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Some New Results on 4-Total Mean Cordial Graphs

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Abstract: Let G be a graph and let $f : V(G) \to \{0, 1, 2, \cdots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign a label $f(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ and f is called a k-total mean cordial labeling of G if $|t_{mf}(i) - t_{mf}(j)| \leq 1$ for all $i, j \in \{0, 1, \cdots, k-1\}$, where $t_{mf}(x)$ denotes the total number of vertices and edges labelled with x, $x \in \{0, 1, 2, \cdots, k-1\}$. A graph with admit a k-total mean cordial labeling is called k-total mean cordial graph. In this paper we investigate the 4-total mean cordial labeling behavior of some graphs which are obtained from stars.

Key Words: k-Total mean cordial labeling, Smarandachely k-total mean cordial labeling, k-total mean cordial labeling graph, Smarandachely k-total mean cordial labeling graph, star, cycle, comb, ladder.

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§1. Introduction

All graphs in this paper are finite, simple and undirected graphs only. Graceful labeling was introduced by Rosa in [15]. Subsequently Graham and Sloan have introduced the notion of harmonious labeling [2]. Motivated by these works several author introduce varies types of graph labeling. The concept of k-total mean cordial labeling has been introduced in [4]. The 4-total mean cordial labeling behavior of several graphs like cycle, complete graph, star, bistar, comb and crown have been investigated in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. In this paper we investigate the 4- total mean cordial labeling behavior of some graphs which are obtained from stars. Let x be any real number. Then $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. Terms are not defined here follow from Harary [3] and Gallian [1].

§2. k-Total Mean Cordial Graph

Definition 2.1 *Let G be a graph. Let $f : V(G) \to \{0, 1, 2, \cdots, k-1\}$ be a function where*

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$k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $f(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called a k-total mean cordial labeling of G if $|t_{mf}(i) - t_{mf}(j)| \leq 1$ for all $i, j \in \{0, 1, 2, \cdots, k-1\}$, where $t_{mf}(x)$ denotes the total number of vertices and edges labelled with x, $x \in \{0, 1, 2, \cdots, k-1\}$. Otherwise, if there exist integers $i, j \in \{0, 1, 2, \cdots, k-1\}$ such that $|t_{mf}(i) - t_{mf}(j)| \geq 2$, such a labeling f is called a Smarandachely k-total mean cordial labeling.

A graph with an admit a k-total mean cordial labeling or a Smarandachely k-total mean cordial labeling is called a k-total mean cordial graph or a Smarandachely k-total mean cordial graph.

§3. Preliminaries

Definition 3.1 *A complete bipartite graph $K_{1,n}$ is called a star. Let $V(K_{1,n}) = \{w, w_i : 1 \leq i \leq n\}$ and $E(K_{1,n}) = \{ww_i : 1 \leq i \leq n\}$, w is called the central vertex of the star $K_{1,n}$.*

Definition 3.2 *A graph obtained from the cycle C_n and star $K_{1,n}$ by identifying the vertex of C_n with these central vertex of $K_{1,n}$ is denoted by $C_n \oplus K_{1,n}$.*

Definition 3.3 *Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. A corona of G_1 with G_2 is the graph $G_1 \odot G_2$ obtained by taking one copy of G_1 , p_1 copies of G_2 and joining the i^{th} vertex of G_1 by an edge to every vertex in the i^{th} copy of G_2 where $1 \leq i \leq p_1$.*

Definition 3.4 *A graph $P_n \odot K_1$ is called a comb. Let P_n be the path $u_1 u_2 \cdots u_n$. Let $V(P_n \odot K_1) = V(P_n) \cup \{v_i : 1 \leq i \leq n\}$ and $E(P_n \odot K_1) = E(P_n) \cup \{u_i v_i : 1 \leq i \leq n\}$.*

Definition 3.5 *A graph $L_n = P_n + K_2$ is called a ladder. Let the vertex set be $V(L_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and the edge set $E(L_n) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i : 1 \leq i \leq n\}$.*

§4. Main Results

Theorem 4.1 *A graph $C_n \oplus K_{1,n}$ is 4-total mean cordial labeling for all $n \geq 3$.*

Proof Let C_n be the cycle $u_1\ u_2 \cdots u_n\ u_1$, $V(C_n \oplus K_{1,n})) = V(C_n) \cup \{w, w_i : 1 \leq i \leq n,\ u_1 = w\}$ and $E(C_n \oplus K_{1,n})) = E(C_n) \cup \{u_1 w_i : 1 \leq i \leq n\}$. Obviously $|V(C_n \oplus K_{1,n})| + |E(C_n \oplus K_{1,n})| = 4n$.

