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Aim and Scope

International Journal of Neutrosophic Science (IJNS) is a peer-review journal publishing high quality experimental and theoretical research in all areas of Neutrosophic and its Applications. IJNS is published quarterly. IJNS is devoted to the publication of peer-reviewed original research papers lying in the domain of neutrosophic sets and systems. Papers submitted for possible publication may concern with foundations, neutrosophic logic and mathematical structures in the neutrosophic setting. Besides providing emphasis on topics like artificial intelligence, pattern recognition, image processing, robotics, decision making, data analysis, data mining, applications of neutrosophic mathematical theories contributing to economics, finance, management, industries, electronics, and communications are promoted. Variants of neutrosophic sets including refined neutrosophic set (RNS). Articles evolving algorithms making computational work handy are welcome.

Topics of Interest

IJNS promotes research and reflects the most recent advances of neutrosophic Sciences in diverse disciplines, with emphasis on the following aspects, but certainly not limited to:

- Neutrosophic sets
- Neutrosophic topolog
- Neutrosophic probabilities
- Neutrosophic theory for machine learning
- Neutrosophic numerical measures
- A neutrosophic hypothesis
- The neutrosophic confidence interval
- Neutrosophic theory in bioinformatics
- and medical analytics
- Neutrosophic tools for deep learning
- Quadripartitioned single-valued
- neutrosophic sets
- Neutrosophic algebra
- Neutrosophic graphs
- Neutrosophic tools for decision making
- Neutrosophic statistics
- Classical neutrosophic numbers
- The neutrosophic level of significance
- The neutrosophic central limit theorem
- Neutrosophic tools for big data analytics
- Neutrosophic tools for data visualization
- Refined single-valued neutrosophic sets

- Applications of neutrosophic logic in image processing
- Neutrosophic logic for feature learning, classification, regression, and clustering
- Neutrosophic knowledge retrieval of medical images
- Neutrosophic set theory for large-scale image and multimedia processing
- Neutrosophic set theory for brain-machine interfaces and medical signal analysis
- Applications of neutrosophic theory in large-scale healthcare data
- Neutrosophic set-based multimodal sensor data
- Neutrosophic set-based array processing and analysis
- Wireless sensor networks Neutrosophic set-based Crowd-sourcing
- Neutrosophic set-based heterogeneous data mining
- Neutrosophic in Virtual Reality
- Neutrosophic and Plithogenic theories in Humanities and Social Sciences
- Neutrosophic and Plithogenic theories in decision making
- Neutrosophic in Astronomy and Space Sciences

Remark on Artificial Intelligence, *humanoid* and *Terminator* scenario: A Neutrosophic way to futurology

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Abstract

This article is an update of our previous article in this SGJ journal, titled: *On Gödel's Incompleteness Theorem, Artificial Intelligence & Human Mind* [7]. We provide some commentary on the latest developments around AI, humanoid robotics, and future scenario. Basically, we argue that a more thoughtful approach to the future is "*techno-realism*."

Keywords: Neutrosophic Logic, Neutrosophic Futurology, artificial intelligence

1.Introduction

Indeed among the futurists, there are people who are so optimistic about the future of mankind with its various technologies, such as Peter Diamandis with his "*Abundance*." But there are also skeptics, predicting "*dystopia,"* like George Orwell's 1984 etc. [4]

At my best, our response is: we must develop a view of technology that is not very optimistic but also not pessimistic, perhaps the right term is: "*Techno-realism*."[3]

We mean this: with a lot of research on robotics, humanoid etc., then emerged developments in the direction of *transhumanism and human-perfection*. [6]

There is already a fortune-telling that AI will be established with psychological and spiritual science, so as to bring up the AI/robotic consciousness. [7]

But lest we become forgetting our past, and building the tower of Babylon.

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For example, last year the world's robotics experts were made yammer because there was a "*tactical-robot"* report developed in one of the labs on campus in South Korea. It means this tactical robot is a robot designed to kill. Then Elon Musk and more than 2000 AI researchers raised petitions to the UN to stop all research on the tactical robotic. [2]

Roughly it's a true story that we can recall, although it is not our intention here to give foretelling that the world would be heading for the *Terminator* movie scenario.... but there's a chance we're heading there.

A Neutrosophic perspective

As an alternative to the above term of "techno-realism", our problem of predicting future technology that is not very optimistic but also not pessimistic, is indeed a Neutrosophic problem.

First, let us discuss a commonly asked question: what is Neutrosophic Logic? Here, we offer a short answer.

Vern Poythress argues that sometimes we need a modification of the basic philosophy of mathematics, in order to re-define and redeem mathematics [8]. In this context, allow us to argue in favor of Neutrosophic logic as a starting point, in lieu of the Aristotelian logic that creates so many problems in real world.

In Neutrosophy, we can connect an idea with its opposite and with its neutral and get common parts, i.e. <A> ∧ <non-A> = nonempty set. This constitutes the common part of the uncommon things! It is true/real—paradox. From neutrosophy, it all began: neutrosophic logic, neutrosophic set, neutrosophic probability, neutrosophic statistics, neutrosophic measures, neutrosophic physics, and neutrosophic algebraic structures [9].

It is true in a restricted case, i.e. Hegelian dialectics considers only the dynamics of opposites (<A> and <anti-A>), but in our everyday life, not only the opposites interact, but the neutrals < neut-A > between them too. For example, if you fight with a man (so you both are the opposites to each other), but neutral people around both of you (especially the police) interfere to reconcile both of you. Neutrosophy considers the dynamics of opposites and their neutrals.

So, neutrosophy means that: <A>, <anti-A> (the opposite of <A>), and < neut-A > (the neutrals between <A> and <anti-A>) interact among themselves. A neutrosophic set is characterized by a truth-membership function (T), an indeterminacy-membership function (I), and a falsity-membership function (F), where T, I, F are subsets of the unit interval [0, 1].

As particular cases we have a single-valued neutrosophic set {when T, I, F are crisp numbers in [0, 1]}, and an interval-valued neutrosophic set {when T, I, F are intervals included in [0, 1]}.

From a different perspective, we can also say that neutrosophic logic is (or "Smarandache logic") a generalization of fuzzy logic based on Neutrosophy (http://fs.unm.edu/NeutLog.txt). A proposition is t true, i indeterminate, and f false, where t, i, and f are real values from the ranges T, I, F, with no restriction on T, I, F, or the sum n = t + i + f. Neutrosophic logic thus generalizes:

- Intuitionistic logic, which supports incomplete theories (for 0 < n < 100 and i = 0, 0 < = t, i, f < = 100);

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- Fuzzy logic (for n = 100 and i = 0, and 0 < = t, i, f < = 100);
- Boolean logic (for n = 100 and i = 0, with t, f either 0 or 100);
- Multi-valued logic (for 0 < = t, i, f < = 100);
- Paraconsistent logic (for n > 100 and i = 0, with both t, f < 100);
- Dialetheism, which says that some contradictions are true (for t = f = 100 and i = 0; some paradoxes can be denoted this way).

Compared with all other logics, neutrosophic logic introduces a percentage of "indeterminacy"—due to unexpected parameters hidden in some propositions. It also allows each component t, i, f to "boil over" 100 or "freeze" under 0. For example, in some tautologies t > 100, called "overtrue." Neutrosophic Set is a powerful structure in expressing indeterminate, vague, incomplete and inconsistent information.

Therefore, from Neutrosophic Logic perspective, "our problem of predicting future technology that is not very optimistic but also not pessimistic" can be rephrased as follows:

(Opposite 1) pessimism – pess-optimism –- optimism (Opposite 2)

While the term pess-optimism may be originated in engineering (perhaps in geotechnical engineering), but it has become one term in urban dictionary, see:

> "A philosophy that encourages forward-thinking optimism with an educated acceptance of a basic level of **pessimism**. **Optimism's** fault is its *naïveté*, while **pessimism's** fault is its *blind jadedness*. We live on Earth and are human. There is, was and will be good and bad."[10].

That would mean a more balanced view of the future (futurology), something between too optimistic view and too pessimistic view. It is our hope that Neutrosophic perspective may shed more light on this wise term of pess-optimism, although for us "*techno-realism*" term may bring more clarity with respective to technology foretelling.

Alternatively, we can also consider a few new terms, such as:

a. *Less-optimism*: somewhat less than optimism, although it is not pessimism.
b. Merging optimism and realism: *opti-realism*. It can be somewhat better term compared to pess-optimism, because *realism* brings a more pragmatic view into the conventional dialogue between pessimism and optimism.

Then may be we can call this new approach: Neutrosophic Futurology.

What about AI fever?

In line with it, a Canadian mathematics professor wrote the following message a few days ago:

> "*I am appalled by the way how computer science damaged humanity. It has*
>
> *Been even worse than nuclear bombs. It destroyed the soul of humanity and*

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I have less than 0% interest in doing anything in this evil field.

Now something more destructive than data mining is coming up. Yes AI,

Probabilistic AI. It says we don't know why but somehow it works. So we

Started to have air plane malfunction because of the AI program failure. "

Of course you can agree or not with the expression of that mathematics professor, but reportedly the employees of Google also demanded strict rules for AI to be freed from weaponry purposes, or called "*weaponized AI* "[1].

Meanwhile, it is known that the development of science and technology has a positive and negative facet as well as the Robotics & AI. Although positive contributions are obvious, but the side effects are spiritual and mental aspects; and it needs to be prepared so that people can still take the positives, for example the planner of robotic Intelligence must have a code of ethics: *Intelligence robotics should not harm or kill humans, rob banks etc*. For other ethical issues of AI, see for example [5].

Are there practical examples of the realism attitude in technology?

If you got free time, read the periodicals around the industry in Japan. There are at least 2 interesting phrases that are worth a study: *Ikigai and Monozukuri.*

The ikigai may be a bit often we hear, meaning: The reason we wake up early, consisting of a balance between passion, work, profession etc.

Then what is Monozukuri? According to a source:

> "*Monozukuri is a Japanese word derived from the word " mono "means product or item and " Zukuri "means the creation, creation or production process. However, this concept has far broader implications than its literal meaning, where there is a creative spirit in delivering superior products as well as the ability to continuously improve the process...* "

What is the implementation? Let's look at 2 simple examples:

A. Sushi: Though simple at a glance, sushi is carefully designed so that the size is a one-stop meal. No more and no less. That is the advantage of many innovations that are typical of Japanese, because they think carefully from the usefulness, size, artistic value of the product. And so on.

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B. Shinkansen: The uniqueness of this train is not only about speed, but also on time (punctual). Even reportedly, the time lag between train sets is less than 5 minutes. And everything is designed by Japanese railway engineers even before there is a personal computer or AI. Then how did they design such an intricate system? Answer: They use dynamic control theory ("*Dynamic control Theory*").

Concluding remarks

Of course this is just a brief comment on a complicated topic that needs to be carefully examined and cautiously thought of.

Let the authors close this article by quoting the sentence of a wise man in the past centuries:

"*Lo, this only have I found, that God hath made man upright; but they have sought out many inventions*."

Wishing you all a happy a new year 2020. Hopefully next year there will be not a robot to greet you. Yes it is indeed a great paradox in the 21st century: "*Robots are increasingly proficient at imitating humans, but many humans live like robots."*- personal quote.

Acknowledgement

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There is No Constant in Physics: a Neutrosophic Explanation

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Abstract

In Neutrosophic Logic, a basic assertion is that there are variations of about everything that we can measure; the variations surround three parameters called T,I,F (truth, indeterminacy, falsehood) which can take a range of values. Similarly, in this paper we consider NL applications in physics constants. Those constants actually all have a window of plus and minus values, relative to the average value of the constant. For example, *speed of light, c, can vary in a window up to +/- 3000 m/s*. Therefore it should be written: 300000 km/s +/- 3 km/s. We also discuss some implications of this new perspective of physics constants, including in gravitation physics etc.

Keywords: Neutrosophic Logic, Physical Neutrosophy, gravitation, physics constants, Michelson-Morley experiment

1.Introduction

For majority of physicists, constants play a fundamental role. Like an anchor for a ship, they allow physicists build theories on the ground of those constants as basic "known" quantities. However, in real experiments, there are always variation of those constants. Moreover, from Neutrosophic Logic perspective, those constants always fluctuate depending on various circumstances.

In Neutrosophic Logic, a basic assertion is that there are variations of about everything that we can measure, the variations surround three parameters called T,I,F (truth, indeterminacy, falsehood) which can take a range of values. Similarly, in this paper we consider NL applications in physics constants. Those constants actually all have a window of plus and minus values, relative to the average value of the constant. For example, *speed of light, c, can vary in a window up to +/- 3000 m/s*.[1] Therefore it should be written: 300000 km/s +/- 3 km/s.

[1] Note by one of us (RNB): "The data from the experiment was recorded in the actual handwritten log books from the actual M-M experiments as **up to** plus and minus 3000 meters per second variation in the measured speed of light. I closely examined all the handwritten logs and lab notes personally. Most of the light speed excursions recorded in the actual log books were smaller than this. I recall calculating the average speed of light excursion to be in the vicinity of 300 meters per second. The apparatus was capable of measuring c to an accuracy of 0.00025 meters per second, as I recall.

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We also discuss some implications of this new perspective of physics constants, including in gravitation physics etc.

It is our hope that this new perspective on physics constants will point to a more substantial and evidence-based approach to physics sciences.

2. Definition

Neutrosophic Logic, as developed by one of us (FS), is generalization of fuzzy logic based on **Neutrosophy**. A proposition is t true, i indeterminate, and f false, where t, i, and f are real values from the ranges T, I, F, with no restriction on T, I, F, or the sum n=t+i+f. Neutrosophic logic thus generalises:

- intuitionistic logic, which supports incomplete theories (for 0<n<100 and i=0, 0<=t,i,f<=100);

- fuzzy logic (for n=100 and i=0, and 0<=t,i,f<=100);

- Boolean logic (for n=100 and i=0, with t,f either 0 or 100);

- multi-valued logic (for 0<=t,i,f<=100);

- paraconsistent logic (for n>100 and i=0, with both t,f<100);

- dialetheism, which says that some contradictions are true (for t=f=100 and i=0; some paradoxes can be denoted this way).

Compared with all other logics, neutrosophic logic introduces a percentage of "indeterminacy" - due to unexpected parameters hidden in some propositions. It also allows each component t,i,f to "boil over" 100 or "freeze" under 0. For example, in some tautologies t>100, called "overtrue".[1]

Neutrosophic Logic allows one to develop new approaches in many fields of science, including a redefinition of physics constants, as will be discussed in the next section.

3. Neutrosophic reasoning: There is no Physics Constant

In accordance with Neutrosophic Logic, actually all physics constants have a window of plus and minus values, relative to the average value of the constant. For example, variation of c is approximately within the range of plus or minus 3000 meters/second.

There may be larger excursions, but we would not expect larger excursions to happen very often. Probability considerations are thus also involved in determining the average value and the statistical extremes for the given constant.

There are also curves which vary according to the materials involved, and the environment. For example, most recently and most importantly, it has been realized that h and h_bar cannot be used for any material other than carbon black (soot).

All other materials must have their thermal emissions curve instrumented. Then the h and h_bar for that material can be calculated. But the values calculated are subject to modifications by the local environment. Unless

Both periodic and stochastic measurements of *speed of light variances* are recorded in the handwritten log books from the M-M experiments. Should be as listed: “variation of c is approximately within the range of plus or minus 3000 meters/second.” No larger excursions were recorded.” See also [10][11].

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the aether environment can be considered and measured, the calculated values of h and h_bar for the given material will not be as reliable as we might prefer. (It depends on the specific application which requires instrumented measurements of the thermal emissions curve of the given material.)

So there should be a way to produce an accurate thermal emissions curve using a neutrosophic approach. Because all thermal emissions curves have extremes from absolute zero to very high heat values. Neutrosophic modifications of Kirchoff's law of "blackbody radiation", and Planck's "constant" would be very useful. (See for instance, a report by Robitaille and Crothers on the flaws of Kirchoff law, [2-4]). It is worth noting here, that from dynamical perspective, Shpenkov argues for a redefinition of Planck constant: “The Planck constant h is the quantity the value of which is equal to the orbital action of the electron on the Bohr first orbit in the hydrogen atom, namely to its orbital moment of momentum P_{orb} multiplied by 2π, or it can be rewritten as: $h=2\pi.\ P_{orb}$.” According to him, Planck constant also has acoustic origin. [5]

There are also physical situations where the variations of the value of one constant, directly alters the values of physically-related constants. The fine structure "constant" is an example of this kind of mutual influence. If the fine structure value changes, it changes the value of e, the charge of the electron. (Which informs us that the charge on the electron is an environmentally influenced Neutrosophic window.) Going the other way, if the value of e changes, it changes the value of the fine structure constant.

Another aspect of this to consider is that some constants-windows may not be perfectly symmetrical, but large on one side of the center value, and small on the other side, and exhibit dependence on the environment, such that under most conditions the value of the given "constant" would live inside the window, while there could be large asymmetrical extremes at other times, depending on the local and non-local environmental parameters of the aether, at the location where we are examining the measured value of the constant.

4. A few applications

At this point, some readers may ask: Can we get an example when a so-called constant has a value, while in another example the same so-called constant gets another value?

Answer: Gravitation is a good example. g changes depending on where and when it is measured. This is used in gravitational prospecting and by the GRACE experiment (NASA) which maps the gravitation variations of the Earth, over time. [6] In ref [6], they show many more data sets and graphical images showing gravitational variations on the Earth.

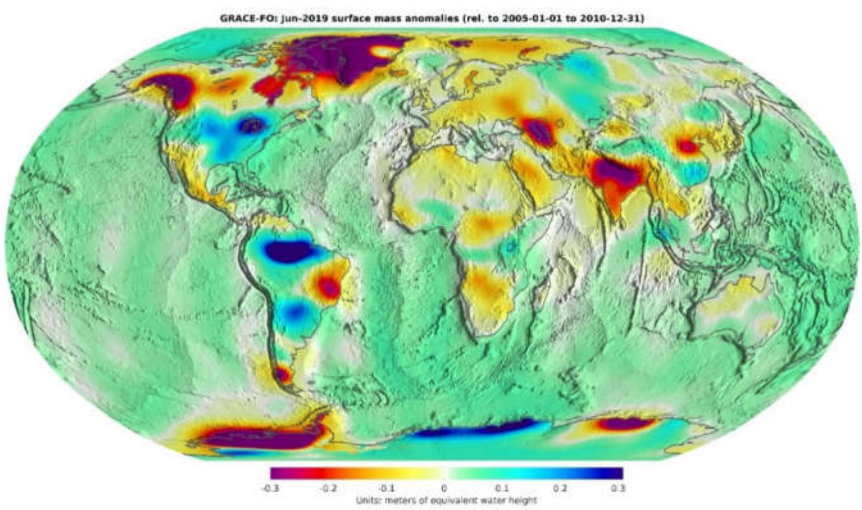

Figure 1. gravitational variations on the Earth. Source: [6]

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Another article contains a good table of measurements of gravitation from 1798, until 2004 [7].

There is also a discussion of the increase in the force due to gravitation of the Earth, showing the dinosaurs would be crushed by their own weight if they were subjected to the gravitational force of the Earth today.

The *gravitational "constant"* is a good one to start with, since the variations can't be denied.

The next best one would be speed of light variations, although these days they refuse to allow one-way measurements of light velocity, because vast numbers of variations show up, depending on the time and place of the measurement. The mainstream insists that the speed of light can only be measured by round-trip measurements. This is because the light going back and forth along the same line results in many of the measured one-way variations in the velocity, being averaged out.

Typically, speed of light experiments cook their books and throw out any large deviations in measured light velocity. This tactic is similar to even more egregious cheating methods which are used by "global warming" and "climate change" advocates, paid for by the oil companies.

The next best one would be variations in Planck's "constant". h and h_bar are only valid for carbon black (soot). Every different material has a different thermal radiation when plotted on a thermal radiation curve. Some examples are displayed by Robitaille and Crothers in some of their presentations on the original "black body" thermal radiation constant known as Kirchoff's law, which was never measured by instrumented experiments, and was accepted as universally valid by Planck, who never did experiments to measure the thermal radiation curves of anything.[2-4]

5. Conclusions

In this article, we discussed how physics constants can vary in a wide range of values, in particular from Neutrosophic Logic perspective. We also discussed some examples, including variation in Earth gravitation measurements, speed of light measurement, and also Planck constant. It is our hope that this short discussion will be found as good impetus for a new direction in physics, more corresponding to experimental data, toward: "*evidence-based physics*." This new direction is in direct contrast to the unfortunate development of theoretical physics in the last 30-40 years with their overreliance on too much abstraction, oversophisticated mathematics, and other fantasies, which often have less and less to do with the actual physics as an empirical science. Two books can be mentioned here in relation to the present situation of physics science, see [8][9].

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A Direct Model for Triangular Neutrosophic Linear Programming

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Abstract

This paper aims to propose a new direct algorithm to solve the neutrosophic linear programming where the variables and right-hand side represented with triangular neutrosophic numbers. The effectiveness of the proposed procedure is illustrated through numerical experiments. The extracted results show that the new algorithm is straightforward and could be useful to guide the modeling and design of a wide range of neutrosophic optimization.

Keywords: Single valued neutrosophic number; Neutrosophic linear programming problem; Linear programming problem.

1. Introduction

Fuzzy set originally introduced by Zadeh [1] in 1965 is a useful tool to capture the imprecision and uncertainty in decision-making [2, 3]. It is characterized by a membership degree between zero and one, and the non-membership degree is equal to one minus the membership degree. The intuitionistic fuzzy set (IFS) theory launched by Atanassov [4], addresses the problem of uncertainty by considering a non-membership function along with the fuzzy membership function on a universal set. The membership degree of an object is complemented with a non-membership degree that gives the extent to which an object does not belong to the IFS such that the sum of the two degrees should be less than or equal to 1.

Neutrosophy has been proposed by Smarandache [5] as a new branch of philosophy, with ancient roots, dealing with "the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra". The fundamental thesis of neutrosophy is that every idea has not only a certain degree of truth, as is generally assumed in many-valued logic contexts but also a falsity and indeterminacy degrees that have to be considered independently from each other. Smarandache seems to understand such "indeterminacy" both in a subjective and an objective sense, i.e., as uncertainty as well as imprecision, vagueness, error, doubtfulness, etc. Neutrosophic set (NS) is a generalization of the fuzzy set [1] and intuitionistic fuzzy set [4] and can deal with uncertain, indeterminate and incongruous information where the indeterminacy is quantified explicitly and truth membership, indeterminacy membership and falsity membership are completely independent. It can effectively describe uncertain, incomplete and inconsistent information and overcomes some limitations of the existing methods in depicting uncertain decision information. In the neutrosophic logic, each proposition is estimated by a triplet viz, truth grade, indeterminacy grade and falsity grade. The indeterministic part of uncertain data, introduced in NS theory, plays an important role in making a proper decision which is not possible by intuitionistic fuzzy set theory. Since indeterminacy always appears in our routine activities, the NS theory can analyze the various situations smoothly. Moreover, some extensions of NSs, including interval neutrosophic set [6], bipolar neutrosophic set [7], single-valued neutrosophic set [8], multi-valued neutrosophic set [9], and neutrosophic linguistic set [10] have been proposed and applied to solve various problems [11-20].

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Linear programming problem (LP) is a method for achieving the best outcome (such as maximum profit or minimum cost) in a mathematical model represented by linear relationships. Decision-making is a process of solving the problem and achieving goals under the asset of constraints, and it is very difficult in some cases due to incomplete and imprecise information. In uncertain linear programming problems, such as approaches using fuzzy and stochastic logics, interval numbers, or uncertain variables, some uncertain linear programming methods have been developed in the existing literature. For example, Bellman and Zadeh [21] introduced fuzzy optimization problems where they have stated that a fuzzy decision can be viewed as the intersection of fuzzy goals and problem constraints. Many researchers such as; Zimmermann [22], Tanaka et al.[17], Campos and Verdegay [23], Rommelfanger et al.[24], Cadenas and Verdegay [25] who were dealing with the concept of solving fuzzy optimization problems, later studied this subject. In the past few years, a growing interest has been shown in Fuzzy optimization. Buckley and Feuring [26] introduced a general class of fuzzy linear programming, called fully fuzzified linear programming (FFLP) problems, where all decision parameters and variables are fuzzy numbers. Fuzzy mathematical programming, using a unified approach, has been studied by [27]. Lodwick and Bachman [28] have studied large scale fuzzy and possible optimization problems. Buckley and Abdalla [29] have considered Monte Carlo methods in the fuzzy queuing theory. Some authors have considered fuzzy linear programming, in which not all parts of the problem were assumed to be fuzzy, e.g., only the right-hand side and the objective function coefficients were fuzzy; or only the variables were fuzzy [30-35]. The fuzzy linear programming problems in which fuzzy numbers represent all the parameters and variables are known as fully fuzzy linear programming (FFLP) problems. FFLP problem with inequality constraints studied in [36-37]. However, the main disadvantage of the solution obtained by the existing methods is that it does not satisfy the constraints exactly i.e., it is not possible to obtain the fuzzy number of the right-hand side of the constraint by putting the obtained solution in the left-hand side of the constraint. Dehghan et al. [38] proposed some practical methods to solve a fully fuzzy linear system (FFLS) that is comparable to the well-known methods. Then they extended a new method employing Linear Programming (LP) for solving square and non-square fuzzy systems. Lotfi et al. [39] applied the concept of the symmetric triangular fuzzy number, obtained a new method for solving FFLP by converting an FFLP into two corresponding LPs. Kumar et al. [40] pointed out the shortcomings of the above methods. To overcome these shortcomings, they proposed a new method for finding the fuzzy optimal solution of FFLP problems with equality constraints. Saberi Najafi and Edalatpanah [41] pointed out the method of [40] needs some corrections to make the model well in general; for other methods see[42-46]. In general, the above existing methods can be applied for the following type of FFLP problems:

i) FFLP Problem with nonnegative fuzzy coefficients and nonnegative fuzzy variables.

ii) FFLP Problem with unrestricted fuzzy coefficients and nonnegative fuzzy variables.

iii) FFLP Problem with nonnegative fuzzy coefficients and unrestricted fuzzy variables.

However, the above mentioned methods can-not deal with indeterminate optimization problems. Furthermore, the existing uncertain linear programming methods are not really meaningful indeterminate programming because these uncertain linear programming methods are generally to turn these optimization models into crisp objective programming models to find unique crisp optimal solutions rather than indeterminate solutions in uncertain situations. However, the unique crisp optimal solutions obtained by existing uncertain linear programming methods may be conservative and relatively insensitive to input uncertainty or the optimization performance may degrade significantly. From an indeterminate viewpoint, an indeterminate optimization problem should contain possible ranges of the optimal solutions (indeterminate intervals) corresponding to various indeterminate ranges to be suitable for indeterminate requirements rather than the unique crisp optimal solution under indeterminate environments. Then, Abdel-Baset et al. [47] and Pramanik [48] proposed neutrosophic linear programming methods based on the neutrosophic set (NS) concept. Also, Abdel-Baset et al. [49] introduced the neutrosophic LP models where their parameters are represented with trapezoidal neutrosophic numbers and presented a technique for solving them. However, it is observed that Abdel-Basset et al.[50] have considered several mathematical incorrect assumptions in their proposed method and hence, it is scientifically incorrect to use this method [51].

So, the main purposes of this paper are (1) to propose a new direct model, including neutrosophic variables and the right-hand side; and (2) to present a solution method for this neutrosophic LP problems. This paper organized as follows: some basic knowledge, concepts of neutrosophic set theory, an arithmetic operation are introduced in Section 2. In Section 3, we present a new algorithm to solve the neutrosophic LP. In Section 4, a numerical example is given to reveal the effectiveness of the proposed model. Finally, some conclusions are drawn in the last section.

2. Preliminaries

In this section, we present some basic definitions and arithmetic operations on neutrosophic sets.

Definition 1 [5]. Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. If the functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are singleton subintervals/subsets in the real standard [0, 1], that is $T_A(x)$: $X \rightarrow [0,1]$, $I_A(x)$: $X \rightarrow [0,1]$, and $F_A(x)$: $X \rightarrow [0,1]$. Then, a Single valued neutrosophic set A is denoted by $A = \{(x, T_A(x), I_A(x), F_A(x)) \mid x \in X\}$ which is called an SVN. Also, SVN satisfies the condition:

$$0 \le T_A(x) + I_A(x) + F_A(x) \le 3.$$

Definition 2 [5]. For SVNSs A and B, $A \subseteq B$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, and $F_A(x) \ge F_B(x)$ for every x in X.

Definition 3 [50]. A triangular neutrosophic number (TNNs) is denoted by $A^{\aleph} = \langle (a^l, a^m, a^u), (\mu, i, \omega) \rangle$ whose the three membership functions for the truth, indeterminacy, and falsity of x can be defined as follows:

$$\tau_{A^{\aleph}}(x) = \begin{cases} \dfrac{(x - a^l)}{(a^m - a^l)}\mu & a^l \le x < a^m, \\ \mu & x = a^m, \\ \dfrac{(a^u - x)}{(a^u - a^m)}\mu & a^m \le x < a^u, \\ 0 & otherwise. \end{cases}$$

$$\iota_{A^{\aleph}}(x) = \begin{cases} \dfrac{(a^m - x)}{(a^m - a^l)}i, & a^l \le x < a^m, \\ i, & x = a^m, \\ \dfrac{(x - a^u)}{(a^u - a^m)}i, & a^m \le x < a^u, \\ 1, & otherwise. \end{cases}$$

$$\nu_{A^{\aleph}}(x)=\begin{cases}\dfrac{\left(a^{m}-x\right)}{\left(a^{m}-a^{l}\right)}\omega, & a^{l}\le x<a^{m},\\ \omega, & x=a^{m},\\ \dfrac{\left(x-a^{u}\right)}{\left(a^{u}-a^{m}\right)}\omega, & a^{m}\le x<a^{u},\\ 1, & otherwise.\end{cases}$$

Where, $0\le \tau_{A^{\aleph}}(x)+\iota_{A^{\aleph}}(x)+\nu_{A^{\aleph}}(x)\le 3, x\in A^{\aleph}$. Additionally, when $a^{l}\ge 0$, $A^{\aleph}$ is called a nonnegative TNN. Similarly, when $a^{l}<0$, $A^{\aleph}$ becomes a negative TNN.

Definition 4 [50]. Suppose $A_1^{\aleph}=<(a_1,b_1,c_1),(\mu_1,\nu_1,\omega_1)>$ and $A_2^{\aleph}=<(a_2,b_2,c_2),(\mu_2,\nu_2,\omega_2)>$ be two TNNs. Then the arithmetic relations are defined as:

$(i)A_1^{\aleph}\oplus A_2^{\aleph}=<(a_1+a_2,b_1+b_2,c_1+c_2),(\mu_1\wedge\mu_2,\nu_1\vee\nu_2,\omega_1\vee\omega_2)>$

$(ii)A_1^{\aleph}-A_2^{\aleph}=<(a_1-c_2,b_1-b_2,c_1-a_2),(\mu_1\wedge\mu_2,\nu_1\vee\nu_2,\omega_1\vee\omega_2)>$

$(iii)A_1^{\aleph}\otimes A_2^{\aleph}=$
$<(a_1a_2,b_1b_2,c_1c_2),(\mu_1\wedge\mu_2,\nu_1\vee\nu_2,\omega_1\vee\omega_2)>,\ \ if\ a_1>0,b_1>0,$

$$(iv)\lambda A_1^{\aleph}=\begin{cases}<(\lambda a_1,\lambda b_1,\lambda c_1),(\mu_1,\nu_1,\omega_1)>, & if\ \ \lambda>0\\ <(\lambda c_1,\lambda b_1,\lambda a_1),(\mu_1,\nu_1,\omega_1)>, & if\ \ \lambda<0\end{cases}$$

3. Proposed method

Consider the following trapezoidal neutrosophic linear programming (TNLP) with *m* constraints and *n* variables;

$$\begin{aligned}&\text{Max (Min) } (c^t\tilde{x})\\ &\text{subject to}\\ &\qquad A\tilde{x}\le\tilde{b},\end{aligned}\qquad (1)$$

$\tilde{x}$ is a non-negative TNN.

