(uv) = 0$ if $D \nmid S$. We say $f$ is an SD-divisor labeling if $f'(uv) = 1$ for all $uv \in E(G)$. Moreover, $G$ is SD-divisor if it admits an SD-divisor labeling.*

**Definition** 1.8 ([5]) *Let $G = (V(G), E(G))$ be a simple graph and a bijection $f : V(G) \to \{1, 2, 3, \cdots, |V(G)|\}$ induces an edge labeling $f' : E(G) \to \{0, 1\}$ such that for any edge $uv$ in $E(G)$, $f'(uv) = 1$ if $D \mid S$ and $f'(uv) = 0$ if $D \nmid S$. The labeling $f$ is called an SD-divisor cordial labeling if $|e_{f'}(0) - e_{f'}(1)| \leq 1$. We say that $G$ is SD-divisor cordial if it admits an SD-divisor cordial labeling.*

Generally, the labeling $f$ in Definition 1.8 is said to be *Smarandachely SD-divisor cordial* if $|e_{f'}(0) - e_{f'}(1)| \geq 2$ and $G$ is said to be a Smarandachely SD-divisor cordial graph. In [5], we introduced two new types of labeling called SD-divisor and SD-divisor cordial labeling. Also, we proved some graphs are SD-divisor. Motivated by the concepts of SD-divisor and SD-divisor cordial labeling, we introduce two new types of labeling called PD-divisor and PD-divisor cordial labeling. In this paper, we define PD-divisibility and PD-divisor pair of numbers and establish some of its properties. Also, we are dealing in PD-divisor labeling of some standard graphs.

## §2. PD-Divisibility and its Properties

First, we define PD-divisibility of two positive integers.

**Definition** 2.1 *Let $a$ and $b$ be the two distinct positive integers, we say that $a$ PD-divides $b$ if $|a-b| \mid ab$. It is denoted by $a \mid_{PD} b$. If $a$ does not PD-divide $b$, then it is denoted by $a \nmid_{PD} b$.*

**Example** 2.2 $4 \mid_{PD} 6$.

**Example** 2.3 $2 \nmid_{PD} 8$.

Notice that

1. From the Examples 2.2 and 2.3, divisibility and PD-divisibility are different concepts.
2. By Definition 2.1, PD-divisibility is not reflexive.
3. From Definition 2.1, $a \mid_{PD} b \Rightarrow |a-b| \mid ab$

$$\Rightarrow |b-a| \mid ba$$
$$\Rightarrow b \mid_{PD} a.$$

Thus, PD-divisibility is symmetric.

4. PD-divisibility is not transitive.

**Example** 2.4 $1 \mid_{PD} 2$ and $2 \mid_{PD} 6$ but $1 \nmid_{PD} 6$.

PD-divisibility is not an equivalence relation.

**Observation** 2.5 Its known that if $k$ and $k+1$ are two consecutive integers, then $k \nmid k+1$ for $k \geq 2$.

**Proposition** 2.6 *1 PD-divides only to the integer 2.*

*Proof* Let $a = 1$ and $b > 1$ be the any positive integer. If $1 \mid_{PD} b$, then $(b-1) \mid b$. This means that two consecutive integers divide. This is possible only if $b = 2$. □

**Proposition** 2.7 *2 PD-divides only to the integers 1, 3, 4 and 6.*

*Proof* Let $a = 2$ and $b$ be the any positive integer. If $2 \mid_{PD} b$, then $|b-2| \mid 2b$. This is possible only if $b = 1, 3, 4$ and 6. □

**Proposition** 2.8 *3 PD-divides only to the integers 2, 4, 6 and 12.*

*Proof* Let $a = 3$ and $b$ be the any positive integer. If $3 \mid_{PD} b$, then $|b-3| \mid 3b$. This is possible only if $b = 2, 4, 6$ and 12. □

**Observation** 2.9 Let $a \geq 2$ be the any positive integer. Then $a-1, a+1, 2a$ and $a(a+1)$ are PD-divisible by $a$.

**Observation** 2.10 Let $a \geq 4$ be the any positive even integer. Then $a-2$, $a-1$, $a+1$, $a+2$, $a+4$, $2a$, $3a$ and $a(a+1)$ are PD-divisible by $a$.

**Proposition** 2.11 *Let $a$ and $b$ be the two consecutive odd integers, then $a \nmid_{PD} b$.*

*Proof* Let $a = 2k+1$ and $b = 2k+3$ for $k \geq 0$. Then $|a-b| = |2k+1-2k-3| = 2$ and $ab = (2k+1)(2k+3)$.

