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Semi-Invariant Submanifolds of (k, μ)-Contact Manifold Admitting Semi-Symmetric Metric Connection

M.S. Siddesha

(Department of Mathematics, Jain Deemed-to-be-University, Global Campus-562112, Karnataka, India)

C.S. Bagewadi

(Department of Mathematics, Kuvempu University, Shankaraghatta, Shimoga-577451, Karnataka, India)

E-mail: mssiddesha@gmail.com, prof_bagewadi@yahoo.co.in

Abstract: In this paper, we define a semi-symmetric metric connection in a (k, μ)-contact manifold and study semi-invariant submanifolds of a (k, μ)-contact manifold endowed with a semi-symmetric metric connection. We determine the integrability conditions of distributions on semi-invariant submanifolds of a (k, μ)-contact manifold with a semi-symmetric metric connection.

Key Words: (k, μ)-contact manifold, semi-invariant submanifold, semi-symmetric metric connection.

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§1. Introduction

The torsion tensor T of a linear connection ∇ on a Riemannian manifold M is given by

$$T(X, Y) = \nabla_X Y - \nabla_Y X - [X, Y], \quad X, Y \in \chi(M). \tag{1.1}$$

The connection ∇ is symmetric if its torsion tensor T vanishes identically, otherwise it is non-symmetric. Again a linear connection ∇ is said to be a semi-symmetric connection if, the torsion T of the connection ∇ satisfies

$$T(X, Y) = \eta(Y)X - \eta(X)Y, \tag{1.2}$$

where η is a 1-form and ϕ is a tensor field of type $(1, 1)$. Further, a connection ∇ is called metric connection on a Riemannian manifold M if

$$(\nabla_X g)(Y, Z) = 0. \tag{1.3}$$

Otherwise, it is non-metric. The connection ∇ is said to be semi-symmetric metric connection

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[2]Corresponding author: M.S. Siddesha, E-mail: mssiddesha@gmail.com.

if it satisfies (1.1)-(1.3). In 1924, Friedmann and Schouten [8] were the pioneers who unveiled the notion of a semisymmetric linear connection on a Riemannian manifold. The subsequent introduction of the concept of a metric connection came about through the work of Hayden [9] in 1932.

In 1981, Bejancu and Papaghiuc introduced the idea of semi-invariant submanifolds, as a generalization of invariant and anti-invariant submanifolds of contact metric manifolds. On the other hand in 1995, Blair, Koufogiorgos and Papantoniou [5] introduced the new class of contact metric manifolds with ξ belonging to (k,μ)-nullity distributions which are known as (k,μ)-contact metric manifolds. In our previous work [12]-[17], we have investigated various categories of submanifolds, such as invariant, slant, and semi-slant submanifolds within these manifolds.

Recently, a noteworthy contribution was made by Avijit Sarkar et al. [10], who engaged in a study focused on semi-invariant submanifolds within the context of (k,μ)-contact manifolds. Moreover, an exploration of semi-invariant submanifolds extended to various classes of almost contact manifolds has attracted attention from several geometers such as [1, 2, 7, 11, 20] and other researchers.

Building upon the insights from the aforementioned research endeavors, this present paper embarks on an investigation into semi-invariant submanifolds within the realm of (k,μ)-contact manifolds with a semi-symmetric metric connection. The paper's structure unfolds as follows: In Section 2, a concise introduction to (k,μ)-contact manifolds sets the stage for the ensuing exploration. Moving into Section 3, the analysis demonstrates that the connection induced on semi-invariant submanifolds of a (k,μ)-contact manifold, endowed with a semi-symmetric metric connection, retains both the attributes of being semi-symmetric and metric. Section 4 is dedicated to laying the groundwork by presenting fundamental outcomes pertinent to semi-invariant submanifolds of (k,μ)-contact manifolds with a semi-symmetric metric connection. Concluding the paper, the final section delves into a comprehensive discussion regarding the integrability conditions governing distributions on semi-invariant submanifolds of (k,μ)-contact manifolds with semi-symmetric metric connection comes into play.