Where $A=\left[a_{ij}\right]_{m\times n}$ is the coefficient matrix, $\tilde{b}=\left[\tilde{b}_1,\tilde{b}_2,\tilde{b}_3,\ldots,\tilde{b}_m\right]^t$ is the triangular neutrosophic available resource vector, $c=\left[c_1,c_2,c_3,\ldots,c_n\right]^t$ is the objective coefficient vector and $\tilde{x}=\left[\tilde{x}_1,\tilde{x}_2,\tilde{x}_3,\ldots,\tilde{x}_n\right]^t$ is the triangular neutrosophic decision variable vector.

The steps of the proposed method are as follows:

Step 1: Assuming $\tilde{b}=<b^l,b^m,b^r;T_{\tilde{b}},I_{\tilde{b}},F_{\tilde{b}}>$, $\tilde{x}=<x^l,x^m,x^r;T_{\tilde{x}},I_{\tilde{x}},F_{\tilde{x}}>$, and using Definition 4, the LP problem (1) can be transformed into the problem (2).

$$
\begin{aligned}
&\text{Max (Min) } \sum_{j=1}^{n} c_j < x_j^l, x_j^m, x_j^r; T_{\tilde{x}_j}, I_{\tilde{x}_j}, F_{\tilde{x}_j} >, \\
&\text{subject to} \\
&\quad \sum_{j=1}^{n} a_{ij} < x_j^l, x_j^m, x_j^r; T_{\tilde{x}_j}, I_{\tilde{x}_j}, F_{\tilde{x}_j} > \ \leq \ < b_i^l, b_i^m, b_i^r; T_{\tilde{b}_i}, I_{\tilde{b}}, F_{\tilde{b}} >, \forall i \\
&\quad\quad < x_j^l, x_j^m, x_j^r; T_{\tilde{x}_j}, I_{\tilde{x}_j}, F_{\tilde{x}_j} > \ \geq 0, \ \ \forall j.
\end{aligned} \tag{2}
$$

Step 2: Using definition 2 -4, and with the following assumptions:

$$\sum_{j=1}^{n} c_j < x_j^l, x_j^m, x_j^r; T_{\tilde{x}_j}, I_{\tilde{x}_j}, F_{\tilde{x}_j} > = < u^l, u^m, u^r; T, I, F >,$$

$$\sum_{j=1}^{n} a_{ij} < x_j^l, x_j^m, x_j^r; T_{\tilde{x}_j}, I_{\tilde{x}_j}, F_{\tilde{x}_j} > = < v^l, v^m, v^r; T, I, F >,$$

the LP problem (2) can be transformed into the problem (3).

$$
\begin{aligned}
&\text{Max (Min) } < u^l, u^m, u^r; T, I, F >, \\
&\text{subject to} \\
&< v^l, v^m, v^r; T, I, F > \ \leq \ < b_i^l, b_i^m, b_i^r; T_{\tilde{b}_i}, I_{\tilde{b}}, F_{\tilde{b}} >, \forall i \\
&T \leq \min(T_{\tilde{b}_i}), I \geq \max(I_{\tilde{b}}), F \geq \max(F_{\tilde{b}}), \\
&T = \min(T_{\tilde{x}_j}), I = \max(I_{\tilde{x}_j}), F = \max(F_{\tilde{x}_j}), \\
&< x_j^l, x_j^m, x_j^r; T_{\tilde{x}_j}, I_{\tilde{x}_j}, F_{\tilde{x}_j} > \ \geq 0.
\end{aligned} \tag{3}
$$

Step 3: By the neutrosophic nature, the LP problem (3) can be transformed into a multi-objective problem (4).

$$
\begin{aligned}
&\text{Max (Min) } u^l, \\
&\text{Max (Min) } u^m, \\
&\text{Max (Min) } u^r,
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
&\text{Max (Min) } \sum_{j=1}^{n} T_{\tilde{x}_j}, \\
&\text{Min (Max)} \sum_{j=1}^{n} I_{\tilde{x}_j}, \\
&\text{Min (Max)} \sum_{j=1}^{n} F_{\tilde{x}_j},
\end{aligned}
$$

$$
\begin{aligned}
&\text{subject to}\\
&v^{l} \le b_i^{l}, \forall i\\
&v^{m} \le b_i^{m}, \forall i\\
&v^{r} \le b_i^{r}, \forall i\\
&\sum_{j=1}^{n} T_{\tilde{x}_j} \le n \min(T_{\tilde{b}_i}),\\
&\sum_{j=1}^{n} I_{\tilde{x}_j} \ge n \max(I_{\tilde{b}}),\\
&\sum_{j=1}^{n} F_{\tilde{x}_j} \ge n \max(F_{\tilde{b}})\\
&x_j^{l} \ge 0, x_j^{m} - x_j^{l} \ge 0, x_j^{r} - x_j^{m} \ge 0,\\
&0 \le T_{\tilde{x}_j} \le 1, 0 \le I_{\tilde{x}_j} \le 1, 0 \le F_{\tilde{x}_j}\ 0 \le 1,\\
&T_{\tilde{x}_j} + I_{\tilde{x}_j} + F_{\tilde{x}_j} \le 3,\\
&T_{\tilde{x}_j} \ge F_{\tilde{x}_j}, T_{\tilde{x}_j} \ge I_{\tilde{x}_j}.
\end{aligned}
$$

Step 4: Using summation of all object functions, the Model (4), obtained in Step 3, can be converted into the crisp linear programming problem as follows:

$$\text{Max (Min) } u^{l} + u^{m} + u^{r} + \sum_{j=1}^{n} T_{\tilde{x}_j} - \sum_{j=1}^{n} I_{\tilde{x}_j} - \sum_{j=1}^{n} F_{\tilde{x}_j}, \tag{5}$$

Subject to: all constraints of Model (4).

Step 4: Find the optimal solution $\tilde{x}$ by solving the crisp linear programming problems obtained in problem (5) and find the neutrosophic optimal value by putting in the objective function.

4. Numerical example

In this section, a numerical example problem has been solved using the proposed method to illustrate the applicability and efficiency of it.

Example 1 .

$$Max\ (\tilde{z}) = 5\tilde{x}_1 + 4\tilde{x}_2$$

subject to

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$$\begin{aligned}
&6\tilde{x}_1+4\tilde{x}_2 \le\ <3,5,6;0.6,0.5,0.6>,\\
&\tilde{x}_1+2\tilde{x}_2 \le\ <5,8,10;0.3,0.6,0.6>,\\
&-\tilde{x}_1+\tilde{x}_2 \le\ <12,15,19;0.6,0.4,0.5>,\\
&\qquad\tilde{x}_2 \le\ <14,17,21;0.8,0.2,0.6>,\\
&\tilde{x}_1,\tilde{x}_2 \ge 0.
\end{aligned} \tag{6}$$

Now. To solve the problem with the proposed method, we have the following steps:

Step 1: Assuming $\tilde{x} =< x^l, x^m, x^r; T_{\tilde{x}}, I_{\tilde{x}}, F_{\tilde{x}} >$, and using Definition 4, the LP problem (4) can be transformed into the problem (7).

$$\begin{aligned}
&\text{Max}\,\tilde{z} = 5 < x_1^l, x_1^m, x_1^r; T_{\tilde{x}_1}, I_{\tilde{x}_1}, F_{\tilde{x}_1} > \oplus 4 < x_2^l, x_2^m, x_2^r; T_{\tilde{x}_2}, I_{\tilde{x}_2}, F_{\tilde{x}_2} >\\
&\text{subject to}\\
&6 < x_1^l, x_1^m, x_1^r; T_{\tilde{x}_1}, I_{\tilde{x}_1}, F_{\tilde{x}_1} > \oplus 4 < x_2^l, x_2^m, x_2^r; T_{\tilde{x}_2}, I_{\tilde{x}_2}, F_{\tilde{x}_2} > \ \le\ < 3,5,6; 0.9, 0.1, 0.2 >,\\
&< x_1^l, x_1^m, x_1^r; T_{\tilde{x}_1}, I_{\tilde{x}_1}, F_{\tilde{x}_1} > \oplus 2 < x_2^l, x_2^m, x_2^r; T_{\tilde{x}_2}, I_{\tilde{x}_2}, F_{\tilde{x}_2} > \ \le\ < 5,8,10; 0.7, 0.2, 0.1 >,\\
&- < x_1^l, x_1^m, x_1^r; T_{\tilde{x}_1}, I_{\tilde{x}_1}, F_{\tilde{x}_1} > \oplus < x_2^l, x_2^m, x_2^r; T_{\tilde{x}_2}, I_{\tilde{x}_2}, F_{\tilde{x}_2} > \ \le\ < 12,15,19; 0.8, 0.3, 0.1 >,\\
&< x_2^l, x_2^m, x_2^r; T_{\tilde{x}_2}, I_{\tilde{x}_2}, F_{\tilde{x}_2} > \ \le\ < 14,17,21; 0.8, 0.2, 0.1 >,\\
&< x_j^l, x_j^m, x_j^r; T_{\tilde{x}_j}, I_{\tilde{x}_j}, F_{\tilde{x}_j} > \ \ge 0,\ \forall j.
\end{aligned} \tag{7}$$

Step 2: Using definition 2 -4, the LP problem (7) can be transformed into the problem (8).

$$\begin{aligned}
&\text{Max}\,\tilde{z} =< 5x_1^l + 4x_2^l, 5x_1^m + 4x_2^m, 5x_1^r + 4x_2^r; T, I, F >\\
&\text{subject to}\\
&< 6x_1^l + 4x_2^l, 6x_1^m + 4x_2^m, 6x_1^r + 4x_2^r; T, I, F > \ \le\ < 3,5,6; 0.9, 0.1, 0.2 >,\\
&< x_1^l + 2x_2^l, x_1^m + 2x_2^m, x_1^r + 2x_2^r; T, I, F > \ \le\ < 5,8,10; 0.7, 0.2, 0.1 >,\\
&< -x_1^r + x_2^l, -x_1^m + x_2^m, -x_1^l + x_2^r; T, I, F > \ \le\ < 12,15,19; 0.8, 0.3, 0.1 >,\\
&< x_2^l, x_2^m, x_2^r; T_{\tilde{x}_2}, I_{\tilde{x}_2}, F_{\tilde{x}_2} > \ \le\ < 14,17,21; 0.8, 0.2, 0.1 >,\\
&< x_j^l, x_j^m, x_j^r; T_{\tilde{x}_j}, I_{\tilde{x}_j}, F_{\tilde{x}_j} > \ \ge 0,\ \forall j.
\end{aligned} \tag{8}$$

Step 3: Using the Step 4 of our proposed method, the Model (8), can be converted into the crisp linear programming problem as follows:

$$\begin{aligned}
&\text{Max}\,\tilde{z} = 5x_1^l + 4x_2^l + 5x_1^m + 4x_2^m + 5x_1^r + 4x_2^r + T_{\tilde{x}_1} + T_{\tilde{x}_2} - I_{\tilde{x}_1} - I_{\tilde{x}_2} - F_{\tilde{x}_1} - F_{\tilde{x}_2}\\
&\text{subject to}\\
&6x_1^l + 4x_2^l \le 3, x_1^l + 2x_2^l \le 5, -x_1^r + x_2^l \le 12, x_2^l \le 14,
\end{aligned} \tag{9}$$

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$$6x_1^m + 4x_2^m \le 5, x_1^m + 2x_2^m \le 8, -x_1^m + x_2^m \le 15, x_2^m \le 17,$$
$$6x_1^r + 4x_2^r \le 6, x_1^r + 2x_2^r \le 10, -x_1^l + x_2^r \le 19, x_2^r \le 21,$$
$$T_{\tilde{x}_1} + T_{\tilde{x}_2} \le 1.4,$$
$$I_{\tilde{x}_1} + I_{\tilde{x}_2} \ge 0.6,$$
$$F_{\tilde{x}_1} + F_{\tilde{x}_2} \ge 0.4,$$
$$x_j^l \ge 0, x_j^m - x_j^l \ge 0, x_j^r - x_j^m \ge 0,$$
$$0 \le T_{\tilde{x}_j} \le 1, 0 \le I_{\tilde{x}_j} \le 1, 0 \le F_{\tilde{x}_j} 0 \le 1,$$
$$T_{\tilde{x}_j} + I_{\tilde{x}_j} + F_{\tilde{x}_j} \le 3,$$
$$T_{\tilde{x}_j} \ge F_{\tilde{x}_j}, T_{\tilde{x}_j} \ge I_{\tilde{x}_j}.$$

Step 4: Using Matlab or any software, we can solve the optimal solution as follows:

$$\tilde{x}_1 = < 0,0,0; 0.6, 0.6, 0.4 >,$$
$$\tilde{x}_2 = < 0.75, 1.25, 1.5; 0.8, 0, 0 >,$$
$$\tilde{z} = < 3,5,6; 0.6, 0.6, 0.4 >.$$

5. Conclusion

In this paper, we proposed a new direct algorithm for solving the linear programming problems, including neutrosophic variables and the right-hand side. In the proposed model, we maximize the degrees of acceptance and minimize indeterminacy and rejection of objectives. Meantime, a numerical example was provided to show the efficiency of the proposed method and illustrate the solution process. The new model not only richens uncertain linear programming methods but also provides a new effective way for handling indeterminate optimization problems

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MBJ-neutrosophic T-ideal on B-algebra

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Abstract

In this paper we define and study the MBJ-neutrosophic T-ideal through diferent concept like union, intersection. further we use the important properties to investigate the MBJ-neutrosophic T-ideal under cartesian product and homomorphic results.

Keywords: B-algebra, MBJ-neutrosophic set, MBJ-neutrosophic T-ideal, Cartesian product, Homomorphism.

1.Introduction

[1] presented the idea of fuzzy set. [2] gave the idea of BCK-algebra. [3] introduced the BCI-algebra and it is obvius that BCK-algebra is a proper sub class of BCI-algebra. [4] wrote on intuitionistic fuzzy G-algebra. [5] introduced the intuitionistic fuzzy sets. [6] provided the idea of interval-valued intuitionistic fuzzy sets. [7] studied the neutrosophic soft subalgebra and investigted many results. [8] initiated the intuitionistic fuzzy structure of B-algebra. [9] discussed the fuzzy dot subalgebra and fuzzy dot ideal of B-algebras. [10] introduced the B-algebra in 2002. [11,12] worked on t-intuitionistic fuzzy sets in fuzzy subgroups and subrings. Barbhuiya [13] studied the t-intuitionistic fuzzy BG-subalgebra. [14] studied ideals with its level subsets. [15] introduced the T-neutrosophic cubic set and deeply investigated this set with significant characterisitics. [16] defined and studied the MBJ-neutrosophic set through subalgebra and S-extension. [17,18] The concept of Neutrosophic set is given by Smarandache with truth, indeterminate and false membership function.

This paper is presented to studied the idea of MBJ-neutrosophic set throught the concept of T-ideal, homomorphic characteristics and cartesian product.

2. Preliminaries

First we cite some definitions which are used to present this paper. [3] An algebra $(Y,*,0)$ of type (2,0) is called a BCI-algebra if it satisfies the following conditions:

i) $(t_1 * t_2) * (t_1 * t_3) \leq (t_3 * t_2)$,

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ii) $t_1 * (t_1 * t_2) \leq t_2$,

iii) $t_1 \leq t_1$,

iv) $t_1 \leq t_2$ and $t_2 \leq t_1 \Rightarrow t_1 = t_2$,

v) $t_1 \leq 0 \Rightarrow t_1 = 0$, where $t_1 \leq t_2$ is defined by $t_1 * t_2 = 0$, for all $t_1, t_2, t_3 \in Y$.

[1] An algebra $(Y,* ,0)$ of type (2,0) is called a BCK-algebra if it satisfies the following conditions:

i) $(t_1 * t_2) * (t_1 * t_3) \leq (t_3 * t_2)$,

ii) $t_1 * (t_1 * t_2) \leq t_2$,

iii) $t_1 \leq t_1$,

iv) $t_1 \leq t_2$ and $t_2 \leq t_1 \Rightarrow t_1 = t_2$,

v) $0 \leq t_1 \Rightarrow t_1 = 0$, where $t_1 \leq t_2$ is defined by $t_1 * t_2 = 0$, for all $t_1, t_2, t_3 \in Y$.

[10] A nonempty set Y with a constant 0 and having binary operation $*$ is called B-algebra if it satisfies the following conditions:

i) $t_1 * t_1 = 0$

ii) $t_1 * 0 = x$

iii) $(t_1 * t_2) * t_3 = (t_1 * (t_3 * (0 * t_2))$ for all $t_1, t_2 \in Y$.

[10] Let K be a nonempty subset of B-algebra Y, then K is called a subalgebra of Y if $t_1 * t_2 \in K$, for all $t_1, t_2 \in K$.

[14] Let Y be a PS-algebra. A fuzzy set B of Y is called a fuzzy PS ideal of Y if it satisfies the following conditions:

i) $\vartheta(0) \geq \vartheta(t_1)$,

ii) $\vartheta(t_1) \geq min\{\vartheta(t_2 * t_1), \vartheta(t_2)\}$, for all $t_1, t_2 \in Y$.

Let Y be a group of objects denoted generally by t_1. Then a fuzzy set B of Y is defined as $B = \{< t_1, \vartheta_B(t_1) > | t_1 \in Y\}$, where $\vartheta_B(t_1)$ is called the existence ship value of t_1 in B and $\vartheta_B(t_1) \in [0,1]$.

A fuzzy set B [14] of PS-algebra Y is called a fuzzy PS subalgebra of Y if $\vartheta(t_1 * t_2) \geq min\{\vartheta(t_1), \vartheta(t_2)\}$, for all $t_1, t_2 \in Y$.

An intuitionistic fuzzy set **(IFS)** [5] B over Y is an object having the form $B = \{\langle t_1, \vartheta_B(t_1), \phi_B(t_1)\rangle | t_1 \in Y\}$, where $\vartheta_B(t_1) | Y \to [0,1]$ and $\phi_B(t_1) | Y \to [0,1]$, with the condition $0 \leq \vartheta_B(t_1) + \phi_B(t_1) \leq 1$, for all $t_1 \in Y$. $\vartheta_B(t_1)$ and $\phi_B(t_1)$ represent the degree of existence and the degree of non-existence of the element t_1 in the set B respectively.

An **IFS** $B = \{\langle t_1, \vartheta_B(t_1), \phi_B(t_1)\rangle | t_1 \in Y\}$ of Y is said to be an **IFID** of Y if it satisfies these three conditions:

(i) $\vartheta_B(0) \geq \vartheta_B(t_1), \phi_B(0) \leq \phi_B(t_1)$,

(ii) $\vartheta_B(t_1) \geq min\{\vartheta_B(t_1 * t_2), \vartheta_B(t_2)\}$,

(iii) $\phi_B(t_1) \leq max\{\phi_B(t_1 * t_2), \phi_B(t_2)\}$, for all $t_1, t_2 \in Y$.

An IVFS [1] B over Y is of the form $B = \{\langle t_1, \hat{\vartheta}_B(t_1)\rangle | t_1 \in Y\}$, where $\hat{\vartheta}_B(t_1)|Y \to D[0.1]$, where D[0,1] is the set of all sub intervals of [0,1]. $\hat{\vartheta}_B(t_1)$ is the interval of the existence value of the element t_1 in B, where $\hat{\vartheta}_B(t_1) = [\hat{\vartheta}_{B\,L}(t_1), \hat{\vartheta}_{B\,U}(t_1)]\ \forall\ t_1 \in Y$.

An IVIFS [6] B over Y is an object of the form $B = \{\langle t_1, \hat{\vartheta}_B(t_1), \hat{\phi}_B(t_1)\rangle | t_1 \in Y\}$ where $\hat{\vartheta}_B(t_1)|Y \to [0,1]$ and $\hat{\phi}_B(t_1)|Y \to [0,1]$, D[0,1] is the set of all subintervals of [0,1]. $\hat{\vartheta}_B(t_1)$ and $\hat{\phi}_B(t_1)$ are the intervals of existence value and non-existence value of element t_1 to B, $\hat{\vartheta}_B(t_1) = [\hat{\vartheta}_{B\,L}(t_1), \hat{\vartheta}_{B\,U}(t_1)]$ and $\hat{\phi}_B(t_1) = [\hat{\phi}_{B\,L}(t_1), \hat{\phi}_{B\,U}(t_1)]$, $\forall\ t_1 \in Y$, with the condition $0 \le \hat{\vartheta}_{B\,U}(t_1) + \hat{\phi}_{B\,U}(t_1) \le 1$.

An IVIFS $\hat{B} = (\hat{\vartheta}_B, \hat{\phi}_B)$ in Y is called an IVIF-ideal [9] of Y when it fulfills these axioms.

i) $\hat{\vartheta}_B(0) \ge \hat{\vartheta}_B(t_1)$ and $\hat{\phi}_B(0) \le \hat{\phi}_B(t_1)$,

ii) $\hat{\vartheta}_B(t_1) \ge rmin\{\hat{\vartheta}_B(t_1 * t_2), \hat{\vartheta}_B(t_2)\}$,

iii) $\hat{\phi}_B(t_1) \le rmax\{\hat{\phi}_B(t_1 * t_2), \hat{\phi}_B(t_2)\}$.

Let $B = (\vartheta_B, \phi_B)$ be an IFS of Y. Let $t \in [0,1]$, then the IFS B^t is called the t-intuitionistic fuzzy subset [11, 12] of Y w.r.t B and is defined as $B^t = \{< t_1, \vartheta_{B^t}(t_1), \phi_{B^t}(t_1) > | t_1 \in Y\} =< \vartheta_{B^t}, \phi_{B^t} >$, where $\vartheta_{B^t}(t_1) = min\{\vartheta_B(t_1), t\}$ and $\vartheta_{B^t}(t_1) = max\{\phi_B(t_1), 1 - t\}$ for all $t_1 \in Y$.

Let **IFS** $B = \{\langle t_1, \vartheta_B(t_1), \phi_B(t_1)\rangle | t_1 \in Y\}$ of Y is said to be an **IFTID** of Y if it satisfies these three conditions:

(i) $\vartheta_B(0) \ge \vartheta_B(t_1), \phi_B(0) \le \phi_B(t_1)$,

(ii) $\vartheta_B(t_1 * t_3) \ge min\{\vartheta_B((t_1 * t_2) * t_3), \vartheta_B(t_2)\}$,

(iii) $\phi_B(t_1 * t_3) \le max\{\phi_B((t_1 * t_2) * t_3), \phi_B(t_2)\}$, for all $t_1, t_2 \in Y$.

An Neutrosophic fuzzy set [17, 18] on Y is defined by $B = \langle B_T, B_I, B_F\rangle | x \in Y$, where $B_T \to [0,1]$ is a truth membership function, $B_I \to [0,1]$, is an indeterminate membership function and $B_F \to [0,1]$ is a false membership function.

Let Y be a non empty set. MBJ-neutrosophic set [16] in Y, is a structure of the form $C = \{\langle M_C t_1, \hat{B}_C t_1, J_C t_1\rangle | t_1 \in Y\}$ where M_C and J_C are fuzzy sets in Y and M_C is a truth membership function,J_C is a false membership function and $\hat{B}$ is interval valued fuzzy set in Y and is an Indeterminate Interval Valued membership function.

3. MBJ-neutrosophic T-ideal of B-algebra

Definition 3.1. Let $C = (M_C, \hat{B}_C, J_C)$ is a MBJ-neutrosphic set of B-algebra Y. Let $t \in [0,1]$ then C is called MBJ-neutrosphic T-ideal (MBJNTID) of Y if it fulfills these assertions:

i) $M_C(0) \ge M_C(t_1)$, $\hat{B}_C(0) \ge \hat{B}_C(t_1)$ and $J_C(0) \le J_C(t_1)$.

ii) $M_C(t_1 * t_3) \ge min\{M_C((t_1 * t_2) * t_3), M_C(t_2)\}$.

iii) $\hat{B}_C(t_1 * t_3) \ge rmin\{\hat{B}_C((t_1 * t_2) * t_3), \hat{B}_C(t_2)\}$.

iv) $J_C(t_1 * t_3) \le max\{J_C((t_1 * t_2) * t_3), J_C(t_2)\}$.

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Theorem 3.1. Let $C = \left(M_C, \hat{B}_C, J_C\right)$ be a MBJNID of a B-algebra Y. If $t_1 * t_2 \leq t_3$, then $M_C(t_1 * x) \geq \min\{M_C(t_3), \hat{B}_C(t_2)\}$, $\hat{B}_C(t_1 * x) \geq rmin\{\hat{B}_C(t_3), \hat{B}_C(t_2)\}$ and $J_C(t_1 * x) \leq \max\{J_C(t_3), J_C(t_2\ )\}$ for all $t_1, t_2, t_3 \in Y$.

Proof. Let $t_1, t_2, t_3 \in Y$ such that $t_1 * t_2 = t_3$, now

$$M_C(t_1 * x) \geq \{min\{M_C((t_1 * t_2) * x), M_C(t_2)\}$$

$$\geq min\{min\{M_C(((t_1 * t_2) * t_3) * x), M_C(t_3)\}, M_C(t_2)\}$$

$$\geq min\{min\{M_C(0), M_C(t_3)\}, M_C(t_2)\}$$

$$\geq min\{M_C(t_3), M_C(t_2)\}$$

$$M_C(t_1 * x) = \min\{M_C(t_3), M_C(t_2)\}$$

and

$$\hat{B}_C(t_1 * x) \geq \{rmin\{\hat{B}_C((t_1 * t_2) * x), \hat{B}_C(t_2)\}$$

$$\geq rmin\{rmin\{\hat{B}_C(((t_1 * t_2) * t_3) * x), \hat{B}_C(t_3)\}, \hat{B}_C(t_2)\}$$

$$\geq rmin\{rmin\{\hat{B}_C(0), \hat{B}_C(t_3)\}, \hat{B}_C(t_2)\}$$

$$\geq rmin\{\hat{B}_C(t_3), \hat{B}_C(t_2)\}$$

$$\hat{B}_C(t_1 * x) = rmin\{\hat{B}_C(t_3), \hat{B}_C(t_2)\}$$

and

$$J_C(t_1 * x) \leq \{max\{J_C((t_1 * t_2) * x), J_C(t_2)\}$$

$$\leq max\{max\{J_C(((t_1 * t_2) * t_3) * x)), J_C(t_3)\}, J_C(t_2)\}$$

$$\leq max\{max\{J_C(0), J_C(t_3)\}, J_C(t_2)\}$$

$$\leq max\{J_C(t_3), J_C(t_2)\}$$

$$J_C(t_1 * x) = max\{J_C(t_3), J_C(t_2)\}.$$

Theorem 3.2. Let $C = (M_C, J_C)$ be a MBJNTID of a B-algebra Y. If $t_1 \leq t_2$ then $M_C(t_1 * x) \geq M_C(t_2)$, $\hat{B}_C(t_1 * x) \geq \hat{B}_C(t_2)$ and $J_C(t_1 * x) \leq J_C(t_2)$ for all $t_1, t_2, t_3 \in Y$.

Proof. Let $t_1, t_2 \in Y$ such that $t_1 \leq t_2$. Then $t_1 * t_2 = 0$, now

$$M_C(t_1 * x) \geq \{min\{M_C((t_1 * t_2) * x), M_C(t_2)\}$$

$$\geq \{min\{M_C(0), M_C(t_2)\}$$

$$M_C(t_1 * x) = M_C(t_2)$$

and

$$\hat{B}_C(t_1 * x) \geq \{rmin\{\hat{B}_C((t_1 * t_2) * x), \hat{B}_C(t_2)\}$$

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$$\geq \{rmin\{\hat{B}_C(0), \hat{B}_C(t_2)\}$$

$$\hat{B}_C(t_1 * x) = \hat{B}_C(t_2)$$

and

$$J_C(t_1 * x) \leq \{max\{J_C((t_1 * t_2) * x), J_C(t_2)\}$$

$$\leq \{max\{J_C(0), J_C(t_2)\}$$

$$J_C(t_1 * x) = J_C(t_2).$$

Hence $M_C(t_1 * x) \geq M_C(t_2)$, $\hat{B}_C(t_1 * x) \geq \hat{B}_C(t_2)$ and $J_C(t_1 * x) \leq J_C(t_2)$ for all $t_1, t_2, t_3 \in Y$.

Theorem 3.3. If C is a MBJNTID, then it fulfills the condition $(M_C((t_1 * (t_2 * t_1)) * x) \geq (M_C(t_2)$, $(\hat{B}_C((t_1 * (t_2 * t_1)) * x) \geq (\hat{B}_C(t_2)$ and $(J_C((t_1 * (t_2 * t_1)) * x) \leq (J_C(t_2)$.

Proof. Let C is a MBJNTID, so

$$M_C((t_1 * (t_2 * t_1)) * x) \geq min\{M_C(((t_1 * (t_2 * t_1)) * t_2) * x), M_C(t_2)\}$$

$$= min\{M_C(0), M_C(t_2)\}$$

$$M_C((t_1 * (t_2 * t_1)) * x) = M_C(t_2)$$

and

$$\hat{B}_C((t_1 * (t_2 * t_1)) * x) \geq rmin\{\hat{B}_C(((t_1 * (t_2 * t_1)) * t_2) * x), \hat{B}_C(t_2)\}$$

$$= rmin\{\hat{B}_C(0), \hat{B}_C(t_2)\}$$

$$\hat{B}_C((t_1 * (t_2 * t_1)) * x) = \hat{B}_C(t_2)$$

and

$$J_C((t_1 * (t_2 * t_1)) * x) \leq max\{J_C(((t_1 * (t_2 * t_1)) * t_2) * x), J_C(t_2)\}$$

$$= max\{J_C(0), J_C(t_2)\}$$

$$J_C((t_1 * (t_2 * t_1)) * x) = J_C(t_2).$$

Hence $(M_C((t_1 * (t_2 * t_1)) * x) \geq (M_C(t_2)$, $(\hat{B}_C((t_1 * (t_2 * t_1)) * x) \geq (\hat{B}_C(t_2)$ and $(J_C((t_1 * (t_2 * t_1)) * x) \leq (J_C(t_2)$.

Theorem 3.4. Let $A = (M_{\mathrm{A}}, \hat{B}_{\mathrm{A}}, J_{\mathrm{A}})$ and $C = (M_C, \hat{B}_{\mathrm{C}}, J_C)$ are two MBJNTIDs of a B-algebra Y. Then the intersection A ∩ C is also MBJNTID of Y.

Proof. Let $t_1, t_2 \in$ A ∩ C, then $t_1, t_2 \in$ A and $t_1, t_2 \in C$.

$$M_{\mathrm{A} \cap C}(0) = M_{\mathrm{A} \cap C}(t_1 * t_1)$$

$$= min\{M_{\mathrm{A}}(t_1 * t_1), M_C(t_1 * t_1)\}$$

$$\geq min\{min\{M_{\mathrm{A}}(t_1), M_{\mathrm{A}}(t_1)\}, min\{M_C(t_1), M_C(t_1)\}\}$$

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$= min\{M_{A}(t_1), M_C(t_1)\}$

$M_{A\cap C}(0) = M_{A\cap C}(t_1)$

and

$\hat{B}_{A\cap C}(0) = \hat{B}_{A\cap C}(t_1 * t_1)$

$= rmin\{\hat{B}_{A}(t_1 * t_1), \hat{B}_C(t_1 * t_1)\}$

$\geq rmin\{rmin\{\hat{B}_{A}(t_1), \hat{B}_{A}(t_1)\}, rmin\{\hat{B}_C(t_1), \hat{B}_C(t_1)\}\}$

$= rmin\{\hat{B}_{A}(t_1), \hat{B}_C(t_1)\}$

$\hat{B}_{A\cap C}(0) = \hat{B}_{A\cap C}(t_1)$

and

$J_{A\cap C}(0) = J_{A\cap C}(t_1 * t_1)$

$= max\{J_C(t_1 * t_1), J_C(t_1 * t_1)\}$

$\leq max\{max\{J_C(t_1), J_C(t_1)\}, max\{J_C(t_1), J_C(t_1)\}\}$

$= max\{J_C(t_1), J_C(t_1)\}$

$J_{A\cap C}(0) = J_{A\cap C}(t_1),$

now

$M_{A\cap C}(t_1 * x) = min\{M_{A}(t_1 * x), M_C(t_1 * x)\}$

$\geq min\{min\{M_{A}((t_1 * t_2) * x), M_{A}(t_2)\}, min\{M_C((t_1 * t_2) * x), M_C(t_2)\}\}$

$= min\{min\{M_{A}((t_1 * t_2) * x), M_C((t_1 * t_2) * x)\}, min\{M_{A}(t_2), M_C(t_2)\}\}$

$= \{min\{M_{A\cap C}((t_1 * t_2) * x), M_{A\cap C}(t_2)\}$

and

$\hat{B}_{A\cap C}(t_1 * x) = rmin\{\hat{B}_{A}(t_1 * x), \hat{B}_C(t_1 * x)\}$

$\geq rmin\{rmin\{\hat{B}_{A}((t_1 * t_2) * x), \hat{B}_{A}(t_2)\}, rmin\{\hat{B}_C((t_1 * t_2) * x), \hat{B}_C(t_2)\}\}$

$= rmin\{rmin\{\hat{B}_{A}((t_1 * t_2) * x), \hat{B}_C((t_1 * t_2) * x)\}, rmin\{\hat{B}_{A}(t_2), \hat{B}_C(t_2)\}\}$

$= \{rmin\{\hat{B}_{A\cap C}((t_1 * t_2) * x), \hat{B}_{A\cap C}(t_2)\}$

and

$J_{A\cap C}(t_1 * x) = max\{J_{A}(t_1 * x), J_C(t_1 * x)\}$

$\leq max\{max\{J_{A}((t_1 * t_2) * x), J_{A}(t_2)\}, max\{J_C((t_1 * t_2) * x), J_C(t_2)\}\}$

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$= max\{max\{J_{A}((t_1 * t_2) * x), J_C((t_1 * t_2) * x)\}, max\{J_{A}(t_2), J_C(t_2)\}\}$

$= max\{J_{A\cap C}((t_1 * t_2) * x), J_{A\cap C}(t_2)\}.$

Hence the intersection $A \cap C$ is MBJNTID of Y.