Clearly, $2 \nmid (2k+1)(2k+3)$. Then $a \nmid_{PD} b$. □

**Proposition** 2.12 *Let $a$ and $b$ be the two consecutive even integers, then $a \mid_{PD} b$.*

*Proof* Let $a = 2k+2$ and $b = 2k+4$ for $k \geq 0$. Then $|a-b| = |2k+2-2k-4| = 2$ and $ab = (2k+2)(2k+4)$.

Clearly, $2 \mid (2k+2)(2k+4)$. Then $a \mid_{PD} b$. □

## §3. PD-Divisor Pair

**Definition** 3.1 *Let $a$ and $b$ be the two distinct positive integers. If $a \mid_{PD} b$, then we say that $(a, b)$ is called PD-divisor pair.*

**Example** 3.2 For $k \geq 1$, $(k, k+1)$ is PD-divisor pair.

Notice that if $l \geq 1$ is any positive integer, then $(lk, l(k+1))$ is PD-divisor pair. We know the following results.

**Proposition** 3.3 *If the pair $(a, b)$ is PD-divisor, then $(ka, kb)$ is PD-divisor pair for $k \geq 1$.*

*Proof* Let $a$ and $b$ the PD-divisor pair. Without loss of generality, we take $a > b$.

$$\begin{aligned}\text{Then,}\quad a \mid_{PD} b &\Rightarrow (a-b) \mid ab\\ &\Rightarrow k(a-b) \mid kab \text{ for } k \geq 1\\ &\Rightarrow (ka-kb) \mid kab\\ &\Rightarrow (ka-kb) \mid k^2ab \Rightarrow ka \mid_{PD} kb.\end{aligned}$$ □

**Proposition** 3.4 *Let $k \geq 3$ be an odd integer. Then $(k+1, k-1)$ is PD-divisor pair.*

*Proof* Let $a = k+1$ and $b = k-1$ for all odd integer $k \geq 3$. Then $|a-b| = |k+1-k+1| = 2$ and $ab = (k+1)(k-1) = k^2-1$.

Clearly $2 \mid k^2-1$. Thus, $(k+1, k-1)$ is PD-divisor pair for all odd integer $k \geq 3$. □

**Proposition** 3.5 *Let $k \geq 2$ be an even integer. Then $(k+1, k-1)$ is not PD-divisor pair.*

*Proof* Let $a = k+1$ and $b = k-1$ for all even integer $k \geq 2$. Then, $|a-b| = |k+1-k+1| = 2$ and $ab = (k+1)(k-1) = k^2-1$.

Clearly, $2 \nmid k^2-1$. Thus, $(k+1, k-1)$ is not PD-divisor pair for all even integer $k \geq 2$. □

**Proposition** 3.6 *Let $k \geq 0$. Then $(2^k, 2^{k+1})$ is PD-divisor pair.*

*Proof* Let $a = 2^k$ and $b = 2^{k+1}$ for $k \geq 0$. Then $|a-b| = |2^k - 2^{k+1}| = 2^k$ and $ab = (2^k)(2^{k+1})$.

Clearly, $|a-b| \mid ab$. Thus, $(2^k, 2^{k+1})$ is PD-divisor pair for $k \geq 0$. □

**Proposition** 3.7 *Let $k \geq 0$. Then $(3^k, 3^{k+1})$ is not PD-divisor pair.*

*Proof* Let $a = 3^k$ and $b = 3^{k+1}$ for $k \geq 0$. Then, $|a-b| = |3^k - 3^{k+1}| = 2 \cdot 3^k$ and $ab = (3^k)(3^{k+1})$.

Clearly, $|a-b| \nmid ab$. Thus, $(3^k, 3^{k+1})$ is not PD-divisor pair for $k \geq 0$. □

**Proposition** 3.8 *Let $l \geq 3$ and $k \geq 0$. Then $(l^k, l^{k+1})$ is not PD-divisor pair.*

*Proof* Let $a = l^k$ and $b = l^{k+1}$ for $l \geq 3$, $k \geq 0$. Then, $|a-b| = |l^k - l^{k+1}| = l^k(l-1)$ and $ab = (l^k)(l^{k+1})$.