Case 1. $n \equiv 1 \pmod 2$.

Let $n = 2r+1$, $r \in \mathbb{N}$. Consider the cycle $C_n : u_1\ u_2 \cdots u_n\ u_1$. Assign the label 0 to the $r+1$ vertices u_1, u_2, $\cdots$, u_{r+1}. Now we assign the label 1 to the r vertices u_{r+2}, u_{r+3}, $\cdots$, u_{2r+1}. Next move to the pendent vertices. We now assign the label 3 to the $2r+1$ vertices w_1, w_2, $\cdots$, w_{2r+1}.

Case 2. $n \equiv 0 \pmod 2$.

Let $n = 2r$, $r \in \mathbb{N}$. Assign the label 2 to the vertex u_1. Next we assign the label 0 to the r vertices u_2, u_3, $\cdots$, u_{r+1}. We now assign the label 1 to the $r-1$ vertices u_{r+2}, u_{r+3}, $\cdots$, u_{2r}. Now we assign the label 0 to the vertex w_1. Next we assign the label 2 to the $r-1$ vertices w_2, w_3, $\cdots$, w_r. Finally we assign the label 3 to the r vertices w_{r+1}, w_{r+2}, $\cdots$, w_{2r}.

This shows that f is a 4-total mean cordial labeling follows from Table 1.

Order of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 2r + 1$	$2r + 1$	$2r + 1$	$2r + 1$	$2r + 1$
$n = 2r$	$2r$	$2r$	$2r$	$2r$

Table 1

This completes the proof. □

Theorem 4.2 *A graph obtained from the comb $P_n \odot K_1$ and star $K_{1,n}$ by identifying the central vertex w of the star with the vertex u_1 of the comb is 4-total mean cordial.*

Proof Let G be the resulting graph. Take the vertex set and edge set of the comb is as in Definition 3.4. Let $V(G) = V(P_n \odot K_1) \cup \{w_i : 1 \leq i \leq n\}$. $E(G) = E(P_n \odot K_1) \cup \{u_1 w_i : 1 \leq i \leq n\}$. Clearly $|V(G)| + |E(G)| = 6n - 1$.

Case 1. $n \equiv 0 \pmod 2$.

Let $n = 2r$, $r \in \mathbb{N}$. Assign the label 2 to the r vertices $u_1, u_2, \cdots, u_r$. Now we assign the label 3 to the r vertices $u_{r+1}, u_{r+2}, \cdots, u_{2r}$. Next we assign the label 0 to the r vertices $v_1, v_2, \cdots, v_r$. We now assign the label 2 to the r vertices $v_{r+1}, v_{r+2}, \cdots, v_{2r}$. Now we assign the label 0 to the $2r$ vertices $w_1, w_2, \cdots, w_{2r}$.

Case 2. $n \equiv 1 \pmod 2$.

Let $n = 2r + 1$, $r \in \mathbb{N}$. Assign the label 2 to the r vertices $u_1, u_2, \cdots, u_r$. Next we assign the label 3 to the $r + 1$ vertices $u_{r+1}, u_{r+2}, \cdots, u_{2r+1}$. Now we assign the label 0 to the $r + 1$ vertices $v_1, v_2, \cdots, v_{r+1}$. We now assign the label 2 to the r vertices $v_{r+2}, v_{r+3}, \cdots, v_{2r+1}$. Next we assign the label 0 to the $2r$ vertices $w_1, w_2, \cdots, w_{2r}$. Finally we assign the label 1 to the vertex w_{2r+1}.

Thus, this vertex labeling f is a 4-total mean cordial labeling follows from Table 2.

n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 2r$	$3r$	$3r$	$3r - 1$	$3r$
$n = 2r + 1$	$3r + 1$	$3r + 1$	$3r + 1$	$3r + 2$

Table 2

This completes the proof. □

Theorem 4.3 *A graph G obtained from the comb $P_n \odot K_1$ and two stars by identifying the vertex u_1 of the comb with the central vertex of one star and u_n with the central vertex of another star is 4-total men cordial.*

Proof Take the vertex set and edge set of the comb is as in Definition 3.4. Let $V(G) = V(P_n \odot K_1) \cup \{x_i, y_i : 1 \leq i \leq n\}$ and $E(G) = E(P_n \odot K_1) \cup \{u_1 x_i, u_n y_i : 1 \leq i \leq n\}$. Note that $|V(G)| + |E(G)| = 8n - 1$.

