International J.Math. Combin. Vol.2(2023), 65-73

Pair Difference Cordial Labeling of Some Trees and Some Graphs Derived From Cube Graph

R. Ponraj

Department of Mathematics, Sri Paramakalyani College, Alwarkurichi–627 412, India

A. Gayathri

Research Scholor, Reg.No:20124012092023

Department of Mathematics, Manonmaniam Sundaranar University, Abhishekapati, Tirunelveli–627 012, India

S. Somasundaram

Department of Mathematics, Manonmaniam Sundaranar University, Tirunelveli– 627 012, India

E-mail: ponrajmaths@gmail.com, gayugayathria555@gmail.com, somutvl@gmail.com

Abstract: In this paper we study the pair difference cordial labeling behavior of some trees and some graphs derived from cube graph.

Key Words: Smarandachely pair difference cordial labeling, pair difference cordial labeling, tree, star, cube, Y-tree, W-tree.

AMS(2010): 05C78.

§1. Introduction

In this paper we consider only finite, undirected and simple graphs.The concept of pair difference cordial labeling of a graph was introduced and studied some properties of pair difference cordial labeling in [4]. By definition, let $L = \{\pm 1, \pm 2, \pm 3, \cdots, \pm\lfloor p/2\rfloor\}$. Consider a mapping $f : V \longrightarrow L$ by assigning different labels in L to the different elements of V when p is even and different labels in L to $p-1$ elements of V and repeating a label for the remaining one vertex when p is odd. Such a labeling is said to be a pair difference cordial labeling if for each edge uv of G there exists a labeling $|f(u)-f(v)|$ such that $\left|\Delta_{f_1} - \Delta_{f_1^c}\right| \leq 1$. Otherwise, it is called a Smarandachely pair difference cordial labeling if $\left|\Delta_{f_1} - \Delta_{f_1^c}\right| \geq 2$, where Δ_{f_1} and $\Delta_{f_1^c}$ respectively denote the numbers of edges labeled or not labeled with 1.

A graph G for which there exists a pair difference cordial labeling or Smarandachely pair difference cordial labeling is called a pair difference cordial graph or Smarandachely pair difference cordial graph. The pair difference cordial labeling behavior of several graphs have been investigated in [4,5,6,7,8,9,10,11]. In this paper we investigate pair difference cordial labeling behavior of some trees and some graphs derived from cube graph.Terms not defined here are follow from [2,3].

[1]Received March 29, 2023, Accepted June 9, 2023.

§2. Preliminaries

Definition 2.1([2]) *Let P_n be the path $a_1a_2a_3\cdots a_n$. A $Y-$tree Y_n is the tree of order $n+1$ whose vertex set is $V(Y_n)=\{a_1,a_2,a_3,\cdots,a_n,a\}$ and the edge set $E(Y_n)=E(P_n)\cup\{a_{n-1}a\}$. In other words Y_n is obtained by attaching the vertex a to the vertex a_{n-1} of P_n.*

Definition 2.2([2]) *A $W-$ graph $W(n)$ is the graph with vertex set*

$$\{c_1,c_2,b,w,d\}\bigcup\{x^1,x^2,x^3,\cdots,x^n\}\bigcup\{y^1,y^2,y^3,\cdots,y^n\}$$

and the edge set

$$\{c_1x^1,c_1x^2,\cdots,c_1x^n\}\bigcup\{c_2y^1,c_2y^2,\cdots,c_2y^n\}\bigcup\{c_1b,c_1w,c_2w,c_2d\}.$$

Definition 2.3([2]) *A $W-$tree $WT(n,k)$ is a graph obtained by taking $k-$ copies of $W-$ graph $W(n)$ and a new vertex a and joining a which in each copy d where $n\geq 2,k\geq 3$.*

Let $V(WT(n,k))=\{a,c_1^i,c_2^i,d^i,x_1^i,x_2^i,x_3^i,\cdots,x_{n+1}^i,y_1^i,y_2^i,y_3^i,\cdots,y_{n+1}^i:1\leq i\leq k\}$, $E(WT(n,k))=\{ac_1^i,ac_2^i,d^ic_1^i,d^ic_2^i,c_1^ix_j^i,c_2^ix_j^i:1\leq i\leq k,1\leq j\leq n\}$. Obviously $WT(n,k)$ has $nk(k+1)+k(n+1)+1$ vertices and $nk(k+1)+k(n+1)$ edges.

