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Table of Content

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Items	Page Number
Table of Contents	2
Editorial Board	4
Aim and Scope	6
Topics of Interest	6
ISSUE 1	
The Concept of Neutrosophic Limits in Real Sequences Huda E. Khalid , Ahmed K. Essa	8-13
Using Moving Averages To Pave The Neutrosophic Time Series Rafif Alhabib , A. A. Salama	14-20
Application of Pentagonal Neutrosophic Number in Shortest Path Problem Avishek Chakraborty	21-28
Linear and Non-Linear Octagonal Neutrosophic Numbers: Its Representation, α -Cut and Applications Muhammad Saqlain , Ali Hamza , Sara Faroon	29-43
Neutrosophic \mathfrak{N} -Ideals (\mathfrak{N} -Subalgebras) of Subtraction Algebra Madeleine Al- Tahan , Bijan Davvaz	44-53
ISSUE 2	
An Approach to Solve the Linear Programming Problem Using Single Valued Trapezoidal Neutrosophic Number Tuhin Bera , Nirmal Kumar Mahapatra	54-66
Utilization of Jaccard Index Measures on Multiple Attribute Group Decision Making under Neutrosophic Environment N. Gayathri , Dr. M. Helen , P. Mounika	67-77
New Technology in Agriculture Using Neutrosophic Soft Matrices with the Help of Score Function Muhammad Naveed Jafar, Muhammad Saqlain, Ahmad Raza Shafiq, Muhammad Khalid, Hamza Akbar, Aamir Naveed	78-88
Image and Inverse Image of Neutrosophic Cubic Sets in UP-Algebras under UP-Homomorphisms Metawee Songsaeng , Aiyared Iampan	89-107
On single valued neutrosophic sets and neutrosophic N-structures: Applications on algebraic structures (hyperstructures) Madeleine Al-Tahan , Bijan Davvaz	108--117

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Aim and Scope

International Journal of Neutrosophic Science (IJNS) is a peer-review journal publishing high quality experimental and theoretical research in all areas of Neutrosophic and its Applications. IJNS is published quarterly. IJNS is devoted to the publication of peer-reviewed original research papers lying in the domain of neutrosophic sets and systems. Papers submitted for possible publication may concern with foundations, neutrosophic logic and mathematical structures in the neutrosophic setting. Besides providing emphasis on topics like artificial intelligence, pattern recognition, image processing, robotics, decision making, data analysis, data mining, applications of neutrosophic mathematical theories contributing to economics, finance, management, industries, electronics, and communications are promoted. Variants of neutrosophic sets including refined neutrosophic set (RNS). Articles evolving algorithms making computational work handy are welcome.

Topics of Interest

IJNS promotes research and reflects the most recent advances of neutrosophic Sciences in diverse disciplines, with emphasis on the following aspects, but certainly not limited to:

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| <input type="checkbox"/> neutrosophic sets | <input type="checkbox"/> Refined single-valued neutrosophic sets |

- ☐ Applications of neutrosophic logic in image processing
- ☐ Neutrosophic logic for feature learning, classification, regression, and clustering
- ☐ Neutrosophic knowledge retrieval of medical images
- ☐ Neutrosophic set theory for large-scale image and multimedia processing
- ☐ Neutrosophic set theory for brain-machine interfaces and medical signal analysis
- ☐ Applications of neutrosophic theory in large-scale healthcare data
- ☐ Neutrosophic set-based multimodal sensor data
- ☐ Neutrosophic set-based array processing and analysis
- ☐ Wireless sensor networks Neutrosophic set-based Crowd-sourcing
- ☐ Neutrosophic set-based heterogeneous data mining
- ☐ Neutrosophic in Virtual Reality
- ☐ Neutrosophic and Plithogenic theories in Humanities and Social Sciences
- ☐ Neutrosophic and Plithogenic theories in decision making
- ☐ Neutrosophic in Astronomy and Space Sciences



The Concept of Neutrosophic Limits in Real Sequences

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Abstract

The theory of neutrosophic limits is the natural step before developing the theory of neutrosophic continuous functions and developing the theory of neutrosophic differentiation. The goal of this work is to construct a new definition of the neutrosophic limits for real sequences using the distance as a linear operator. Some new theorems are presented to cover the theoretical notions of this topic and an illustrative example is presented to help the reader understanding the notions of this article.

Keywords: Neutrosophic Limits, Neutrosophic Convergence, Real Sequences.

1. Introduction

The essential concepts that should be extended in neutrosophic theory are the fuzzy limits, fuzzy continuity, and fuzzy derivatives. Neutrosophic researchers must extend the usual concept of the conventional limit of a sequence. Furthermore, the main differences between neutrosophic theory and fuzzy theory are that any notion in neutrosophic theory say $\langle A \rangle$ together with its counteractive $\langle antiA \rangle$ and with their spectrum of neutralities $\langle neutA \rangle$ in between them (i.e. ideas supporting neither $\langle A \rangle$ nor $\langle antiA \rangle$). The $\langle neutA \rangle$ and $\langle antiA \rangle$ ideas together are referred to as $\langle nonA \rangle$. Neutrosophic logic is generalization of the fuzzy logic. In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^{-}0, 1^{+}[$ [1]. The neutrosophic logic was established by F. Smarandache [2]. In this paper, the authors introduce pioneering work called neutrosophic limits for real-sequences using the linear operator. The first improvement in the topic about neutrosophic limits was by F. Smarandache in his first edition of neutrosophic calculus [3], the second attempt to determine the concept of functional limits, introduce the binomial factorial theorem, and specify the indeterminate values of neutrosophic calculus were by Huda E. Khalid et al. [4,14]. There are many additional works by Huda E. Khalid concerning specifying the type of indeterminacy regarded to the (over, off, under) neutrosophic theory [5,6,7,10], also she worked to improve the concept of neutrosophic geometric programming [8,9,11,12,13], those previous works did not cover the limits and the continuity of real sequences which regards as the basic concepts of mathematical analysis. This paper represents the first attempt in this direction.

2. Basic concepts

2.1. Definition [15]

Suppose $R = (-\infty, \infty)$, $R^+ = [0, \infty)$ and $R^{++} = (0, \infty)$, assume that $r \in R^+$ and let $l = \{a_i \in R; i = 1, 2, \dots\}$ be a sequence of real numbers.

2.2. Definition [15]

(a) A real number " a " is called an r -limit of a sequence l (it is denoted by $a = r - \lim_{i \rightarrow \infty} a_i$ or $a = r - \lim l$) if for any $\varepsilon \in R^{++}$, the inequality $|a - a_i| < r + \varepsilon$ is valid for almost all a_i , i.e., there is such n that for any $i > n$, we have $|a - a_i| < r + \varepsilon$.

(b) A sequence l that has an r -limit is called r -convergent and it is denoted by $l \rightarrow r^a$. Informally, a is an r -limit of a sequence l for an arbitrarily small ε , the distance between a and all but a finite number of elements from l is smaller than $r + \varepsilon$. In other words, a number a is an r -limit of a sequence l if for any $\varepsilon \in R^{++}$, almost all a_i belong to the interval $(a - r - \varepsilon, a + r + \varepsilon)$. Hence, r becomes a measure of convergence for l .

2.3. Compare Between Fuzzy and Neutrosophic Logics [1,7]

The neutrosophic connections have a better truth-value definition approach to the real-world systems than the fuzzy connections. They are defined on triple non-standard subsets included in the non-standard interval $]^{-0}, 1^{+}[$, while in fuzzy theory they are defined on the interval $[0, 1]$. n_sup is not limited to 1, but it's aggrandized to a monad $m(3^+)$; similarly, n_inf maybe as low as $m(-0)$, not as 0. A paradox, which is simultaneously true and false, cannot be evaluated in fuzzy logic, because the sum of the combinations should add up to 1, but it is allowed in neutrosophic theory because of the thought of contradictory in neutrosophic logic may be (1,1,1). In dissenting to the fuzzy theory, if an assumption $\langle A \rangle$ is $t\%$ true, doesn't necessarily mean it is $(100 - t)\%$ false. A better tactic is $t\%$ true, $i\%$ indeterminate, and $f\%$ false, as in intuitionistic fuzzy logic, whereas $t \in T$, $i \in I$, $f \in F$. More general, with $n_sup \leq 3^+$ and $n_inf \geq -0$.

3. Neutrosophic Limits of Sequences

This section presents basic steps to define the neutrosophic limits for real sequences using linear operator.

3.1. The Behavior of limits in Neutrosophic Environment

To put forward the concept of neutrosophic convergence, one must compare the differences between the fuzzy convergent in Definition 2.2 with the neutrosophic notion, the condition $|a - a_i| < r + \varepsilon$ given in Definition 2.2 is not enough to maintain the neutrosophic convergent of any real number a to a sequence of real numbers l . In fact, we need two additional conditions related with $p(a, a_i) < r + \varepsilon$. Any neutrosophic convergence need three joints, the first hinge is the neutrosophically convergent which is somehow likely fuzzy convergent but this indeed could not be enough to lead to neutrosophic convergence, the second hinge is $anti(a)$ which must exist and satisfy $|anti(a) - a_i| > r + \varepsilon$, and $neut(a)$ will be the final hinge. Logically with respect to the neutrosophic notion, $anti(a)$ should satisfy the divergent from the sequence l , while $neut(a)$ is any real number that is neither convergent to l nor divergent, in other words, $neut(a)$ is situated in the location between a & $anti(a)$, so the distance between $neut(a)$ and any element $a_i \in l$ could not be less than $r + \varepsilon$ and could not be greater than $r + \varepsilon$. Consequently, the distance between $neut(a)$ and any element of l must be equal to $r + \varepsilon$ and this only happens when $neut(a)$ becomes as sequence of neutralities say $c = \{c_1, c_2, c_3, \dots\}$, where c_i is $neut(a)$ corresponding to the value of $a_i \in l$. The above inspired us the following definition

3.2. Definition

Let a be a real number, b a real number or an interval of real numbers, and c be a set of real numbers. Then a is called an r – neutrosophic limit of a sequence l , referred to as $a = r - N \lim_{i \rightarrow \infty} a_i$ or denoted by $a = r - N \lim l$, if for any $\varepsilon \in R^{++}$ the following inequalities are satisfied together:

$$1- |a - a_i| < r + \varepsilon,$$

$$2- |b - a_i| > r + \varepsilon \quad \text{where } b = \text{anti}(a) \text{ [i.e. } b \text{ is the opposite or negation of } a \text{]},$$

$$3- |c_i - a_i| = r + \varepsilon \quad \text{where } c = \text{neut}(a) \text{ [i.e. here } c = \{c_1, c_2, c_3, \dots\}, c_i \text{ is } \text{neut}(a) \text{ corresponding to the value of } a_i \text{] also } \text{neut}(a) \text{ means neither } a \text{ nor } b.$$

Note that $p(a, a_i) < r + \varepsilon$ is valid for almost all a_i , (i.e. there is such n that for any $i > n$, we have $p(a, a_i) < r + \varepsilon$). Also $p(b, a_i) > r + \varepsilon$ is valid for almost all elements of a_i , (i.e. there is such m that for any $i > m$, we have $p(b, a_i) > r + \varepsilon$). Moreover, for the neutral of a there is a set of elements $c = \{c_1, c_2, c_3, \dots\}$ that satisfies the inequality $p(c_i, a_i) = r + \varepsilon$ (i.e. c_1 represents the $\text{neut}(a)$ corresponding to the element $a_1 \in l$, c_2 represent the $\text{neut}(a)$ corresponding to the element $a_2 \in l$ and so on). In this case, the sequence l is r -neutrosophically convergent to a and it is denoted by $l \xrightarrow{N} r a$.

In other words, the element a is $r - N\text{limit}$ of a sequence l if for an arbitrarily small $\varepsilon \in R^{++}$, the distance between a and all but a finite number of elements from l is less than $r + \varepsilon$, at the same time there is another element b (it could be an interval $b = (b_1, b_2)$) that the distance between b and all but finite number of elements from l is greater than $r + \varepsilon$, simultaneously for any element of l say a_i there is a corresponding element c_i that the distance between a_i & c_i is equal to $r + \varepsilon$.

It is obvious that almost all a_i belong to the interval $(a - r - \varepsilon, a + r + \varepsilon)$, at the same time those almost all a_i do not belong to the interval $(b - r - \varepsilon, b + r + \varepsilon)$ while for each element $a_i \in l$ there is one and only one element of a sequence c satisfy $a_i = c_i - r - \varepsilon$.

4. Some Theoretical Results for Neutrosophic Limits

This section presents some results on neutrosophic limits.

4.1. Theorem

Let a, b be any two real numbers, and $c = \{c_1, c_2, c_3, \dots\}$ be a set of real numbers, suppose that $r \in R^+$, and let $l = \{a_i \in R; i = 1, 2, \dots\}$ be a sequence of real numbers. If $r = 0$ (i.e. $a = 0 - N \text{ limit } l$), then for any $\varepsilon \in R^{++}$ the following inequalities hold:

$$|a - c_i| < 2\varepsilon$$

$$|b - c_i| < 2\varepsilon$$

Proof

Apply $r = 0$ to the inequalities of Definition (3.2), we get

$$|a - a_i| < \varepsilon \tag{1}$$

$$|b - a_i| > \varepsilon \tag{2}$$

$$|c_i - a_i| = \varepsilon \tag{3}$$

From equality (3),

$$c_i - a_i = \mp \varepsilon \rightarrow a_i = c_i \pm \varepsilon \quad (4)$$

Return to the inequality $|a - a_i| < \varepsilon$ and by using the definition of absolute value, we will get

$$0 < a - a_i < \varepsilon \ \& \ -\varepsilon < -(a - a_i) < 0$$

Applying (4) to the above inequalities, will get the following inequalities,

$$0 < a - c_i < 2\varepsilon \ \& \ -2\varepsilon < -(a - c_i) < 0,$$

which imply that, $|a - c_i| < 2\varepsilon$,

Track the same above way to prove that,

$$|b - c_i| < 2\varepsilon.$$

The proof is complete.

4.2. Theorem

Let $a = r - N \lim l$, then $a = q - N \lim l$, at $q - r = \varepsilon_1$ where $\varepsilon_1 \in R^{++}$.

Proof

Since $a = r - N \lim l$, the following inequalities are holding together:

$$|a - a_i| < r + \varepsilon \quad (5)$$

$$|b - a_i| > r + \varepsilon \quad (6)$$

$$|c_i - a_i| = r + \varepsilon \quad (7)$$

Since $q - r = \varepsilon_1$, it follows that $q > r$ and hence, $q + \varepsilon > r + \varepsilon$. But from inequality (5) we have,

$$r + \varepsilon > |a - a_i| \rightarrow |a - a_i| < q + \varepsilon.$$

Therefore, the first inequality of Definition (3.2) is holds for q .

From equality (7) we have, $r = |c_i - a_i| - \varepsilon$

$$\Rightarrow q - \varepsilon_1 = |c_i - a_i| - \varepsilon \Rightarrow |c_i - a_i| = q - \varepsilon_1 + \varepsilon,$$

Reset $-\varepsilon_1 + \varepsilon$ as ε_2

$\therefore |c_i - a_i| = q + \varepsilon_2$, which means that the equality of Definition (3.2) is holding for q .

For the second inequality of Definition (3.2),

$$|b - a_i| > r + \varepsilon = q - \varepsilon_1 + \varepsilon,$$

reset $-\varepsilon_1 + \varepsilon$ as $\varepsilon_2 \Rightarrow |b - a_i| > q + \varepsilon_2$.

So, all restricted conditions of Definition (3.2) are holds for q .

$$\therefore a = q - N \lim l,$$

4.3. Note

1- We can re-express Definition (3.2) in the following alternative way.

A number a represents the neutrosophic limit for sequence l if the triple (a, b, c) satisfies

$$a = r - N \lim l \quad (a \text{ is an } r - N \text{ limit of } l \text{ for some } r \in R^+)$$

$$b \neq r - N \lim l \quad (b \text{ is not } r - N \text{ limit of } l)$$

$$c \neq r - N \lim l \quad (c \text{ is neither } r - N \text{ limit of } l \text{ nor not } r - N \text{ limit of } l).$$

2- A sequence l is neutrosophically convergent to a if l has a neutrosophic limit to a .

5. Numerical Example

Let $a = 1$ and $l = \left\{ \frac{1}{i} \right\}$ where i is the set of all-natural numbers. Test if a is $2 - N \lim$ of the sequence l . What about the convergence of l to $a = 2.5$?

Solution:

Here $a = 1$, $r = 2$, and let $\varepsilon = 0.1$, it is obvious that $p(1, a_i) < r + \varepsilon \Rightarrow p(1, a_i) < 2.1$ for almost all elements of l . The value of $b = anti(a)$ for $a = 1, r = 2, \varepsilon = 0.1$ is $b = (3.1, \infty)$, while the sequence $c = neut(a) = \{3.1, 2.6, 2.4333, \dots\}$, i.e. $p(3.1, a_1) = 2.1 \Rightarrow a_1 = 1$ which is the first element in l , $p(2.6, a_2) = 2.1 \Rightarrow a_2 = 0.5$ which is the second element in l , also $p(2.4333, a_3) = 2.1 \Rightarrow a_3 = 0.3333 = \frac{1}{3}$ the third element in l , it is clear that the sequence $c = neut(a)$ satisfies the equality $|c_i - a_i| = r + \varepsilon$, here $r + \varepsilon = 2.1$. Consequently, $a = 1$ is $2 - N \lim$ of l (i.e. $1 = 2 - N \lim l$). But $a = 2.5$ is not $2 - N \lim$ of l since at $\varepsilon = 0.1$ this implies that $\left| 2.5 - \frac{1}{i} \right| > 2.1$ for almost all values of i except $i = 1$ and $i = 2$, this is a contradict to the first inequality of Definition (3.2), (i.e. $2.5 \neq 2 - N \lim l$).

6. Conclusions

It is well known that many processes of mathematics, such as differentiation and integration, demand the use of limits. This paper presented the definition of neutrosophic limits of real sequence l using the distance as a linear operator. Moreover, two theorems for the sake of the theoretical part of this paper were proved. The notions of this paper came as necessary first step to develop the notion of neutrosophic differentiation in the real space R and the neutrosophic convergent.

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Using Moving Averages To Pave The Neutrosophic Time Series

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Abstract

In this paper, we are using moving averages to pave the Neutrosophic time series. similar to use moving averages to pave the classical time series. the difference, here we are dealing with inaccurate data and values of the time series. in the Neutrosophic time series, each unit of time(t) corresponds to a range of values instead of a single value. Finally, we find that the Neutrosophic time series provide an accurate description of the behavior of the series better than in the classic. Therefore, can predict the future of the series as accurately as possible.

Keywords: Time Series, Neutrosophic logic, Neutrosophic Time Series, moving averages.

1.Introduction

A time series is a set of data arranged in chronological order, the data of this series are associated to each other in the general case, and this correlation gives us reliable future forecasts. also define it as a set of consecutive values (observations) that describe the evolution of a phenomenon over time. say about this time series that it is **neutrosophic time series (NTS)**, if some or all of its values (its observations) are not explicitly specific, such as being a range of values instead of one value [9]. That is, successive observations "that describe the evolution of a phenomenon with time", some or all of it is not precisely defined.

The neutrosophic logic was established by F. Smarandache in 1995. It is a new branch of philosophy, presented as a generalization for the fuzzy logic [1] and as a generalization for the intuitionistic fuzzy logic [3]. Where presented it as a type of formal logic that aims at explaining the truth, falsehood, and neutral propositions.

The fundamental concepts of neutrosophic set, introduced by Smarandache in [4,5,6,8], Salama and rafif et al. in [7,9,10,11,12,13,14,15,16,17], provides a new foundation for dealing with issues that have indeterminate data. The idea of basic neutrosophic statistics was developed by F. Smarandache [2]. The idea of inferential neutrosophic statistics and neutrosophic statistics was developed by Aslam [18,19]. Singh and Hong presented the time series dataset in to Neutrosophic series using three different memberships are truth-membership, indeterminacy-membership and falsity-membership[23]. In this paper, the authors provide a method for paving the Neutrosophic time series using "forward, backward and central" moving averages. This is done similarly to that in classical logic, with the difference that dealing with a Neutrosophic series rather than a classic series.

The aim of this paper is to demonstrate the method of "paving time series using moving averages" within the framework of the Neutrosophic logic. Where find that dealing with the Neutrosophic series is better than the classic

series, because the Neutrosophic series provides a more accurate and comprehensive description of the time series data set and thus describe the series behavior better than it is in the classic. Which provides a better environment and base for future forecasting.

Moving averages mean dividing the series into a number of equal and overlapping divisions, and replacing each section with a number (mean, median, or others).

2. The forward moving averages of a Neutrosophic time series:

Symbolize the moving averages of a Neutrosophic time series with the symbol Nm_t .

If denote to the Neutrosophic time series with the symbol NQ_t , the forward moving average is given by:

$$Nm_t = \frac{1}{n} [\sum_{i=t}^{t+n-1} NQ_i] \quad (2,1)$$

Where n is the degree of the moving average.

For example, for $n = 3$ we have:

$$Nm_t = \frac{1}{3} [\sum_{i=t}^{t+2} NQ_i] = \frac{1}{3} [NQ_t + NQ_{t+1} + NQ_{t+2}]$$

Therefore:

$$Nm_1 = \frac{1}{3} [NQ_1 + NQ_2 + NQ_3]$$

$$Nm_2 = \frac{1}{3} [NQ_2 + NQ_3 + NQ_4]$$

And so on..

2.1. Example:

Let's calculate the forward triple moving average for the following Neutrosophical time series, a series that represents the humidity recorded during seven days in Homs:

t	NQ_t	Nm_t
1	[50, 52]	$=\frac{1}{3} [[50, 52] + [51, 53] + [49, 50]] = [50, 51.67]$
2	[51, 53]	$=\frac{1}{3} [[51, 53] + [49, 50] + [55, 58]] = [51.67, 53.67]$
3	[49, 50]	$=[54.67, 56.67]$
4	[55, 58]	$=[58, 61]$
5	[60, 62]	$=[61, 64]$
6	[59, 63]	
7	[64, 67]	

Table (1): Calculating the forward moving average

3. Backward Moving Averages of a Neutrosophic Time Series:

The Backward moving average for a Neutrosophic time series is given by the following relationship:

$$Nm_t = \frac{1}{n} \left[\sum_{i=t}^{t-n+1} NQ_i \right] \quad (3,1)$$

Where n is the degree of the moving average.

For example, for n = 3 we have:

$$Nm_t = \frac{1}{3} \left[\sum_{i=t}^{t-2} NQ_i \right] = \frac{1}{3} [NQ_t + NQ_{t-1} + NQ_{t-2}]$$

Therefore:

$$Nm_3 = \frac{1}{3} [NQ_3 + NQ_2 + NQ_1]$$

$$Nm_4 = \frac{1}{3} [NQ_4 + NQ_3 + NQ_2]$$

And so on..

3.1. Example:

Let's calculate the Backward triple moving average for the Neutrosophical time series in the previous example (2.1):

t	NQ_t	Nm_t
1	[50 , 52]	
2	[51 , 53]	
3	[49 , 50]	$=\frac{1}{3} [[50 , 52] + [51 , 53] + [49 , 50]] = [50 , 51.67]$
4	[55 , 58]	$=\frac{1}{3} [[51 , 53] + [49 , 50] + [55 , 58]] = [51.67 , 53.67]$
5	[60 , 62]	$=[54.67 , 56.67]$
6	[59 , 63]	$=[58 , 61]$
7	[64 , 67]	$=[61 , 64]$

Table 2: Calculating the Backward moving average

4. Central moving average of a Neutrosophic time series:

The central moving average of a Neutrosophic time series is given by the following relationship:

$$Nm_t = \frac{1}{n} \left[\sum_{i=t-(n-1)/2}^{t+(n-1)/2} NQ_i \right] \quad (4,1)$$

Where n is the degree of the moving average.

For example, for n = 3 we have:

$$Nm_t = \frac{1}{3} \left[\sum_{i=t-1}^{t+1} NQ_i \right] = \frac{1}{3} [NQ_{t-1} + NQ_t + NQ_{t+1}]$$

Therefore:

$$Nm_2 = \frac{1}{3} [NQ_1 + NQ_2 + NQ_3]$$

$$Nm_3 = \frac{1}{3} [NQ_2 + NQ_3 + NQ_4]$$

For example, for $n = 5$ we have:

$$Nm_t = \frac{1}{5} \left[\sum_{i=t-2}^{t+2} NQ_i \right]$$

And so on..

4.1. Example:

Let's calculate the central triple moving average for the Neutrosophical time series in the previous example (2.1):

t	NQ_t	Nm_t
1	[50 , 52]	
2	[51 , 53]	= [50 , 51.67]
3	[49 , 50]	= [51.67 , 53.67]
4	[55 , 58]	= [54.67 , 56.67]
5	[60 , 62]	= [58 , 61]
6	[59 , 63]	= [61 , 64]
7	[64 , 67]	

Table 3: Calculating the central moving average

5. Note:

Through the previous three tables (1),(2),(3) we find that:

- When using the forward moving average, lose from the end of the series a number of values equal to $(n-1)$ value, (in our example the degree of Moving average $n = 3$, so lost two values from the end of the series).
- When using the backward moving average, lose from the beginning of the series a number of values equal to $(n-1)$ value, (in our example the degree of Moving average $n = 3$, so lost two values from the beginning of the series).

- When using the central moving average, lose from the beginning of the series and at the end of it a number of values equal to $(n-1)/2$ value, (in our example the degree of moving average $n = 3$, therefore lost a value from the beginning of the series and a value from the end of the series).

6. Graphical representation of the time series:

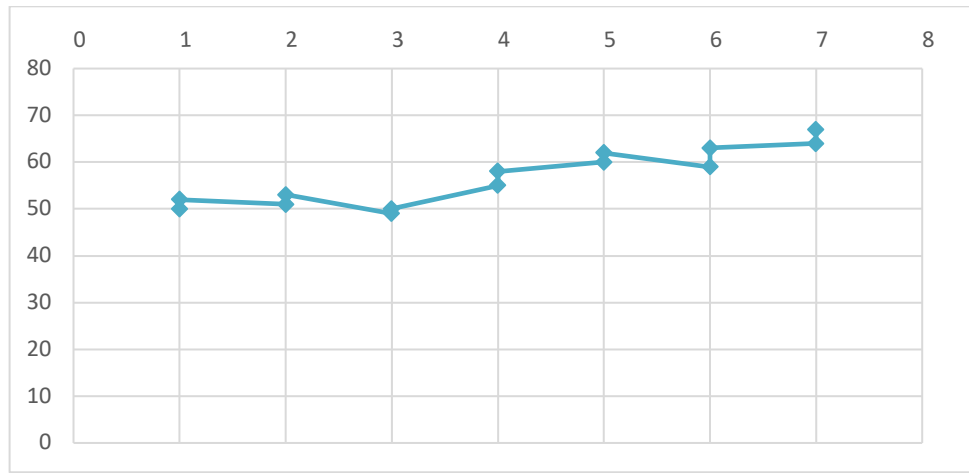


Figure 1. Before paving

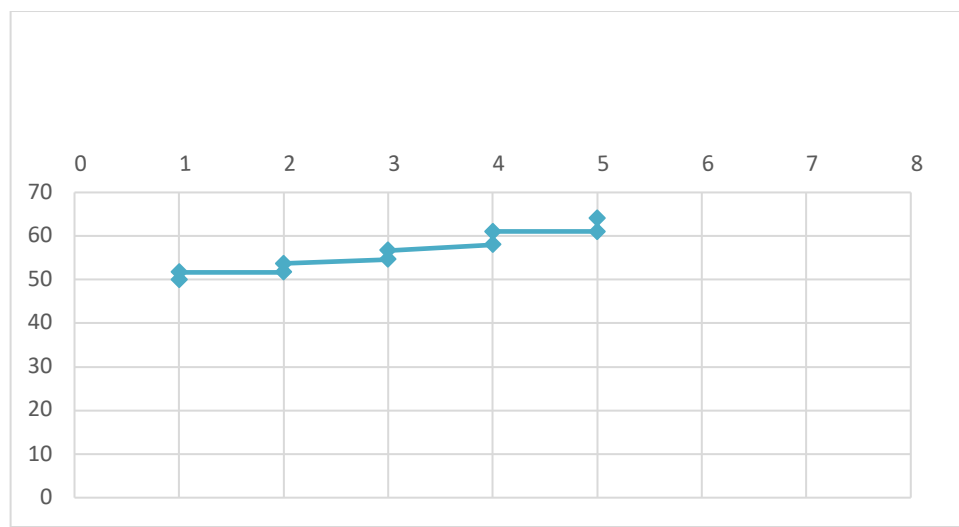


Figure 2. After paving

- **conclude from Figures (1) and (2):** That the graph line in the series after paving has become smoother than the graph line in series before paving .

- The diagram of the series did not merge "before and after the paving " into a single diagram until the Neutrosophical values appear clearly and do not cause any confusion.
- Was used the forward moving average in the graph.

7. Conclusion

conclude that the use of the Neutrosophic time series and paving them using the moving averages method provides us with a complete and accurate description of the behavior of the time series, which facilitates the prediction process for the future of this series, as well as the prediction process is performed accurately and with the least possible errors.

In the near future, we are looking forward to studying the seasonal, periodic and random changes of neutrosophic time series, as well as the method of eliminating the seasonal effect of neutrosophic time series.

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Application of Pentagonal Neutrosophic Number in Shortest Path Problem

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Abstract: Real-human kind issues have distinct sort of ambiguity and among them; one of the critical troubles is solving the shortest path problem. In this contribution, we applied the developed score function and accuracy function of pentagonal neutrosophic number (PNN) into a shortage path selection problem. Further, a time dependent and heuristic cost function related shortest path algorithm is considered here in PNN area and solved it utilizing an influx of dissimilar rational & pioneer thinking. Lastly, estimation of total ideal time of the graph reflects the importance of this noble work.

Keywords: PNN, Score and accuracy function, shortest path algorithm.

Introduction: The perception of fuzzy set was first manifested by Professor Zadeh [1] in 1965 to grip the uncertain data. Since then, the conception of fuzziness plays a most important feature to solve engineering and statistical problem. As the researches goes on, researchers from different areas published several articles in this areas and they extended the idea of fuzzy set in various fields according to their need. Recently, researchers invented the perceptions of pentagonal [2], hexagonal [3], heptagonal [4] fuzzy numbers and they applied it in distinct areas like operation research based EOQ, EPQ model, game theory, transportation problem etc. Further, in 1986 Prof. Atanassov [5] demonstrated the idea of intuitionistic fuzzy set which was the extension of Zadeh's fuzzy set. Here, he considered both membership and non-membership function instead of Zadeh's membership function. After that, a basic question grows up into our mind that how can we construct the mathematical model to deal with the idea of vagueness? Different kinds of methodologies were devised by using the researchers to describe intricately the conceptions of some new unsure parameters and to handle these complex problems, the selection makers placed forth their numerous thoughts in disjunctive areas. Later, Smarandache [6] in 1998 developed the remarkable idea of neutrosophic set which contains three disjunctive membership function namely i) truth ii) hesitation and iii) falsity. Actually, this is the extension of intuitionistic fuzzy set and general Zadeh's fuzzy set. As researches goes on, researchers introduced triangular [7], trapezoidal [8] and very recently pentagonal [9] neutrosophic set and its classification in different cases. Recently, de-neutrosophic technique of pentagonal neutrosophic number [10], score function based application [11] and MCGDM problem [12] has been illustrated by Chakraborty et. al. Also, Chakraborty [13, 14] manifested the concept of cylindrical neutrosophic number in research domain and applied it in graph theory and MCGDM problem. A few novel works [15-25] are also comes out recently in neutrosophic area which plays an essential role in distinct fields like MCGDM, mathematical modeling, neutro-algebra, cryptography, linear programming, and topological spaces. In this current area, Shortest path search problem is one of the important problem in neutrosophic domain. Recently, Kumar et al [26] developed neutrosophic shortest path, [27] interger valued neutrosophic shortest path, [28] Gaussian based shortest path and [29] weighted arc length based shortest path in neutrosophic area. Also, Broumi et al [30] manifested bellman shaped shortest path problem which plays a vital role in graph theory research.

This paper deals with the conception of pentagonal neutrosophic number in different aspect. Nowadays, researchers are very much interested in doing shortest path problem in artificial intelligence problem in PNN environment. Here, we consider a shortest path problem in PNN area where we utilize the idea of our developed score and accuracy function for solving the problem.

1.1 Motivation

The idea of vagueness performs an important position in construction of mathematical modeling, economic problem and social real lifestyles hassle and so on. If, anyone consider a shortest path problem in PNN area then

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how he/she will managed it and solve it? How we can relate PNN and crispificaton result ? From this aspect we actually try to develop this research article.

1.2 Novelties

Only some of the articles are published in PNN area till now. Although it can be applied in many fields and compute the results there. Our main focus is to apply the established PNN number in different areas.

- (i) Develop score and accuracy function.
- (ii) Application of our established score function in shortest path problem.

2. Preliminaries

2.1 Definition: Fuzzy Set: [1] Set \tilde{M} called as a fuzzy set when represented by the pair $(x, \mu_{\tilde{M}}(x))$ and thus stated as $\tilde{M} = \{(x, \mu_{\tilde{M}}(x)) : x \in X, \mu_{\tilde{M}}(X) \in [0,1]\}$ where $x \in$ the crisp set X and $\mu_{\tilde{M}}(X) \in$ the interval $[0,1]$.

2.2 Definition: Intuitionistic Fuzzy Set (IFS): [5] An fuzzy set [2] \tilde{S}_F in the universal discourse X , symbolized widely by x is referred as Intuitionistic set if $\tilde{S}_F = \{(x; [\gamma(x), \delta(x)]) : x \in X\}$, where $\gamma(x): X \rightarrow [0,1]$ is termed as the certainty membership function which specify the degree of confidence, $\delta(x): X \rightarrow [0,1]$ is termed as the uncertainty membership function which specify the degree of indistinctness.

$$\gamma(x), \delta(x) \text{ exhibits the following the relation } 0 \leq \gamma(x) + \delta(x) \leq 1.$$

2.3 Definition: Neutrosophic Set: [6] A set $\tilde{N}e_M$ in the universal discourse X , figuratively represented by x named as a neutrosophic set if $\tilde{N}e_M = \{(x; [\lambda_{\tilde{N}e_M}(x), \pi_{\tilde{N}e_M}(x), \sigma_{\tilde{N}e_M}(x)]) : x \in X\}$, where $\lambda_{\tilde{N}e_M}(x): X \rightarrow]-0,1+[$ is stated as the truthness function, which designates the degree of confidence, $\pi_{\tilde{N}e_M}(x): X \rightarrow]-0,1+[$ is stated as the hesitation function, which designates the degree of indistinctness, and $\sigma_{\tilde{N}e_M}(x): X \rightarrow]-0,1+[$ is stated as the falseness function, which designates the degree of deceptiveness on the decision taken by the decision maker.

$\lambda_{\tilde{N}e_M}(x), \pi_{\tilde{N}e_M}(x)$ & $\sigma_{\tilde{N}e_M}(x)$ displays the following relation:

$$-0 \leq \text{Sup} \{\lambda_{\tilde{N}e_M}(x)\} + \text{Sup} \{\pi_{\tilde{N}e_M}(x)\} + \text{Sup} \{\sigma_{\tilde{N}e_M}(x)\} \leq 3 +$$

2.4 Definition: Single-Valued Neutrosophic Set: [7] A Neutrosophic set $\tilde{N}e_M$ in the definition 2.3 is assumed as a Single-Valued Neutrosophic Set ($\tilde{S}N\tilde{e}_M$) if x is a single-valued independent variable. $\tilde{S}N\tilde{e}_M = \{(x; [\lambda_{\tilde{S}N\tilde{e}_M}(x), \pi_{\tilde{S}N\tilde{e}_M}(x), \sigma_{\tilde{S}N\tilde{e}_M}(x)]) : x \in X\}$, where $\lambda_{\tilde{S}N\tilde{e}_M}(x), \pi_{\tilde{S}N\tilde{e}_M}(x)$ & $\sigma_{\tilde{S}N\tilde{e}_M}(x)$ signified the notion of correct, indefinite and incorrect memberships function respectively. If three points d_0, e_0 & f_0 exists for which $\lambda_{\tilde{S}N\tilde{e}_M}(d_0) = 1, \pi_{\tilde{S}N\tilde{e}_M}(e_0) = 1$ & $\sigma_{\tilde{S}N\tilde{e}_M}(f_0) = 1$, then the $\tilde{S}N\tilde{e}_M$ is termed neut-normal.

