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Pair Difference Cordial Labeling of Double Cone, Double Step Grid, Double Arrow and Shell Related Graphs

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Abstract: In this paper we investigate the pair difference cordial labeling behaviour of double cone,double step grid,sun flower, shell graph and double arrow graph.

Key Words: Pair difference cordial labeling, double cone, double step grid, sun flower, shell graph, double arrow graph, Smarandachely pair difference cordial labeling, Smarandachely pair difference cordial labeling graph.

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§1. Introduction

In this paper we consider only finite, undirected and simple graphs. Cordial labeling was introduced by Cachit [1] in the year 1987. Also cordial related labeling technique was studied in [1,2,3,4,5,6,7,8,9,10,11]. In this sequal the notion of pair difference cordial labeling of a graph was introduced in [14], which is defined as follows:

Let $G = (V, E)$ be a (p, q) graph and let

$$\rho = \begin{cases} \frac{p}{2}, & \text{if } p \text{ is even} \\ \frac{p-1}{2}, & \text{if } p \text{ is odd} \end{cases}$$

and $L = \{\pm 1, \pm 2, \pm 3, \cdots, \pm \rho\}$ be the set of labels. Consider a mapping $f : V \longrightarrow L$ by assigning different labels in L to the different elements of V when p is even and different labels in L to $p-1$ elements of V and repeating a label for the remaining one vertex when p is odd. Such a labeling is said to be a pair difference cordial labeling if for each edge uv of G there exists a labeling $|f(u) - f(v)|$ such that $\left|\Delta_{f_1} - \Delta_{f_1^c}\right| \leq 1$, where Δ_{f_1} and $\Delta_{f_1^c}$ respectively denote the number of edges labeled with 1 and number of edges not labeled with 1. A graph G for which there exists a pair difference cordial labeling is called a pair difference cordial graph.

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Generally, let $H \prec G$ be a typical subgraph of G. If there is a pair difference cordial labeling on graph $G-H$. Then, we say G is Smarandachely pair difference cordial labeling on H and G is called a Smarandachely pair difference cordial labeling graph on H. Particularly, if $H=\emptyset$ such a Smarandachely pair difference cordial labeling is nothing else but a pair difference cordial labeling on G.

The pair difference cordial labeling behavior of several graphs like path, cycle, star, wheel, triangular snake, alternate triangular snake, butterfly etc have been investigated in [14-24]. In this paper we investigate the pair difference cordial labeling behavior of double cone,double step grid,sun flower, shell graph and double arrow graph. Terms not defined here are follow from Gallian [12] and Harary [13].

§2. Preliminaries

Definition 2.1([13]) *The subdivision graph $S(G)$ of a graph G is obtained by replacing each edge uv by a path uvw.*

Definition 2.2([12]) *Take the tha paths $P_n, P_n, P_{n-1}, \cdots, P_2$ on $n, n, n-2, n-4, \cdots, 4, 2$ vertices and arrange them centrally horizontal where n is even and $n \neq 2$. A graph obtained by joining vertical vertices of given successive paths is known as a double step grid of size n. It is denoted by DSt_n.*

For illustration, DSt_n is shown in Figure 1.

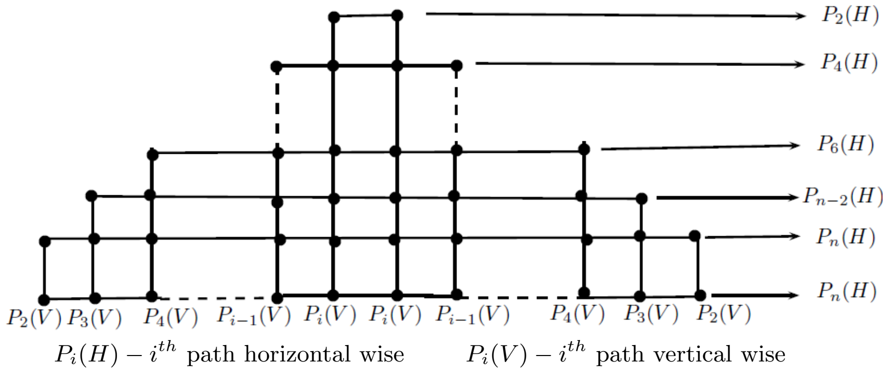

Figure 1

Definition 2.3([13]) *Double arrow graphs obtained from $P_n \times P_n$ by joinin two vertices u, v with first and last copy of the path P_n.Let $a_{i,j}$ be the vertices of prism $P_n \times P_n$.*