Definition 2.4([3]) *Let G be the graph and $G_1,G_2,G_3,\cdots,G_n;n\geq 2$ be n copies of the graph G. Then the graph obtained by adding an edge from G_i to G_{i+1}, $i=1,2,3,\cdots,n-1)$ is called path union of graph G.*

Definition 2.5([3]) *Let $G_1,G_2,G_3,\cdots,G_n$ be any $n-$ graphs. A graph obtained by replacing each vertex of $K_{1,n}$ except the apex vertex by the graph $G_1,G_2,G_3,\cdots,G_n$ is known as an open star of graphs which is denoted by $OS(G_1,G_2,G_3,\cdots,G_n)$. If $G_1=G_2=G_3=\cdots=G_n=G$ then it is denoted by $OS(n.G)$.*

Definition 2.6([3]) *A hypercube is an $n-$ dimensional analogue of a square $(n=2)$ and a cube $(n=3)$ which is also known as an $n-$ cube or $n-$ dimensional cube which is denoted by Q_n.*

§3. Graphs Obtained From Trees

Theorem 3.1 *A Y-tree is pair difference cordial for all values of $n\geq 3$.*

Proof Take the vertex set and edge set from Definition 2.1. The proof is divided into the following 4 cases.

Case 1. $n\equiv 0(mod 4)$.

Assign the labels $1,2,-1,-2$ respectively to the vertices a_1,a_2,a_3,a_4 and allocate the values $3,4,-3,-4$ individually to the vertices a_5,a_6,a_7,a_8. Net we put the labels $5,6,-5,-6$ separately to the vertices a_9,a_{10},a_{11},a_{12} and assign the labels $7,8,-7,-8$ respectively to the vertices $a_{13},a_{14},a_{15},a_{16}$. Proceeding like this process until we reach the vertex a_n. Finally assign the label -1 to the vertex a.

In this case $\Delta_{f_1} = \Delta_{f_1^c} = \frac{n}{2}$.

Case 2. $n \equiv 1(mod4)$.

Assign the labels as in Case 1 to the vertices $a_i, 1 \leq i \leq n-1)$. And then, assign the labels $\frac{n+1}{2}, -(\frac{n+1}{2})$ to the vertices a_n, a. Then $\Delta_{f_1} = \frac{n+1}{2}, \Delta_{f_1^c} = \frac{n-1}{2}$.

Case 3. $n \equiv 2(mod4)$.

Assign the labels as in Case 1 to the vertices $a_i, 1 \leq i \leq n-2)$. Lastly assign the labels $\frac{n}{2}, -(\frac{n}{2}), \frac{n-2}{2}$ to the vertices a_{n-1}, a_n, a.

In this case $\Delta_{f_1} = \Delta_{f_1^c} = \frac{n}{2}$.

Case 4. $n \equiv 3(mod4)$.

Assign the labels as in Case 1 to the vertices $a_i, 1 \leq i \leq n-3)$. Finally assign the labels $\frac{n-1}{2}, \frac{n+1}{2}, -(\frac{n-1}{2}), -(\frac{n+1}{2})$ to the vertices a_{n-2}, a_{n-1}, a_n, a. Then $\Delta_{f_1} = \frac{n-1}{2}, \Delta_{f_1^c} = \frac{n+1}{2}$. □

Theorem 3.2 *The W-tree $WT(2,n)$ is not pair difference cordial for all values of $n \geq 3$.*

Proof A $WT(2,n)$ has $7n+3$ vertices and $7n+2$ edges. Our proof is divided into 2 cases following.

Case 1. n is even.

The maximum possible of $\Delta_{f_1} = 4n$. Then $\Delta_{f_1^c} \geq 7n+2-4n.\Delta_{f_1^c} - \Delta_{f_1} \geq 3n+2 > 1$.

Case 2. n is odd.

The maximum possible of $\Delta_{f_1} = 4n+1$. Then $\Delta_{f_1^c} \geq 7n+2-4n-1$. $\Delta_{f_1^c} - \Delta_{f_1} \geq 3n+1 > 1$.