$\tilde{S}C\tilde{S}_M$ is called neut-convex indicating that $\tilde{S}C\tilde{S}_M$ is a subset of a real line by meeting the resulting conditions:

- i. $\lambda_{\tilde{S}N\tilde{e}_M}(\delta d_1 + (1 - \delta)d_2) \geq \min\{\lambda_{\tilde{S}N\tilde{e}_M}(d_1), \lambda_{\tilde{S}N\tilde{e}_M}(d_2)\}$
- ii. $\pi_{\tilde{S}N\tilde{e}_M}(\delta d_1 + (1 - \delta)d_2) \leq \max\{\pi_{\tilde{S}N\tilde{e}_M}(d_1), \pi_{\tilde{S}N\tilde{e}_M}(d_2)\}$
- iii. $\sigma_{\tilde{S}N\tilde{e}_M}(\delta d_1 + (1 - \delta)d_2) \leq \max\{\sigma_{\tilde{S}N\tilde{e}_M}(d_1), \sigma_{\tilde{S}N\tilde{e}_M}(d_2)\}$

where d_1 & $d_2 \in \mathbb{R}$ and $\delta \in [0,1]$

2.5 Definition: Single-Valued Pentagonal Neutrosophic Number: [9] A Single-Valued Pentagonal Neutrosophic Number (\tilde{M}) is defined as, $\tilde{S}s = \{(s^1, t^1, u^1, v^1, w^1); \mu\}, [(s^2, t^2, u^2, v^2, w^2); \theta], [(s^3, t^3, u^3, v^3, w^3); \eta]\}$,

where $\mu, \theta, \eta \in [0,1]$. The truthness function ($\mu_{\tilde{S}s}$): $\mathbb{R} \rightarrow [0, \mu]$, the indeterminacy function ($\theta_{\tilde{S}s}$): $\mathbb{R} \rightarrow [\theta, 1]$ and the falsity function ($\eta_{\tilde{S}s}$): $\mathbb{R} \rightarrow [\eta, 1]$ are given as:

$$\mu_{\tilde{S}s}(x) = \begin{cases} \mu_{\tilde{S}s1}(x) s^1 \leq x < t^1 \\ \mu_{\tilde{S}s12}(x) t^1 \leq x < u^1 \\ \mu_{\tilde{S}s12}(x) x = u^1 \\ \mu_{\tilde{S}s2}(x) u^1 \leq x < v^1 \\ \mu_{\tilde{S}s2}(x) v^1 \leq x < w^1 \\ 0 & \text{otherwise} \end{cases}, \quad \theta_{\tilde{S}s}(x) = \begin{cases} \theta_{\tilde{S}s1}(x) s^2 \leq x < t^2 \\ \theta_{\tilde{S}s12}(x) t^2 \leq x < u^2 \\ \theta_{\tilde{S}s12}(x) x = u^2 \\ \theta_{\tilde{S}s2}(x) u^2 \leq x < v^2 \\ \theta_{\tilde{S}s2}(x) v^2 \leq x < w^2 \\ 1 & \text{otherwise} \end{cases}$$

$$\eta_{\widetilde{ss}}(x) = \begin{cases} \eta_{\widetilde{ss}l1}(x) s^3 \leq x < t^3 \\ \eta_{\widetilde{ss}l2}(x) t^3 \leq x < u^3 \\ \vartheta & x = u^3 \\ \eta_{\widetilde{ss}r2}(x) u^3 \leq x < v^3 \\ \eta_{\widetilde{ss}r1}(x) v^3 \leq x < w^3 \\ 1 & \text{otherwise} \end{cases}$$

3. Score Function: Score function actually relates any uncertain number and the crisp number in our real world. A score function is defined and developed in PNN $\widetilde{W}_{Pen} = (w_1, w_2, w_3, w_4, w_5; \pi, \mu, \sigma)$ as,

Here, $(1 + \pi)$ is the beneficiary portions of PNN membership function and $(1 - \mu - \sigma)$ is the hesitation portions of PNN membership function. Also, we have the mean of the components as, $\frac{(w_1 + w_2 + w_3 + w_4 + w_5)}{5}$

Thus, Score function is described as $\widetilde{SC}_{Pen} = \frac{1}{15} (w_1 + w_2 + w_3 + w_4 + w_5) \times (2 + \pi - \mu - \sigma)$,

Accuracy function is described as $\widetilde{AC}_{Pen} = \frac{1}{15} (w_1 + w_2 + w_3 + w_4 + w_5) \times (2 + \pi + \mu + \sigma)$

3.1 Relationship between any two PNN:

Let us consider any two PNN defined as follows

$$W_{Pen1} = (\pi_{Pen1}, \mu_{Pen1}, \sigma_{Pen1}), W_{Pen2} = (\pi_{Pen2}, \mu_{Pen2}, \sigma_{Pen2})$$

- 1) $SC_{Pen1} > SC_{Pen2}, W_{Pen1} > W_{Pen2}$
- 2) $SC_{Pen1} < SC_{Pen2}, W_{Pen1} < W_{Pen2}$
- 3) $SC_{Pen1} = SC_{Pen2}, W_{Pen1} = W_{Pen2}$

Then, if $AC_{Pen1} > AC, W_{Pen1} > W_{Pen2}$

$$AC_{Pen1} < AC_{Pen2}, W_{Pen1} < W_{Pen2}$$

$$AC_{Pen1} = AC_{Pen2}, W_{Pen1} = W_{Pen2}$$

4. Shortest Path Search Algorithm under PNN Environment:

Here we consider a problem in PNN environment to compute the shortest path in a very simple way. Shortest path Search Algorithm is one of the best and popular skills used in path finding and graph traversals. Many games and web- based graphs are used here to compute the shortest path very efficiently. This Algorithm finds the shortest path through the search space using the hemistich function. It uses a best first search graph algorithm and finds a least cost path from current node to destination node. Consider a weighted graph in PNN area [10] whose weights and heuristic cost function are given as a pentagonal neutrosophic number with multiple nodes and we want to reach the target node to starting node as quick as possible. It defined a heuristic cost function $S(n) = p(n) + h(n)$ where $S(n)$ = estimated cost of the cheapest solution, $p(n)$ = cost to reach node n from the starting position, $h(n)$ = estimated heuristic cost. Time and cost these are hesitation factors in case of real life problem. Here we consider the functions $p(n), h(n)$ are both pentagonal neutrosophic number. This algorithm expands less search tree and computes the optimal result faster.

4.1 Algorithm

Step1: Convert all the PNN into crisp number using the established score value Section (3).

Step2: Placed the starting node to open list

Step 3: Check whether the open list is empty or not, if the list is empty then stop the process.

Step 4: choose the node from the open list, which has the least value of estimation function $S(n)$, if node "n" is target node then back to success and stop.

Step 5: Expand node "n" and produce all of its successes and put "n" in the closed list.

- For each successes "n", check whether "n" is already in the open or closed list.
- If not then compute evaluation function for "n" and placed it into open list.

Doi : [10.5281/zenodo.3732659](https://doi.org/10.5281/zenodo.3732659)

Step 6: Else if node “n” is already in open and closed then it should be attached to the back pointer which reflects the lowest $g(n')$ value.

Step 7: Stop.

4.2 Flowchart:

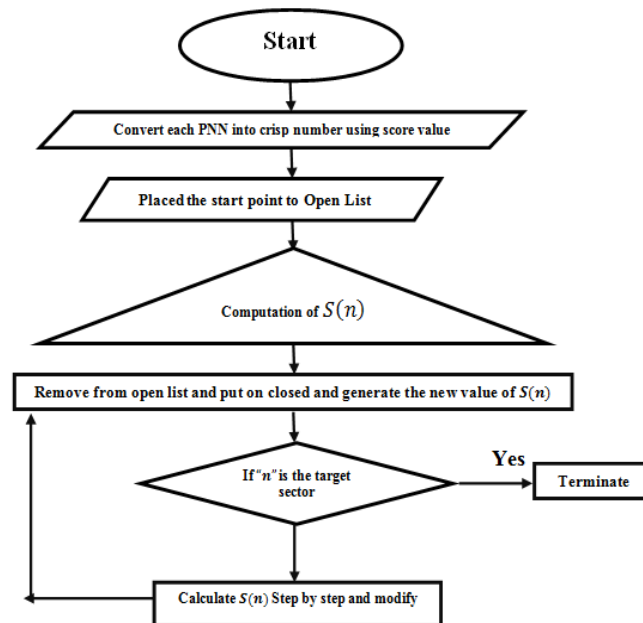
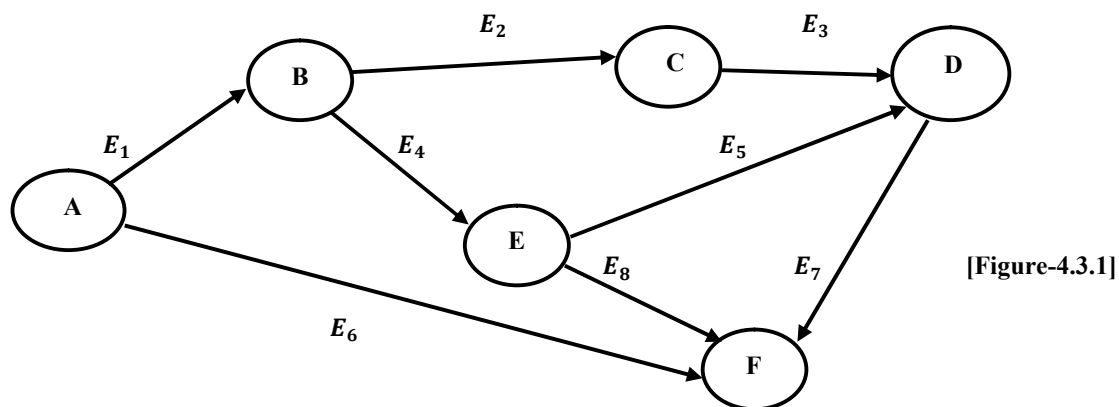


Figure 4.2.1: Flowchart for the problem

4.3 Illustrative Example: Find the shortest path from A to F of the following graph in PNN environment.

Edges	Optimistic Time	Stage	Heuristic Value
E_1	$< (2,3,4,5,6; 0.4,0.5,0.6) >$	A	$< (0.3,0.7,1.2,1.5,2; 0.6,0.5,0.3) >$
E_2	$< (3,4,5,6,7; 0.6,0.3,0.4) >$	B	$< (0.5,0.8,1.4,2,2.4; 0.6,0.7,0.4) >$
E_3	$< (1,2,2.5,3,3.5; 0.6,0.4,0.5) >$	C	$< (0.2,0.4,0.6,0.8,1; 0.8,0.6,0.5) >$
E_4	$< (0,0.5,1,3,5; 0.3,0.2,0.6) >$	D	$< (0.8,1.3,1.8,2.4,3; 0.3,0.4,0.5) >$
E_5	$< (1.5,2,2.5,3,4.5; 0.3,0.4,0.3) >$	E	$< (0.7,1.5,2,2.5,3; 0.7,0.4,0.4) >$
E_6	$< (2,3,3.5,4,4.5; 0.7,0.2,0.4) >$		

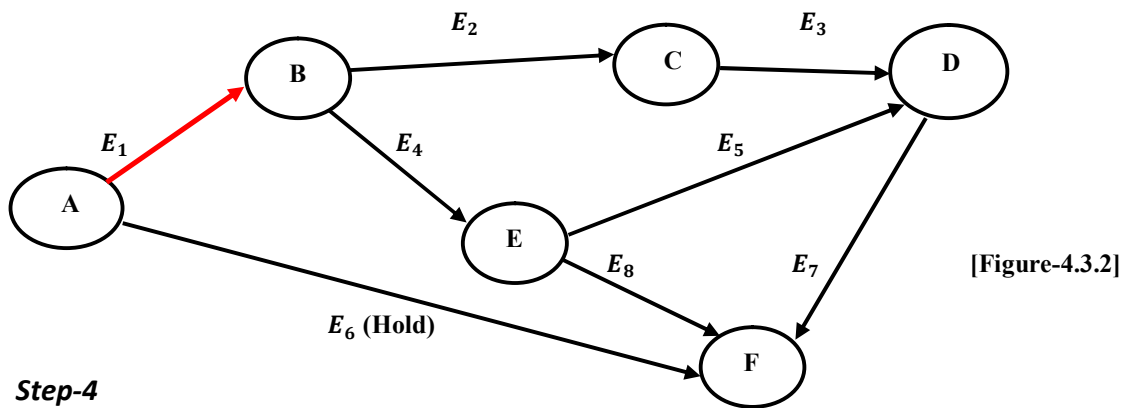
E_7	$< (3,3.5,4,4.5,5; 0.6,0.2,0.3) >$				F	$< (0.6,0.9,1.3,1.8,2.4; 0.5,0.2,0.3) >$		
E_8	$< (0.3,0.4,0.45,0.5,0.6; 0.6,0.3,0.4) >$							
Connection	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8
Edges	$A-B$	$B-C$	$C-D$	$B-E$	$E-D$	$A-F$	$D-F$	$E-F$

Step-1 Network Diagram:**Step-2 Crispification using the established Score function (3)**

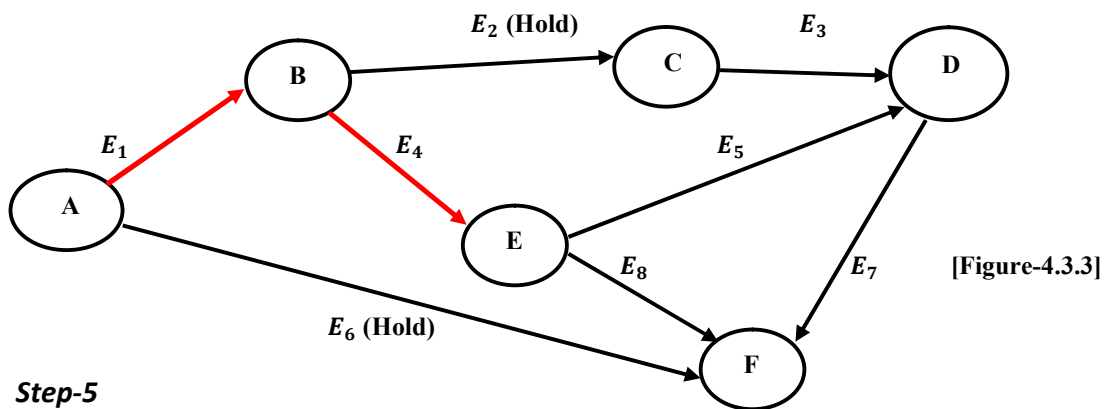
Edges	Optimistic Time		Stage	Heuristic Value
E_1	1.73		A	0.68
E_2	2.83		B	0.71
E_3	1.36		C	0.34
E_4	0.95		D	0.87
E_5	1.44		E	1.23
E_6	2.38		F	0.93
E_7	2.80			
E_8	0.28			

Step-3 Here, A is the starting node

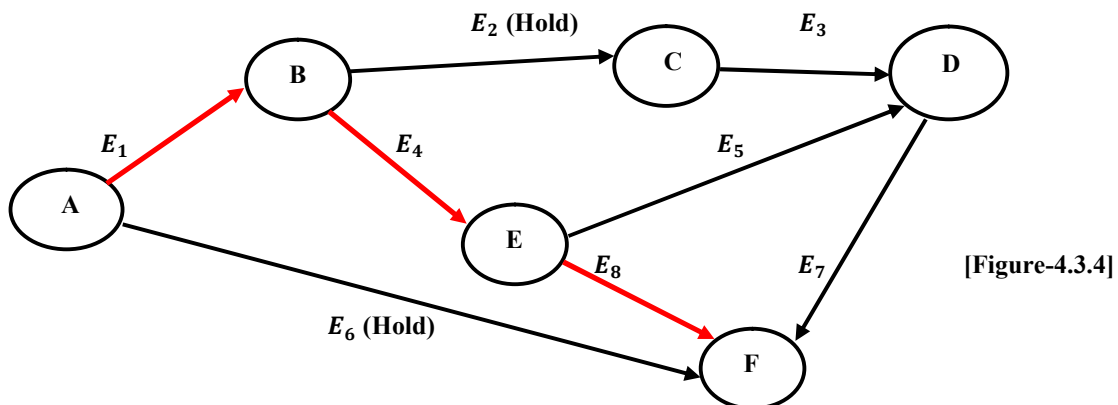
$A \rightarrow$	$S(n) = p(n) + h(n) = 0 + 0.68 = 0.68$
$A \rightarrow B$	$S(n) = 1.73 + 0.71 = 2.44$
$A \rightarrow F$	$S(n) = 2.38 + 0.93 = 3.31$ (hold)



$A \rightarrow B \rightarrow C$	$S(n) = 1.73 + 2.83 + 0.34 = 4.9$ (hold)
$A \rightarrow B \rightarrow E$	$S(n) = 1.73 + 0.95 + 1.23 = 3.91$



$A \rightarrow B \rightarrow E \rightarrow F$	$S(n) = 1.73 + 0.95 + 0.28 + 0.93 = 3.89$
$A \rightarrow B \rightarrow E \rightarrow D$	$S(n) = 1.73 + 0.95 + 1.44 + 0.87 = 4.99$ (hold)



SO, the final shortest path is $A \rightarrow B \rightarrow E \rightarrow F$ and optimal cost is = 2.96 unit.

5. Conclusion and future research scope

The concept of PNN has an adequate scope of utilization in various studies in different domain. In this research article, we strongly erect the perception of score and accuracy function from different aspects. Additionally, we consider a shortest path problem in PNN environment and resolve the problem applying the idea of score function. Since, there may be no such articles is until now hooked up in PNN area, for this reason we cannot done comparison study of our work with the other established methods.

Further, researchers can immensely apply this idea of neutrosophic number in numerous flourishing research fields like engineering problem, mobile computing problems, diagnoses problem, realistic mathematical modeling, social media problem etc.

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Linear and Non-Linear Octagonal Neutrosophic Numbers: Its Representation, α -Cut and Applications

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Abstract

In this paper, the primarily focus is to extend the concept of Octagonal Neutrosophic Numbers (ONN) since these numbers provide a wide range of applications while dealing with more fluctuations in the linguistic environment. Firstly, mathematical notions and definitions of Linear, Symmetric and Asymmetric types are proposed. Secondly, α – Cut is defined. Finally, a case study is done by using the TOPSIS technique of MCDM.

Keywords: Accuracy Function, Neutrosophic Numbers, Octagonal Neutrosophic Numbers (ONN), MCDM, TOPSIS.

1.Introduction

Researchers and mathematicians all over the world developed important analytical skills and problem-solving strategies to assess a broad range of issues in human resource, medicine, selection problems etc. But the most challenging issues were related to the problems which were more qualitative rather than quantitative in nature.

Thus, the need to handle uncertain situations and vagueness in practical as well as theoretical problems led the researchers to the development of theories like fuzzy, neutrosophic set theory. The neutrosophic sets (NSs) [1] reflect on the truth membership, indeterminacy membership, and falsity membership concurrently, which is more practical and adequate than FSs and IFSs in selection problems, that are uncertain, incomplete, and inconsistent.

The idea of triangular, trapezoidal and pentagonal neutrosophic numbers having membership function which are dependent and independent was given by [2-4]. Single-valued neutrosophic sets are an extension of NSs which were introduced by Wang et al. [8]. Ye [9] introduced, simplified neutrosophic sets, and Peng et al. [8,9] define their novel operations and aggregation operators. Finally, there are different extensions of NSs, such as interval neutrosophic set [10], bipolar neutrosophic sets [11], and multi-valued neutrosophic sets [12].

Smarandache and many other researchers [13-20] also discussed the various extension of neutrosophic sets in TOPSIS and MCDM. Saqlain et.al. [21] proposed a new algorithm along with a new decision-making environment. Many other novel approaches were also used by many researches [22-42] in decision makings. Some Fundamental properties and applications of triangular and pentagonal neutrosophic numbers are proposed by [43-47]. With the concept of octagonal neutrosophic numbers, decision maker can deal with more fluctuations because they have more edges as compared to pentagonal numbers. In this current epoch, the neutrosophic numbers can be converted into fuzzy numbers and the ability to deal with fluctuations will be increased.

1.1 Motivation

Different researchers had already published a lot of articles on neutrosophic arena, as they applied and extended this concept in different fields such as MCDM. The conception on neutrosophic octagonal number is totally new. An important issue is that if someone wants to take Linear ONN, then what should its representation be? How should we define membership, indeterminacy and non-membership functions? From this point of view, ONN is a good choice for a decision maker in a practical scenario.

1.2 The Paper Presentation

In this paper, the concept of Octagonal Neutrosophic Numbers ONN is extended.

- Formulation of Linear, Non-Linear, Linear symmetric, Non-Linear symmetric Octagonal Neutrosophic Numbers.
- Defining the α – cut of each type.
- A case study of personal selection.

1.3 Structure of Paper

The article is structured as shown in the following Figure.

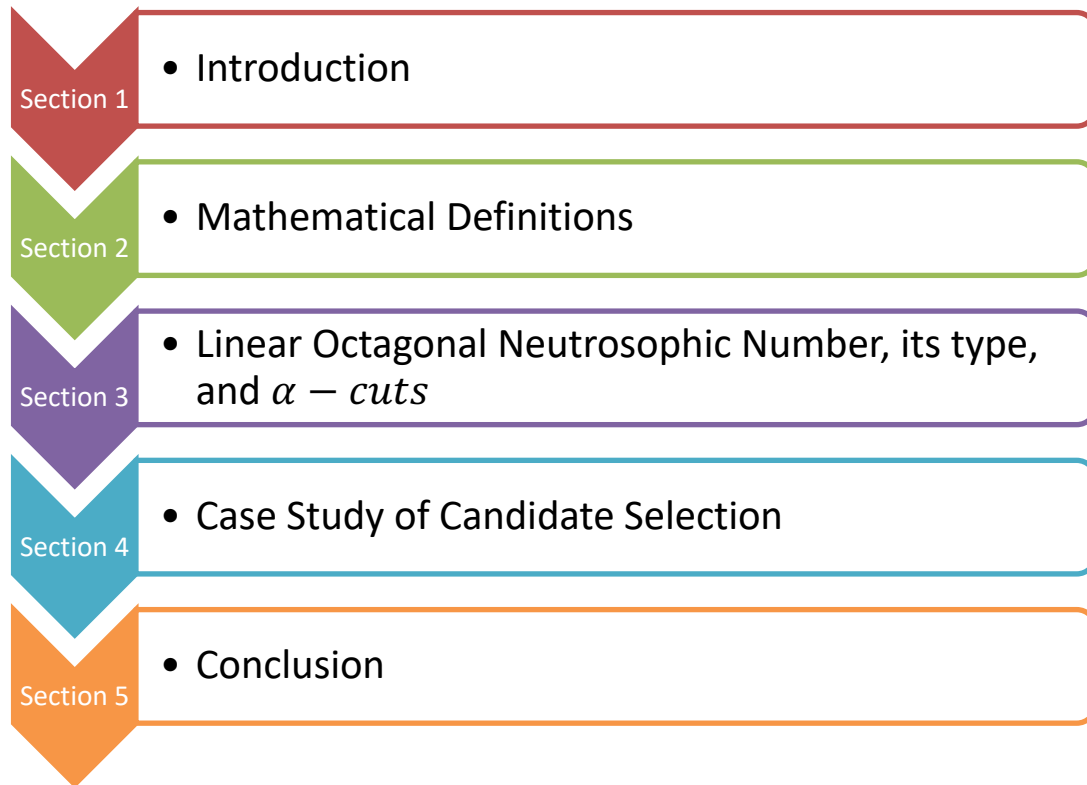


Figure 1: Pictorial view for the structure of the article

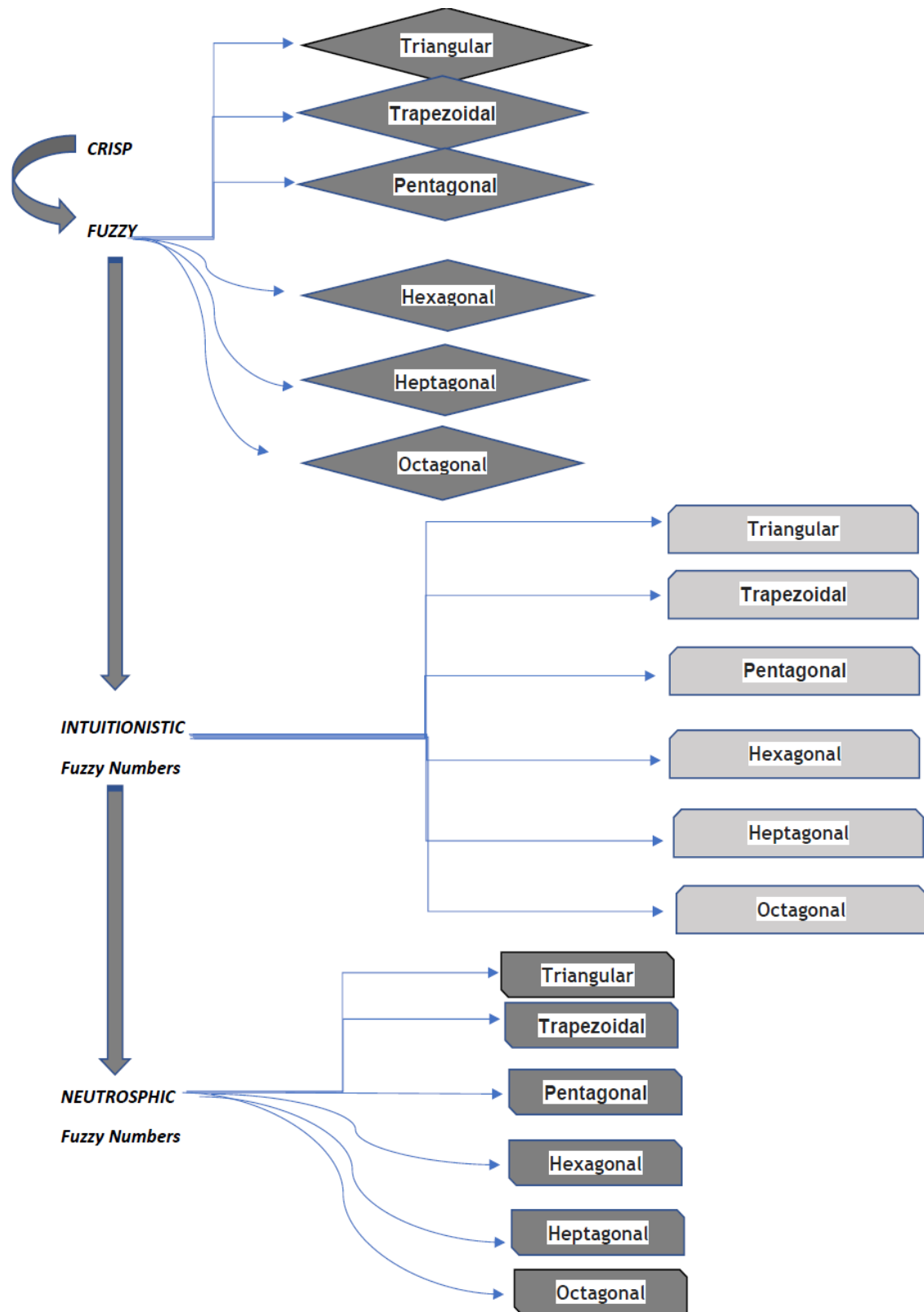


Figure 2. Flow chart of the three types, fuzzy, Intuitionistic fuzzy, and neutrosophic logic numbers

2. Mathematical Definitions

In this section, we present necessary definitions that are used throughout the paper.

Definition 2.1: Neutrosophic Set [1]: A set \widehat{nA} is neutrosophic if $\widehat{nA} = \{ \langle x; [T_{\widehat{nA}}(x), I_{\widehat{nA}}(x), F_{\widehat{nA}}(x)] \rangle : x \in X \}$, where $T_{\widehat{nA}}(x) : \rightarrow [0,1]$ be a truth membership function $I_{\widehat{nA}}(x)$ be a indeterminacy membership function and $F_{\widehat{nA}}(x)$ is falsity membership function $T_{\widehat{nA}}(x), I_{\widehat{nA}}(x), F_{\widehat{nA}}(x)$ exhibits the following relation:

$$0^- \leq T_{\widehat{nA}}(x), I_{\widehat{nA}}(x), F_{\widehat{nA}}(x) \leq 3^+$$

Definition 2.2: Triangular Neutrosophic Number [2]: Triangular single value neutrosophic number is given as $\mathcal{A}'_{\text{Neu}} = (p_1, p_2, p_3; r_1, r_2, r_3)$ whose truth, indeterminacy and falsity membership is given as:

$$\mathcal{T}_{\mathcal{A}'_{\text{Neu}}}(x) = \begin{cases} \frac{x-p_1}{p_1-p_2} & \text{for } p_1 \leq x < p_2 \\ 1 & \text{when } x = p_2 \\ \frac{p_3-x}{p_3-p_2} & \text{for } p_2 < x \leq p_3 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathcal{I}_{\mathcal{A}'_{\text{Neu}}}(x) = \begin{cases} \frac{q_2-x}{q_2-q_1} & \text{for } q_1 \leq x < q_2 \\ 0 & \text{when } x = q_2 \\ \frac{x-q_2}{q_3-q_2} & \text{for } q_2 < x \leq q_3 \\ 1 & \text{otherwise} \end{cases}$$

$$\mathcal{F}_{\mathcal{A}'_{\text{Neu}}}(x) = \begin{cases} \frac{x-p_1}{p_1-p_2} & \text{for } p_1 \leq x < p_2 \\ 1 & \text{when } x = p_2 \\ \frac{p_3-x}{p_3-p_2} & \text{for } p_2 < x \leq p_3 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Where } 0 \leq \mathcal{T}_{\mathcal{A}'_{\text{Neu}}}(x) + \mathcal{I}_{\mathcal{A}'_{\text{Neu}}}(x) + \mathcal{F}_{\mathcal{A}'_{\text{Neu}}}(x) \leq 3, x \in \mathcal{A}'_{\text{Neu}}.$$

And the parametric foam of this type is $(\mathcal{A}'_{\text{Neu}})_{\alpha, \beta, \gamma} = [\mathcal{T}_{\text{Neu1}}(\alpha), \mathcal{T}_{\text{Neu2}}(\alpha); \mathcal{I}_{\text{Neu1}}(\beta), \mathcal{I}_{\text{Neu2}}(\beta); \mathcal{F}_{\text{Neu1}}(\gamma), \mathcal{F}_{\text{Neu2}}(\gamma)]$, where, $\mathcal{T}_{\text{Neu1}}(\alpha) = p_1 + \alpha(p_2 - p_1)$, $\mathcal{T}_{\text{Neu2}}(\alpha) = p_3 - \alpha(p_3 - p_2)$, $\mathcal{I}_{\text{Neu1}}(\beta) = q_2 - \beta(q_2 - q_1)$, $\mathcal{I}_{\text{Neu2}}(\beta) = q_2 + \beta(q_3 - q_2)$, $\mathcal{F}_{\text{Neu1}}(\gamma) = r_2 - \gamma(r_2 - r_1)$, $\mathcal{F}_{\text{Neu2}}(\gamma) = r_2 + \gamma(r_3 - r_2)$, Here $0 < \alpha \leq 1, 0 < \beta \leq 1, 0 < \gamma \leq 1$ and $0 < \alpha + \beta + \gamma \leq 3$

Definition 2.3: Trapezoidal Neutrosophic Number [3]: Let \mathcal{X} be the universe of discourse, a trapezoidal neutrosophic set $\tilde{\mathcal{A}}$ in \mathcal{X} is defined by: $\tilde{\mathcal{N}} = \{ \langle \mathcal{X}, \mathcal{T}_{\mathcal{N}}(\mathcal{X}), \mathcal{I}_{\mathcal{N}}(\mathcal{X}), \mathcal{F}_{\mathcal{N}}(\mathcal{X}) \rangle | x \in \mathcal{X} \}$, where $\mathcal{T}_{\mathcal{N}}(\mathcal{X}) \subset [0,1]$, $\mathcal{I}_{\mathcal{N}}(\mathcal{X}) \subset [0,1]$, $\mathcal{F}_{\mathcal{N}}(\mathcal{X}) \subset [0,1]$ are consider as three trapezoidal number, $\mathcal{T}_{\mathcal{N}}(\mathcal{X}) = (\mathcal{t}_{\mathcal{N}}^1(x), \mathcal{t}_{\mathcal{N}}^2(x), \mathcal{t}_{\mathcal{N}}^3(x), \mathcal{t}_{\mathcal{N}}^4(x)) : \mathcal{X} \mapsto [0,1]$, $\mathcal{I}_{\mathcal{N}}(\mathcal{X}) = (\mathcal{i}_{\mathcal{N}}^1(x), \mathcal{i}_{\mathcal{N}}^2(x), \mathcal{i}_{\mathcal{N}}^3(x), \mathcal{i}_{\mathcal{N}}^4(x)) : \mathcal{X} \mapsto [0,1]$, $\mathcal{F}_{\mathcal{N}}(\mathcal{X}) = (\mathcal{f}_{\mathcal{N}}^1(x), \mathcal{f}_{\mathcal{N}}^2(x), \mathcal{f}_{\mathcal{N}}^3(x), \mathcal{f}_{\mathcal{N}}^4(x)) : \mathcal{X} \mapsto [0,1]$ with the condition $0 \leq \mathcal{t}_{\mathcal{N}}^4(x) + \mathcal{i}_{\mathcal{N}}^4(x) + \mathcal{f}_{\mathcal{N}}^4(x) \leq 3, x \in \mathcal{X}$.

Definition 2.4: Pentagonal Neutrosophic Number [4]: Pentagonal neutrosophic number ($\tilde{\mathcal{S}}$) for single valued is defined as $\tilde{\mathcal{S}} = \langle [\dot{m}^1, \ddot{n}^1, \ddot{o}^1, \dot{p}^1, \dot{q}^1; \pi], [\dot{m}^2, \ddot{n}^2, \ddot{o}^2, \dot{p}^2, \dot{q}^2; \rho], [\dot{m}^3, \ddot{n}^3, \ddot{o}^3, \dot{p}^3, \dot{q}^3; \epsilon] \rangle$ where $\pi, \rho, \epsilon \in [0,1]$. The truth membership function $(\mathcal{T}_{\mathcal{S}}) : \mathbb{R} \mapsto [0, \pi]$, the indeterminacy membership function $(\mathcal{I}_{\mathcal{S}}) : \mathbb{R} \mapsto [\rho, 1]$ and falsity membership function $(\mathcal{F}_{\mathcal{S}}) : \mathbb{R} \mapsto [6, 1]$ and given as:

$$\mathcal{T}_{\hat{s}}(x) = \begin{cases} \mathcal{T}_{\hat{s}t1}(x) \dot{m}^1 \leq x < \dot{n}^1 \\ \mathcal{T}_{\hat{s}t2}(x) \dot{n}^1 \leq x < \dot{o}^1 \\ \mu & x = \dot{o}^1 \\ \mathcal{T}_{\hat{s}t1}(x) \dot{o}^1 \leq x < \dot{p}^1 \\ \mathcal{T}_{\hat{s}t1}(x) \dot{p}^1 \leq x < \dot{q}^1 \\ 0 & \text{otherwise} \end{cases} \quad \mathcal{J}_{\hat{s}}(x) = \begin{cases} \mathcal{J}_{\hat{s}t1}(x) \dot{m}^2 \leq x < \dot{n}^2 \\ \mathcal{J}_{\hat{s}t2}(x) \dot{n}^2 \leq x < \dot{o}^2 \\ \theta & x = \dot{o}^2 \\ \mathcal{J}_{\hat{s}t1}(x) \dot{o}^2 \leq x < \dot{p}^2 \\ \mathcal{J}_{\hat{s}t1}(x) \dot{p}^2 \leq x < \dot{q}^2 \\ 1 & \text{otherwise} \end{cases} \quad \varepsilon_{\hat{s}}(x) = \begin{cases} \varepsilon_{\hat{s}t1}(x) \dot{m}^3 \leq x < \dot{n}^3 \\ \varepsilon_{\hat{s}t2}(x) \dot{n}^3 \leq x < \dot{o}^3 \\ \theta & x = \dot{o}^3 \\ \varepsilon_{\hat{s}t1}(x) \dot{o}^3 \leq x < \dot{p}^3 \\ \varepsilon_{\hat{s}t1}(x) \dot{p}^3 \leq x < \dot{q}^3 \\ 1 & \text{otherwise} \end{cases}$$

Where $\langle [\dot{m}^1 < \dot{n}^1 < \dot{o}^1 < \dot{p}^1 < \dot{q}^1; \pi], [\dot{m}^2 < \dot{n}^2 < \dot{o}^2 < \dot{p}^2 < \dot{q}^2; \rho], [\dot{m}^3 < \dot{n}^3 < \dot{o}^3 < \dot{p}^3 < \dot{q}^3; \delta] \rangle$

Definition 2.5: Octagonal Neutrosophic Number [ONN] A Neutrosophic Number denoted by \hat{S} is defined as,

$$\hat{S} = \langle [(\Omega, \eta, \varepsilon_v, \varepsilon, \mathcal{K}, \acute{o}, \mathfrak{z}): \Theta], [(\Omega^1, \eta^1, \varepsilon^1, \mathfrak{v}^1, \varepsilon^1, \mathcal{K}^1, \acute{o}^1, \mathfrak{z}^1): \Psi], [(\Omega^2, \eta^2, \varepsilon^2, \mathfrak{v}^2, \varepsilon^2, \mathcal{K}^2, \acute{o}^2, \mathfrak{z}^2): \delta] \rangle \text{ Where } \Theta, \Psi, \delta \in [0, 1].$$

The truth membership function $(\Theta_{\hat{s}}): \mathbb{R} \rightarrow [0, 1]$,

the indeterminacy membership function $(\Psi_{\hat{s}}): \mathbb{R} \rightarrow [\delta, 1]$,

and the falsity membership function $(Y_{\hat{s}}): \mathbb{R} \rightarrow [\delta, 1]$ are given as follows:

$$\Theta_{\hat{s}}(x) = \begin{cases} \Theta_{\hat{s}0}(x) & \Omega \leq x < \eta \\ \Theta_{\hat{s}1}(x) & \eta \leq x < \varepsilon \\ \Theta_{\hat{s}2}(x) & \varepsilon \leq x < \mathfrak{v} \\ \Theta_{\hat{s}3}(x) & \mathfrak{v} \leq x < \varepsilon \\ \delta & x = \varepsilon \\ \Theta_{\hat{s}3}(x) & \varepsilon \leq x < \mathcal{K} \\ \Theta_{\hat{s}2}(x) & \mathcal{K} \leq x < \acute{o} \\ \Theta_{\hat{s}1}(x) & \acute{o} \leq x < \mathfrak{z} \\ 0 & \text{otherwise} \end{cases} \quad \Psi_{\hat{s}}(x) = \begin{cases} \Psi_{\hat{s}0}(x) & \Omega^1 \leq x < \eta^1 \\ \Psi_{\hat{s}1}(x) & \eta^1 \leq x < \varepsilon^1 \\ \Psi_{\hat{s}2}(x) & \varepsilon^1 \leq x < \mathfrak{v}^1 \\ \Psi_{\hat{s}3}(x) & \mathfrak{v}^1 \leq x < \varepsilon^1 \\ \delta & x = \varepsilon^1 \\ \Psi_{\hat{s}3}(x) & \varepsilon^1 \leq x < \mathcal{K}^1 \\ \Psi_{\hat{s}2}(x) & \mathcal{K}^1 \leq x < \acute{o}^1 \\ \Psi_{\hat{s}1}(x) & \acute{o}^1 \leq x < \mathfrak{z}^1 \\ 1 & \text{otherwise} \end{cases} \quad Y_{\hat{s}}(x) = \begin{cases} Y_{\hat{s}0}(x) & \Omega^2 \leq x < \eta^2 \\ Y_{\hat{s}1}(x) & \eta^2 \leq x < \varepsilon^2 \\ Y_{\hat{s}2}(x) & \varepsilon^2 \leq x < \mathfrak{v}^2 \\ Y_{\hat{s}3}(x) & \mathfrak{v}^2 \leq x < \varepsilon^2 \\ \delta & x = \varepsilon^2 \\ Y_{\hat{s}3}(x) & \varepsilon^2 \leq x < \mathcal{K}^2 \\ Y_{\hat{s}2}(x) & \mathcal{K}^2 \leq x < \acute{o}^2 \\ Y_{\hat{s}1}(x) & \acute{o}^2 \leq x < \mathfrak{z}^2 \\ 1 & \text{otherwise} \end{cases}$$

Where $\hat{S} = \langle [(\Omega < \eta < \varepsilon < \mathfrak{v} < \varepsilon < \mathcal{K} < \acute{o} < \mathfrak{z}): \Theta], [(\Omega^1 < \eta^1 < \varepsilon^1 < \mathfrak{v}^1 < \varepsilon^1 < \mathcal{K}^1 < \acute{o}^1 < \mathfrak{z}^1): \Psi], [(\Omega^2 < \eta^2 < \varepsilon^2 < \mathfrak{v}^2 < \varepsilon^2 < \mathcal{K}^2 < \acute{o}^2 < \mathfrak{z}^2): \delta] \rangle$

3. The definition of [LONN], Representation and Examples had been Presented

In this section, we discuss its representations, and investigate its properties.

Definition 3.1: Linear Octagonal Neutrosophic Number [LONN]

Let $\hat{\mathcal{A}} = (\Omega, \eta, \varepsilon, \mathfrak{v}; \varepsilon, \mathcal{K}, \acute{o}, \mathfrak{z})$ be a linear octagonal neutrosophic number. It should satisfy the following conditions:

1. $\mu_{\hat{\mathcal{A}}}(\mathbf{x})$ is continuous function between the interval Ω to \mathfrak{z} for truthiness.
2. $\mu_{\hat{\mathcal{A}}}(\mathbf{x})$ is a non-increasing continuous function between the interval of Ω to ε for truthiness.
3. $\mu_{\hat{\mathcal{A}}}(\mathbf{x})$ is a non-decreasing continuous function between the interval ε to \mathfrak{z} for truthiness.
4. $\mu_{\hat{\mathcal{A}}}(\mathbf{x})$ is continuous function between the interval Ω^1 to \mathfrak{z}^1 for falsity.
5. $\mu_{\hat{\mathcal{A}}}(\mathbf{x})$ is a non-decreasing continuous function between the interval of Ω^1 to ε^1 for falsity.
6. $\mu_{\hat{\mathcal{A}}}(\mathbf{x})$ is a non-increasing continuous function between the interval of ε^1 to \mathfrak{z}^1 for falsity.
7. $\mu_{\hat{\mathcal{A}}}(\mathbf{x})$ is continuous function between the interval Ω^2 to \mathfrak{z}^2 for indeterminacy.
8. $\mu_{\hat{\mathcal{A}}}(\mathbf{x})$ is a non-increasing continuous function between the interval of Ω^2 to ε^2 for indeterminacy.
9. $\mu_{\hat{\mathcal{A}}}(\mathbf{x})$ is a non-decreasing continuous function between the interval of ε^2 to \mathfrak{z}^2 for indeterminacy.