Definition 2.4([12]) *The graph $C_n + 2K_1$ is called the double cone graph.*

Definition 2.5([12]) *The sunflower graph SF_n is obtained by taking a wheel $W_n = C_n + K_1$ where C_n is the cycle $a_1a_2a_3\cdots a_na_1$, $V(K_1) = \{a\}$ and the new vertices $b_1, b_2, b_3, \cdots, b_n$ where b_i is join by the vertices $b_ib_{i+1}(modn)$.*

Definition 2.6([12]) *A shell graph is defined as a cycle $C_n : a_1a_2a_3\cdots a_na_1$ with $(n-3)$ chords* *sharing a common end point called the apex. Shell graph are denoted as $C_{(n,n-3)}$. A shell S_n is also called fan F_{n-1}.*

Definition 2.7([25]) *An ice cream graph is obtained by combining a shell graph and a path P_2 graaph keeping a_1 and a_n common where $n > 3$ sharing common end point called the apex vertex a_0. It is denoted by IC_n.*

§3. Main Results

Theorem 3.1 *A double step grid DSt_n is pair difference cordial for all even values of $n \geq 4$.*

Proof First we consider the paths $P_2, P_3, P_4, \cdots, P_{\frac{n}{2}}$ from left to right. Assign the labels $1, 2$ respectively to the vertices of the path P_2 from top to bottom and assign the labels $3, 4, 5$ respectively to the vertices of the path P_3 from bottom to top. Now assign the labels $6, 7, 8, 9$ to the vertices of the path P_4 from top to bottom and assign the labels $10, 11, 12, 13, 14$ to the vertices of the path P_5 from bottom to top. Proceeding like this until we reach the path $P_{\frac{n}{2}}$.

Next, we consider the paths $P_2, P_3, P_4, \cdots, P_{\frac{n}{2}}$ from right to left. Assign the labels $-1, -2$ respectively to the vertices of the path P_2 from top to bottom and assign the labels $-3, -4, -5$ respectively to the vertices of the path P_3 from bottom to top. Now assign the labels $-6, -7, -8, -9$ to the vertices of the path P_4 from top to bottom and assign the labels $-10, -11, -12, -13, -14$ to the vertices of the path P_5 from bottom to top. Proceeding like this until we reach the path $P_{\frac{n}{2}}$.

For illustration, DSt_8 is shown in Figure 2.

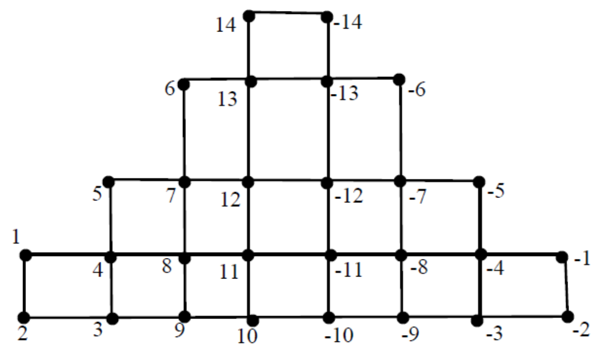

Figure 2

This completes the proof. □

Theorem 3.2 *A double cone graph DC_n is not pair difference cordial for all values of $n \geq 3$.*

Proof There are two cases arises.

Case 1. n is even.

The maximum possible number of edges get the label 1 is $\Delta f_1 = \frac{n}{2} + 2 + 2$ where $\frac{n}{2}$ edges from cycle, 2 edges from edges end with K_1 and next 2 edges from edges end with another K_1.

Therefore $\Delta f_1 = \frac{n}{2} + 4$ and $|E(DC_n)| = 3n$. This implies that $\Delta {f_1}^c = 3n - \frac{n}{2} + 4 = \frac{5n+8}{2}$. Hence $|\Delta f_1 - \Delta {f_1}^c| = 2n > 1$, which is a contradiction.

Case 2. n is odd.