Assign the label 0 to the n vertices $u_1, u_2, \cdots, u_n$. Next we assign the label 1 to the n vertices $v_1, v_2, \cdots, v_n$. Now we assign the label 3 to the n vertices $x_1, x_2, \cdots, x_n$. Finally we assign the label 3 to the n vertices $y_1, y_2, \cdots, y_n$.

Clearly $t_{mf}(0) = 2n - 1$, $t_{mf}(1) = t_{mf}(2) = t_{mf}(3) = 2n$. □

Theorem 4.4 *A graph obtained from the comb $P_n \odot K_1$ and three stars by identifying the vertex u_1 of the comb with the central vertex of one star, u_n with the central vertex of second star and v_1 with the central vertex of third star is 4-total mean cordial.*

Proof Let G be the resulting graph. Take the vertex set and edge set of the comb is as in Definition 3.4. Let $V(G) = V(P_n \odot K_1) \cup \{x_i, y_i, z_i : 1 \leq i \leq n\}$ and $E(G) = E(P_n \odot K_1) \cup \{u_1x_i, u_ny_i, v_1z_i : 1 \leq i \leq n\}$. Clearly $|V(G)| + |E(G)| = 10n - 1$.

Case 1. $n \equiv 0 \pmod 2$.

Let $n = 2r$, $r \in \mathbb{N}$. Assign the label 0 to the $2r$ vertices $u_1, u_2, \cdots, u_{2r}$. Next we assign the label 2 to the $2r$ vertices $v_1, v_2, \cdots, v_{2r}$. Now we assign the label 3 to the $2r$ vertices $x_1, x_2, \cdots, x_{2r}$. Then we assign the label 1 to the r vertices $y_1, y_2, \cdots, y_r$. We now assign the label 3 to the r vertices $y_{r+1}, y_{r+2}, \cdots, y_{2r}$. Now we assign the label 0 to the r vertices $z_1, z_2, \cdots, z_r$. Finally we assign the label 3 to the r vertices $z_{r+1}, z_{r+2}, \cdots, z_{2r}$.

Case 2. $n \equiv 1 \pmod 2$.

Let $n = 2r + 1$, $r \in \mathbb{N}$. Assign the label 0 to the $2r + 1$ vertices $u_1, u_2, \cdots, u_{2r+1}$. Now we assign the label 2 to the $2r + 1$ vertices $v_1, v_2, \cdots, v_{2r+1}$. Next we assign the label 3 to the $2r + 1$ vertices $x_1, x_2, \cdots, x_{2r+1}$. We now assign the label 1 to the r vertices $y_1, y_2, \cdots, y_r$. Then we assign the label 3 to the $r + 1$ vertices $y_{r+1}, y_{r+2}, \cdots, y_{2r+1}$. Now we assign the label 3 to the r vertices $z_1, z_2, \cdots, z_r$. Finally we assign the label 0 to the $r + 1$ vertices $z_{r+1}, z_{r+2}, \cdots, z_{2r+1}$.

Thus, the vertex labeling f is a 4-total mean cordial labeling follows from Table 3.

n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 2r$	$5r - 1$	$5r$	$5r$	$5r$
$n = 2r + 1$	$5r + 2$	$5r + 2$	$5r + 3$	$5r + 2$

Table 3

This completes the proof. □

Theorem 4.5 *A graph G obtained from the comb $P_n \odot K_1$ and four stars by identifying the vertex u_1 of the comb with the central vertex of one star, u_n with the central vertex of second star, v_1 with the central vertex of third star and v_n with the central vertex of fourth star is 4-total mean cordial.*

Proof Take the vertex set and edge set of the comb is as in Definition 3.4. Let $V(G) = V(P_n \odot K_1) \cup \{w_i, x_i, y_i, z_i : 1 \leq i \leq n\}$ and $E(G) = E(P_n \odot K_1) \cup \{u_1w_i, u_nx_i, v_1y_i, v_nz_i : 1 \leq i \leq n\}$. Note that $|V(G)| + |E(G)| = 12n - 1$.