§2. (k,μ)-Contact Manifolds

A contact manifold is a $C^\infty-(2n+1)$ manifold $\tilde{M}^{2n+1}$ equipped with a global 1-form η such that $\eta\wedge(d\eta)^n\neq 0$ everywhere on $\tilde{M}^{2n+1}$. Given a contact form η it is well known that there exists a unique vector field ξ, called the characteristic vector field of η, such that $\eta(\xi)=1$ and $d\eta(X,\xi)=0$ for every vector field X on $\tilde{M}^{2n+1}$. A Riemannian metric is said to be associated metric if there exists a tensor field ϕ of type $(1,1)$ such that

$$\phi^2 = -I+\eta\otimes\xi, \quad \eta(\xi)=1, \quad \phi\xi=0, \quad \eta\cdot\phi=0, \tag{2.1}$$

$$g(\phi X,\phi Y) = g(X,Y)-\eta(X)\eta(Y), \quad g(X,\xi)=\eta(X), \tag{2.2}$$

for all vector fields $X,Y\in T\tilde{M}$. Then the structure (ϕ,ξ,η,g) on $\tilde{M}^{2n+1}$ is called a contact metric structure and the manifold $\tilde{M}^{2n+1}$ equipped with such a structure is called a contact metric manifold [4].

Given a contact metric manifold $\tilde{M}^{2n+1}(\phi, \xi, \eta, g)$, we define a $(1,1)$ tensor field h by $h = \frac{1}{2}\mathcal{L}_\xi \phi$, where $\mathcal{L}$ denotes Lie differentiation. Then h is symmetric and satisfies $h\phi = -\phi h$. Thus, if λ is an eigenvalue of h with eigenvector X, $-\lambda$ is also an eigenvalue with eigenvector ϕX. Also we have $Tr \cdot h = Tr \cdot \phi h = 0$ and $h\xi = 0$. Moreover, if $\tilde{\nabla}$ denotes the Riemannian connection of g, then the following relation holds:

$$\tilde{\nabla}_X \xi = -\phi X - \phi h X. \tag{2.3}$$

It is seen that the vector field ξ is a Killing vector with respect to g if and only if $h = 0$. In this case the manifold becomes a K-contact manifold. A K-contact structure on $\tilde{M}$ gives rise to an almost complex structure on the product $\tilde{M}^{2n+1} \times R$. If this almost complex structure is integrable, the contact metric manifold is said to be Sasakian. Equivalently, a contact metric manifold is Sasakian if and only if

$$\tilde{R}(X,Y)\xi = \eta(Y)X - \eta(X)Y,$$

holds for all X, Y, where $\tilde{R}$ denotes the curvature tensor of the manifold $\tilde{M}$.

The (k, μ)-nullity distribution of a contact metric manifold $\tilde{M}^{2n+1}(\phi, \xi, \eta, g)$ is a distribution [5]

$$\begin{aligned} &N(k,\mu) : p \rightarrow N_p(k,\mu) = \{Z \in T_pM : \\ &R(X,Y)Z = k[g(Y,Z)X - g(X,Z)Y] + \mu[g(Y,Z)hX - g(X,Z)hY]\}. \end{aligned}$$

for any $X, Y \in T_p\tilde{M}$. Hence if the characteristic vector field ξ belongs to the (k, μ)-nullity distribution, then we have

$$R(X,Y)\xi = k[\eta(Y)X - \eta(X)Y + \mu[\eta(Y)hX - \eta(X)hY]. \tag{2.4}$$

Thus a contact metric manifold satisfying the relation (2.4) is called a (k, μ)-contact metric manifold. In particular, if $\mu = 0$, then the notion of (k, μ)-nullity distribution reduces to the notion of k-nullity distribution, introduced by Tanno [19]. A (k, μ)-contact metric manifold is Sasakian if $k = 1$. In a (k, μ)-contact metric manifold the following relations hold [5]:

$$h^2 = (k-1)\phi^2, \quad k \le 1, \tag{2.5}$$

$$(\tilde{\nabla}_X \phi)(Y) = g(X + hX, Y)\xi - \eta(Y)(X + hX). \tag{2.6}$$

§3. Semi-Invariant Submanifolds

In this section, we introduce the notion of semi-invariant submanifold of a (k, μ)-contact manifold which generalizes the notion of both invariant and anti-invariant submanifolds.