The maximum possible number of edges get the label 1 is $\Delta f_1 = \frac{n+1}{2} + 2 + 2$ where $\frac{n+1}{2}$ edges from cycle, 2 edges from edges end with K_1 and next 2 edges from edges end with another K_1. Therefore $\Delta f_1 = \frac{n+1}{2} + 4$ and $|E(DC_n)| = 3n$. This gives that $\Delta {f_1}^c = 3n - \frac{n+1}{2} + 4 = \frac{5n+9}{2}$. Hence $|\Delta f_1 - \Delta {f_1}^c| = 2n > 1$, which is a contradiction.

Hence, a double cone graph DC_n is not pair difference cordial for all values of $n \geq 3$. $\square$

Theorem 3.3 *A double arrow graph DA_n is pair difference cordial for all values of $n \geq 2$.*

Proof Take the vertex set and edge set from Definition 2.3. There are two cases arises.

Case 1. n is even.

Consider the $\frac{n}{2}^{th}$ row. That is consider the vertices $a_{\frac{n}{2},1}$, $a_{\frac{n}{2},2}$, $a_{\frac{n}{2},3}, \cdots, a_{\frac{n}{2},n}$. Assign the labels $1, 2, 3, \cdots, n$ to the vertices $a_{\frac{n}{2},1}$, $a_{\frac{n}{2},2}$, $a_{\frac{n}{2},3}, \cdots, a_{\frac{n}{2},n}$ respectively and next consider the $\frac{n-2}{2}^{th}$ row. Assign the labels $n+1, n+2, n+3, \cdots, 2n$ to the vertices $a_{\frac{n-2}{2},1}$, $a_{\frac{n-2}{2},2}$, $a_{\frac{n-2}{2},3}, \cdots, a_{\frac{n-2}{2},n}$. Now assign the labels $2n+1, 2n+2, 2n+3, \cdots, 3n$ to the vertices $a_{\frac{n-4}{2},1}$, $a_{\frac{n-4}{2},2}$, $a_{\frac{n-4}{2},3}, \cdots, a_{\frac{n-4}{2},n}$ respectively. Proceeding like this until we reach the first row. Note that the vertices $a_{1,1}$, $a_{1,2}$, $a_{1,3}, \cdots, a_{1,n}$ gets the labels $\frac{n}{2}+1, \frac{n}{2}+2, \frac{n}{2}+3, \cdots, \frac{n}{2}+n$.

Consider the $\frac{n+2}{2}^{th}$ row. That is consider the vertices $a_{\frac{n+2}{2},1}$, $a_{\frac{n+2}{2},2}$, $a_{\frac{n+2}{2},3}, \cdots, a_{\frac{n+2}{2},n}$. Assign the labels $-1, -2, -3, \cdots, -n$ to the vertices $a_{\frac{n+2}{2},1}$, $a_{\frac{n+2}{2},2}$, $a_{\frac{n+2}{2},3}, \cdots, a_{\frac{n+2}{2},n}$ respectively and next consider the $\frac{n+4}{2}^{th}$ row. Assign the labels $-(n+1), -(n+2), -(n+3), \cdots, -2n$ to the vertices $a_{\frac{n+4}{2},1}$, $a_{\frac{n+4}{2},2}$, $a_{\frac{n+4}{2},3}, \cdots, a_{\frac{n+4}{2},n}$. Now assign the labels $-(2n+1), -(2n+2), -(2n+3), \cdots, -3n$ to the vertices $a_{\frac{n+6}{2},1}$, $a_{\frac{n+6}{2},2}$, $a_{\frac{n+6}{2},3}, \cdots, a_{\frac{n+6}{2},n}$ respectively. Proceeding like this until we reach the n^{th} row. Note that the vertices $a_{n,1}$, $a_{n,2}$, $a_{n,3}, \cdots, a_{n,n}$ gets the labels

$$-(\frac{n}{2}+1), -(\frac{n}{2}+2), -(\frac{n}{2}+3), \cdots, -(\frac{n}{2}+n),$$

and finally assign the labels $\frac{n}{2}+n$,$-(\frac{n}{2}+n)$ to the vertices u, v respectively.

Case 2. n is odd.