3.2 Linear ONN with symmetry

Let $\mathcal{A}_{LS} = (\Omega, \eta, \varepsilon, \nu, \varepsilon, \mathbb{K}, \acute{o}, \mathfrak{z})$ be a linear ONN with the following membership functions.

$$\text{Truthiness} = T_L(\mathcal{X}) = \begin{cases} 0 & x < \Omega \\ \mathcal{K} \left(\frac{x - \Omega}{\eta - \Omega} \right) & \Omega \leq x \leq \eta \\ \mathcal{K} & \eta \leq x \leq \varepsilon \\ \mathcal{K} + (1 - \mathcal{K}) \left(\frac{x - \varepsilon}{\nu - \varepsilon} \right) & \varepsilon \leq x \leq \nu \\ 1 & \nu \leq x \leq \varepsilon \\ \mathcal{K} + (1 - \mathcal{K}) \left(\frac{\mathbb{K} - x}{\mathbb{K} - \varepsilon} \right) & \varepsilon \leq x \leq \mathbb{K} \\ \mathcal{K} & \mathbb{K} \leq x \leq \acute{o} \\ \mathcal{K} \left(\frac{\mathfrak{z} - x}{\mathfrak{z} - \acute{o}} \right) & \acute{o} \leq x \leq \mathfrak{z} \\ 0 & x > \mathfrak{z} \end{cases}$$

With $0 < \mathcal{K} < 1$

$$\text{Falsity} = F_L(\mathcal{X}) = \begin{cases} 0 & x < \Omega^1 \\ \mathcal{K} \left(\frac{x - \Omega^1}{\eta^1 - \Omega^1} \right) & \Omega^1 \leq x \leq \eta^1 \\ \mathcal{K} & \eta^1 \leq x \leq \varepsilon^1 \\ \mathcal{K} + (1 - \mathcal{K}) \left(\frac{x - \varepsilon^1}{\nu^1 - \varepsilon^1} \right) & \varepsilon^1 \leq x \leq \nu^1 \\ 1 & \nu^1 \leq x \leq \varepsilon^1 \\ \mathcal{K} + (1 - \mathcal{K}) \left(\frac{\mathbb{K}^1 - x}{\mathbb{K}^1 - \varepsilon^1} \right) & \varepsilon^1 \leq x \leq \mathbb{K}^1 \\ \mathcal{K} & \mathbb{K}^1 \leq x \leq \acute{o}^1 \\ \mathcal{K} \left(\frac{\mathfrak{z}^1 - x}{\mathfrak{z}^1 - \acute{o}^1} \right) & \acute{o}^1 \leq x \leq \mathfrak{z}^1 \\ 0 & x > \mathfrak{z}^1 \end{cases}$$

$$\text{Indeterminacy} = I_L(\mathcal{X}) = \begin{cases} 0 & x < \Omega^2 \\ \mathcal{K} \left(\frac{x - \Omega^2}{\eta^2 - \Omega^2} \right) & \Omega^2 \leq x \leq \eta^2 \\ \mathcal{K} & \eta^2 \leq x \leq \varepsilon^2 \\ \mathcal{K} + (1 - \mathcal{K}) \left(\frac{x - \varepsilon^2}{\nu^2 - \varepsilon^2} \right) & \varepsilon^2 \leq x \leq \nu^2 \\ 1 & \nu^2 \leq x \leq \varepsilon^2 \\ \mathcal{K} + (1 - \mathcal{K}) \left(\frac{\mathbb{K}^2 - x}{\mathbb{K}^2 - \varepsilon^2} \right) & \varepsilon^2 \leq x \leq \mathbb{K}^2 \\ \mathcal{K} & \mathbb{K}^2 \leq x \leq \acute{o}^2 \\ \mathcal{K} \left(\frac{\mathfrak{z}^2 - x}{\mathfrak{z}^2 - \acute{o}^2} \right) & \acute{o}^2 \leq x \leq \mathfrak{z}^2 \\ 0 & x > \mathfrak{z}^2 \end{cases}$$

3.3 α -cut of Linear ONN with symmetry:

α -cut can be express as: $\mathcal{A}_{\alpha} = \{x \in \bar{\mathcal{X}} | T_L(\mathcal{X}), I_L(\mathcal{X}), F_L(\mathcal{X}) \geq \alpha\}$

$$\text{Truthiness} = T_L(\mathcal{X}) = \begin{cases} \mathcal{A}_{1L}(\alpha) = \Omega + \frac{\alpha}{b_1}(\eta - \Omega) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_1] \\ \mathcal{A}_{2L}(\alpha) = \eta + \frac{1-\alpha}{1-\acute{b}_2}(\epsilon - \eta) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_2] \\ \mathcal{A}_{3L}(\alpha) = \epsilon + \frac{1-\alpha}{1-\acute{b}_3}(\vartheta - \epsilon) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_3] \\ \mathcal{A}_{4L}(\alpha) = \vartheta + \frac{1-\alpha}{1-\acute{b}_4}(\xi - \vartheta) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_4] \\ \mathcal{A}_{3R}(\alpha) = \mathbb{K} - \frac{\alpha}{\acute{b}_4}(\mathbb{K} - \epsilon) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_4] \\ \mathcal{A}_{2R}(\alpha) = \acute{\alpha} - \frac{\alpha}{\acute{b}_3}(\acute{\alpha} - \mathbb{K}) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_3] \\ \mathcal{A}_{1R}(\alpha) = 3 - \frac{\alpha}{\acute{b}_2}(3 - \acute{\alpha}) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_2] \end{cases}$$

There we have $\mathcal{A}_{1L}(\alpha)$, $\mathcal{A}_{2L}(\alpha)$, $\mathcal{A}_{3L}(\alpha)$, $\mathcal{A}_{4L}(\alpha)$ are increasing and $\mathcal{A}_{3R}(\alpha)$, $\mathcal{A}_{2R}(\alpha)$, $\mathcal{A}_{1R}(\alpha)$ are decreasing.

$$\text{Falsity} = F_L(\mathcal{X}) = \begin{cases} \mathcal{A}_{1L}(\alpha) = \Omega^1 + \frac{\alpha}{b_1}(\eta^1 - \Omega^1) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_1] \\ \mathcal{A}_{2L}(\alpha) = \eta^1 + \frac{1-\alpha}{1-\acute{b}_2}(\epsilon^1 - \eta^1) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_2] \\ \mathcal{A}_{3L}(\alpha) = \epsilon^1 + \frac{1-\alpha}{1-\acute{b}_3}(\vartheta^1 - \epsilon^1) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_3] \\ \mathcal{A}_{4L}(\alpha) = \vartheta^1 + \frac{1-\alpha}{1-\acute{b}_4}(\xi^1 - \vartheta^1) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_4] \\ \mathcal{A}_{3R}(\alpha) = \mathbb{K}^1 - \frac{\alpha}{\acute{b}_4}(\mathbb{K}^1 - \epsilon^1) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_4] \\ \mathcal{A}_{2R}(\alpha) = \acute{\alpha}^1 - \frac{\alpha}{\acute{b}_3}(\acute{\alpha}^1 - \mathbb{K}^1) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_3] \\ \mathcal{A}_{1R}(\alpha) = 3^1 - \frac{\alpha}{\acute{b}_2}(3^1 - \acute{\alpha}^1) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_2] \end{cases}$$

$$\text{Indeterminacy} = I_L(\mathcal{X}) = \begin{cases} \mathcal{A}_{1L}(\alpha) = \eta^2 + \frac{\alpha}{b_1}(\epsilon^2 - \eta^2) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_1] \\ \mathcal{A}_{2L}(\alpha) = \epsilon^2 + \frac{1-\alpha}{1-\acute{b}_2}(\vartheta^2 - \epsilon^2) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_2] \\ \mathcal{A}_{3L}(\alpha) = \vartheta^2 + \frac{1-\alpha}{1-\acute{b}_3}(\xi^2 - \vartheta^2) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_3] \\ \mathcal{A}_{4L}(\alpha) = \xi^2 + \frac{1-\alpha}{1-\acute{b}_4}(\mathbb{K}^2 - \xi^2) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_4] \\ \mathcal{A}_{3R}(\alpha) = \acute{\alpha}^2 - \frac{\alpha}{\acute{b}_3}(\acute{\alpha}^2 - \mathbb{K}^2) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_3] \\ \mathcal{A}_{2R}(\alpha) = 3^2 - \frac{\alpha}{\acute{b}_2}(3^2 - \acute{\alpha}^2) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_2] \\ \mathcal{A}_{1R}(\alpha) = \overline{3^2} - \frac{\alpha}{\acute{b}_2}(\overline{3^2} - \overline{\acute{\alpha}^2}) \text{ for } \alpha \in [\acute{\alpha}, \acute{b}_2] \end{cases}$$

3.4 Non-Linear ONN with symmetry: Let $\mathcal{A}_{LS} = (\Omega, \eta, \epsilon, a_1, a_2, \mathbb{K}, \acute{\alpha}, 3)_{(\acute{n}_1, \acute{n}_2, \acute{m}_1, \acute{m}_2)}$ be a non linear ONN and its membership function are:

$$\begin{aligned}
\text{Truthiness} = T_{NL}(\mathcal{X}) &= \begin{cases} 0 & x < \Omega \\ \ell \left(\frac{x-\Omega}{\eta-\Omega} \right)^{\dot{n}_1} & \Omega \leq x \leq \eta \\ \ell & \eta \leq x \leq \xi \\ \ell + (1-\ell) \left(\frac{x-\xi}{\vartheta-\xi} \right)^{\dot{n}_2} & \xi \leq x \leq \vartheta \\ 1 & \vartheta \leq x \leq \varepsilon \\ \ell + (1-\ell) \left(\frac{\mathbb{K}-x}{\mathbb{K}-\varepsilon} \right)^{\dot{m}_1} & \varepsilon \leq x \leq \mathbb{K} \\ \ell & \mathbb{K} \leq x \leq \acute{o} \\ \ell \left(\frac{\acute{o}-x}{\acute{o}-\mathbb{K}} \right)^{\dot{m}_2} & \acute{o} \leq x \leq \mathbb{K} \\ 0 & x > \mathbb{K} \end{cases} \\
\text{Falsity} = F_{NL}(\mathcal{X}) &= \begin{cases} 0 & x < \Omega^1 \\ \ell \left(\frac{x-\Omega^1}{\eta^1-\Omega^1} \right)^{\dot{m}_1} & \Omega^1 \leq x \leq \eta^1 \\ \ell & \eta^1 \leq x \leq \xi^1 \\ \ell + (1-\ell) \left(\frac{x-\xi^1}{\vartheta^1-\xi^1} \right)^{\dot{m}_2} & \xi^1 \leq x \leq \vartheta^1 \\ 1 & \vartheta^1 \leq x \leq \varepsilon^1 \\ \ell + (1-\ell) \left(\frac{\mathbb{K}^1-x}{\mathbb{K}^1-\varepsilon^1} \right)^{\dot{n}_1} & \varepsilon^1 \leq x \leq \mathbb{K}^1 \\ \ell & \mathbb{K}^1 \leq x \leq \acute{o}^1 \\ \ell \left(\frac{\acute{o}^1-x}{\acute{o}^1-\mathbb{K}^1} \right)^{\dot{n}_2} & \acute{o}^1 \leq x \leq \mathbb{K}^1 \\ 0 & x > \mathbb{K}^1 \end{cases} \\
\text{Indeterminacy} = I_{NL}(\mathcal{X}) &= \begin{cases} 0 & x < \Omega^2 \\ \ell \left(\frac{x-\Omega^2}{\eta^2-\Omega^2} \right)^{\dot{m}_1} & \Omega^2 \leq x \leq \eta^2 \\ \ell & \eta^2 \leq x \leq \xi^2 \\ \ell + (1-\ell) \left(\frac{x-\xi^2}{\vartheta^2-\xi^2} \right)^{\dot{m}_2} & \xi^2 \leq x \leq \vartheta^2 \\ 1 & \vartheta^2 \leq x \leq \varepsilon^2 \\ \ell + (1-\ell) \left(\frac{\mathbb{K}^2-x}{\mathbb{K}^2-\varepsilon^2} \right)^{\dot{n}_1} & \varepsilon^2 \leq x \leq \mathbb{K}^2 \\ \ell & \mathbb{K}^2 \leq x \leq \acute{o}^2 \\ \ell \left(\frac{\acute{o}^2-x}{\acute{o}^2-\mathbb{K}^2} \right)^{\dot{n}_2} & \acute{o}^2 \leq x \leq \mathbb{K}^2 \\ 0 & x > \mathbb{K}^2 \end{cases}
\end{aligned}$$

3.5 α -cut of Non-Linear ONN with symmetry

α -cut of LONNS can be expressed by: $\mathcal{A}_{\alpha} = \{x \in \bar{\mathcal{X}} | T_{NL}(\mathcal{X}), I_{NL}(\mathcal{X}), F_{NL}(\mathcal{X}) \geq \alpha\}$

$$\begin{aligned}
\text{Truthiness} = T_{NL}(\mathcal{X}) &= \begin{cases} \mathcal{A}_{1L}(\alpha) = \Omega + \left(\frac{\alpha}{b_1}\right)^{\dot{n}_1} (\eta - \Omega) \text{ for } \alpha \in [\dot{o}, \dot{b}_1] \\ \mathcal{A}_{2L}(\alpha) = \eta + \left(\frac{1-\alpha}{1-b_2}\right)^{\dot{n}_2} (\xi - \eta) \text{ for } \alpha \in [\dot{b}_2, 1] \\ \mathcal{A}_{3L}(\alpha) = \xi + \left(\frac{1-\alpha}{1-b_3}\right)^{\dot{n}_3} (\nu - \xi) \text{ for } \alpha \in [\dot{b}_3, 1] \\ \mathcal{A}_{4L}(\alpha) = \nu + \left(\frac{1-\alpha}{1-b_4}\right)^{\dot{n}_4} (\varepsilon - \nu) \text{ for } \alpha \in [\dot{b}_4, 1] \\ \mathcal{A}_{3R}(\alpha) = \mathbb{K} - \left(\frac{\alpha}{b_4}\right)^{\dot{m}_1} (\mathbb{K} - \varepsilon) \text{ for } \alpha \in [\dot{o}, \dot{b}_4] \\ \mathcal{A}_{2R}(\alpha) = \acute{o} - \left(\frac{\alpha}{b_3}\right)^{\dot{m}_2} (\acute{o} - \mathbb{K}) \text{ for } \alpha \in [\dot{o}, \dot{b}_3] \\ \mathcal{A}_{1R}(\alpha) = 3 - \left(\frac{\alpha}{b_2}\right)^{\dot{m}_3} (3 - \acute{o}) \text{ for } \alpha \in [\dot{o}, \dot{b}_2] \end{cases} \\
\text{Falsity} = F_{NL}(\mathcal{X}) &= \begin{cases} \mathcal{A}_{1L}(\alpha) = \Omega^1 + \frac{\alpha^{\dot{m}_1}}{b_1} (\eta^1 - \Omega^1) \text{ for } \alpha \in [\dot{o}, \dot{b}_1] \\ \mathcal{A}_{2L}(\alpha) = \eta^1 + \frac{1-\alpha}{1-b_2} (\xi^1 - \eta^1) \text{ for } \alpha \in [\dot{o}, \dot{b}_2] \\ \mathcal{A}_{3L}(\alpha) = \xi^1 + \frac{1-\alpha}{1-b_3} (\nu^1 - \xi^1) \text{ for } \alpha \in [\dot{o}, \dot{b}_3] \\ \mathcal{A}_{4L}(\alpha) = \nu^1 + \frac{1-\alpha}{1-b_4} (\varepsilon^1 - \nu^1) \text{ for } \alpha \in [\dot{o}, \dot{b}_4] \\ \mathcal{A}_{3R}(\alpha) = \mathbb{K}^1 - \frac{\alpha^{\dot{n}_1}}{b_4} (\mathbb{K}^1 - \varepsilon^1) \text{ for } \alpha \in [\dot{b}_4, 1] \\ \mathcal{A}_{2R}(\alpha) = \acute{o}^1 - \frac{\alpha^{\dot{n}_2}}{b_3} (\acute{o}^1 - \mathbb{K}^1) \text{ for } \alpha \in [\dot{b}_3, 1] \\ \mathcal{A}_{1R}(\alpha) = 3^1 - \frac{\alpha^{\dot{n}_3}}{b_2} (3^1 - \acute{o}^1) \text{ for } \alpha \in [\dot{b}_2, 1] \end{cases} \\
\text{Indeterminacy} = I_{NL}(\mathcal{X}) &= \begin{cases} \mathcal{A}_{1L}(\alpha) = \eta^2 + \frac{\alpha^{\dot{m}_1}}{b_1} (\xi^2 - \eta^2) \text{ for } \alpha \in [\dot{o}, \dot{b}_1] \\ \mathcal{A}_{2L}(\alpha) = \xi^2 + \frac{1-\alpha}{1-b_2} (\nu^2 - \xi^2) \text{ for } \alpha \in [\dot{o}, \dot{b}_2] \\ \mathcal{A}_{3L}(\alpha) = \nu^2 + \frac{1-\alpha}{1-b_3} (\varepsilon^2 - \nu^2) \text{ for } \alpha \in [\dot{o}, \dot{b}_3] \\ \mathcal{A}_{4L}(\alpha) = \varepsilon^2 + \frac{1-\alpha}{1-b_4} (\mathbb{K}^2 - \varepsilon^2) \text{ for } \alpha \in [\dot{o}, \dot{b}_4] \\ \mathcal{A}_{3R}(\alpha) = \acute{o}^2 - \frac{\alpha^{\dot{n}_1}}{b_3} (\acute{o}^2 - \mathbb{K}^2) \text{ for } \alpha \in [\dot{b}_4, 1] \\ \mathcal{A}_{2R}(\alpha) = 3^2 - \frac{\alpha^{\dot{n}_2}}{b_3} (3^2 - \acute{o}^2) \text{ for } \alpha \in [\dot{b}_3, 1] \\ \mathcal{A}_{1R}(\alpha) = \overline{3^2} - \frac{\alpha^{\dot{n}_3}}{b_2} (\overline{3^2 - \acute{o}^2}) \text{ for } \alpha \in [\dot{b}_2, 1] \end{cases}
\end{aligned}$$

The increasing functions are $\mathcal{A}_{1L}(\alpha), \mathcal{A}_{2L}(\alpha), \mathcal{A}_{3L}(\alpha), \mathcal{A}_{4L}(\alpha)$ with respect to α and the decreasing functions are $\mathcal{A}_{1R}(\alpha), \mathcal{A}_{2R}(\alpha), \mathcal{A}_{3R}(\alpha)$ with respect to α . $\mathcal{A}_{1L}(\alpha), \mathcal{A}_{1L}(\alpha), \mathcal{A}_{1L}(\alpha), \mathcal{A}_{1L}(\alpha)$ with respect to α .

$$\begin{aligned}
\text{Truthiness} = T_{NL}(\mathcal{X}) &= \begin{cases} 0 & x < \Omega \\ \mathcal{P} \left(\frac{x-\Omega}{\eta-\Omega} \right)^{n_1} & \Omega \leq x \leq \eta \\ k & \eta \leq x \leq \xi \\ k - (k - \mathcal{P}) \left(\frac{x-\xi}{a_1-\xi} \right)^{n_2} & \xi \leq x \leq a_1 \\ 1 & a_1 \leq x \leq a_2 \\ k - (k - r) \left(\frac{\mathcal{K}-x}{\mathcal{K}-a_2} \right)^{m_1} & a_2 \leq x \leq \mathcal{K} \\ k & \mathcal{K} \leq x \leq \acute{o} \\ r \left(\frac{\mathfrak{z}-x}{\mathfrak{z}-\acute{o}} \right)^{m_2} & \acute{o} \leq x \leq \mathfrak{z} \\ 0 & x > \mathfrak{z} \end{cases} \\
\text{Falsity} = F_{NL}(\mathcal{X}) &= \begin{cases} 0 & x < \Omega^1 \\ q \left(\frac{x-\Omega^1}{\eta^1-\Omega^1} \right)^{n_1} & \Omega^1 \leq x \leq \eta^1 \\ w & \eta^1 \leq x \leq \xi^1 \\ w - (w - q) \left(\frac{x-\xi^1}{b_1^1-\xi^1} \right)^{n_2} & \xi^1 \leq x \leq a_1^1 \\ 1 & b_1^1 \leq x \leq a_2^1 \\ w - (w - s) \left(\frac{\mathcal{K}^1-x}{\mathcal{K}^1-b_2^1} \right)^{m_1} & b_2^1 \leq x \leq \mathcal{K}^1 \\ w & \mathcal{K}^1 \leq x \leq \acute{o}^1 \\ s \left(\frac{\mathfrak{z}^1-x}{\mathfrak{z}^1-\acute{o}^1} \right)^{m_2} & \acute{o}^1 \leq x \leq \mathfrak{z}^1 \\ 0 & x > \mathfrak{z}^1 \end{cases} \\
\text{Indeterminacy} = I_{NL}(\mathcal{X}) &= \begin{cases} 0 & x < \Omega^2 \\ \mathcal{Y} \left(\frac{x-\Omega^2}{\eta^2-\Omega^2} \right)^{n_1} & \Omega^2 \leq x \leq \eta^2 \\ \mathcal{X} & \eta^2 \leq x \leq \xi^2 \\ \mathcal{X} - (\mathcal{X} - \mathcal{Y}) \left(\frac{x-\xi^2}{c_1^2-\xi^2} \right)^{n_2} & \xi^2 \leq x \leq a_1^2 \\ 1 & c_1^2 \leq x \leq a_1^2 \\ \mathcal{X} - (\mathcal{X} - Z) \left(\frac{\mathcal{K}^2-x}{\mathcal{K}^2-c_2^2} \right)^{m_1} & c_2^2 \leq x \leq \mathcal{K}^2 \\ Z & \mathcal{K}^2 \leq x \leq \acute{o}^2 \\ Z \left(\frac{\mathfrak{z}^2-x}{\mathfrak{z}^2-\acute{o}^2} \right)^{m_2} & \acute{o}^2 \leq x \leq \mathfrak{z}^2 \\ 0 & x > \mathfrak{z}^2 \end{cases}
\end{aligned}$$

4. Case Study

To demonstrate the feasibility and productiveness of the proposed method, here is the most useful real-life problem. Suppose we have three different persons which have different degree, experience and number of publications. How can we select the person who has more potential to deal with situations?

Numerical Example: Suppose that U is the universe. Suppose that the HR which is responsible for recruiting, interviewing and placing workers, wants to hire a new person in company. Three different persons (A, B, C) apply for this opportunity, which have different degrees, experiences and publications. On the base of choice parameters $\{\mathbb{C}_1, \mathbb{C}_2, \mathbb{C}_3\}$ we have to select the best one.

A	B	C
$\{C_1(0.72,0.35,0.71,0.77,0.41,0.73,0.77,0.81)$ $(0.93,0.83,0.63,0.88,0.94,0.99,0.96,0.60)$ $(0.86,0.95,0.99,0.97,0.94,0.93,0.95,0.91)\}$	$\{C_1(0.33,0.73,0.34,0.25,0.26,0.74,0.45,0.29)$ $(0.33,0.46,0.59,0.79,0.85,0.79,0.74,0.86)$ $(0.88,0.83,0.55,0.75,0.98,0.64,0.96,0.90)\}$	$\{C_1(0.23,0.33,0.63,0.56,0.45,0.35,0.73,0.28)$ $(0.76,0.55,0.69,0.34,0.24,0.63,0.95,0.91)\}$ $(0.94,0.73,0.95,0.95,0.48,0.94,0.96,0.74)\}$
$\{C_2(0.35,0.65,0.36,0.54,0.33,0.65,0.43,0.56)$ $(0.75,0.45,0.95,0.38,0.68,0.79,0.57,0.13)$ $(0.96,0.99,0.78,0.79,0.97,0.36,0.97,0.95)\}$	$\{C_2(0.25,0.55,0.36,0.54,0.33,0.65,0.43,0.56)$ $(0.93,0.83,0.83,0.58,0.84,0.69,0.76,0.80)$ $(0.96,0.99,0.98,0.99,0.97,0.76,0.87,0.95)\}$	$\{C_2(0.23,0.65,0.26,0.54,0.63,0.65,0.41,0.59)$ $(0.75,0.45,0.85,0.38,0.78,0.79,0.67,0.13)$ $(0.98,0.89,0.88,0.79,0.97,0.96,0.87,0.85)\}$
$\{C_3(0.24,0.33,0.44,0.55,0.56,0.34,0.45,0.89)$ $(0.35,0.46,0.58,0.79,0.85,0.71,0.64,0.96)$ $(0.84,0.73,0.85,0.95,0.98,0.84,0.96,0.94)\}$	$\{C_3(0.24,0.33,0.74,0.35,0.46,0.54,0.85,0.19)$ $(0.88,0.86,0.58,0.85,0.85,0.61,0.64,0.86)$ $(0.98,0.93,0.95,0.95,0.98,0.84,0.66,0.84)\}$	$\{C_3(0.24,0.23,0.44,0.25,0.26,0.34,0.85,0.89)$ $(0.35,0.44,0.78,0.79,0.75,0.71,0.94,0.96)$ $(0.74,0.63,0.95,0.95,0.98,0.94,0.98,0.94)\}$

[In this above, matrix (C_1, C_2, C_3) is mentioned in the row and persons (A,B,C) are mentioned in the column]

STEP 1: Defuzzify the Octagonal Neutrosophic number by using Accuracy Function [21]

$$D^{TNO_N} = \left(\frac{\Omega + \eta + \epsilon + \nu + \epsilon + \zeta + \delta + \alpha + \beta}{8} \right), D^{INO_N} = \left(\frac{\Omega^1 + \eta^1 + \epsilon^1 + \nu^1 + \epsilon^1 + \zeta^1 + \delta^1 + \alpha^1 + \beta^1}{8} \right), D^{FNO_N} = \left(\frac{\Omega^2 + \eta^2 + \epsilon^2 + \nu^2 + \epsilon^2 + \zeta^2 + \delta^2 + \alpha^2 + \beta^2}{8} \right)$$

Then the neutrosophic soft matrix is

Criteria	\mathcal{A}_1	\mathcal{A}_2	\mathcal{A}_3
\mathcal{C}_1	(0.6,0.8,0.9)	(0.4,0.6,0.8)	(0.4,0.5,0.9)
\mathcal{C}_2	(0.4,0.5,0.7)	(0.4,0.7,0.9)	(0.4,0.6,0.9)
\mathcal{C}_3	(0.4,0.6,0.9)	(0.4,0.7,0.9)	(0.4,0.7,0.9)

STEP 2: For normalized aggregate fuzzy decision matrix.

$$\bar{r}_{ij} = \left(\frac{\bar{a}_{ij}}{c_{ij}}, \frac{\bar{b}_{ij}}{c_{ij}}, \frac{\bar{c}_{ij}}{c_{ij}} \right)$$

Criteria	\mathcal{A}_1	\mathcal{A}_2	\mathcal{A}_3
\mathcal{C}_1	(0.6,0.8,1.0)	(0.5,0.7,1.0)	(0.4,0.5,1.0)
\mathcal{C}_2	(0.5,0.7,1.0)	(0.4,0.7,1.0)	(0.4,0.6,1.0)
\mathcal{C}_3	(0.4,0.6,1.0)	(0.4,0.7,1.0)	(0.4,0.7,1.0)

Aggregate decision matrix for criteria weighting

$$\bar{W}_1 = (0.3, 0.4, 0.5) \quad , \quad \bar{W}_2 = (0.5, 0.6, 0.7) \quad \bar{W}_3 = (0.1, 0.2, 0.3)$$

STEP 3. Weighted normalized fuzzy decision matrix. $\bar{P}_{ij} = \bar{r}_{ij}$ multiply-by w_j

Criteria	\mathcal{A}_1	\mathcal{A}_2	\mathcal{A}_3
\mathcal{C}_1	(0.1,0.32,0.5)	(0.1,0.28,0.5)	(0.1,0.2,0.5)
\mathcal{C}_2	(0.2,0.42,0.7)	(0.2,0.42,0.7)	(0.2,0.36,0.7)
\mathcal{C}_3	(0.04,0.12,0.3)	(0.04,0.14,0.3)	(0.04,0.14,0.3)

STEP 4. Find FNIS AND FPIS

$$\mathcal{A}^+ = (\mathcal{P}_1^+, \mathcal{P}_2^+, \mathcal{P}_3^+ \dots \mathcal{P}_n^+)$$

$$\mathcal{P}_j^+ = \max (\mathcal{P}_{ij3}) \quad i=1,2,\dots,m \quad , \quad j=1,2,3,\dots,n$$

$$\mathcal{A}^- = (\mathcal{P}_1^-, \mathcal{P}_2^-, \mathcal{P}_3^- \dots \mathcal{P}_n^-)$$

$$\mathcal{P}_j^- = \min (\mathcal{P}_{ij3}) \quad i=1,2,\dots,m \quad , \quad j=1,2,3,\dots,n$$

$$\mathcal{A}^+ = (\mathcal{P}_1^+(0.5,0.5,0.5), \mathcal{P}_2^+(0.7,0.7,0.7), \mathcal{P}_3^+(0.3,0.3,0.3))$$

$$\mathcal{A}^- = (\mathcal{P}_1^-(0.1,0.1,0.1), \mathcal{P}_2^-(0.2,0.2,0.2), \mathcal{P}_3^-(0.04,0.04,0.04))$$

$$\text{Now by } d(\bar{x}, \bar{y}) = \sqrt{\frac{1}{3}(\alpha_1 - \alpha_2)^2 + (\beta_1 - \beta_2)^2 + (c_1 - c_2)^2}$$

Positive Ideal Solution

Criteria	\mathcal{A}_1	\mathcal{A}_2	\mathcal{A}_3
\mathcal{C}_1	0.253	0.263	0.288
\mathcal{C}_2	0.330	0.330	0.349
\mathcal{C}_3	0.182	0.176	0.176

Negative Ideal Solution

Criteria	\mathcal{A}_1	\mathcal{A}_2	\mathcal{A}_3
\mathcal{C}_1	0.050	0.041	0.023
\mathcal{C}_2	0.063	0.063	0.046
\mathcal{C}_3	0.012	0.015	0.015

STEP 5. Now calculate distance between each weighted alternative

$$\bar{d}_i^* = \sum_{j=1}^n \check{d}(v_{ij}, v_j^*), \bar{d}_i^- = \sum_{j=1}^n \check{d}(v_{ij}, v_j^-)$$

$$\bar{d}_1^* = 0.765 \quad \bar{d}_1^- = 0.125$$

$$\bar{d}_2^* = 0.769 \quad \bar{d}_2^- = 0.119$$

$$\bar{d}_3^* = 0.813 \quad \bar{d}_3^- = 0.084$$

STEP 6. Closeness coefficient

$$\bar{C}C_i = \frac{\bar{d}_i^-}{\bar{d}_i^- + \bar{d}_i^+}$$

$$\bar{C}C_1 = \frac{0.125}{0.125 + 0.765} = 0.140$$

$$\bar{C}C_2 = \frac{0.119}{0.119 + 0.769} = 0.134$$

$$\bar{C}C_3 = \frac{0.084}{0.084 + 0.813} = 0.093$$

Strategy	Result value	Rank
$\bar{C}C_1$	0.140	1
$\bar{C}C_2$	0.134	2
$\bar{C}C_3$	0.093	3

Clearly

$\mathcal{A}_1 > \mathcal{A}_2 > \mathcal{A}_3$, The person deserves this post is \mathcal{A}_1 .

5. Conclusions

In this article, types of octagonal neutrosophic number (Linear, Non-Linear, Symmetric, Asymmetric) are proposed and their α – cuts were also derived. Octagonal Neutrosophic Numbers are very useful in solving multi criteria decision making MCDM problems from daily life since this number can deal with more fluctuations. To discuss the applicability and productiveness in daily life issues a case study was done using TOPSIS technique of MCDM. In which firstly numbers were converted from octagonal to fuzzy using accuracy function and then used in

the existing method. In forthcoming work, we'll propose the aggregate operators of Octagonal Neutrosophic Numbers and their matrix notions with operations.

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Neutrosophic \mathfrak{N} -Ideals (\mathfrak{N} -Subalgebras) of Subtraction Algebra

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Abstract

The connection between neutrosophy and algebra has been of great interest with respect to many researchers. The objective of this paper is to provide a connection between neutrosophic \mathfrak{N} -structures and subtraction algebras. In this regard, we introduce the concept of neutrosophic \mathfrak{N} -ideals in subtraction algebra. Moreover, we study its properties and find a necessary and sufficient condition for a neutrosophic \mathfrak{N} -structure to be a neutrosophic \mathfrak{N} -ideal.

Keywords: Subtraction algebra, \mathfrak{N} -structure, Neutrosophic \mathfrak{N} -ideal, Level set.

1. Introduction

Neutrosophic sets were introduced by Florentin Smarandache [11] as a new mathematical tool for dealing with uncertainty. They can be viewed as a generalization of the fuzzy sets that were introduced in 1965 by Lotfi Zadeh [14]. Where Zadeh defined fuzzy sets as mathematical model of vagueness in which an element belongs to a given set to some degree that is a number between 0 and 1 (both inclusive). Neutrosophy is a base of neutrosophic logic which is an extension of fuzzy logic where indeterminacy is included [13]. In neutrosophic logic [10], each proposition is estimated to have the degree of truth in a subset T , the degree of indeterminacy in a subset I , and the degree of falsity in a subset F . The study of neutrosophic sets and their properties have a great importance in the sense of applications as well as in understanding the fundamentals of uncertainty. Some related work can be found in [1, 2, 3, 12].

A crisp set A in a universe X can be defined in the form of its membership function $\mu_A: X \rightarrow \{0,1\}$ where $\mu_A(x) = 1$ if $x \in A$ and $\mu_A(x) = 0$ if $x \notin A$. A single valued neutrosophic set is an example of neutrosophic set which has many applications [10]. A new function, which is called *negative-valued function*, was introduced by Jun et al. [5] and they used it to construct \mathfrak{N} -structures. Some work related to neutrosophic \mathfrak{N} -structures can be found in [6, 7]. Schein [9] considered systems of the form $(\Phi, \boxminus, \backslash)$, where Φ is a set of functions closed under the composition " \boxminus " of functions and the set theoretic subtraction " \backslash " and hence (Φ, \backslash) is a *subtraction algebra*. Jun et al. [4] introduced the concept of ideals in subtraction algebras and discussed the properties of these ideals. Some researchers worked on combining the notions of neutrosophic sets and subtraction algebra. For example, Ibrahim et al. introduced neutrosophic subtraction algebra (semigroups) and presented some results about them. Moreover, Park [8] discussed neutrosophic ideals of subtraction algebras by using single valued neutrosophic sets.

In this paper, we apply the concept of neutrosophic \mathfrak{N} -structures in subtraction algebras. And it is organized as follows: After an Introduction, in Section 2 and Section 3, we present some basic results about neutrosophic \mathfrak{N} -structures as well as about subtraction algebras that are used throughout the paper. In Section 4, we introduce neutrosophic \mathfrak{N} -ideals (\mathfrak{N} -subalgebras) of subtraction algebra and prove that the intersection, the product, the homomorphic preimage, and onto homomorphic image are neutrosophic \mathfrak{N} -ideals. Finally, in Section 5, we prove a necessary and sufficient condition for \mathfrak{N} -structures to be neutrosophic \mathfrak{N} -ideals by introducing the (α, β, γ) -level sets.

2. Neutrosophic \mathfrak{N} -structures

In this section, we present some basic results about neutrosophic \mathfrak{N} -structures. For more details about neutrosophy, we refer to [5, 6, 7].

Definition 2.1. [5] Let S be a non-empty set. A function from $S \rightarrow [-1,0]$ is called a negative-valued function (\mathfrak{N} -function) from S to $[-1,0]$.

Definition 2.2. [7] Let S be a non-empty set. A neutrosophic \mathfrak{N} -structure over S is defined as follows:

$$S_N = \left\{ \frac{x}{(T_N, I_N, F_N)} : x \in S \right\},$$

where T_N, I_N, F_N are \mathfrak{N} -functions on S which are called the negative truth membership function, the negative indeterminacy membership function and the negative falsity membership function, respectively, on S . It is clear that for any \mathfrak{N} -structure S_N over S , $-3 \leq T_N(x) + I_N(x) + F_N(x) \leq 0$ for all $x \in S$.

Definition 2.3. [7] Let $S_N = \left\{ \frac{x}{(T_N, I_N, F_N)} : x \in S \right\}$ and $S_M = \left\{ \frac{x}{(T_M, I_M, F_M)} : x \in S \right\}$ be \mathfrak{N} -structures over S .

- (1) S_N is called a neutrosophic \mathfrak{N} -substructure of S_M , denoted as $S_N \subseteq S_M$, if for all $x \in S$, $T_N(x) \geq T_M(x)$, $I_N(x) \leq I_M(x)$, $F_N(x) \geq F_M(x)$.
If $S_N \subseteq S_M$ and $S_M \subseteq S_N$, we say that $S_N = S_M$.

- (2) The union of S_N and S_M is defined to be the \mathfrak{N} -structure over S :

$$S_{N \cup M} = \left\{ \frac{x}{(T_{N \cup M}, I_{N \cup M}, F_{N \cup M})} : x \in S \right\},$$

where $T_{N \cup M}(x) = T_N(x) \wedge T_M(x)$, $I_{N \cup M}(x) = I_N(x) \vee I_M(x)$, and $F_{N \cup M}(x) = F_N(x) \wedge F_M(x)$ for all $x \in S$.

- (3) The intersection of S_N and S_M is defined to be the \mathfrak{N} -structure over S :

$$S_{N \cap M} = \left\{ \frac{x}{(T_{N \cap M}, I_{N \cap M}, F_{N \cap M})} : x \in S \right\},$$

where $T_{N \cap M}(x) = T_N(x) \vee T_M(x)$, $I_{N \cap M}(x) = I_N(x) \wedge I_M(x)$, and $F_{N \cap M}(x) = F_N(x) \vee F_M(x)$ for all $x \in S$.

- (4) The complement of S_N is defined to be the \mathfrak{N} -structure over S :

$$S_{N^c} = \left\{ \frac{x}{(T_{N^c}, I_{N^c}, F_{N^c})} : x \in S \right\},$$

where $T_{N^c} = -1 - T_N(x)$, $I_{N^c} = -1 - I_N(x)$, and $F_{N^c} = -1 - F_N(x)$ for all $x \in S$.