A non degenerated submanifold M of a (k, μ)-contact manifold is called a semi-invariant submanifold, if there exists a pair of orthogonal distributions $\{D, D^{\perp}\}$ on M such that

(i) $TM = D \oplus D^{\perp} \oplus < \xi >$;

(ii) The distribution D is invariant under ϕ, that is $\phi D_x = D_x$, for each $x \in M$;

(iii) The distribution $D^{\perp}$ is anti-invariant under ϕ, that is $\phi D_x^{\perp} \subset T_x^{\perp} M$, for each $x \in M$.

A semi-invariant submanifold becomes invariant (resp. anti-invariant) submanifold if $D_x^{\perp} = 0$ (resp. $D_x = 0$) for all $x \in M$. Further, a submanifold which is neither invariant nor anti-invariant is called a proper semi-invariant submanifold.

We denote by $\tilde{\nabla}$ the Levi-Civita connection on $\tilde{M}$ with respect to induced metric g. Then the Gauss and Weingarten formulas are given by

$$\tilde{\nabla}_X Y = \nabla_X Y + \sigma(X, Y), \tag{3.1}$$

$$\tilde{\nabla}_X N = -A_N X + \nabla_X^{\perp} N, \tag{3.2}$$

for any tangent vector fields X, Y and the normal vector field N on M, where σ, A, ∇ and $\nabla^{\perp}$ are the second fundamental form, the shape operator, induced connection on M and the normal connection respectively. If the second fundamental form σ is identically zero, then the manifold is said to be totally geodesic. The second fundamental form σ and the shape operator A_N are related by

$$g(\sigma(X, Y), N) = g(A_N X, Y).$$

Now, we define a semi-symmetric metric connection $\bar{\bar{\nabla}}$ in a (k, μ)-contact manifold $\tilde{M}$ by

$$\bar{\bar{\nabla}}_X Y = \tilde{\nabla}_X Y + \eta(Y) X - g(X, Y)\xi. \tag{3.3}$$

for all $X, Y \in T\tilde{M}$.

Proposition 3.1 *Let M be a semi-invariant submanifold of a (k, μ)-contact manifold $\tilde{M}$ with a semi-symmetric metric connection. Then*

$$(\bar{\bar{\nabla}}_X \phi) Y = g(X + hX, Y)\xi - \eta(Y)(X + hX) + g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{3.4}$$

$\forall\ X, Y \in \Gamma(TM)$.

Proof By virtue of (2.6) and (3.3), the proposition follows after having done similar computations as in the proof of Theorem 3 in [20]. □

Proposition 3.2 *Let M be a semi-invariant submanifold of a (k, μ)-contact manifold $\tilde{M}$ with a semi-symmetric metric connection. Then*

$$\bar{\bar{\nabla}}_X \xi = -\phi X - \phi h X + X - \eta(X)\xi, \tag{3.5}$$

$\forall\ X, Y \in \Gamma(TM)$.

Proof By virtue of (3.3) and (2.3), the proposition follows after having done similar computations as in the proof of Theorem 4 in [20]. □

Theorem 3.3 *The connection induced on a semi-invariant submanifold of a (k,μ)-contact manifold that admits a semi-symmetric metric connection also admits a semi-symmetric metric connection.*

Proof Let $\bar{\nabla}$ be the induced connection with respect to the unit normal N on semi-invariant submanifold M of a (k,μ)-contact manifold with semi-symmetric metric connection $\bar{\bar{\nabla}}$. Then,

$$\bar{\bar{\nabla}}_X Y = \bar{\nabla}_X Y + m(X,Y), \tag{3.6}$$

where m is a tensor field of type $(0,2)$ on semi-invariant submanifold M.