Theorem 3.5. Let $A = (M_A, \hat{B}_A, J_A)$ and $C = (M_C, \hat{B}_C, J_C)$ are two MBJNTIDs of a B-algebra Y. Then the union $A \cup C$ is also MBJNTID of Y.

Proof. Let $t_1, t_2 \in A \cup C$, then $t_1, t_2 \in A$ and $t_1, t_2 \in C$.

$M_{A\cup C}(0) = M_{A\cup C}(t_1 * t_1)$

$= max\{M_A(t_1 * t_1), M_C(t_1 * t_1)\}$

$\geq max\{max\{M_A(t_1), M_A(t_1)\}, max\{M_C(t_1), M_C(t_1)\}\}$

$= max\{M_A(t_1), M_C(t_1)\}$

$M_{A\cup C}(0) = M_{A\cup C}(t_1)$

and

$\hat{B}_{A\cup C}(0) = \hat{B}_{A\cup C}(t_1 * t_1)$

$= rmax\{\hat{B}_A(t_1 * t_1), \hat{B}_C(t_1 * t_1)\}$

$\geq rmax\{rmax\{\hat{B}_A(t_1), \hat{B}_A(t_1)\}, rmax\{\hat{B}_C(t_1), \hat{B}_C(t_1)\}\}$

$= rmax\{\hat{B}_A(t_1), \hat{B}_C(t_1)\}$

$\hat{B}_{A\cup C}(0) = \hat{B}_{A\cup C}(t_1)$

and

$J_{A\cup C}(0) = J_{A\cup C}(t_1 * t_1)$

$= min\{J_A(t_1 * t_1), J_C(t_1 * t_1)\}$

$\leq min\{min\{J_A(t_1), J_A(t_1)\}, min\{J_C(t_1), J_C(t_1)\}\}$

$= min\{J_A(t_1), J_C(t_1)\}$

$J_{A\cup C}(0) = J_{A\cup C}(t_1)$

now

$M_{A\cup C}(t_1 * x) = max\{M_A(t_1 * x), M_C(t_1 * x)\}$

$\geq max\{max\{M_A((t_1 * t_2) * x), M_A(t_2)\}, max\{M_C((t_1 * t_2) * x), M_C(t_2)\}\}$

$= max\{max\{M_A((t_1 * t_2) * x), M_C((t_1 * t_2) * x)\}, max\{M_A(t_2), M_C(t_2)\}\}$

$= \{max\{M_{A\cup C}((t_1 * t_2) * x), M_{A\cup C}(t_2)\}$

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and

$$\hat{B}_{\mathrm{A}\cup C}(t_1 * x) = rmax\{\hat{B}_{\mathrm{A}}(t_1 * x), \hat{B}_C(t_1 * x)\}$$

$$\geq rmax\{rmax\{\hat{B}_{\mathrm{A}}((t_1 * t_2) * x), \hat{B}_{\mathrm{A}}(t_2)\}, rmax\{\hat{B}_C((t_1 * t_2) * x), \hat{B}_C(t_2)\}\}$$

$$= rmax\{rmax\{\hat{B}_{\mathrm{A}}((t_1 * t_2) * x), \hat{B}_C((t_1 * t_2) * x)\}, rmax\{\hat{B}_{\mathrm{A}}(t_2), \hat{B}_C(t_2)\}\}$$

$$= \{rmax\{\hat{B}_{\mathrm{A}\cup B}((t_1 * t_2) * x), \hat{B}_{\mathrm{A}\cup B}(t_2)\}$$

and

$$J_{\mathrm{A}\cup C}(t_1 * x) = min\{J_{\mathrm{A}}(t_1 * x), J_C(t_1 * x)\}$$

$$\leq min\{min\{J_{\mathrm{A}}((t_1 * t_2) * x), J_{\mathrm{A}}(t_2)\}, min\{J_C((t_1 * t_2) * x), J_C(t_2)\}\}$$

$$= min\{min\{J_{\mathrm{A}}((t_1 * t_2) * x), J_C((t_1 * t_2) * x)\}, min\{J_{\mathrm{A}}(t_2), J_C(t_2)\}\}$$

$$= \{min\{J_{\mathrm{A}\cup C}((t_1 * t_2) * x), J_{\mathrm{A}\cup C}(t_2)\}.$$

Hence the union A $\cup$ C is also MBJNTID of Y.

Theorem 3.6. Let A and C are the MBJ-neutrosophic sets in Y. The cartesian product $A \times C : Y \times Y \to [0,1]$ is defined by $M_A \times M_C(t_1, t_2) = max\{M_A(t_1), M_C(t_2)\}$, $\hat{B}_A \times \hat{B}_C(t_1, t_2) = rmax\{\hat{B}_A(t_1), \hat{B}_C(t_2)\}$ and $J_A \times J_C(t_1, t_2) = min\{J_A(t_1), J_C(t_2)\}$ for all $t_1, t_2 \in Y$. Let A and C are two MBJNTIDs of Y, then $A \times C$ is a MBJNTID of $Y \times Y$.

Proof. For any $t_1, t_2 \in Y \times Y$, we have $M_{\mathrm{A}} \times M_C(0,0) = min\{M_{\mathrm{A}}(0), M_C(0)\} \geq min\{M_{\mathrm{A}}(t_1), M_C(t_2)\} = M_{\mathrm{A}} \times M_C(t_1, t_2)$, $\hat{B}_A \times \hat{B}_C(0,0) = rmin\{\hat{B}_A(0), \hat{B}_C(0)\} \geq rmin\{\hat{B}_A(t_1), \hat{B}_C(t_2)\} = \hat{B}_A \times \hat{B}_C(t_1, t_2)$ and $J_{\mathrm{A}} \times J_C(0,0) = max\{J_{\mathrm{A}}(0), J_C(0)\} \leq max\{J_{\mathrm{A}}(t_1), J_C(t_2)\} = J_{\mathrm{A}} \times J_C(t_1, t_2)$. That is $M_{\mathrm{A}} \times M_C(0,0) \geq M_{\mathrm{A}} \times M_C(t_1, t_2)$, $\hat{B}_A \times \hat{B}_C(0,0) \geq \hat{B}_A \times \hat{B}_C(t_1, t_2)$ and $J_{\mathrm{A}} \times J_C(0,0) \leq J_{\mathrm{A}} \times J_C(t_1, t_2)$.

Now let $x, (t_1, t_2)$ and $(y_1, y_2) \in Y \times Y$. Then, $(M_{\mathrm{A}} \times M_C)(t_1 * x, t_2 * x) = min\{M_{\mathrm{A}}(t_1 * x), M_C(t_2 * x)\} \geq min\{min\{M_{\mathrm{A}}((t_1 * y_1) * x), M_{\mathrm{A}}(y_1)\}, min\{M_C((t_2 * y_2) * x), M_C(y_2)\}\} = min\{min\{M_{\mathrm{A}}((t_1 * y_1) * x), M_C((t_2 * y_2) * x)\}, min\{M_{\mathrm{A}}(y_1), M_C(y_2)\}\} = min\{(M_{\mathrm{A}} \times M_C)((((t_1 * y_1) * x), ((t_2 * y_2) * x)), (M_{\mathrm{A}} \times M_C)(y_1, y_2)\} = min\{(M_{\mathrm{A}} \times M_C)((t_1 * x, t_2 * x) * (y_1, y_2)), (M_{\mathrm{A}} \times M_C)(y_1, y_2)$. That is $(M_{\mathrm{A}} \times M_C)(t_1 * x, t_2 * x) \geq min\{(M_{\mathrm{A}} \times M_C)((t_1 * x, t_2 * x) * (y_1, y_2)), (M_{\mathrm{A}} \times M_C)(y_1, y_2)$ and $(\hat{B}_A \times \hat{B}_C)(t_1 * x, t_2 * x) = rmin\{\hat{B}_A(t_1 * x), \hat{B}_C(t_2 * x)\} \geq rmin\{rmin\{\hat{B}_A((t_1 * y_1) * x), \hat{B}_A(y_1)\}, rmin\{\hat{B}_C((t_2 * y_2) * x), \hat{B}_C(y_2)\}\} = rmin\{rmin\{\hat{B}_A((t_1 * y_1) * x), \hat{B}_C((t_2 * y_2) * x)\}, rmin\{\hat{B}_A(y_1), \hat{B}_C(y_2)\}\} = rmin\{(\hat{B}_A \times \hat{B}_C)((((t_1 * y_1) * x), ((t_2 * y_2) * x)), (\hat{B}_A \times \hat{B}_C)(y_1, y_2)\} = rmin\{(\hat{B}_A \times \hat{B}_C)((t_1 * x, t_2 * x) * (y_1, y_2)), (\hat{B}_A \times \hat{B}_C)(y_1, y_2)$. That is $(\hat{B}_A \times \hat{B}_C)(t_1 * x, t_2 * x) \geq rmin\{(\hat{B}_A \times \hat{B}_C)((t_1 * x, t_2 * x) * (y_1, y_2)), (\hat{B}_A \times \hat{B}_C)(y_1, y_2)$ and $(J_A \times J_C)(t_1 * x, t_2 * x) = max\{J_A(t_1 * x), J_C(t_2 * x)\} \leq max\{max\{J_A((t_1 * y_1) * x), J_A(y_1)\}, max\{J_C((t_2 * y_2) * x), J_C(y_2)\}\} = max\{max\{J_A((t_1 * y_1) * x), J_C((t_2 * y_2) * x)\}, max\{J_A(y_1), J_C(y_2)\}\} = max\{(J_A \times J_C)((((t_1 * y_1) * x), ((t_2 * y_2) * x)), (J_A \times J_C)(y_1, y_2)\} = max\{(J_A \times J_C)((t_1 * x, t_2 * x) * (y_1, y_2)), (J_A \times J_C)(y_1, y_2)$.
That is $(J_A \times J_C)(t_1 * x, t_2 * x) \leq max\{(J_A \times J_C)((t_1 * x, t_2 * x) * (y_1, y_2)), (J_A \times J_C)(y_1, y_2)$.

Theorem3.7. Let A and C are two MBJNs in Y such that A $\times$ C is an MBJNTID of $Y \times Y$, then
1. $M_{\mathrm{A}}(0) \geq M_{\mathrm{A}}(t_1)$, $\hat{B}_{\mathrm{A}}(0) \geq \hat{B}_{\mathrm{A}}(t_1)$, $J_{\mathrm{A}}(0) \leq J_{\mathrm{A}}(t_1)$ and $M_C(0) \geq M_C(t_1)$, $\hat{B}_C(0) \geq \hat{B}_C(t_1)$, $J_C(0) \leq J_C(t_1)$ for all $t_1 \in Y$.

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2. If $M_A(0) \geq M_A(t_1)$ then $M_C(0) \geq M_A(t_1)$ and $M_C(0) \geq M_C(t_1)$ for all $t_1 \in Y$ also if $\hat{B}_A(0) \geq \hat{B}_A(t_1)$ then $\hat{B}_C(0) \geq \hat{B}_A(t_1)$ and $\hat{B}_C(0) \geq \hat{B}_C(t_1)$ for all $t_1 \in Y$ also if $J_A(0) \leq J_A(t_1)$ then $J_C(0) \leq J_A(t_1)$ and $J_C(0) \leq J_C(t_1)$ for all $t_1 \in Y$.

3. If $M_C(0) \geq M_C(t_1)$ then $M_A(0) \geq M_A(t_1)$ and $M_A(0) \geq M_C(t_1)$ for all $t_1 \in Y$ also if $\hat{B}_C(0) \geq \hat{B}_C(t_1)$ then $\hat{B}_A(0) \geq \hat{B}_A(t_1)$ and $\hat{B}_A(0) \geq \hat{B}_C(t_1)$ for all $t_1 \in Y$ also if $J_C(0) \leq J_C(t_1)$ then $J_A(0) \leq J_A(t_1)$ and $J_A(0) \leq J_C(t_1)$ for all $t_1 \in Y$.

Proof. 1. Suppose $M_A(t_1) > M_A(0)$ or $M_C(t_1) > M_C(0)$ for all $t_1 \in Y$ $(M_A \times M_C)(t_1, t_1) = \min\{M_A(t_1), M_C(t_1)\} > \min\{M_A(0), M_C(0)\} = (M_A \times M_C)(0,0)$. Thus $(M_A \times M_C)(t_1, t_1) > (M_A \times M_C)(0,0)$ for all $\in Y$, which is the contradiction to $(M_A \times M_C)$ is a MBJNTID of $Y \times Y$. Therefore $M_A(0) \geq M_A(t_1)$ and $M_C(0) \geq M_C(t_1)$ for all $t_1 \in Y$, also $\hat{B}_A(t_1) > \hat{B}_A(0)$ or $\hat{B}_C(t_1) > \hat{B}_C(0)$ for all $t_1 \in Y$, $(\hat{B}_A \times \hat{B}_C)(t_1, t_1) = rmin\{\hat{B}_A(t_1), \hat{B}_C(t_1)\} > rmin\{\hat{B}_A(0), \hat{B}_C(0)\} = (\hat{B}_A \times \hat{B}_C)(0,0)$. Thus $(\hat{B}_A \times \hat{B}_C)(t_1, t_1) > (\hat{B}_A \times \hat{B}_C)(0,0)$ for all $\in Y$, which is the contradiction to $(\hat{B}_A \times \hat{B}_C)$ is a MBJNTID of $Y \times Y$. Therefore $\hat{B}_A(0) \geq \hat{B}_A(t_1)$ and $\hat{B}_C(0) \geq \hat{B}_C(t_1)$ for all $t_1 \in Y$ and also $J_A(t_1) < J_A(0)$ or $J_C(t_1) < J_C(0)$ for all $t_1 \in Y$. $(J_A \times J_C)(t_1, t_1) = \max\{J_A(t_1), J_C(t_1)\} > max\{J_A(0), J_C(0)\} = (J_A \times J_C)(0,0)$. Thus $(J_A \times J_C)(t_1, t_1) < (J_A \times J_C)(0,0)$ for all $\in Y$, which is the contradiction to $(J_A \times J_C)$ is a MBJNTID of $Y \times Y$. Therefore $J_A(0) \leq J_A(t_1)$ and $J_C(0) \leq J_C(t_1)$ for all $t_1 \in Y$.

2. Suppose $M_C(0) < M_A(t_1)$ or$M_A(0) < M_A(t_1)$ for all $t_1 \in Y$. Then $(M_A \times M_C)(0,0) = min\{M_A(0), M_C(0)\} = M_C(0)$ and $(M_A \times M_C)(t_1, t_1) = min\{M_A(t_1), M_C(t_1)\} > M_C(0) = (M_A \times M_C)(0,0)$. This implies $(M_A \times M_C)(t_1, t_1) = (M_A \times M_C)(0,0)$. Which is the contradiction to $(M_A \times M_C)$ is a MBJNTID of $Y \times Y$. Hence if $M_A(0) \geq M_A(t_1)$ then $M_C(0) \geq M_A(t_1)$ and $M_C(0) \geq M_C(t_1)$ for all $t_1 \in Y$. Now Suppose $\hat{B}_C(0) < \hat{B}_A(t_1)$ or $\hat{B}_A(0) < \hat{B}_A(t_1)$ for all $t_1 \in Y$. Then $(\hat{B}_A \times \hat{B}_C)(0,0) = rmin\{\hat{B}_A(0), \hat{B}_C(0)\} = \hat{B}_C(0)$ and $(\hat{B}_A \times \hat{B}_C)(t_1, t_1) = rmin\{\hat{B}_A(t_1), \hat{B}_C(t_1)\} > \hat{B}_C(0) = (\hat{B}_A \times M_C)(0,0)$. This implies $(\hat{B}_A \times \hat{B}_C)(t_1, t_1) = (\hat{B}_A \times \hat{B}_C)(0,0)$. Which is the contradiction to $(\hat{B}_A \times \hat{B}_C)$ is a MBJNTID of $Y \times Y$. Hence if $\hat{B}_A(0) \geq \hat{B}_A(t_1)$ then $\hat{B}_C(0) \geq \hat{B}_A(t_1)$ and $\hat{B}_C(0) \geq \hat{B}_C(t_1)$ for all $t_1 \in Y$. Now suppose $J_C(0) > J_A(t_1)$ or $J_A(0) > J_A(t_1)$ for all $t_1 \in Y$. Then $(J_A \times J_C)(0,0) = max\{J_A(0), J_C(0)\} = J_C(0)$ and $(J_A \times J_C)(t_1, t_1) = max\{J_A(t_1), J_C(t_1)\} > J_C(0) = (J_A \times J_C)(0,0)$. This implies $(J_A \times J_C)(t_1, t_1) = (J_A \times J_C)(0,0)$. Which is the contradiction to $(J_A \times J_C)$ is a MBJNTID of $Y \times Y$. Hence if $J_A(0) \leq J_A(t_1)$ then $J_C(0) \leq J_A(t_1)$ and $J_C(0) \leq J_C(t_1)$ for all $t_1 \in Y$.

3. The proof is quit same to 2.

Theorm 3.8. Let $\Gamma: Y \to X$ is a B-homomorphism of B-algebra. If $C = (M_C, \hat{B}_C, J_C)$ is a MBJNTID of X then the pre-image $\Gamma^{-1}(C) = (\Gamma^{-1}(M_C), \Gamma^{-1}(\hat{B}_C),\ \Gamma^{-1}(J_C))$ of B under Γ is a MBJNTID in Y.

Proof. For any $t_1 \in Y$,

1. $\Gamma^{-1}(M_C)(t_1) = M_C(\Gamma(t_1)) \geq M_C(0) = M_C(\Gamma(0)) = \Gamma^{-1}(M_C)(0)$, $\Gamma^{-1}(\hat{B}_C)(t_1) = \hat{B}_C(\Gamma(t_1)) \geq \hat{B}_C(0) = \hat{B}_C(\Gamma(0)) = \Gamma^{-1}(\hat{B}_C)(0)$ and $\Gamma^{-1}(J_C)(t_1) = J_C(\Gamma(t_1)) \leq J_C(0) = J_C(\Gamma(0)) = \Gamma^{-1}(J_C)(0)$.

2. $\Gamma^{-1}(M_C)(t_1 * x) = M_C(\Gamma(t_1 * x)) \geq \min\{M_C((\Gamma(t_1) * \Gamma(t_2)) * x), M_C(\Gamma(t_2))\} = \min\{M_C(\Gamma((t_1 * t_2) * x), M_C(\Gamma(t_2))\} = \min\{\Gamma^{-1}(\tau)_B((t_1 * t_2) * x), \Gamma^{-1}(M_C)(t_2)\}$, also $\Gamma^{-1}(\hat{B}_C)(t_1 * x) = \hat{B}_C(\Gamma(t_1 * x)) \geq rmin\{\hat{B}_C((\Gamma(t_1) * \Gamma(t_2)) * x), \hat{B}_C(\Gamma(t_2))\} = rmin\{\hat{B}_C(\Gamma((t_1 * t_2) * x), \hat{B}_C(\Gamma(t_2))\} = rmin\{\Gamma^{-1}(\hat{B}_C)((t_1 * t_2) * x), \Gamma^{-1}(\hat{B}_C)(t_2)\}$ and $\Gamma^{-1}(J_C)(t_1 * x) = J_C(\Gamma(t_1 * x)) \leq \max\{J_C(\Gamma(((t_1) * \Gamma(t_2)) * x), J_C(\Gamma(t_2))\} = max\ \{J_C(\Gamma((t_1 * t_2) * x), J_C(\Gamma(t_2))\} = max\{\Gamma^{-1}(J_C)((t_1 * t_2) * x), \Gamma^{-1}(\sigma)(t_2)\}$.

Theorem 3.9. Let $\Gamma: Y \to X$ be an epimorphism of B-algebra. Then $C = (M_C, \hat{B}_C, J_C)$ is a MBJNTID of X, if $\Gamma^{-1}(C) = (\Gamma^{-1}(M_C), (\Gamma^{-1}(\hat{B}_C), \Gamma^{-1}(J_C))$ of B under Γ is a MBJNTID in Y.

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Proof. For any $t_1 \in X$, there exist $a \in Y$ such that $\Gamma(a) = t_1$. Then $M_C(t_1) = M_C(\Gamma(a)) = \Gamma^{-1}(M_C)(a) \geq \Gamma^{-1}(M_C)(0) = M_C(\Gamma(0)) = M_C(0)$, $\hat{B}_C(t_1) = \hat{B}_C(\Gamma(a)) = \Gamma^{-1}(\hat{B}_C)(a) \geq \Gamma^{-1}(\hat{B}_C)(0) = \hat{B}_C(\Gamma(0)) = \hat{B}_C(0)$ and $J_C(t_1) = J_C(\Gamma(a)) = \Gamma^{-1}(J_C)(a) \leq \Gamma^{-1}(J_C)(0) = J_C(\Gamma(0)) = J_C(0)$. Now let $t_1, t_2 \in X$ then $\Gamma(a) = t_1$ and $\Gamma(b) = t_2$ for some $a, b \in Y$, now $M_C(t_1 * x) = M_C(\Gamma(a * x)) = \Gamma^{-1}(M_C)(a * x) \geq min\{\Gamma^{-1}(M_C((a * b) * x)), \Gamma^{-1}(M_C(b))\} = min\{M_C(\Gamma((a * b) * x)), M_C(\Gamma(b))\} = min\{M_C(\Gamma(a) * \Gamma(b)) * x), M_C(\Gamma(b))\} = min\{M_C((t_1 * t_2) * x), M_C(t_2)\}$ and $\hat{B}_C(t_1 * x) = \hat{B}_C(\Gamma(a * x)) = \Gamma^{-1}(\hat{B}_C)(a * x) \geq rmin\{\Gamma^{-1}(\hat{B}_C((a * b) * x)), \Gamma^{-1}(\hat{B}_C(b))\} = rmin\{\hat{B}_C(\Gamma((a * b) * x)), \hat{B}_C(\Gamma(b))\} = rmin\{\hat{B}_C(\Gamma(a) * \Gamma(b)) * x), \hat{B}_C(\Gamma(b))\} = rmin\{\hat{B}_C((t_1 * t_2) * x), \hat{B}_C(t_2)\}$ and $J_C(t_1 * x) = J_C(\Gamma(a * x)) = \Gamma^{-1}(J_C)(a * x) \leq max\{\Gamma^{-1}(J_C((a * b) * x)), \Gamma^{-1}(J_C(b))\} = max\{J_C(\Gamma((a * b) * x)), J_C(\Gamma(b))\} = max\{J_C((\Gamma(a) * \Gamma(b) * x)), J_C(\Gamma(b))\} = max\{J_C(((t_1 * t_2) * x)), J_C(t_2)\}$. Hence $C = (M_C, \hat{B}_C, J_C)$ is MBJ-neutrosphic T-ideal of Y.

7. Conclusion

In this paper, we studied the MBJ neutrosophic T-ideal, cartesian product of MBJ neutrosophic T-ideals and homomorphism of MBJ neutrosophic T-ideal through significent properties and results. This paper will give us new direction to use MBJ neutrosphic set in different atmosphere. In future, work can be done on MBJ-neutrosophic T-normal ideal,MBJ-neutrosophic cubic T-BMBJ ideal.

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A New Score Function of Pentagonal Neutrosophic Number and its Application in Networking Problem

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Abstract: Pentagonal neutrosophic number is an extended version of single typed neutrosophic number. Real-humankind problems have different sort of ambiguity in nature and amongst them; one of the important problems is solving the networking problem. In this contribution, the conception of pentagonal neutrosophic number has been focused in a distinct framework of reference. Here, we develop of a new score function and its estimation have been formulated in different perspectives. Further, a time computing based networking problem is considered here in pentagonal neutrosophic arena and solved it using an influx of dissimilar logical & innovative thinking. Lastly, computation of total completion time of the problem reflects the impotency of this noble work.

Keywords: Pentagonal neutrosophic number, Networking problem, Score function.

Introduction:

Researchers though have various fields to work on but hesitant theory is one of the vital topics in today's world to deal with. Professor Zadeh [1] was first to familiarize with the fuzzy set theory (in 1965) to handle hesitant idea. The theory of fuzziness has a leading feature to solve clear soundly engineering and statistical problem. Applying the uncertainty theory, plentiful varieties of realistic problem can be solved, networking problem, decision making problem, influence on social science, etc. Pertaining the concept of Zadeh's research paper, Atanassov [2] created phenomenally the intuitionistic fuzzy set where he meticulously elucidates the concept of membership and non-membership function. With the going researches triangular [3], trapezoidal [4], pentagonal [5], hexagonal [6] fuzzy numbers are constructed in indefinite environment. The notion of triangular intuitionistic fuzzy set was put forth by Liu & Yuan [7]. The elementary idea of trapezoidal intuitionistic fuzzy set in research arena was constructed by Ye [8]. Unsurprisingly a basic question ascends onto our mind that how can mathematical model deal with the idea of vagueness? Different sorts of methodologies have been devised by the researchers to describe intricately the conceptions of some new uncertain parameters and to handle these complicated problems, the decision makers put forth their various ideas in disjunctive areas. F. Smarandache [9] in 1998 germinated the notion of having neutrosphic set holding three different fundamental element (i) truth, (ii) indeterminate, and (iii) falsity. Each and every attributes of the neutrosophic sets are very relevant factors to our real life models. Afterwards, Wang et al. [10] progressed with single typed neutrosophic set which serves the solution to any sort of complicated problem in a very efficient way. Later on, Chakraborty et al. [11, 12] abstracted the concept of triangular and trapezoidal neutrosophic numbers and functionalized it in diversified real life problem fruitfully. Also, Maity et al. [13] constructed ranking and defuzzification using totally dissimilar sort of attributes. Bosc and Pivert [14] fostered the concept of bipolarity to deal with human decision making problem on the base of positive and negative sides. Lee [15] continued the expound the theory of bipolar fuzzy set into their research article. Broadening the hypothesis into

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groups and semi-groups structure field was done by Kang and Kang [16]. With ongoing researches Deli et al. [17] build up with the concept of bipolar neutrosophic number and applied it in the field of decision-making associated problem. Broumi et al. [18] put forward the concept of bipolar neutrosophic graph theory and successively Ali and Smarandache [19] proposed the perception of the indeterminate complex neutrosophic set. Chakraborty [20] acquainted us with bipolar number in distinctive aspects. Sequentially, the notion of use of operators in bipolar neutrosophic set was put forward by Wang et al. [21]. He applied in decision related problem. Researchers dealing with the evaluation of any scientific decision, multi-criteria decision making (MCDM) problem is at the utmost concerned. Nowadays utilization of group of criteria is more likely agreeable. Application of MCDM has a wider aspect in disjunctive fields under numerous skepticism frameworks. Researchers show their enthusiasm against problems relating to multi-criteria group decision making (MCGDM) problem. Several applications and progressions on neutrosophic theory could be found under multi-criteria decision making problem, moving with literature surveys showed in [22-25], graph theory [26-30], optimization systems [31-33] etc. On the recent times, Abdel [34] structured the view point of plithogenic set which had a vast significant in uncertain field in research area. Correspondingly, Chakraborty [35, 36] established the outset of cylindrical neutrosophic number and applied it in networking planning problem, MCDM problem and in minimal spanning tree problem. Recently, the concept of pentagonal fuzzy number was first incubated by R. Helen [37]. Later on, utilizing this concept, Christi [38] established pentagonal intuitionistic number and solved transportation problem very proficiently. Chakraborty [39, 40] set forth the idea of pentagonal neutrosophic number and its application in transportation problem and graphical research area. Also, Ye [41] manifested the idea of Single valued neutrosophic minimal spanning tree and clustering method & Mandal & Basu [42] focused on similarity measure based spanning tree problems in neutrosophic arena. Mullai et.al [43] ignited the minimum spanning tree problem & Broumi et.al [44] introduced shortest path problem in neutrosophic graphs. Also, Broumi et.al [45] manifested neutrosophic shortest path to solve Dijkstra algorithm & some published articles [46-47] are addressed here related with neutrosophic domain which plays an essential role in research arena.

This paper deals with the conception of pentagonal neutrosophic number in different aspect. Nowadays researchers are very much interested in doing networking problem in neutrosophic domain. In this article, we consider a networking based PERT problem in pentagonal neutrosophic where we utilize the idea of our developed score function for solving the problem.

1.1 Motivation

The idea of vagueness plays an essential role in construction of mathematical modeling, economic problem and social real life problem etc. Now there will be a vital point that if someone considers pentagonal neutrosophic number in networking domain then what will be the final solution and the critical path? How should we convert a pentagonal neutrosophic number into crisp number? From this aspect we actually try to develop this research article.

1.2 Novelties

Till date lots of research works are already published under neutrosophic environment. Under numerous fields researches have established formulas to work on. Although many fields are unknown and works are still going on. Our job is to give a try on developing new ideas on unfamiliar points.

(i) To develop score and accuracy function.

(ii) Usage of our function in networking problem.

2. Preliminaries

Definition 2.1: Fuzzy Set: **[1]** Set $\widetilde{\mathrm{M}}$ called as a fuzzy set when represented by the pair $\left(\mathrm{x}, \mu_{\widetilde{\mathrm{M}}}(\mathrm{x})\right)$ and thus stated as $\widetilde{\mathrm{M}}=\left\{\left(\mathrm{x}, \mu_{\widetilde{\mathrm{M}}}(\mathrm{x})\right): \mathrm{x} \in \mathrm{X}, \mu_{\widetilde{M}}(\mathrm{X}) \in [0,1]\right\}$ where $x \in$ the crisp set X and $\mu_{\widetilde{\mathrm{M}}}(\mathrm{X}) \in$ the interval$[0,1]$.

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Definition 2.2: Intuitionistic Fuzzy Set (IFS): [2] An fuzzy set [2] $\tilde{S}_F$ in the universal discourse X, symbolized widely by x is referred as Intuitionistic set if $\tilde{S}_F = \{\langle x; [\gamma(x), \delta(x)]\rangle : x \in X\}$, where $\gamma(x): X \to [0,1]$ is termed as the certainty membership function which specify the degree of confidence, $\delta(x): X \to [0,1]$ is termed as the uncertainty membership function which specify the degree of indistinctness.