Therefore, a wheel $WT(2,n)$ is not pair difference cordial. □

§4. Graphs Obtained From Cube

Theorem 4.1 *The path union of $n-$ copies of Q_3 is pair difference cordial for all values of $n \geq 2$.*

Proof Let G be the graph obtained by joining $n-$ copies of the cube Q_3. Let

$$\begin{aligned} V(G) &= \{x_{i1}, y_{i1}, x_{i2}, y_{i2}, x_{i3}, y_{i3}, x_{i4}, y_{i4} : 1 \leq i \leq n\}, \\ E(G) &= \{x_{i1}x_{i2}, x_{i2}x_{i3}, x_{i3}x_{i4}, x_{i1}x_{i4}, y_{i1}y_{i2}, y_{i2}y_{i3}, y_{i3}y_{i4}, y_{i1}y_{i4} : 1 \leq i \leq n\} \\ & \quad \bigcup \{x_{ij}y_{ij} : 1 \leq i \leq n, 1 \leq j \leq 4\}. \end{aligned}$$

Obviously, G has $8n$ vertices and $13n-1$ edges. Our proof is divided into 2 cases following.

Case 1. n is even.

When $n = 2$, Assign the labels $1, 2, 3, 4, -1, -2, -3, -4$ respectively to the vertices x_{11}, x_{12},

$x_{13}, x_{14}, y_{11}, y_{12}, y_{13}, y_{14}$ and assign the labels $5, 6, 7, 8, -5, -6, -7, -8$ respectively to the vertices $x_{21}, x_{22}, x_{23}, x_{24}, y_{21}, y_{22}, y_{23}, y_{24}$.

If $n \geq 4$, define a map ψ from the vertex set $V(G)$ to the set $\{\pm1, \pm2, \cdots, \pm4n\}$ by

$$\begin{aligned}
\psi(x_{i1}) &= 8i-7, & i &= 1,3,5,\cdots,n-1,\\
\psi(x_{i2}) &= 8i-6, & i &= 1,3,5,\cdots,n-1,\\
\psi(x_{i3}) &= 8i-5, & i &= 1,3,5,\cdots,n-1,\\
\psi(x_{i4}) &= 8i-4, & i &= 1,3,5,\cdots,n-1,\\
\psi(y_{i1}) &= -(8i-7), & i &= 1,3,5,\cdots,n-1,\\
\psi(y_{i2}) &= -(8i-6), & i &= 1,3,5,\cdots,n-1,\\
\psi(y_{i3}) &= -(8i-5), & i &= 1,3,5,\cdots,n-1,\\
\psi(y_{i4}) &= -(8i-4), & i &= 1,3,5,\cdots,n-1,\\
\psi(x_{i1}) &= 8i-2, & i &= 2,4,6,\cdots,n,\\
\psi(x_{i2}) &= 8i-3, & i &= 2,4,6,\cdots,n,\\
\psi(x_{i3}) &= 8i-1, & i &= 2,4,6,\cdots,n,\\
\psi(x_{i4}) &= 8i, & i &= 2,4,6,\cdots,n,\\
\psi(y_{i1}) &= -(8i-2), & i &= 2,4,6,\cdots,n,\\
\psi(y_{i2}) &= -(8i-3), & i &= 2,4,6,\cdots,n,\\
\psi(y_{i3}) &= -(8i-1), & i &= 2,4,6,\cdots,n,\\
\psi(y_{i4}) &= -(8i), & i &= 2,4,6,\cdots,n.
\end{aligned}$$

Case 2. n is odd.