Case 1. $n \equiv 0 \pmod 2$.

Let $n = 2r$, $r \in \mathbb{N}$. Assign the label 0 to the $2r$ vertices $u_1, u_2, \cdots, u_{2r}$. Next we assign the label 0 to the r vertices v_1, v_2, $\cdots$, v_r. We now assign the label 1 to the r vertices v_{r+1}, v_{r+2}, $\cdots$, v_{2r}. Next we assign the label 3 to the $2r$ vertices w_1, w_2, $\cdots$, w_{2r}. Now we assign the label 3 to the $2r$ vertices x_1, x_2, $\ldots$, x_{2r}. Then we assign the label 1 to the $2r$ vertices y_1, y_2, $\cdots$, y_{2r}. Finally we assign the label 3 to the $2r$ vertices z_1, z_2, $\cdots$, z_{2r}.

Case 2. $n \equiv 1 \pmod 2$.

Let $n = 2r + 1$, $r \in \mathbb{N}$. We now assign the label 0 to the $2r$ vertices u_1, u_2, $\cdots$, u_{2r}. Next we assign the label 0 to the $r + 1$ vertices v_1, v_2, $\cdots$, v_{r+1}. We now assign the label 1 to the r vertices v_{r+2}, v_{r+3}, $\cdots$, v_{2r+1}. Next we assign the label 3 to the $2r + 1$ vertices w_1, w_2, $\cdots$, w_{2r+1}. Now we assign the label 3 to the $2r + 1$ vertices x_1, x_2, $\cdots$, x_{2r+1}. Then we assign the label 1 to the $2r + 1$ vertices y_1, y_2, $\cdots$, y_{2r+1}. Finally we assign the label 3 to the $2r + 1$ vertices z_1, z_2, $\cdots$, z_{2r+1}.

Thus, this vertex labeling f is a 4-total mean cordial labeling follows from Table 4.

n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 2r$	$6r - 1$	$6r$	$6r$	$6r$
$n = 2r + 1$	$6r + 3$	$6r + 2$	$6r + 3$	$6r + 3$

Table 4

This completes the proof. □

Theorem 4.6 *A graph obtained from the ladder L_n and star $K_{1,n}$ by identifying the vertex u_1 of the ladder with the central vertex of star is* 4*-total mean cordial.*

Proof Let G be the resulting graph. Take the vertex set and edge set of the ladder is as in Definition 3.5. Let $V(G) = V(L_n) \cup \{w_i : 1 \leq i \leq n\}$ and $E(G) = E(L_n) \cup \{u_1 w_i : 1 \leq i \leq n\}$. Obviously $|V(G)| + |E(G)| = 7n - 2$.

Case 1. $n \equiv 0 \pmod 4$.
Let $n = 4r$, $r \in \mathbb{N}$. Assign the label 0 to the r vertices u_1, u_2, $\cdots$, u_r. Next we assign the label 1 to the r vertices u_{r+1}, u_{r+2}, $\cdots$, u_{2r}. Now we assign the label 2 to the r vertices u_{2r+1}, u_{2r+2}, $\cdots$, u_{3r}. We now assign the label 3 to the r vertices u_{3r+1}, u_{3r+2}, $\cdots$, u_{4r}. Now we assign the label 0 to the r vertices v_1, v_2, $\cdots$, v_r. We now assign the label 1 to the r vertices v_{r+1}, v_{r+2}, $\cdots$, v_{2r}. Next we assign the label 2 to the r vertices v_{2r+1}, v_{2r+2}, $\cdots$, v_{3r}. We now assign the label 3 to the r vertices v_{3r+1}, v_{3r+2}, $\cdots$, v_{4r}. Now we assign the label 0 to the $r+1$ vertices w_1, w_2, $\cdots$, w_{r+1}. Then we assign the label 1 to the r vertices w_{r+2}, w_{r+3}, $\cdots$, w_{2r+1}. We now assign the label 3 to the $2r - 1$ vertices w_{2r+2}, w_{2r+3}, $\cdots$, w_{4r}.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r + 1$, $r \in \mathbb{N}$. As in Case 1 assign the label to the vertices u_i, v_i, w_i $(1 \leq i \leq 4r)$. Now we assign the labels 3, 0, 1 to the vertices u_{4r+1}, v_{4r+1}, w_{4r+1}.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r + 2$, $r \in \mathbb{N}$. Label the vertices u_i, v_i, w_i $(1 \leq i \leq 4r + 1)$ as in Case 2. Next we assign the labels 3, 0, 2 to the vertices u_{4r+2}, v_{4r+2}, w_{4r+2}.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r+3$, $r \in \mathbb{N}$. In this case assign the label for the vertices u_i, v_i, w_i $(1 \le i \le 4r+2)$ as in Case 3. We now assign the labels 3, 0, 2 to the vertices u_{4r+3}, v_{4r+3}, w_{4r+3}.