Using (3.1) and (3.3), we have

$$\bar{\nabla}_X Y + m(X,Y) = \nabla_X Y + \sigma(X,Y) + \eta(Y)X - g(X,Y)\xi.$$

Equating tangential and normal components of the above equation, we get

$$\begin{aligned} m(X,Y) &= \sigma(X,Y). \\ \nabla_X Y &= \nabla^*_X Y + \eta(Y)X - g(X,Y)\xi. \end{aligned} \tag{3.7}$$

Thus, $\bar{\nabla}$ is also semi-symmetric metric connection. □

Now, the Gauss and Weingarten formulae for semi-invariant submanifolds of a (k,μ)-contact manifold with a semi-symmetric metric connection are given by

$$\bar{\bar{\nabla}}_X Y = \bar{\nabla}_X Y + \sigma(X,Y), \tag{3.8}$$

$$\bar{\bar{\nabla}}_X N = (-A_N + \eta(N))X + \bar{\nabla}^{\perp}_X N, \tag{3.9}$$

for all $X,Y \in \Gamma(TM)$ and $N \in \Gamma(T^{\perp}M)$, where σ and A_N are the second fundamental form and Weingarten endomorphism associated with N, and are related by

$$g(\sigma(X,Y),N) = g((-A_N + \eta(N))X, Y).$$

For $X \in \Gamma(TM)$, $N \in \Gamma(T^{\perp}M)$, we can write

$$X = PX + QX + \eta(X)\xi, \tag{3.10}$$

$$\phi N = BN + CN, \tag{3.11}$$

where P and Q are the projection operators of TM to D and $D^{\perp}$ respectively, and BN (resp. CN) denote the tangential (resp. normal) component of ϕN.

§4. Basic Results

Lemma 4.1 *Let M be a semi-invariant submanifold of a (k,μ)-contact manifold $\tilde{M}$ with a* *semi-symmetric metric connection, then we have*

$$(\bar{\bar{\nabla}}_X\phi)Y = (\bar{\nabla}_X P)Y + \sigma(X,PY) - A_{QY}X + \eta(N)X + (\bar{\nabla}_X Q)Y - B\sigma(X,Y) - C\sigma(X,Y), \quad (4.1)$$

$$(\bar{\bar{\nabla}}_X\phi)N = (\bar{\nabla}_X B)N + \sigma(X,BN) - A_{CN}X + P(-A_N)X + Q(-A_N)X + (\bar{\nabla}_X C)N + P\eta(N)X + Q\eta(N)X. \quad (4.2)$$

Proof Using (3.10), (3.11), the Gauss and Weingarten formulae, necessary arrangements are made to obtain the desired. □

We state the next Lemma whose proof is straightforwardly deduced by applying (3.4) in (4.1) and (4.2), hence omitted.

Lemma 4.2 *Let M be a semi-invariant submanifold of a (k,μ)-contact manifold $\tilde{M}$ with a semi-symmetric metric connection, then we have*

$$(\bar{\nabla}_X P)Y - A_{QY}X + \eta(N)X - B\sigma(X,Y) = g(X+hX,Y)\xi - \eta(Y)(X+hX) - \eta(Y)PX, \quad (4.3)$$

$$(\bar{\nabla}_X Q)Y + \sigma(X,PY) - C\sigma(X,Y) = -\eta(Y)QX, \quad (4.4)$$

$$(\bar{\nabla}_X B)N - A_{CN}X\sigma(X,PY) - C\sigma(X,Y) = 0, \quad (4.5)$$

$$(\bar{\nabla}_X C)N + \sigma(X,BN) + Q(-A_N X + \eta(N)X) = 0, \quad (4.6)$$

$$g(PX,Y) = 0, \quad (4.7)$$

$$g(QX,Y) = 0, \quad (4.8)$$

for all $X,Y \in \Gamma(TM)$, $N \in \Gamma(T^{\perp}M)$.