Define a map $\psi : V(G) \to \{\pm1, \pm2, \cdots, \pm4n\}$ by

$$\begin{aligned}
\psi(x_{i1}) &= 8i-7, & i &= 1,3,5,\cdots,n,\\
\psi(x_{i2}) &= 8i-6, & i &= 1,3,5,\cdots,n,\\
\psi(x_{i3}) &= 8i-5, & i &= 1,3,5,\cdots,n,\\
\psi(x_{i4}) &= 8i-4, & i &= 1,3,5,\cdots,n,\\
\psi(y_{i1}) &= -(8i-7), & i &= 1,3,5,\cdots,n,\\
\psi(y_{i2}) &= -(8i-6), & i &= 1,3,5,\cdots,n,\\
\psi(y_{i3}) &= -(8i-5), & i &= 1,3,5,\cdots,n,\\
\psi(y_{i4}) &= -(8i-4), & i &= 1,3,5,\cdots,n,\\
\psi(x_{i1}) &= 8i-2, & i &= 2,4,6,\cdots,n-1,\\
\psi(x_{i2}) &= 8i-3, & i &= 2,4,6,\cdots,n-1,\\
\psi(x_{i3}) &= 8i-1, & i &= 2,4,6,\cdots,n-1,
\end{aligned}$$

$$\psi(x_{i4}) = 8i, \qquad i = 2, 4, 6, \cdots, n-1,$$
$$\psi(y_{i1}) = -(8i-2), \qquad i = 2, 4, 6, \cdots, n-1,$$
$$\psi(y_{i2}) = -(8i-3), \qquad i = 2, 4, 6, \cdots, n-1,$$
$$\psi(y_{i3}) = -(8i-1), \qquad i = 2, 4, 6, \cdots, n-1,$$
$$\psi(y_{i4}) = -(8i), \qquad i = 2, 4, 6, \cdots, n-1.$$

Table 2 given below establishes that this vertex labeling f is a pair difference cordial.

Nature of n	$\Delta_{f_1^c}$	Δ_{f_1}
n is odd	$\frac{13n-1}{2}$	$\frac{13n-1}{2}$
n is even	$\frac{13n}{2}$	$\frac{13n-2}{2}$

This completes the proof. □

Theorem 4.2 *A graph obtained by joining two copies of Q_3 by path P_n is pair difference cordial for all values of $n \geq 4$.*

Proof Let G be the graph obtained by joining two copies of Q_3 by path P_n with

$$V(G) = \{x_{i1}, y_{i1}, x_{i2}, y_{i2}, x_{i3}, y_{i3}, x_{i4}, y_{i4} : 1 \leq i \leq 2\} \bigcup \{z_k : 1 \leq k \leq n-2\},$$
$$E(G) = E(Q_3) \bigcup \{z_i z_{i+1} : 1 \leq i \leq n-2\} \bigcup \{z_1 y_{14}, z_{n-2} x_{11}\}.$$

Obviously, G has $n+14$ vertices and $n+23$ edges.

Case 1. $n \equiv 0(mod 4)$.

Assign labels $1, 2, 3, 4, 5, 6, 7, 8$ respectively to vertices $x_{11}, x_{12}, x_{13}, x_{14}, y_{11}, y_{12}, y_{13}, y_{14}$ and assign the labels $-1, -2, -3, -4, -5, -6, -7, -8$ respectively to the vertices $x_{21}, x_{22}, x_{23}, x_{24}, y_{21}, y_{22}, y_{23}, y_{24}$.

Assign the labels $9, 10, -9, -10$ respectively to the vertices z_1, z_2, z_3, z_4 and allocate the values $11, 12, -11, -12$ individually to the vertices z_5, z_6, z_7, z_8. Net we put the labels $5, 6, -5, -6$ separately to the vertices $z_9, z_{10}, z_{11}, z_{12}$ and assign the labels $7, 8, -7, -8$ respectively to the vertices $z_{13}, z_{14}, z_{15}, z_{16}$. Proceeding like this process until we reach the vertex z_{n-4}. Finally assign the labels $\frac{n+14}{2}, -(\frac{n+14}{2})$ to the vertex z_{n-3}, z_{n-2}.

Case 2. $n \equiv 1(mod 4)$.

Assign the labels as in Case 1 to the vertices $x_{ij}, y_{ij}, 1 \leq i \leq 2, 1 \leq j \leq 4,$) and $z_k, 1 \leq k \leq n-5$. And then, assign the labels $\frac{n+13}{2}, -(\frac{n+13}{2}), -(\frac{n+11}{2})$ to the vertices $z_{n-4}, z_{n-3}, z_{n-2}$.

Case 3. $n \equiv 2(mod 4)$.