$\gamma(x), \delta(x)$ exhibits the following the relation

$$0 \leq \gamma(x) + \delta(x) \leq 1.$$

2.3Defination: Neutrosophic Set: [9] A set $\widetilde{Ne}_M$ in the universal discourse X, figuratively represented by x named as a neutrosophic set if $\widetilde{Ne}_M = \{\langle x; [\lambda_{\widetilde{Ne}_M}(x), \pi_{\widetilde{Ne}_M}(x), \sigma_{\widetilde{Ne}_M}(x)]\rangle : x \in X\}$, where $\lambda_{\widetilde{Ne}_M}(x): X \to [0,1]$ is stated as the certainty membership function, which designates the degree of confidence, $\pi_{\widetilde{Ne}_M}(x): X \to [0,1]$ is stated as the uncertainty membership, which designates the degree of indistinctness, and $\sigma_{\widetilde{Ne}_M}(x): X \to [0,1]$ is stated as the untruthful membership, which designates the degree of deceptiveness on the decision taken by the decision maker.

$\lambda_{\widetilde{Ne}_M}(x), \pi_{\widetilde{Ne}_M}(x) \& \sigma_{\widetilde{Ne}_M}(x)$ displays the following relation:

$$-0 \leq \lambda_{\widetilde{Ne}_M}(x) + \pi_{\widetilde{Ne}_M}(x) + \sigma_{\widetilde{Ne}_M}(x) \leq 3+.$$

2.4 Definition: Single-Valued Neutrosophic Set: [10] A Neutrosophic set $\widetilde{Ne}_M$ in the definition 2.3 is assumed as a Single-Valued Neutrosophic Set $(\widetilde{SNe}_M)$ if x is a single-valued independent variable. $\widetilde{SNeA} = \{\langle x; [\lambda_{\widetilde{SNe}_M}(x), \pi_{\widetilde{SNe}_M}(x), \sigma_{\widetilde{SNe}_M}(x)]\rangle : x \in X\}$, where $\lambda_{\widetilde{SNe}_M}(x), \pi_{\widetilde{SNe}_M}(x) \& \sigma_{\widetilde{SNe}_M}(x)$ signified the notion of correct, indefinite and incorrect memberships function respectively.

If three points $d_0, e_0 \& f_0$ exists for which $\lambda_{\widetilde{SNe}_M}(d_0) = 1, \pi_{\widetilde{SNe}_M}(e_0) = 1 \& \sigma_{\widetilde{SNe}_M}(f_0) = 1$, then the $\widetilde{SNe}_M$is termed neut-normal.

$\widetilde{SCS}_M$ is called neut-convex indicating that $\widetilde{SCS}_M$ is a subset of a real line by meeting the resulting conditions:

i. $\lambda_{\widetilde{SNe}_M}\langle \delta d_1 + (1-\delta)d_2\rangle \geq min\langle \lambda_{\widetilde{SNe}_M}(d_1), \lambda_{\widetilde{SNe}_M}(d_2)\rangle$
ii. $\pi_{\widetilde{SNe}_M}\langle \delta d_1 + (1-\delta)d_2\rangle \leq max\langle \pi_{\widetilde{SNe}_M}(d_1), \pi_{\widetilde{SNe}_M}(d_2)\rangle$
iii. $\sigma_{\widetilde{SNe}_M}\langle \delta d_1 + (1-\delta)d_2\rangle \leq max\langle \sigma_{\widetilde{SNe}_M}(d_1), \sigma_{\widetilde{SNe}_M}(d_2)\rangle$

where $d_1 \& d_2 \in \mathbb{R}$ and $\delta \in [0,1]$

2.5 Definition: Single-Valued Pentagonal Neutrosophic Number: A Single-Valued Pentagonal Neutrosophic Number $(\widetilde{M})$ is demarcated as $\widetilde{Ss} = \langle [(s^1, t^1, u^1, v^1, w^1); \mu], [(s^2, t^2, u^2, v^2, w^2); \theta], [(s^3, t^3, u^3, v^3, w^3); \eta] \rangle$, where $\mu, \theta, \eta \in [0,1]$. The correct membership function$(\mu_{\widetilde{Ss}}): \mathbb{R} \to [0, \mu]$, the indefinite membership function $(\theta_{\widetilde{Ss}}): \mathbb{R} \to [\theta, 1]$ and the incorrect membership function $(\eta_{\widetilde{Ss}}): \mathbb{R} \to [\eta, 1]$ are given as:

$$\mu_{\widetilde{Ss}}(x) = \begin{cases} \mu_{\widetilde{Ssl1}}(x)\; s^1 \leq x < t^1 \\ \mu_{\widetilde{Ssl2}}(x)\; t^1 \leq x < u^1 \\ \mu \qquad x = u^1 \\ \mu_{\widetilde{Ssr2}}(x) u^1 \leq x < v^1 \\ \mu_{\widetilde{Ssr1}}(x) v^1 \leq x < w^1 \\ 0 \quad otherwise \end{cases}, \quad \theta_{\widetilde{Ss}}(x) = \begin{cases} \theta_{\widetilde{Ssl1}}(x)\; s^2 \leq x < t^2 \\ \theta_{\widetilde{Ssl2}}(x)\; t^2 \leq x < u^2 \\ \vartheta \qquad x = u^2 \\ \theta_{\widetilde{Ssr2}}(x) u^2 \leq x < v^2 \\ \theta_{\widetilde{Ssr1}}(x) v^2 \leq x < w^2 \\ 1 \quad otherwise \end{cases}$$

$$\eta_{\widetilde{Ss}}(x) = \begin{cases} \eta_{\widetilde{Ssl1}}(x)\ s^3 \leq x < t^3 \\ \eta_{\widetilde{Ssl2}}(x)\ t^3 \leq x < u^3 \\ \vartheta \qquad x = u^3 \\ \eta_{\widetilde{Ssr2}}(x) u^3 \leq x < v^3 \\ \eta_{\widetilde{Ssr1}}(x) v^3 \leq x < w^3 \\ 1 \quad otherwise \end{cases}$$

3. Proposed Score Function:

Score function utterly relies upon the value of exact membership indicator degree, inexact membership indicator degree and hesitancy membership indicator degree for a pentagonal neutrosophic number. The fundamental use of score function is to drag the judgment of conversion of pentagonal neutrosophic number to real number. A score function is developed for any Pentagonal Single typed Neutrosophic Number (PSNN).

$$\tilde{P}_{Pen} = (P_1, P_2, P_3, P_4, P_5; \alpha, \beta, \gamma)$$

Score function is described as $\tilde{S}_{Pen} = \frac{1}{15}\{(P_1 + P_2 + P_3 + P_4 + P_5) \times (2 + \alpha - \beta - \gamma)\}$

Here, $\tilde{S}_{Pen} \in [0,1]$

3.1 Relationship between any two pentagonal neutrosophic fuzzy numbers:

Let us consider any two pentagonal neutrosophic fuzzy number defined as follows

$P_{Pen1} = (\alpha_{Pen1}, \beta_{Pen1}, \gamma_{Pen1})$,$P_{Pen2} = (\alpha_{Pen2}, \beta_{Pen2}, \gamma_{Pen2})$

1) $S_{Pen1} > S_{Pen2}$,$P_{Pen1} > P_{Pen2}$
2) $S_{Pen1} < S_{Pen2}$,$P_{Pen1} < P_{Pen2}$
3) $S_{Pen1} = S_{Pen2}$,$P_{Pen1} = P_{Pen2}$

Table 3.1: Numerical Examples

Pentagonal Neutrosophic Number (P_{Pen})	Score Value (S_{Pen})	Ordering
$P_{Pen1} =< (0.3,0.4,0.5,0.6,0.7; 0.4,0.7,0.6) >$	0.1833	$P_{Pen2} > P_{Pen3} > P_{Pen4} > P_{Pen1}$
$P_{Pen2} =< (0.3,0.35,0.45,0.55,0.7; 0.6,0.5,0.4) >$	0.2663	
$P_{Pen3} =< (0.25,0.3,0.4,0.5,0.7; 0.6,0.5,0.5) >$	0.2293	
$P_{Pen4} =< (0.4,0.45,0.5,0.6,0.7; 0.3,0.5,0.6) >$	0.2120	

4. PERT in Pentagonal Neutrosophic Environment and the Proposed Model

PERT system or Project Evaluation and Review Technique are a project managing scheme which is used to plan, arrange, systemize and equalize tasks amongst a project. These techniques basically examine the minimum time required in finishing the total task and also calculate the time required in completion of each task for the given project.

PERT arrangement entails the specified steps:

1. Identification of specified activities and milestones.

2. In determination of accurate sequence of the activities.

3. In construction of a network map.

4. Evaluation of time needed for ac task.

5. Determination of the critical path.

6. Updating the PERT chart on progression with the project.

The chief purpose of PERT chart is to simplify and to decrease both time and cost of completion of any decision forming project. This method is proposed for wide-ranging, one-time, non-routine difficult projects having high degree of dependence. In projects, where series of tasks are present, some are always executed successively while rest are accomplished matching with other activities. In case of new projects having huge uncertainty in technology and networking system PERT is essentially used. For handling the uncertainties, triangular neutrosophic setting For PERT activity duration has been introduced.

The three time estimations for activity duration are:

Optimistic Time($\breve{o_t}$): In general, the optimistic time requires minimum time for completing the activities and it is considered with three standards deviations from mean and approximately there is 1% chance for the activity to complete within time.

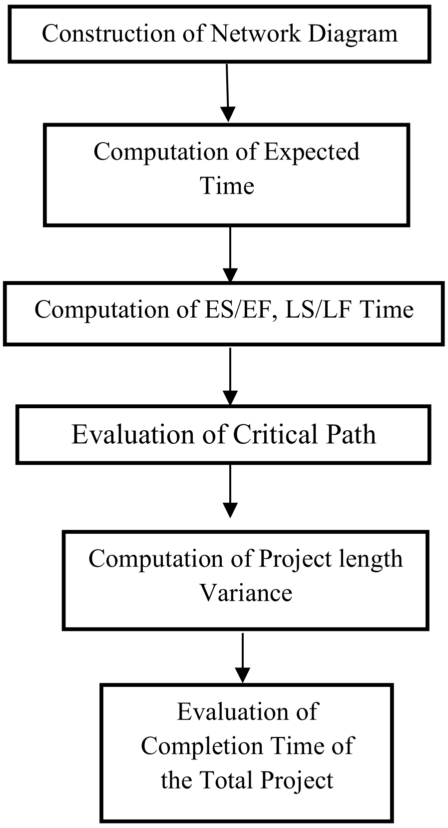
Pessimistic time($\breve{p}_t$): . It is known for tasks taking the longest time. Here also the three standards deviations are used.

Most Likely time($\widetilde{m_t}$): – The completion time in general status for most likely have the highest probability and is absolutely different from the projected time

We choose all the different three activities duration for the model put forward as in triangular neutrosophic number. Score value $R(\widetilde{S}_{Pen}, 0) = \frac{1}{15}(P_1 + P_2 + P_3 + P_4 + P_5) \times (2 + \alpha - \beta - \gamma)$ is presented to attain the pentagonal neutrosophic number$(P_1, P_2, P_3, P_4, P_5; \alpha, \beta, \gamma)$. By the use of formulas the expected time $E_t = \frac{(o_t + 4m_t + p_t)}{6}$ and the standard deviation $\sigma_t = \frac{(p_t - o_t)}{6}$ is calculated where o_t, p_t, m_t denotes the optimistic time, pessimistic time and most likely time respectively for all crisp value. For the add-on calculations of latest time, critical path and float CPM method is used. Considering the forward pass with zero starting time, the first event progresses from left to right and reaches to the final event. Let us assume j, k for any activity, the earliest time event of j is ES_j therefore $S_k = ES_j + t_{jk}$. There might be a case where in an event more than one activity enters then the earliest time is calculated as $ES_k = \max\{ES_j + t_{kj}\}$ for all activities radiating from node j to k. Backward pass starts with the final node and calculation progresses from right to left till the initial event. Let us assume j, k for any activity, the latest finished time event of j is LF_j therefore, $F_j = LF_k - t_{jk}$. There might be a case where in an event more than one activity enters then the latest finish time is calculated as $LF_j = \min\{LF_k - t_{kj}\}$ for all activities radiating from node k to i. Once the critical path is calculated, computation of project length variance is done which is sum of the variances of all critical activities. After that standard normal variable $Z = \frac{T_{sd} - T_{ex}}{\sigma}$ is computed where

T_{sd} is the schedules time given for a project to complete and , T_{ex} is the expected project length duration. By the use of normal curve, the probability of project completion within the definite time can be approximated.

Flowchart-



4.1 Illustrative Example:

Activity	Description	Predecessors	Optimistic Time	Pessimistic Time	Most Likely Time
A	Selection of Manager and Other Marketing members	———	$< 0.5,1.8,2.5,3,4.4; 0.5,0.5,0.8 >$	$< 1.4,2,2.6,3,4.5; 0.4,0.5,0.7 >$	$< 2.2,2.8,3.8,4.6,5.5; 0.7,0.5,0.6 >$
B	Choice of Market Areas	———	$< 0.7,1.6,2.8,3.5,4.8; 0.6,0.6,0.3 >$	$< 2,3,4,5,6; 0.6,0.7,0.6 >$	$< 1.5,2,2.5,3,4; 0.6,0.5,0.7 >$
C	Selection of Marketing Products	A	$< 1,2,3,4,5; 0.7,0.6,0.4 >$	$< 2,2.5,3,3.5,4.5; 0.8,0.6,0.8 >$	$< 1.8,2.6,3.6,4.2,5.5; 0.6,0.5,0.4 >$
D	Ultimate Planning and Master-plan	B	$< 2.5,3,4,4.5,6; 0.6,0.5,0.7 >$	$< 0.8,1.8,2.5,3.6,5; 0.4,0.6,0.7 >$	$< 1,1.8,2.6,3.6,5; 0.6,0.7,0.5 >$
E	Training Schedule	B	$< 1.5,2.5,3,4,5.5; 0.7,0.6,0.4 >$	$< 2,3,4,5,6; 0.8,0.6,0.7 >$	$< 1.4,2.2,2.8,3.5,4.5; 0.6,0.7,0.5 >$
F	Delay Times	C,D	$< 2,2.5,3.5,4,5.5; 0.7,0.6,0.4 >$	$< 1.6,2.4.3.2.4.5.5.6; 0.8,0.7,0.5 >$	$< 2.2,2.8,3.6,4.5,5.5; 0.7,0.4,0.6 >$

G	Estimation of the total time	E	$< 1.6,2,2.5,3,4.5; 0.6,0.7,0.5 >$	$< 2,2.5,3.5,4,5.5; 0.8,0.4,0.6 >$	$< 2.6,3.5,4,5,6; 0.7,0.5,0.5 >$

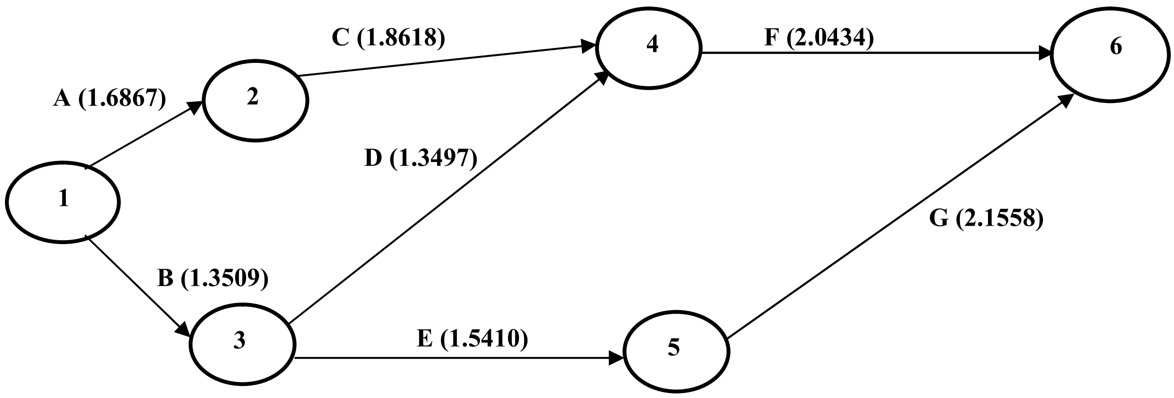
Draw the project network and find the probability that the project is completed in 5.6 days?

Step-1

Optimistic Time(o_t)	Pessimistic Time(p_t)	Most Likely Time(m_t)	$E_{jk} = \frac{o_t + 4m_t + p_t}{6}$	${\sigma_{jk}}^2 = \left(\frac{p_t - o_t}{6}\right)^2$
0.9760	1.0800	2.0160	1.6867	0.0003
1.5187	1.7333	1.2133	1.3509	0.0013
1.7000	1.4467	2.0060	1.8618	0.0018
1.8667	1.0047	1.3067	1.3497	0.0206
1.8700	2.0000	1.3440	1.5410	0.0005
1.9833	1.8453	2.1080	2.0434	0.0005
1.2693	2.1000	2.3913	2.1558	0.0192

Step-2

Network Diagram



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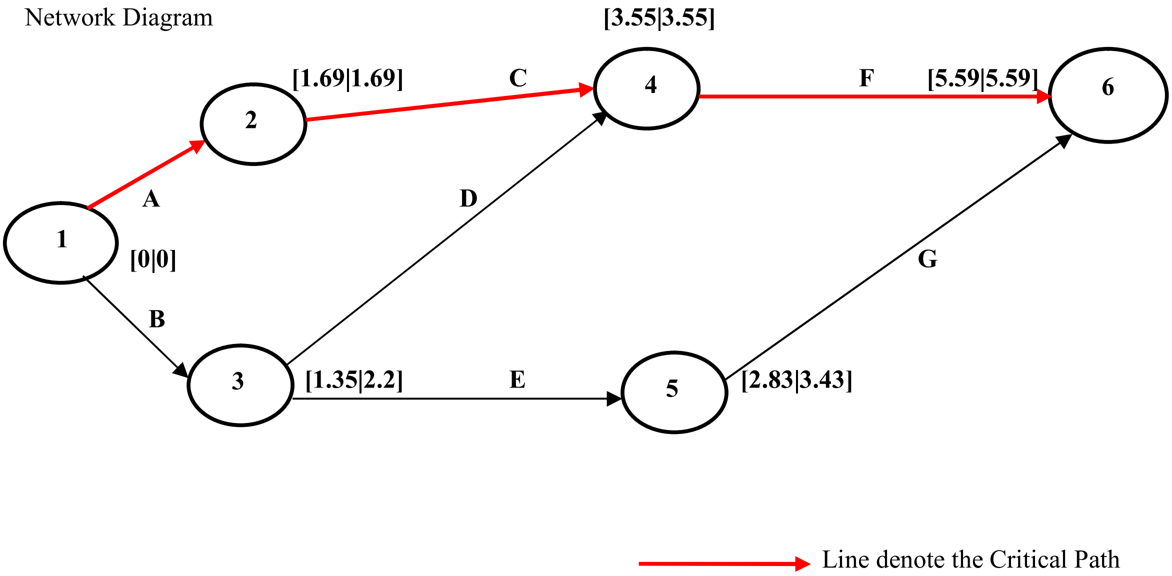
Step-3

Network Diagram



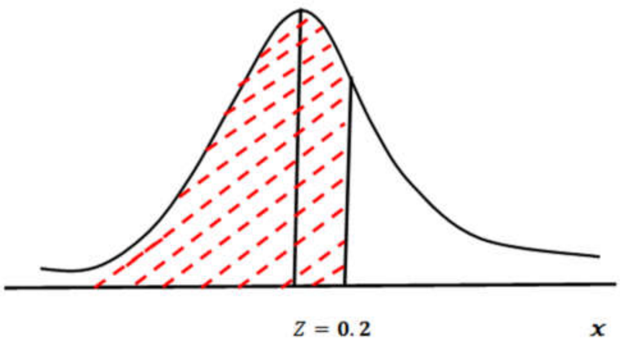
So, expected project duration-5.59 days

Critical path- 1→2→4→6

Project length variance $\sigma^2 = 0.0026$, Standard deviation-0.051

Probability that the project will be finished within 5.6 days is $P\left(z \leq \frac{5.6-5.59}{0.051}\right) = P(z \leq 0.2)$

$Area\ under\ the\ normal\ curve\ P(z \leq 0.2) = 0.5 + \emptyset(0.2)$ = 0.5793



Normal Distribution Curve

5. Conclusion and future research scope

The idea of pentagonal neutrosophic number is intriguing, competent and has an ample scope of utilization in various research domains. In this research article, we vigorously erect the perception of pentagonal neutrosophic number from different aspects. We introduced a score function here in pentagonal neutrosophic domain. Additionally, we consider a networking problem in neutrosophic environment and solve the problem utilizing the

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idea of score function. Since, there is no such articles is till now established in pentagonal networking neutrosophic arena, thus we cannot compare our work with other methods.

Further, researchers can immensely apply this idea of neutrosophic number in numerous flourishing research fields like engineering problem, mobile computing problems, diagnoses problem, realistic mathematical modeling, social media problem etc.

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A Single Valued neutrosophic Inventory Model with Neutrosophic Random Variable

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Abstract

This paper presents the problematic period of neutrosophic inventory in an inaccurate and unsafe mixed environment. The purpose of this paper is to present demand as a neutrosophic random variable. For this model, a new method is developed for determining the optimal sequence size in the presence of neutrosophic random variables. Where to get optimality by gradually expressing the average value of integration. The newsvendor problem is used to describe the proposed model.

Keywords: Neutrosophic set, Neutrosophic random variable, Triangle neutrosophic numbers, single period neutrosophic inventory

1.Introduction

A single-period inventory (Buffa E.S. and Sarin R.K. 1987) is one of those elementary models in which only a single procurement is being made. There is a wide application of this model on production management system, like stocking seasonal items (Christmas trees, woolen materials), perishable goods, spare parts, etc. To develop the methodology of this model, we consider the well-known newsboy problem, in which the decision-maker wants to know the optimal number of newspapers to be purchased daily to maximize his expected profit. In a real situation, the daily demand of the newspapers may vary day to day. Either due to lack of historical data or abundance of information it is worthwhile to consider a distribution for demand. Recently some researchers considered the demand as a fuzzy number only (Kao C. and Hsu W.K. 2002). In (Ishii H. and Konno T. 1998) the newsboy problem has been redefined considering shortage cost as fuzzy number and demand as random variable. However, no attempt is made to define the demand in mixed environment, where fuzziness and randomness both appears simultaneously. Thus, we consider the demand as a fuzzy random variable involving imprecise probabilities since the probability of a fuzzy event is a fuzzy number (Chakraborthy D. 2002).

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The concept of fuzzy random variable and its fuzzy expectation has been presented by (Kwakernaak H. 1978) and later by Puri and Ralescue (Puri M.L. andRalescu D.A. 1986). Further, recently the notation of a fuzzy random variable has also been considered in (Feng Y., Hu L. and Shu H. 2001). In (Smarandache F. 1998) proposed concept of neutrosophic set which is generalization of classical set, fuzzy set, intuitionistic fuzzy set and so on. In the neutrosophic set, for an element x of the universe, the membership functions independently indicates the truth-membership degree, indeterminacy-membership degree, and false-membership degree of the element x belonging to the neutrosophic set. Also, fuzzy, intuitionistic and neutrosophic models have been studied by (Wang H, Smarandache F, Y, Zhang Q 2010). In a multiple-attribute decision-making problem the decision makers need to rank the given alternatives and the ranking of alternatives with neutrosophic numbers is many is many difficult because neutrosophic numbers are not ranked by ordinary methods as real numbers. However it is possible with score functions , aggregation operation , distance measure and so on. In section 2 of this paper, the neutrosophic random variable and its neutrosophic expectation are defined, a brief overview of the integration of graded mean representation of triangular neutrosophic number discussed later. Next, in section 3 a single-valued neutrosophic inventory problem of neutrosophic random variable demand is formulated and methodology is developed. Section 4 handles the numerical example of the proposed model.

2. Preliminaries

In this section, the basic definitions involving neutrosophic set, single valued neutrosophic sets and triangularneutrosophic number are outlined.

Definition 1. (Smarandache F. 1998) Let E be a universe. A neutrosophic set A over E is defined by, $A = \{\langle x, (T_A(x), I_A(x), F_A(x))\rangle : x \in E\}$ where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are called truth-membership function, indeterminacy-membership function and falsity-membership function respectively. They are respectively defined by, $T_A: E \square]^-0,1^+[$, $I_A: E \longrightarrow]^-0,1^+[$, $F_A : E \longrightarrow]^-0,1^+[$ such that $0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Definition 2. (Wang H, Smarandache F. Zhang Y.Q 2010) Let E be a universe. A single valued neutrosophic set (SVN-set) over E, but the truth-membership function, indeterminacy-membership function and falsity-membership function are respectively defined by $T_A: E \square]^-0,1^+[$, $I_A: E \longrightarrow]^-0,1^+[$, $F_A : E \longrightarrow]^-0,1^+[$ such that $0 \leq T_A(x), I_A(x), F_A(x) \leq 3$.

Definition 3. (Subas Y. 2015) Let $w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \in [0, 1]$ be any real numbers, $a_i, b_i, c_i \in R$ and $a_i \leq b_i \leq c_i (i = 1,2,3)$.Then single valued neutrosophic number (SVN-number) $\tilde{a} = \langle ((a_1, b_1, c_1), w_{\tilde{a}}), ((a_2, b_2, c_2), u_{\tilde{a}}), ((a_3, b_3, c_3), y_{\tilde{a}})\rangle$, is a special neutrosophic set on the set of real number R, whose truth-membership function $\mu_{\tilde{a}}$, indeterminacy-membership function $v_{\tilde{a}}$ and falsity-membership function $\lambda_{\tilde{a}}$ are respectively defined by

$\mu_{\tilde{a}}: R \to [0, w_{\tilde{a}}]$, $v_{\tilde{a}}: R \to [u_{\tilde{a}}, 1]$, $\lambda_{\tilde{a}}: R \to [y_{\tilde{a}}, 1]$.

Definition 4. (Subas Y. 2015)A single valued triangular neutrosopic number $\langle (a, b, c); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$, is a special neutrosopic set on the real number set R , whose truth-membership, indeterminacy -membership and falsity-membership are given as follows;

$$\mu_{\tilde{a}}(x) = \begin{cases} (x-a)w_{\tilde{a}}/(b-a), & (a \leq x < b) \\ (c-x)w_{\tilde{a}}/(c-b), & (b \leq x \leq c) \\ 0, & otherwise \end{cases}$$

$$v_{\tilde{a}}(x) = \begin{cases} (b-x+u_{\tilde{a}}(x-a))/(b-a), & (a \leq x < b) \\ (x-b+u_{\tilde{a}}(c-x))/(c-b), & (b \leq x \leq c) \\ 0, & otherwise \end{cases}$$

$$\lambda_{\tilde{a}}(x) = \begin{cases} \frac{(b-x+y_{\tilde{a}}(x-a))}{(b-a)}, & (a \le x < b) \\ \frac{(x-b+y_{\tilde{a}}(c-x))}{(c-b)}, & (b \le x \le c) \\ 0, & otherwise \end{cases}$$

Definition 5. (Deli I, Subas y) Let $\tilde{a} = \langle((a_1, b_1, c_1), w_{\tilde{a}}), ((a_2, b_2, c_2), u_{\tilde{a}}), ((a_3, b_3, c_3), y_{\tilde{a}})\rangle$ be a $SVN - Number$. Then $\langle\alpha, \beta, \gamma\rangle$ – cut set of the $SVN - Number\tilde{a}$, denoted by $\tilde{a}_{\langle\alpha,\beta,\gamma\rangle}$ is defined as ,$\tilde{a}_{\langle\alpha,\beta,\gamma\rangle} = \{x/\mu_{\tilde{a}}(x) \ge \alpha,\ v_{\tilde{a}}(x) \le \beta, \lambda_{\tilde{a}}(x) \le \gamma, x \in R\}$, which satisfies the conditions as follows: $0 \le \alpha \le w_{\tilde{a}}$, $u_{\tilde{a}} \le \beta \le 1$, $y_{\tilde{a}} \le \gamma \le 1$ and $0 \le \alpha + \gamma + \beta \le 3$.

Clearly, any $\langle\alpha, \beta, \gamma\rangle$ –cut set $\tilde{a}_{\langle\alpha,\beta,\gamma\rangle}$ of a $SVN - Number\tilde{a}$ is a crisp subset of a real number set R.

Definition 6.(Deli I, Subas y) Let $\tilde{a} = \langle((a_1, b_1, c_1), w_{\tilde{a}}), ((a_2, b_2, c_2), u_{\tilde{a}}), ((a_3, b_3, c_3), y_{\tilde{a}})\rangle$ be a $SVN - Number$. Then α– cut set of the $SVN - Number\tilde{a}$, denoted by $\tilde{a}_\alpha$ is defined as ,$\tilde{a}_\alpha = \{x/\mu_{\tilde{a}}(x) \ge \alpha, x \in R\}$,where $\alpha \in [0, w_{\tilde{a}}]$.

Clearly, any α –cut set of a $SVN - Number\tilde{a}$ is a crisp subset of a real number set R. Also any α –cut set of a SVN-Number $\tilde{a}$for truth-membership function is a closed interval, denoted by $\tilde{a}_\alpha = [L_{\tilde{a}}(\alpha), R_{\tilde{a}}(\alpha)]$.

Definition 7.(Deli I, Subas y) Let $\tilde{a} = \langle((a_1, b_1, c_1), w_{\tilde{a}}), ((a_2, b_2, c_2), u_{\tilde{a}}), ((a_3, b_3, c_3), y_{\tilde{a}})\rangle$ be a $SVN - Number$. Then β– cut set of the $SVN - Number\tilde{a}$, denoted by $\tilde{a}^\beta$ is defined as ,$\tilde{a}^\beta = \{x/v_{\tilde{a}}(x) \le \beta, x \in R\}$, where $\beta \in [u_{\tilde{a}}, 1]$.

Clearly, any β –cut set of a $SVN - Number\tilde{a}$ is a crisp subset of a real number set R.. Also any β –cut set of a SVN-Number $\tilde{a}$for indeterminacy-membership function is a closed interval, denoted by $\tilde{a}^\beta = [L'_{\tilde{a}}(\beta), R'_{\tilde{a}}(\beta)]$.

Definition 8.(Deli I, Subas y) Let $\tilde{a} = \langle((a_1, b_1, c_1), w_{\tilde{a}}), ((a_2, b_2, c_2), u_{\tilde{a}}), ((a_3, b_3, c_3), y_{\tilde{a}})\rangle$ be a $SVN - Number$. Then γ– cut set of the $SVN - Number\tilde{a}$, denoted by $\gamma_{\tilde{a}}$ is defined as ,$\gamma_{\tilde{a}} = \{x/\lambda_{\tilde{a}}(x) \le \gamma, x \in R\}$, where $\gamma \in [y_{\tilde{a}}, 1]$.

Clearly, any γ –cut set of a $SVN - Number\tilde{a}$ is a crisp subset of a real number set R. Also any γ –cut set of a SVN-Number $\tilde{a}$for indeterminacy-membership function is a closed interval, denoted by $\gamma_{\tilde{a}} = [L''_{\tilde{a}}(\gamma), R''_{\tilde{a}}(\gamma)]$.

3. Cuts and neutrosophicgraded mean integration:

α, β and γ-cuts, expectation of neutrosophic random variable and neutrosophic graded mean are introduced in this section.