Lemma 4.3 *Let M be a semi-invariant submanifold of a (k,μ)-contact manifold $\tilde{M}$ with a semi-symmetric metric connection such that $\xi \in \Gamma(TM)$, we have*

$$\bar{\nabla}_X\xi = -\phi X - \phi hX + X - \eta(X)\xi, \quad \sigma(X,\xi) = 0. \quad (4.9)$$

$$\bar{\nabla}_\xi\xi = 0, \quad \sigma(\xi,\xi) = 0, \quad A_N\xi = 0. \quad (4.10)$$

Proof Using (3.3) and (3.5) we have (4.10). In addition, we get

$$o = g(\sigma(X,\xi),N) = g(\sigma(\xi,X),N) = g(A_N\xi,X).$$

This completes the proof. □

§5. Integrability of Distributions

In this section, we study the integrability of all the distributions involved in the definition of semi-invariant submanifolds.

For all $X, Y \in \Gamma(D)$, we have

$$g([X,Y],\xi) = g(\nabla_X Y,\xi) - g(\nabla_Y X,\xi).$$

Using (3.1) in above, we get

$$g([X,Y],\xi) = g(\tilde{\nabla}_X Y,\xi) - g(\tilde{\nabla}_Y X,\xi). \tag{5.1}$$

Taking account of (3.3) in (5.1) and using (3.5), we obtain

$$g([X,Y],\xi) = g(Y,\phi X + \phi hX) - g(X,\phi Y + \phi hY)$$

and so, $g([X,Y],\xi) \neq 0$. This leads to the following:

Theorem 5.4 *Let M be a semi-invariant submanifold of a (k,μ)-contact manifold $\tilde{M}$ with a semi-symmetric metric connection such that dim$(D) \neq 0$. Then the distribution D is not integrable.*

Theorem 5.5 *Let M be a semi-invariant submanifold of a (k,μ)-contact manifold $\tilde{M}$ with a semi-symmetric metric connection. Then the distribution $D\oplus <\xi>$ is integrable if and only if*

$$\sigma(X,\phi Y) = \sigma(\phi X, Y),$$

for all $X, Y \in \Gamma(D\oplus <\xi>)$.

Proof For $X, Y \in \Gamma(D)$, we have

$$\phi([X,Y]) = \phi(\nabla_X Y - \nabla_Y X).$$

Using (3.1) in the above equation, we get

$$\phi([X,Y]) = \phi(\tilde{\nabla}_X Y - \tilde{\nabla}_Y X). \tag{5.2}$$

Making use of relation (3.3) in (5.2), we get

$$\phi([X,Y]) = \bar{\bar{\tilde{\nabla}}}_X \phi Y - (\bar{\bar{\tilde{\nabla}}}_X \phi)Y - \eta(Y)\phi X - \bar{\bar{\tilde{\nabla}}}\phi X + (\bar{\bar{\tilde{\nabla}}}_Y \phi)X - \eta(X)\phi Y. \tag{5.3}$$

Taking account of (3.8) in (5.3) and using (3.4), we obtain

$$\begin{aligned}\phi([X,Y]) &= \bar{\nabla}_X \phi Y + \sigma(X,\phi Y) + \bar{\nabla}_Y \phi X + \sigma(Y,\phi X) \\ &\quad - g(hX,Y)\xi + \eta(Y)(X + hX) + g(hY,X)\xi - \eta(X)(Y + hY) + 2g(\phi X, Y),\end{aligned}$$

where $\phi([X,Y])$ shows the component of $\nabla_X Y$ from the orthogonal complementary distribution of $D\oplus <\xi>$ in M. Then, $[X,Y]\in\Gamma(D\oplus<\xi>)$ if and only if $\sigma(X,\phi Y)=\sigma(Y,\phi X)$. □