Clearly, $l-1 \nmid l^{k+1}$ for $l \geq 3$ and $k \geq 0$. Thus, $(l^k, l^{k+1})$ is not a PD-divisor pair for $l \geq 3$ and $k \geq 0$. □

**Definition** 3.9 *Let $S$ be a set of any distinct positive integers. Then $S$ is said to be PD-divisor set if every pair of integers in $S$ is PD-divisor.*

we always use notation $[n] = \{1, 2, \cdots, n\}$ in this paper.

**Example** 3.10 [2] is PD-divisor set.

## §4. PD-Divisor Labeling of Graphs

Now, we introduce two new types of labeling called PD-divisor and PD-divisor cordial labeling. Given a bijection $f : V \to \{1, 2, \cdots, n\}$, we associate two integers $P = f(u)f(v)$ and $D = |f(u) - f(v)|$ with every edge $uv$ in $E$.

**Definition** 4.1 *Let $G = (V(G), E(G))$ be a simple graph and a bijection $f : V(G) \to \{1, 2, 3, \cdots, |V(G)|\}$ induces an edge labeling $f' : E(G) \to \{0, 1\}$ such that for any edge $uv$ in $E(G)$, $f'(uv) = 1$ if $D \mid P$ and $f'(uv) = 0$ if $D \nmid P$. We say $f$ is an PD-divisor labeling if $f'(uv) = 1$ for all $uv \in E(G)$. Moreover, $G$ is PD-divisor if it admits an PD-divisor labeling.*

**Example** 4.2 Consider the following graph $G$.

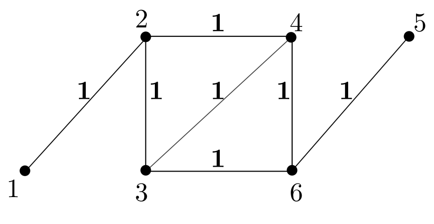


**Figure** 1

We see that $e_{f'}(1) = 7$. Hence $G$ is PD-divisor.

**Definition** 4.2 *Let $G = (V(G), E(G))$ be a simple graph and a bijection $f : V(G) \to \{1, 2, 3, ..., |V(G)|\}$ induces an edge labeling $f' : E(G) \to \{0, 1\}$ such that for any edge $uv$ in $E(G)$, $f'(uv) = 1$ if $D \mid P$ and $f'(uv) = 0$ if $D \nmid P$. The labeling $f$ is called an PD-divisor cordial labeling if $|e_{f'}(0) - e_{f'}(1)| \leq 1$. We say that $G$ is PD-divisor cordial if it admits an PD-divisor cordial labeling.*

**Example** 4.3 Consider the the labeling of $G$ in Figure 3.

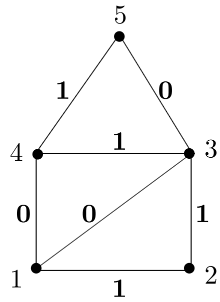


**Figure** 2

We see that $e_{f'}(0) = 3$ and $e_{f'}(1) = 4$. Thus $|e_{f'}(0) - e_{f'}(1)| \leq 1$ and hence $G$ is PD-divisor cordial.

Now, we prove path and some path related graphs are PD-divisor. Also, we prove some standard graphs such as star, cycle, complete, complete bipartite and wheel graphs are not PD-divisor.

**Theorem** 4.5 *A path $P_n$ is PD-divisor.*

*Proof* Let $v_1, v_2, \cdots, v_n$ be the vertices of path $P_n$. Let $V(P_n) = \{v_i : 1 \leq i \leq n\}$ and $E(P_n) = \{v_i v_{i+1} : 1 \leq i \leq n-1\}$. Therefore, $P_n$ is of order $n$ and size $n-1$. Define $f : V(P_n) \to \{1, 2, 3, \cdots, n\}$ to be

$$f(v_i) = i, \quad 1 \leq i \leq n.$$

From the above labeling pattern we get, $e_{f'}(1) = n - 1$. Hence, $P_n$ is PD-divisor. □

**Example** 4.6 Consider the labeling of $P_8$ in Figure 3.

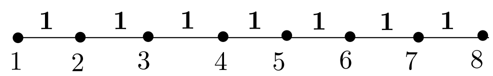


**Figure** 3

Here $e_{f'}(1) = 7$. Hence, $P_8$ is PD-divisor.