Definition 1. Let F^N be the set of all neutrosophic numbers.The α-cut,β-cut and γ-cut of $w_{a^N}, u_{a^N}, and y_{a^N}$ in F^N is closed interval of any $\alpha, \beta, \gamma \in F^N$ .The addition and scalar multiplication on F^N are defined by the following;

$[a, b]_\alpha^N = a_\alpha^N + b_\alpha^N, [\lambda a]_\alpha^N = \lambda a_\alpha^N, \lambda \in R, \quad \alpha \in [0,1];$

$[a, b]_\beta^N = a_\beta^N + b_\beta^N, [\lambda a]_\beta^N = \lambda a_\beta^N, \lambda \in R, \quad \beta \in [0,1];$

$[a, b]_\gamma^N = a_\gamma^N + b_\gamma^N, [\lambda a]_\gamma^N = \lambda a_\gamma^N, \lambda \in R, \ \gamma \in [0,1]; a^N, b^N \in F^N$and

$[a, b]_{\alpha+\beta+\gamma}^N = a_{\alpha+\beta+\gamma}^N + b_{\alpha+\beta+\gamma}^N, [\lambda a]_{\alpha+\beta+\gamma}^N = \lambda a_{\alpha+\beta+\gamma}^N, \ \lambda \in R, \alpha + \beta + \gamma \in [0,3].$

A metric on F^N is defined by,

$$d^N(a^N, b^N) = \frac{1}{2}\int_0^1 ((|a_\alpha^- - b_\alpha^-|)^2 + (|a_\alpha^+ + b_\alpha^+|)^2)\, d^N\alpha, \qquad \forall a^N, b^N \in F^N$$

$$d^N(a^N, b^N) = \frac{1}{2}\int_0^1 \left((|a_\beta^- - b_\beta^-|)^2 + (|a_\beta^+ + b_\beta^+|)^2\right) d^N\beta, \qquad \forall a^N, b^N \in F^N$$

$$d^N(a^N, b^N) = \frac{1}{2}\int_0^1 \left((|a_\gamma^- - b_\gamma^-|)^2 + (|a_\gamma^+ + b_\gamma^+|)^2\right) d^N\gamma, \ \forall a^N, b^N \in F^N \text{ and}$$

$$d^N(a^N, b^N) = \frac{1}{2}\int_0^1 \left((|a_{\alpha+\beta+\gamma}^- - b_{\alpha+\beta+\gamma}^-|)^2 + (|a_{\alpha+\beta+\gamma}^+ + b_{\alpha+\beta+\gamma}^+|)^2\right) d^N(\alpha+\beta+\gamma),$$
$$\forall a^N, b^N \in F^N, (\alpha+\beta+\gamma) \in [0,3].$$

Where a_α^- ,a_α^+ are lower and upper end point of a_α and (F^N, d^N) is a complete neutrosophic metric space.

Let (Ω^N, A^N, P^N) be a complete neutrosophic probability space. A neutrosophic random variable (n.r.v) is a borel measurable function $X^N: (\Omega^N, A^N, P^N) \to (F^N, d^N)$.

If $X^N$is a n.r.v, then $[X]_\alpha^N = [X_\alpha^-, X_\alpha^+]^N, [X]_\beta^N = [X_\beta^-, X_\beta^+]^N, [X]_\gamma^N = [X_\gamma^-, X_\gamma^+]^N$, and $[X]_{\alpha+\beta+\gamma}^N = [X_{\alpha+\beta+\gamma}^-, X_{\alpha+\beta+\gamma}^+]^N, \alpha+\beta+\gamma \in [0,3]$ is neutrosophic random closed interval set and $X_{\alpha+\beta+\gamma}^-$, $X_{\alpha+\beta+\gamma}^+$ are real valued neutrosophic random variables.The expectation of a n.r.v X^N is defined by

$$a_{\alpha+\beta+\gamma} = [EX]_{\alpha+\beta+\gamma}^N = E^N[X_{\alpha+\beta+\gamma}] = [E^N(X_{\alpha+\beta+\gamma}^-), E^N(X_{\alpha+\beta+\gamma}^+)]$$

for $\alpha+\beta+\gamma \in [0,3]$, $\alpha, \beta, \gamma \in [0,1]$.

Definition 2. For a neutrosophic random variable

$X^N = \{(X_{\alpha+\beta+\gamma}^-, X_{\alpha+\beta+\gamma}^+)^N / 0 \le \alpha+\beta+\gamma \le 3\}$the expectation of $X^N$is defined by

$$E^N X^N = \int X^N dP^N = \left\{\int X_{\alpha+\beta+\gamma}^- dP^N, \int X_{\alpha+\beta+\gamma}^+ dP^N / 0 \le \alpha+\beta+\gamma \le 3\right\}.$$

If $X^N$is a discrete neutrosophic random variable, such that $P^N(X^N = x_i^N) = p_i^N, i = 1,2,3, \ldots$, then its neutrosophic expected is given as,

$EX^N = \sum_{i=1}^{\infty} x_i^N p_i^N$, for$EX^N \in F^Nand[EX^N]_0 = \int X_0 dP^N = [EX_0^-, EX_0^+]^N$, for

$\alpha+\beta+\gamma = 0$.

To achieve computational efficiency the method of discovering a neutrosophic number becomes a graded representation of the average integration.

A generalized neutrosophic number $T_A + I_A + F_A$ is explained as any neutrosophic subset of the real line R, whose membership function $\mu_{T_A+I_A+F_A}^N(u)$ satisfies as follows,

i)$\mu_{T_A+I_A+F_A}^N(u)$is continuous neutrosophic mapping from R to [0,1],

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ii) $\mu^{N}_{T_A+I_A+F_A}(u) = 0$, $\infty < u \leq \underline{a}$,

iii) $\mu^{N}_{L}(u) = L(u)$ is strictly increasing on $[\underline{a}, a]$,

iv) $\mu^{N}_{T_A+I_A+F_A}(u) = w, u = a$,

v) $\mu^{N}_{R}(u) = R(u)$ is strictly decreasing on $[a, \bar{a}]$, where $0<w\leq 3$ and $a, \bar{a}, \underline{a}$ are real numbers.

This type of generalized neutrosophic number is a triangular neutrosophic number, and its denoted by $T_A+I_A+F_A= (a, \bar{a}, \underline{a}, w)^{N}_{LR}$. When w=1, this kind of generalized neutrosophic number is called normal neutrosophic number and its characterization, $T_A+I_A+F_A= (a, \bar{a}, \underline{a})^{N}_{LR}$.

The graded mean $\alpha+\beta+\gamma$ –level value of $T_A+I_A+F_A = (a, \bar{a}, \underline{a})^{N}_{LR}$ is

$(\alpha+\beta+\gamma)\,[L(\alpha+\beta+\gamma)+R(\alpha+\beta+\gamma)]/2$. Therefore the graded mean integration characterization of generalized triangular neutrosophic number $T_A+I_A+F_A$ is,

$$G(T_A+I_A+F_A) = \frac{1}{2}\frac{\left[\int_0^{w_{a^N}} \alpha\left[L_{a^N}(\alpha)+R_{a^N}(\alpha)\right]d\alpha + \int_{u_{a^N}}^{1}(1-\beta)\left[L'_{a^N}(\beta)+R'_{a^N}(\beta)\right]d\beta + \int_{y_{a^N}}^{1}(1-\gamma)\left[L''_{a^N}(\gamma)+R''_{a^N}(\gamma)\right]d\gamma\right]}{\left[\int_0^{w_{a^N}}\alpha d\alpha + \int_0^{1-u_{a^N}}\beta d\beta + \int_0^{1-y_{a^N}}\gamma d\gamma\right]}$$

$$\int_0^{w_{a^N}} \alpha\left[L_{a^N}(\alpha)+R_{a^N}(\alpha)\right]d\alpha = \int_0^{w_{a^N}}\left[(a+c)+\frac{(2b-b-c)\alpha}{w_{a^N}}\right]\alpha\, d\alpha,$$

$$\left\{where\left[L_{a^N}(\alpha)+R_{a^N}(\alpha)\right] = \left[\frac{(w_{a^N}-\alpha)a+\alpha b}{w_{a^N}}, \frac{(w_{a^N}-\alpha)c+\alpha b}{w_{a^N}}\right]\right\}$$

$$= \left[\frac{(a+c)\alpha^2}{2}+\frac{(2b-a-c)\alpha^3}{3w_{a^N}}\right]_0^{w_{a^N}}$$

$$= \left[\frac{(a+c)w_{a^N}{}^2}{2}+\frac{(2b-a-c)w_{a^N}{}^3}{3w_{a^N}}\right]$$

$$= \frac{(3a+3c+4b-2a-2c)\,w_{a^N}{}^2}{6}$$

$$= \frac{(a+4b+c)w_{a^N}{}^2}{6} \qquad (1)$$

$$\int_{u_{a^N}}^{1}(1-\beta)\left[L'_{a^N}(\beta)+R'_{a^N}(\beta)\right]d\beta = \int_{u_{a^N}}^{1}\left[(a+c)+\frac{(2b-a-c)(1-\beta)}{1-u_{a^N}}\right](1-\beta)\, d\beta,$$

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$$\left\{where\left[L'_{a^N}(\beta)+R'_{a^N}(\beta)\right]=\left[\frac{(1-\beta)b+(\beta-u_{a^N})a}{1-u_{a^N}},\frac{(1-\beta)b+(\beta-u_{a^N})c}{1-u_{a^N}}\right]\right\}$$

$$=\left[\frac{(a+c)(1-\beta)^2}{2}+\frac{(2b-a-c)(1-\beta)^3}{3(1-u_{a^N})}\right]_{u_{a^N}}^{1}$$

$$=\left[\frac{(a+c)(1-u_{a^N})^2}{2}+\frac{(2b-a-c)(1-u_{a^N})^3}{3(1-u_{a^N})}\right]$$

$$=\frac{(3a+3c+4b-2a-2c)\,(1-u_{a^N})^2}{6}$$

$$=\frac{(a+4b+c)(1-u_{a^N})^2}{6} \qquad (2)$$

$$\int_{y_{a^N}}^{1}(1-\gamma)\left[L''_{a^N}(\gamma)+R''_{a^N}(\gamma)\right]d\gamma=\int_{y_{a^N}}^{1}\left[(a+c)+\frac{(2b-a-c)(1-\gamma)}{1-y_{a^N}}\right](1-\gamma)\,d\gamma,$$

$$\left\{where\left[L''_{a^N}(\gamma)+R''_{a^N}(\gamma)\right]=\left[\frac{(1-\gamma)b+(\gamma-y_{a^N})a}{1-y_{a^N}},\frac{(1-\gamma)b+(\gamma-y_{a^N})c}{1-y_{a^N}}\right]\right\}$$

$$=\left[\frac{(a+c)(1-\gamma)^2}{2}+\frac{(2b-a-c)(1-\gamma)^3}{3(1-y_{a^N})}\right]_{y_{a^N}}^{1}$$

$$=\left[\frac{(a+c)(1-y_{a^N})^2}{2}+\frac{(2b-a-c)(1-y_{a^N})^3}{3(1-y_{a^N})}\right]$$

$$=\frac{(3a+3c+4b-2a-2c)\,(1-y_{a^N})^2}{6}$$

$$=\frac{(a+4b+c)(1-y_{a^N})^2}{6} \qquad (3)$$

$$\int_{0}^{w_{a^N}}\alpha d\alpha=\left(\frac{\alpha^2}{2}\right)_0^{w_{a^N}}=\frac{{w_{a^N}}^2}{2} \qquad (4)$$

$$\int_{0}^{1-u_{a^N}}\beta d\beta=\left(\frac{\beta^2}{2}\right)_0^{1-u_{a^N}}=\frac{(1-u_{a^N})^2}{2} \qquad (5)$$

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$$\int_0^{1-y_{\alpha^N}} \gamma d\gamma = \left(\frac{\gamma^2}{2}\right)_0^{1-y_{\alpha^N}} = \frac{(1-y_{\alpha^N})^2}{2} \quad (6)$$

Substitute the equation (1), (2), (3), (4), (5) and (6) in graded mean integration ,we get

$$G(T_A + I_A + F_A) = \frac{1}{2}\frac{\frac{(a+4b+c)(w_{\alpha^N}{}^2+(1-u_{\alpha^N})^2+(1-y_{\alpha^N})^2)}{6}}{\frac{w_{\alpha^N}{}^2+(1-u_{\alpha^N})^2+(1-y_{\alpha^N})^2}{2}} = \frac{a+4b+c}{6}$$

4. Formulation of a Problem and Methodology

4.1. Single-Valued Neutrosophic Inventory Problem

The single-valued neutrosophic inventory model of time independent profit maximizing neutrosophic costs can be thought of as a classic newsvendor problem releases where should a newsvendor buy the approximate number of newspapers for his corner newspaper shop such that he eventually reached the maximum expected profit.

Consider the item you can buy at the beginning of the period and after the end of the period, it is either used or sold at a price below the purchases price. Let,

b^N = Neutrosophic unit price of purchased product (independent number of item purchased),

a^N = Neutrosophic unit retail price of the products ($a^N < b^N$),

h^N = Neutrosophic holding cost per each item after the end of the period ($h^N < a^N$) (After

theend of the period a single price can be considered),

s^N = Neutrosophic price of one product per defect,

and the demand Υ^N as a neutrosophic random variable with an order pair is given as $\{(y_1^N, p_1^N), (y_2^N, p_2^N), (y_3^N, p_3^N), \dots, (y_n^N, p_n^N)\}$. If y_k^N products are purchased at the beginning of the period, then the neutrosophicprofit function P^N is given by,

$$P^N(y_k^N, \Upsilon^N) = \begin{cases} a^N y_i^N - b^N y_k^N - s^N(y_k^N - y_i^N), y_i^N \le y_k^N \\ (a^N - b^N) y_k^N - s(y_i^N - y_k^N), y_i^N \ge y_k^N \end{cases}$$

For some $i = 1$ ton.

As the neutrosophicdemand Υ^N is a neutrosophic random variable, so its neutrosophicprofit function P^N is also a neutrosophic random variable and obviously its total neutrosophic expected cost EP^N becomes a unique neutrosophic number.Therefore the neutrosophic total expected profit ($E^N P^N$) is determined by,

$$E^N P^N(y_k^N, \Upsilon^N) = \sum_{i=1}^{k} [a^N y_i^N - b^N y_k^N - s^N(y_k^N - y_i^N)] p_i^N + \sum_{i=k+1}^{n} [(a^N - b^N) y_k^N - s^N(y_i^N - y_k^N)] p_i^N$$

4.2 Mathematical Model with Neutrosophic Random Variable

We consider the problem of stocks described above in a period when demand is seen as a neutrosophic random variable. The data are not accurate for neutrosophic probabilities so for simplicity, all data sets and probabilities are considered as triangular neutrosophic numbers $\left(\underline{y_i}, y_i, \overline{y_i}\right)^N$ and $\left(\underline{p_i}, p_i, \overline{p_i}\right)^N$ for $i = 1$ to n, respectively.

Now, the neutrosophic expected profit function $E^N P^N = \left(\underline{EP}, EP, \overline{EP}\right)^N$ is given by

$$E^N P^N = \sum_{i=1}^{k} [a^N y_i^N - b^N y_k^N - s^N(y_k^N - y_i^N)] p_i^N + \sum_{i=k+1}^{n} [(a^N - b^N) y_k^N - s^N(y_i^N - y_k^N)] p_i^N$$

$$= \sum_{i=1}^{k} [(a^N + s^N) y_i^N - (b^N + s^N) y_k^N] p_i^N + \sum_{i=k+1}^{n} [(a^N - b^N + s^N) y_k^N - s^N y_i^N] p_i^N$$

where,

$$(EP)^N = E^N[P^N_{\alpha+\beta+\gamma=3}]$$

$$= \sum_{i=1}^{k} [(a^N + s^N) y_i^N - (b^N + s^N) y_k^N] p_i^N + \sum_{i=k+1}^{n} [(a^N - b^N + s^N) y_k^N - s^N y_i^N] p_i^N$$

$$= \sum_{i=1}^{k} [(a^N + s^N) y_i^N p_i^N - (b^N + s^N) y_k^N p_i^N] + \sum_{i=k+1}^{n} [(a^N - b^N + s^N) y_k^N p_i^N - s^N y_i^N p_i^N]$$

$$\underline{(EP)}^N = E^N[P^N_{\alpha+\beta+\gamma=0^-}]$$

$$= \sum_{i=1}^{k} \left[(a^N + s^N)\underline{y_i^N p_i^N} - (b^N + s^N)\underline{y_k^N p_i^N}\right] + \sum_{i=k+1}^{n} \left[(a^N - b^N + s^N)\underline{y_k^N p_i^N} - s^N \underline{s^N y_i^N p_i^N}\right]$$

$$\overline{(EP)}^N = E^N[P^N_{\alpha+\beta+\gamma=3}]$$

$$= \sum_{i=1}^{k} \left[(a^N + s^N)\overline{y_i^N p_i^N} - (b^N + s^N)\overline{y_k^N p_i^N}\right] + \sum_{i=k+1}^{n} \left[(a^N - b^N + s^N)\overline{y_k^N p_i^N} - s^N \overline{y_i^N p_i^N}\right]$$

Here the $(\alpha + \beta + \gamma)$ −level set of the neutrosophic number $E^N P^N$ are considered as follows,

$$[EP]^N_{\alpha+\beta+\gamma} = E^N[P_{\alpha+\beta+\gamma}]^N = \left[E^N(P^N_{\alpha+\beta+\gamma})^-, E^N(P^N_{\alpha+\beta+\gamma})^+\right], 0 \le \alpha + \beta + \gamma \le 3$$

and we get $\alpha + \beta + \gamma - \mathrm{cut}$ interval with different neutrosophic number $E^N P^N$ for $\alpha + \beta + \gamma$ between 0 and 3.The membership function of this neutrosophic number $E^N P^N$ is defined by,

$$\mu_{E^N P^N}(u) = \begin{cases} L(u), \underline{E^N P^N} \le u \le E^N P^N \\ R(u), E^N P^N \le u \le \overline{E^N P^N} \\ 0, \quad \text{otherwise} \end{cases}$$

where $L(u)$ is left continuous function from $\left[\underline{E^N P^N}, E^N P^N\right]$ to $[0,1]$and $R(u)$ is the right continuous function from $\left[E^N P^N, \overline{E^N P^N}\right]$ to $[0,1]$.Now, we use the method of indicating the amount of neutrosophic number that are summed based on the integral value.of graded mean $\alpha + \beta + \gamma - \mathbf{level}$, we find out a lost representative of the unique neutrosophic number $E^N P^N$ is

$G(E^N P^N) =$

$$\frac{1}{2}\frac{\left[\int_0^{w_{a^N}} \alpha\left[L_{a^N}(\alpha)+R_{a^N}(\alpha)\right]d\alpha+\int_{u_{a^N}}^{1}(1-\beta)\left[L'_{a^N}(\beta)+R'_{a^N}(\beta)\right]d\beta+\int_{y_{a^N}}^{1}(1-\gamma)\left[L''_{a^N}(\gamma)+R''_{a^N}(\gamma)\right]d\gamma\right]}{\left[\int_0^{w_{a^N}}\alpha d\alpha+\int_0^{1-u_{a^N}}\beta d\beta+\int_0^{1-y_{a^N}}\gamma d\gamma\right]}$$

and we get,

$$G(E^N P^N) = \frac{\underline{E^N P^N} + E^N P^N + \overline{E^N P^N}}{6}$$

Moreover the neutrosophic optimal order quantity y_k^N, we have

$$G\left(E^N P^N\left(y_k^N, \Upsilon^N\right)\right) > G\left(E^N P^N\left(y_{k-1}^N, \Upsilon^N\right)\right) \quad (7)$$

$$\text{and} G\left(E^N P^N\left(y_k^N, \Upsilon^N\right)\right) > G\left(E^N P^N\left(y_{k+1}^N, \Upsilon^N\right)\right) \quad (8)$$

The result (7) and (8), are gives us the neutrosophic optimal order quantity y_k^N, which satisfies as following,

$$\text{lowerlimit} < \frac{a^N - b^N + s^N}{a^N + b^N + s^N} < \text{upperlimit}, \quad (9)$$

where,

$$\text{lowerlimit} = \frac{4\left(y_k^N - y_{k-1}^N\right)\sum_{i=1}^{k-1} p_i^N + \left(\underline{y_k^N} - \underline{y_{k-1}^N}\right)\sum_{i=1}^{k-1}\underline{p_i^N} + \left(\overline{y_k^N} - \overline{y_{k-1}^N}\right)\sum_{i=1}^{k-1}\overline{p_i^N}}{4\left(y_k^N - y_{k-1}^N\right)\sum_{i=1}^{n} p_i^N + \left(\underline{y_k^N} - \underline{y_{k-1}^N}\right)\sum_{i=1}^{n}\underline{p_i^N} + \left(\overline{y_k^N} - \overline{y_{k-1}^N}\right)\sum_{i=1}^{n}\overline{p_i^N}}$$

and

$$\text{upperlimit} = \frac{4\left(y_{k+1}^N - y_k^N\right)\sum_{i=1}^{k} p_i^N + \left(\underline{y_{k+1}^N} - \underline{y_{k-1}^N}\right)\sum_{i=1}^{k-1}\underline{p_i^N} + \left(\overline{y_{k+1}^N} - \overline{y_k^N}\right)\sum_{i=1}^{k}\overline{p_i^N}}{4\left(y_{k+1}^N - y_k^N\right)\sum_{i=1}^{n} p_i^N + \left(\underline{y_{k+1}^N} - \underline{y_{k-1}^N}\right)\sum_{i=1}^{n}\underline{p_i^N} + \left(\overline{y_{k+1}^N} - \overline{y_k^N}\right)\sum_{i=1}^{n}\overline{p_i^N}}$$

Therefore, we conclude that the neutrosophic optimal order quantity can be determined over a single-valued neutrosophic inventory model when requirements are mentioned in a mixed environment, it is more accurate and less certain that it is more realistic for an inventory. Then if an inaccurate probability is specified for the search data set, we give a numerical example of how to get the neutrosophic optimal order quantity.

5. Numerical Example

To illustrate this model, suppose a newsvendor cannot pay in cash for a day if he needs more papers. Let the neutrosophic purchase price $b^N = 5$, the neutrosophic price $a^N = 8$, the neutrosophic holding cost $h^N = 3$, and the neutrosophic shortage price $s^N = 6$.The daily neutrosophic demand for this section is unknown, but based on experience and previous sales dates, you can set neutrosophic probabilities for different search levels of neutrosophic demand. The neutrosophic demand and neutrosophic probability are given in the first table. Now, using our methodology you can find the neutrosophic optimal order quantity y_k^N (say) from second table.

Table 1:

Neutrosophic demand	Neutrosophic probabilities
(23,24,25) (22,24,26) (21,24,27)	(0.037,0.04,0.048) (0.032,0.04,0.053) (0.027,0.04,0.058)
(29,30,31) (28,30,32) (27,30,33)	(0.134,0.16,0.150) (0.129,0.16,0.155) (0.124,0.16,0.160)
(35,36,37) (34,36,38) (33,36,39)	(0.260,0.30,0.305) (0.255,0.30,0.310) (0.250,0.30,0.315)
(41,42,43) (40,42,44) (39,42,45)	(0.169,0.20,0.205) (0.164,0.20,0.210) (0.159,0.20,0.215)
(47,48,49) (46,48,50) (45,48,51)	(0.069,0.10,0.105) (0.064,0.10,0.110) (0.059,0.10,0.115)
(53,54,55) (52,54,56) (51,54,57)	(0.070,0.10,0.104) (0.065,0.10,0.109) (0.060,0.10,0.114)
(59,60,61) (58,60,62) (57,60,63)	(0.071,0.10,0.103) (0.066,0.10,0.108) (0.067,0.10,0.113)

Table 2:

y_k^N	(21,24,27)	(27,30,33)	(33,36,39)	(39,42,45)	(45,48,51)	(51,54,57)
$4(y_{k+1}^N - y_k^N)\sum_{i=1}^{k} p_i^N$	0.96	4.8	12	16.8	19.2	21.6
$(\underline{y_{k+1}^N} - \underline{y_k^N})\sum_{i=1}^{k} \underline{p_i^N}$	0.162	0.906	2.41	3.36	3.71	4.074
$(\overline{y_{k+1}^N} - \overline{y_{k+1}^N})\sum_{i=1}^{k} \overline{p_i^N}$	0.348	1.308	3.198	4.488	5.178	5.862
A	1.47	7.014	17.608	24.68	28.118	31.554
$4(y_{k+1}^N - y_k^N)\sum_{i=1}^{n} p_i^N$	24	24	24	24	24	24
$(\underline{y_{k+1}^N} - \underline{y_k^N})\sum_{i=1}^{n} \underline{p_i^N}$	4.44	4.44	4.44	4.44	4.44	4.44
$(\overline{y_{k+1}^N} - \overline{y_{k+1}^N})\sum_{i=1}^{n} \overline{p_i^N}$	6.54	6.54	6.54	6.54	6.54	6.54
B	34.98	34.98	34.98	34.98	34.98	34.98
$\frac{a^N - b^N + s^N}{a^N + b^N + s^N}$	0.5294	0.5294	0.5294	0.5294	0.5294	0.5294
$\frac{A}{B}$	0.0420	0.2005	0.5034	0.7055	0.8038	0.9021

Let A and B be the numerator and denominator of the upper limit of (9) respectively. Then the neutrosophic optimal order quantity y_k^N is gives $(39,42,45)$, this means that for newsvendor it is better to buy about 42 newspapers in order to maximize the expected daily profit.

6. Conclusion

This paper, introduces the development of stochastic neutrosophic inventory models using neutrosophic random variables to get more realistic information, where fixed value is invalid. Here, a single-valued neutrosophic inventory models are discussed when there are inaccuracies and uncertainties in inventory system. This aggregation can be extended to other neutrosophic inventory models.

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Multiplicative Interpretation of Neutrosophic Cubic Set on B-Algebra

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Abstract

Purpose of this paper is to interpret the multiplication of neutrosophic cubic set. Here we define the notation of ɤ-multiplication of neutrosophic cubic set and study it with the help of neutrosophic cubic M-subalgebra, neutrosophic cubic normal ideal and neutrosophic cubic closed normal ideal. We also study ɤ-multiplication under homomorphism and cartesian product through significant characteristics.

Keywords: B-algebra, Neutrosophic cubic set, ɤ-Multiplication, Cartesian product, Homomorphism.

1.Introduction

Theory of existing and non-existing value was first introduced by Zadeh [1,2]. Cubic set was defined by Jun et al. [3] in 2012, which was the modern form of interval-valued fuzzy set. Cubic set with the help of subalgebras, ideals and closed ideals of *B*-algebra was studied by Senapati et al. [4]. After the defing of *BCK*-algebra and *BCI*-algebra by Imai et al. [5] and Iseki [6], cubic set through subalgebras and q-ideals in BCK/BCI-algebra was investigated by Jun et al. [7, 8]. Notion of M-subalgebra on G-algebra is introduced and analyzed by Khalid et al. [9]. Interval-valued fuzzy set on *B*-algebra was studied by Senapati et al. [10,11]. Intuitionistic fuzzy translation and multiplication of G-algebra were deeply studied by by Khalid et al. [19]. Neutrosophic cubic set is the extended form of interval valued intuitionistic fuzzy theory with indeterminacy was introduced by Smarandache [12]. Neutrosophic logics and neutrosophic probability gave the new idea of research were interpret by Smarandache [13]. Neutrosophic cubic was introduced by Jun et al. [14]. Neutrosophic cubic point, (α,β)-fuzzy ideals and neutrosophic cubic (α,β)-ideals were analyzed by Gulistan et al. [15]. A new idea of normal ideal and closed normal ideal under neutrosophic cubic set was given and investigated by Khalid et al. [16]. Neutrosophic cubic set was investigated by Jun et al. [17]. PS fuzzy ideals were studied by Priya et al. [18]. Rosenfeld's fuzzy subgroup was studied by Biswas [20]. B-homomorphism was deeply studied by Neggers et al. [21]. Neutrosophic soft cubic subalgebra was extensively studied by Khalid et al. [22]. A B-algebra is an important logical class of algebra was defined by Neggers et al. [23]. T-Neutrosophic Cubic Set was defined and deeply investigated by Khalid et al. [24].

In this paper, we define ɤ-multiplication of neutrosophic cubic set and investigate the neutrosophic cubic M-subalgebra, neutrosophic cubic normal ideal (NCNID) and neutrosophic cubic closed normal ideal (NCCNID) under ɤ-multiplication with the help of P-intersection, P-union etc. We also study the cartesian product and

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homomorphism of ɤ-multiplication of neutrosophic cubic normal ideal (ɤMNCNID) and ɤ-multiplication of neutrosophic cubic closed normal ideal (ɤMNCCNID) with important results.

2. Preliminaries

Definition 2.1 [19] A nonempty set X with a constant 0 and $*$ is said to be B-algebra if it fulfills these conditions:

1: $ţ * ţ = 0$,

2: $ţ * 0 = 0$, for all $ţ \in X$.

3: $(ţ * ŧ) * t = ţ * (t * (0 * ŧ))\ \forall\ ţ, ŧ, t \in X$.

Definition 2.2 [21] A nonempty subset K of B-algebra X is called a subalgebra of Y if $ţ * ŧ \in K\ \forall\ ţ, ŧ \in K$, a mapping $f : X \to Y$ of B-algebra is called B-homomorphism if $f(ţ * ŧ) = f(ţ) * f(ŧ)\ \forall\ ţ, ŧ \in X$.

Definition 2.3 [1] Let X be a collection of elements like ţ. Then a FS J in X is defined as $J = \{< ţ, \nu_J(ţ) > | ţ \in X\}$, where $\mu_J(ţ)$ is called the existenceship value of ţ in J and $\nu_J(ţ) \in [0,1]$.

For a family $J_i = \{< ţ, \nu_{J_i}(ţ) > | ţ \in X\}$ of FSs in X, where $i \in k$ and k is index set, Then join ($\vee$) and meet ($\wedge$) are as follows:

$$\bigvee_{i \in k} J_i = (\bigvee_{i \in k} \nu_{J_i})(ţ) = \sup\{\nu_{J_i} | i \in k\}$$

and

$$\bigwedge_{i \in k} J_i = (\bigwedge_{i \in k} \nu_{J_i})(ţ) = \inf\{\nu_{J_i} | i \in k\},$$

respectively, $\forall\ ţ \in X$.

Definition 2.4 [2] An IVFS B is of the form $B = \{< ţ, \tilde{\nu}_B(ţ) > | ţ \in X\}$, where $\tilde{\nu}_B | X \to D[0,1]$, here $D[0,1]$ is the collection of all subintervals of $[0,1]$. The intervals $\tilde{\nu}_B(ţ) = [\nu_B^-(ţ), \nu_B^+(ţ)]\ \forall\ ţ \in X$ denote the degree of existence of ţ to the set B, also $\tilde{\nu}_B^c = [1 - \nu_B^-(ţ), 1 - \nu_B^+(ţ)]$ shows the complement of $\tilde{\nu}_B$.

For a family $B_i = \{< ţ, \tilde{\nu}_B(ţ) > | ţ \in X\}$ of IVFSs in X where k is an index set and $i \in k$, the union $G = \bigcup_{i \in k} \tilde{\nu}_{B_i}(ţ)$ and the intersection $F = \bigcap_{i \in k} \tilde{\nu}_{B_i}(ţ)$ are defined below:

$$G(ţ) = \mathrm{rsup}\{\tilde{\nu}_{B_i}(ţ) i \in k\}$$

and

$$F(ţ) = \mathrm{rinf}\{\tilde{\nu}_{B_i}(ţ) | i \in k\},$$

respectively, $\forall\ ţ \in X$.

Definition 2.5 [20] Consider two elements $D_1, D_2 \in D[0,1]$. If $D_1 = [ţ_1^-, ţ_1^+]$ and $D_2 = [ţ_2^-, ţ_2^+]$, then $\mathrm{rmax}(D_1, D_2) = [\max(ţ_1^-, ţ_2^-), \max(ţ_1^+, ţ_2^+)]$ which is denoted by $D_1 \vee^r D_2$ and $\mathrm{rmin}(D_1, D_2) = [\min(ţ_1^-, ţ_2^-), \min(ţ_1^+, ţ_2^+)]$ which is denoted by $D_1 \wedge^r D_2$. Thus, if $D_i = [ţ_i^-, ţ_2^+] \in D[0,1]$ for $i = 1,2,3, \ldots$, then we define $\mathrm{rsup}_i(D_i) = [\sup_i(ţ_i^-), \sup_i(ţ_i^+)]$, i.e., $\vee_i^r D_i = [\vee_i ţ_i^-, \vee_i ţ_i^+]$. Similarly we define $\mathrm{rinf}_i(D_i) = [\inf_i(ţ_i^-), \inf_i(ţ_i^+)]$, i.e., $\wedge_i^r D_i = [\wedge_i ţ_i^-, \wedge_i ţ_i^+]$. Now we call $D_1 \geq D_2 \Leftarrow ţ_1^- \geq ţ_2^-$ and $ţ_1^+ \geq ţ_2^+$. Similarly the relations $D_1 \leq D_2$ and $D_1 = D_2$ are defined.

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Definition 2.6 [19] A fuzzy set $B = \{< ţ, \nu_B(ţ) > | ţ \in X\}$ is called a fuzzy subalgebra of X if $\nu_B(ţ * ŧ) \geq \min\{\nu_B(ţ), \nu_B(ŧ)\}\ \forall\ ţ, ŧ \in X$.