Theorem 5.6 *Let M be a semi-invariant submanifold of a (k,μ)-contact manifold $\tilde{M}$ with a semi-symmetric metric connection. Then the distribution $D^{\perp}$ is integrable.*

Proof For all $X,Y\in\Gamma(D^{\perp})$, we have

$$g([X,Y],\xi)=g(\nabla_X Y,\xi)-g(\nabla_Y X,\xi).$$

Using (3.1) in the above equation, we get

$$g([X,Y],\xi)=g(\tilde{\nabla}_X Y,\xi)-g(\tilde{\nabla}_Y X,\xi). \tag{5.4}$$

Taking account of (3.3) in (5.4) and using (3.5), we obtain

$$g([X,Y],\xi)=g(Y,\phi X+\phi hX)-g(X,\phi Y+\phi hY),$$

which implies that $g([X,Y],\xi)=0$. So $\eta([X,Y])=0$. Then, we have $[X,Y]\in\Gamma(D^{\perp})$. □

Theorem 5.7 *Let M be a semi-invariant submanifold of a (k,μ)-contact manifold $\tilde{M}$ with a semi-symmetric metric connection. Then the distribution $D^{\perp}\oplus<\xi>$ is integrable if and only if*

$$A_{\phi X}Y=A_{\phi Y}X,$$

for all $X,Y\in\Gamma(D^{\perp}\oplus<\xi>)$.

Proof For $X,Y\in\Gamma(D^{\perp})$, we have

$$\phi([X,Y])=\phi(\nabla_X Y-\nabla_Y X).$$

Using (3.1) in above, we get

$$\phi([X,Y])=\phi(\tilde{\nabla}_X Y-\tilde{\nabla}_Y X). \tag{5.5}$$

Making use of relation (3.3) in (5.5), we get

$$\phi([X,Y])=\bar{\bar{\nabla}}_X\phi Y-(\bar{\bar{\nabla}}_X\phi)Y-\eta(Y)\phi X-\bar{\bar{\nabla}}\phi X+(\bar{\bar{\nabla}}_Y\phi)X-\eta(X)\phi Y. \tag{5.6}$$

Taking account of (3.9) in (5.6) and using (3.4), we obtain

$$\begin{aligned}\phi([X,Y]) \;=\; & A_{\phi X}Y-A_{\phi Y}X+\bar{\nabla}^{\perp}_X\phi Y-\bar{\nabla}^{\perp}_Y\phi X+g(hY,X)\xi-g(hX,Y)\xi\\ & +\eta(Y)(X+hX)-\eta(X)(Y+hY).\end{aligned}$$

Then ,we get

$$[X,Y]\in\Gamma(D^{\perp}\oplus<\xi>)\Rightarrow A_{\phi X}Y=A_{\phi Y}X.$$

Conversely

$$\begin{aligned}\phi^2([X,Y]) &= \phi\{A_{\phi X}Y - A_{\phi Y}X + \bar{\nabla}^{\perp}_X \phi Y - \bar{\nabla}^{\perp}_Y \phi X + g(hY,X)\xi - g(hX,Y)\xi \\ &\quad +\eta(Y)(X+hX) - \eta(X)(Y+hY)\}.\end{aligned}$$

Making use of (2.1) and using the equality $A_{\phi X}Y = A_{\phi Y}X$, the above equation can be written as

$$[X,Y] = \phi(\bar{\nabla}^{\perp}_X \phi Y) - \phi(\bar{\nabla}^{\perp}_Y \phi X) + \eta(Y)\phi(X+hX) - \eta(X)\phi(Y+hY).$$

Thus, $[X,Y] \in \Gamma(D^{\perp} \oplus < \xi >)$. Hence the distribution $D^{\perp} \oplus < \xi >$ is integrable. □

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References

[1] P. Alegre, Semi-invariant submanifolds of Lorentzian Sasakian manifolds, *Demonstratio Mathematica*, 44(2) (2011), 391-406.