Assign the labels as in case 1 to the vertices $x_{ij}, y_{ij}, 1 \leq i \leq 2, 1 \leq j \leq 4,$) and $z_k, 1 \leq k \leq n-6$. Lastly assign the labels $\frac{n+12}{2}, \frac{n+14}{2}, -(\frac{n+12}{2}), -(\frac{n+14}{2})$ to the vertices $z_{n-5}, z_{n-4}, z_{n-3}, z_{n-2}$.

Case 4. $n \equiv 3(mod4)$.

Assign the labels as in case 1 to the vertices $x_{ij}, y_{ij}, 1 \leq i \leq 2, 1 \leq j \leq 4,)$ and $z_k, 1 \leq k \leq n-7$. Finally assign the labels $\frac{n+12}{2}, \frac{n+14}{2}, -(\frac{n+12}{2}), -(\frac{n+14}{2}), -(\frac{n+12}{2})$ to the vertices $z_{n-6}, z_{n-5}, z_{n-4}, z_{n-3}, z_{n-2}$.

The Table 3 given below establishes that this vertex labeling f is a pair difference cordial.

Nature of n	Δ_{f_1}	$\Delta_{f_1^c}$
$n \equiv 0 \pmod 4$	$\frac{n+24}{2}$	$\frac{n+22}{2}$
$n \equiv 1 \pmod 4$	$\frac{n+23}{2}$	$\frac{n+23}{2}$
$n \equiv 2 \pmod 4$	$\frac{n+24}{2}$	$\frac{n+22}{2}$
$n \equiv 3 \pmod 4$	$\frac{n+23}{2}$	$\frac{n+23}{2}$

This completes the proof. □

Theorem 4.3 *An $S(n.Q_3)$ is pair difference cordial for all even n.*

Proof Our proof is divided into 2 cases following.

Case 1. $n \equiv 0(mod4)$.

Define a map $\psi : V(G) \to \{\pm 1, \pm 2, \cdots, \pm 4n\}$ by

$$\begin{aligned}
\psi(x) &= 1, \\
\psi(x_{i1}) &= 4i-3, & 1 \leq i \leq \frac{n}{2}, \\
\psi(x_{i2}) &= 4i-2, & 1 \leq i \leq \frac{n}{2}, \\
\psi(x_{i3}) &= 4i-1, & 1 \leq i \leq \frac{n}{2}, \\
\psi(x_{i4}) &= 4i, & 1 \leq i \leq \frac{n}{2}, \\
\psi(y_{i1}) &= -(4i-3), & 1 \leq i \leq \frac{n}{2}, \\
\psi(y_{i2}) &= -(4i-2), & 1 \leq i \leq \frac{n}{2}, \\
\psi(y_{i3}) &= -(4i-1), & 1 \leq i \leq \frac{n}{2}, \\
\psi(y_{i4}) &= -4i, & 1 \leq i \leq \frac{n}{2}, \\
\psi(x_{(\frac{n}{2}+2i-1)1}) &= 2n+4i-3, & 1 \leq i \leq \frac{n}{4}, \\
\psi(x_{(\frac{n}{2}+2i-1)2}) &= 2n+4i-2, & 1 \leq i \leq \frac{n}{4}, \\
\psi(x_{(\frac{n}{2}+2i-1)3}) &= 2n+4i-1, & 1 \leq i \leq \frac{n}{4}, \\
\psi(x_{(\frac{n}{2}+2i-1)4}) &= 2n+4i, & 1 \leq i \leq \frac{n}{4}, \\
\psi(y_{(\frac{n}{2}+2i-1)1}) &= 2n+4i+4, & 1 \leq i \leq \frac{n}{4},
\end{aligned}$$