Definition 2.7 [14] Let X be a nonempty set. A NCS is $P_k = (B, \Lambda)$, where $B = \{\langle ţ; B_T(ţ), B_I(ţ), B_F(ţ)\rangle | ţ \in X\}$ is an interval neutrosophic set in X and $\Lambda = \{\langle ţ; \lambda_T(ţ), \lambda_I(ţ), \lambda_F(ţ)\rangle | ţ \in X\}$ is a neutrosophic set in X.

Definition 2.8 [3] Let U be a universe and cubic set in U, we mean a structure $\{ţ, \bar{\nu}_A(ţ), \lambda_A(ţ) |\ ţ \in U\}$ in which $\bar{\nu}_A$ is an IVF set in U and λ_A is a fuzzy set in U. A cubic set $A = \{ţ, \bar{\nu}_A(ţ), \lambda_A(ţ) |\ ţ \in U\}$ is simply denoted by C(U), which is the set of all cubic sets in U.

Definition 2.9 [3] Let $C = \{\langle ţ, C(ţ), \lambda(ţ)\rangle\}$ be a cubic set, where $C(ţ)$ is an IVFS in Y, $\lambda(ţ)$ is a fuzzy set in Y. Then A is cubic subalgebra under $*$ if it fulfills these axioms:

C1: $C(ţ * ŧ) \geq \mathrm{rmin}\{C(ţ), C(ŧ)\}$,

C2: $\lambda(ţ * ŧ) \leq \max\{\lambda(ţ), \lambda(ŧ)\}\ \forall\ ţ, ŧ \in X$.

Definition 2.10 [18] A fuzzy set $B = \{< ţ, \nu_B(ţ) > | ţ \in X\}$ is called a fuzzy ideal of X if

(i) $\nu_B(0) \geq \nu_B(ţ)$,

(ii) $\nu_B(ţ) \geq \min\{\nu_B(ţ * ŧ), \nu_B(ŧ)\}\ \forall\ ţ, ŧ \in X$.

Definition 2.11 [14] For any $C_i = (A_I, F_I)$, where $A_i = \{\langle t_1; A_{iT}(ţ), A_{iI}(ţ), A_{iF}(ţ)\rangle | ţ \in Y\}$, $F_i = \{\langle t_1; F_{iT}(ţ), F_{iI}(ţ), F_{iF}(ţ)\rangle | ţ \in Y\}$ for $i \in k$, then

P-union: $\bigcup_{\substack{P \\ i\in k}} C_i = (\bigcup_{i\in k} A_I, \bigvee_{i\in k} F_i)$,

P-intersection: $\bigcap_{\substack{P \\ i\in k}} C_i = (\bigcap_{i\in k} A_i, \bigwedge_{i\in k} F_i)$,

R-union: $\bigcup_{\substack{R \\ i\in k}} C_i = (\bigcup_{i\in k} A_I, \bigwedge_{i\in k} F_i)$,

R-intersection: $\bigcap_{\substack{R \\ i\in k}} C_i = (\bigcap_{i\in k} A_i, \bigvee_{i\in k} F_i)$.

Definition 2.12 [16] A NCS $Ʀ = (R_{T,I,F}, \lambda_{T,I,F})$ of X is called a NCNID of X if it fulfills following axioms:

N3. $R_{T,I,F}(0) \geq R_{T,I,F}(ţ * \alpha)$ and $\lambda_{T,I,F}(0) \leq \lambda_{T,I,F}(ţ * \alpha)$,

N4. $R_{T,I,F}(ţ * \alpha) \geq \mathrm{rmin}\{R_{T,I,F}((ţ * \alpha) * (ŧ * \beta)), R_{T,I,F}(ŧ * \beta)\}$,

N5. $\lambda_{T,I,F}(ţ * \alpha) \leq \max\{\lambda_{T,I,F}(ţ * \alpha) * (ŧ * \beta)), \lambda_{T,I,F}(ŧ * \beta)\}, \forall\ ţ, ŧ$ X and $\alpha, \beta \in [0,1]$.

Let $Ʀ = \{R_{T,I,F}, \lambda_{T,I,F}\}$ be a NCS X then it is called NCCNID of X if it fulfills N4, N5 and N6: $R_{T,I,F}(0 * (ţ * \alpha)) \geq R_{T,I,F}(ţ * \alpha)$ and $\lambda_{T,I,F}(0 * (ţ * \alpha)) \leq \lambda_{T,I,F}(ţ * \alpha)$, $\forall\ ţ \in X$ and $\alpha \in [0,1]$.

Definition 2.13 [16] Let $Ʀ = (R_{T,I,F}, \lambda_{T,I,F})$ and $\mathcal{B} = (B_{T,I,F}, \upsilon_{T,I,F})$ are two NCSs of X and Y respectively. The Cartesian product $Ʀ \times \mathcal{B} = (X \times Y, R_{T,I,F} \times B_{T,I,F}, \lambda_{T,I,F} \times \upsilon_{T,I,F})$ is defined by $(R_{T,I,F} \times B_{T,I,F})(ţ * \alpha, ŧ * \beta) = \mathrm{rmin}\{R_{T,I,F}(ţ * \alpha), B_{T,I,F}(ŧ * \beta)\}$ and $(\lambda_{T,I,F} \times \upsilon_{T,I,F})(ţ * \alpha, ŧ * \beta) = \max\{\lambda_{T,I,F}(ţ * \alpha), \upsilon_{T,I,F}(ŧ * \beta))\}$, where $R_{T,I,F} \times B_{T,I,F} \mid X \times Y \to D[0,1]$ and $\lambda_{T,I,F} \times \upsilon_{T,I,F} \mid X \times Y \to [0,1]\ \forall\ (ţ, ŧ) \in X \times Y$ and $\alpha, \beta \in [0,1]$.

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Definition 2.14 [16] A neutrosophic cubic subset $Ɽ \times Ƒ = (X \times Y, R_{T,I,F} \times F_{T,I,F}, \lambda_{T,I,F} \times \mu_{T,I,F})$ is called a NCNID if satisfies these conditions:

1. $(R_{T,I,F} \times F_{T,I,F})(0,0) \geq (R_{T,I,F} \times F_{T,I,F})((ţ * \alpha), (ŧ * \beta))$ and $(\lambda_{T,I,F} \times \mu_{T,I,F})(0,0) \leq (\lambda_{T,I,F} \times \mu_{T,I,F})((ţ * \alpha), (ŧ * \beta)) \ \forall\ (ţ, ŧ) \in X \times Y$ and $\alpha, \beta \in [0,1]$.

2. $(R_{T,I,F} \times F_{T,I,F})(ţ_1 * \alpha, ŧ_1 * \beta) \geq \text{rmin}\{(R_{T,I,F} \times F_{T,I,F})((ţ_1 * \alpha, ŧ_1 * \beta) * (ţ_2 * \alpha, ŧ_2 * \beta)), (R_{T,I,F} \times F_{T,I,F})(ţ_2 * \alpha, ŧ_2 * \beta)\}$.

3.$(\lambda_{T,I,F} \times \mu_{T,I,F})(ţ_1 * \alpha, ŧ_1 * \beta) \leq \max\{(\lambda_{T,I,F} \times \mu_{T,I,F})((ţ_1 * \alpha, ŧ_1 * \beta)(ţ_2 * \alpha, ŧ_2 * \beta)), (\lambda_{T,I,F} \times \mu_{T,I,F})(ţ_2 * \alpha, ŧ_2 * \beta)\}$ and $Ɽ \times Ƒ$ is closed normal ideal if it satisfies 2, 3, and 4. $(R_{T,I,F} \times F_{T,I,F})((0,0) * (ţ_1 * \alpha, ŧ_1 * \beta)) \geq (R_{T,I,F} \times F_{T,I,F})(ţ * \alpha, ŧ * \beta)$ and $(\lambda_{T,I,F} \times \mu_{T,I,F})((0,0) * (ţ * \alpha, ŧ * \beta)) \leq (\lambda_{T,I,F} \times \mu_{T,I,F})(ţ * \alpha, ŧ * \beta)\ \ \forall\ \ (ţ_1, ŧ_1)$ and $(ţ_2, ŧ_2) \in X \times Y$ and $\alpha, \beta \in [0,1]$.

Definition 2.15 [9] Let $\tilde{\mathcal{F}}_k = (A_{e_i}, \Lambda_{e_i})$ be a neutrosophic soft cubic set, where Y is subalgebra. Then $\tilde{\mathcal{F}}_k$ is NSCMSU under binary operation $*$ where $t_1, t_2 \in Y$ and $\alpha, \beta \in [0,1]$ if it fulfills these conditions:

$A^{\varrho}_{e_i}((t_1 * \alpha) * (t_2 * \beta)) \geq \text{rmin}\{A^{\varrho}_{e_i}(t_1 * \alpha), A^{\varrho}_{e_i}(t_2 * \beta)\}$ and $\lambda^{\varrho}_{e_i}((t_1 * \alpha) * (t_2 * \beta)) \leq \max\{\lambda^{\varrho}_{e_i}(t_1 * \alpha), \lambda^{\varrho}_{e_i}(t_2 * \beta)\}$.

3. ɤ-Multiplication of Neutrosophic Cubic Normal Ideal and Closed Normal Ideal

Definition 3.1. Let $Ƕ = (H_{T,I,F}, \lambda_{T,I,F})$ be a NCS of X and $ɤ \in [0,1]$. An object of the form $Ƕ^{M}_{ɤ} = ({}^{M}_{ɤ}H^{Ƕ}_{T,I,F}, {}^{M}_{ɤ}\lambda^{Ƕ}_{T,I,F})$ is called neutrosophic cubic ɤ multiplication of Ƕ X if it fulfills following axioms:

$${}^{M}_{ɤ}H^{Ƕ}_{T}(x) = ɤ.\, H^{Ƕ}_{T}(x), \qquad {}^{M}_{ɤ}\lambda^{Ƕ}_{T}(x) = ɤ.\, \lambda^{Ƕ}_{T}(x),$$

$${}^{M}_{ɤ}H^{Ƕ}_{I}(x) = ɤ.\, H^{Ƕ}_{I}(x), \qquad {}^{M}_{ɤ}\lambda^{Ƕ}_{I}(x) = ɤ.\, \lambda^{Ƕ}_{I}(x),$$

$${}^{M}_{ɤ}H^{Ƕ}_{F}(x) = ɤ.\, H^{Ƕ}_{F}(x), \qquad {}^{M}_{ɤ}\lambda^{Ƕ}_{F}(x) = ɤ.\, \lambda^{Ƕ}_{F}(x).$$

For convinience we use ${}^{M}_{ɤ}H^{Ƕ}_{T,I,F} = ɤ.\, H^{Ƕ}_{T,I,F}(x)$ and ${}^{M}_{ɤ}\lambda^{Ƕ}_{T,I,F} = ɤ.\, \lambda^{Ƕ}_{T,I,F}(x)$.

Theorem 3.1 A ɤ-multilplication of NCCNID of B-algebra X is also a ɤ-multilplication of NCMSU of X.

Proof. Suppose $Ƕ = \{H_{T,I,F}, \lambda_{T,I,F}\}$ be a NCCNID of X, then for any $ţ \in X$, we have ${}^{M}_{ɤ}H_{T,I,F}(0 * (ţ * \alpha)) = ɤ.\, H_{T,I,F}(0 * (ţ * \alpha)) \geq ɤ.\, H_{T,I,F}(ţ * \alpha)$ and ${}^{M}_{ɤ}\lambda_{T,I,F}(0 * (ţ * \alpha)) = ɤ.\, \lambda_{T,I,F}(0 * (ţ * \alpha)) \leq ɤ.\, \lambda_{T,I,F}(ţ * \alpha)$. Now by N4, N6, and through proposition 3.3 of article M subalgebra, we know that ${}^{M}_{ɤ}H_{T,I,F}((ţ * \alpha) * (ŧ * \beta)) = ɤ.\, H_{T,I,F}((ţ * \alpha) * (ŧ * \beta)) \geq ɤ.\, \text{rmin}\{H_{T,I,F}(((ţ * \alpha) * (ŧ * \beta)) * (0 * (ŧ * \beta))), H_{T,I,F}(0 * (ŧ * \beta))\} = ɤ.\, \text{rmin}\{H_{T,I,F}(ţ * \alpha), H_{T,I,F}(0 * (ŧ * \beta))\} \geq ɤ.\, \text{rmin}\{H_{T,I,F}(ţ * \alpha), H_{T,I,F}(ŧ * \beta)\} = \text{rmin}\{ɤ.\, H_{T,I,F}(ţ * \alpha), ɤ.\, H_{T,I,F}(ŧ * \beta)\} = \text{rmin}\{{}^{M}_{ɤ}H_{T,I,F}(ţ * \alpha), {}^{M}_{ɤ}H_{T,I,F}(ŧ * \beta)\}$ and ${}^{M}_{ɤ}\lambda_{T,I,F}((ţ * \alpha) * (ŧ * \beta)) = ɤ.\, \lambda_{T,I,F}((ţ * \alpha) * (ŧ * \beta)) \leq ɤ.\, \max\{\lambda_{T,I,F}(((ţ * \alpha) * (ŧ * \beta)) * (0 * (ŧ * \beta))), \lambda_{T,I,F}(0 * (ŧ * \beta))\} = ɤ.\, \max\{\lambda_{T,I,F}(ţ * \alpha), \lambda_{T,I,F}(0 * (ŧ * \beta))\} \leq ɤ.\, \max\{\lambda_{T,I,F}(ţ * \alpha), \lambda_{T,I,F}(ŧ * \beta)\} = \max\{ɤ.\, \lambda_{T,I,F}(ţ * \alpha), ɤ.\, \lambda_{T,I,F}(ŧ * \beta)\} = \max\{{}^{M}_{ɤ}\lambda_{T,I,F}(ţ * \alpha), {}^{M}_{ɤ}\lambda_{T,I,F}(ŧ * \beta)\}$. Hence, ɤMNCCNID is ɤMNCMSU of X.

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Proposition 3.1 Every ɤ-multiplication of NCCNID is a ɤ-multiplication NCNID but the converse is not true in general.

Theorem 3.2 The R-intersection of any set of ɤMNCNIDs of X is also a ɤMNCNID of X.

Proof. Let $\text{Ƕ}_i = \{H^i_{T,I,F}, \lambda^i_{T,I,F}\}$, where $i \in k$, be a ɤMNCNID of X and $\text{ţ}, \text{ŧ} \in X$. Then

$$(\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})(0) = \text{rinf}\ {}^{M}_{\text{ɤ}}H^i_{T,I,F}(0) = \text{rinf}\ H^i_{T,I,F}(0).\text{ɤ}$$

$$\geq \text{rinf}H^i_{T,I,F}(\text{ţ} * \alpha).\text{ɤ} = \text{rinf}\ {}^{M}_{\text{ɤ}}H^i_{T,I,F}(\text{ţ} * \alpha)$$

$$= (\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})(\text{ţ} * \alpha)$$

$$\Rightarrow (\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})(0) \geq (\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})(\text{ţ} * \alpha)$$

and

$$(\vee^{M}_{\text{ɤ}}\lambda^i_{T,I,F})(0) = \sup{}^{M}_{\text{ɤ}}\lambda^i_{T,I,F}(0) = \sup\lambda^i_{T,I,F}(0).\text{ɤ}$$

$$\leq \sup\lambda^i_{T,I,F}(\text{ţ} * \alpha).\text{ɤ} = \sup{}^{M}_{\text{ɤ}}\lambda^i_{T,I,F}(\text{ţ} * \alpha)$$

$$= (\vee^{M}_{\text{ɤ}}\lambda^i_{T,I,F})(\text{ţ} * \alpha)$$

$$\Rightarrow (\vee^{M}_{\text{ɤ}}\lambda^i_{T,I,F})(0) \leq (\vee^{M}_{\text{ɤ}}\lambda^i_{T,I,F})(\text{ţ} * \alpha),$$

now

$$(\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})(\text{ţ} * \alpha) = \text{rinf}{}^{M}_{\text{ɤ}}H^i_{T,I,F}(\text{ţ} * \alpha) = \text{rinf}\ H^i_{T,I,F}(\text{ţ} * \alpha).\text{ɤ}$$

$$\geq \text{rinf}\{\text{rmin}\{H^i_{T,I,F}((\text{ţ} * \alpha) * (\text{ŧ} * \beta)), H^i_{T,I,F}(\text{ŧ} * \beta)\}\}.\text{ɤ}$$

$$= \text{rmin}\{\text{rinf}H^i_{T,I,F}((\text{ţ} * \alpha) * (\text{ŧ} * \beta)).\text{ɤ}, \text{rinf}H^i_{T,I,F}(\text{ŧ} * \beta).\text{ɤ}\}$$

$$= \text{rmin}\{\text{rinf}{}^{M}_{\text{ɤ}}H^i_{T,I,F}((\text{ţ} * \alpha) * (\text{ŧ} * \beta)), \text{rinf}{}^{M}_{\text{ɤ}}H^i_{T,I,F}(\text{ŧ} * \beta)\}$$

$$= \text{rmin}\{(\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})((\text{ţ} * \alpha) * (\text{ŧ} * \beta)), (\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})((\text{ŧ} * \beta))\} \Rightarrow (\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})(\text{ţ} * \alpha) \geq \text{rmin}\{(\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})((\text{ţ} * \alpha) * (\text{ŧ} * \beta)), (\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})(\text{ŧ} * \beta)\}$$

and

$$(\vee^{M}_{\text{ɤ}}\lambda^i_{T,I,F})(\text{ţ} * \alpha) = \sup{}^{M}_{\text{ɤ}}\lambda^i_{T,I,F}(\text{ţ} * \alpha) = \sup\lambda^i_{T,I,F}((\text{ţ} * \alpha).\text{ɤ}$$

$$\leq \sup\{\max\{\lambda^i_{T,I,F}((\text{ţ} * \alpha) * (\text{ŧ} * \beta)), \lambda^i_{T,I,F}(\text{ŧ} * \beta)\}\}.\text{ɤ}$$

$$= \max\{\sup\lambda^i_{T,I,F}((\text{ţ} * \alpha) * (\text{ŧ} * \beta)).\text{ɤ}, \sup\lambda^i_{T,I,F}(\text{ŧ} * \beta).\text{ɤ}\}$$

$$= \max\{\sup{}^{M}_{\text{ɤ}}\lambda^i_{T,I,F}((\text{ţ} * \alpha) * (\text{ŧ} * \beta)), \sup{}^{M}_{\text{ɤ}}\lambda^i_{T,I,F}(\text{ŧ} * \beta)\}$$

$$= \max\{(\vee^{M}_{\text{ɤ}}\lambda^i_{T,I,F})((\text{ţ} * \alpha) * (\text{ŧ} * \beta)), (\vee^{M}_{\text{ɤ}}\lambda^i_{T,I,F})(\text{ŧ} * \beta)\}$$

$$\Rightarrow (\vee^{M}_{\text{ɤ}}\lambda^i_{T,I,F})(\text{ţ} * \alpha) \leq \max\{(\vee^{M}_{\text{ɤ}}\lambda^i_{T,I,F})((\text{ţ} * \alpha) * (\text{ŧ} * \beta)), (\vee^{M}_{\text{ɤ}}\lambda^i_{T,I,F})(\text{ŧ} * \beta)\},$$

which show that R-intersection is a ɤMNCNID of X.

Theorem 3.3. The R-intersection of any set of ɤMNCCNIDs of X is also a ɤ-multiplication of NCCNID of X.

Proof**.** We can prove this theorem as Theorem 3.2.

Theorem 3.4. Let $Ћ = \{H_{T,I,F}, \lambda_{T,I,F}\}$ be a NCS of X. Then ɤMNCNID of Ћ is a NCNID of X iff ${}^{M}_{ɤ}H^{-}_{T,I,F}$, ${}^{M}_{ɤ}H^{+}_{T,I,F}$ and ${}^{M}_{ɤ}\lambda_{T,I,F}$ are fuzzy ideals of X.

Proof. Suppose that $ţ, ȶ \in X$. Since ${}^{M}_{ɤ}H^{-}_{T,I,F}(0) = H^{-}_{T,I,F}(0).ɤ \geq H^{-}_{T,I,F}(ţ*\alpha).ɤ = {}^{M}_{ɤ}H^{-}_{T,I,F}(ţ*\alpha)$, ${}^{M}_{ɤ}H^{+}_{T,I,F}(0) = H^{+}_{T,I,F}(0).ɤ \geq H^{+}_{T,I,F}(ţ*\alpha).ɤ = {}^{M}_{ɤ}H^{-}_{T,I,F}(ţ*\alpha)$, therefore, $H_{T,I,F}(0) \geq H_{T,I,F}(ţ*\alpha)$, also ${}^{M}_{ɤ}\lambda_{T,I,F}(0) = \lambda_{T,I,F}(0).ɤ \leq \lambda_{T,I,F}(ţ*\alpha).ɤ = {}^{M}_{ɤ}\lambda_{T,I,F}(ţ*\alpha)$. Suppose that ${}^{M}_{ɤ}H^{-}_{T,I,F}$, ${}^{M}_{ɤ}H^{+}_{T,I,F}$ and ${}^{M}_{ɤ}\lambda_{T,I,F}$ are ɤ-multiplication of fuzzy ideals of X. Then ${}^{M}_{ɤ}H_{T,I,F}(ţ*\alpha) = H_{T,I,F}(ţ*\alpha).ɤ = \{H^{-}_{T,I,F}(ţ*\alpha), H^{+}_{T,I,F}(ţ*\alpha)\}.ɤ \geq [\min\{H^{-}_{T,I,F}((ţ*\alpha)*(ȶ*\beta)), H^{-}_{T,I,F}((ȶ*\beta))\}, \min\{H^{+}_{T,I,F}((ţ*\alpha)*(ȶ*\beta)), H^{+}_{T,I,F}(ȶ*\beta)\}.ɤ = r\min\{[H^{-}_{T,I,F}\big((ţ*\alpha)*(ȶ*\beta)\big), H^{+}_{T,I,F}\big((ţ*\alpha)*(ȶ*\beta)\big)], [H^{-}_{T,I,F}\big((ȶ*\beta)\big), H^{+}_{T,I,F}(ȶ*\beta)]\}.ɤ = r\min\{H_{T,I,F}((ţ*\alpha)*(ȶ*\beta)), H_{T,I,F}(ȶ*\beta)\}.ɤ = r\min\{H_{T,I,F}((ţ*\alpha)*(ȶ*\beta)).ɤ, H_{T,I,F}(ȶ*\beta).ɤ\} = r\min\{{}^{M}_{ɤ}H_{T,I,F}((ţ*\alpha)*(ȶ*\beta)), {}^{M}_{ɤ}H_{T,I,F}(ȶ*\beta)\}$ and ${}^{M}_{ɤ}\lambda_{T,I,F}(ţ*\alpha) \leq \max\{{}^{M}_{ɤ}\lambda_{T,I,F}((ţ*\alpha)*(ȶ*\beta)), {}^{M}_{ɤ}\lambda_{T,I,F}(ȶ*\beta)\}$. Therefore ɤMNCNID of Ћ is a NCNID of X.

Conversely, assume that ɤMNCNID Ћ is a NCNID of X. For any $ţ, ȶ \in X$, we have $\{{}^{M}_{ɤ}H^{-}_{T,I,F}(ţ*\alpha), {}^{M}_{ɤ}H^{+}_{T,I,F}(ţ*\alpha)\} = \{H^{-}_{T,I,F}(ţ*\alpha).ɤ, H^{+}_{T,I,F}(ţ*\alpha).ɤ\} = \{H^{-}_{T,I,F}(ţ*\alpha), H^{+}_{T,I,F}(ţ*\alpha)\}.ɤ = H_{T,I,F}(ţ*\alpha).ɤ = {}^{M}_{ɤ}H_{T,I,F}(ţ*\alpha) = r\min\{{}^{M}_{ɤ}H_{T,I,F}\big((ţ*\alpha)*(ȶ*\beta)\big), {}^{M}_{ɤ}H_{T,I,F}(ȶ*\beta)\} = r\min\{[{}^{M}_{ɤ}H^{-}_{T,I,F}\big((ţ*\alpha)*(ȶ*\beta)\big), {}^{M}_{ɤ}H^{+}_{T,I,F}((ţ*\alpha)*(ȶ*\beta))], [{}^{M}_{ɤ}H^{-}_{T,I,F}(ȶ*\beta), {}^{M}_{ɤ}H^{+}_{T,I,F}(ȶ*\beta)]\} = [\min\{H^{-}_{T,I,F}((ţ*\alpha)*(ȶ*\beta)).ɤ, H^{-}_{T,I,F}(ȶ*\beta).ɤ\}, \min\{H^{+}_{T,I,F}((ţ*\alpha)*(ȶ*\beta)).ɤ, H^{+}_{T,I,F}(ȶ*\beta).ɤ\} = [\min\{{}^{M}_{ɤ}H^{-}_{T,I,F}((ţ*\alpha)*(ȶ*\beta)), {}^{M}_{ɤ}H^{-}_{T,I,F}(ȶ*\beta)\}, \min\{{}^{M}_{ɤ}H^{+}_{T,I,F}((ţ*\alpha)*(ȶ*\beta)), {}^{M}_{ɤ}H^{+}_{T,I,F}(ȶ*\beta)\}]$. Thus, ${}^{M}_{ɤ}H^{-}_{T,I,F}(ţ*\alpha) \geq \min\{{}^{M}_{ɤ}H^{-}_{T,I,F}((ţ*\alpha)*(ȶ*\beta)), {}^{M}_{ɤ}H^{-}_{T,I,F}(ȶ*\beta)\}, {}^{M}_{ɤ}H^{+}_{T,I,F}(ţ*\alpha) \geq \min\{{}^{M}_{ɤ}H^{+}_{T,I,F}((ţ*\alpha)*(ȶ*\beta)), {}^{M}_{ɤ}H^{+}_{T,I,F}(ȶ*\beta)\}$ and ${}^{M}_{ɤ}\lambda_{T,I,F}(ţ*\alpha) \leq \max\{{}^{M}_{ɤ}\lambda_{T,I,F}((ţ*\alpha)*(ȶ*\beta)), {}^{M}_{ɤ}\lambda_{T,I,F}(ȶ*\beta)\}$. Hence, ${}^{M}_{ɤ}H^{-}_{T,I,F}$, ${}^{M}_{ɤ}H^{+}_{T,I,F}$ and ${}^{M}_{ɤ}\lambda_{T,I,F}$ are fuzzy ideals of X.

Theorem 3.5. For a NCNID $Ћ = \{H_{T,I,F}, \lambda_{T,I,F}\}$ of X, the following statements are valid:

1. If $(ţ*\alpha)*(ȶ*\beta) \leq z*\gamma$, then ${}^{M}_{ɤ}H_{T,I,F}(ţ*\alpha) \geq r\min\{{}^{M}_{ɤ}H_{T,I,F}(ȶ*\beta), {}^{M}_{ɤ}H_{T,I,F}(z*\gamma)\}$ and ${}^{M}_{ɤ}\lambda_{T,I,F}(ţ*\alpha) \leq \max\{{}^{M}_{ɤ}\lambda_{T,I,F}(ȶ*\beta), {}^{M}_{ɤ}\lambda_{T,I,F}(z*\gamma)\}$,

2. If $(ţ*\alpha) \leq (ȶ*\beta)$, then ${}^{M}_{ɤ}H_{T,I,F}(ţ*\alpha) \geq {}^{M}_{ɤ}H_{T,I,F}(ȶ*\beta)$ and ${}^{M}_{ɤ}\lambda_{T,I,F}(ţ*\alpha) \leq {}^{M}_{ɤ}\lambda_{T,I,F}(ȶ*\beta)$ $\forall$ $ţ, ȶ, z \in$ X and $\alpha, \beta, \gamma \in [0,1]$.

Proof. 1. Assume that $ţ, ȶ, z \in X$ such that $(ţ*\alpha)*(ȶ*\beta) \leq (z*\gamma)$. Then $((ţ*\alpha)*(ȶ*\beta))*(z*\gamma) = 0$ and thus ${}^{M}_{ɤ}H_{T,I,F}(ţ*\alpha) = H_{T,I,F}(ţ*\alpha).ɤ \geq r\min\{H_{T,I,F}((ţ*\alpha)*(ȶ*\beta)), H_{T,I,F}(ȶ*\beta)\}.ɤ \geq r\min\{r\min\{H_{T,I,F}(((ţ*\alpha)*(ȶ*\beta))*(z*\gamma)), H_{T,I,F}(z*\gamma)\}, H_{T,I,F}(ȶ*\beta)\}.ɤ = r\min\{r\min\{H_{T,I,F}(0), H_{T,I,F}(z*\gamma)\}, H_{T,I,F}(ȶ*\beta)\}.ɤ = r\min\{H_{T,I,F}(ȶ*\beta).ɤ, H_{T,I,F}(z*\gamma).ɤ\} = r\min\{{}^{M}_{ɤ}H_{T,I,F}(ȶ*\beta), {}^{M}_{ɤ}H_{T,I,F}(z*\gamma)\}$ and ${}^{M}_{ɤ}\lambda_{T,I,F}(ţ*\alpha) = \lambda_{T,I,F}(ţ*\alpha).ɤ \leq \max\{\lambda_{T,I,F}((ţ*\alpha)*(ȶ*\beta)), \lambda_{T,I,F}(ȶ*\beta)\}.ɤ \leq \max\{\max\{\lambda_{T,I,F}(((ţ*\alpha)*(ȶ*\beta))*(z*\gamma)), \lambda_{T,I,F}(z*\gamma)\}, \lambda_{T,I,F}(ȶ*\beta)\}.ɤ = \max\{\max\{\lambda_{T,I,F}(0), \lambda_{T,I,F}(z*\gamma)\}, \lambda_{T,I,F}(ȶ*\beta)\}.ɤ = \max\{\lambda_{T,I,F}(ȶ*\beta).ɤ, \lambda_{T,I,F}(z*\gamma).ɤ\} = \max\{{}^{M}_{ɤ}\lambda_{T,I,F}(ȶ*\beta), {}^{M}_{ɤ}\lambda_{T,I,F}(z*\gamma)\}$.

2. Again, take $ţ, ȶ \in X$ and $\alpha, \beta \in [0,1]$, such that $(ţ*\alpha) \leq (ȶ*\beta)$. Then $(ţ*\alpha)*(ȶ*\beta) = 0$ and thus ${}^{M}_{ɤ}H_{T,I,F}(ţ*\alpha) = H_{T,I,F}(ţ*\alpha).ɤ \geq rmin\{H_{T,I,F}((ţ*\alpha)*(ȶ*\beta)), H_{T,I,F}(ȶ*\beta)\}.ɤ = rmin\{H_{T,I,F}(0), H_{T,I,F}(ȶ*\beta)\}.ɤ = H_{T,I,F}(ȶ*\beta).ɤ = {}^{M}_{ɤ}H_{T,I,F}(ȶ*\beta), so\ {}^{M}_{ɤ}H_{T,I,F}(ţ*\alpha) \geq {}^{M}_{ɤ}H_{T,I,F}(ȶ*\beta)$ and ${}^{M}_{ɤ}\lambda_{T,I,F}(ţ*\alpha) = \lambda_{T,I,F}(ţ*\alpha).ɤ \leq$

$max\{\lambda_{T,I,F}((ţ*\alpha)*(ƫ*\beta)), \lambda_{T,I,F}(ƫ*\beta)\}.ɤ = \max\{\lambda_{T,I,F}(0), \lambda_{T,I,F}(ƫ*\beta)\}.ɤ = \lambda_{T,I,F}(ƫ*\beta).ɤ = {}^{M}_{ɤ}\lambda_{T,I,F}(ƫ*\beta)$, so ${}^{M}_{ɤ}\lambda_{T,I,F}(ţ*\alpha) \leq {}^{M}_{ɤ}\lambda_{T,I,F}(ƫ*\beta)$.

Theorem 3.6. Let ${}^{M}_{ɤ}Ƕ$ of $Ƕ = \{H_{T,I,F}, \lambda_{T,I,F}\}$ is a NCNID of X. $\forall$ $ţ, ƫ \in X$ and $\alpha, \beta \in [0,1]$, then Ƕ is a NCMSU of X.