[2] C.S. Bagewadi, D. Nirmala and M.S. Siddesha, Semi-invariant submanifolds of $(\mathrm{LCS})_n$-manifold, *Communications of the Korean Mathematical Society*, 33(4) (2018), 1331-1339.

[3] A. Bejancu and N. Papaghuic, Semi-invariant submanifolds of a Sasakian manifold, *An. Sti. Univ. AL I CUZA Iasi*, 27 (1981), 163-170.

[4] D.E. Blair, Contact manifolds in Riemannian geometry, *Lecture notes in Math.*, 509, Springer-Verlag, Berlin, (1976).

[5] D.E. Blair, T. Koufogiorgos and B.J. Papantoniou, Contact metric manifolds satisfyng a nullity condition, *Israel J. Math.*, 91 (1995), 189-214.

[6] B.Y. Chen, Geometry of submanifolds and its applications, *Science University of Tokyo*, Tokyo, 1981.

[7] U.C. De And A.A. Shaikh, Non-existence of proper semi-Invariant submanifolds of a Lorentzian para-Sasakian manifold, *Bull. Malaysian Math. Soc.* (Second Series), 22 (1999) 179-183.

[8] A. Friedmann and J.A. Schouten, Uber die geometrie der halbsymmetrischen Ubertragung, *Math. Zeitscr.*, 21 (1924), 211-223.

[9] H.A. Hayden, Subspaces of a space with torsion, *Proc. London Math. Soc.*, 34 (1932), 27-50.

[10] A. Sarkar, M.S. Ali and D. Biswas, On submanifolds of (k,μ)-contact metric manifolds, *Dhaka Univ. J. Sci.*, 61(1) 2013, 93-96.

[11] M.H. Shahid, On semi-invariant submanifolds of a nearly Sasakian manifold, *Indian J. Pure and Applied Math.*, 95(10) (1993), 571-580.

[12] M.S. Siddesha and C.S. Bagewadi, Submanifold of a (k, μ)-Contact manifold, *CUBO A Mat. J.*, 18 (01) (2016), 59-68.

[13] M.S. Siddesha and C.S. Bagewadi, On some classes of invariant submanifolds (k, μ)-contact manifold, *J. of Inform. and Mat. Sci.*, 9(1) (2017), 13-26.

[14] M.S. Siddesha and C.S. Bagewadi, Invariant submanifolds of (k, μ)-contact manifolds admitting quarter symmetric metric connection, *International J. of Math. Trends and Tech.*, 34(2) (2016), 48-53.

[15] M.S. Siddesha and C.S. Bagewadi, On slant submanifolds of (k, μ)-contact manifold, *Differential Geometry-Dynamical Systems*, 18 (2016), 123-131.

[16] M.S. Siddesha and C.S. Bagewadi, Slant submanifolds of a conformal (k, μ)-contact manifold, *International J.Math. Combin.*, 3 (2017), 39-50.

[17] M.S. Siddesha and C.S. Bagewadi, Semi-slant submanifolds of (k, μ)-contact manifold, *Commun. Fac. Sci. Univ. Ser. A_1 Math. Stat.*, 67(2) (2018), 116-125.

[18] A. Sharafuddin and S.I. Hussain, Semi-symmetric metric connection in almost contact manifolds, *Tensor, N.S.*, 30 (1976), 133-139.

[19] S. Tanno, Ricci curvatures of contact Riemannian manifolds, *Tohoku Math. J.*, 40(1988), 441-448.

[20] A.T. Vanli, R. Sari, On semi-invariant submanifolds of a generalized kenmotsu manifold admitting a semi-symmetric metric connection, *Acta Universitatis Apulensis*, 43 (2015), 79-92.

[21] K. Yano and M. Kon, *Structures on Manifolds*, World Scientific Publishing, 1984.