Consider the $\frac{n-1}{2}^{th}$ row. That is consider the vertices $a_{\frac{n-1}{2},1}$, $a_{\frac{n-1}{2},2}$, $a_{\frac{n-1}{2},3}, \cdots, a_{\frac{n-1}{2},n}$. Assign the labels $1, 2, 3, \cdots, n$ to the vertices $a_{\frac{n-1}{2},1}$, $a_{\frac{n-1}{2},2}$, $a_{\frac{n-1}{2},3}, \cdots, a_{\frac{n-1}{2},n}$ respectively and next consider the $\frac{n-3}{2}^{th}$ row. Assign the labels $n+1, n+2, n+3, \cdots, 2n$ to the vertices $a_{\frac{n-3}{2},1}$, $a_{\frac{n-3}{2},2}$, $a_{\frac{n-3}{2},3}, \cdots, a_{\frac{n-3}{2},n}$. Now assign the labels $2n+1, 2n+2, 2n+3, \cdots, 3n$ to the vertices $a_{\frac{n-5}{2},1}$, $a_{\frac{n-5}{2},2}$, $a_{\frac{n-5}{2},3}, \cdots, a_{\frac{n-5}{2},n}$ respectively. Proceeding like this until we reach the first row. Note that the vertices $a_{1,1}$, $a_{1,2}$, $a_{1,3}, \cdots, a_{1,n}$ gets the labels

$$\frac{n}{2}+1, \frac{n}{2}+2, \frac{n}{2}+3, \cdots, \frac{n}{2}+n.$$

Consider the $\frac{n+1}{2}^{th}$ row. That is consider the vertices $a_{\frac{n+1}{2},1}$, $a_{\frac{n+1}{2},2}$, $a_{\frac{n+1}{2},3}, \cdots, a_{\frac{n+1}{2},n}$. Assign the labels $-1, -2, -3, \cdots, -n$ to the vertices $a_{\frac{n+1}{2},1}$, $a_{\frac{n+1}{2},2}$, $a_{\frac{n+1}{2},3}, \cdots, a_{\frac{n+1}{2},n}$ respectively and next consider the $\frac{n+3}{2}^{th}$ row. Assign the labels $-(n+1), -(n+2), -(n+3), \cdots, -2n$ to the vertices $a_{\frac{n+3}{2},1}$, $a_{\frac{n+3}{2},2}$, $a_{\frac{n+3}{2},3}, \cdots, a_{\frac{n+3}{2},n}$. Now assign the labels $-(2n+1), -(2n+2), -(2n+3), \cdots, -3n$ to the vertices $a_{\frac{n+5}{2},1}$, $a_{\frac{n+5}{2},2}$, $a_{\frac{n+5}{2},3}, \cdots, a_{\frac{n+5}{2},n}$ respectively. Proceeding like this until we reach the $(n-1)^{th}$ row. Note that the vertices $a_{n-1,1}$, $a_{n-1,2}$, $a_{n-1,3}, \cdots, a_{n-1,n}$ gets the labels$-(\frac{n-1}{2}+1), -(\frac{n-1}{2}+2), -(\frac{n-1}{2}+3), \cdots, -(\frac{n-1}{2}+n)$. Finally assign the labels

$$(\frac{n-1}{2}+n+1), (\frac{n-1}{2}+n+2), (\frac{n-1}{2}+n+3), \cdots, (\frac{n-1}{2}+\frac{n-1}{2}),$$
$$-(\frac{n-1}{2}+n+1), -(\frac{n-1}{2}+n+2), -(\frac{n-1}{2}+n+3), \cdots, -(\frac{n-1}{2}+\frac{n-3}{2})$$

respectively and assign the labels $\frac{n}{2}+n-1, -(\frac{n-1}{2}+\frac{n-1}{2})$ to the vertices u, v.

Table 1 given below establishes that this vertex labeling is a pair difference cordial labeling of DA_n for all values of $n \geq 2$.

Nature of n	Δ_{f_1}	$\Delta_{f_1^c}$
n is even	n^2	n^2
n is odd	n^2	n^2

Table 1

For illustration, DA_5 is shown in Figure 3.