Proof. Assume that ${}^{M}_{ɤ}Ƕ$ is a NCNID of X, $\forall$ $ţ, ƫ \in X$ and $\alpha, \beta \in [0,1]$. Then $ɤ.H_{T,I,F}((ţ*\alpha)*(ƫ*\beta)) = {}^{M}_{ɤ}H_{T,I,F}((ţ*\alpha)*(ƫ*\beta)) \geq rmin\{{}^{M}_{ɤ}H_{T,I,F}((ƫ*\beta)*((ţ*\alpha)*(ƫ*\beta))), {}^{M}_{ɤ}H_{T,I,F}(ƫ*\beta)\} = rmin\{{}^{M}_{ɤ}H_{T,I,F}(0), {}^{M}_{ɤ}H_{T,I,F}(ƫ*\beta)\} \geq rmin\{{}^{M}_{ɤ}H_{T,I,F}(ţ*\alpha), {}^{M}_{ɤ}H_{T,I,F}(ƫ*\beta)\} = rmin\{H_{T,I,F}(ţ*\alpha).ɤ, H_{T,I,F}(ƫ*\beta).ɤ\} = rmin\{H_{T,I,F}(ţ*\alpha), H_{T,I,F}(ƫ*\beta)\}.ɤ \Rightarrow H_{T,I,F}((ţ*\alpha)*(ƫ*\beta)) \geq rmin\{H_{T,I,F}(ţ*\alpha), H_{T,I,F}(ƫ*\beta)\}$ and $ɤ.\lambda_{T,I,F}((ţ*\alpha)*(ƫ*\beta)) = {}^{M}_{ɤ}\lambda_{T,I,F}((ţ*\alpha)*(ƫ*\beta)) \leq max\{{}^{M}_{ɤ}\lambda_{T,I,F}((ƫ*\beta)*((ţ*\alpha)*(ƫ*\beta))), {}^{M}_{ɤ}\lambda_{T,I,F}(ƫ*\beta)\} = max\{{}^{M}_{ɤ}\lambda_{T,I,F}(0), {}^{M}_{ɤ}\lambda_{T,I,F}(ƫ*\beta)\} \leq max\{{}^{M}_{ɤ}\lambda_{T,I,F}(ţ*\alpha), {}^{M}_{ɤ}\lambda_{T,I,F}(ƫ*\beta)\} = max\{\lambda_{T,I,F}(ţ*\alpha).ɤ, \lambda_{T,I,F}(ƫ*\beta).ɤ\} = max\{\lambda_{T,I,F}(ţ*\alpha), \lambda_{T,I,F}(ƫ*\beta)\}.ɤ \Rightarrow \lambda_{T,I,F}((ţ*\alpha)*(ƫ*\beta)) \leq max\{\lambda_{T,I,F}(ţ*\alpha), \lambda_{T,I,F}((ƫ*\beta))\}$. Hence, $Ƕ\{H_{T,I,F}, \lambda_{T,I,F}\}$ is a NCMSU of X.

4. ɤ-MULTIPLICATION UNDER HOMOMORPHISM

Theorem 4.1. Suppose that $\Gamma|X \to Y$ is a homomorphic mapping of PS-algebra. If ${}^{M}_{ɤ}Ƕ$ of $Ƕ = (H_{T,I,F}, \lambda_{T,I,F})$ is a NCNID of Y, then pre-image $\Gamma^{-1}({}^{M}_{ɤ}Ƕ) = (\Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F}), \Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F}))$ of ${}^{M}_{ɤ}Ƕ$ under Γ of X is a NCNID of X.

Proof. For all $ţ \in X$ and $\alpha \in [0,1]$, $\Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F})(ţ*\alpha) = {}^{M}_{ɤ}H_{T,I,F}(\Gamma(ţ*\alpha)) = H_{T,I,F}(\Gamma(ţ*\alpha)).ɤ \leq H_{T,I,F}(\Gamma(0)).ɤ = {}^{M}_{ɤ}H_{T,I,F}(\Gamma(0)) = \Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F})(0)$ and $\Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F})(ţ*\alpha) = {}^{M}_{ɤ}\lambda_{T,I,F}(\Gamma(ţ*\alpha)) = \lambda_{T,I,F}(\Gamma(ţ*\alpha)).ɤ \geq \lambda_{T,I,F}(\Gamma(0)).ɤ = {}^{M}_{ɤ}\lambda_{T,I,F}(\Gamma(0)) = \Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F})(0)$.

Let $ţ, ƫ \in X$, $\Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F})(ţ*\alpha) = {}^{M}_{ɤ}H_{T,I,F}(\Gamma(ţ*\alpha)) = H_{T,I,F}(\Gamma(ţ*\alpha)).ɤ \geq rmin\{H_{T,I,F}(\Gamma(ţ*\alpha)*\Gamma(ƫ*\beta)), H_{T,I,F}(\Gamma(ƫ*\beta))\}.ɤ = rmin\{H_{T,I,F}(\Gamma((ţ*\alpha)*(ƫ*\beta))), H_{T,I,F}(\Gamma(ƫ*\beta))\}.ɤ = rmin\{\Gamma^{-1}(H_{T,I,F}((ţ*\alpha)*(ƫ*\beta)).ɤ), \Gamma^{-1}(H_{T,I,F}(ƫ*\beta).ɤ)\} = rmin\{\Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F}((ţ*\alpha)*(ƫ*\beta))), \Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F}(ƫ*\beta))\}$ and $\Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F})(ţ*\alpha) = {}^{M}_{ɤ}\lambda_{T,I,F}(\Gamma(ţ*\alpha)) = \lambda_{T,I,F}(\Gamma(ţ*\alpha)).ɤ \leq max\{\lambda_{T,I,F}(\Gamma(ţ*\alpha)*\Gamma(ƫ*\beta)), \lambda_{T,I,F}(\Gamma(ƫ*\beta))\}.ɤ = max\{\lambda_{T,I,F}(\Gamma((ţ*\alpha)*(ƫ*\beta))), \lambda_{T,I,F}(\Gamma(ƫ*\beta))\}.ɤ = max\{\Gamma^{-1}(\lambda_{T,I,F}((ţ*\alpha)*(ƫ*\beta)).ɤ), \Gamma^{-1}(\lambda_{T,I,F}(ƫ*\beta).ɤ)\} = max\{\Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F}((ţ*\alpha)*(ƫ*\beta)), \Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F}(ƫ*\beta))\}$. Hence, $\Gamma^{-1}({}^{M}_{ɤ}Ƕ) = (\Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F}), \Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F}))$ is a NCNID of X.

Theorem 4.2. Let $\Gamma|X \to Y$ be a homomorphic mapping of B-algebra. If ${}^{M}_{ɤ}Ƕ_i$ of $Ƕ_i = (H^{i}_{T,I,F}, \lambda^{i}_{T,I,F})$ is a NCNID of Y where $i \in k$, then the pre-image $\Gamma^{-1}(\bigcap_{i\in k} {}^{M}_{ɤ}Ƕ^{i}_{T,I,F}) = (\Gamma^{-1}(\bigcap_{i\in k} {}^{M}_{ɤ}H^{i}_{T,I,F}), \Gamma^{-1}(\bigcap_{i\in k} {}^{M}_{ɤ}\lambda^{i}_{T,I,F}))$ is a NCNID of X.

Proof. We can prove this theorem through Theorem 3.2 and Theorem 4.1.

Theorem 4.3. Let $\Gamma|X \to Y$ is an epimorphic mapping of B-algebra.Then ${}^{M}_{ɤ}Ƕ = ({}^{M}_{ɤ}H_{T,I,F}, {}^{M}_{ɤ}\lambda_{T,I,F})$ is a NCNID of Y, if pre-image $\Gamma^{-1}({}^{M}_{ɤ}Ƕ) = (\Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F}), \Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F}))$ of ${}^{M}_{ɤ}Ƕ$ under Γ of X is a NCNID of X

Proof. For any $ƫ \in Y$, $ţ \in X$ and $\alpha, \beta \in [0,1]$ such that $(ƫ*\beta) = \Gamma(ţ*\alpha)$. Then ${}^{M}_{ɤ}H_{T,I,F}(ƫ*\beta) = {}^{M}_{ɤ}H_{T,I,F}(\Gamma(ţ*\alpha)) = \Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F})(ţ*\alpha) = \Gamma^{-1}(H_{T,I,F})(ţ*\alpha).ɤ \geq \Gamma^{-1}(H_{T,I,F})(0).ɤ = H_{T,I,F}(\Gamma(0)).ɤ = H_{T,I,F}(0).ɤ = {}^{M}_{ɤ}H_{T,I,F}(0)$ and ${}^{M}_{ɤ}\lambda_{T,I,F}(ƫ*\beta) = {}^{M}_{ɤ}\lambda_{T,I,F}(\Gamma(ţ*\alpha)) = \Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F})(ţ*\alpha) = \Gamma^{-1}(\lambda_{T,I,F})(ţ*\alpha).ɤ \leq \Gamma^{-1}(\lambda_{T,I,F})(0).ɤ = \lambda_{T,I,F}(\Gamma(0)).ɤ = \lambda_{T,I,F}(0).ɤ = {}^{M}_{ɤ}\lambda_{T,I,F}(0)$.

Assume $ƫ_1, ƫ_2 \in Y$. Then $\Gamma(ţ_1*\alpha) = ƫ_1*\beta$ and $\Gamma(ţ_2*\alpha) = ƫ_2*\beta$ for some $ţ_1, ţ_2 \in X$ and $\alpha, \beta \in [0,1]$. Thus ${}^{M}_{ɤ}H_{T,I,F}(ƫ_1*\beta) = {}^{M}_{ɤ}H_{T,I,F}(\Gamma(ţ_1*\alpha)) = \Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F})(ţ_1*\alpha) = \Gamma^{-1}(H_{T,I,F})(ţ_1*\alpha).ɤ \geq rmin\{\Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F})((ţ_1*$

$\alpha) * (ţ_2 * \alpha)), \Gamma^{-1}({}_{ɤ}^{M}H_{T,I,F})(ţ_2 * \alpha)\}.ɤ = rmin\{{}_{ɤ}^{M}H_{T,I,F}(\Gamma((ţ_1 * \alpha) * (ţ_2 * \alpha))), H_{T,I,F}(\Gamma(ţ_2 * \alpha))\}.ɤ = rmin\{H_{T,I,F}(\Gamma(ţ_1 * \alpha) * \Gamma(ţ_2 * \alpha)), H_{T,I,F}(\Gamma(ţ_2 * \alpha))\} = rmin\{H_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta)), H_{T,I,F}(ŧ_2 * \beta)\}.ɤ = rmin\{H_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta)).ɤ, H_{T,I,F}(ŧ_2 * \beta).ɤ\} = rmin\{{}_{ɤ}^{M}H_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta)), {}_{ɤ}^{M}H_{T,I,F}(ŧ_2 * \beta)\}$ and ${}_{ɤ}^{M}\lambda_{T,I,F}(ŧ_1 * \beta) = {}_{ɤ}^{M}\lambda_{T,I,F}(\Gamma(ţ_1 * \alpha)) = \Gamma^{-1}({}_{ɤ}^{M}\lambda_{T,I,F})(ţ_1 * \alpha) = \Gamma^{-1}(\lambda_{T,I,F})(ţ_1 * \alpha).ɤ \leq max\{\Gamma^{-1}({}_{ɤ}^{M}\lambda_{T,I,F})((ţ_1 * \alpha) * (ţ_2 * \alpha)), \Gamma^{-1}({}_{ɤ}^{M}\lambda_{T,I,F})(ţ_2 * \alpha)\}.ɤ = max\{{}_{ɤ}^{M}\lambda_{T,I,F}(\Gamma((ţ_1 * \alpha) * (ţ_2 * \alpha))), \lambda_{T,I,F}(\Gamma(ţ_2 * \alpha))\}.ɤ = max\{\lambda_{T,I,F}(\Gamma(ţ_1 * \alpha) * \Gamma(ţ_2 * \alpha)), \lambda_{T,I,F}(\Gamma(ţ_2 * \alpha))\} = max\{\lambda_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta)), \lambda_{T,I,F}(ŧ_2 * \beta)\}.ɤ = max\{\lambda_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta)).ɤ, \lambda_{T,I,F}(ŧ_2 * \beta).ɤ\} = max\{{}_{ɤ}^{M}\lambda_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta)), {}_{ɤ}^{M}\lambda_{T,I,F}(ŧ_2 * \beta)\}$.
Hence, ${}_{ɤ}^{M}Ƕ = ({}_{ɤ}^{M}H_{T,I,F}, {}_{ɤ}^{M}\lambda_{T,I,F})$ is a NCNID of Y.

5. ɤ-MULTIPLICATION OF CARTESIAN PRODUCT

Theorem 5.1. Let ${}_{ɤ}^{M}Ƕ = ({}_{ɤ}^{M}H_{T,I,F}, {}_{ɤ}^{M}\lambda_{T,I,F})$ and ${}_{ɤ}^{M}Ƒ = ({}_{ɤ}^{M}F_{T,I,F}, {}_{ɤ}^{M}\mu_{T,I,F})$ are NCNIDs of X and Y respectively. Then ${}_{ɤ}^{M}Ƕ \times {}_{ɤ}^{M}Ƒ$ is a neutrosophic cubic normal ideal of $X \times Y$.

Proof. For any $(ţ, ŧ) \in X \times Y$ and $\alpha, \beta \in [0,1]$. We have $({}_{ɤ}^{M}H_{T,I,F} \times {}_{ɤ}^{M}F_{T,I,F})(0,0) = ɤ.(H_{T,I,F} \times F_{T,I,F})(0,0) = ɤ.rmin\{H_{T,I,F}(0), F_{T,I,F}(0)\} \geq ɤ.rmin\{H_{T,I,F}(ţ * \alpha), F_{T,I,F}(ŧ * \beta)\} = rmin\{H_{T,I,F}(ţ * \alpha).ɤ, F_{T,I,F}(ŧ * \beta).ɤ\} = rmin\{{}_{ɤ}^{M}H_{T,I,F}(ţ * \alpha), {}_{ɤ}^{M}F_{T,I,F}(ŧ * \beta)\} = ({}_{ɤ}^{M}H_{T,I,F} \times {}_{ɤ}^{M}F_{T,I,F})(ţ * \alpha, ŧ * \beta)$ and $({}_{ɤ}^{M}\lambda_{T,I,F} \times {}_{ɤ}^{M}\mu_{T,I,F})(0,0) = ɤ.(\lambda_{T,I,F} \times \mu_{T,I,F})(0,0) = ɤ.max\{\lambda_{T,I,F}(0), \mu_{T,I,F}(0)\} \leq ɤ.max\{\lambda_{T,I,F}(ţ * \alpha), \mu_{T,I,F}(ŧ * \beta)\} = max\{\lambda_{T,I,F}(ţ * \alpha).ɤ, \mu_{T,I,F}(ŧ * \beta).ɤ\} = max\{{}_{ɤ}^{M}\lambda_{T,I,F}(ţ * \alpha), {}_{ɤ}^{M}\mu_{T,I,F}(ŧ * \beta)\} = ({}_{ɤ}^{M}\lambda_{T,I,F} \times {}_{ɤ}^{M}\mu_{T,I,F})(ţ * \alpha, ŧ * \beta)$.

Let $(ţ_1, ŧ_1), (ţ_2, ŧ_2) \in X \times Y$ and $\alpha, \beta \in [0,1]$. Then $({}_{ɤ}^{M}H_{T,I,F} \times {}_{ɤ}^{M}F_{T,I,F})(ţ_1 * \alpha, ŧ_1 * \beta) = ɤ.(H_{T,I,F} \times F_{T,I,F})(ţ_1 * \alpha, ŧ_1 * \beta) = ɤ.rmin\{H_{T,I,F}(ţ_1 * \alpha), F_{T,I,F}(ŧ_1 * \beta)\} \geq ɤ.rmin\{rmin\{H_{T,I,F}((ţ_1 * \alpha) * (ţ_2 * \alpha)), H_{T,I,F}(ţ_2 * \alpha)\}, rmin\{F_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta)), F_{T,I,F}(ŧ_2 * \beta)\}\} = ɤ.rmin\{rmin\{H_{T,I,F}((ţ_1 * \alpha) * (ţ_2 * \alpha)), F_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta))\}, rmin\{H_{T,I,F}(ţ_2 * \alpha), F_{T,I,F}(ŧ_2 * \beta)\}\} = ɤ.rmin\{(H_{T,I,F} \times F_{T,I,F})((ţ_1 * \alpha) * (ţ_2 * \alpha), (ŧ_1 * \beta) * (ŧ_2 * \beta)), (H_{T,I,F} \times F_{T,I,F})((ţ_2 * \alpha), (ŧ_2 * \beta))\} = rmin\{(H_{T,I,F} \times F_{T,I,F})((ţ_1 * \alpha, ŧ_1 * \beta) * (ţ_2 * \alpha, ŧ_2 * \beta)).ɤ, (H_{T,I,F} \times F_{T,I,F})(ţ_2 * \alpha, ŧ_2 * \beta).ɤ\} = rmin\{({}_{ɤ}^{M}R_{T,I,F} \times {}_{ɤ}^{M}F_{T,I,F})((ţ_1 * \alpha, ŧ_1 * \beta) * (ţ_2 * \alpha, ŧ_2 * \beta)), ({}_{ɤ}^{M}R_{T,I,F} \times {}_{ɤ}^{M}F_{T,I,F})(ţ_2 * \alpha, ŧ_2 * \beta)\}$ and $({}_{ɤ}^{M}\lambda_{T,I,F} \times {}_{ɤ}^{M}\mu_{T,I,F})(ţ_1 * \alpha, ŧ_1 * \beta) = ɤ.(\lambda_{T,I,F} \times \mu_{T,I,F})(ţ_1 * \alpha, ŧ_1 * \beta) = ɤ.max\{\lambda_{T,I,F}(ţ_1 * \alpha), \mu_{T,I,F}(ŧ_1 * \beta)\} \leq ɤ.max\{max\{\lambda_{T,I,F}((ţ_1 * \alpha) * (ţ_2 * \alpha)), \lambda_{T,I,F}(ţ_2 * \alpha)\}, max\{\mu_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta)), F_{T,I,F}(ŧ_2 * \beta)\}\} = ɤ.max\{max\{\lambda_{T,I,F}((ţ_1 * \alpha) * (ţ_2 * \alpha)), \mu_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta))\}, max\{\lambda_{T,I,F}(ţ_2 * \alpha), \mu_{T,I,F}(ŧ_2 * \beta)\}\} = ɤ.max\{(\lambda_{T,I,F} \times \mu_{T,I,F})((ţ_1 * \alpha) * (ţ_2 * \alpha), (ŧ_1 * \beta) * (ŧ_2 * \beta)), (\lambda_{T,I,F} \times \mu_{T,I,F})((ţ_2 * \alpha), (ŧ_2 * \beta))\} = max\{(\lambda_{T,I,F} \times \mu_{T,I,F})((ţ_1 * \alpha, ŧ_1 * \beta) * (ţ_2 * \alpha, ŧ_2 * \beta)).ɤ, (\lambda_{T,I,F} \times \mu_{T,I,F})(ţ_2 * \alpha, ŧ_2 * \beta).ɤ\} = max\{({}_{ɤ}^{M}\lambda_{T,I,F} \times {}_{ɤ}^{M}\mu_{T,I,F})((ţ_1 * \alpha, ŧ_1 * \beta) * (ţ_2 * \alpha, ŧ_2 * \beta)), ({}_{ɤ}^{M}\lambda_{T,I,F} \times {}_{ɤ}^{M}\mu_{T,I,F})(ţ_2 * \alpha, ŧ_2 * \beta)\}$. Hence, ${}_{ɤ}^{M}Ƕ \times {}_{ɤ}^{M}Ƒ$ is a neutrosophic cubic normal ideal of $X \times Y$.

Theorem 5.2. Let ${}_{ɤ}^{M}Ƕ = ({}_{ɤ}^{M}H_{T,I,F}, {}_{ɤ}^{M}\lambda_{T,I,F})$ and ${}_{ɤ}^{M}Ƒ = ({}_{ɤ}^{M}F_{T,I,F}, {}_{ɤ}^{M}\mu_{T,I,F})$ are two ɤ-multiplications of neutrosophic cubic closed normal ideals of X and Y respectively. Then ${}_{ɤ}^{M}Ƕ \times {}_{ɤ}^{M}Ƒ$ is a NCCNID of $X \times Y$.

Proof. By Proposition 3.1 and Theorem 5.1, ${}_{ɤ}^{M}Ƕ \times {}_{ɤ}^{M}Ƒ$ is NCNID. Now, $({}_{ɤ}^{M}H_{T,I,F} \times {}_{ɤ}^{M}F_{T,I,F})((0,0) * (ţ * \alpha, ŧ * \beta)) = (H_{T,I,F} \times F_{T,I,F})((0,0) * (ţ * \alpha, ŧ * \beta)).ɤ = (H_{T,I,F} \times F_{T,I,F})(0 * (ţ * \alpha), 0 * (ŧ * \beta)).ɤ = ɤ.\mathrm{rmin}\{\mathrm{H}_{T,I,F}(0 * (ţ * \alpha)), \mathrm{F}_{T,I,F}(0 * (ŧ * \beta))\} \geq ɤ.\mathrm{rmin}\{\mathrm{H}_{T,I,F}(ţ * \alpha), \mathrm{F}_{T,I,F}(ŧ * \beta)\} = \mathrm{rmin}\{\mathrm{H}_{T,I,F}(ţ * \alpha).ɤ, \mathrm{F}_{T,I,F}(ŧ * \beta).ɤ\} = \mathrm{rmin}\{{}_{ɤ}^{M}\mathrm{H}_{T,I,F}(ţ * \alpha), {}_{ɤ}^{M}\mathrm{F}_{T,I,F}(ŧ * \beta)\} = ({}_{ɤ}^{M}\mathrm{H}_{T,I,F} \times {}_{ɤ}^{M}\mathrm{F}_{T,I,F})(ţ * \alpha, ŧ * \beta)$ and $({}_{ɤ}^{M}\lambda_{T,I,F} \times {}_{ɤ}^{M}\mu_{T,I,F})((0,0) * (ţ * \alpha, ŧ * \beta)) = (\lambda_{T,I,F} \times \mu_{T,I,F})((0,0) * (ţ * \alpha, ŧ * \beta)).ɤ = (\lambda_{T,I,F} \times \mu_{T,I,F})(0 * (ţ * \alpha), 0 * (ŧ * \beta)).ɤ = ɤ.\max\{\lambda_{T,I,F}(0 * (ţ * \alpha)), \mu_{T,I,F}(0 * (ŧ * \beta))\} \leq ɤ.\max\{\lambda_{T,I,F}(ţ * \alpha), \mu_{T,I,F}(ŧ * \beta)\} = \max\{\lambda_{T,I,F}(ţ * \alpha).ɤ, \mu_{T,I,F}(ŧ * \beta).ɤ\} = \max\{{}_{ɤ}^{M}\lambda_{T,I,F}(ţ * \alpha), {}_{ɤ}^{M}\mu_{T,I,F}(ŧ * \beta)\} = ({}_{ɤ}^{M}\lambda_{T,I,F} \times {}_{ɤ}^{M}\mu_{T,I,F})(ţ * \alpha, ŧ * \beta)$. Hence, ${}_{ɤ}^{M}Ƕ \times {}_{ɤ}^{M}Ƒ$ is a neutrosophic cubic closed normal ideal of $X \times Y$.

6. Conclusion

In this paper, the notion of ɤ-multiplication of neutrosophic cubic set was introduced and ɤ-multiplication was studies by several useful results. This study will provide the base for further work like t-neutrosophic soft cubic and intuitionistic soft cubic set etc

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Neutrosophy for physiological data compression: in particular by neural nets using deep learning

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Abstract: We would like to show the small distance in neutropsophy applications in sciences and humanities, has both finally consider as a terminal user a human. The pace of data production continues to grow, leading to increased needs for efficient storage and transmission. Indeed, the consumption of this information is preferably made on mobile terminals using connections invoiced to the user and having only reduced storage capacities. Deep learning neural networks have recently exceeded the compression rates of algorithmic techniques for text. We believe that they can also significantly challenge classical methods for both audio and visual data (images and videos).

To obtain the best physiological compression, i.e. the highest compression ratio because it comes closest to the specificity of human perception, we propose using a neutrosophical representation of the information for the entire compression-decompression cycle. Such a representation consists for each elementary information to add to it a simple neutrosophical number which informs the neural network about its characteristics relative to compression during this treatment. Such a neutrosophical number is in fact a triplet (t,i,f) representing here the belonging of the element to the three constituent components of information in compression; 1° t = the true significant part to be preserved, 2° i = the inderterminated redundant part or noise to be eliminated in compression and 3° f = the false artifacts being produced in the compression process (to be compensated). The complexity of human perception and the subtle niches of its defects that one seeks to exploit requires a detailed and complex mapping that a neural network can produce better than any other algorithmic solution, and networks with deep learning have proven their ability to produce a detailed boundary surface in classifiers.

Keywords: Compression – physiological data compression – neural nets – neutrosophic – deep learning

1 Introduction

Here we would like to emphase the incidence of the human aspect present in every application, and therefore in any theory. In the end, every development address directly or indirectly on a more close or further relation a human user. The physical specificities of the human being as a user, also on the cognitive side, are quite well defined and relatively homogeneous. Therefore the applications, developments and theories can be suited, tailored, to take into account those characteristics for optimization. In our current case, we consider an improvement to data compression adapted to human perception such as in hearing of seeing. Based on the closeness of human cognition/perception with artificial intelligence inspired by neural networks, we propose a narrower adaptation to the human receiver through the highly-performing deep-learning methods with now adding the potential of the 3-state approach of neutrosophy, that we believe is closer to human cognition than classical binary logic, and also closer to the compression process itself.

More and more data is produced, then transmitted and stored. In both cases a compression of this data allows to optimize these operations in cost and time. The main types of data are text, sound, images and videos, in ascending order of sizem as well as compression potential. Currently videos are the most used by the majority of people. This data is often accessed on mobile devices such as smartphones that communicate through a user-charged connection- Also, their solid-state memory storage is more expensive than traditional hard disks. Thus, more efficient compression methods would bring significant advantages on both levels (transmission and storage).

Two main categories of compression are considered according to usage: lossless compression and lossy compression. Lossless compression fully restores initial information after decompression. A theoretical limit to the compression ratio is given by Shannon's theorem [1]. Lossy compression produces better compression rates at the cost of information loss: decompression produces only a degraded version of the original information. However, losses can be intelligently directed mainly towards limitations of human perception (and also to less relevant data). This type of compression. Physiological (adapted to the imperfections of the human physiology), is used when the information is intended for a human being who will receive it through his perceptive system, mainly auditory and visual. Thus a certain deterioration of the message is admissible, and it can even pass completely unnoticed. Sometimes some perceptible degradation is even tolerable and higher compression rates can be achieved.

In such lossy compression the decompressed information differs from the original information in three ways: (1) some initial elements important to the human receptor have disappeared, (2) other elements have also disappeared but are not important to the human receptor, and, as a result of treatments performed to best eliminate this second category, (3) parasitic elements (called artifacts) are unfortunately introduced into the decompressed information. In the first case there is degradation by loss of content, in the second case there is compression by voluntary deletion of insignificant content, and in the third case there is degradation by disturbance. A good compression technique is characterized by maximizing category 2 with simultaneous minimization of categories 1 and 3. Often compression methods allow compression and degradation to be exchanged according to the objectives sought, favouring one or the other.

Text compression exploits the existing dependency between one symbol and the following ones (character, word, sentence, paragraph). The audio compression also, and more often performs a transformation to the frequency domain where this redundancy is more apparent, and therefore easier to exploit. For images, the redundancy to be eliminated is sought spatially and sometimes hierarchically, but a frequency transformation is often exploited too. Finally for the videos a temporal redundancy is also exploited, an image of a film varying only little from the previous one.

Neural networks, especially those with a large number of layers (deep learning), most often act by detection, classification, transformation of representation space when it is not directly by coding and compression. Thus they reproduce the major features of compression methods. In particular for the image processing they realize an automatic identification of the most significant characteristics which is the typical operation of an encoding with optimal compression.

2 Current research

Although little has been found in the literature, recent research on deep multilayer neural networks, known as deep learning, has also focused on compression techniques and new methods have been proposed for both lossless and lossy compression. In general, these networks have the potential to discover relatively hidden relationships and exploit them to reduce redundancy as compression requires. In the second category their advantage would also be to better exploit the limits of human perception and thus to adjust compression as close as possible both to what is most important for the observer and to that, while being parasitic, disturbs him the least. The adaptation to the physiology of human perception can be pushed further than with algorithmic methods. Another positive point is their ability to perform sub-optimal processing at a much lower cost than full processing (algorithmic), and even to exploit fast vector or matrix processors existing in graphics rendering systems present even on low-end mobile phones. Builders of such graphics systems have begun to produce such architectures optimized also for deep learning, counting on the rapid emergence of this field.

In the most demanding field of lossless text compression, the method proposed by Tatwawadi [2] exceeds existing compression methods for complex corpuses such as chromosome data with a compression ratio of 82.5% versus 76.2% for the most used gZip compressor (free version with GNU license of Zip, derived from LWZ, itself an extension of LZ78 due to Jacob Ziv[3] and my prominent former colleague Abraham Lempel at HP Labs). Let us also note that the best result achieved for the compression of the Wikipedia encyclopedia, compression ratio of 84.6%, is obtained by another deep learning neural network technique choosing the best result among 2000 competing compressors [4], of which the one proposed by this reference is significantly close at 83.2%.

For the most commonly used type of compression, lossy compression, few experiments have been performed. In the case of images, preluding the largest category in quantity: video, mixed techniques are also used. They consist in mixing a preprocessing by neural networks with a usual image compression algorithm such as JPEG. For test images the similarity to the original was measured for an identical compression ratio, the method proposed by Harshavardhan [5] produces the best quality, the similarity measurement varying from 0.82 to 0.89 with an average of 0.85 against 0.75 for JPEG. This method also has the advantage of low computing costs and parallel implementation (distribution over several processors).

3 Proposal

In addition to the use of deep learning neural networks for lossy and lossless compression, we propose the introduction of a treatment based on a neutrosophical approach for lossy physiological compression. By using the model of neutrosophical representation of the data to be compressed, deep learning can be directed towards a more optimal treatment in the sense of the maximisation (2) and the minimisations (1 and 3) mentioned above, thus taking better account of the physiology of human perception and making the most of its weaknesses.

Note that Kuwar [6] has already successfully experimented with the use of convolutional deep learning neural networks to reduce artifacts introduced by JPEG compression in the frequency domain.

A neutrosophical approach is to consider information no longer in a precise and binary way, but fuzzy and ternary. By fuzzy we refer to fuzzy logic [7] where a value is not 0 or 1 but may be 0 with some membership function and 1 with some other membership function. By ternary we mean a representation of three states: true state, false state and undetermined or neutral state. This last state, neutral, gave its name to neutrosophy, which was originally a philosophy, but which developed by extending its central logical aspect into a mathematical one. This philosophy corresponds more to the human way of thinking, and this mathematics is more general than that of binary logic (the one of first order predicates) as well as that of fuzzy logic (and includes them as special cases). We return in more details to neutrosophy in the next section.

Indeed, if human cognition is neutrosphic, that is, its way of thinking consistently implies the use of a neutral state, then so is its perception. All human reflection is not only rational, i.e. logical, but is also influenced by affective aspects. Every reflection is tinged with judgment, both in its result and first of all in its premises and steps. Behind every elementary idea there is closely associated an affect, which is a value judgment: I like it, I do not like it or I am indifferent, that is to say neutral. The affect or judgment is neutrosophical in nature, ternary, with an important neutral aspect because it is frequent.

In any act of human perception, part of the information continues its way into the human brain system, while another part is more or less actively rejected and the rest is ineffective, neutral. It is also so in the various subsequent processing steps.This is in fact a semantic extraction (not to say a compression) seeking to isolate the relevant according to the context by rejecting what is determined not significant according to this same context, and the rest which is declared neutral.

We mentioned earlier the three aspects of compression on a physiological basis which is to detect the most significant, to ignore the neutral, but also to reject noise, the non-relevant. The orientation towards these three functions - detection, deactivation (passive), rejection (active) - of learning (and preception and cognition and compression) can be produced by a neutrosphic representation of the information throughout the processing by the neural network according to the three neutrosophic states: meaning, to be reject or neutral. Thus the network will organize itself more efficiently and more rapidly towards the three joint functions of optimal physiological compression: selection of what is significant for the human receptor, rejection of disturbuing artefacts produced by extreme compression in the non-physiologically perceptible, and elimination of all that is neutral for the human receptor. Then the mechanism of compression can be pushed to its limits given human perception.