Definition 2.4. [7] Let X, Y be non-empty sets, $f: X \rightarrow Y$ be any function, and $X_N = \left\{ \frac{x}{(T_N, I_N, F_N)} : x \in X \right\}$,

$Y_M = \left\{ \frac{y}{(T_M, I_M, F_M)} : y \in Y \right\}$ be \mathfrak{N} -structures over X, Y respectively. Then

- (1) the \mathfrak{N} -structure $X_{f^{-1}(M)} = \left\{ \frac{x}{(T_{f^{-1}(M)}, I_{f^{-1}(M)}, F_{f^{-1}(M)})} : x \in X \right\}$ over X is defined as follows:

$$T_{f^{-1}(M)}(x) = T_M(f(x)), I_{f^{-1}(M)}(x) = I_M(f(x)), \text{ and } F_{f^{-1}(M)}(x) = F_M(f(x));$$

- (2) the \mathfrak{N} -structure $Y_{f(N)} = \left\{ \frac{y}{(T_{f(N)}, I_{f(N)}, F_{f(N)})} : y \in Y \right\}$ over Y is defined as follows:

$$T_{f(N)}(y) = \begin{cases} \bigwedge_{f(x)=y} T_M(x) & \text{if } f^{-1}(y) \neq \emptyset; \\ 0 & \text{otherwise.} \end{cases}, \quad I_{f(N)}(y) = \begin{cases} \bigvee_{f(x)=y} I_M(x) & \text{if } f^{-1}(y) \neq \emptyset; \\ -1 & \text{otherwise.} \end{cases},$$

and

$$F_{f(N)}(y) = \begin{cases} \bigwedge_{f(x)=y} F_M(x) & \text{if } f^{-1}(y) \neq \emptyset; \\ 0 & \text{otherwise.} \end{cases}.$$

Remark 2.1. Let X, Y be non-empty sets, $f: X \rightarrow Y$ be any onto function, and $X_N = \left\{ \frac{x}{(T_N, I_N, F_N)} : x \in X \right\}$, $Y_M = \left\{ \frac{y}{(T_M, I_M, F_M)} : y \in Y \right\}$ be \mathfrak{N} -structures over X, Y respectively. Then for all $y \in Y$

$$T_{f(N)}(y) = \bigwedge_{f(x)=y} T_M(x), \quad I_{f(N)}(y) = \bigvee_{f(x)=y} I_M(x), \quad F_{f(N)}(y) = \bigwedge_{f(x)=y} F_M(x).$$

3. Subtraction algebra

In this section, we present some results related to subtraction algebra that are used throughout the paper. For more details, we refer to [4, 9, 15].

Definition 3.1. [15] An algebra $(X, -)$ is called a subtraction algebra if the single binary operation “ $-$ ” satisfies the following identities: for any $x, y, z \in X$,

- (1) $x - (y - x) = x$;
- (2) $x - (x - y) = y - (y - x)$;
- (3) $(x - y) - z = (x - z) - y$.

We introduce an order relation “ \leq ” on subtraction algebras: $a \leq b$ if and only if $a - b = 0$; where $0 = a - a$ is an element that does not depend on the choice of $a \in X$.

It is clear that $a - 0 = a$ and $0 - a = 0$ for all $a \in X$.

Example 3.1. Let $A_1 = \{0, 1\}$. Then $(A_1, -_1)$ is a subtraction algebra defined in Table 1.

Table 1. The subtraction algebra $(A_1, -_1)$

$-_1$	0	1
0	0	0
1	1	0

Example 3.2. Let $A_2 = \{0, a, b, c\}$. Then $(A_2, -_2)$ is a subtraction algebra defined in Table 2.

Table 2. The subtraction algebra $(A_2, -_2)$

$-_2$	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

Definition 3.3. [4] A non-empty subset A of a subtraction algebra X is called a subalgebra of X if for all $a, b \in A$, $a - b \in A$.

Definition 3.4. [4] A non-empty subset A of a subtraction algebra X is called an ideal of X if it satisfies the following conditions.

- (1) $a - x \in A$ for all $a \in A$ and $x \in X$;
- (2) for all $a, b \in A$, whenever $a \vee b$ exists in X then $a \vee b \in A$.

Remark 3.1. Every ideal of a subtraction algebra is a subalgebra. But the converse may not hold.

We illustrate Remark 3.1 by Example 3.3.

Example 3.3. Let $(A_2, -_2)$ be the subtraction algebra in Example 3.2. Then $\{0, c\}$ is a subalgebra of A_2 that is not an ideal of A_2 . This is clear as $c - a = b \notin \{0, c\}$.

Example 3.4. Let $A_3 = \{0, d, e\}$. Then $(A_3, -_3)$ is a subtraction algebra defined in Table 3.

Table 3. The subtraction algebra $(A_3, -_3)$

$-_3$	0	d	e
0	0	0	0
d	d	0	d
e	e	e	0

Example 3.5. Let $(A_3, -_3)$ be the subtraction algebra in Example 3.3. Then $\{0\}, \{0, d\}, \{0, e\}, A_3$ are the only subalgebras of A_3 . Moreover, every subalgebra of A_3 is an ideal of A_3 .

Definition 3.5. Let $(X, -_1), (Y, -_2)$ be subtraction algebras and $f: X \rightarrow Y$ be a function. Then

- (1) f is a homomorphism if $f(x -_1 y) = f(x) -_2 f(y)$ and $f(x \vee y) = f(x) \vee f(y)$.
- (2) f is an isomorphism if f is a bijective homomorphism. In this case, we say that X and Y are isomorphic subtraction algebras and we write $X \cong Y$.

Example 3.6. Let $(X, -)$ be a subtraction algebra and Let S be any subalgebra of Let X . Then $f: S \rightarrow X$ defined as $f(x) = x$ is a homomorphism.

Example 3.7. Let $(X, -_1), (Y, -_2)$ be subtraction algebras and $f: X \rightarrow Y$ be defined as $f(x) = 0$. Then f is a homomorphism.

4. Operations on neutrosophic \mathfrak{N} -ideals (\mathfrak{N} -subalgebra) of subtraction algebra

In this section, we introduce neutrosophic \mathfrak{N} -ideals (\mathfrak{N} -subalgebras) of subtraction algebra, present some examples, and study different operations on them.

Definition 4.1. Let $(X, -)$ be a subtraction algebra. An \mathfrak{N} -structure X_N over X is called a neutrosophic \mathfrak{N} -subalgebra of X if the following conditions hold for all $x, y \in X$.

$$T_N(x - y) \leq T_N(x) \vee T_N(y), I_N(x - y) \geq I_N(x) \wedge I_N(y), \text{ and } F_N(x - y) \leq F_N(x) \vee F_N(y).$$

Definition 4.2. Let $(X, -)$ be a subtraction algebra. An \mathfrak{N} -structure X_N over X is called a neutrosophic \mathfrak{N} -ideal of X if the following conditions hold.

- (1) $T_N(x - y) \leq T_N(x)$, $I_N(x - y) \geq I_N(x)$, and $F_N(x - y) \leq F_N(x)$ for all $x, y \in X$,

(2) if $x \vee y$ exists in X then

$$T_N(x \vee y) \leq T_N(x) \vee T_N(y), I_N(x \vee y) \geq I_N(x) \wedge I_N(y), \text{ and } F_N(x \vee y) \leq F_N(x) \vee F_N(y).$$

Example 4.1. Let $(X, -)$ be any subtraction algebra and $t, i, f \in [-1, 0]$. Then $X_N = \left\{ \frac{x}{(t,i,f)} : x \in S \right\}$ is a neutrosophic \aleph -ideal of X . We call this neutrosophic \aleph -ideal as **constant neutrosophic \aleph -ideal**.

Example 4.2. Let $(X, -)$ be any non-trivial subtraction algebra and $t, i, f \in [-1, 0]$ with $(t, i, f) \neq (0, -1, 0)$. Then $X_N = \left\{ \frac{0}{(t,i,f)}, \frac{x}{(0,-1,0)} : x \in X - \{0\} \right\}$ is a neutrosophic \aleph -ideal of X .

Corollary 4.1. Let $(X, -)$ be any non-trivial subtraction algebra (i.e., $X \neq \emptyset$). Then X has at least two neutrosophic \aleph -ideals.

Proof. The proof follows from Example 4.1 and Example 4.2.

Remark 4.1. Let $(X, -)$ be a subtraction algebra. Then every neutrosophic \aleph -ideal of X is a neutrosophic \aleph -subalgebra of X . But the converse may not hold.

We illustrate Remark 4.1 by Example 4.3.

Example 4.3. Let $(A_2, -_2)$ be the subtraction algebra defined in Example 3.2 and define A_{2N} as follows:

$$A_{2N} = \left\langle \frac{0}{(-0.8, -0.1, -0.7)}, \frac{a}{(-0.4, -0.5, -0.6)}, \frac{b}{(-0.4, -0.5, -0.6)}, \frac{c}{(-0.8, -0.1, -0.7)} \right\rangle.$$

Then A_{2N} is a neutrosophic \aleph -subalgebra of A_2 that is not neutrosophic \aleph -ideal of A_2 .

Remark 4.2. The results in this section are also valid for neutrosophic \aleph -subalgebras. But we restrict our proof to neutrosophic \aleph -ideals.

Proposition 4.1. Let $(X, -)$ be a subtraction algebra and X_N be a neutrosophic \aleph -ideal (\aleph -subalgebra) of X . Then for all $x \in X$, $T_N(0) \leq T_N(x)$, $I_N(0) \geq I_N(x)$, and $F_N(0) \leq F_N(x)$.

Proof. Since $0 = x - x$ for all $x \in X$, it follows that $T_N(0) = T_N(x - x) \leq T_N(x)$, $I_N(0) = I_N(x - x) \geq I_N(x)$, and $F_N(0) = F_N(x - x) \leq F_N(x)$.

Theorem 4.1. Let $(X, -)$ be a subtraction algebra. Then X_N and X_{N^c} are neutrosophic \aleph -ideals (\aleph -subalgebras) of X if and only if X_N is the constant neutrosophic \aleph -ideal of X .

Proof. It is clear that if X_N is the constant neutrosophic \aleph -ideal of X then X_N and X_{N^c} are neutrosophic \aleph -ideals (\aleph -subalgebras) of X .

Let X_N and X_{N^c} be neutrosophic \aleph -ideals (\aleph -subalgebras) of X . Proposition 4.1 asserts that $T_N(0) \leq T_N(x)$ and $T_{N^c}(0) \leq T_{N^c}(x)$, $I_N(0) \geq I_N(x)$ and $I_{N^c}(0) \geq I_{N^c}(x)$, and $F_N(0) \leq F_N(x)$ and $F_{N^c}(0) \leq F_{N^c}(x)$. The latter implies that

$$\begin{aligned} T_N(0) &\leq T_N(x) \text{ and } -1 - T_N(0) \leq -1 - T_N(x), \\ I_N(0) &\geq I_N(x) \text{ and } -1 - I_N(0) \geq -1 - I_N(x), \\ F_N(0) &\leq F_N(x) \text{ and } -1 - F_N(0) \leq -1 - F_N(x). \end{aligned}$$

We get now that $T_N(x) = T_N(0)$, $I_N(x) = I_N(0)$, and $F_N(x) = F_N(0)$. Thus, X_N is the constant neutrosophic \aleph -ideal of X .

Proposition 4.2. Let $(X, -)$ be a subtraction algebra and X_N, X_M be neutrosophic \aleph -ideals (\aleph -subalgebras) of X . Then $X_{N \cap M}$ is a neutrosophic \aleph -ideal (\aleph -subalgebra) of X .

Proof. Let $x, y \in X$. Then

$$\begin{aligned} T_{N \cap M}(x - y) &= T_N(x - y) \vee T_M(x - y) \leq T_N(x) \vee T_M(x) = T_{N \cap M}(x); \\ I_{N \cap M}(x - y) &= I_N(x - y) \wedge I_M(x - y) \geq I_N(x) \wedge I_M(x) = I_{N \cap M}(x); \\ F_{N \cap M}(x - y) &= F_N(x - y) \vee F_M(x - y) \leq F_N(x) \vee F_M(x) = F_{N \cap M}(x). \end{aligned}$$

Suppose that $x \vee y$ exists in X . Then

$$\begin{aligned} T_{N \cap M}(x \vee y) &= T_N(x \vee y) \vee T_M(x \vee y) \leq T_N(x) \vee T_N(y) \vee T_M(x) \vee T_M(y) = T_{N \cap M}(x) \vee T_{N \cap M}(y); \\ I_{N \cap M}(x \vee y) &= I_N(x \vee y) \wedge I_M(x \vee y) \geq I_N(x) \wedge I_N(y) \wedge I_M(x) \wedge I_M(y) = I_{N \cap M}(x) \wedge I_{N \cap M}(y); \\ F_{N \cap M}(x \vee y) &= F_N(x \vee y) \vee F_M(x \vee y) \leq F_N(x) \vee F_N(y) \vee F_M(x) \vee F_M(y) = F_{N \cap M}(x) \vee F_{N \cap M}(y). \end{aligned}$$

Therefore, $X_{N \cap M}$ is a neutrosophic \aleph -ideal (\aleph -subalgebra) of X .

Corollary 4.2. Let $(X, -)$ be a subtraction algebra and X_{N_i} be a neutrosophic \aleph -ideal (\aleph -subalgebra) of X for $i = 1, 2, \dots, n$. Then $X_{\cap_{i=1}^n N_i}$ is a neutrosophic \aleph -ideal (\aleph -subalgebra) of X .

Remark 4.3. Let $(X, -)$ be a subtraction algebra and X_N, X_M be neutrosophic \aleph -ideals (\aleph -subalgebras) of X . Then $X_{N \cup M}$ may not be a neutrosophic \aleph -ideal (\aleph -subalgebra) of X .

We illustrate Remark 4.3 by Example 4.4.

Example 4.4. Let $(A_2, -_2)$ be the subtraction algebra defined in Example 3.2 and define the neutrosophic \aleph -ideals of X A_{2N}, A_{2M} as follows:

$$\begin{aligned} A_{2N} &= \left\langle \frac{0}{(-0.8, -0.1, -0.7)}, \frac{a}{(-0.8, -0.1, -0.7)}, \frac{b}{(-0.4, -0.5, -0.6)}, \frac{c}{(-0.4, -0.5, -0.6)} \right\rangle, \\ A_{2M} &= \left\langle \frac{0}{(-0.8, -0.2, -0.7)}, \frac{a}{(-0.4, -0.3, -0.6)}, \frac{b}{(-0.8, -0.2, -0.7)}, \frac{c}{(-0.4, -0.3, -0.6)} \right\rangle. \end{aligned}$$

Then

$$A_{2NUM} = \left\langle \frac{0}{(-0.8, -0.1, -0.7)}, \frac{a}{(-0.8, -0.1, -0.7)}, \frac{b}{(-0.8, -0.2, -0.7)}, \frac{c}{(-0.4, -0.3, -0.6)} \right\rangle$$

is not a neutrosophic \aleph -ideal of X as $-0.3 = I_{NUM}(c) = I_{NUM}(a \vee b) \not\geq I_{NUM}(a) \wedge I_{NUM}(b) = -0.2$.

Proposition 4.3. Let $(X, -_1), (Y, -_2)$ be subtraction algebras and X_N, Y_M be neutrosophic \aleph -ideals (\aleph -subalgebras) of X, Y respectively. Then $(X \times Y)_{N \times M}$ is a neutrosophic \aleph -ideal (\aleph -subalgebra) of $X \times Y$. Here, for all $(x, y) \in X \times Y$, $T_{N \times M}((x, y)) = T_N(x) \vee T_M(y)$, $I_{N \times M}((x, y)) = I_N(x) \wedge I_M(y)$, and $F_{N \times M}((x, y)) = F_N(x) \vee F_M(y)$.

Proof. The proof is straightforward.

Example 4.5. Let $(A_1, -_1)$ be the subtraction algebra defined in Example 3.1 and define A_{1N} as follows:

$$A_{1N} = \left\langle \frac{0}{(-0.8, -0.1, -0.7)}, \frac{1}{(-0.4, -0.5, -0.6)} \right\rangle.$$

Then $(A_1 \times A_1)_{N \times N} = \langle \frac{(0,0)}{(-0.8,-0.1,-0.7)}, \frac{(0,1)}{(-0.4,-0.5,-0.6)}, \frac{(1,0)}{(-0.4,-0.5,-0.6)}, \frac{(1,1)}{(-0.4,-0.5,-0.6)} \rangle$ is a neutrosophic \mathfrak{N} -ideal for the subtraction algebra $(A_1 \times A_1, -)$ presented in Table 4.

Table 4. The subtraction algebra $(A_1 \times A_1, -)$

$-$	(0,0)	(0,1)	(1,0)	(1,1)
(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
(0,1)	(0,1)	(0,0)	(0,1)	(0,0)
(1,0)	(1,0)	(1,0)	(0,0)	(0,0)
(1,1)	(1,1)	(1,0)	(0,1)	(0,0)

Theorem 4.2. Let $(X, -_1)$, $(Y, -_2)$ be subtraction algebras, X_N , Y_M be neutrosophic \mathfrak{N} -ideals (\mathfrak{N} -subalgebras) of X, Y respectively, and $f: X \rightarrow Y$ be a homomorphism. Then $X_{f^{-1}(M)}$ is a neutrosophic \mathfrak{N} -ideal (\mathfrak{N} -subalgebra) of X .

Proof. Let $x, a \in X$. Then

$$\begin{aligned} T_{f^{-1}(M)}(x - a) &= T_M(f(x - a)) = T_M(f(x) - f(a)) \leq T_M(f(x)) = T_{f^{-1}(M)}(x), \\ I_{f^{-1}(M)}(x - a) &= I_M(f(x - a)) = I_M(f(x) - f(a)) \geq I_M(f(x)) = I_{f^{-1}(M)}(x), \\ F_{f^{-1}(M)}(x - a) &= F_M(f(x - a)) = F_M(f(x) - f(a)) \leq F_M(f(x)) = F_{f^{-1}(M)}(x). \end{aligned}$$

Suppose that $x \vee a$ exists in X . Then

$$\begin{aligned} T_{f^{-1}(M)}(x \vee a) &= T_M(f(x \vee a)) = T_M(f(x) \vee f(a)) \leq T_M(f(x)) \vee T_M(f(a)) = T_{f^{-1}(M)}(x) \vee T_{f^{-1}(M)}(a), \\ I_{f^{-1}(M)}(x \vee a) &= I_M(f(x \vee a)) = I_M(f(x) \vee f(a)) \geq I_M(f(x)) \wedge I_M(f(a)) = I_{f^{-1}(M)}(x) \wedge I_{f^{-1}(M)}(a), \\ F_{f^{-1}(M)}(x \vee a) &= F_M(f(x \vee a)) = F_M(f(x) \vee f(a)) \leq F_M(f(x)) \vee F_M(f(a)) = F_{f^{-1}(M)}(x) \vee F_{f^{-1}(M)}(a). \end{aligned}$$

Therefore, $X_{f^{-1}(M)}$ is a neutrosophic \mathfrak{N} -ideal of X .

Example 4.6. Let $(X, -)$ be a subtraction algebra, S a subalgebra of X , and $X_N = \left\{ \frac{x}{(T_N, I_N, F_N)} : x \in X \right\}$ a neutrosophic \mathfrak{N} -ideal of X . Then by Theorem 4.1 and by taking $f: S \rightarrow X$ as $f(x) = x$ for all $x \in S$ we get that S_N is a neutrosophic \mathfrak{N} -ideal of S . Where $S_N = \left\{ \frac{x}{(T_N, I_N, F_N)} : x \in S \right\}$

Theorem 4.3. Let $(X, -_1)$, $(Y, -_2)$ be subtraction algebras, X_N , Y_M be neutrosophic \mathfrak{N} -ideals (\mathfrak{N} -subalgebras) of X, Y respectively, and $f: X \rightarrow Y$ be a surjective homomorphism. Then $Y_{f(N)}$ is a neutrosophic \mathfrak{N} -ideal (\mathfrak{N} -subalgebra) of Y .

Proof. Let $y, b \in Y$. Since f is surjective, it follows that $T_{f(N)}(y - b) = \Lambda_{y-b=f(x)} T_N(x)$. Moreover, there exist $a \in X$ such that $b = f(a)$. We have that $T_{f(N)}(y) = \Lambda_{y=f(x)} T_N(x) = T_N(x')$ for some $x' \in X$ with $f(x') = y$. We get now that $y - b = f(x') - f(a) = f(x' - a)$. The latter implies that $T_{f(N)}(y - b) \leq T_N(x' - a) \leq T_N(x') = T_{f(N)}(y)$. Similarly, we get that $F_{f(N)}(y - b) \leq F_{f(N)}(y)$.

$I_{f(N)}(y - b) = \vee_{f(x)=y-b} I_N(x)$. Moreover, there exists $a \in X$ such that $b = f(a)$. We have that $I_{f(N)}(y) = \vee_{f(x)=y} I_N(x) = I_N(x')$ for some $x' \in X$ with $f(x') = y$. We get now that $y - b = f(x') - f(a) = f(x' - a)$. The latter implies that $I_{f(N)}(y - b) \geq I_N(x' - a) \geq I_N(x') = I_{f(N)}(y)$.

Suppose that $y \vee b \in Y$. We prove that $T_{f(N)}(y \vee b) \leq T_{f(N)}(y) \vee T_{f(N)}(b)$ and $I_{f(N)}(y \vee b) \geq I_{f(N)}(y) \wedge I_{f(N)}(b)$, $F_{f(N)}(y \vee b) \leq F_{f(N)}(y) \vee F_{f(N)}(b)$ are done similarly.

We have $T_{f(N)}(y \vee b) = \bigwedge_{y \vee b = f(x)} T_N(x)$. Moreover, there exists $a \in X$ such that $b = f(a)$. We have that $T_{f(N)}(y) = \bigwedge_{y = f(x)} T_N(x) = T_N(x')$ and $T_{f(N)}(b) = \bigwedge_{b = f(x)} T_N(x) = T_N(a)$ for some $x', a \in X$ with $f(x') = y$, $f(a) = b$. We get now that $y \vee b = f(x') \vee f(a) = f(x' \vee a)$. The latter implies that $T_{f(N)}(y \vee b) \leq T_N(x' \vee a) \leq T_N(x') \vee T_N(a) = T_{f(N)}(y) \vee T_{f(N)}(b)$.

5. Level sets and neutrosophic \mathfrak{N} -ideals (\mathfrak{N} -subalgebra) of subtraction algebra

In this section, we define (α, β, γ) -level sets of X_N and study their relation with \mathfrak{N} -ideals of X .

Let X_N be a neutrosophic \mathfrak{N} -structure over X and let $\alpha, \beta, \gamma \in [-1, 0]$ be such that $-3 \leq \alpha + \beta + \gamma \leq 0$. Then (α, β, γ) -level set of X_N is defined as follows:

$$\{x \in X : T_N(x) \leq \alpha, I_N(x) \geq \beta, F_N(x) \leq \gamma\}.$$

Remark 5.1. The results in this section are also valid for neutrosophic \mathfrak{N} -subalgebras (instead of ideal we have subalgebra). But we restrict our proof to neutrosophic \mathfrak{N} -ideals.

Proposition 5.1. Let $(X, -)$ be a subtraction algebra, $\alpha \in [-1, 0]$, and X_N a neutrosophic \mathfrak{N} -ideal of X . Then T_N^α is either an empty set or an ideal of X . Here, $T_N^\alpha = \{x \in X : T_N(x) \leq \alpha\}$ is either an empty set or an ideal of X .

Proof. Let $x, y \in T_N^\alpha \neq \emptyset$. Since $T_N(x - y) \leq T_N(x) \leq \alpha$, it follows that $x - y \in T_N^\alpha$. Suppose that $x \vee y$ exists in X . Then $T_N(x \vee y) \leq T_N(x) \vee T_N(y) \leq \alpha$. Thus, $x \vee y \in T_N^\alpha$. Therefore, T_N^α is an ideal of X .

Proposition 5.2. Let $(X, -)$ be a subtraction algebra, $\beta \in [-1, 0]$, and X_N a neutrosophic \mathfrak{N} -ideal of X . Then I_N^β is either an empty set or an ideal of X . Here, $I_N^\beta = \{x \in X : I_N(x) \geq \beta\}$.

Proof. The proof is similar to that of Proposition 5.1.

Proposition 5.3. Let $(X, -)$ be a subtraction algebra, $\gamma \in [-1, 0]$, and X_N a neutrosophic \mathfrak{N} -ideal of X . Then F_N^γ is either an empty set or an ideal of X . Here, $F_N^\gamma = \{x \in X : F_N(x) \leq \gamma\}$.

Proof. The proof is similar to that of Proposition 5.1.

Corollary 5.1. Let $(X, -)$ be a subtraction algebra, $\alpha, \beta, \gamma \in [-1, 0]$, and X_N a neutrosophic \mathfrak{N} -ideal of X . Then the (α, β, γ) -level set of X_N is either an empty set or an ideal of X .

Proof. We have the (α, β, γ) -level set of X_N is $T_N^\alpha \cap I_N^\beta \cap F_N^\gamma$. And by Propositions 5.1, 5.2, and 5.3, we have T_N^α , I_N^β , and F_N^γ are either empty sets or ideals of X . Thus, the (α, β, γ) -level set of X_N is either empty or an intersection of ideals of X and hence, it is an ideal.

Lemma 5.1. Let $(X, -)$ be a subtraction algebra, $\alpha, \beta, \gamma \in [-1, 0]$, and X_N an \mathfrak{N} -structure over X . If every non-empty (α, β, γ) -level set of X_N is an ideal of X then X_N a neutrosophic \mathfrak{N} -ideal of X .

Proof. Let $x, a \in X$. Then there exist $\alpha', \beta', \gamma' \in [-1, 0]$ such that $T_N(a) = \alpha'$, $I_N(a) = \beta'$, $F_N(a) = \gamma'$. Then a is in the $(\alpha', \beta', \gamma')$ -level set of X_N which is an ideal of X . The latter implies that $a - x$ is in the $(\alpha', \beta', \gamma')$ -level set of X_N . We get now that $T_N(a - x) \leq \alpha' = T_N(a)$, $I_N(a - x) \geq \beta' = I_N(a)$, $F_N(a - x) \leq \gamma' = F_N(a)$.

Suppose that $x \vee a \in X$. Then there exist $\alpha', \beta', \gamma', \alpha'', \beta'', \gamma'' \in [-1, 0]$ such that $T_N(a) = \alpha', I_N(a) = \beta', F_N(a) = \gamma', T_N(x) = \alpha'', I_N(x) = \beta'', F_N(x) = \gamma''$. Let $\alpha = \alpha' \vee \alpha'', \beta = \beta' \wedge \beta'', \gamma = \gamma' \vee \gamma''$. Then a, x are in the (α, β, γ) -level set of X_N which is an ideal of X . The latter implies that $a \vee x$ is in the (α, β, γ) -level set of X_N . Thus, $T_N(a \vee x) \leq \alpha = T_N(a) \vee T_N(x), I_N(a \vee x) \geq \beta = I_N(a) \wedge I_N(x), F_N(a \vee x) \leq \gamma = F_N(a) \vee F_N(x)$.

Therefore, X_N is a neutrosophic \aleph -ideal of X .

Theorem 5.1. Let $(X, -)$ be a subtraction algebra, $\alpha, \beta, \gamma \in [-1, 0]$, and X_N an \aleph -structure over X . Then X_N is a neutrosophic \aleph -ideal of X if and only if every non-empty (α, β, γ) -level set of X_N is an ideal of X .

Proof. The proof follows from Corollary 5.1 and Lemma 5.1.

Theorem 5.2. Let $(X, -)$ be a subtraction algebra, $\alpha, \beta, \gamma \in [-1, 0]$, and X_N an \aleph -structure over X . Then the following statements are equivalent.

- (1) X_N is a neutrosophic \aleph -ideal of X ;
- (2) $T_N^\alpha, I_{N\beta},$ and F_N^γ are either empty sets or ideals of X ;
- (3) Every non-empty (α, β, γ) -level set of X_N is an ideal of X .

Proof. **(1) \Rightarrow (2):** If X_N is a neutrosophic \aleph -ideal of X then by Propositions 5.1, 5.2, and 5.3, we have $T_N^\alpha, I_{N\beta},$ and F_N^γ are ideals of X .

(2) \Rightarrow (3): If $T_N^\alpha, I_{N\beta},$ and F_N^γ are ideals of X then the (α, β, γ) -level set of X_N is intersection of ideals of X (intersection of $T_N^\alpha, I_{N\beta},$ and F_N^γ) and hence, it is an ideal of X .

(3) \Rightarrow (1): By Lemma 5.1.

Theorem 5.3. Let $(X, -)$ be a subtraction algebra and $\alpha, \beta, \gamma \in [-1, 0]$ with $(\alpha, \beta, \gamma) \neq (0, -1, 0)$. Then every ideal of X is an (α, β, γ) -level set of a neutrosophic \aleph -ideal of X .

Proof. Let I be an ideal of X and $X_N = \left\{ \frac{x}{(T_N, I_N, F_N)} : x \in X \right\}$ be \aleph -structure of X defined as follows.

$$(T_N(x), I_N(x), F_N(x)) = \begin{cases} (\alpha, \beta, \gamma) & \text{if } x \in I; \\ (0, -1, 0) & \text{otherwise.} \end{cases}$$

Let $\alpha', \beta', \gamma' \in [-1, 0]$. Then the $(\alpha', \beta', \gamma')$ -level set of X_N is given as follows:

$$\begin{cases} I & \text{if } \alpha \leq \alpha' < 0, -1 < \beta' \leq \beta, \gamma \leq \gamma' < 0; \\ X & (\alpha', \beta', \gamma') = (0, -1, 0) \\ \emptyset & \text{otherwise.} \end{cases}$$

And it is either an empty set or an ideal of X . Therefore, by Lemma 5.1, X_N is a neutrosophic \aleph -ideal of X .

6. Conclusion

In this paper, we combined the notions of \aleph -structures, neutrosophy, and subtraction algebra to introduce \aleph -ideals (\aleph -subalgebras) of subtraction algebras. Some operations on the defined notions were discussed. Moreover, the (α, β, γ) -level sets were introduced and used to find a necessary and sufficient condition for \aleph -structures to be neutrosophic \aleph -ideals (\aleph -subalgebras).

For future work, it would be interesting to check whether there is a relation between our results about \aleph -ideals (\aleph -subalgebras) of subtraction algebras and the results related to single valued neutrosophic subtraction algebras discussed by Chul Hwan Park in [8].

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An Approach to Solve the Linear Programming Problem Using Single Valued Trapezoidal Neutrosophic Number

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Abstract

While making a decision, the neutrosophic set theory includes the uncertainty part beside certainty part (i.e., Yes or No). In the present uncertain socio-economic fashion, this pattern is highly acceptable and hence, the limitations of fuzzy set and intuitionistic fuzzy set are overcome with neutrosophic set theory. The present study provides a modified structure of linear programming problem (LP-problem) and its solution approach in neutrosophic sense. A special type of neutrosophic set defined over the set of real number, viz., single valued trapezoidal neutrosophic number (SVTN-number) is adopted here as the coefficients of the objective function, right-hand side coefficients and decision variables itself of an LP-problem. In order to solve such problem, a parameter based ranking function of SVTN-number is newly constructed from the geometrical configuration of the trapezium. It plays a key role in the development of the solution algorithm. An LP-problem is normally solved under the asset of some given constraints. Besides that, there may be some hidden parameters (e.g., awareness level of nearer society for the smooth run of a clinical pharmacy, ruined structure of road to be met a profit from a bus, etc) of an LP-problem and these affect the solution badly when experts ignore them. This study makes an attempt to solve an LP-problem by giving importance to all these to attain a fair outcome. The efficiency of the proposed concept is illustrated in a real field. A real example is stated and is solved numerically under the present view.

Keywords: Neutrosophic set; Single valued trapezoidal neutrosophic (SVTN) number; Linear programming problem in neutrosophic sense; Simplex method.

1 Introduction

Due to complex diversity and vague atmosphere in the present socio-economic scenario, making a decision on several events are being complicated day in day out. It is almost impossible to draw a decision in a straight way due to incomplete and imprecise information available in the respective ground. Work pressure, diverted mind, measurement errors, limited attention, lack of knowledge, time bounding pressure, the narrow scope of placement at the end of academic, etc will force the experts to have such information. So, decision makers focus to develop the concepts of decision making and optimization in an uncertain way. This results in the exploration of fuzzy set by Zadeh [30] and intuitionistic fuzzy set by Atanassov [2]. But these logics can't manage the situations involving indeterminacy. There are many practical facts like in sports game, the role of elector in the casting of poll, making decisions in different sectors, etc wherein one may predict three kinds of outcomes. Smarandache [27] studied this kind of facts more precisely and he then introduced the notion of the neutrosophic set (NS), a generalisation of intuitionistic fuzzy set. Each object in NS is characterised by a triplet, viz., truth-membership value, indeterminacy-membership value and falsity-membership value. Each of neutrosophic triplet is quantified explicitly and is independent in nature. The indeterministic part of uncertain data plays an important role to make a proper decision which is out of scope in intuitionistic fuzzy set theory.

The ranking technique of fuzzy number, intuitionistic fuzzy number, neutrosophic number play an important role in developing different multi-attributive decision making, optimization, mathematical structures and others. Gani and Ponnalagu [13] defined a method based on intuitionistic fuzzy linear programming for investment strategy. Li [21] developed a ratio ranking technique of triangular intuitionistic fuzzy numbers and applied it to MADM problems. Yao and Wu [29] brought a ranking method of fuzzy numbers based on the decomposition principle and signed distance while Rao and Shankar [25] developed so with an area method using the circumference of the centroid. Mukherjee and Basu [22] applied a fuzzy ranking method for solving assignment problems with fuzzy costs. Roy and Das [26] solved a neutrosophic multi-objective production planning problem. Deli and Subas [11] applied a ranking method of trapezoidal neutrosophic number in MADM problems. Biswas et al. [9] have proposed an approach for multi-attribute group decision making problems under single-valued neutrosophic environment. Hussian et al. [14] have solved the neutrosophic

linear programming problem by transforming it into a crisp programming model. Pramanik [24] has put a new direction to solve a neutrosophic multi-objective programming by extending Zimmermann's approach. Several approaches [3-8, 12] are seen to optimise LP-problems under real neutrosophic climate. Chakraborty [10] established a score function of pentagonal neutrosophic number and applied it on a net working problem. Khalid [15-20] handled neutrosophic geometric programming in several directions. Mullai et al. [23] developed an inventory model with neutrosophic random variable.

Decision makers generally solve an LP-problem based on some constraints provided to them. Furthermore, there are a number of parameters of an LP-problem such as, but not limited to, the awareness level of nearer society for the smooth run of a clinical pharmacy, degree of redemption from Govt. tax on raw materials used in industry to meet a profit, degree of ruined economy of society for inhaling situation of the nearer market, degree of road condition in driving a bus to get a profit, etc. While solving an LP-problem, experts ignore these facts and so the outcome is not fair as a whole. To conquer this limitation, a ranking function of SVTN-number in term of a parameter graded in $[0,1]$ is derived here from the geometrical configuration of the trapezium. This parameter describes an additional character of an LP-problem which is modified here and thus several situations (i.e., for different grades) of respective problem are managed nicely.

This study extends the concept of LP-problem from crisp sense to neutrosophic sense. Based on a ranking function newly brought here, an efficient solution algorithm is developed to solve such a problem. The efficiency of the present thought is measured in the real field. The paper is organised as follows.

Some preliminary definitions are remembered in Section 2. Section 3 provides a ranking function of SVTN-number, and an LP-problem is given from the point of neutrosophic view. In Section 4, an algorithm is developed towards solving such problem. The method has been illustrated with the help of a real life example in Section 5. Finally, the present work and its future aspect and limitation are given in Section 6.

2 Preliminaries

Some necessary definitions and results are stated below to make out the main results.

2.1 Definition [1]

A fuzzy number P is designed by a pair of bounded functions $P^L(\alpha), P^R(\alpha), \alpha \in [0, 1]$ where P^L is monotone increasing, left continuous and P^R is monotone decreasing, right continuous with $P^L(\alpha) \leq P^R(\alpha)$.

A trapezoidal fuzzy number is displayed by $P = (m_0, n_0, \gamma, \delta)$ where $[m_0, n_0]$ is interval defuzzifier and $\gamma(> 0), \delta(> 0)$ are respectively left fuzziness, right fuzziness and $(m_0 - \gamma, n_0 + \delta)$ is the support of P . Its membership function is defined as :

$$P(x) = \begin{cases} \frac{1}{\gamma}(x - m_0 + \gamma), & m_0 - \gamma \leq x \leq m_0, \\ 1, & x \in [m_0, n_0], \\ \frac{1}{\delta}(n_0 - x + \delta), & n_0 \leq x \leq n_0 + \delta, \\ 0, & \text{elsewhere.} \end{cases}$$

In parametric form $P^L(\alpha) = m_0 - \gamma + \gamma\alpha, P^R(\alpha) = n_0 + \delta - \delta\alpha$.

2.2 Definition [27]

An NS P over the universe U is defined by a triplet namely truth-membership value μ_P , indeterminacy-membership value ν_P and falsity-membership value η_P where $\mu_P, \nu_P, \eta_P : U \rightarrow]^{-}0, 1^{+}[$. Thus P is displayed as : $P = \{< x, \mu_P(x), \nu_P(x), \eta_P(x) > : x \in U\}$ with $^{-}0 \leq \sup \mu_P(x) + \sup \nu_P(x) + \sup \eta_P(x) \leq 3^{+}$. Here $1^{+} = 1 + \varepsilon$, where 1 is its standard part and ε is its non-standard part. Similarly $^{-}0 = 0 - \varepsilon$, where 0 is its standard part and ε is its non-standard part.

This concept was primarily viewed in philosophical sense. But it is difficult to use NS with value from real standard or nonstandard subset of $]^{-}0, 1^{+}[$ in real field. To overcome this, NS with value from the subset of $[0,1]$ is considered.

2.3 Definition [28]

A single valued neutrosophic (SVN) set M over a universe U is an NS where the components of each triplet are real standard elements of $[0, 1]$. Thus an SVN-set M is executed as : $M = \{< x, \mu_M(x), \nu_M(x), \eta_M(x) > : x \in U \text{ and } \mu_M(x), \nu_M(x), \eta_M(x) \in [0, 1]\}$ such that $0 \leq \sup \mu_M(x) + \sup \nu_M(x) + \sup \eta_M(x) \leq 3$.

2.4 Definition [11]

Let $p_i, q_i, s_i, t_i \in \mathbf{R}$ (the set of all real numbers) with ordered as $p_i \leq q_i \leq s_i \leq t_i$ ($i = 1, 2, 3$) and $w_{\tilde{m}}, u_{\tilde{m}}, y_{\tilde{m}} \in [0, 1]$. Then a SVN-number $\tilde{m} = \langle ([p_1, q_1, s_1, t_1]; w_{\tilde{m}}), ([p_2, q_2, s_2, t_2]; u_{\tilde{m}}), ([p_3, q_3, s_3, t_3]; y_{\tilde{m}}) \rangle$ is a special SVN-set on \mathbf{R} whose truth value, indeterminacy value, falsity value are respectively defined by the mappings $\mu_{\tilde{m}} : \mathbf{R} \rightarrow [0, w_{\tilde{m}}], \nu_{\tilde{m}} : \mathbf{R} \rightarrow [u_{\tilde{m}}, 1], \eta_{\tilde{m}} : \mathbf{R} \rightarrow [y_{\tilde{m}}, 1]$ and they are respectively given as :

$$\begin{cases} g_{\mu}^l(x), & p_1 \leq x \leq q_1, \\ w_{\tilde{m}}, & q_1 \leq x \leq s_1, \\ g_{\mu}^r(x), & s_1 \leq x \leq t_1, \\ 0, & \text{otherwise.} \end{cases} \quad \begin{cases} g_{\nu}^l(x), & p_2 \leq x \leq q_2, \\ u_{\tilde{m}}, & q_2 \leq x \leq s_2, \\ g_{\nu}^r(x), & s_2 \leq x \leq t_2, \\ 1, & \text{otherwise.} \end{cases} \quad \begin{cases} g_{\eta}^l(x), & p_3 \leq x \leq q_3, \\ y_{\tilde{m}}, & q_3 \leq x \leq s_3, \\ g_{\eta}^r(x), & s_3 \leq x \leq t_3, \\ 1, & \text{otherwise.} \end{cases}$$

The functions $g_{\mu}^l : [p_1, q_1] \rightarrow [0, w_{\tilde{m}}], g_{\nu}^r : [s_2, t_2] \rightarrow [u_{\tilde{m}}, 1], g_{\eta}^r : [s_3, t_3] \rightarrow [y_{\tilde{m}}, 1]$ are continuous and non-decreasing which satisfy : $g_{\mu}^l(p_1) = 0, g_{\mu}^l(q_1) = w_{\tilde{m}}, g_{\nu}^r(s_2) = u_{\tilde{m}}, g_{\nu}^r(t_2) = 1, g_{\eta}^r(s_3) = y_{\tilde{m}}, g_{\eta}^r(t_3) = 1$. The functions $g_{\mu}^r : [s_1, t_1] \rightarrow [0, w_{\tilde{m}}], g_{\nu}^l : [p_2, q_2] \rightarrow [u_{\tilde{m}}, 1], g_{\eta}^l : [p_3, q_3] \rightarrow [y_{\tilde{m}}, 1]$ are continuous and non-increasing which satisfy : $g_{\mu}^r(s_1) = w_{\tilde{m}}, g_{\mu}^r(t_1) = 0, g_{\nu}^l(p_2) = 1, g_{\nu}^l(q_2) = u_{\tilde{m}}, g_{\eta}^l(p_3) = 1, g_{\eta}^l(q_3) = y_{\tilde{m}}$.