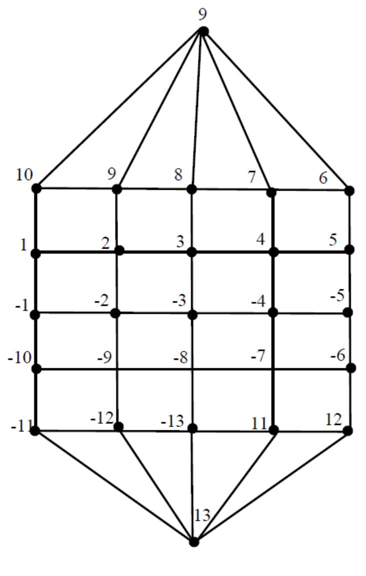

Figure 3

This completes the proof. □

Theorem 3.4 *A sunflower graph SF_n is pair difference cordial for all values of $n \geq 3$.*

Proof Take the vertex set and edge set from Definition 2.5. There are two cases arises.

Case 1. n is even.

Assign the label 2 to the vertex a. Next assign the labels $1, 3, 5, \cdots, n-1$ to the vertices $a_1, a_2, a_3, \cdots, a_{\frac{n}{2}}$ respectively and assign the labels $2, 4, 6, \cdots, n$ respectively to the vertices $b_1, b_2, b_3, \cdots, b_{\frac{n}{2}}$.

Now we assign the labels $-1, -3, -5, \cdots, -(n-1)$ to the vertices $a_{\frac{n+2}{2}}, a_{\frac{n+4}{2}}, a_{\frac{n+6}{2}}, \cdots, a_n$ respectively and assign the labels $-2, -4, -6, \cdots, -n$ respectively to the vertices $b_{\frac{n+2}{2}}$, $b_{\frac{n+4}{2}}$, $b_{\frac{n+6}{2}}, \cdots, b_n$.

Case 2. n is odd.

Assign the label 2 to the vertex a. Next assign the labels $1, 3, 5, \cdots, n$ to the vertices $a_1, a_2, a_3, \cdots, a_{\frac{n+1}{2}}$ respectively and assign the labels $2, 4, 6, \cdots, n-1$ respectively to the vertices $b_1, b_2, b_3, \cdots, b_{\frac{n-1}{2}}$.

Now we assign the labels $-1, -3, -5, \cdots, -(n)$ to the vertices $a_{\frac{n+3}{2}}$, $a_{\frac{n+5}{2}}$, $a_{\frac{n+7}{2}}, \cdots$, a_n respectively and assign the labels $-2, -4, -6, \cdots, -(n-1)$ respectively to the vertices $b_{\frac{n+1}{2}}$, $b_{\frac{n+3}{2}}$, $b_{\frac{n+5}{2}}, \cdots, b_n$.

Notices that Table 2 given below establishes that this vertex labeling is a pair difference cordial labeling of SF_n for all values of $n \geq 3$.

Nature of n	Δ_{f_1}	$\Delta_{f_1^c}$
n is even	$2n$	$2n$
n is odd	$2n$	$2n$

Table 2

This completes the proof. □

Theorem 3.5 *A shell graph $C_{(n,n-3)}$ is pair difference cordial for all values of $n \geq 3$.*

Proof Let us take vertex set and edge set from Definition 2.6 Assign the labels $1, 2, 3, \cdots, \frac{n}{2}$ respectively to the vertices $a_1, a_2, a_3, \cdots, a_{\frac{n}{2}}$ and assign the labels $-1, -2, -3, \cdots, -\frac{n}{2}$ to the vertices $a_{\frac{n+2}{2}}, a_{\frac{n+4}{2}}, a_{\frac{n+6}{2}}, \cdots, a_n$ respectively. □

Theorem 3.6 *A butterfly graph with shell order m, m is pair difference cordial for all values of $m \geq 3$.*

Proof Assign the labels $1, 2, 3, \cdots, n$ respectively to the vertices $a_1, a_2, a_3, \cdots, a_n$ and assign the labels $-1, -2, -3, \cdots, -n$ to the vertices $a_1, a_2, a_3, \cdots, a_n$ respectively. □

Theorem 3.7 *A graph obtained by joining two copies of shell graph by a path of arbitrary length is pair difference cordial.*

Proof Let G be the graph obtained by joining two copies of shell graph by a path of length. Let $a_1, a_2, a_3, \cdots, a_n$ be the successive vertices of 1^{st} copy of shell graph and let $b_1, b_2, b_3, \cdots, b_n$ be the successive vertices of 2^{nd} copy of shell graph. Let $c_1, c_2, c_3, \cdots, c_k$ be the successive vertices of path P_k with $c_1 = a_1$ and $c_k = b_1$.