This vertex labeling f is a 4-total mean cordial labeling follows from Table 5.

Order of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 4r$	$7r$	$7r$	$7r-1$	$7r-1$
$n = 4r+1$	$7r+1$	$7r+2$	$7r+1$	$7r+1$
$n = 4r+2$	$7r+3$	$7r+3$	$7r+3$	$7r+3$
$n = 4r+3$	$7r+5$	$7r+4$	$7r+5$	$7r+5$

Table 5

This completes the proof. □

Theorem 4.7 *A graph G obtained from the ladder L_n and two stars $K_{1,n}$ by identifying the vertex u_1 with the central vertex of one star and u_n with the central vertex of another star is 4-total mean cordial.*

Proof Take the vertex set and edge set of the ladder is as in Definition 3.5. Let $V(G) = V(L_n) \cup \{x_i, y_i : 1 \le i \le n\}$ and $E(G) = E(L_n) \cup \{u_1x_i, u_ny_i : 1 \le i \le n\}$. Note that $|V(G)| + |E(G)| = 9n - 2$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r \in \mathbb{N}$. Assign the label 0 to the r vertices u_1, u_2, $\cdots$, u_r. Now we assign the label 1 to the r vertices u_{r+1}, u_{r+2}, $\cdots$, u_{2r}. We now assign the label 2 to the r vertices u_{2r+1}, u_{2r+2}, $\cdots$, u_{3r}. Next assign the label 3 to the r vertices u_{3r+1}, u_{3r+2}, $\cdots$, u_{4r}. We now assign the label 0 to the r vertices v_1, v_2, $\cdots$, v_r. Next we assign the label 1 to the r vertices v_{r+1}, v_{r+2}, $\cdots$, v_{2r}. Now we assign the label 2 to the r vertices v_{2r+1}, v_{2r+2}, $\cdots$, v_{3r}. We now assign the label 3 to the r vertices v_{3r+1}, v_{3r+2}, $\cdots$, v_{4r}. Next we assign the label 0 to the $2r+1$ vertices x_1, x_2, $\cdots$, x_{2r+1}. Now we assign the label 2 to the $2r-1$ vertices x_{2r+2}, x_{2r+3}, $\cdots$, x_{4r}. Then we assign the label 1 to the $2r$ vertices y_1, y_2, $\cdots$, y_{2r}. Finally we assign the label 3 to the $2r$ vertices y_{2r+1}, y_{2r+2}, $\cdots$, y_{4r}.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r+1$, $r \in \mathbb{N}$. Label the vertices u_i, v_i, x_i, y_i $(1 \le i \le 4r)$ as in Case 1. Next we assign the labels 3, 1, 0, 1 to the vertices u_{4r+1}, v_{4r+1}, x_{4r+1} y_{4r+1}.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r+2$, $r \in \mathbb{N}$. In this case assign the label for the vertices u_i, v_i, x_i, y_i $(1 \le i \le 4r+1)$ as in Case 2. We now assign the labels 3, 1, 0, 1 to the vertices u_{4r+2}, v_{4r+2}, x_{4r+2} y_{4r+2}.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r+3$, $r \in \mathbb{N}$. As in Case 3, we assign the label to the vertices u_i, v_i, x_i, y_i

$(1 \leq i \leq 4r+2)$. Finally we assign the labels 3, 1, 0, 1 to the vertices u_{4r+3}, v_{4r+3}, x_{4r+3} y_{4r+3}.

Thus, the vertex labeling f is a 4-total mean cordial labeling follows from Table 6.