**Theorem** 4.7 *A comb $P_n \odot K_1$ is PD-divisor.*

*Proof* Let $v_1, v_2, \cdots, v_n$ be the vertices of path $P_n$. Let $V(P_n \odot K_1) = \{v_i, u_i : 1 \leq i \leq n\}$ and $E(P_n \odot K_1) = \{v_i v_{i+1} : 1 \leq i \leq n-1\} \bigcup \{v_i u_i : 1 \leq i \leq n\}$. Therefore, $P_n \odot K_1$ is of order $2n$ and size $2n - 1$. Define $f : V(P_n \odot K_1) \to \{1, 2, 3, \cdots, 2n\}$ to be $f(v_i) = 2i$, $1 \leq i \leq n$, $f(u_i) = 2i - 1$, $1 \leq i \leq n$.

From the above labeling pattern we get, $e_{f'}(1) = 2n - 1$. So $P_n \odot K_1$ is PD-divisor. □

**Example** 4.8 Consider the labeling of $P_6 \odot K_1$ in Figure 4.

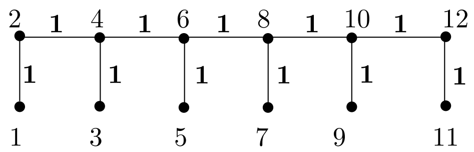


**Figure** 4

Here, $e_{f'}(1) = 11$. Hence, $P_6 \odot K_1$ is PD-divisor.

**Theorem** 4.9 *A double comb $P_n \odot 2K_1$ is PD-divisor.*

*Proof* Let $v_1, v_2, \cdots, v_n$ be the vertices of path $P_n$. Let $V(P_n \odot 2K_1) = \{v_i, u_i, w_i : 1 \leq i \leq n\}$ and $E(P_n \odot 2K_1) = \{v_iv_{i+1} : 1 \leq i \leq n-1\} \bigcup \{v_iu_i : 1 \leq i \leq n\} \bigcup \{v_iw_i : 1 \leq i \leq n\}$. Therefore, $P_n \odot 2K_1$ is of order $3n$ and size $3n-1$. Define $f : V(P_n \odot 2K_1) \rightarrow \{1, 2, 3, \cdots, 3n\}$ to be

$$\begin{aligned} f(v_{2i-1}) &= 6i-4, \quad 1 \leq i \leq \lceil \frac{n}{2} \rceil, \\ f(v_{2i}) &= 6i-2, \quad 1 \leq i \leq \lfloor \frac{n}{2} \rfloor, \\ f(u_i) &= 3i, \quad 1 \leq i \leq n, \\ f(w_{2i-1}) &= 6i-5, \quad 1 \leq i \leq \lceil \frac{n}{2} \rceil, \\ f(w_{2i}) &= 6i-1, \quad 1 \leq i \leq \lfloor \frac{n}{2} \rfloor. \end{aligned}$$

From the above labeling pattern we get, $e_{f'}(1) = 3n-1$. Hence, $P_n \odot 2K_1$ is PD-divisor.□

**Example** 4.10 Consider the labeling of $P_8 \odot 2K_1$ in Figure 5.

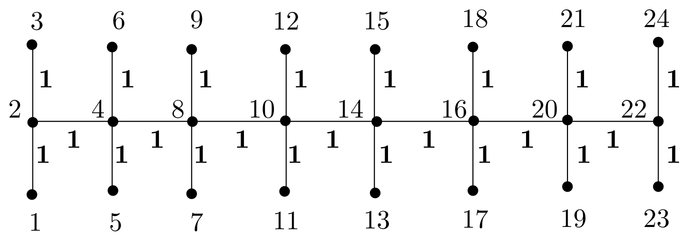


**Figure** 5

Here $e_{f'}(1) = 23$. Hence, $P_8 \odot 2K_1$ is PD-divisor.

**Theorem** 4.11 *A crown $C_n \odot K_1$ is PD-divisor.*

*Proof* Let $v_1, v_2, \cdots, v_n$ be the vertices of cycle $C_n$. Let $V(C_n \odot K_1) = \{v_i, u_i : 1 \leq i \leq n\}$ and $E(C_n \odot K_1) = \{v_iv_{i+1} : 1 \leq i \leq n-1\} \bigcup \{v_nv_1, v_iu_i : 1 \leq i \leq n\}$. Therefore, $C_n \odot K_1$ is of order $2n$ and size $2n$. Define $f : V(C_n \odot K_1) \to \{1, 2, 3, \cdots, 2n\}$ to be

$$
\begin{aligned}
f(v_i) &= 4i-2, \quad 1 \le i \le \left\lceil \frac{n}{2} \right\rceil, \\
f(v_{n+1-i}) &= 4i, \qquad 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor, \\
f(u_i) &= 4i-3, \quad 1 \le i \le \left\lceil \frac{n}{2} \right\rceil, \\
f(u_{n+1-i}) &= 4i-1, \quad 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor.
\end{aligned}
$$