If $[p_1, q_1, s_1, t_1] = [p_2, q_2, s_2, t_2] = [p_3, q_3, s_3, t_3]$ in \tilde{m} , it is reduced to a SVTN-number. Thus $\tilde{n} = \langle ([p, q, s, t]; w_{\tilde{n}}, u_{\tilde{n}}, y_{\tilde{n}}) \rangle$ is a SVTN-number.

2.5 Definition [6]

A neutrosophic set of the form $\tilde{m} = \langle ([p_1, q_1, \delta_1, \xi_1]; w_{\tilde{m}}), ([p_2, q_2, \delta_2, \xi_2]; u_{\tilde{m}}), ([p_3, q_3, \delta_3, \xi_3]; y_{\tilde{m}}) \rangle$ and defined on \mathbf{R} is called a SVN-number. $\delta_i (> 0)$ are the left spreads, $\xi_i (> 0)$ are the right spreads and $[p_i, q_i]$ are the modal intervals for degree of truth, indeterminacy, falsity-membership for $i = 1, 2, 3$ respectively in \tilde{m} and $w_{\tilde{m}}, u_{\tilde{m}}, y_{\tilde{m}} \in [0, 1]$. The three neutrosophic components are designed as :

$$\begin{aligned} T_{\tilde{m}}(x) &= \begin{cases} \frac{1}{\delta_1} w_{\tilde{m}}(x - p_1 + \delta_1), & p_1 - \delta_1 \leq x \leq p_1, \\ w_{\tilde{m}}, & x \in [p_1, q_1], \\ \frac{1}{\xi_1} w_{\tilde{m}}(q_1 - x + \xi_1), & q_1 \leq x \leq q_1 + \xi_1, \\ 0, & \text{elsewhere.} \end{cases} \\ I_{\tilde{m}}(x) &= \begin{cases} \frac{1}{\delta_2} (p_2 - x + u_{\tilde{m}}(x - m_2 + \delta_2)), & p_2 - \delta_2 \leq x \leq p_2, \\ u_{\tilde{m}}, & x \in [p_2, q_2], \\ \frac{1}{\xi_2} (x - q_2 + u_{\tilde{m}}(q_2 - x + \xi_2)), & q_2 \leq x \leq q_2 + \xi_2, \\ 1, & \text{elsewhere.} \end{cases} \\ F_{\tilde{m}}(x) &= \begin{cases} \frac{1}{\delta_3} (p_3 - x + y_{\tilde{m}}(x - p_3 + \delta_3)), & p_3 - \delta_3 \leq x \leq p_3, \\ y_{\tilde{m}}, & x \in [p_3, q_3], \\ \frac{1}{\xi_3} (x - q_3 + y_{\tilde{m}}(q_3 - x + \xi_3)), & q_3 \leq x \leq q_3 + \xi_3, \\ 1, & \text{elsewhere.} \end{cases} \end{aligned}$$

Here \tilde{m} consists of three pairs $(T_{\tilde{m}}^l, T_{\tilde{m}}^u), (I_{\tilde{m}}^l, I_{\tilde{m}}^u), (F_{\tilde{m}}^l, F_{\tilde{m}}^u)$ of bounded and continuous functions so that

- (i) $T_{\tilde{m}}^l, I_{\tilde{m}}^u, F_{\tilde{m}}^u$ are monotone non-decreasing and $T_{\tilde{m}}^u, I_{\tilde{m}}^l, F_{\tilde{m}}^l$ are monotone non-increasing.
- (ii) $T_{\tilde{m}}^l(r) \leq T_{\tilde{m}}^u(r), I_{\tilde{m}}^l(r) \geq I_{\tilde{m}}^u(r), F_{\tilde{m}}^l(r) \geq F_{\tilde{m}}^u(r), r \in [0, 1]$.

A SVN-number \tilde{m} is transformed into a SVTN-number when three modal intervals in \tilde{m} are all equal. Thus $\tilde{q} = \langle ([m_0, n_0, \delta_1, \xi_1]; w_{\tilde{q}}), ([m_0, n_0, \delta_2, \xi_2]; u_{\tilde{q}}), ([m_0, n_0, \delta_3, \xi_3]; y_{\tilde{q}}) \rangle$ is a SVTN-number.

The truth, indeterminacy and falsity-membership values of a SVTN-number differ with respect to their corresponding height only by Definition 2.4. But to compare the various SVTN-numbers in a more flexible way, both supports (i.e. the bases of trapeziums) and heights of neutrosophic components are allowed to differ in Definition 2.5. So, it is the more generalisation of Definition 2.4.

3 Ranking technique of SVTN-number

The score value of SVTN-number is evaluated here from geometrical view and its properties are studied. Then a linear ranking function is defined with this score value.

3.1 Definition

The maximum heights of three neutrosophic components of a SVTN-number (proposed in Definition 2.5) are all taken as one (i.e., 1) to have a linear ranking function. Thus three components of a SVTN-number $\tilde{p} = \langle [p, q, \delta_1, \xi_1], [p, q, \delta_2, \xi_2], [p, q, \delta_3, \xi_3] \rangle$ are designed respectively as follows.

$$\mu_{\tilde{p}}(y) = \begin{cases} \frac{1}{\delta_1}(y - p + \delta_1), & p - \delta_1 \leq y \leq p, \\ 1, & x \in [p, q], \\ \frac{1}{\xi_1}(q - y + \xi_1), & q \leq y \leq q + \xi_1, \\ 0, & \text{elsewhere.} \end{cases}$$

$$\nu_{\tilde{p}}(y) = \begin{cases} \frac{1}{\delta_2}(p - y), & p - \delta_2 \leq y \leq p, \\ 0, & y \in [p, q], \\ \frac{1}{\xi_2}(y - q), & q \leq y \leq q + \xi_2, \\ 1, & \text{elsewhere.} \end{cases}$$

$$\eta_{\tilde{p}}(y) = \begin{cases} \frac{1}{\delta_3}(p - y), & p - \delta_3 \leq y \leq p, \\ 0, & x \in [p, q], \\ \frac{1}{\xi_3}(y - q), & q \leq y \leq q + \xi_3, \\ 1, & \text{elsewhere.} \end{cases}$$

Consider two SVTN-numbers $\tilde{a} = \langle [a, b, \omega_1, \lambda_1], [a, b, \omega_2, \lambda_2], [a, b, \omega_3, \lambda_3] \rangle$ and $\tilde{c} = \langle [c, d, \xi_1, \kappa_1], [c, d, \xi_2, \kappa_2], [c, d, \xi_3, \kappa_3] \rangle$. Then,

(i) Addition :

$$\tilde{a} + \tilde{c} = \langle [a + c, b + d, \omega_1 + \xi_1, \lambda_1 + \kappa_1], [a + c, b + d, \omega_2 + \xi_2, \lambda_2 + \kappa_2], [a + c, b + d, \omega_3 + \xi_3, \lambda_3 + \kappa_3] \rangle.$$

(ii) Scalar multiplication : For any real number x ,

$$x\tilde{a} = \langle [xa, xb, x\omega_1, x\lambda_1], [xa, xb, x\omega_2, x\lambda_2], [xa, xb, x\omega_3, x\lambda_3] \rangle \text{ if } x > 0.$$

$$x\tilde{a} = \langle [xb, xa, -x\omega_1, -x\lambda_1], [xb, xa, -x\omega_2, -x\lambda_2], [xb, xa, -x\omega_3, -x\lambda_3] \rangle \text{ if } x < 0.$$

(iii) If $a = b = \omega_i = \lambda_i = 0$ for all i in \tilde{a} , then it is called a zero SVTN-number and is denoted by

$$\tilde{0} = \langle [0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0, 0] \rangle.$$

3.1.1 Product of two SVTN-numbers

Let $\tilde{a} = \langle [a, b, \omega_1, \lambda_1], [a, b, \omega_2, \lambda_2], [a, b, \omega_3, \lambda_3] \rangle$ and $\tilde{c} = \langle [c, d, \kappa_1, \zeta_1], [c, d, \kappa_2, \zeta_2], [c, d, \kappa_3, \zeta_3] \rangle$ be two svtn-numbers. Their product $\tilde{a} \cdot \tilde{c}$ is defined as :

$$\begin{aligned} \tilde{a} \cdot \tilde{c} = & \langle [ac, bd, a\kappa_1 + c\omega_1 - \omega_1\kappa_1, b\zeta_1 + d\lambda_1 + \lambda_1\zeta_1], \\ & [ac, bd, a\kappa_2 + c\omega_2 - \omega_2\kappa_2, b\zeta_2 + d\lambda_2 + \lambda_2\zeta_2], \\ & [ac, bd, a\kappa_3 + c\omega_3 - \omega_3\kappa_3, b\zeta_3 + d\lambda_3 + \lambda_3\zeta_3] \rangle \\ & \text{when } b + \lambda_i > 0 \text{ and } d + \zeta_i > 0, \forall i = 1, 2, 3. \end{aligned} \quad (1)$$

$$\begin{aligned} \tilde{a} \cdot \tilde{c} = & \langle [ad, bc, -a\zeta_1 + d\omega_1 + \omega_1\zeta_1, -b\kappa_1 + c\lambda_1 - \lambda_1\kappa_1], \\ & [ad, bc, -a\zeta_2 + d\omega_2 + \omega_2\zeta_2, -b\kappa_2 + c\lambda_2 - \lambda_2\kappa_2], \\ & [ad, bc, -a\zeta_3 + d\omega_3 + \omega_3\zeta_3, -b\kappa_3 + c\lambda_3 - \lambda_3\kappa_3] \rangle \\ & \text{when } b + \lambda_i < 0, \text{ but } d + \zeta_i > 0, \forall i = 1, 2, 3. \end{aligned} \quad (2)$$

$$\begin{aligned} \tilde{a} \cdot \tilde{c} = & \langle [bd, ac, -b\zeta_1 - d\lambda_1 - \lambda_1\zeta_1, -a\kappa_1 - c\omega_1 + \omega_1\kappa_1], \\ & [bd, ac, -b\zeta_2 - d\lambda_2 - \lambda_2\zeta_2, -a\kappa_2 - c\omega_2 + \omega_2\kappa_2], \\ & [bd, ac, -b\zeta_3 - d\lambda_3 - \lambda_3\zeta_3, -a\kappa_3 - c\omega_3 + \omega_3\kappa_3] \rangle \\ & \text{when } b + \lambda_i < 0 \text{ and } d + \zeta_i < 0, \forall i = 1, 2, 3. \end{aligned} \quad (3)$$

Suppose $b + \lambda_3 < 0, d + \zeta_3 > 0$ only but $b + \lambda_k > 0, d + \zeta_k > 0$ or $b + \lambda_k < 0, d + \zeta_k < 0$ for $k = 1, 2$ (i.e., others keep same sign) then the product is also defined from above as the components in neutrosophic triplet are independent in nature. More precisely, the product of falsity components in $\tilde{a} \cdot \tilde{c}$ follows the 2nd rule whereas the product of truth and indeterminacy components in $\tilde{a} \cdot \tilde{c}$ follow either 1st rule or 3rd rule.

3.1.2 Geometrical representation of SVTN-number

The SVTN-number in different looks and their comparison are now presented geometrically by means of Definition 2.4 (Figure 1), Definition 2.5 (Figure 2), Definition 3.1 (Figure 3) respectively. In the present study,

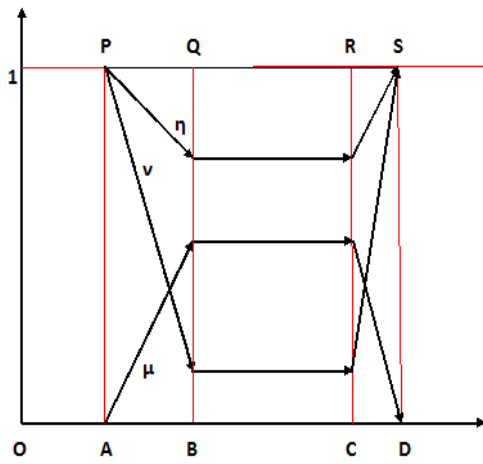


Figure 1 : SVTN-number

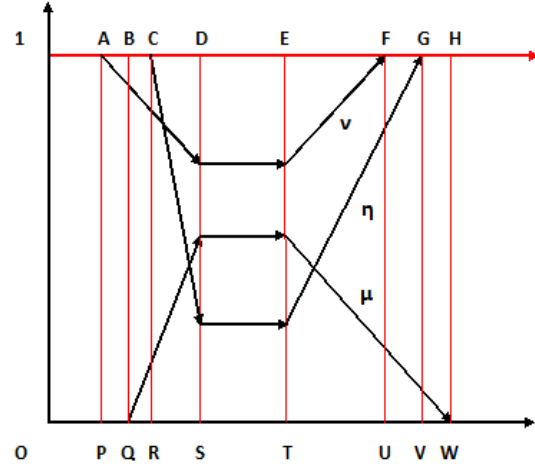


Figure 2 : SVTN-number

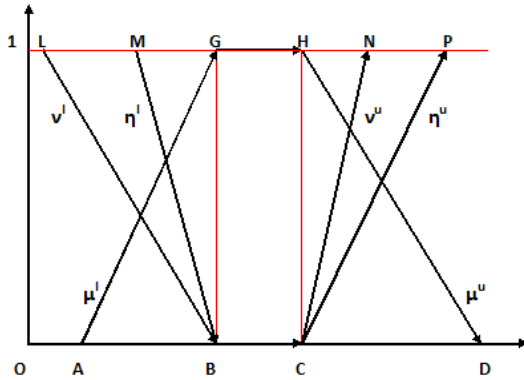


Figure 3 : SVTN-number

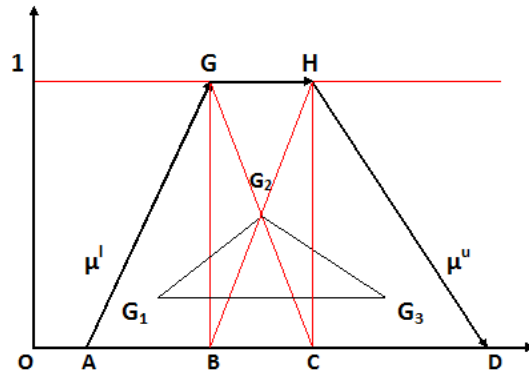


Figure 4 : Truth value

the components of neutrosophic triplet differ by their supports only. Figure 4 represents the truth value of SVTN-number alone by Definition 3.1.

3.2 Score of SVTN-number

Let, Figure 4 represent a SVTN-number $\tilde{p} = \langle [p, q, \delta_1, \zeta_1], [p, q, \delta_2, \zeta_2], [p, q, \delta_3, \zeta_3] \rangle$. Let the trapeziums $AGHD$, $LBCN$ and $MBCP$ correspond respectively the truth value, indeterminacy value and falsity value. Two perpendicular lines BG and CH are drawn. Then each trapezium is divided into two triangles and one rectangle/square. Now let us consider each trapezium separately.

The trapezium $AGHD$ (see Figure 5) consists of two triangles ABG , CHD and one rectangle $BCHG$. With respect to the co-ordinate of vertices of trapezium $AGHD$ corresponding to \tilde{p} , the centroid of triangles ABG and CHD are $G_1(p - \frac{\delta_1}{3}, \frac{1}{3})$ and $G_3(q + \frac{\zeta_1}{3}, \frac{1}{3})$ respectively. Also the centroid of rectangle $BCHG$ is the intersecting point G_2 of its two diagonals i.e., $G_2(\frac{p+q}{2}, \frac{1}{2})$. For contrary, suppose G_1, G_2, G_3 are collinear. Then the area of triangle formed with these points as vertices will be zero i.e.,

$$\frac{1}{3}(\frac{p+q}{2} - q - \frac{\zeta_1}{3}) + \frac{1}{2}(q + \frac{\zeta_1}{3} - p + \frac{\delta_1}{3}) + \frac{1}{3}(p - \frac{\delta_1}{3} - \frac{p+q}{2}) = 0 \quad (4)$$

$$\Rightarrow \delta_1 + \zeta_1 = 3(p - q) < 0, \text{ (using Def. 3.1)}$$

It is a contradiction to the hypothesis that $\delta_1 + \zeta_1 > 0$. Hence G_1, G_2, G_3 are non-collinear and a triangle can be formed with these points as vertices. The centroid of this triangle is $G'(\frac{9p+9q-2\delta_1+2\zeta_1}{18}, \frac{7}{18})$. The centroid points G_1, G_2, G_3 are the balancing points for triangle ABG , rectangle $BCHG$ and triangle CHD respectively. But as the centroid G' is much more balancing point for the two triangles and one rectangle as a whole, so this point is taken to construct the ranking function.

The trapezium $LBCN$ consists of two triangles LBG, HCN and one rectangle $BCHG$. With respect to the co-ordinate of vertices of trapezium $LBCN$ corresponding to \tilde{p} , the centroid of the triangles LBG and HCN are $G_4(p - \frac{\delta_2}{3}, \frac{2}{3})$ and $G_5(q + \frac{\zeta_2}{3}, \frac{2}{3})$ respectively. The centroid of the rectangle $BCHG$ is $G_2(\frac{p+q}{2}, \frac{1}{2})$. Here G_4, G_2, G_5 are non-collinear for the same fact stated above. So the centroid $G''(\frac{9p+9q-2\delta_2+2\zeta_2}{18}, \frac{11}{18})$ of the triangle with vertices G_4, G_2, G_5 is taken to construct the ranking function.

Finally, the trapezium $MBCP$ consists of two triangles MBG, PCH and the rectangle $BCHG$. With respect to the co-ordinate of vertices of trapezium $MBCP$ corresponding to \tilde{p} , the centroid of the triangles MBG and PCH are $G_6(p - \frac{\delta_3}{3}, \frac{2}{3})$ and $G_7(q + \frac{\zeta_3}{3}, \frac{2}{3})$ respectively. By similar argument as above, the centroid $G'''(\frac{9p+9q-2\delta_3+2\zeta_3}{18}, \frac{11}{18})$ of a triangle of vertices G_6, G_2, G_7 is considered to form the ranking function.

We now define the score of \tilde{p} corresponding to truth value, indeterminacy value and falsity value respectively as :

$$S_\mu(\tilde{p}) = \frac{7}{18}(\frac{9p+9q-2\delta_1+2\zeta_1}{18}), S_\nu(\tilde{p}) = \frac{11}{18}(\frac{9p+9q-2\delta_2+2\zeta_2}{18}), S_\eta(\tilde{p}) = \frac{11}{18}(\frac{9p+9q-2\delta_3+2\zeta_3}{18}).$$

For an arbitrary parameter γ lying in $[0, 1]$ and for any natural number n , the γ -weighted score function of \tilde{p} is denoted by $S_\gamma(\tilde{p})$ and is defined as :

$$\begin{aligned} S_\gamma(\tilde{p}) &= S_\mu(\tilde{p})\gamma^n + S_\nu(\tilde{p})(1 - \gamma^n) + S_\eta(\tilde{p})(1 - \gamma^n) \\ &= \frac{1}{324}[7(9p + 9q - 2\delta_1 + 2\zeta_1)\gamma^n + 11(9p + 9q - 2\delta_2 + 2\zeta_2)(1 - \gamma^n) \\ &\quad + 11(9p + 9q - 2\delta_3 + 2\zeta_3)(1 - \gamma^n)] \end{aligned} \quad (5)$$

3.2.1 Proposition

The γ -weighted score of a SVTN-number obeys the following the norms.

(i) It is linear i.e., $S_\gamma(\tilde{c} \pm \tilde{d}) = S_\gamma(\tilde{c}) \pm S_\gamma(\tilde{d})$ and $S_\gamma(\pi\tilde{c}) = \pi S_\gamma(\tilde{c})$, π being any real number and \tilde{c}, \tilde{d} are two SVTN-numbers.

(ii) $S_\gamma(\tilde{d} - \tilde{d}) = S_\gamma(\tilde{0})$.

(iii) $S_\gamma(\tilde{c})$ is monotone increasing or decreasing or constant according as $S_\mu(\tilde{c}) >$

$S_\nu(\tilde{c}) + S_\eta(\tilde{c})$ or $S_\mu(\tilde{c}) < S_\nu(\tilde{c}) + S_\eta(\tilde{c})$ or $S_\mu(\tilde{c}) = S_\nu(\tilde{c}) + S_\eta(\tilde{c})$ respectively.

Proof. (i) Let $\tilde{c} = \langle [a, b, \omega_1, \lambda_1], [a, b, \omega_2, \lambda_2], [a, b, \omega_3, \lambda_3] \rangle$ and $\tilde{d} = \langle [x, y, \kappa_1, \zeta_1], [x, y, \kappa_2, \zeta_2], [x, y, \kappa_3, \zeta_3] \rangle$ be two SVTN-numbers. Then,

$$-\tilde{d} = \langle [-y, -x, \zeta_1, \kappa_1], [-y, -x, \zeta_2, \kappa_2], [-y, -x, \zeta_3, \kappa_3] \rangle$$

$$\tilde{c} + \tilde{d} = \langle [a+x, b+y, \omega_1+\kappa_1, \lambda_1+\zeta_1], [a+x, b+y, \omega_2+\kappa_2, \lambda_2+\zeta_2], [a+x, b+y, \omega_3+\kappa_3, \lambda_3+\zeta_3] \rangle,$$

$$\tilde{c} - \tilde{d} = \langle [a-y, b-x, \omega_1+\zeta_1, \lambda_1+\kappa_1], [a-y, b-x, \omega_2+\zeta_2, \lambda_2+\kappa_2], [a-y, b-x, \omega_3+\zeta_3, \lambda_3+\kappa_3] \rangle;$$

Now,

$$S_\gamma(\tilde{c}) = \frac{1}{324}[7(9a+9b-2\omega_1+2\lambda_1)\gamma^n + 11(9a+9b-2\omega_2+2\lambda_2)(1-\gamma^n) + 11(9a+9b-2\omega_3+2\lambda_3)(1-\gamma^n)],$$

$$S_\gamma(\tilde{d}) = \frac{1}{324}[7(9x+9y-2\kappa_1+2\zeta_1)\gamma^n + 11(9x+9y-2\kappa_2+2\zeta_2)(1-\gamma^n) + 11(9x+9y-2\kappa_3+2\zeta_3)(1-\gamma^n)],$$

$$S_\gamma(\tilde{c} + \tilde{d}) = \frac{1}{324}[7(9\overline{a+x} + 9\overline{b+y} - 2\overline{\omega_1+\kappa_1} + 2\overline{\lambda_1+\zeta_1})\gamma^n + 11(9\overline{a+x} + 9\overline{b+y} - 2\overline{\omega_2+\kappa_2} + 2\overline{\lambda_2+\zeta_2})(1-\gamma^n) + 11(9\overline{a+x} + 9\overline{b+y} - 2\overline{\omega_3+\kappa_3} + 2\overline{\lambda_3+\zeta_3})(1-\gamma^n)],$$

$$S_\gamma(\tilde{c} - \tilde{d}) = \frac{1}{324}[7(9\overline{a-y} + 9\overline{b-x} - 2\overline{\omega_1+\zeta_1} + 2\overline{\lambda_1+\kappa_1})\gamma^n + 11(9\overline{a-y} + 9\overline{b-x} - 2\overline{\omega_2+\zeta_2} + 2\overline{\lambda_2+\kappa_2})(1-\gamma^n) + 11(9\overline{a-y} + 9\overline{b-x} - 2\overline{\omega_3+\zeta_3} + 2\overline{\lambda_3+\kappa_3})(1-\gamma^n)];$$

Hence the result is.

(ii) Here, $-\tilde{d} = \langle [-y, -x, \zeta_1, \kappa_1], [-y, -x, \zeta_2, \kappa_2], [-y, -x, \zeta_3, \kappa_3] \rangle$.

$$\tilde{d} - \tilde{d} = \langle [x-y, y-x, \kappa_1+\zeta_1, \zeta_1+\kappa_1], [x-y, y-x, \kappa_2+\zeta_2, \zeta_2+\kappa_2], [x-y, y-x, \kappa_3+\zeta_3, \zeta_3+\kappa_3] \rangle;$$

$$S_\gamma(\tilde{d} - \tilde{d}) = \frac{1}{324}[7(9\overline{x-y} + 9\overline{y-x} - 2\overline{\kappa_1+\zeta_1} + 2\overline{\zeta_1+\kappa_1})\gamma^n + 11(9\overline{x-y} + 9\overline{y-x} - 2\overline{\kappa_2+\zeta_2} + 2\overline{\zeta_2+\kappa_2})(1-\gamma^n) + 11(9\overline{x-y} + 9\overline{y-x} - 2\overline{\kappa_3+\zeta_3} + 2\overline{\zeta_3+\kappa_3})(1-\gamma^n)] = 0 = S_\gamma(\tilde{0});$$

This ends (ii).

(iii)

$$\begin{aligned} S_\gamma(\tilde{c}) &= \gamma^n S_\mu(\tilde{c}) + (1 - \gamma^n) S_\nu(\tilde{c}) + (1 - \gamma^n) S_\eta(\tilde{c}) \\ \frac{dS_\gamma(\tilde{c})}{d\gamma} &= n\gamma^{n-1}[S_\mu(\tilde{c}) - (S_\nu(\tilde{c}) + S_\eta(\tilde{c}))] \end{aligned}$$

$\frac{dS_\gamma(\tilde{c})}{d\gamma} >, <, = 0$ when $[S_\mu(\tilde{c}) - (S_\nu(\tilde{c}) + S_\eta(\tilde{c}))] >, <, = 0$ respectively as $\gamma \geq 0$. This meets the fact.

3.3 Definition

Let $\text{SVTN}(\mathbf{R})$ be the set of all SVTN-numbers defined over \mathbf{R} . For $\gamma \in [0, 1]$, a mapping $f_\gamma : \text{SVTN}(\mathbf{R}) \rightarrow \mathbf{R}$ is called a ranking function and it is defined as : $f_\gamma(\tilde{c}) = S_\gamma(\tilde{c})$ for $\tilde{c} \in \text{SVTN}(\mathbf{R})$. The order of $\tilde{x}, \tilde{w} \in \text{SVTN}(\mathbf{R})$ is defined as :

$$\begin{aligned} S_\gamma(\tilde{x}) > S_\gamma(\tilde{w}) &\Leftrightarrow \tilde{x} >_{f_\gamma} \tilde{w} \quad (\text{i.e., } \tilde{x} > \tilde{w} \text{ with respect to } f_\gamma), \quad S_\gamma(\tilde{x}) < S_\gamma(\tilde{w}) \Leftrightarrow \tilde{x} <_{f_\gamma} \tilde{w}, \\ S_\gamma(\tilde{x}) = S_\gamma(\tilde{w}) &\Leftrightarrow \tilde{x} =_{f_\gamma} \tilde{w}. \end{aligned}$$

4 Linear programming in neutrosophic sense

Here, we shall extend the concept of crisp LP-problem under the neutrosophic environment. First we recall the structure of crisp LP-problem.

$$\begin{aligned} \text{Max } z &= cx \\ \text{such that } Ax &= b, \quad x = (x_1, x_2, \dots, x_n)^t, \quad x_i \geq 0 \end{aligned}$$

where $c = (c_1, c_2, \dots, c_n)$, $b = (b_1, b_2, \dots, b_n)^t$ and $A = [p_{ij}]_{m \times n}$ with c_j, b_j, p_{ij} all real.

The concept of LP-problem is now modified by considering the coefficients of the variables in the objective function, the right hand side coefficients in the constraints and in the decision variables regarded as SVTN-numbers. Thus a LP-problem in neutrosophic sense is designed as follows :

$$\begin{aligned} \text{Max } \tilde{z} &=_{f_\gamma} \tilde{c}\tilde{x} \\ \text{such that } A\tilde{x} &=_{f_\gamma} \tilde{b}, \quad \tilde{x} \geq_{f_\gamma} \tilde{0} \end{aligned} \quad (6)$$

where $\tilde{b} \in (\text{SVTN}(\mathbf{R}))^m$, $\tilde{x} \in (\text{SVTN}(\mathbf{R}))^n$, $A \in \mathbf{R}^{m \times n}$, $\tilde{c}^t \in (\text{SVTN}(\mathbf{R}))^n$ and f_γ is a ranking function.

4.1 Definition

1. $\tilde{x} \in (\text{SVTN}(\mathbf{R}))^n$ satisfying the constraints of (6) is called a feasible solution to (6).
2. If $\tilde{c}\tilde{x}^* \geq_{f_\gamma} \tilde{c}\tilde{x}$ holds for all solutions \tilde{x} to (6), then \tilde{x}^* is an optimal solution to (6).
3. For the modified LP-problem (6), consider $\text{rank}(A, \tilde{b}) = \text{rank}(A) = m$. The columns of A is partitioned as $[B, N]$ where $B_{m \times m}$ and N are respectively called basis and non-basis matrix. Clearly $\text{rank}(B) = m$. Then, a feasible solution $\tilde{x} = (\tilde{x}_B, \tilde{x}_N)^t$ to (6) obtained by setting $\tilde{x}_B =_{f_\gamma} B^{-1}\tilde{b}$, $\tilde{x}_N =_{f_\gamma} \tilde{0}$ is called a neutrosophic basic feasible solution (NBFS). The component \tilde{x}_B and \tilde{x}_N are respectively called basic variable and nonbasic variable.
4. \tilde{x} is non-degenerate NBFS when all components of $\tilde{x}_B >_{f_\gamma} \tilde{0}$. For \tilde{x} being degenerate NBFS, at least one component of $\tilde{x}_B =_{f_\gamma} \tilde{0}$.

4.1.1 Note

In the modified LP-problem (6), let $A = [p_{ij}]_{m \times n} = [p_1, p_2, \dots, p_n]$ where each $p_k = (p_{1k}, p_{2k}, \dots, p_{mk})^t$ is m component column vector. Taking partition on the columns of A , let $B_{m \times m}$ be the basis matrix. Suppose $w_k = (w_{1k}, w_{2k}, \dots, w_{mk})^t$ is a set of m component scalars required to represent any column p_k of A as a linear combination of the column vectors of basis matrix B i.e., $p_k = Bw_k$.

5 Simplex method for modified LP-problem

The modified LP-problem (6) can be put as follows :

$$\begin{aligned} \text{Max } \tilde{z} &=_{f_\gamma} \tilde{c}_B\tilde{x}_B + \tilde{c}_N\tilde{x}_N \\ \text{such that } B\tilde{x}_B + N\tilde{x}_N &=_{f_\gamma} \tilde{b} \\ \tilde{x}_B, \tilde{x}_N &\geq_{f_\gamma} \tilde{0} \end{aligned}$$

where $\tilde{x}_B, \tilde{x}_N, B, N$ all are signified already. We have then,

$$\tilde{x}_B + B^{-1}N\tilde{x}_N =_{f_\gamma} B^{-1}\tilde{b} \quad (7)$$

$$\begin{aligned} \Rightarrow \tilde{c}_B\tilde{x}_B + \tilde{c}_B B^{-1}N\tilde{x}_N &=_{f_\gamma} \tilde{c}_B B^{-1}\tilde{b} \\ \Rightarrow \tilde{z} - \tilde{c}_N\tilde{x}_N + \tilde{c}_B B^{-1}N\tilde{x}_N &=_{f_\gamma} \tilde{c}_B B^{-1}\tilde{b} \\ \Rightarrow \tilde{z} + (\tilde{c}_B B^{-1}N - \tilde{c}_N)\tilde{x}_N &=_{f_\gamma} \tilde{c}_B B^{-1}\tilde{b} \end{aligned} \quad (8)$$

Assuming $\tilde{x}_N =_{f_\gamma} \tilde{0}$, we get $\tilde{x}_B =_{f_\gamma} B^{-1}\tilde{b}$ by (7) and $\tilde{z} =_{f_\gamma} \tilde{c}_B B^{-1}\tilde{b}$ by (8). The LP-problem (6) is thus arranged in the following table (Table 1).

Table 1 : Tabular form of modified LP-problem.

	\tilde{c}_j	\tilde{c}_B	\tilde{c}_N	
	\tilde{z}	\tilde{x}_B	\tilde{x}_N	R.H.S
\tilde{x}_B	0	1	$B^{-1}N$	$B^{-1}\tilde{b}$
\tilde{z}	1	0	$\tilde{c}_B B^{-1}N - \tilde{c}_N$	$\tilde{c}_B B^{-1}\tilde{b}$

All the required information are met by Table 1 to proceed the simplex method. The cost row in Table 1 is $\tilde{\chi}_j =_{f_\gamma} (\tilde{c}_B B^{-1}p_j - \tilde{c}_j)_{p_j \notin B}$ which implies $\tilde{\chi}_j =_{f_\gamma} (\tilde{z}_j - \tilde{c}_j)$ for non-basic variables.

5.1 Theorem

A non-degenerate NBFS $(\tilde{x}_B, \tilde{x}_N) = (B^{-1}\tilde{b}, \tilde{0})$ is optimal to the modified LP-problem (6) if and only if $\tilde{z}_j - \tilde{c}_j \geq_{f_\gamma} \tilde{0}, \forall j = 1, \dots, n$.

Proof. Let $\tilde{x}^* = (\tilde{x}_B^t, \tilde{x}_N^t)^t$ be an NBFS to (1) where $\tilde{x}_B = B^{-1}\tilde{b}, \tilde{x}_N = \tilde{0}$. Let \tilde{z}^* be the objective function corresponding to \tilde{x}^* . Then $\tilde{z}^* =_{f_\gamma} \tilde{c}_B\tilde{x}_B =_{f_\gamma} \tilde{c}_B B^{-1}\tilde{b}$. Let \tilde{z} be the objective function corresponding to another feasible solution $\tilde{x} = [\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n]^t$ to LP-problem (6), then $B\tilde{x}_B + N\tilde{x}_N =_{f_\gamma} \tilde{b} =_{f_\gamma} A\tilde{x}$ and objective function is :

$$\tilde{z} =_{f_\gamma} \tilde{c}_B\tilde{x}_B + \tilde{c}_N\tilde{x}_N =_{f_\gamma} \tilde{c}_B B^{-1}\tilde{b} - \sum_{p_j \notin B} (\tilde{c}_B B^{-1}p_j - \tilde{c}_j)\tilde{x}_j =_{f_\gamma} \tilde{z}^* - \sum_{p_j \notin B} (\tilde{z}_j - \tilde{c}_j)\tilde{x}_j$$

Clearly, the solution is optimal if and only if $\tilde{z}_j - \tilde{c}_j \geq_{f_\gamma} \tilde{0} \forall j = 1, \dots, n$.

5.2 Theorem

For any NBFS to the modified LP-problem (6), if there is some column not in basis for which $\tilde{z}_k - \tilde{c}_k <_{f_\gamma} \tilde{0}$ and $w_{ik} \leq 0; i = 1, 2, \dots, m$, then LP-problem attains an unbounded solution.

Proof. Let \tilde{x}_B be a basic solution for the problem (6). Arranging the constraints,

$$\begin{aligned} B\tilde{x}_B + N\tilde{x}_N &= \tilde{b} \\ \Rightarrow \tilde{x}_B + B^{-1}N\tilde{x}_N &= B^{-1}\tilde{b} \\ \Rightarrow \tilde{x}_B + B^{-1}\sum_j (p_j\tilde{x}_j) &= B^{-1}\tilde{b}, p_j \text{ s are the columns of } N \\ \Rightarrow \tilde{x}_B + \sum_j (B^{-1}p_j\tilde{x}_j) &= B^{-1}\tilde{b} \\ \Rightarrow \tilde{x}_B + \sum_j (w_j\tilde{x}_j) &= \tilde{w}_0, \text{ for } p_j = Bw_j, p_j \notin B \\ \Rightarrow \tilde{x}_{B_i} + \sum_j (w_{ij}\tilde{x}_j) &= \tilde{w}_{i0}; i = 1, \dots, m; j = 1, \dots, n \\ \Rightarrow \tilde{x}_{B_i} = \tilde{w}_{i0} - \sum_j (w_{ij}\tilde{x}_j); &i = 1, \dots, m; j = 1, \dots, n \end{aligned}$$

If \tilde{x}_k enters into the basis, then $\tilde{x}_k >_{f_\gamma} \tilde{0}$ and $\tilde{x}_j =_{f_\gamma} \tilde{0}$ for $j \neq B_i \cup k$ (B_i being a column of B). Since $w_{ik} \leq 0$ ($i = 1, \dots, m$), so $\tilde{w}_{i0} - w_{ik}\tilde{x}_k \geq_{f_\gamma} \tilde{0}$. Hence, basic solution remains feasible at present and for that

feasible solution, objective function is :

$$\begin{aligned}\tilde{z}^* &=_{f_\gamma} \tilde{c}_B \tilde{x}_B + \tilde{c}_N \tilde{x}_N =_{f_\gamma} \sum_{i=1}^m \tilde{c}_{B_i} (\tilde{w}_{i0} - w_{ik} \tilde{x}_k) + \tilde{c}_k \tilde{x}_k \\ &=_{f_\gamma} \sum_{i=1}^m \tilde{c}_{B_i} \tilde{w}_{i0} - \left(\sum_{i=1}^m \tilde{c}_{B_i} w_{ik} - \tilde{c}_k \right) \tilde{x}_k \\ &=_{f_\gamma} \tilde{c}_B \tilde{w}_0 - (\tilde{c}_B y_k - \tilde{c}_k) \tilde{x}_k =_{f_\gamma} \tilde{z} - (\tilde{z}_k - \tilde{c}_k) \tilde{x}_k\end{aligned}$$

Thus $\tilde{z}^* =_{f_\gamma} \tilde{z} - (\tilde{z}_k - \tilde{c}_k) \tilde{x}_k$. It implies $\tilde{z}^* >_{f_\gamma} \tilde{z}$, as $\tilde{z}_k - \tilde{c}_k <_{f_\gamma} \tilde{0}$.

Hence the LP-problem attains an unbounded solution.

5.3 Simplex algorithm for solving modified LP-problem

While applying simplex method to solve an LP-problem studied here, it is always assumed that the initial solution is feasible. It will be optimised through some iterations. Following steps are practiced :

Step 1. For maximization problem, go to Step 1 directly. Otherwise, convert it into a maximization problem by changing the sign of all price vectors \tilde{c}_j .