4 Some information about neutrosphy

Recent trends in the use of neutrosophy can be found in the reference work edited by Smanradache and Pramanik entitled "New trends in neutrosophical theory and its applications" [8].

As previously mentioned, the inspiration for the neutrosophical representation of reality derives from the philosophy called neutrosophy. This representation is general and makes possible to unify in particular the various apparently very distinct logic variants: classical logic, also called binary logic or Bool's logic, fuzzy logic and its various varieties, and itself, a three-state logic, characterized by a neutral state [9].

In summary, instead of a logical value with two crisp states 0 or 1, the neutrosophical approach considers a representation by a triplet (t.i,f) where these three real values t, i and f represent the equivalent of probabilities for truth (t), indetermination or neutral state (i) and falsity (f) respectively. We prefer to speak of belonging functions according to the vocabulary used in fuzzy logic. These three values are between 0 and 1. Thus, the two classical binary logic values 0 and 1 are represented respectively by (0,0,1) and (1,0,0). Now a simple probability p of having the value 1 and therefore (1-p) of having the value 0 is represented by (p,0,1-p). In this particular case the neutrosophical representation mainly brings a general formulation (just like for a binary value), and thus it also makes possible to represent this conception which it encompasses in its generality (also for fuzzy logic and its numerous varieties of which the so-called "intuitionistic" one).

Operations on neutrosophical triplets, preferably called simple neutrosophical numbers (for simple value, in the sense of mono, a single triplet), can be defined in various ways, either by using arithmetic operators (e.g. multiplication for logical-AND) or functional operators (such as minimum, maximum, etc). For example the complement of (t,i,f) can be defined as (f,1-i,t), but other conventions may be more appropriate depending on the applications. Let us return now on the preceding case of an operator with two operands, as the logical AND mentioned before, let us consider this time the logical OR, i.e. the union together. For two simple neutrosophical numbers A and B represented by the triplets (t_a,i_a,f_a) and (t_b,i_b,f_b) then their union A OR B will be the triplet $(\max(t_a,t_b), \max(i_a,i_b), \min(f_a,f_b))$.

Although a neural network is self-organisîng according to a learning algorithm cleverly elaborated and parameterised to use internally representations adapted to the problem to be solved, in particular it can carry out a change of reference, a projection and other operations that can be geometrically illustrated. This autonomous organisation is however costly in terms of learning time but also in terms of the quality of the performance produced. If, for a given application, it is known that a representation is generally chosen for powerful classical algorithms, then it is highly likely that the network will choose a similar representation, a relatively close one. Indeed often for a specific application it is preferable to start from a network pre-trained on a problem either more general or rather close, which precisely means to start from a relatively appropriate representation.

Then the neutrosophical representation can be precisely one of these representations appropriate for a class of problems considered. And we postulate that this is the case for lossy physiological compression.

5 Physiological compression based on neutrosophical representation

As highlighted in the presentation of our proposal, lossy cuts are essentially adjusted in the case of a human receptor. It can be for example an audio content, an image, or a video sequence. At the end of the processing, storage and transmission chain there is a human user who will receive the message after decompression. This message carries a semantic content which is the most important, it must be perceived by the human receiver through his capacities of perception, mainly auditory and visual. However, human perceptual systems are not perfect and have a number of defects (more than a hundred for vision, the best known of which are optical illusions) that are generally not disturbing for the proper perception of common messages.

These defects have the advantage that they can be exploited in the overall organization of the compression process. And this in two ways: first, it is useless to transmit what cannot be perceived and it should therefore be deleted during compression. But also the involuntary residues of compression, the artefacts, must be organized to preferentially be located in these "blind" zones, orat least the less sensitive ones.

By training on appropriate examples, neural networks can learn to optimize these two situations, just as they are able to detect what is important, for example in speech recognition or artificial vision.

As we are in the presence of three components of the original information: (1) the significant, (2) the insignificant neutral and (3) the disturbing, and as we want to exploit this decomposition for compression, more exactly to realize this decomposition in those three components (or close to). Then to increase the performance of the neural network and facilitate its learning it is preferable to work entirely according to these three components, that is to say according to a neutrosophical paradigm. So the network does not have to discover this helping organization; we give it as a starting and working orientation.

For example, consider the case of image compression. The first step would be a pre-treatment to transform it into a neutrosophically represented image for the purpose of physiological perception. For each pixel of the image conventionally represented by an RGB triplet (red, green, blue), it would be productive to convert it in a more significant color space like HLS (Hue, Luminosity, Saturation) then to add the neutrosophical triplet (t,i,f). The three neutrosophical components t, i and f would then have the significance of probabilities of belonging to the meaningful part of the image, respectively to the potentially irrelevant one and most drastically compressible part, but also to a potential artifact zone. These three values t, i and f could be initialized by specific networks of significant content detection, semantic "noise" zones, and zones potentially tainted with compression artifacts. An additional (pre)treatment would be to move in the frequency domain (other representation) by a transformation of the FFT type, even it can mean letting the network treat jointly the two domains: the spatial and the frequency one.

The Fig. 1 illustrates examples of artifacts produced by compression. Note the "shadow" of the balloon in the sky, and similarly the edges of the star after magnification.

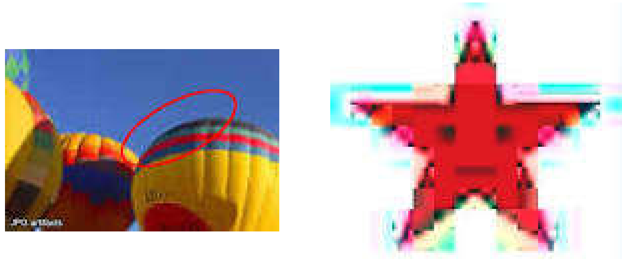

Figure 1: Examples of compression artifacts

Such an artifact is more visible when located in a quiet area of the image. This area can be identified by a neural network using deep learning and corrected during decompression at the cost of a small additional amount of data. It can potentially be partly counterbalanced before, during compression. Image segmentation and other content extraction operations performed efficiently by neural networks exploited for artificial vision can help to delimit the most significant areas and by complement (or preferably by specific processing) the least significant areas.

Finally, this triple process of accentuation of the significant, reduction of the non-significant as well as compensation of the artefacts produced (or to occur) can be carried out in a dynamic way throughout the compression process. Thus the neutrosophical representation of the information is exploited throughout the processing of compression, conducted in a physiologically adapted manner. In total, it must be possible to achieve a higher compression ratio than with other methods because it will include physiological aiming (thanks to neutrosophical representation) at the heart of the compression process.

Conclusion

On the basis of works demonstrating the performance of neural networks using the deep learning paradigm for lossless text and lossy image compression, we recommend to preferentially consider such solutions for other types of compression whatever audio and video as well.

In such cases of data reduction high compression ratios can be achieved when the receptor is human and the compression can be adjusted to its physiological abilities of perception. We postulate that neural networks of the deep learning kind are the most optimal solution to such a multi-criteria fine tuning (problem equivalent to multi-criteria classification).

Moreover, we propose to use a neutrosophic representation of the information throughout the compression-decompression process because its three components of belonging forming the triplet (t, i ,f) make it possible to direct such networks towards the triple representation both characteristic of lossy compression and the physiology of human perception, from which the former actually derives. The first component t then corresponds to the information truely significant for the human receptor (and in a relatively equivalent manner for a network working on recognition applications), the second noted i corresponds to what can be the most highly compressed because it is the least significant (neutral) according to human physiology (and also, by making a complementary use of the detection capabilities of the network, as content), and respectively the third noted f denoting as falsity all that is artifact potentially produced by the compression process itself (which can be learned by a network).

The whole process of compression-decompression is to be considered in the neutrosphical representation to leave to the network the possibility of exploiting compression on a physiological basis at best.

We believe that this is realistic in the short term because of the hardware vector processing accelerators available, even in mobile phones, and that higher compression ratios, respectively at equal ratio better fidelity, can be achieved.

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Plithogenic set for multi-variable data analysis

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Abstract

he m-polar and multi-dimensional data sets given a platform to deal with multi--valued attributes. In this case, a problem addressed that sometimes the attributes may contain many types of opposites, non--opposites and neutrals values as for example Rainbow. One of the best examples is sports data sets where each time the value of an attribute changes several time towards the given team, the opposition of the given team as well as draw conditions. The precise representation of these types of data sets and their mathematical analysis are crucial tasks for the research communities. The current paper tried to develop new mathematical set theories for precise representation and analysis of sports data via plithogenic set and its mathematical algebra.

Keywords: Plithogenic data, multi-valued, data, Neutrosophic set, Knowledge representation

1.Introduction

The theory of neutrosophic set is introduced by Smarandache [1] considering the concept of fuzzy sets [2] for handling acceptation, rejection and uncertain part. This theory is recently utilized for data analysis and processing tasks [3]. This given a well established platform to analyze the n-valued data sets [4-5] based on their acceptance, rejection and uncertain part. In this process, the researchers addressed that there are several data sets that contains many types of dynamics, opposites, neutrals as well as non--opposites sides. One of the best suitable example is

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sports data sets which contain many types of opposite, non--opposite, neutral side for the given match. Same time the behavior of the crowd, address of any person contains many types of dynamics. This type of data set and its representation is called as ``Plithogenic set'' by Smarandache in 2017 [6-7]. One of the suitable examples is student CGPA is based on more than several papers performance. For example, a faculty says "The student (x1) is intelligent". This proposition is statement can be based on more than three or four subject performance according to the experts as: Science (whose attribute values are: mathematics, physics, anatomy), Literature (whose attribute values are: poetry, novel), and Arts (whose only attribute value is: sculpture). Nowadays online TV channel pack is based on multi-valued different types of attributes and customers select the particular group which fulfills the requirement. In these cases, difficult for the users to select the subscribe the particular pack for the given channel. To resolve this issue current paper focuses on plithogenic set, its uses and applications is discussed for better understanding.

The Plitho--geny word coined from: crowd, a large number of events, to be blended or mixed together. It means the plithogeny may utilize as genesis or formation, development of many static or dynamic attributes at the same time. In this way plithogeny set contains contradictory, neutrals, non-contradictory multi-valuedd attributes to represent the particular event or knowledge. Hence it is hybrid of many opposites, neutrals and non-opposites attributes connected or non--connected to represent the particular concept. A continuum process of merging, and splitting, or integration and disintegration as happened in chemistry, opinion mining, social synchrony. Same time the psychological view that conscious, unconscious, Aconscious, Optimism, Pesimism is plithogenic view that used to change several times based on our degree of perception. One of the suitable examples is win, loss and draw of any matches fluctuate several times for the same match. The precise representation of this data set is one of the crucial tasks for the researchers when this large number of dynamic data generated at each moment which can be done using plithogeny set. To achieve this goal, the current paper focuses on dealing with these types of multi--attributes and its algebra for further applications in knowledge processing tasks. One of the suitable examples is the performance of a cricket player is based on plithogenic set which increases several times, decrease several time and may consistent. It is difficult to measure the performance of a player for the selectors at a given phase

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of time. To deal with these types of issues current paper works on plithogenic set and its properties.

Other parts are composed as follows: Section 2 contains preliminaries about neutrosophic set to measure the linguistics. Section 3 contains introduction of plithogenic set and its graphical applications. Section 4 contains conclusions and references.

2. Background

2.1 Neutrosophic Set:

The neutrosophic set consisting reptile functions namely truth, indeterminacy and false, (T, I, F), independently. Each of these values lies between 0 and 1 and does not depend on them. The boundary conditions of sum of these membership degrees $0 \leq T + I + F \leq 3$. In this 0 is hold for the universal false cases and 3 are the universal truth cases three memberships.

i.e. $\lambda = \{\langle x : T, I, F \rangle : x \in \xi\}$......................

This set contains triplet having true, a false and indeterminacy membership values which can be charaterized independently, T_N, F_N , I_N, independently in [0,1]. It can be abbreviate as follows:

$$N = \{<k;\ T_N(k), I_N(k),\ F_N(k)>: k \in \xi;\ T_N(k), I_N(k),\ F_N(k) \in \xi]^{-}0,\ 1^{+}[\} \quad \ldots\ldots(1)$$

There is no restriction on the sum of $T_N(k)$,$I_N(k)$ and $F_N(k)$.

$$\text{So } 0^{-} \leq T_N(k) + I_N(k) + F_N(k) \leq 3^{+}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

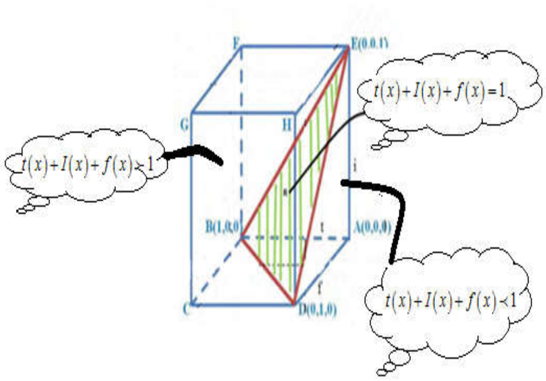

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Figure 1. The graphical visualization of neutrosophic environment

2.2. Linguistic neutrosophic set.

A single valued neutrosophic linguistic set A (abbr. SVNLS)in ξcan bedefined as

$$N_{SVNLS} = \left\{ \left\langle x, \left[s_{\theta(x)}, s_{\rho(x)} \right], \left(T_N(k), I_N(k), F_N(k) \right) \right\rangle \middle| k \in \xi \right\} (37)$$

where $s_{\theta(x)}, s_{\rho(x)} \in \hat{s}$, $T_N(k) \subseteq [0,1]$, $I_N(k) \subseteq [0,1]$, $F_N(k) \subseteq [0,1]$ with the condition $0 \le T_N(k) + I_N(k) + F_N(k) \le 3$, for any x $\in \xi$. $\left[s_{\theta(x)}, s_{\rho(x)} \right]$ is an uncertain linguistic term,The function $T_N(x)$, $I_N(x)$, $F_N(x)$ express, respectively, the degree of truth - membership, the degree of indeterminacy - membership and the degree of falsehood - membership. Of the element x in ξbelonging to the linguistic term $\left[s_{\theta(x)}, s_{\rho(x)} \right]$.

In this process the author address that there are several attributes which changes several times in opposites, neutral and non-opposites side. One of the best suitable example is a cricket match which changes several times. The precise analysis of these types of uncertainty in multi-valued attributes. The current paper focused on utilizing the properties of plithogenic set in the next sextion.

2.3. Plithogenic Set

Let ξbe a universe of discourse, P be a subset of this universe of discourse, "a" be a multi-valued attribute, V the range of the multi-valued attribute, "d" be the known (fuzzy, intuitionistic fuzzy, or neutrosophic) degree of appurtenance with regard to some generic of element x's attribute value to the set P, and c the (fuzzy, intuitionistic fuzzy, neutrosophic) degree of contradiction (dissimilarity) between attribute values as (<*A*, Neutral *A*, Anti *A*>; <*B*, Neutral *B*, Anti *B*>; <*C*, Neutral *C*, Anti *C*>)

Then (P, a, V, d, c) is named a Plithogenic Set.

A plithogenic set is a set P whose each element x is characterized by many attribute values.

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A generic element $x \in P$ is therefore characterized by all attribute's values in $V = \{v_1, v_2, \ldots, v_n\}$, for $n \geq 1$.

The precise representation of plithogenic operators, a contradiction (dissimilarity) degree function c(. , .) between the attribute values is implemented. Each plithogenic operator is a linear combinations of the fuzzy t-norm and fuzzy t-conorm:

$c: V \times V \rightarrow [0, 1]$ is the contradiction degree function between the values $v_1$and v_2, noted as $c(v_1, v_2)$, and satisfying the following axioms:

$c(v_1, v_1) = 0,$

$c(v_1, v_2) = c(v_2, v_1),$ commutativity.

Into the set V, in general, one has a dominant attribute value (the most important attribute value in V) that is established by each expert upon the application needed to solve.

It means the plithogenic set may contains four or more then four-valued attribute. In this case, the plithogenic aggregation operators (intersection, union, complement, inclusion, equality) are based on contradiction degrees between attributes' values. Same time the union and intersections can be used as fuzzy operators' *t-norm* and *t-conorm*. The current paper tried to illustrate these two operators using an illustrative example.

4. Illustration

Plithogenic set is an extension of the classical set, fuzzy set, intuitionistic fuzzy set, and neutrosophic set, since in all these four types of sets a generic element x is characterized by one attribute only (appurtenance), which has one single attribute value (membership – in classical and fuzzy sets), two attribute values (membership and nonmembership – in intuitionistic fuzzy set), and three attribute values (membership, indeterminacy, and nonmembership – in neutrosophic set).

Example: Let us suppose want to represent the performance of a cricketer like Virat Kohli and Rohit sharma which is collected on 30 Jan 2020 [8]. It is difficult to measure them just by

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one parameter. In this case the expert require more than one attributes and its changes as shown below:

Table 1. The Batting performance of Virat Kohli in various format

	Mat	Inns	NO	Runs	HS	Ave	BF	SR	100	50	4s	6s	Ct	St
Tests	84	141	10	7202	254*	54.97	12457	57.81	27	22	805	22	80	0
ODIs	245	236	39	11792	183	59.85	12626	93.39	43	57	1109	120	126	0
T20Is	81	75	21	2783	94*	51.53	2012	138.32	0	24	256	76	41	0
First-class	116	189	17	9451	254*	54.94	16360	57.76	34	30	1118	37	111	0
List A	279	269	42	13234	183	58.29	14162	93.44	47	65	1273	144	144	0
T20s	280	265	50	8889	113	41.34	6605	134.57	5	64	806	286	128	0

It can be observed that from Table 1 the VIrat Kohli selected after measurement of more than 116 matches in First Class with 9451runs, 54 average, 57 SR and 34 hundreds. This performance can be founded same in ODI and Test after selection. He has faced almost similar balls in Tests and one day with same Average and Double strike rate. It shows Virat Kohli plays based on situation of the game in consistent or mesokurtic behavior as per his performance data.

Table 2. The Batting performance of Rohit Sharma various format

	Mat	Inns	NO	Runs	HS	Ave	BF	SR	100	50	4s	6s	Ct	St
Tests	32	53	7	2141	212	46.54	3613	59.25	6	10	216	52	31	0
ODIs	224	217	32	9115	264	49.27	10250	88.92	29	43	817	244	77	0
T20Is	107	99	14	2713	118	31.91	1957	138.63	4	20	242	124	40	0
First-class	92	143	16	7118	309*	56.04	NA	NA	23	30	NA	NA	73	0
List A	295	284	40	11357	264	46.54	NA	NA	32	56	NA	NA	101	0
T20s	327	314	46	8582	118	32.02	6422	133.63	6	59	760	358	131	0

It can be observed that from Table 2 that, Rohit Sharma selected after measurement of more than 92 matches in First Class with 7118 runs, 56 average, and 23 hundreds. His performance is below in Test and ODI as per First Claas. However his performance is better in ODI and T20 game when compared to Test as per current data. It shows his performance is leptokurtic behavior as per data.

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The problem arises with selectors to choose a particular player which can perform as per Virat Kohli and Rohit Sharma. In this case finding the minimum level and maximum level data is one of the major issues for the selection committee. To resolve this issue the current paper utilizes min and max of t-norms and t conforms operators in plithogenic set.

Case (i) The selection committee first try to get one of the suitable batsman who have perform maximum level in each format as per record of Virat Kohli or Rohit Sharma in this case intersection operator i.e. t-co-norms will provide as shown in Table 3.

Table 3. The Maximum level of Batting performance either Virat Kohli or Rohit Sharma in various format

	Mat	Inns	NO	Runs	HS	Ave	BF	SR	100	50	4s	6s	Ct	St
Tests	84	141	10	7202	254*	54.97	12457	59.25	27	22	805	52	80	0
ODIs	245	236	39	11792	264	59.85	12626	93.39	43	57	1109	244	126	0
T20Is	107	99	14	2783	118	51.53	2012	138.63	4	24	256	124	41	0
First-class	116	189	17	9451	309*	56.04	16360	57.76	34	30	1118	37	111	0
List A	295	284	42	13234	264	58.29	14162	93.44	47	65	1273	144	144	0
T20s	327	314	50	8889	118	41.34	6605	134.57	6	64	806	358	131	0

The Table 3 represents that, a batsman has faced more than 9000 balls with 50 average in each format having strike rate more than 60 can be selected.

In case this data is more higher level and none of candidate can be selected. In this case the selection committee need a minimum level data which predict the common performance of Virat Kohli and Rohit Sharma. It will be minimum to select a batsman and later groomed. To achieve this goal, the t-norm can be utilized in plithogenic set shown in Table 4.

Table 4. The Maximum Common Batting performance of Virat Kohli and Rohit Sharma in various format

	Mat	Inns	NO	Runs	HS	Ave	BF	SR	100	50	4s	6s	Ct	St
Tests	32	53	7	2141	212	46.54	3613	57.81	6	10	216	52	31	0
ODIs	224	217	32	9115	183	49.27	10250	88.92	29	43	817	244	77	0
T20Is	81	75	14	2713	94*	31.91	1957	138.63	0	24	242	124	40	0
First-class	92	143	16	7118	254*	54.94	NA	NA	23	30	NA	NA	73	0

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List A	279	284	40	11357	183	56.54	NA	NA	32	56	NA	NA	101	0
T20s	327	265	56	8582	113	32.02	6422	133.63	5	59	760	286	128	0

The Table 4 represents that, a batsman played more than 90 match in First Class with 7000 Runs and 50 average can be selected who can later perform in Test and One day as per Virat and Rohit Sharma.

It can be observed that, the current paper provides a solution to deal with multi-valued attributes. However the current paper does not focus on dominant attributes in the plithogenic set which affect the decision.

The current paper shows that set of multi-valued and different types of data set can be done using plithogenic set and its operator. It can be observed that in above-given data sets Virat Kohli performance oversees the Rohit sharma on some entries, on some entries they are equal whereas some entries it is less. These data can fluctuate or change in opposite, neutral and non-opposite direction. These types of data sets can be handled by plithogenic set and its operations. Same time plithogenic set provides different levels to set the parameters for selection based on expert requirements rather than single-valued. In future, author work will focus on introducing a new graphical structure for the visualization of plithogeny set.

4. Conclusions:

In this paper, the author provides an initial way to establish plithogenic set with an example. It will helpful for the researchers of data analysis and neutrosophic researchers to deal with uncertainty in multi-valued attributes.

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Three Possible Applications of Neutrosophic Logic in Fundamental and Applied Sciences

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Abstract

In Neutrosophic Logic, a basic assertion is that there are variations of about everything that we can measure; the variations surround three parameters called T,I,F (truth, indeterminacy, falsehood) which can take a range of values. This paper shortly reviews the links among aether and matter creation from the perspective of Neutrosophic Logic. Once we accept the existence of aether as physical medium, then we can start to ask on what causes matter ejection, as observed in various findings related to quasars etc. One particular cosmology model known as VMH (variable mass hypothesis) has been suggested by notable astrophysicists like Halton Arp and Narlikar, and the essence of VMH model is matter creation processes in various physical phenomena. Nonetheless, matter creation process in Nature remains a big mystery for physicists, biologists and other science researchers. To this problem Neutrosophic Logic offers a solution. We also discuss two other possible applications of Neutrosophic Logic. In short, Neutrosophic Logic may prove useful in offering resolution to long standing conflicts.

Keywords: Neutrosophic Logic, Physical Neutrosophy, aether, matter creation, integrative medicine

1.Introduction

Matter creation process in Nature remains a big mystery for physicists, biologists and other science researchers. To this problem Neutrosophic Logic offers a solution, along solutions to two other problems, namely the point particle assumption in Quantum Electrodynamics and also in resolving the old paradigm conflict between Western approach to medicine and Eastern approach.

In short, Neutrosophic Logic may prove useful in offering resolution to long standing conflicts. See also our previous papers on this matter. [29-30]

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2. Matter creation processes

Physicists throughout many centuries have debated over the physical existence of aether medium. Since its inception by Isaac Newton and later on Anton Mesmer (Franz Anton Mesmer 1734 – 1815), many believed that it is needed because otherwise there is no way to explain interaction at a distance in a vacuum space. We need medium of interaction, of which has been called by various names, such as: quantum vacuum, zero point field, etc.
Nonetheless, modern physicists would answer: no, it is not needed, especially after Special Relativity theory. Some would even say that aether has been removed even since Maxwell'sd theory, but it is not true : James Clark Maxwell initially suggested a mechanical model of aether vortices in his theory [26-28]. Regardless of those debates, both approaches (with or without assuming aether) are actually resulting in the same empirical results [9].

The famous Michelson-Morley experiments were thought to give null result to aether hypothesis, and historically it was the basis of Einstein's STR. Nonetheless, newer discussions proved that the evidence was rather ambiguous, from MM data itself. Especially after Dayton Miller experiments of aether drift were reported, more and more data came to support aether hypothesis, although many physicists would prefer a new terms such as physical vacuum or superfluid vacuum. See [21]-[25].

Once we accept the existence of aether as physical medium, then we can start to ask on what causes matter ejection, as observed in various findings related to quasars etc. One particular cosmology model known as VMH (variable mass hypothesis) has been suggested by notable astrophysicists like Halton Arp and Narlikar, and the essence of VMH model is matter creation processes in various physical phenomena. Nonetheless, matter creation process in Nature remains a big mystery for physicists, biologists and other science researchers. To this problem Neutrosophic Logic offers a solution.

Although we can start with an assumption of aether medium is composed of particle-antiparticle pairs, which can be considered as a model based on Dirac's new aether by considering vacuum fluctuation (see Sinha, Sivaram, Sudharsan.) [5][6] Nonetheless, we would prefer to do a simpler assumption as follows:

Let us assume that under certain conditions that aether can transform using Bose condensation process to become "*unmatter*", a transition phase of material, which then it sublimates into matter (solid, gas, liquid). Unmatter can also be considered as "pre-physical matter."

Summarizing our idea, it is depicted in the following block diagram:[1]

[1] The matter creation scheme as outlined here is different from Norman & Dunning-Davies's argument: "Energy may be derived at a quantum of 0.78 MeV to artificially create the resonant oscillatory condensations of a neutroid, then functioning as a Poynting vortex to induce a directionalized scalar wave of that quantum toward that vortical receptive surface." See R.L. Norman & J. Dunning-Davies, Energy and matter creation: The Poynting Vortex, 2019, vixra.org/1910.0241

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Aether → bose condensation → "unmatter" (pre-physical matter) → sublimation → ordinary matter/particle

Diagram 1. How aether becomes ordinary matter

Actually the term "unmatter" can be viewed as a solution from perspective of Neutrosophic Logic. A bit of history of unmatter term may be useful here:

> "*The word 'Unmatter' was coined by one of us (F. Smarandache) and published in 2004 in three papers on the subject. Unmatter is formed by combinations of matter and antimatter that bound together, or by long-range mixture of matter and antimatter forming a weakly-coupled phase. The idea of unparticle was first considered by F. Smarandache in 2004, 2005 and 2006, when he uploaded a paper on CERN web site and he published three papers about what he called 'unmatter', which is a new form of matter formed by matter and antimatter that bind together. Unmatter was introduced in the context of 'neutrosophy' (Smarandache, 1995) and 'paradoxism' (Smarandache, 1980), which are based on combinations of opposite entities 'A' and 'antiA' together with their neutralities 'neutA' that are in between."*[2] See also Smarandache [13].

Nonetheless, in this paper, unmatter is considered as a transition state (pre-physical) from aether to become ordinary matter/particle, see also [14].
Moreover, superfluid model of dark matter has been discussed by some authors [7].

As one more example/case of our proposed scheme of transition from aether to matter, see a recent paper [18]. See the illustrations at pages 5 and 6 of [18] regarding the physically observed properties of the Galactic Center (GC), which are obviously completely different from the imaginary "black hole" model.

The mapping of the magnetic field structures of the Core is a profile of a torus, as we have previously suggested. Page 5 also illustrates the relation between Sag A and Sag B and the space in between them.

These illustrations are also relevant to *matter creation* at the galactic scale. Also note the gamma ray distributions in [18], which are relevant to matter destruction processes. Electrical discharges such as lightning, stars, and galaxies, all produce gamma rays. Gamma ray resonance dissociates atomic matter back into the aether at the rate of 6,800,000,000 **horsepower** of energy liberated per gram of matter dissociated per second. And where does all that energy go? Back into creating new matter. It's a never-ending cycle, and infinitely Universe-wide.

3. Towards QED without renormalization

[2] http://fs.unm.edu/unmatter.htm

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One problem in theoretical physics is how to do away with infinity and divergence in QED without renormalization. As we know, renormalization group theory was hailed as cure in order to solve infinity problem in QED theory.

For instance, a quote from Richard Feynman goes as follows:

> "*What the three Nobel Prize winners did, in the words of Feynman, was "to get rid of the infinities in the calculations. The infinities are still there, but now they can be skirted around . . . We have designed a method for sweeping them under the rug."[*19]

And Paul Dirac himself also wrote with similar tune:

> "Hence most physicists are very satisfied with the situation. They say: "Quantum electrodynamics is a good theory, and we do not have to worry about it any more." I must say that I am very dissatisfied with the situation, because this so-called "good theory" does involve neglecting infinities which appear in its equations, neglecting them in an arbitrary way. This is just not sensible mathematics. Sensible mathematics involves neglecting a quantity when it turns out to be small—not *neglecting it just because it is infinitely great and you do not want* it!"[20]

Here we submit a viewpoint that the problem begins with assumption of point particle in classical and quantum electrodynamics. Therefore, a solution shall be sought in developing fluidic Electrodynamics [10], i.e. by using fluid particle, or perhaps we can call it "*fluidicle*." It is hoped that a fluidicle can remove the infinity problem caused by divergence. And fluidicle can be viewed as a solution from perspective of Neutrosophic Logic.

4. Another application: Resolution to conflicting paradigms in medicine

It is well known by most medicine practitioners, that Western approach to medicine is based on "*curing*" or "attacking" a disease, one by one. This is called germ theory: one cure for one disease (Pasteur). On the opposite side, Eastern medicine is based in particular on ancient wisdom of returning the balance of the body, in other words: to *harmonize* our body and our live with nature. Although those two approaches in medicine and healthcare have caused so many conflicts and misunderstandings, actually it is possible to do a dialogue between them.

From Neutrosophic Logic perspective, a resolution to the above conflicting paradigms can be found in developing novel approaches which appreciate both traditions in medicine, or we may call such an approach: "*curemony*," i.e. by at the same time curing a disease and restoring balance and returning harmony in one's *body-mind-spirit as a whole.*

Although we don't mention here specific case example, in general speaking we can mention:

a. in HGH therapy, it is known that nutrition can affect the well-being of body [12],
b. in the same way *Epigenetics* admits the role of external factors into the genes.
c. We can also mention that psoriasis –a skin problem- can be related to stress and other emotions, which suggests a plausible new term: *psychodermatology*.[11]

All of these examples seem to suggest relational aspect within human being, among *mind-body-spirit*, just like what Eastern medicine emphasizes all along. In some literature, such a dialogue between Western and Eastern medicine approaches can be considered as *integrative medicine*, but actually it goes far deeper that just "integrative", it is more like rethinking the "isolate and solve" attitude of Western scientists, toward more "*relational biology*." And the

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concept of systems biology or relational biology have become new terms in recent years. See also recent literatures in this subject [15][16][17].

Hopefully many more approaches can be developed in the direction as mentioned above.

5. Conclusions

In this paper, we discussed three possible applications of Neutrosophic Logic in the field of matter creation processes etc. For instance, a redefinition of term "unmatter" is proposed here, where under certain conditions, aether can transform using Bose condensation process to become "*unmatter*", a transition phase of material, which then it sublimates into matter (solid, gas, liquid). Unmatter can also be considered as "pre-physical matter." Moreover, a transition phase between fluid and particle (or fluidicle) is considered necessary in order to solve the "point particle" assumption which cause the divergence problem in QED. And for the third application of NL, we consider a dialogue is possible between Eastern and Western approaches to medicine.
Further researches are recommended in the above directions.

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