There are two cases arises.

Case 1. k is odd.

Assign the labels $1, 2, -1, -2$ respectively to the vertices c_1, c_2, c_3, c_4 and assign the labels $3, 4, -3, -4$ to the vertices c_5, c_6, c_7, c_8 respectively. Next assign the labels $1, 2, -1, -2$ respectively to the vertices $c_9, c_{10}, c_{11}, c_{12}$ and assign the labels $5, 6, -5, -6$ to the vertices $c_{13}, c_{14}, c_{15}, c_{16}$ respectively. Proceeding like this until we reach c_{n-1}.

Now, we assign the labels

$$\frac{k+1}{2}, \frac{k+3}{2}, \frac{k+5}{2}, \cdots, \frac{2n+k-1}{2}$$

respectively to these vertices $a_2, a_3, a_4, \cdots, a_{n-1}$ and

$$-\frac{k+1}{2}, -\frac{k+3}{2}, -\frac{k+5}{2}, \cdots, -\frac{2n+k-1}{2}$$

respectively to the vertices $b_2, b_3, b_4, \cdots, b_{n-1}$. Finally assign the label $-\frac{k-1}{2}$ to the vertex c_n.

Case 2. k is even.

Assign the labels $1, 2, -1, -2$ respectively to the vertices c_1, c_2, c_3, c_4 and assign the labels $3, 4, -3, -4$ to the vertices c_5, c_6, c_7, c_8 respectively. Next assign the labels $1, 2, -1, -2$ respectively to the vertices $c_9, c_{10}, c_{11}, c_{12}$ and assign the labels $5, 6, -5, -6$ to the vertices $c_{13}, c_{14}, c_{15}, c_{16}$ respectively. Proceeding like this until we reach c_n.

Now, we assign the labels

$$\frac{k+2}{2}, \frac{k+4}{2}, \frac{k+6}{2}, \cdots, \frac{2n+k-2}{2}$$

respectively to these vertices $a_2, a_3, a_4, \cdots, a_{n-1}$ and

$$-\frac{k+2}{2}, -\frac{k+4}{2}, -\frac{k+6}{2}, \cdots, -\frac{2n+k-2}{2}$$

respectively to the vertices $b_2, b_3, b_4, \cdots, b_{n-1}$ and get the result. □

Theorem 3.8 *An ice cream graph IC_n is pair difference cordial for $n \geq 3$.*

Proof Take the vertex set and edge set from Definition 2.5. There are two cases arises.

Case 1. n is even.

Assign the labels

$$1, 2, 3, \cdots, \frac{n}{2}$$

to the vertices $a_1, a_2, a_3, \cdots, a_{\frac{n}{2}}$ and assign the labels

$$-1, -2, -3, \cdots, -\frac{n}{2}$$

to the vertices $a_{\frac{n+2}{2}}$, $a_{\frac{n+4}{2}}$,$a_{\frac{n+6}{2}}, \cdots, a_n$. Finally assign the labels $\frac{n+2}{2}$,$-\frac{n+2}{2}$ to the vertices

v_0, v.

Case 2. n is odd.

Assign the labels

$$1, 2, 3, \cdots, \frac{n+1}{2}$$

to the vertices $a_1, a_2, a_3, \cdots, a_{\frac{n+1}{2}}$ and assign the labels

$$-1, -2, -3, \cdots, -\frac{n-1}{2}$$

to the vertices $a_{\frac{n+3}{2}}, a_{\frac{n+5}{2}}, a_{\frac{n+7}{2}}, \cdots, a_n$. Finally assign the labels $\frac{n+1}{2}$,1 to the vertices v_0, v.

For illustration, IC_5 is shown in Figure 4.

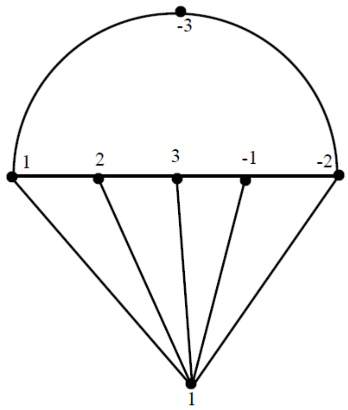

Figure 4

This completes the proof. □

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