Step 2. Introduce slack variables to convert all ' \leq ' type inequations into equations. Consider the costs of all slack variables to $\tilde{0}$.

Step 3. Calculate a NBFS to the problem of the form $\tilde{x}_B = B^{-1} \tilde{b} = \tilde{w}_0$ and $\tilde{x}_N = \tilde{0}$ and the respective objective function as $\tilde{z} =_{f_\gamma} \tilde{c}_B B^{-1} \tilde{b} =_{f_\gamma} \tilde{c}_B \tilde{w}_0$.

Step 4. Assume $\tilde{\chi}_B =_{f_\gamma} \tilde{z}_B - \tilde{c}_B =_{f_\gamma} \tilde{0}$ for each basic variable and in the present iteration, calculate $\tilde{\chi}_j =_{f_\gamma} \tilde{z}_j - \tilde{c}_j =_{f_\gamma} \tilde{c}_B B^{-1} p_j - \tilde{c}_j$ for each non-basic variable. The present solution will be optimal, if $\tilde{z}_j - \tilde{c}_j \geq_{f_\gamma} \tilde{0}, \forall j$.

Step 5. If $\tilde{\chi}_j =_{f_\gamma} \tilde{z}_j - \tilde{c}_j <_{f_\gamma} \tilde{0}$ for some non-basic variables then compute $\tilde{\chi}_k = \min\{\tilde{\chi}_j\}$. If $w_{ik} < 0$ for all $i = 1, \dots, m$, then the given problem attains unbounded solution and so terminate the iteration. Otherwise determine

$$\frac{\tilde{w}_{r0}}{w_{rk}} = \min\left\{\frac{\tilde{w}_{i0}}{w_{ik}} : w_{ik} > 0; i = 1, \dots, m\right\}.$$

to find out the index of the variable \tilde{x}_{B_r} to be removed from the present basis.

Step 6. Modify \tilde{w}_{i0} by replacing $\tilde{w}_{i0} - \frac{\tilde{w}_{r0}}{w_{rk}} w_{ik}$ for $i \neq r$ and \tilde{w}_{r0} by $\frac{\tilde{w}_{r0}}{w_{rk}}$.

Step 7. Develop new basis and perform Step 4, Step 5 repeatedly until the optimality is reached.

Step 8. Find the optimal solution and the optimal value of objective function.

6 Numerical Example

A real life problem is stated here and is solved numerically by use of proposed concept. For simplicity, we define the γ -weighted score function for $n = 1$ in rest of the paper.

6.1 Example

For business purpose, Mr. X wishes to drive his two lorries (L_1, L_2) in two different routes (R_1, R_2). The route R_1 is assigned for the lorry L_1 and route R_2 for L_2 . He likes to allow a maximum of Rs. $300\tilde{b}_1$ for fuel charge and at most Rs. $320\tilde{b}_2$ for the salary of staffs in a week. The consumed fuel charge is Rs. 23/hr for L_1 and Rs. 25/hr for L_2 . The salary of staffs is estimated Rs. 30/hr for L_1 and Rs. 40/hr for L_2 . Such type of variation of fuel charge and salary estimation are due to road condition, mileage of lorry, distance, road tax, time bounds and different business angles. This results a profit approximately Rs. \tilde{c}_1 /hr from L_1 and Rs. \tilde{c}_2 /hr from L_2 . Now suggest him what time can he allow to run his lorries in two routes depending on these criteria so that the maximum profit will be met as a whole in a week.

The problem can be summarised in the following table (Table 2):

Table 2 : Summarisation of Example 6.1

Expenditure \Downarrow	Route : R_1	R_2	Available cost / week \Downarrow
Fuel charge	Rs. 23/hr	Rs. 25/hr	Rs. $300\tilde{b}_1$
Staff salary	Rs. 30/hr	Rs. 40/hr	Rs. $320\tilde{b}_2$
Profit/hr \Rightarrow	Rs. \tilde{c}_1	Rs. \tilde{c}_2	

Let the lorry L_1 will run for \tilde{x}_1 hr in route R_1 and the lorry L_2 will run for \tilde{x}_2 hr in route R_2 in a week. The problem will be then designed as follows :

$$\begin{aligned} \text{Max } \tilde{z} &=_{f_\gamma} \tilde{c}_1\tilde{x}_1 + \tilde{c}_2\tilde{x}_2 \\ \text{such that } &23\tilde{x}_1 + 25\tilde{x}_2 \leq_{f_\gamma} 300\tilde{b}_1 \\ &30\tilde{x}_1 + 40\tilde{x}_2 \leq_{f_\gamma} 320\tilde{b}_2 \\ &\tilde{x}_1, \tilde{x}_2 \geq_{f_\gamma} \tilde{0} \end{aligned}$$

It is a modified LP-problem in neutrosophic sense with a pre-assigned $\gamma = 0.4$ where $\tilde{c}_1, \tilde{c}_2, \tilde{b}_1, \tilde{b}_2$ are all SVTN-numbers given as follows :

$$\begin{aligned} \tilde{c}_1 &= \langle [4, 7, 1, 3], [4, 7, 3, 4], [4, 7, 2, 1] \rangle, \\ \tilde{c}_2 &= \langle [6, 8, 4, 10], [6, 8, 5, 1], [6, 8, 3, 2] \rangle, \\ \tilde{b}_1 &= \langle [10, 12, 3, 7], [10, 12, 6, 12], [10, 12, 4, 15] \rangle, \\ \tilde{b}_2 &= \langle [8, 22, 2, 18], [8, 22, 4, 25], [8, 22, 7, 30] \rangle. \end{aligned}$$

Rewriting the given constraints by introducing slack variables,

$$\begin{aligned} 23\tilde{x}_1 + 25\tilde{x}_2 + \tilde{x}_3 &=_{f_\gamma} 300\tilde{b}_1 \\ 30\tilde{x}_1 + 40\tilde{x}_2 + \tilde{x}_4 &=_{f_\gamma} 320\tilde{b}_2 \\ \tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 &\geq_{f_\gamma} \tilde{0} \end{aligned}$$

The first revised simplex table is given in the following table (Table 3).

Table 3 : First revised simplex table.

$\tilde{c}_j \Rightarrow$	\tilde{c}_1	\tilde{c}_2	$\tilde{0}$	$\tilde{0}$	
$\tilde{x}_B \Downarrow$	\tilde{x}_1	\tilde{x}_2	\tilde{x}_3	\tilde{x}_4	R.H.S
\tilde{x}_3	23	25	1	0	$300\tilde{b}_1$
\tilde{x}_4	30	40	0	1	$320\tilde{b}_2 \rightarrow$
$\tilde{z} \Rightarrow$	$\tilde{c}_1^{(1)}$	$\tilde{c}_2^{(1)} \uparrow$	$\tilde{c}_3^{(1)}$	$\tilde{c}_4^{(1)}$	

where $\tilde{c}_1^{(1)} = -\tilde{c}_1$, $\tilde{c}_2^{(1)} = -\tilde{c}_2$ and $S_\gamma(\tilde{c}_3^{(1)}) = S_\gamma(\tilde{c}_4^{(1)}) = S_\gamma(\tilde{0})$.

Now $S_\gamma(\tilde{c}_1^{(1)}) = \frac{1}{324}(1457\gamma - 2178) < 0$, $S_\gamma(\tilde{c}_2^{(1)}) = \frac{1}{324}(1696\gamma - 2662) < 0$ and $\tilde{c}_1^{(1)} >_{f_\gamma} \tilde{c}_2^{(1)}$ for $\gamma = 0.4$. So \tilde{x}_2 enters in the basis.

Further $S_\gamma(300\tilde{b}_1/25) = \frac{12}{324}(4730 - 3288\gamma)$, $S_\gamma(320\tilde{b}_2/40) = \frac{8}{324}(6908 - 4794\gamma)$. For $\gamma = 0.4$, $(300\tilde{b}_1/25) >_{f_\gamma} (320\tilde{b}_2/40)$ and so the leaving variable is \tilde{x}_4 . The second revised simplex table is (Table 4):

Table 4 : Second revised simplex table.

$\tilde{c}_j \Rightarrow$	\tilde{c}_1	\tilde{c}_2	$\tilde{0}$	$\tilde{0}$	
$\tilde{x}_B \Downarrow$	\tilde{x}_1	\tilde{x}_2	\tilde{x}_3	\tilde{x}_4	R.H.S
\tilde{x}_3	17/4	0	1	-5/8	$100(3\tilde{b}_1 - 2\tilde{b}_2) \rightarrow$
\tilde{x}_2	3/4	1	0	1/40	$8\tilde{b}_2$
$\tilde{z} \Rightarrow$	$\tilde{c}_1^{(2)} \uparrow$	$\tilde{c}_2^{(2)}$	$\tilde{c}_3^{(2)}$	$\tilde{c}_4^{(2)}$	$8\tilde{b}_2\tilde{c}_2$

where $S_\gamma(\tilde{c}_2^{(2)}) = S_\gamma(\tilde{c}_3^{(2)}) = S_\gamma(\tilde{0})$ and $\tilde{c}_1^{(2)} = \frac{3}{4}\tilde{c}_2 - \tilde{c}_1$, $\tilde{c}_4^{(2)} = \frac{1}{40}\tilde{c}_2$.

Then $S_\gamma(\tilde{c}_1^{(2)}) = \frac{1}{1296}(-726 + 740\gamma)$, $S_\gamma(\tilde{c}_4^{(2)}) = \frac{1}{12960}(2662 - 1696\gamma)$. For $\gamma = 0.4$, clearly $S_\gamma(\tilde{c}_1^{(2)}) < 0$, $S_\gamma(\tilde{c}_4^{(2)}) > 0$. So \tilde{x}_1 enters in the basis.

Further $S_\gamma((300\tilde{b}_1 - 200\tilde{b}_2)/\frac{17}{4}) = \frac{100}{1377}(374 - 276\gamma)$, $S_\gamma(8\tilde{b}_2/\frac{3}{4}) = \frac{32}{972}(6908 - 4794\gamma)$. So the leaving variable is \tilde{x}_3 for $\gamma = 0.4$. The final revised table is (Table 5):

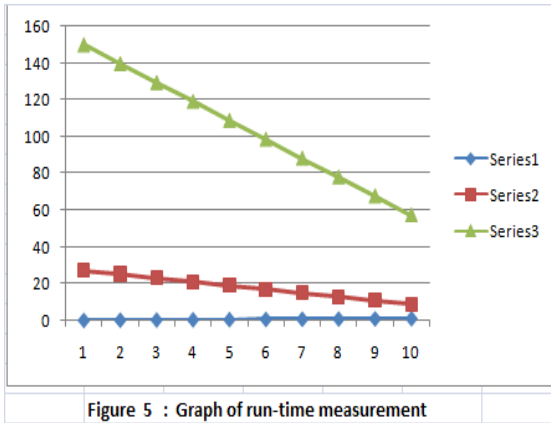


Figure 5 : Graph of run-time measurement

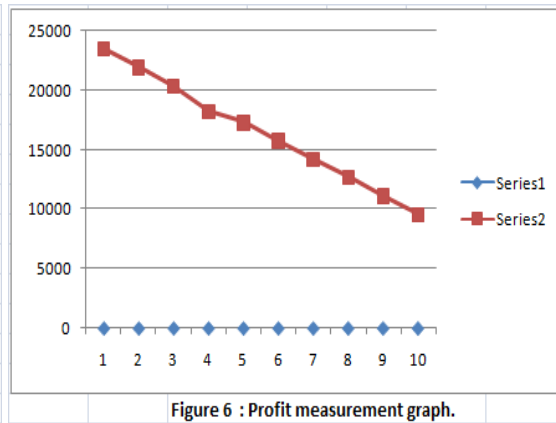


Figure 6 : Profit measurement graph.

Table 5 : Final revised simplex table.

$\tilde{c}_j \Rightarrow$	\tilde{c}_1	\tilde{c}_2	$\tilde{0}$	$\tilde{0}$	
$\tilde{x}_B \Downarrow$	\tilde{x}_1	\tilde{x}_2	\tilde{x}_3	\tilde{x}_4	R.H.S
\tilde{x}_1	1	0	4/17	-5/34	$400(3\tilde{b}_1 - 2\tilde{b}_2)/17$
\tilde{x}_2	0	1	-3/17	23/170	$(736\tilde{b}_2 - 900\tilde{b}_1)/17$
$\tilde{z} \Rightarrow$	$\tilde{c}_1^{(3)}$	$\tilde{c}_2^{(3)}$	$\tilde{c}_3^{(3)}$	$\tilde{c}_4^{(3)}$	$\frac{400}{17}(3\tilde{b}_1 - 2\tilde{b}_2)\tilde{c}_1 + \frac{1}{17}(736\tilde{b}_2 - 900\tilde{b}_1)\tilde{c}_2$

where $S_\gamma(\tilde{c}_1^{(3)}) = S_\gamma(\tilde{c}_2^{(3)}) = S_\gamma(\tilde{0})$ and $\tilde{c}_3^{(3)} = \frac{1}{17}(4\tilde{c}_1 - 3\tilde{c}_2)$, $\tilde{c}_4^{(3)} = \frac{1}{170}(23\tilde{c}_2 - 25\tilde{c}_1)$. Then $S_\gamma(\tilde{c}_3^{(3)}) = \frac{1}{5508}(726 - 740\gamma) > 0$ and $S_\gamma(\tilde{c}_4^{(3)}) = \frac{1}{55080}(6776 - 2583\gamma) > 0$ for $\gamma = 0.4$.

Thus the optimality arises. The optimal solution is : $\tilde{x}_1 = 400(3\tilde{b}_1 - 2\tilde{b}_2)/17$, $\tilde{x}_2 = (736\tilde{b}_2 - 900\tilde{b}_1)/17$ and so, $\text{Max } \tilde{z} =_{f_\gamma} \frac{400}{17}(3\tilde{b}_1 - 2\tilde{b}_2)\tilde{c}_1 + \frac{1}{17}(736\tilde{b}_2 - 900\tilde{b}_1)\tilde{c}_2$.

6.1.1 Result and discussion

At optimality stage, different optimal values of Example 6.1 for different γ is displayed in the following table (Table 6).

Here $S_\gamma(\tilde{x}_1) = \frac{100}{1377}(374 - 276\gamma)$, $S_\gamma(\tilde{x}_2) = \frac{2}{1377}(103411 - 71148\gamma)$ and $S_\gamma(\tilde{z}) = \frac{1}{324}(7605114 - 5014498\gamma)$.

Table 6 : Optimal values for different γ .

γ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
\tilde{x}_1	27.16	25.16	23.15	21.15	19.14	17.14	15.13	13.13	11.13	9.12
\tilde{x}_2	150.2	139.86	129.53	119.2	108.86	98.53	88.19	77.86	67.53	57.19
\tilde{z}	23473	21925	20377	18230	17282	15734	14186	12639	11091	9543

The value for $\gamma = 1$ is excluded in Table 6, as it terminates the iteration in Table 4. There $S_\gamma(\tilde{c}_1^{(2)}) = \frac{1}{1296}(-726 + 740\gamma) > 0$ for $\gamma = 0.9811$ approximately. It is then suggested to run the lorry L_2 in route R_2 only to meet a profit. Thus, it is clear from Table 6 that the character γ plays a vital role to determine the optimal solution in modified LP-problem. With respect to different γ , it is seen that \tilde{x}_1 , \tilde{x}_2 , \tilde{z} are all monotone decreasing functions. This γ is here signified as the level of ruination of road. It is one of the factors determining the profit of owner from the lorry. In Figure 5, Series 2 and Series 3 represent the weekly run time of two lorries L_1 and L_2 respectively. Figure 6 deals the weekly profit (Series 2) of Mr. X. In both graphical presentation, Series 1 measures the level of ruination of road i.e., different γ .

7 Conclusion

The present study deals a modified structure of crisp LP-problem in the parlance of SVTN-number. An approach is taken to solve such problem by developing an efficient algorithm. A new ranking technique plays a key role to develop this algorithm and also to establish some well known theories. The proposed concept is illustrated by solving a real life problem. A discussion of result obtained is performed and is presented graphically.

This concept will assist the industrialists, directors of management institutes, marketing supervisors to manage the various uncertain situations and complexities. They can reach at a fair end as the present notion helps to solve a LP-problem with respect to the provided constraints and its hidden states together. Several linear, non-linear programming problem, multi criteria decision making and also many mathematical frame works may be enlighten by this attempt.

The ranking function is innovated here by taking the maximum height of each trapezium. It may be allowed within $[0,1]$ in future.

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Utilization of Jaccard Index Measures on Multiple Attribute Group Decision Making under Neutrosophic Environment

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Abstract

In this paper, we introduce the concept of Jaccard index measures under the neutrosophic environment to make the right decision in multiple attributes. Here, we insinuate two Jaccard index measures based on distance and the included weighted Jaccard of two vectors between the neutrosophic environment. Then, we determine the Multiple Attribute group decision-making method (in short MAGDM) based on the Jaccard index measures under the neutrosophic environment and also we compare the applications of the proposed MAGDM method in the neutrosophic environment. Finally, certain descriptive examples are on hand to verify the residential handle and to express its practicality and effectiveness.

Keywords: Jaccard index measure, Neutrosophic vague set, MAGDM.

1.INTRODUCTION

In 1999, Smarandache [28] presents another part of the theory known as neutrosophy, which studies the origin, nature, and scope of neutralities, as well as their interactions with various ideational spectra. The neutrosophic set is the generalization of the classic set, fuzzy sets [35], interval-valued fuzzy set [29], intuitionistic fuzzy set [5], interval-valued intuitionistic fuzzy set [4], paraconsistent set, dialetheist set, paradoxical set, and tautological set. A neutrosophic set has three basic components such that truth-membership, indeterminacy-membership, and falsity-membership, and they are independent [28], for more informations on the neutrosophic theory we refere the readers to [36-39].

Vague sets have been presented by Gau and Buehrar in 1993 as an extension of the fuzzy set theory [20]. It is considered as an effective tool to deal with uncertainty since it gives more data when contrasted with fuzzy sets [30]. A vague set is defined by a truth-membership function t_v and a false-membership function f_v [17,18].

Shawkat Alkhazaleh [27] in 2015 presented the idea of the neutrosophic vague set as a combination of neutrosophic set and vague set. Neutrosophic vague theory is an effective tool to process incomplete, indeterminate and inconsistent information. In 2019, Hashim et al. [21] developed a new generalized mathematical model called interval neutrosophic vague sets which are a combination of vague sets and interval neutrosophic sets and a generalization of interval neutrosophic vague sets. Al-Quran and Hassan [1,2, 3] in 2018 presented and gave more application on neutrosophic vague soft under decision making.

In 2013, Ye [33] introduces the Multi-attribute decision-making method using the correlation coefficient under a single-valued neutrosophic environment. It is one of the most significant angles in the executive's science which can deliver significant financial benefits in an assortment of fields, such as manufacturing domain [17], disaster assessment, company investment management. For decision-making problems in engineering practice, the decision information is generally incomplete and indeterminate [34]. To apply them to multi-criteria decision-making problems with simplified neutrosophic information. Recently, Chakraborty et al. developed a multi-criteria decision-making problem for different used in the bipolar neutrosophic domain [12]. Furthermore, Abdel basset develop multi-criteria decision-making problem under a hybrid neutrosophic set [24].

The similarity measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be easily selected as well. In fact, the degree of similarity between the objects under study plays an important role. In vector space, especially the Jaccard similarity measures [9,10,11,13] are often used in information retrieval, citation analysis, and automatic classification. Ye [32] proposed the Jaccard, Dice, and cosine similarity measures between trapezoidal intuitionistic fuzzy numbers (TIFNs) that are treated as continuous and applied them to multicriteria group decision-making problems. In 2014 Ye [32] developed three vector similarity measures between single valued neutrosophic sets as a generalization of the Jaccard, Dice, and cosine similarity measures between two vectors. Furthermore, in 2016, Mehmet and Deli developed a multi-criteria decision making for bipolar neutrosophic sets based on Jaccard vector similarity measures and applied to a numerical example in order to confirm the practicality and accuracy of the proposed method [23]. In the paper, we using Jaccard index measures which are more efficiency, further this method will give better result.

1.1. Motivation

A significant issue then arises if one considers a neutrosophic vague number: what will be a Jaccard index neutrosophic vague measures and a weighted Jaccard index neutrosophic vague measures? How should we utilize a Jaccard index neutrosophic vague measure in MAGDM? In light of this point of view, we built up the subject of this exploration article. We succeeded in producing an illustration example.

1.2. Novelties

Various works have just been distributed right now setting. Analysts have just built up a few definitions and applications in different fields. In any case, many interesting outcomes are as yet obscure. Our work aimed to create thoughts for those obscure viewpoints:

- (i) Introduction of a Jaccard index measures of neutrosophic vague set and its definition.
- (ii) Application in a Jaccard index measures in MAGDM.

1.3. The structure of the paper

The paper is organized as follows: In section 1, we have discussed the introduction and literature review. In section 2, contains the preliminaries section. In section 3, the concept of the neutrosophic vague set, a Jaccard index measures and its properties. In section 4, we introduce the algorithm to solve a Multiple Attribute Group Decision-Making (MAGDM). The practical problem is considered in section 5. The compression of the result has been done with two more research in section 6. The conclusions are written in section 7.

2. PRELIMINARIES

Definition.2.1 ^[27]

A Vague set V on the universe of discourse X written as $A = \{ \langle x, t_A(x), 1-f_A(x) \rangle \mid x \in X \}$, is characterized by a truth-membership function t_v , and a false-membership function f_v , as follows:

$$t_v: U \rightarrow [0,1], f_v: U \rightarrow [0,1], \text{ and } t_v + f_v \leq 1$$

Definition.2.2 ^[27]

Let A and B be vague sets of the form $A = \{ \langle x, t_A(x), 1-f_A(x) \rangle \mid x \in X \}$ and

$B = \{ \langle x, t_B(x), 1-f_B(x) \rangle \mid x \in X \}$. Then

- i. $A \subseteq B$ if and only if $t_A(x) \leq t_B(x)$ and $1-f_A(x) \leq 1-f_B(x)$.
- ii. $A=B$ if and only if $A \subseteq B$ and $B \subseteq A$.
- iii. $A^c = \{ \langle x, 1-f_A(x), t_A(x) \rangle \mid x \in X \}$.
- iv. $A \cup B = \{ \langle x, \max(t_A(x), t_B(x)), \max(1-f_A(x), 1-f_B(x)) \rangle \mid x \in X \}$.
- v. $A \cap B = \{ \langle x, \min(t_A(x), t_B(x)), \min(1-f_A(x), 1-f_B(x)) \rangle \mid x \in X \}$.

Definition.2.3 ^[27]

A neutrosophic set A on the universe of discourse X is defined as

$$A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle, x \in X \} \text{ where } T, I, F: X \rightarrow]0,1]^+ \text{ and } 0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+.$$

Definition.2.4 ^[27]

A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X written as $A_{NV} = \{ \langle x; T_{ANV}(x), I_{ANV}(x), F_{ANV}(x) \rangle; x \in X \}$ whose truth-membership, indeterminacy-membership and false-membership functions is defined as:

$$T_{ANV}(x) = [T^-, T^+], I_{ANV}(x) = [I^-, I^+], F_{ANV} = [F^-, F^+] \text{ where}$$

- 1) $T^+ = 1 - F^-$
- 2) $F^+ = 1 - T^-$ and
- 3) $0 \leq T^- + I^- + F^- \leq 2^+$.

Definition.2.5 ^[25]

Let X be a universe of discourse. A bipolar neutrosophic set A_{BNS} in X is defined as an object of the form

$$A_{BNS} = \{ \langle x; T^+(x), I^+(x), F^+(x), T^-(x), I^-(x), F^-(x) \rangle; x \in X \}$$

Where $T^+, F^+, I^+ : X \rightarrow [1, 0]$ and $T^-, F^-, I^- : X \rightarrow [-1, 0]$

3. Jaccard Index Measure of Neutrosophic Vague Sets.

Definition.3.1

Let $A_{NV} = \{ \langle x; T_{ANV}(x), I_{ANV}(x), F_{ANV}(x) \rangle; x \in X \}$ and $B_{NV} = \{ \langle x; T_{BNV}(x), I_{BNV}(x), F_{BNV}(x) \rangle; x \in X \}$ two neutrosophic vague set in X . Then the two Jaccard index measure of A_{NV} and B_{NV} are proposed based on distance and the included weighted Jaccard index measure of two vectors, respectively as follows:

Jaccard index measure based on distance

$$J_{NV}(A_{NV}, B_{NV}) = \sum_{i=1}^n \left(\frac{[T_{ANV}^+(x_i).T_{BNV}^+(x_i)] + [T_{ANV}^-(x_i).T_{BNV}^-(x_i)] + [I_{ANV}^+(x_i).I_{BNV}^+(x_i)] + [I_{ANV}^-(x_i).I_{BNV}^-(x_i)] + [F_{ANV}^+(x_i).F_{BNV}^+(x_i)] + [F_{ANV}^-(x_i).F_{BNV}^-(x_i)]}{\left([T_{ANV}^+(x_i)]^2 + [T_{BNV}^+(x_i)]^2 + [T_{ANV}^-(x_i)]^2 + [T_{BNV}^-(x_i)]^2 + [I_{ANV}^+(x_i)]^2 + [I_{BNV}^+(x_i)]^2 + [I_{ANV}^-(x_i)]^2 + [I_{BNV}^-(x_i)]^2 + [F_{ANV}^+(x_i)]^2 + [F_{BNV}^+(x_i)]^2 + [F_{ANV}^-(x_i)]^2 + [F_{BNV}^-(x_i)]^2 \right)} - \left([T_{ANV}^+(x_i).T_{BNV}^-(x_i)] + [T_{ANV}^-(x_i).T_{BNV}^+(x_i)] + [I_{ANV}^+(x_i).I_{BNV}^-(x_i)] + [I_{ANV}^-(x_i).I_{BNV}^+(x_i)] + [F_{ANV}^+(x_i).F_{BNV}^-(x_i)] + [F_{ANV}^-(x_i).F_{BNV}^+(x_i)] \right) \right) \quad (5)$$

Weighted Jaccard index measure based on two vectors

$$WJ_{NV}(A_{NV}, B_{NV}) = \sum_{i=1}^n (w_i) \left(\frac{[T_{ANV}^+(x_i).T_{BNV}^+(x_i)] + [T_{ANV}^-(x_i).T_{BNV}^-(x_i)] + [I_{ANV}^+(x_i).I_{BNV}^+(x_i)] + [I_{ANV}^-(x_i).I_{BNV}^-(x_i)] + [F_{ANV}^+(x_i).F_{BNV}^+(x_i)] + [F_{ANV}^-(x_i).F_{BNV}^-(x_i)]}{\left([T_{ANV}^+(x_i)]^2 + [T_{BNV}^+(x_i)]^2 + [T_{ANV}^-(x_i)]^2 + [T_{BNV}^-(x_i)]^2 + [I_{ANV}^+(x_i)]^2 + [I_{BNV}^+(x_i)]^2 + [I_{ANV}^-(x_i)]^2 + [I_{BNV}^-(x_i)]^2 + [F_{ANV}^+(x_i)]^2 + [F_{BNV}^+(x_i)]^2 + [F_{ANV}^-(x_i)]^2 + [F_{BNV}^-(x_i)]^2 \right)} - \left([T_{ANV}^+(x_i).T_{BNV}^-(x_i)] + [T_{ANV}^-(x_i).T_{BNV}^+(x_i)] + [I_{ANV}^+(x_i).I_{BNV}^-(x_i)] + [I_{ANV}^-(x_i).I_{BNV}^+(x_i)] + [F_{ANV}^+(x_i).F_{BNV}^-(x_i)] + [F_{ANV}^-(x_i).F_{BNV}^+(x_i)] \right) \right) \quad (6)$$

According to the above definition 3.1, the two Jaccard index measures $J(A_{NV}, B_{NV})$ for NVs satisfy the following properties (p1)–(p3):

- (p1) $0 \leq J_{NV}(A_{NV}, B_{NV}) \leq 1$;
- (p2) $J_{NV}(A_{NV}, B_{NV}) = J_{NV}(B_{NV}, A_{NV})$;
- (p3) If $A_{NV} = B_{NV}$, then $J_{NV}(A_{NV}, B_{NV}) = 1$.

Proof. Firstly, we prove the properties (p1)–(p3) of $J(A_{NV}, B_{NV})$

(p1) It is clear that $J_{NV}(A_{NV}, B_{NV}) \geq 0$

We have to proof $J_{NV}(A_{NV}, B_{NV}) \leq 1$ By the inequality

$$2ab \leq a^2 + b^2.$$

$$\sum_{i=1}^n \left(\begin{aligned} & [T_{ANV}^+(x_i).T_{BNV}^+(x_i)] + [T_{ANV}^-(x_i).T_{BNV}^-(x_i)] + [I_{ANV}^+(x_i).I_{BNV}^+(x_i)] + [I_{ANV}^-(x_i).I_{BNV}^-(x_i)] \\ & + [F_{ANV}^+(x_i).F_{BNV}^+(x_i)] + [F_{ANV}^-(x_i).F_{BNV}^-(x_i)] \end{aligned} \right) \leq$$

$$\sum_{i=1}^n \left(\begin{aligned} & \left([T_{ANV}^+(x_i)]^2 + [T_{BNV}^+(x_i)]^2 + [T_{ANV}^-(x_i)]^2 + [T_{BNV}^-(x_i)]^2 + [I_{ANV}^+(x_i)]^2 + [I_{BNV}^+(x_i)]^2 \right. \\ & \left. + [I_{ANV}^-(x_i)]^2 + [I_{BNV}^-(x_i)]^2 + [F_{ANV}^+(x_i)]^2 + [F_{BNV}^+(x_i)]^2 + [F_{ANV}^-(x_i)]^2 + [F_{BNV}^-(x_i)]^2 \right) \\ & - \left([T_{ANV}^+(x_i).T_{BNV}^+(x_i)] + [T_{ANV}^-(x_i).T_{BNV}^-(x_i)] + [I_{ANV}^+(x_i).I_{BNV}^+(x_i)] + [I_{ANV}^-(x_i).I_{BNV}^-(x_i)] \right. \\ & \left. + [F_{ANV}^+(x_i).F_{BNV}^+(x_i)] + [F_{ANV}^-(x_i).F_{BNV}^-(x_i)] \right) \end{aligned} \right)$$

$$\sum_{i=1}^n \left(\begin{aligned} & \frac{[T_{ANV}^+(x_i).T_{BNV}^+(x_i)] + [T_{ANV}^-(x_i).T_{BNV}^-(x_i)] + [I_{ANV}^+(x_i).I_{BNV}^+(x_i)] + [I_{ANV}^-(x_i).I_{BNV}^-(x_i)]}{\left([T_{ANV}^+(x_i)]^2 + [T_{BNV}^+(x_i)]^2 + [T_{ANV}^-(x_i)]^2 + [T_{BNV}^-(x_i)]^2 + [I_{ANV}^+(x_i)]^2 + [I_{BNV}^+(x_i)]^2 \right.} \\ & \left. + [I_{ANV}^-(x_i)]^2 + [I_{BNV}^-(x_i)]^2 + [F_{ANV}^+(x_i)]^2 + [F_{BNV}^+(x_i)]^2 + [F_{ANV}^-(x_i)]^2 + [F_{BNV}^-(x_i)]^2 \right) \\ & - \left([T_{ANV}^+(x_i).T_{BNV}^+(x_i)] + [T_{ANV}^-(x_i).T_{BNV}^-(x_i)] + [I_{ANV}^+(x_i).I_{BNV}^+(x_i)] + [I_{ANV}^-(x_i).I_{BNV}^-(x_i)] \right. \\ & \left. + [F_{ANV}^+(x_i).F_{BNV}^+(x_i)] + [F_{ANV}^-(x_i).F_{BNV}^-(x_i)] \right) \end{aligned} \right) \leq 1$$

$$\therefore J_{NV}(ANV, BNV) \leq 1$$

Hence, $0 \leq J_{NV}(ANV, BNV) \leq 1$ holds.

(p2) It is clear that

$$J_{NV}(ANV, BNV) = J_{NV}(BNV, ANV),$$

\therefore It is true.

(p3) if $ANV = BNV$,

$$(T_{ANV}^+(x_i), T_{ANV}^-(x_i); I_{ANV}^+(x_i), I_{ANV}^-(x_i); F_{ANV}^+(x_i), F_{ANV}^-(x_i)) =$$

$$(T_{BNV}^+(x_i), T_{BNV}^-(x_i); I_{BNV}^+(x_i), I_{BNV}^-(x_i); F_{BNV}^+(x_i), F_{BNV}^-(x_i))$$

where $i = 1, 2, 3, \dots, n$.

Here, ANV and BNV considered as two vectors so ,

$$\|ANV\| = \|BNV\| \text{ where}$$

$$\|ANV\| = \sqrt{[T_{ANV}^+(x_i)]^2 + [T_{ANV}^-(x_i)]^2 + [I_{ANV}^+(x_i)]^2 + [I_{ANV}^-(x_i)]^2 + [F_{ANV}^+(x_i)]^2 + [F_{ANV}^-(x_i)]^2}$$

$$\|BNV\| = \sqrt{[T_{BNV}^+(x_i)]^2 + [T_{BNV}^-(x_i)]^2 + [I_{BNV}^+(x_i)]^2 + [I_{BNV}^-(x_i)]^2 + [F_{BNV}^+(x_i)]^2 + [F_{BNV}^-(x_i)]^2}$$

And there exist $\frac{ANV \cdot BNV}{\|ANV\| \cdot \|BNV\|}$

$$= \frac{[T_{ANV}^+(x_i).T_{BNV}^+(x_i)] + [T_{ANV}^-(x_i).T_{BNV}^-(x_i)] + [I_{ANV}^+(x_i).I_{BNV}^+(x_i)] + [I_{ANV}^-(x_i).I_{BNV}^-(x_i)]}{\left([T_{ANV}^+(x_i)]^2 + [T_{ANV}^-(x_i)]^2 + [I_{ANV}^+(x_i)]^2 + [I_{ANV}^-(x_i)]^2 + [F_{ANV}^+(x_i)]^2 + [F_{ANV}^-(x_i)]^2 \right) \cdot \left([T_{BNV}^+(x_i)]^2 + [T_{BNV}^-(x_i)]^2 + [I_{BNV}^+(x_i)]^2 + [I_{BNV}^-(x_i)]^2 + [F_{BNV}^+(x_i)]^2 + [F_{BNV}^-(x_i)]^2 \right)}$$

$$= 1.$$

Hence,

$$J_{NV}(ANV, BNV) = 1.$$

Thus, we have proved.

If we consider the weighted Jaccard index measure between ANV and BNV are proposed, respectively, as follow:

$WJ_{NV}(A_{NV}, B_{NV})$

$$= \sum_{i=1}^n (w_i) \left(\frac{[T_{A_{NV}}^+(x_i).T_{B_{NV}}^+(x_i)] + [T_{A_{NV}}^-(x_i).T_{B_{NV}}^-(x_i)] + [I_{A_{NV}}^+(x_i).I_{B_{NV}}^+(x_i)] + [I_{A_{NV}}^-(x_i).I_{B_{NV}}^-(x_i)] + [F_{A_{NV}}^+(x_i).F_{B_{NV}}^+(x_i)] + [F_{A_{NV}}^-(x_i).F_{B_{NV}}^-(x_i)]}{\left([T_{A_{NV}}^+(x_i)]^2 + [T_{B_{NV}}^+(x_i)]^2 + [T_{A_{NV}}^-(x_i)]^2 + [T_{B_{NV}}^-(x_i)]^2 + [I_{A_{NV}}^+(x_i)]^2 + [I_{B_{NV}}^+(x_i)]^2 + [I_{A_{NV}}^-(x_i)]^2 + [I_{B_{NV}}^-(x_i)]^2 + [F_{A_{NV}}^+(x_i)]^2 + [F_{B_{NV}}^+(x_i)]^2 + [F_{A_{NV}}^-(x_i)]^2 + [F_{B_{NV}}^-(x_i)]^2 \right)} - \left([T_{A_{NV}}^+(x_i).T_{B_{NV}}^+(x_i)] + [T_{A_{NV}}^-(x_i).T_{B_{NV}}^-(x_i)] + [I_{A_{NV}}^+(x_i).I_{B_{NV}}^+(x_i)] + [I_{A_{NV}}^-(x_i).I_{B_{NV}}^-(x_i)] + [F_{A_{NV}}^+(x_i).F_{B_{NV}}^+(x_i)] + [F_{A_{NV}}^-(x_i).F_{B_{NV}}^-(x_i)] \right) \right)$$

Where $w_i \in [0,1]$, and $\sum_{i=1}^n w_i = 1$ for $i = 1, 2, \dots, n$.

It is obvious that the two weighted Jaccard index measures $WJ(A_{NV}, B_{NV})$ also satisfy the following properties (p1)-(p3):

(p1) $0 \leq WJ_{NV}(A_{NV}, B_{NV}) \leq 1$;

(p2) $WJ_{NV}(A_{NV}, B_{NV}) = WJ_{NV}(B_{NV}, A_{NV})$;

(p3) If $A_{NV} = B_{NV}$, then $WJ_{NV}(A_{NV}, B_{NV}) = 1$.

We can easily prove the properties (p1)-(p3) for $WJ_{NV}(A_{NV}, B_{NV})$ by a similar proof process.

4. MAGDM Method Based on the Jaccard Index Measures

For an MAGDM problem, let $G = \{g_1, g_2, \dots, g_m\}$ be a set of m alternatives and $A = \{A_1, A_2, \dots, A_n\}$ be a set of n attributes. The weight vector of the attributes A_j ($j = 1, 2, \dots, n$) is

$\omega_A = (\omega_{A1}, \omega_{A2}, \dots, \omega_{An})^T$, satisfying $\omega_{Aj} \in [0, 1]$, and $\sum_{j=1}^n \omega_{Aj} = 1$ for $j = 1, 2, \dots, n$. Assume that $EX = \{EX_1, EX_2, \dots, EX_y\}$ is a group of specialists and their corresponding weight vector is $\omega_E = (\omega_{E1}, \omega_{E2}, \dots, \omega_{Ey})^T$, satisfying $\omega_{Ek} \in [0, 1]$, and $\sum_{k=1}^y \omega_{Ek} = 1$. Each specialist can dole out the truth-degree, falsity-degree, and indeterminacy-degree to each attribute A_j ($j = 1, 2, \dots, n$) on the choices g_i ($i = 1, 2, \dots, m$) according to the neutrosophic environment respectively.

Therefore, we can established in NVs decision matrix

$D^k = (d_{i,j}^k)_{m \times n} = [D_1^k, D_2^k, \dots, D_m^k]^T$, is an NVs for $T_{NV}, I_{NV}, F_{NV} \in [0,1]$

Then, we apply the Jaccard index measures of neutrosophic vague set (in short NVs) to solve MAGDM problems

4.1. Algorithm to solve MAGDM problem.

Step 1: We establish the Ns, BNs, NVs matrix $G_H^* = (g_{i,j}^*)_{4 \times 3} G_i^*$ ($i=1,2,3,4$) as follows:

$$G_H^k = (g_{i,j}^k)_{m \times n} = [G_1^k, G_2^k, \dots, G_m^k]^T$$

Step 2: Calculate the weighted Jaccard index measures values by Eq.(2,4,6) using H.

Step 3: Calculate the overall weighted Jaccard index measure values considering the corresponding weight of each expert to evaluate the alternatives G_i ($i = 1, 2, \dots, m$), as follows:

$$J_H(D^k, G_i) = \sum_{k=1}^y \omega_{Ek} J_H(D^k, G_i) \quad (7)$$

$$WJ_H(D^k, G_i) = \sum_{k=1}^y \omega_{Ek} J_H(D^k, G_i) \quad (8)$$

Where $\omega_{Ek} \in [0,1]$ and $\sum_{k=1}^y \omega_{Ek} = 1$.