$$\psi(y_{(\frac{n}{2}+2i-1)2}) = 2n + 4i + 3, \qquad 1 \le i \le \frac{n}{4},$$
$$\psi(y_{(\frac{n}{2}+2i-1)3}) = 2n + 4i + 2, \qquad 1 \le i \le \frac{n}{4},$$
$$\psi(y_{(\frac{n}{2}+2i-1)4}) = 2n + 4i + 1, \qquad 1 \le i \le \frac{n}{4},$$
$$\psi(x_{(\frac{n}{2}+2i)1}) = -(2n + 4i - 3), \qquad 1 \le i \le \frac{n}{4},$$
$$\psi(x_{(\frac{n}{2}+2i)2}) = -(2n + 4i - 2), \qquad 1 \le i \le \frac{n}{4},$$
$$\psi(x_{(\frac{n}{2}+2i)3}) = -(2n + 4i - 1), \qquad 1 \le i \le \frac{n}{4},$$
$$\psi(x_{(\frac{n}{2}+2i)4}) = -(2n + 4i), \qquad 1 \le i \le \frac{n}{4},$$
$$\psi(y_{(\frac{n}{2}+2i)1}) = -(2n + 4i + 4), \qquad 1 \le i \le \frac{n}{4},$$
$$\psi(y_{(\frac{n}{2}+2i)2}) = -(2n + 4i + 3), \qquad 1 \le i \le \frac{n}{4},$$
$$\psi(y_{(\frac{n}{2}+2i)3}) = -(2n + 4i + 2), \qquad 1 \le i \le \frac{n}{4},$$
$$\psi(y_{(\frac{n}{2}+2i)4}) = -(2n + 4i + 1), \qquad 1 \le i \le \frac{n}{4},$$

Case 2. $n \equiv 1(mod4)$.

Define a map ψ from the vertex set $V(G)$ to the set $\{\pm 1, \pm 2, \cdots, \pm 4n\}$ by

$$\psi(x) = 3,$$
$$\psi(x_{i1}) = 4i - 3, \qquad 1 \le i \le \frac{n+2}{2},$$
$$\psi(x_{i2}) = 4i - 2, \qquad 1 \le i \le \frac{n+2}{2},$$
$$\psi(x_{i3}) = 4i - 1, \qquad 1 \le i \le \frac{n+2}{2},$$
$$\psi(x_{i4}) = 4i, \qquad 1 \le i \le \frac{n+2}{2},$$
$$\psi(y_{i1}) = -(4i - 3), \qquad 1 \le i \le \frac{n+2}{2},$$
$$\psi(y_{i2}) = -(4i - 2), \qquad 1 \le i \le \frac{n+2}{2},$$
$$\psi(y_{i3}) = -(4i - 1), \qquad 1 \le i \le \frac{n+2}{2},$$
$$\psi(y_{i4}) = -4i, \qquad 1 \le i \le \frac{n+2}{2},$$
$$\psi(x_{(\frac{n}{2}+2i-1)1}) = 2n + 4i + 1, \qquad 1 \le i \le \frac{n-2}{4},$$
$$\psi(x_{(\frac{n}{2}+2i-1)2}) = 2n + 4i + 2, \qquad 1 \le i \le \frac{n-2}{4},$$
$$\psi(x_{(\frac{n}{2}+2i-1)3}) = 2n + 4i + 3, \qquad 1 \le i \le \frac{n-2}{4},$$

$$\psi(x_{(\frac{n}{2}+2i-1)4}) = 2n+4i+4, \qquad 1 \le i \le \frac{n-2}{4},$$
$$\psi(y_{(\frac{n}{2}+2i-1)1}) = 2n+4i+5, \qquad 1 \le i \le \frac{n-2}{4},$$
$$\psi(y_{(\frac{n}{2}+2i-1)2}) = 2n+4i+6, \qquad 1 \le i \le \frac{n-2}{4},$$
$$\psi(y_{(\frac{n}{2}+2i-1)3}) = 2n+4i+7, \qquad 1 \le i \le \frac{n-2}{4},$$
$$\psi(y_{(\frac{n}{2}+2i-1)4}) = 2n+4i+8, \qquad 1 \le i \le \frac{n-2}{4},$$
$$\psi(x_{(\frac{n}{2}+2i)1}) = -(2n+4i+1), \qquad 1 \le i \le \frac{n-2}{4},$$
$$\psi(x_{(\frac{n}{2}+2i)2}) = -(2n+4i+2), \qquad 1 \le i \le \frac{n-2}{4},$$
$$\psi(x_{(\frac{n}{2}+2i)3}) = -(2n+4i+3), \qquad 1 \le i \le \frac{n-2}{4},$$
$$\psi(x_{(\frac{n}{2}+2i)4}) = -(2n+4i+4), \qquad 1 \le i \le \frac{n-2}{4},$$
$$\psi(y_{(\frac{n}{2}+2i)1}) = -(2n+4i+5), \qquad 1 \le i \le \frac{n-2}{4},$$
$$\psi(y_{(\frac{n}{2}+2i)2}) = -(2n+4i+6), \qquad 1 \le i \le \frac{n-2}{4},$$
$$\psi(y_{(\frac{n}{2}+2i)3}) = -(2n+4i+7), \qquad 1 \le i \le \frac{n-2}{4},$$
$$\psi(y_{(\frac{n}{2}+2i)4}) = -(2n+4i+8), \qquad 1 \le i \le \frac{n-2}{4},$$