Size of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n=4r$	$9r$	$9r-1$	$9r-1$	$9r$
$n=4r+1$	$9r+2$	$9r+1$	$9r+2$	$9r+2$
$n=4r+2$	$9r+4$	$9r+4$	$9r+4$	$9r+4$
$n=4r+3$	$9r+6$	$9r+7$	$9r+6$	$9r+6$

Table 6

This completes the proof. □

Theorem 4.8 *A graph obtained from the ladder L_n and three stars $K_{1,n}$ by identifying the vertex u_1 with the central vertex of one star, u_n with the central vertex of second star and v_1 with the central vertex of third star is 4-total mean cordial.*

Proof We denote G as the resulting graph. Take the vertex set and edge set of the ladder is as in Definition 3.5. Let $V(G) = V(L_n) \cup \{x_i, y_i, z_i : 1 \leq i \leq n\}$ and $E(G) = E(L_n) \cup \{u_1x_i, u_ny_i, v_1z_i : 1 \leq i \leq n\}$. Clearly $|V(G)| + |E(G)| = 11n - 2$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r \in \mathbb{N}$. Assign the label 0 to the r vertices $u_1, u_2, \cdots, u_r$. Next we assign the label 1 to the r vertices $u_{r+1}, u_{r+2}, \cdots, u_{2r}$. Now we assign the label 2 to the r vertices u_{2r+1}, $u_{2r+2}, \cdots, u_{3r}$. We now assign the label 3 to the r vertices $u_{3r+1}, u_{3r+2}, \cdots, u_{4r}$. Now we assign the label 0 to the r vertices $v_1, v_2, \cdots, v_r$. We now assign the label 1 to the r vertices $v_{r+1}, v_{r+2}, \cdots, v_{2r}$. Next we assign the label 2 to the r vertices $v_{2r+1}, v_{2r+2}, \cdots, v_{3r}$. We now assign the label 3 to the r vertices $v_{3r+1}, v_{3r+2}, \cdots, v_{4r}$. Next we assign the label 0 to the $3r+1$ vertices $x_1, x_2, \cdots, x_{3r+1}$. Now we assign the label 2 to the $r-1$ vertices x_{3r+2}, x_{3r+3}, $\cdots, x_{4r}$. Then we assign the label 1 to the r vertices $y_1, y_2, \cdots, y_r$. We now assign the label 3 to the $3r$ vertices $y_{r+1}, y_{r+2}, \cdots, y_{4r}$. Finally we assign the label 2 to the $4r$ vertices z_1, z_2, $\cdots, z_{4r}$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r+1$, $r \in \mathbb{N}$. In this case we assign the label to the vertices u_i, v_i, x_i, y_i, z_i $(1 \leq i \leq 4r)$ as in Case 1. We now assign the labels 3, 0, 0, 1, 1 to the vertices u_{4r+1}, v_{4r+1}, x_{4r+1} y_{4r+1}, z_{4r+1}.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r+2$, $r \in \mathbb{N}$. As in Case 2, we assign the label to the vertices u_i, v_i, x_i, y_i z_i $(1 \leq i \leq 4r+1)$. Finally we assign the labels 3, 1, 0, 2, 2 to the vertices u_{4r+2}, v_{4r+2}, x_{4r+2}, y_{4r+2}, z_{4r+2}.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r + 3$, $r \in \mathbb{N}$. Label the vertices u_i, v_i, x_i, y_i, z_i $(1 \le i \le 4r + 2)$ as in Case 3. Next we assign the labels 3, 0, 0, 2, 1 to the vertices u_{4r+3}, v_{4r+3}, x_{4r+3}, y_{4r+3}, z_{4r+3}.

This vertex labeling f is a 4-total mean cordial labeling follows from Table 7.

n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 4r$	$11r$	$11r - 1$	$11r - 1$	$11r$
$n = 4r + 1$	$11r + 3$	$11r + 2$	$11r + 2$	$11r + 2$
$n = 4r + 2$	$11r + 5$	$11r + 5$	$11r + 5$	$11r + 5$
$n = 4r + 3$	$11r + 8$	$11r + 8$	$11r + 7$	$11r + 8$

Table 7

This completes the proof. □

Theorem 4.9 *A graph G obtained from the ladder L_n and four stars by identifying the vertex u_1 with the central vertex of one star, u_n with the central vertex of second star, v_1 with the central vertex of third star and v_n with the central vertex of fourth star is 4-total mean cordial.*