Step 4: Rank all alternatives according to the value of $WJ_H(D^k, G_i)$ or $J_H(D^k, G_i)$ and

select the better choice. The greater value of a Jaccard index measure, is the better alternative.

Step 5: End.

5. Practical example

Let us consider the decision making problem. There is a speculation organization, which needs to put an aggregate of cash in the best choices. There is a board with four potential alternatives to invest the money. (1) G_1 is a motor company; (2) G_2 is a pump company; (3) G_3 is an arms company; (4) G_4 is a furniture company. The investment company must make a decision according to three attributes given below: (1) A_1 is the growth analysis; (2) A_2 is the risk analysis; (3) A_3 is the environmental impact analysis. Then, the weight vector of the attributes is given by are 0.35, 0.25 and 0.40. Thus, when the four possible alternatives with respect to the above three attributes are evaluated by the expert, we can get the accompanying the neutrosophic vague decision matrix:

$$D^1 = \begin{bmatrix} D_1^1 \\ D_2^1 \\ D_3^1 \\ D_4^1 \end{bmatrix} = \begin{bmatrix} [< 0.4, 0.2 >, < 0.2, 0.7 >, < 0.3, 0.4 >], [< 0.3, 0.2 >, < 0.1, 0.1 >, < 0.2, 0.3 >], [< 0.4, 0.3 >, < 0.2, 0.2 >, < 0.4, 0.5 >] \\ [< 0.5, 0.6 >, < 0.7, 0.2 >, < 0.9, 0.1 >], [< 0.4, 0.5 >, < 0.2, 0.2 >, < 0.4, 0.3 >], [< 0.3, 0.1 >, < 0.4, 0.5 >, < 0.8, 0.1 >] \\ [< 0.6, 0.4 >, < 0.2, 0.2 >, < 0.4, 0.5 >], [< 0.5, 0.6 >, < 0.7, 0.7 >, < 0.8, 0.4 >], [< 0.3, 0.5 >, < 0.4, 0.4 >, < 0.4, 0.3 >] \\ [< 0.3, 0.2 >, < 0.4, 0.4 >, < 0.5, 0.7 >], [< 0.4, 0.3 >, < 0.4, 0.4 >, < 0.8, 0.9 >], [< 0.4, 0.8 >, < 0.3, 0.3 >, < 0.4, 0.5 >] \end{bmatrix},$$

$$D^2 = \begin{bmatrix} D_1^2 \\ D_2^2 \\ D_3^2 \\ D_4^2 \end{bmatrix} = \begin{bmatrix} [< 0.3, 0.4 >, < 0.5, 0.8 >, < 0.2, 0.4 >], [< 0.2, 0.3 >, < 0.3, 0.4 >, < 0.5, 0.1 >], [< 0.6, 0.4 >, < 0.3, 0.4 >, < 0.5, 0.4 >] \\ [< 0.4, 0.3 >, < 0.5, 0.6 >, < 0.6, 0.8 >], [< 0.5, 0.1 >, < 0.4, 0.3 >, < 0.6, 0.7 >], [< 0.5, 0.9 >, < 0.2, 0.3 >, < 0.6, 0.7 >] \\ [< 0.1, 0.3 >, < 0.4, 0.6 >, < 0.8, 0.5 >], [< 0.4, 0.3 >, < 0.2, 0.3 >, < 0.4, 0.3 >], [< 0.5, 0.6 >, < 0.3, 0.5 >, < 0.8, 0.6 >] \\ [< 0.6, 0.1 >, < 0.6, 0.5 >, < 0.6, 0.3 >], [< 0.6, 0.5 >, < 0.1, 0.3 >, < 0.5, 0.2 >], [< 0.3, 0.6 >, < 0.2, 0.1 >, < 0.3, 0.6 >] \end{bmatrix},$$

$$D^3 = \begin{bmatrix} D_1^3 \\ D_2^3 \\ D_3^3 \\ D_4^3 \end{bmatrix} = \begin{bmatrix} [< 0.3, 0.1 >, < 0.1, 0.6 >, < 0.2, 0.3 >], [< 0.4, 0.6 >, < 0.2, 0.4 >, < 0.4, 0.5 >], [< 0.5, 0.7 >, < 0.1, 0.5 >, < 0.3, 0.4 >] \\ [< 0.3, 0.7 >, < 0.6, 0.5 >, < 0.6, 0.3 >], [< 0.3, 0.4 >, < 0.1, 0.1 >, < 0.3, 0.4 >], [< 0.4, 0.3 >, < 0.5, 0.6 >, < 0.7, 0.2 >] \\ [< 0.5, 0.3 >, < 0.4, 0.5 >, < 0.6, 0.4 >], [< 0.4, 0.5 >, < 0.5, 0.6 >, < 0.6, 0.5 >], [< 0.4, 0.2 >, < 0.6, 0.5 >, < 0.2, 0.4 >] \\ [< 0.4, 0.3 >, < 0.5, 0.3 >, < 0.4, 0.6 >], [< 0.5, 0.2 >, < 0.5, 0.3 >, < 0.5, 0.6 >], [< 0.5, 0.6 >, < 0.4, 0.2 >, < 0.3, 0.2 >] \end{bmatrix}$$

Then, the developed MAGDM approach can be applied to this decision- making problem using the following steps:

Step 1: we can compute the Jaccard index measures G_i ($i=1,2,3,4$) by using by Eq.(5) as follows:

$$G^* = \begin{bmatrix} G_1^* \\ G_2^* \\ G_3^* \\ G_4^* \end{bmatrix} = \begin{bmatrix} [< 0.5, 0.6 >, < 0.1, 0.6 >, < 0.2, 0.3 >], [< 0.4, 0.6 >, < 0.1, 0.1 >, < 0.2, 0.1 >], [< 0.6, 0.7 >, < 0.1, 0.2 >, < 0.3, 0.4 >] \\ [< 0.5, 0.7 >, < 0.5, 0.2 >, < 0.6, 0.1 >], [< 0.5, 0.6 >, < 0.1, 0.1 >, < 0.3, 0.3 >], [< 0.5, 0.9 >, < 0.2, 0.3 >, < 0.6, 0.1 >] \\ [< 0.5, 0.3 >, < 0.2, 0.2 >, < 0.4, 0.4 >], [< 0.4, 0.5 >, < 0.2, 0.3 >, < 0.4, 0.3 >], [< 0.5, 0.6 >, < 0.3, 0.4 >, < 0.2, 0.3 >] \\ [< 0.6, 0.3 >, < 0.4, 0.3 >, < 0.4, 0.3 >], [< 0.6, 0.5 >, < 0.1, 0.3 >, < 0.5, 0.2 >], [< 0.5, 0.8 >, < 0.2, 0.1 >, < 0.3, 0.2 >] \end{bmatrix}$$

Step 2: We calculate the Jaccard index measures values dependent on the distance between D_i^k

Equation (1) as follows:

$$J_{NVs}(D^1, G_i) = \{ J_{NVs}(D_1^1, G_1^*), J_{NVs}(D_2^1, G_2^*), J_{NVs}(D_3^1, G_3^*), J_{NVs}(D_4^1, G_4^*) \} = \{ 0.7844, 0.7088, 0.7409, 0.9409 \}$$

$$J_{NVs}(D^2, G_i) = \{ J_{NVs}(D_1^2, G_1^*), J_{NVs}(D_2^2, G_2^*), J_{NVs}(D_3^2, G_3^*), J_{NVs}(D_4^2, G_4^*) \} = \{ 0.9550, 0.9847, 0.9122, 0.9688 \}$$

$$J_{NVs}(D^3, G_i) = \{ J_{NVs}(D_1^3, G_1^*), J_{NVs}(D_2^3, G_2^*), J_{NVs}(D_3^3, G_3^*), J_{NVs}(D_4^3, G_4^*) \} = \{ 0.9245, 0.9066, 0.9455, 0.9813 \}$$

Similarly, we can calculate the weighted Jaccard index measures values dependent on the two vectors between by equation (6) as follows:

$$WJ_{NVs}(D^1, G_i) = \{ WJ_{NVs}(D_1^1, G_1^*), WJ_{NVs}(D_2^1, G_2^*), WJ_{NVs}(D_3^1, G_3^*), WJ_{NVs}(D_4^1, G_4^*) \} = \{ 0.9732, 0.9644, 0.9673, 0.9432 \}$$

$$WJ_{NVs}(D^2, G_i) = \{ WJ_{NVs}(D_1^2, G_1^*), WJ_{NVs}(D_2^2, G_2^*), WJ_{NVs}(D_3^2, G_3^*), WJ_{NVs}(D_4^2, G_4^*) \} = \{ 0.8468, 0.8615, 0.9750, 0.9584 \}$$

$$WJ_{NVs}(D^3, G_i) = \{ WJ_{NVs}(D_1^3, G_1^*), WJ_{NVs}(D_2^3, G_2^*), WJ_{NVs}(D_3^3, G_3^*), WJ_{NVs}(D_4^3, G_4^*) \} = \{ 0.7807, 0.8500, 0.9827, 0.8366 \}.$$

Step 3: Considering the relating weight $\omega_E = (0.37, 0.33, 0.3)^T$ of the specialists to assess the alternatives G_i ($i = 1, 2, 3, 4$), we can calculate the overall weighted Jaccard index measure values depends on distance by Equation (7) as follows:

$$J_{NVs}(D^k, G_1) = 0.37 \times J_{NVs}(D_1^1, G_1^*) + 0.33 \times J_{NVs}(D_1^2, G_1^*) + 0.3 \times J_{NVs}(D_1^3, G_1^*) = 0.8064$$

$$J_{NVs}(D^k, G_2) = 0.37 \times J_{NVs}(D_2^1, G_2^*) + 0.33 \times J_{NVs}(D_2^2, G_2^*) + 0.3 \times J_{NVs}(D_2^3, G_2^*) = 0.8568$$

$$J_{NVs}(D^k, G_3) = 0.37 \times J_{NVs}(D_3^1, G_3^*) + 0.33 \times J_{NVs}(D_3^2, G_3^*) + 0.3 \times J_{NVs}(D_3^3, G_3^*) = 0.9331$$

$$J_{NVs}(D^k, G_4) = 0.37 \times J_{NVs}(D_4^1, G_4^*) + 0.33 \times J_{NVs}(D_4^2, G_4^*) + 0.3 \times J_{NVs}(D_4^3, G_4^*) = 0.8597$$

Similarly, we can calculate the overall weighted Jaccard index measures values based on the two vectors between by equation (8) as follows:

$$WJ_{NVs}(D^k, G_1) = 0.8737 \quad WJ_{NVs}(D^k, G_2) = 0.8961 \quad WJ_{NVs}(D^k, G_3) = 0.9744 \quad WJ_{NVs}(D^k, G_4) = 0.9162$$

Step 4: According to the above values of $J_{NVs}(D^k, G_i)$ and ($i=1,2,3,4$), the distance value of both the Jaccard index measure and the weighted Jaccard index measure values based on two vectors, the ranking orders: $G_3 > G_4 > G_2 > G_1$ are same. As indicated by the most extreme value of Jaccard index measures, the alternative G_3 is the better decision.

6. Related Comparison

Further comparison, table 6.1 show the MAGDM results based on the Jaccard index measures of NVs proposed in this paper and the neutrosophic set and bipolar neutrosophic set were proposed Jaccard index in the relevant paper [23][32]. Here, we utilizing Enq (8) for the two neutrosophic set and bipolar neutrosophic set respectively.

MAGDM METHOD	JACCARD INDEX	RANKING ORDER	THE BEST ALTERNATIVE
$J_{NVs}(D^k, G_i)$	0.8064, 0.8568, 0.9331, 0.8597	$G_3 > G_4 > G_2 > G_1$	G_3
$WJ_{NVs}(D^k, G_i)$	0.8737, 0.8961, 0.9744, 0.9162	$G_3 > G_4 > G_2 > G_1$	G_3
$WJ_{NS}(D^k, G_i)$	0.8003, 0.7961, 0.8447, 0.8147	$G_3 > G_4 > G_2 > G_1$	G_3
$WJ_{BNS}(D^k, G_i)$	0.8700, 0.7456, 0.8940, 0.8957	$G_3 > G_4 > G_2 > G_1$	G_3

Table 6.1. Decision results based on neutrosophic environment MAGDM method

Obviously, from the result of table 6.1, ranking orders and best alternatives based on the new method based on this paper is consistent with the result provided by Mehmet and Irfan [23]. Compared with the [23,32] the calculation

process of the Jaccard index for MAGDM proposed in this paper is relatively compared to neutrosophic set and bipolar neutrosophic set based on the Jaccard index for MAGDM in [23, 32]. The above comparisons demonstrate that this paper present a new concept for solving decision-making problems is more efficient under a neutrosophic environment.

7. Conclusion

In this paper we develop MAGDM method and gave its application under the neutrosophic environment and also to show the exhibit effectiveness of the proposed method, we utilized an illustration example. There are many similarity measures utilized in the decision-making problem but we have utilized a Jaccard similarity measure to show that the proposed method can effectively solve decision-making problems with NVs information. Furthermore, researchers can be extended to study some new correlation coefficients between NVs and their MAGDM.

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New Technology in Agriculture Using Neutrosophic Soft Matrices with the Help of Score Function

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Abstract

Uncertainty is a big problem in our routine life. Many theories were developed to handle uncertain environments. This paper approaches the concept of neutrosophic soft matrices (NSM) and multiple types of NSM to achieve solutions to a possible problem and provide ideas to tackle other problems relating to uncertainties. Here, NSM has been utilized to demonstrate the performance of different farmers, and further score function has been implemented to solve a possible application of decision making in agriculture. It explains the selection of the best farmer by scientific experts through an algorithm in this paper. The selection based upon the better production of crop and nature, fertilizer, pesticides, etc. are used as attributes, which will contribute to the performance of each farmer. Finally, combining the attributes, which will help us achieve a conclusion to determine the best farmer.

Keywords: Neutrosophic Soft Set, NSM, Agriculture, Decision Making, Score function.

1. Introduction

Many researchers use different tools to solve the uncertainties and problematic issues in different fields like engineering, business administration, environment, medical sciences, etc., which are unable to solve, by using standard mathematical tools. To overcome the difficulties of standard tools, researchers work to apply different tools to deal with uncertain problems. Some of these are fuzzy sets, intuitionistic fuzzy sets, neutrosophic sets, etc. In 1965 Lotfi. A. Zadeh [1] proposed a wonderful theory named the fuzzy set theory to deal with uncertain issues. Further, in 1975, a more advanced interval-valued fuzzy set (IVFS) was proposed by Yang [2], which has a wide range than a simple fuzzy set. In 1982, Pawalk initiated another wonderful theory named rough set theory [3]. After that, the intuitionistic fuzzy set theory was coined by Atanassov [4] in 1983. In 1995 neutrosophic fuzzy set was proposed by Florentine Smarandache [5]. After that, in 1999, Molodtsov [6] developed the soft set theory, which is a major mathematical operator when dealing with decision-making problems in a vague environment. It has wide applications such as decision making in the medical field, economics, and social sciences. In 2001, Maji et al. [7] extended the Molodtsov [6] theory and defined different basics of soft sets. Later in 2004, Maji et al. [8] proposed the idea of intuitionistic fuzzy soft sets. Cagman [9] coined the idea of fuzzy soft matrices in 2010. Fuzzy soft matrices have a wide range of applications in decision-making problems. However, simple matrix theory fails when sometimes dealing with uncertainty problems. After that, in 2012, Das and Chetia [10] introduced intuitionistic fuzzy soft matrices (abbr. IFSMs) with

different types of products and axioms on these products. Then, Mondal et al. [11, 12] introduced different types of IFSMs. In some real-world problems, we need to go with different tools to solve the uncertainty and abstruse problems, and furthermore, if we talk about intuitionistic fuzzy soft sets, we just have to deal with truth and false-membership values for a proper description of an object. The intuitionistic fuzzy sets can only handle the unclear info regarding both truth and false membership values. The neutrosophic soft set is a complete family of all neutrosophic sets, which is the generalized form of an intuitionistic fuzzy soft set with truth-value, indeterminate value, and false value. In 2013 P.K.Maji [13] defined different operators on neutrosophic fuzzy soft set and applied soft sets in unpredictable problems. In 2014, Broumi, et al. [14] defined interval-valued NSS and its applications in decision-making problems. In the same year, Broumi et al. [15] proposed different relations on interval-valued neutrosophic soft sets. It leads to decision making in various fields of life. In 2014, Broumi, et al. [16] applied different mappings on neutrosophic soft expert sets. In the same year, Irfan Deli and Broumi[17] defined neutrosophic soft matrices and used soft matrices in decision-making. Applications were requiring decision-making, having multiple selection criteria. Researchers applied these techniques in decision making in different fields of life. In 2019, Jafar et al. [18] applied intuitionistic soft set in medical diagnose. In 2019, Jafar et al. [19] worked on Sanchez's identification by trapezoidal fuzzy number. In 2020, Jafar et al. [20] discussed the application of soft-set relation and soft matrix in medical diagnosis by Sanchez. In 2019, Riaz et al. [21] studied the hardness of the water in laundry based on the fuzzy logic controller. The selection of smartphones in Pakistan decision making by Saqlain et al. [22] in 2018. In 2019, Saqlain et al. [23] predicted about 2019 Cricket world cup by TOPSIS Technique. Researchers [24-26] applied different strategies for problem solving and selection. In 2009, Mustafa et al. [27] applied fuzzy logic on sorting and grading in agriculture. After in 2013, Papageorgion et al. [28] proposed yield prediction using the fuzzy cognitive map. In 2014, Virgin and Riganabanu [29] proposed an application of an interval-valued fuzzy soft matrix for the detection of diseases in plants. In 2017, Neamatollah et al. [30] proposed an optimal cropping pattern of agriculture on the fuzzy system. In 2018, Mota et al. [31] defined fuzzy validity measures and their applications of decision making in agricultural engineering. In the same year, 2018, Loganathan and Pushpalatha [32] proposed an application in agriculture using fuzzy matrices. In 2019, Savarimuthu and Mahalaksmi [33] defined T-Conorm operators on IFSM and proposed its applications in agriculture. For more information on neutrosophic theory and their application, we refer the readers to the following references [34-37]. In this paper, the concept of neutrosophic fuzzy soft matrices, different types of fuzzy neutrosophic soft matrices, and some operators on soft matrices and its application in agriculture has been demonstrated. Several applications, studied under neutrosophic soft matrices, fuzzy neutrosophic matrices and neutrosophic fuzzy matrices, have been worked on and are being worked on as we speak. Conclusively, neutrosophic fuzzy soft matrices have been used in decision making to approach the desired result. The goal is to display the usage of the said concepts and ideas in the possible application of agriculture. In section 2, the discussion is about the soft set and its different types. In

section 3, the discussion is about methodology, which is used in the application of neutrosophic soft matrices. In section 4 and 5, we discussed the algorithm and real-life example of an NSM.

2 Preliminaries

In this section, some basics will be under discussion; to understand the concepts of paper, you must know the following.

2.1 Soft Set [20]

Let K be a set of alternatives, and D is a set of attributes. Let $P(K)$ denotes the set of all subsets of K and A is a subset of D . Then (F, A) is called a soft set over K where F is a mapping given by $F: A \rightarrow P(K)$. In fact soft set (F, A) is a family of subsets of K . For $e \in A$, (F, A) is defined as

$$(F, A) = \{F(e) \in P(K) : e \in D, F(e) = \emptyset \text{ if } e \notin A\}$$

Example 2.1

Let $K = \{f_1, f_2, f_3, f_4\}$ be a set of houses of different colors (paints) and $D = \{\text{Yellow}(e_1), \text{Green}(e_2), \text{Sky blue}(e_3)\}$ is a set of attributes. If $A = \{e_1, e_3\} \subseteq D$. Let $F_A(e_1) = \{f_1, f_2, f_4\}$ and $F_A(e_3) = \{f_1, f_3, f_4\}$ then the soft set $(F_A, D) = \{(e_1, \{f_1, f_2, f_4\}), (e_3, \{f_1, f_3, f_4\})\}$, which describes the colors of houses. We write the soft set as follows

K	Yellow(e_1)	Green(e_2)	Sky blue(e_3)
f_1	1	0	1
f_2	1	0	0
f_3	0	0	1
f_4	1	0	1

2.2 Fuzzy Soft Set [21]

Let K be a universe, and D be a set of attributes and any set $A \subseteq D$. Let $P(K)$ denotes the set of all fuzzy sets of K . A set (F_A, D) is said to be fuzzy soft set over K such that F_A is a mapping given by $F_A: D \rightarrow P(K)$ such that $F_A(e) = \varphi$ if $e \notin A$ where φ is a null fuzzy set.

Example 2.2

See example 2.1, we give membership value in 0 or 1, but in FSS we choose membership value from interval $[0, 1]$ instead of crisp numbers 0 and 1. Then

$$(F_A, D) = F_A(e_1) = \{(f_1, 0.5), (f_2, 0.3), (f_4, 0.8)\}$$

$F_A(e_3) = \{(f_1, 0.2), (f_3, 0.9), (f_4, 0.6)\}$ is the fuzzy soft sets describe the colors of houses.

K	Yellow(e_1)	Green(e_2)	Sky blue(e_3)
f_1	0.5	0.0	0.2

f_2	0.3	0.0	0.0
f_3	0.0	0.0	0.9
f_4	0.8	0.0	0.6

2.3 Fuzzy Soft Matrices (FSM) [18]

Let (F_A, D) be a fuzzy soft set and $K \times D$ is defined by a relation $G_A = \{(f, e): e \in A, f \in F_A(e)\}$. The function of G_A is written by $a_{GA}: K \times D \rightarrow [0, 1]$ where $a_{GA}(f, e) \in [0, 1]$ is the membership value.

If $[a_{ij}] = a_{ij}(f_i, e_j)$ then, the matrix is

$$[a_{ij}]_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Which is soft matrix of soft set (F, D) over K called the fuzzy soft matrix (FSS).

Example 2.3

Let $K = \{f_1, f_2, f_3, f_4, f_5\}$ is a universal set and $D = \{e_1, e_2, e_3, e_4\}$ is a set of all attributes then

$A = \{e_1, e_2, e_4\} \subseteq D$ Then the soft set $(F_D, A) = \{F_D(e_1), F_D(e_2), F_D(e_4)\}$ where

$$F_D(e_1) = \{(f_1, 0.5), (f_2, 0.3), (f_3, 0.1), (f_4, 0.2), (f_5, 0.8)\}$$

$$F_D(e_2) = \{(f_1, 0.3), (f_2, 0.6), (f_3, 0.4), (f_4, 0.9), (f_5, 0.2)\}$$

$$F_D(e_4) = \{(f_1, 0.2), (f_2, 0.7), (f_3, 0.1), (f_4, 0.3), (f_5, 0.5)\}.$$

Then, the soft matrix $[a_{ij}]$ is

$$[a_{ij}] = \begin{bmatrix} 0.5 & 0.3 & 0.0 & 0.2 \\ 0.3 & 0.6 & 0.0 & 0.7 \\ 0.1 & 0.4 & 0.0 & 0.1 \\ 0.2 & 0.9 & 0.0 & 0.3 \\ 0.8 & 0.2 & 0.0 & 0.5 \end{bmatrix}$$

2.4 Neutrosophic Soft Set (NSS) [17]

Suppose K be a universe with an element in K denoted by f and D be a set of attributes. A neutrosophic set N over K is characterized by a truthiness T_A , indeterminacy I_A , and a falsity value F_A where T_A , I_A and F_A are real standard subsets of $[0, 1]$. And $f_N: D \rightarrow N(K)$

$$A = \{(e, \{< f, (T_A(f), I_A(f), F_A(f))\} >): f \in U, e \in D, T_A(f), I_A(f), F_A(f) \in [0, 1]\}$$

There is no restriction on the sum of $T_A(f), I_A(f), F_A(f)$. $0 \leq T_A(f) + I_A(f) + F_A(f) \leq 3^+$.

Example 2.4

Let $K = \{f_1, f_2, f_3, f_4\}$ be a set of houses of different colors (paints) and $D = \{\text{Yellow}(e_1), \text{Green}(e_2), \text{Sky blue}(e_3)\}$ is a set of attributes. If $B = \{e_1, e_3\} \subseteq D$.

Let $(F_B, D) = F_B(e_1) = \{(f_1, 0.5, 0.2, 0.3), (f_2, 0.3, 0.4, 0.2), (f_4, 0.4, 0.1, 0.3)\}$

$F_B(e_3) = \{(f_1, 0.2, 0.4, 0.3), (f_3, 0.5, 0.1, 0.4), (f_4, 0.6, 0.3, 0.1)\}$ Which describes the colors of houses. We write the neutrosophic soft set as follows

K	Yellow(e_1)	Green(e_2)	Sky blue(e_3)
f_1	0.5,0.2,0.3	0.0,0.0,0.0	0.2,0.4,0.3
f_2	0.3,0.4,0.2	0.0,0.0,0.0	0.0,0.0,0.0
f_3	0.0,0.0,0.0	0.0,0.0,0.0	0.5,0.1,0.4
f_4	0.4,0.1,0.3	0.0,0.0,0.0	0.6,0.3,0.1

2.5 Neutrosophic Soft Matrix (NSM)

Suppose $K = \{\dot{f}_1, \dot{f}_2, \dot{f}_3, \dots\}$ be the universe and $D = \{e_1, e_2, e_3, \dots\}$ be a set of attributes and $A \subseteq D$. A set (F, A) be an NFSS over K. Then the subset of $K \times D$

Is defined as $R_A = \{(\dot{f}, e); e \in A, \dot{f} \in F_A(e)\}$ which is the relation form of (F_A, D) . The truthiness, indeterminacy and falsity values are:

$$T_{R_A}: K \times D \rightarrow [0, 1], \quad I_{R_A}: K \times D \rightarrow [0, 1], \quad F_{R_A}: K \times D \rightarrow [0, 1]$$

$T_{R_A}(f, e) \in [0, 1]$, $I_{R_A}(f, e) \in [0, 1]$, $F_{R_A}(f, e) \in [0, 1]$ are the truthiness, indeterminacy, and falsity of $f \in K$ for each $e \in D$?

If $[(T_{ij}, I_{ij}, F_{ij})] = [T_{ij}(\dot{f}_i, e_j), I_{ij}(\dot{f}_i, e_j), F_{ij}(\dot{f}_i, e_j)]$ then

$$[(T_{ij}, I_{ij}, F_{ij})]_{m \times n} = \begin{bmatrix} \langle T_{11}, I_{11}, F_{11} \rangle & \langle T_{12}, I_{12}, F_{12} \rangle & \cdots & \langle T_{1n}, I_{1n}, F_{1n} \rangle \\ \langle T_{21}, I_{21}, F_{21} \rangle & \langle T_{22}, I_{22}, F_{22} \rangle & \cdots & \langle T_{2n}, I_{2n}, F_{2n} \rangle \\ \vdots & \vdots & \vdots & \vdots \\ \langle T_{m1}, I_{m1}, F_{m1} \rangle & \langle T_{m2}, I_{m2}, F_{m2} \rangle & \cdots & \langle T_{mn}, I_{mn}, F_{mn} \rangle \end{bmatrix}$$

That is called $m \times n$ order neutrosophic soft matrix over K.

Example 2.5

Let $K = \{\dot{f}_1, \dot{f}_2, \dot{f}_3, \dot{f}_4, \dot{f}_5\}$ is a universal set and $D = \{e_1, e_2, e_3, e_4\}$ is a set of all attributes then

$A = \{e_1, e_2, e_4\} \subseteq D$ Then the soft set $(F_D, A) = \{F_D(e_1), F_D(e_2), F_D(e_4)\}$ where

$$F_D(e_1) = \{(f_1, 0.5, 0.2, 0.2), (f_2, 0.3, 0.4, 0.1), (f_3, 0.1, 0.6, 0.3), (f_4, 0.2, 0.3, 0.5), (f_5, 0.2, 0.1, 0.4)\}$$

$$F_D(e_2) = \{(f_1, 0.3, 0.5, 0.1), (f_2, 0.6, 0.2, 0.2), (f_3, 0.4, 0.3, 0.3), (f_4, 0.3, 0.1, 0.5), (f_5, 0.2, 0.5, 0.1)\}$$

$$F_D(e_4) = \{(f_1, 0.2, 0.3, 0.5), (f_2, 0.7, 0.1, 0.1), (f_3, 0.1, 0.6, 0.3), (f_4, 0.3, 0.4, 0.1), (f_5, 0.5, 0.2, 0.2)\}.$$

Then, the neutrosophic soft matrix is

$$[T_{ij}, I_{ij}, F_{ij}] = \begin{bmatrix} (0.5, 0.2, 0.2) & (0.3, 0.5, 0.1) & (0.0, 0.0, 0.0) & (0.2, 0.3, 0.5) \\ (0.3, 0.4, 0.1) & (0.6, 0.2, 0.2) & (0.0, 0.0, 0.0) & (0.7, 0.1, 0.1) \\ (0.1, 0.6, 0.3) & (0.4, 0.3, 0.3) & (0.0, 0.0, 0.0) & (0.1, 0.6, 0.3) \\ (0.2, 0.3, 0.5) & (0.3, 0.1, 0.5) & (0.0, 0.0, 0.0) & (0.3, 0.4, 0.1) \\ (0.2, 0.1, 0.4) & (0.2, 0.5, 0.1) & (0.0, 0.0, 0.0) & (0.5, 0.2, 0.2) \end{bmatrix}$$

2.6 Complement of Neutrosophic Soft Matrices

Suppose $A = [T_{ij}, I_{ij}, F_{ij}] \in NSM_{m \times n}$. Then the complement of A is denoted by A° and is defined as $A^\circ = [F_{ij}, 1 - I_{ij}, T_{ij}]$ for all i and j.

Example 2.6

See example 2.5

$$A^\circ = [F_{ij}, 1 - I_{ij}, T_{ij}] = \begin{bmatrix} (0.2, 0.8, 0.5) & (0.1, 0.5, 0.3) & (0.0, 0.0, 0.0) & (0.5, 0.7, 0.2) \\ (0.1, 0.6, 0.3) & (0.2, 0.8, 0.6) & (0.0, 0.0, 0.0) & (0.1, 0.9, 0.7) \\ (0.3, 0.4, 0.1) & (0.3, 0.7, 0.4) & (0.0, 0.0, 0.0) & (0.3, 0.4, 0.1) \\ (0.5, 0.7, 0.2) & (0.5, 0.9, 0.3) & (0.0, 0.0, 0.0) & (0.1, 0.6, 0.3) \\ (0.4, 0.9, 0.2) & (0.1, 0.5, 0.2) & (0.0, 0.0, 0.0) & (0.2, 0.8, 0.5) \end{bmatrix}$$

2.7 Addition of Neutrosophic Soft Matrices

If $A = [(T_{ij}^A, I_{ij}^A, F_{ij}^A)] \in NSM_{m \times n}$, $B = [(T_{ij}^B, I_{ij}^B, F_{ij}^B)] \in NSM_{m \times n}$ then $C = [(T_{ij}^C, I_{ij}^C, F_{ij}^C)] \in NSM_{m \times n}$. Then the addition of A and B as

$$A + B = C = \left(\max(T_{ij}^A, T_{ij}^B), \frac{I_{ij}^A + I_{ij}^B}{2}, \min(F_{ij}^A, F_{ij}^B) \right) \text{ for all } i \text{ and } j.$$

2.8 Subtraction of Neutrosophic Soft Matrices

If $A = [(T_{ij}^A, I_{ij}^A, F_{ij}^A)] \in NSM_{m \times n}$, $B = [(T_{ij}^B, I_{ij}^B, F_{ij}^B)] \in NSM_{m \times n}$ then $C = [(T_{ij}^C, I_{ij}^C, F_{ij}^C)] \in NSM_{m \times n}$. Then subtraction of A and B as $A - B = C = (T_{ij}^A - T_{ij}^B, I_{ij}^A - I_{ij}^B, F_{ij}^A - F_{ij}^B)$ for all i and j.

3 Neutrosophic Soft Matrix Application in Agriculture

3.1 Value Matrix

Suppose $A = [(T_{ij}^A, I_{ij}^A, F_{ij}^A)] \in NSM_{m \times n}$ then, A is called the value of NSM denoted by $V(A)$ and is defined by

$$V(A) = [(T_{ij}^A + I_{ij}^A - F_{ij}^A)] \text{ for all } i \text{ and } j, \text{ respectively. Where } i = 1, 2, 3 \dots m \text{ and } j = 1, 2, 3 \dots n.$$

3.2 Score Matrix

If $A = [(T_{ij}^A, I_{ij}^A, F_{ij}^A)] \in NSM_{m \times n}$, $B = [(T_{ij}^B, I_{ij}^B, F_{ij}^B)] \in NSM_{m \times n}$. Then, the score matrix of A and B is denoted by $S_{(A,B)}$ and is defined as $S_{(A,B)} = V(A) - V(B)$.

3.3 Total Score

If $A = [(T_{ij}^A, I_{ij}^A, F_{ij}^A)] \in NSM_{m \times n}$, $B = [(T_{ij}^B, I_{ij}^B, F_{ij}^B)] \in NSM_{m \times n}$. Then, their corresponding value matrix be $V(A), V(B)$ and their score matrix be $S_{(A,B)}$. Then the total score for each u_i in U as

$$S_i = \sum_{j=1}^n (V(A) - V(B))$$

Methodology

Let K is a set of farmers who produces a quality of wheat for better health of human beings to be chosen as the best farmer. This whole process and selection of the farmers will be made by agriculture experts, use of natural resources, fertilizers and pesticides will be taken into account. Suppose D is a set, which consists of parameters relative to the harvested products by farmers. First of all, compute $NFSS(F_A, D)$ over K show the farmers' selection by agricultural experts T , Where F_A is a mapping $F_A: D \rightarrow N(K)$, is the collection of all neutrosophic subsets of K . Now, computation of another $NFSS(G_B, D)$ over K demonstrate farmers' selection by the agriculture experts from another field Z , Where G_B is a mapping $G_B: D \rightarrow N(K)$, is the assortment of all neutrosophic subsets of K . Now develop the matrices A and B relative to the neutrosophic soft sets (F_A, D) and (G_B, D) . Also, compute the complement matrices A° and B° from the complements of neutrosophic soft set $(F_A, D)^\circ$ and set $(G_B, D)^\circ$, respectively. After this, calculate $A + B$, the greater membership value of farmers that will be judged by the experts. Also, calculate $A^\circ + B^\circ$, the maximum membership value of non-selected farmers. Now calculate value matrices $V(A + B)$ and $V(A^\circ + B^\circ)$ and score matrix $S_{((A+B), (A^\circ+B^\circ))}$ and the total score S_i for each farmer in K . At last $S_K = \max(S_i)$ determines that the farmer f_k is selected as the best farmer by the experts.

4. Algorithm

Step 1: Compute the neutrosophic soft set (F_A, D) , (G_B, D) and then find the NSMs A and B corresponding to the (F_A, D) and (G_B, D) respectively.

Step 2: Compute the neutrosophic soft complement sets $(F_A, D)^\circ$, $(G_B, D)^\circ$ and compute the NSMs A° and B° corresponding to the $(F_A, D)^\circ$ and $(G_B, D)^\circ$ respectively.

Step 3: Calculate $(A + B)$, $(A^\circ + B^\circ)$, $V(A + B)$, $V(A^\circ + B^\circ)$ and $S_{((A+B), (A^\circ+B^\circ))}$.

Step 4: Calculate the total score S_i for each f_i in K .

Step 5: Find $S_k = \max(S_i)$, and then conclude the best farmer f_k has the maximum value.

Step 6: If S_k has more than one value, then repeat the step 1 and so repeat the complete process.

5 Application in Decision Making

As traditional mathematical methods are limited for solving problems, so researchers use different techniques for problems involving decision-making. Neutrosophic soft matrix (NSM) is one of those techniques that is used in this paper as this tool is used by many researchers to solve their MCDM (Multi-Criteria Decision Making) problems. Let us try to solve an MCDM problem by using NSM. Suppose (F_A, D) and (G_B, D) are two NSS showing the set of four farmers who are selected from the universal set $K = \{f_1, f_2, f_3, f_4\}$ by the experts T and

Z. Suppose $D = \{e_1, e_2, e_3\}$ be the set of attributes representing the different manures like nature, fertilizer, pesticides, etc. will be considered to choose the best farmer by examining wheat that is better for human health.