From the above labeling pattern we get, $e_{f'}(1) = 2n$. Hence, $C_n \odot K_1$ is PD-divisor. □

**Example** 4.12 Consider the labeling of $C_{11} \odot K_1$ in Figure 6.

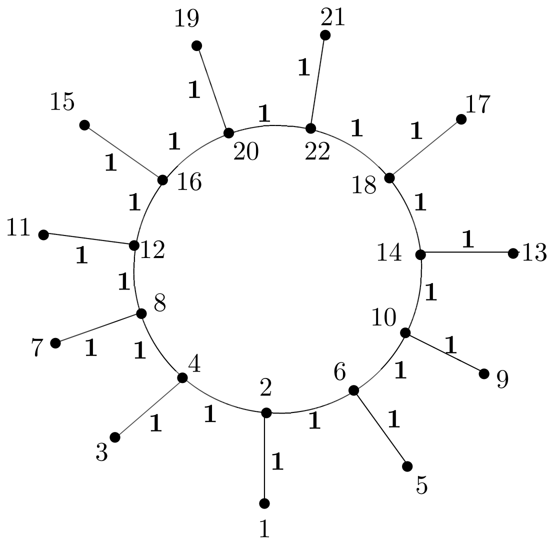


**Figure** 6

Here, $e_{f'}(1) = 22$. Hence, $C_{11} \odot K_1$ is PD-divisor.

Next, we will investigate whether the star graph $K_{1,n}$ is PD-divisor or not. Clearly, $K_{1,1}$ and $K_{1,2}$ are PD-divisor, and also $K_{1,3}$ is PD-divisor in the following labeling.

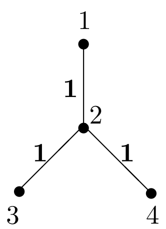


**Figure** 7

Next, we will prove that $K_{1,n}$ is not PD-divisor for $n \ge 4$.

**Theorem** 4.13 *For $n \ge 4$, the star graph $K_{1,n}$ is not PD-divisor.*

*Proof* Consider the set $\{1, 2, \cdots, n+1\}$, $n \geq 4$. Let $v$ be the central vertex of $K_{1,n} (n \geq 4)$.

If we label 1 to $v$ and other numbers to the end vertices of $K_{1,n}$, then it follows from Proposition 2.6, 1 does not PD-divide $3, 4, 5, \cdots, n+1$.

If we label 2 to $v$ and other numbers to the end vertices of $K_{1,n}$, then it follows from Proposition 2.7, 2 does not PD-divide $5, 7, 8, \cdots, n+1$.

Suppose, we label $n \geq 3$ to $v$. Since any one of the end vertex has the label 1, then it follows from Proposition 2.6, 1 does not PD-divide to the label of $v$.

Thus, $K_{1,n}$ is not PD-divisor for $n \geq 4$. □

**Theorem** 4.14 *If $\delta(G) \geq 2$, then $G$ is not PD-divisor.*

*Proof* Suppose $G$ is PD-divisor. Let $v$ be the vertex of degree $\delta(G) \geq 2$, which is labeled with 1. Then, any one of the $\delta$ adjacent vertices of $v$ must have the labels other than 2, say $w$.

From Proposition 2.6, it follows that 1 does not PD-divide the label of $w$. This is contradiction to $G$ is PD-divisor. □

**Remark** 4.15 *If $\delta(G) = 1$, then it is not necessary that $G$ is PD-divisor from Theorem* 4.13 *its follows.*

**Corollary** 4.16 *For $n \geq 3$, the cycle graph $C_n$ is not PD-divisor.*

*Proof* Since $\delta(C_n) \geq 2$ for $n \geq 3$, the result follows from Theorem 4.14. □

**Corollary** 4.17 *For $n \geq 3$, the complete graph $K_n$ is not PD-divisor.*

*Proof* Since $\delta(K_n) \geq 2$ for $n \geq 3$, the result follows from Theorem 4.14. □

**Corollary** 4.18 *For $m, n \geq 2$, the complete bipartite graph $K_{m,n}$ is not PD-divisor.*

*Proof* Since $\delta(K_{m,n}) \geq 2$ for $m, n \geq 2$, the result follows from Theorem 4.14. □

**Corollary** 4.19 *The wheel graph $W_{n+1} (n \geq 2)$ is not PD-divisor.*

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