Proof Take the vertex set and edge set of the ladder is as in Definition 3.5. Let $V(G) = V(L_n) \cup \{w_i, x_i, y_i, z_i : 1 \le i \le n\}$ and $E(G) = E(L_n) \cup \{u_1w_i, u_nx_i, v_1y_i, v_nz_i : 1 \le i \le n\}$. Obviously $|V(G)| + |E(G)| = 13n - 2$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r \in \mathbb{N}$. Assign the label 0 to the $4r$ vertices $u_1, u_2, \cdots, u_{4r}$. Next we assign the label 2 to the $4r$ vertices $v_1, v_2, \cdots, v_{4r}$. We now assign the label 0 to the $2r + 1$ vertices $w_1, w_2, \cdots, w_{2r+1}$. Now we assign the label 1 to the $2r - 1$ vertices $w_{2r+2}, w_{2r+3}, \cdots, w_{4r}$. Next we assign the label 1 to the $r + 1$ vertices $x_1, x_2, \cdots, x_{r+1}$. Then we assign the label 3 to the $3r - 1$ vertices $x_{r+2}, x_{r+3}, \cdots, x_{4r}$. We now assign the label 0 to the $r - 1$ vertices $y_1, y_2, \cdots, y_{r-1}$. Next we assign the label 1 to the $2r + 1$ vertices $y_r, y_{r+1}, \cdots, y_{3r}$. Now we assign the label 3 to the r vertices $y_{3r+1}, y_{3r+2}, \cdots, y_{4r}$. Finally we assign the label 3 to the $4r$ vertices $z_1, z_2, \cdots, z_{4r}$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r + 1$, $r \in \mathbb{N}$. Label the vertices u_i, v_i, w_i, x_i, y_i, z_i $(1 \le i \le 4r)$ as in Case 1. Next we assign the labels 0, 2, 2, 2, 3, 3 to the vertices u_{4r+1}, v_{4r+1}, w_{4r+1}, x_{4r+1}, y_{4r+1}, z_{4r+1}.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r + 2$, $r \in \mathbb{N}$. As in Case 1, we assign the label to the vertices u_i, v_i, w_i, x_i, y_i z_i $(1 \le i \le 4r)$. Finally we assign the labels 0, 0, 2, 2, 1, 0, 2, 3, 3, 1, 3, 3 to the vertices u_{4r+1}, u_{4r+2}, v_{4r+1}, v_{4r+2}, w_{4r+1}, w_{4r+2}, x_{4r+1}, x_{4r+2}, y_{4r+1}, y_{4r+2}, z_{4r+1}, z_{4r+2}.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r + 3$, $r \in \mathbb{N}$. In this case we assign the label to the vertices u_i, v_i, w_i, x_i, y_i, z_i $(1 \le i \le 4r + 2)$ as in Case 3. Finally we assign the labels 0, 2, 1, 3, 0, 3 to the vertices u_{4r+3}, v_{4r+3}, w_{4r+3}, x_{4r+3}, y_{4r+3}, z_{4r+3}.

This vertex labeling f is a 4-total mean cordial labeling follows from Table 8.

Order of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 4r$	$13r$	$13r$	$13r - 1$	$13r - 1$
$n = 4r + 1$	$13r + 2$	$13r + 3$	$13r + 3$	$13r + 3$
$n = 4r + 2$	$13r + 6$	$13r + 6$	$13r + 6$	$13r + 6$
$n = 4r + 3$	$13r + 9$	$13r + 10$	$13r + 9$	$13r + 9$

Table 8

This completes the proof. □

Example 4.1 A 4-total mean cordial labeling of the graph G obtained from Theorem 4.9 with $n = 5$ is given in Figure 1.

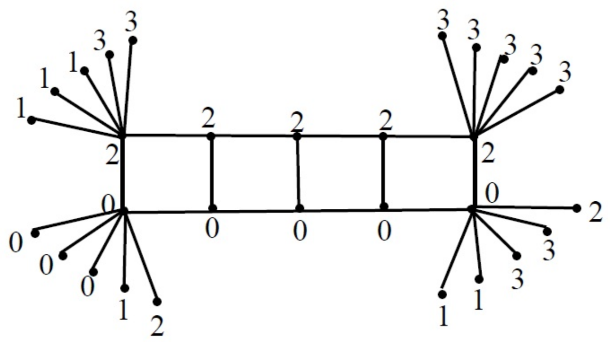

Figure 1

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