In both cases $\Delta_{f_1} = \Delta_{f_1^c} = \frac{13n}{2}$. □

A pair difference cordial labeling of $S(6.Q_3)$ is shown in Figure 1.

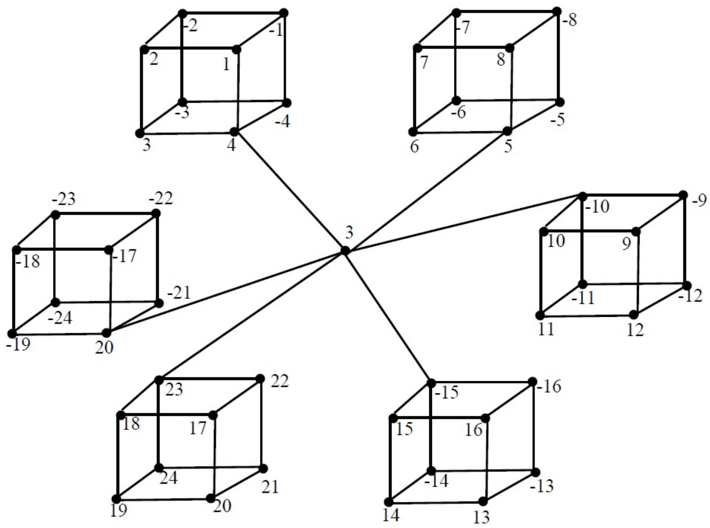

Figure 1

References

[1] Gallian J.A, A Dynamic survey of graph labeling, *The Electronic Journal of Combinatorics.*, 19, (2016).

[2] Harary F, *Graph Theory*, Addision Wesley, New Delhi, 1969.

[3] Jekil A. Gadhiya, Shanti S. Khunti and Mehul P. Rupani, Product cordial labeling of hyper cube related graphs, *Malaya Journal of Matematik*, Vol.8(4), (2020), 1991–1994.

[4] Ponraj R., Gayathri A. and Somasundaram S., Pair difference cordial labeling of graphs, *J.Math. Comp.Sci.*, Vol.11(3), (2021), 2551–2567.

[5] Ponraj R., Gayathri A. and Somasundaram S., Pair difference cordiality of some snake and butterfly graphs, *Journal of Algorithms and Computation*, Vol.53(1), (2021), 149–163.

[6] Ponraj,R., Gayathri,A., and Somasundaram,S., Pair difference cordial graphs obtained from the wheels and the paths, *J. Appl. and Pure Math.*, Vol.3 No. 3-4, (2021), pp. 97–114.

[7] Ponraj R., Gayathri A. and Somasundaram S., Pair difference cordiality of some graphs derived from ladder graph, *J.Math. Comp.Sci.*, Vol.11 No 5, (2021), 6105–6124.

[8] Ponraj R., Gayathri A. and Somasundaram S., Some pair difference cordial graphs, *Ikonion Journal of Mathematics*, Vol.3(2), (2021), 17–26.

[9] Ponraj R., Gayathri A. and Somasundaram S., Pair difference cordial labeling of planar grid and mangolian tent, *Journal of Algorithms and Computation*, Vol.53(2) (2021), 47–56.

[10] Ponraj R., Gayathri A. and Somasundaram S., Pair difference cordiality of some special graphs, *J. Appl. and Pure Math.*, Vol.3 No. 5-6, (2021), pp. 263–274.

[11] Ponraj R., Gayathri A. and Soma Sundaram S., Pair difference cordiality of mirror graph, shadow graph and splitting graph of certain graphs, *Maltepe Journal of Mathematics*, Vol.4, 1(2022), 24–32.