Step 1: Construction of Neutrosophic Soft Sets

$$(F_A, D) = \{F_A(e_1), F_A(e_2), F_A(e_3)\}$$

$$F_A(e_1) = \{(f_1, 0.5, 0.2, 0.2), (f_2, 0.4, 0.3, 0.1), (f_3, 0.3, 0.5, 0.2), (f_4, 0.6, 0.2, 0.1)\}$$

$$F_A(e_2) = \{(f_1, 0.2, 0.4, 0.3), (f_2, 0.7, 0.2, 0.1), (f_3, 0.2, 0.5, 0.3), (f_4, 0.4, 0.5, 0.1)\}$$

$$F_A(e_3) = \{(f_1, 0.6, 0.1, 0.2), (f_2, 0.5, 0.3, 0.2), (f_3, 0.3, 0.5, 0.2), (f_4, 0.7, 0.1, 0.2)\}.$$

$$(G_B, D) = \{G_B(e_1), G_B(e_2), G_B(e_3)\}$$

$$G_B(e_1) = \{(f_1, 0.6, 0.3, 0.1), (f_2, 0.4, 0.3, 0.2), (f_3, 0.2, 0.6, 0.1), (f_4, 0.6, 0.2, 0.1)\}$$

$$G_B(e_2) = \{(f_1, 0.5, 0.3, 0.2), (f_2, 0.7, 0.2, 0.1), (f_3, 0.4, 0.3, 0.2), (f_4, 0.6, 0.2, 0.1)\}$$

$$G_B(e_3) = \{(f_1, 0.4, 0.2, 0.3), (f_2, 0.2, 0.5, 0.3), (f_3, 0.6, 0.2, 0.2), (f_4, 0.7, 0.2, 0.1)\}.$$

These are neutrosophic soft matrices of above soft sets:

$$A = \begin{bmatrix} (0.5, 0.2, 0.2) & (0.2, 0.4, 0.3) & (0.6, 0.1, 0.2) \\ (0.4, 0.3, 0.1) & (0.7, 0.2, 0.1) & (0.5, 0.3, 0.2) \\ (0.3, 0.5, 0.2) & (0.2, 0.5, 0.3) & (0.3, 0.5, 0.2) \\ (0.6, 0.2, 0.1) & (0.4, 0.5, 0.1) & (0.7, 0.1, 0.2) \end{bmatrix}$$

$$B = \begin{bmatrix} (0.6, 0.3, 0.1) & (0.5, 0.3, 0.2) & (0.4, 0.2, 0.4) \\ (0.4, 0.3, 0.2) & (0.7, 0.2, 0.1) & (0.2, 0.5, 0.3) \\ (0.2, 0.6, 0.1) & (0.4, 0.3, 0.2) & (0.6, 0.2, 0.2) \\ (0.1, 0.5, 0.2) & (0.6, 0.2, 0.1) & (0.7, 0.2, 0.1) \end{bmatrix}$$

Step 2

Then, the neutrosophic soft complement matrices are

$$A^\circ = \begin{bmatrix} (0.2, 0.8, 0.5) & (0.3, 0.6, 0.2) & (0.2, 0.9, 0.6) \\ (0.1, 0.7, 0.4) & (0.1, 0.8, 0.7) & (0.2, 0.7, 0.5) \\ (0.2, 0.5, 0.3) & (0.3, 0.5, 0.2) & (0.2, 0.5, 0.3) \\ (0.1, 0.8, 0.6) & (0.1, 0.5, 0.4) & (0.2, 0.9, 0.7) \end{bmatrix}$$

$$B^\circ = \begin{bmatrix} (0.1, 0.7, 0.6) & (0.2, 0.7, 0.5) & (0.3, 0.8, 0.4) \\ (0.2, 0.7, 0.4) & (0.1, 0.8, 0.7) & (0.3, 0.5, 0.2) \\ (0.1, 0.4, 0.2) & (0.2, 0.7, 0.4) & (0.2, 0.8, 0.6) \\ (0.2, 0.5, 0.1) & (0.1, 0.8, 0.6) & (0.1, 0.8, 0.7) \end{bmatrix}$$

Step 3: Construction of Value Matrix.

$$(A + B) = \begin{bmatrix} (0.6, 0.25, 0.1) & (0.5, 0.35, 0.2) & (0.6, 0.15, 0.2) \\ (0.4, 0.3, 0.1) & (0.7, 0.2, 0.1) & (0.5, 0.4, 0.2) \\ (0.3, 0.55, 0.1) & (0.4, 0.4, 0.2) & (0.6, 0.35, 0.2) \\ (0.6, 0.35, 0.1) & (0.6, 0.35, 0.1) & (0.7, 0.15, 0.1) \end{bmatrix}$$

$$(A^\circ + B^\circ) = \begin{bmatrix} (0.2, 0.75, 0.5) & (0.3, 0.65, 0.2) & (0.3, 0.85, 0.4) \\ (0.2, 0.7, 0.4) & (0.1, 0.8, 0.7) & (0.3, 0.6, 0.2) \\ (0.2, 0.6, 0.2) & (0.3, 0.6, 0.2) & (0.2, 0.65, 0.3) \\ (0.2, 0.65, 0.1) & (0.1, 0.65, 0.4) & (0.2, 0.85, 0.71) \end{bmatrix}$$

$$V(A + B) = \begin{bmatrix} 0.75 & 0.65 & 0.55 \\ 0.6 & 0.8 & 0.7 \\ 0.75 & 0.6 & 0.75 \\ 0.85 & 0.85 & 0.75 \end{bmatrix}$$

$$V(A^\circ + B^\circ) = \begin{bmatrix} 0.45 & 0.75 & 0.75 \\ 0.5 & 0.2 & 0.7 \\ 0.6 & 0.7 & 0.55 \\ 0.75 & 0.35 & 0.35 \end{bmatrix}$$

Now calculate score matrix as

$$S_{((A+B), (A^\circ+B^\circ))} = \begin{bmatrix} 0.3 & -0.1 & -0.2 \\ 0.1 & 0.6 & 0 \\ 0.15 & -0.1 & 0.2 \\ 0.1 & 0.5 & 0.4 \end{bmatrix}$$

Step 4: To find the Total Score

$$\text{Total Score : } \begin{bmatrix} 0.0 \\ 0.7 \\ 0.25 \\ 1.0 \end{bmatrix}$$

Step 5: Best Selection using highest score

$$S_k = 1.0$$

As we can see above the last value is maximum so the farmer h_4 has gain more score so the farmer h_4 is selected as best farmer by the experts.

6. Conclusion

In this paper, the concept of neutrosophic soft matrices has been described, and the application of some new operations has been tested through neutrosophic soft matrices. A possible application has been tackled through the usage of NSM, which will not only prove useful by itself but will help researchers to solve other problems of uncertainties through similar procedures. The following paper demonstrated a new solution procedure to solve neutrosophic soft sets based on real-life decision-making problems. This procedure proves quite feasible in many real-life scenarios where ease of decision-making is the goal in mind.

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Image and Inverse Image of Neutrosophic Cubic Sets in UP-Algebras under UP-Homomorphisms

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Abstract

The concept of a neutrosophic cubic set in a UP-algebra was introduced by Songsaeng and Iampan [Neutrosophic cubic set theory applied to UP-algebras, 2019]. In this paper, we define the image and inverse image of a neutrosophic cubic set in a non-empty set under any function and study the image and inverse image of a neutrosophic cubic UP-subalgebra (resp., neutrosophic cubic near UP-filter, neutrosophic cubic UP-filter, neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal) of a UP-algebra under some UP-homomorphisms.

Keywords: UP-algebra, UP-homomorphism, neutrosophic cubic UP-subalgebra, neutrosophic cubic near UP-filter, neutrosophic cubic UP-filter, neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal

1 Introduction

The type of the logical algebra, a UP-algebra was introduced by Iampan.^[7] Later Somjanta et al.^[27] studied a fuzzy UP-subalgebra, a fuzzy UP-ideal and a fuzzy UP-filter of a UP-algebra. Guntasow et al.^[5] studied a fuzzy translation of a fuzzy set in a UP-algebra. Kesorn et al.^[17] studied an intuitionistic fuzzy set in a UP-algebra. Kaijjae et al.^[16] studied anti-fuzzy UP-ideals and anti-fuzzy UP-subalgebras. Tanamoon et al.^[36] studied a Q -fuzzy set in a UP-algebra. Sripaeng et al.^[34] studied an anti Q -fuzzy UP-ideal and an anti Q -fuzzy UP-subalgebra of a UP-algebra. Dokkhamdang et al.^[4] studied a generalized fuzzy set in a UP-algebra. Songsaeng and Iampan^{[28][29]} studied an \mathcal{N} -fuzzy UP-algebra and a fuzzy proper UP-filter of a UP-algebra. Senapati et al.^{[24][25]} studies a cubic set and an interval-valued intuitionistic fuzzy structure in a UP-algebra.

A fuzzy set f in a non-empty set A is a function from A to the closed interval $[0, 1]$. The concept of a fuzzy set in a non-empty set was first introduced by Zadeh.^[38] The fuzzy set theory developed by Zadeh and others have found many applications in the domain of mathematics and other domains. Zadeh^[39] introduced an interval-value fuzzy sets. The concept of a neutrosophic set was introduced by Smarandache^[26] in 1999. Wang et al.^[37] introduced the concept of an interval-valued neutrosophic set in 2005. Jun et al.^[13] introduced the concept of an interval-valued neutrosophic set in a BCK/BCI-algebra. The concept of a neutrosophic \mathcal{N} -structure in a semigroup was introduced by Khan et al.^[19] in 2017. Jun et al.^[14] applied the concept of a neutrosophic \mathcal{N} -structure to a BCK/BCI-algebra in 2017. Songsaeng and Iampan^{[31][33]} applied the concept of a neutrosophic set to a UP-algebra. Ibrahim et. al.^[10] introduced the concept of a neutrosophic subtraction algebra and a neutrosophic subtraction semigroup, and Al-Tahan and Davvaz^[1] introduced the concept of a neutrosophic \mathcal{N} -ideal of a subtraction algebra in 2020.

A neutrosophic cubic set which is the generalized form of fuzzy sets, cubic sets and neutrosophic sets was introduced by Jun et al.^[15] in 2017. Iqbal et al.^[11] introduced the concept of a neutrosophic cubic subalgebra and a neutrosophic cubic closed ideal of a B-algebra in 2016. Songsaeng and Iampan^[30] introduced the concept of a neutrosophic cubic set in a UP-algebra in 2020. Khalid et. al.^[18] applied the concept of a multiplicative interpretation of a neutrosophic cubic set to a B-algebra in 2020.

From literature review, we will study the image and inverse image of neutrosophic cubic UP-subalgebras (resp., neutrosophic cubic near UP-filters, neutrosophic cubic UP-filters, neutrosophic cubic UP-ideals, neutrosophic cubic strong UP-ideals) under some UP-homomorphisms.

2 Basic concepts and preliminary notes on a UP-algebra

Before the study, we will review the definition of a UP-algebra.

Definition 2.1. ^[7] An algebra $X = (X, \circ, 0)$ of type $(2, 0)$ is said to be a *UP-algebra*, where X is a non-empty set, \circ is a binary operation on X , and 0 is a fixed element of X if it holds the followings:

(UP-1) (for all $x, y, z \in X$) $((y \circ z) \circ ((x \circ y) \circ (x \circ z))) = 0$,

(UP-2) (for all $x \in X$) $(0 \circ x = x)$,

(UP-3) (for all $x \in X$) $(x \circ 0 = 0)$, and

(UP-4) (for all $x, y \in X$) $(x \circ y = 0, y \circ x = 0 \Rightarrow x = y)$.

From ^[7] we already know that the concept of a UP-algebra is a generalization of a KU-algebra (see ^[21]).

Example 2.2. ^[23] Let Y be a universal set and let $\Omega \in \mathcal{P}(Y)$, where $\mathcal{P}(Y)$ means the power set of Y . Let $\mathcal{P}_\Omega(Y) = \{A \in \mathcal{P}(Y) \mid \Omega \subseteq A\}$. Define a binary operation \circ on $\mathcal{P}_\Omega(Y)$ by putting $A \circ B = B \cap (A^C \cup \Omega)$ for all $A, B \in \mathcal{P}_\Omega(Y)$, where A^C means the complement of a subset A . Then $(\mathcal{P}_\Omega(Y), \circ, \Omega)$ is a UP-algebra. Let $\mathcal{P}^\Omega(Y) = \{A \in \mathcal{P}(Y) \mid A \subseteq \Omega\}$. Define a binary operation \bullet on $\mathcal{P}^\Omega(Y)$ by putting $A \bullet B = B \cup (A^C \cap \Omega)$ for all $A, B \in \mathcal{P}^\Omega(Y)$. Then $(\mathcal{P}^\Omega(Y), \bullet, \Omega)$ is a UP-algebra. In particular, $(\mathcal{P}(Y), \circ, \emptyset)$ and $(\mathcal{P}(Y), \bullet, X)$ are UP-algebras.

Example 2.3. ^[4] Let \mathbb{N}_0 be the set of all natural numbers with zero. Define two binary operations \cdot and $*$ on \mathbb{N}_0 by

$$(\text{for all } m, n \in \mathbb{N}_0) \left(m \cdot n = \begin{cases} n & \text{if } m < n, \\ 0 & \text{otherwise} \end{cases} \right)$$

and

$$(\text{for all } m, n \in \mathbb{N}_0) \left(m * n = \begin{cases} n & \text{if } m > n \text{ or } m = 0, \\ 0 & \text{otherwise} \end{cases} \right).$$

Then $(\mathbb{N}_0, \cdot, 0)$ and $(\mathbb{N}_0, *, 0)$ are UP-algebras.

For more examples of a UP-algebra, see ^[2,3,8,22,25]

In a UP-algebra $X = (X, \circ, 0)$, the followings are valid (see ^[7,8]).

$$(\text{for all } x \in X) (x \circ x = 0), \quad (2.1)$$

$$(\text{for all } x, y, z \in X) (x \circ y = 0, y \circ z = 0 \Rightarrow x \circ z = 0), \quad (2.2)$$

$$(\text{for all } x, y, z \in X) (x \circ y = 0 \Rightarrow (z \circ x) \circ (z \circ y) = 0), \quad (2.3)$$

$$(\text{for all } x, y, z \in X) (x \circ y = 0 \Rightarrow (y \circ z) \circ (x \circ z) = 0), \quad (2.4)$$

$$(\text{for all } x, y \in X) (x \circ (y \circ x) = 0), \quad (2.5)$$

$$(\text{for all } x, y \in X) ((y \circ x) \circ x = 0 \Leftrightarrow x = y \circ x), \quad (2.6)$$

$$(\text{for all } x, y \in X) (x \circ (y \circ y) = 0), \quad (2.7)$$

$$(\text{for all } a, x, y, z \in X) ((x \circ (y \circ z)) \circ (x \circ ((a \circ y) \circ (a \circ z)))) = 0, \quad (2.8)$$

$$(\text{for all } a, x, y, z \in X) (((a \circ x) \circ (a \circ y)) \circ z) \circ ((x \circ y) \circ z) = 0, \quad (2.9)$$

$$(\text{for all } x, y, z \in X) (((x \circ y) \circ z) \circ (y \circ z) = 0), \quad (2.10)$$

$$(\text{for all } x, y, z \in X) (x \circ y = 0 \Rightarrow x \circ (z \circ y) = 0), \quad (2.11)$$

$$(\text{for all } x, y, z \in X) (((x \circ y) \circ z) \circ (x \circ (y \circ z))) = 0, \text{ and} \quad (2.12)$$

$$(\text{for all } a, x, y, z \in X) (((x \circ y) \circ z) \circ (y \circ (a \circ z))) = 0. \quad (2.13)$$

From ^[7] the binary relation \leq on a UP-algebra $X = (X, \circ, 0)$ is defined as follows:

$$(\text{for all } x, y \in X) (x \leq y \Leftrightarrow x \circ y = 0).$$

In a UP-algebra, 5 types of special subsets are defined as follows.

Definition 2.4. [5][7][27] A non-empty subset A of a UP-algebra $X = (X, \circ, 0)$ is said to be

- (1) a *UP-subalgebra* of X if (for all $x, y \in A$)($x \circ y \in A$).
- (2) a *near UP-filter* of X if
 - (i) the constant 0 of X is in A , and
 - (ii) (for all $x, y \in X$)($y \in A \Rightarrow x \circ y \in A$).
- (3) a *UP-filter* of X if
 - (i) the constant 0 of X is in A , and
 - (ii) (for all $x, y \in X$)($x \circ y \in A, x \in A \Rightarrow y \in A$).
- (4) a *UP-ideal* of X if
 - (i) the constant 0 of X is in A , and
 - (ii) (for all $x, y, z \in X$)($x \circ (y \circ z) \in A, y \in A \Rightarrow x \circ z \in A$).
- (5) a *strong UP-ideal* of X if
 - (i) the constant 0 of X is in A , and
 - (ii) (for all $x, y, z \in X$)($(z \circ y) \circ (z \circ x) \in A, y \in A \Rightarrow x \in A$).

Guntasow et al. [5] and Iampar [6] proved that the concept of a UP-subalgebra is a generalization of a near UP-filter, a near UP-filter is a generalization of a UP-filter, a UP-filter is a generalization of a UP-ideal, and a UP-ideal is a generalization of a strong UP-ideal. Moreover, they proved that the only strong UP-ideal of a UP-algebra X is X .

Definition 2.5. [7] Let $(X, \circ, 0_X)$ and $(Y, \bullet, 0_Y)$ be two UP-algebras. A function f from X to Y is said to be a *UP-homomorphism* if

$$(\text{for all } x, y \in X)(f(x \circ y) = f(x) \bullet f(y)).$$

A UP-homomorphism $f: X \rightarrow Y$ is said to be a *UP-epimorphism* if f is surjective, a *UP-monomorphism* if f is injective, and a *UP-isomorphism* if f is bijective.

Theorem 2.6. [9] Let X and Y be two UP-algebras with fixed elements of 0_X and 0_Y , respectively, and let $f: X \rightarrow Y$ be a UP-homomorphism. Then the followings hold:

- (1) $f(0_X) = 0_Y$, and
- (2) (for all $x_1, x_2 \in X$)($x_1 \leq x_2 \Rightarrow f(x_1) \leq f(x_2)$).

In 1965, the concept of a fuzzy set in a non-empty set was introduced by Zadeh [38] with the following definition.

Definition 2.7. A *fuzzy set* (briefly, FS) in a non-empty set X (or a fuzzy subset of X) is defined to be a function $\lambda: X \rightarrow [0, 1]$, where $[0, 1]$ is the unit segment of the real line. Denote by $[0, 1]^X$ the collection of all FSs in X . Define a binary relation \leq on $[0, 1]^X$ as follows:

$$(\text{for all } \lambda, \mu \in [0, 1]^X)(\lambda \leq \mu \Leftrightarrow (\text{for all } x \in X)(\lambda(x) \leq \mu(x))). \quad (2.14)$$

Definition 2.8. [27] Let λ be a FS in a non-empty set X . The *complement* of λ , denoted by λ^C , is defined by

$$(\text{for all } x \in X)(\lambda^C(x) = 1 - \lambda(x)). \quad (2.15)$$

Definition 2.9. [20] Let $\{\lambda_j \mid j \in J\}$ be a family of FSs in a non-empty set X . We define the *join* and the *meet* of $\{\lambda_j \mid j \in J\}$, denoted by $\bigvee_{j \in J} \lambda_j$ and $\bigwedge_{j \in J} \lambda_j$, respectively, as follows:

$$(\text{for all } x \in X)((\bigvee_{j \in J} \lambda_j)(x) = \sup_{j \in J} \{\lambda_j(x)\}), \quad (2.16)$$

$$(\text{for all } x \in X)((\bigwedge_{j \in J} \lambda_j)(x) = \inf_{j \in J} \{\lambda_j(x)\}). \quad (2.17)$$

In particular, if λ and μ be FSs in X , we have the join and meet of λ and μ as follows:

$$(\text{for all } x \in X)((\lambda \vee \mu)(x) = \max\{\lambda(x), \mu(x)\}), \quad (2.18)$$

$$(\text{for all } x \in X)((\lambda \wedge \mu)(x) = \min\{\lambda(x), \mu(x)\}), \quad (2.19)$$

respectively.

An *interval number* we mean a close subinterval $\hat{a} = [a^-, a^+]$ of $[0, 1]$, where $0 \leq a^- \leq a^+ \leq 1$. The interval number $\hat{a} = [a^-, a^+]$ with $a^- = a^+$ is denoted by \mathbf{a} . Denote by $\text{int}[0, 1]$ the set of all interval numbers.

Definition 2.10. ^[15] Let $\{\hat{a}_j \mid j \in J\}$ be a family of interval numbers. We define the *refined infimum* and the *refined supremum* of $\{\hat{a}_j \mid j \in J\}$, denoted by $\text{rinf}_{j \in J} \hat{a}_j$ and $\text{rsup}_{j \in J} \hat{a}_j$, respectively, as follows:

$$\text{rinf}_{j \in J} \{\hat{a}_j\} = [\inf_{j \in J} \{a_j^-\}, \inf_{j \in J} \{a_j^+\}], \quad (2.20)$$

$$\text{rsup}_{j \in J} \{\hat{a}_j\} = [\sup_{j \in J} \{a_j^-\}, \sup_{j \in J} \{a_j^+\}]. \quad (2.21)$$

In particular, if $\hat{a}_1, \hat{a}_2 \in \text{int}[0, 1]$, we define the *refined minimum* and the *refined maximum* of \hat{a}_1 and \hat{a}_2 , denoted by $\text{rmin}\{\hat{a}_1, \hat{a}_2\}$ and $\text{rmax}\{\hat{a}_1, \hat{a}_2\}$, respectively, as follows:

$$\text{rmin}\{\hat{a}_1, \hat{a}_2\} = [\min\{a_1^-, a_2^-\}, \min\{a_1^+, a_2^+\}], \quad (2.22)$$

$$\text{rmax}\{\hat{a}_1, \hat{a}_2\} = [\max\{a_1^-, a_2^-\}, \max\{a_1^+, a_2^+\}]. \quad (2.23)$$

Definition 2.11. ^[15] Let $\hat{a}_1, \hat{a}_2 \in \text{int}[0, 1]$. We define the symbols “ \succeq ”, “ \preceq ”, “ $=$ ” in case of \hat{a}_1 and \hat{a}_2 as follows:

$$\hat{a}_1 \succeq \hat{a}_2 \Leftrightarrow a_1^- \geq a_2^- \text{ and } a_1^+ \geq a_2^+, \quad (2.24)$$

and similarly we may have $\hat{a}_1 \preceq \hat{a}_2$ and $\hat{a}_1 = \hat{a}_2$. To say $\hat{a}_1 \succ \hat{a}_2$ (resp., $\hat{a}_1 \prec \hat{a}_2$) we mean $\hat{a}_1 \succeq \hat{a}_2$ and $\hat{a}_1 \neq \hat{a}_2$ (resp., $\hat{a}_1 \preceq \hat{a}_2$ and $\hat{a}_1 \neq \hat{a}_2$).

Definition 2.12. ^[39] Let $\hat{a} \in \text{int}[0, 1]$. The *complement* of \hat{a} , denoted by \hat{a}^C , is defined by the interval number

$$\hat{a}^C = [1 - a^+, 1 - a^-]. \quad (2.25)$$

In the $\text{int}[0, 1]$, the followings are valid (see ^[35]).

$$(\text{for all } \hat{a} \in \text{int}[0, 1])(\hat{a} \succeq \hat{a}), \quad (2.26)$$

$$(\text{for all } \hat{a} \in \text{int}[0, 1])(\hat{a}^C)^C = \hat{a}, \quad (2.27)$$

$$(\text{for all } \hat{a} \in \text{int}[0, 1])(\text{rmax}\{\hat{a}, \hat{a}\} = \hat{a} \text{ and } \text{rmin}\{\hat{a}, \hat{a}\} = \hat{a}), \quad (2.28)$$

$$(\text{for all } \hat{a}_1, \hat{a}_2 \in \text{int}[0, 1])(\text{rmax}\{\hat{a}_1, \hat{a}_2\} = \text{rmax}\{\hat{a}_2, \hat{a}_1\} \text{ and } \text{rmin}\{\hat{a}_1, \hat{a}_2\} = \text{rmin}\{\hat{a}_2, \hat{a}_1\}), \quad (2.29)$$

$$(\text{for all } \hat{a}_1, \hat{a}_2 \in \text{int}[0, 1])(\text{rmax}\{\hat{a}_1, \hat{a}_2\} \succeq \hat{a}_1 \text{ and } \hat{a}_2 \succeq \text{rmin}\{\hat{a}_1, \hat{a}_2\}), \quad (2.30)$$

$$(\text{for all } \hat{a}_1, \hat{a}_2 \in \text{int}[0, 1])(\hat{a}_1 \succeq \hat{a}_2 \Leftrightarrow \hat{a}_1^C \preceq \hat{a}_2^C), \quad (2.31)$$

$$(\text{for all } \hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4 \in \text{int}[0, 1])(\hat{a}_1 \succeq \hat{a}_2, \hat{a}_3 \succeq \hat{a}_4 \Rightarrow \text{rmin}\{\hat{a}_1, \hat{a}_3\} \succeq \text{rmin}\{\hat{a}_2, \hat{a}_4\}), \quad (2.32)$$

$$(\text{for all } \hat{a}_1, \hat{a}_2, \hat{a}_3 \in \text{int}[0, 1])(\hat{a}_1 \succeq \hat{a}_2, \hat{a}_3 \succeq \hat{a}_2 \Leftrightarrow \text{rmin}\{\hat{a}_1, \hat{a}_3\} \succeq \hat{a}_2), \quad (2.33)$$

$$(\text{for all } \hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4 \in \text{int}[0, 1])(\hat{a}_1 \succeq \hat{a}_2, \hat{a}_3 \succeq \hat{a}_4 \Rightarrow \text{rmax}\{\hat{a}_1, \hat{a}_3\} \succeq \text{rmax}\{\hat{a}_2, \hat{a}_4\}), \quad (2.34)$$

$$(\text{for all } \hat{a}_1, \hat{a}_2, \hat{a}_3 \in \text{int}[0, 1])(\hat{a}_2 \succeq \hat{a}_1, \hat{a}_2 \succeq \hat{a}_3 \Leftrightarrow \hat{a}_2 \succeq \text{rmax}\{\hat{a}_1, \hat{a}_3\}), \quad (2.35)$$

$$(\text{for all } \hat{a}_1, \hat{a}_2 \in \text{int}[0, 1])(\hat{a}_1 \succeq \hat{a}_2 \Leftrightarrow \text{rmin}\{\hat{a}_1, \hat{a}_2\} = \hat{a}_2), \quad (2.36)$$

$$(\text{for all } \hat{a}_1, \hat{a}_2 \in \text{int}[0, 1])(\hat{a}_1 \succeq \hat{a}_2 \Leftrightarrow \text{rmax}\{\hat{a}_1, \hat{a}_2\} = \hat{a}_1), \quad (2.37)$$

$$(\text{for all } \hat{a}_1, \hat{a}_2 \in \text{int}[0, 1])(\text{rmin}\{\hat{a}_1^C, \hat{a}_2^C\} = \text{rmax}\{\hat{a}_1, \hat{a}_2\}^C), \quad (2.38)$$

$$(\text{for all } \hat{a}_1, \hat{a}_2 \in \text{int}[0, 1])(\text{rmax}\{\hat{a}_1^C, \hat{a}_2^C\} = \text{rmin}\{\hat{a}_1, \hat{a}_2\}^C), \quad (2.39)$$

$$(\text{for all } \hat{a}_1, \hat{a}_2, \hat{a}_3 \in \text{int}[0, 1])(\hat{a}_1 \preceq \text{rmax}\{\hat{a}_2, \hat{a}_3\} \Leftrightarrow \hat{a}_1^C \succeq \text{rmin}\{\hat{a}_2^C, \hat{a}_3^C\}), \quad (2.40)$$

$$(\text{for all } \hat{a}_1, \hat{a}_2, \hat{a}_3 \in \text{int}[0, 1])(\hat{a}_1 \succeq \text{rmax}\{\hat{a}_2, \hat{a}_3\} \Leftrightarrow \hat{a}_1^C \preceq \text{rmin}\{\hat{a}_2^C, \hat{a}_3^C\}), \quad (2.41)$$

$$(\text{for all } \hat{a}_1, \hat{a}_2, \hat{a}_3 \in \text{int}[0, 1])(\hat{a}_1 \preceq \text{rmin}\{\hat{a}_2, \hat{a}_3\} \Leftrightarrow \hat{a}_1^C \succeq \text{rmax}\{\hat{a}_2^C, \hat{a}_3^C\}), \text{ and} \quad (2.42)$$

$$(\text{for all } \hat{a}_1, \hat{a}_2, \hat{a}_3 \in \text{int}[0, 1])(\hat{a}_1 \succeq \text{rmin}\{\hat{a}_2, \hat{a}_3\} \Leftrightarrow \hat{a}_1^C \preceq \text{rmax}\{\hat{a}_2^C, \hat{a}_3^C\}). \quad (2.43)$$

In 1975, the concept of an interval-valued fuzzy set in a non-empty set was first introduced by Zadeh ^[38] with the following definition.

Definition 2.13. An *interval-valued fuzzy set* (briefly, IVFS) in a non-empty set X is an arbitrary function $A : X \rightarrow \text{int}[0, 1]$. Let $IVFS(X)$ stands for the set of all IVFS in X . For every $A \in IVFS(X)$ and $x \in X$, $A(x) = [A^-(x), A^+(x)]$ is said to be the *degree of membership* of an element x to A , where A^-, A^+ are FSs in X which are called a *lower fuzzy set* and an *upper fuzzy set* in X , respectively. For simplicity, we denote $A = [A^-, A^+]$.

Definition 2.14. ^[15] Let A and B be IVFSs in a non-empty set X . We define the symbols “ \subseteq ”, “ \supseteq ”, “ $=$ ” in case of A and B as follows:

$$A \subseteq B \Leftrightarrow (\text{for all } x \in X)(A(x) \preceq B(x)), \quad (2.44)$$

and similarly we may have $A \supseteq B$ and $A = B$.

Definition 2.15. ^[39] Let A be an IVFS in a non-empty set X . The *complement* of A , denoted by A^C , is defined as follows: $A^C(x) = A(x)^C$ for all $x \in X$, that is,

$$(\text{for all } x \in X)(A^C(x) = [1 - A^+(x), 1 - A^-(x)]). \quad (2.45)$$

We note that $A^{C^-}(x) = 1 - A^+(x)$ and $A^{C^+}(x) = 1 - A^-(x)$ for all $x \in X$.

Definition 2.16. ^[39] Let $\{A_j \mid j \in J\}$ be a family of IVFSs in a non-empty set X . We define the *intersection* and the *union* of $\{A_j \mid j \in J\}$, denoted by $\cap_{j \in J} A_j$ and $\cup_{j \in J} A_j$, respectively, as follows:

$$(\text{for all } x \in X)((\cap_{j \in J} A_j)(x) = \text{rinf}_{j \in J} \{A_j(x)\}), \quad (2.46)$$

$$(\text{for all } x \in X)((\cup_{j \in J} A_j)(x) = \text{rsup}_{j \in J} \{A_j(x)\}). \quad (2.47)$$

We note that

$$(\text{for all } x \in X)((\cap_{j \in J} A_j)^-(x) = (\wedge_{j \in J} A_j^-(x)) = \inf_{j \in J} \{A_j^-(x)\})$$

and

$$(\text{for all } x \in X)((\cap_{j \in J} A_j)^+(x) = (\wedge_{j \in J} A_j^+(x)) = \inf_{j \in J} \{A_j^+(x)\}).$$

Similarly,

$$(\text{for all } x \in X)((\cup_{j \in J} A_j)^-(x) = (\vee_{j \in J} A_j^-(x)) = \sup_{j \in J} \{A_j^-(x)\})$$

and

$$(\text{for all } x \in X)((\cup_{j \in J} A_j)^+(x) = (\vee_{j \in J} A_j^+(x)) = \sup_{j \in J} \{A_j^+(x)\}).$$

In particular, if A_1 and A_2 are IVFSs in X , we have the intersection and the union of A_1 and A_2 as follows:

$$(\text{for all } x \in X)((A_1 \cap A_2)(x) = \text{rmin}\{A_1(x), A_2(x)\}), \quad (2.48)$$

$$(\text{for all } x \in X)((A_1 \cup A_2)(x) = \text{rmax}\{A_1(x), A_2(x)\}). \quad (2.49)$$

In 1999, the concept of a neutrosophic set in a non-empty set was introduced by Smarandache ^[26] with the following definition.

Definition 2.17. A *neutrosophic set* (briefly, NS) in a non-empty set X is a structure of the form:

$$\Lambda = \{(x, \lambda_T(x), \lambda_I(x), \lambda_F(x)) \mid x \in X\}, \quad (2.50)$$

where $\lambda_T : X \rightarrow [0, 1]$ is a *truth membership function*, $\lambda_I : X \rightarrow [0, 1]$ is an *indeterminate membership function*, and $\lambda_F : X \rightarrow [0, 1]$ is a *false membership function*. For our convenience, we will denote a NS as $\Lambda = (X, \lambda_T, \lambda_I, \lambda_F) = (X, \lambda_{T,I,F}) = \{(x, \lambda_T(x), \lambda_I(x), \lambda_F(x)) \mid x \in X\}$.

Definition 2.18. ^[26] Let Λ be a NS in a non-empty set X . The NS $\Lambda^C = (X, \lambda_T^C, \lambda_I^C, \lambda_F^C)$ in X is said to be the *complement* of Λ in X .

In 2005, the concept of an interval neutrosophic set in a non-empty set was introduced by Wang et al. ^[37] with the following definition.

Definition 2.19. An *interval-valued neutrosophic set* (briefly, IVNS) in a non-empty set X is a structure of the form:

$$\mathbf{A} := \{(x, A_T(x), A_I(x), A_F(x)) \mid x \in X\}, \quad (2.51)$$

where A_T , A_I and A_F are IVFSs in X , which are called an *interval truth membership function*, an *interval indeterminacy membership function* and an *interval falsity membership function*, respectively. For our convenience, we will denote a IVNS as $\mathbf{A} = (X, A_T, A_I, A_F) = (X, A_{T,I,F}) = \{(x, A_T(x), A_I(x), A_F(x)) \mid x \in X\}$.

Definition 2.20. ^[37] Let $\mathbf{A} = (X, A_T, A_I, A_F)$ be an IVNS in a non-empty set X . The IVNS $\mathbf{A}^C = (X, A_T^C, A_I^C, A_F^C)$ in X is said to be the *complement* of \mathbf{A} in X .

In 2012, the concept of a cubic set in a non-empty set was introduced by Jun et al.^[12] with the following definition.

Definition 2.21. A *cubic set* (briefly, CS) in a non-empty set X is a structure of the form:

$$\mathbf{C} = \{(x, A(x), \lambda(x)) \mid x \in X\}, \quad (2.52)$$

where A is an IVFS in X and λ is a FS in X . For our convenience, we will denote a CS as $\mathbf{C} = (X, A, \lambda) = \{(x, A(x), \lambda(x)) \mid x \in X\}$.

In 2017, Jun et al.^[15] introduced the concept of a neutrosophic cubic set with the following definition.

Definition 2.22. A neutrosophic cubic set (briefly, NCS) in a non-empty set X is a pair $\mathcal{A} = (\mathbf{A}, \Lambda)$, where $\mathbf{A} = (X, A_T, A_I, A_F)$ is an IVNS in X and $\Lambda = (X, \lambda_T, \lambda_I, \lambda_F)$ is a neutrosophic set in X . For simplicity, we denote $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$. A NCS $\mathcal{A} = (\mathbf{A}, \Lambda)$ in a non-empty set X is said to be *constant* if $A_T, A_I, A_F, \lambda_T, \lambda_I$, and λ_F are constant functions. The complement of a NCS $\mathcal{A} = (\mathbf{A}, \Lambda)$ is defined to be the NCS $\mathcal{A}^C = (\mathbf{A}^C, \Lambda^C)$.

In 2020, the concepts of a neutrosophic cubic UP-subalgebra, a neutrosophic cubic near UP-filter, a neutrosophic cubic UP-filter, a neutrosophic cubic UP-ideal, and a neutrosophic cubic strong UP-ideal of a UP-algebra were introduced by Songsaeng and Iampan^[30] with the following definition.

Definition 2.23. A NCS $\mathcal{A} = (\mathbf{A}, \Lambda)$ in a UP-algebra $X = (X, \circ, 0)$ is said to be

(1) a *neutrosophic cubic UP-subalgebra* of X if

$$(\text{for all } x, y \in X) \begin{pmatrix} A_T(x \circ y) \succeq \text{rmin}\{A_T(x), A_T(y)\} \\ A_I(x \circ y) \preceq \text{rmax}\{A_I(x), A_I(y)\} \\ A_F(x \circ y) \succeq \text{rmin}\{A_F(x), A_F(y)\} \end{pmatrix}, \quad (2.53)$$

$$(\text{for all } x, y \in X) \begin{pmatrix} \lambda_T(x \circ y) \leq \max\{\lambda_T(x), \lambda_T(y)\} \\ \lambda_I(x \circ y) \geq \min\{\lambda_I(x), \lambda_I(y)\} \\ \lambda_F(x \circ y) \leq \max\{\lambda_F(x), \lambda_F(y)\} \end{pmatrix}. \quad (2.54)$$

(2) a *neutrosophic cubic near UP-filter* of X if

$$(\text{for all } x \in X) \begin{pmatrix} A_T(0) \succeq A_T(x) \\ A_I(0) \preceq A_I(x) \\ A_F(0) \succeq A_F(x) \end{pmatrix}, \quad (2.55)$$

$$(\text{for all } x \in X) \begin{pmatrix} \lambda_T(0) \leq \lambda_T(x) \\ \lambda_I(0) \geq \lambda_I(x) \\ \lambda_F(0) \leq \lambda_F(x) \end{pmatrix}, \quad (2.56)$$

$$(\text{for all } x, y \in X) \begin{pmatrix} A_T(x \circ y) \succeq A_T(y) \\ A_I(x \circ y) \preceq A_I(y) \\ A_F(x \circ y) \succeq A_F(y) \end{pmatrix}, \quad (2.57)$$

$$(\text{for all } x, y \in X) \begin{pmatrix} \lambda_T(x \circ y) \leq \lambda_T(y) \\ \lambda_I(x \circ y) \geq \lambda_I(y) \\ \lambda_F(x \circ y) \leq \lambda_F(y) \end{pmatrix}. \quad (2.58)$$

(3) a *neutrosophic cubic UP-filter* of X if it holds the followings: (2.55), (2.56), and

$$(\text{for all } x, y \in X) \begin{pmatrix} A_T(y) \succeq \text{rmin}\{A_T(x \circ y), A_T(x)\} \\ A_I(y) \preceq \text{rmax}\{A_I(x \circ y), A_I(x)\} \\ A_F(y) \succeq \text{rmin}\{A_F(x \circ y), A_F(x)\} \end{pmatrix}, \quad (2.59)$$

$$(\text{for all } x, y \in X) \begin{pmatrix} \lambda_T(y) \leq \max\{\lambda_T(x \circ y), \lambda_T(x)\} \\ \lambda_I(y) \geq \min\{\lambda_I(x \circ y), \lambda_I(x)\} \\ \lambda_F(y) \leq \max\{\lambda_F(x \circ y), \lambda_F(x)\} \end{pmatrix}. \quad (2.60)$$

(4) a *neutrosophic cubic UP-ideal* of X if it holds the followings: (2.55), (2.56), and

$$(\text{for all } x, y, z \in X) \begin{pmatrix} A_T(x \circ z) \succeq \text{rmin}\{A_T(x \circ (y \circ z)), A_T(y)\} \\ A_I(x \circ z) \preceq \text{rmax}\{A_I(x \circ (y \circ z)), A_I(y)\} \\ A_F(x \circ z) \succeq \text{rmin}\{A_F(x \circ (y \circ z)), A_F(y)\} \end{pmatrix}, \quad (2.61)$$

$$(\text{for all } x, y, z \in X) \begin{pmatrix} \lambda_T(x \circ z) \leq \max\{\lambda_T(x \circ (y \circ z)), \lambda_T(y)\} \\ \lambda_I(x \circ z) \geq \min\{\lambda_I(x \circ (y \circ z)), \lambda_I(y)\} \\ \lambda_F(x \circ z) \leq \max\{\lambda_F(x \circ (y \circ z)), \lambda_F(y)\} \end{pmatrix}. \quad (2.62)$$

(5) a *neutrosophic cubic strong UP-ideal* of X if it holds the followings: (2.55), (2.56), and

$$(\text{for all } x, y, z \in X) \begin{pmatrix} A_T(x) \succeq \text{rmin}\{A_T((z \circ y) \circ (z \circ x)), A_T(y)\} \\ A_I(x) \preceq \text{rmax}\{A_I((z \circ y) \circ (z \circ x)), A_I(y)\} \\ A_F(x) \succeq \text{rmin}\{A_F((z \circ y) \circ (z \circ x)), A_F(y)\} \end{pmatrix}, \quad (2.63)$$

$$(\text{for all } x, y, z \in X) \begin{pmatrix} \lambda_T(x) \leq \max\{\lambda_T((z \circ y) \circ (z \circ x)), \lambda_T(y)\} \\ \lambda_I(x) \geq \min\{\lambda_I((z \circ y) \circ (z \circ x)), \lambda_I(y)\} \\ \lambda_F(x) \leq \max\{\lambda_F((z \circ y) \circ (z \circ x)), \lambda_F(y)\} \end{pmatrix}. \quad (2.64)$$

Songsaeng and Iampan³⁰ proved that the concept of a neutrosophic cubic UP-subalgebra is a generalization of a neutrosophic cubic near UP-filter, a neutrosophic cubic near UP-filter is a generalization of a neutrosophic cubic UP-filter, a neutrosophic cubic UP-filter is a generalization of a neutrosophic cubic UP-ideal, and a neutrosophic cubic UP-ideal is a generalization of a neutrosophic cubic strong UP-ideal. Moreover, they proved that a neutrosophic cubic strong UP-ideal and a constant NCS coincide.

3 Homomorphic properties of a NCSs in a UP-algebra

In this section, the image and inverse image of a NCS are defined and some results are studied.

Definition 3.1. Let f be a function from a non-empty set X into a non-empty set Y and $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ be a NCS in X . Then the image of \mathcal{A} under f is defined as a NCS $f(\mathcal{A}) = (f(A)_{T,I,F}, f(\lambda)_{T,I,F})$ in Y , where

$$f(A)_T(y) = \begin{cases} \text{rsup}_{x \in f^{-1}(y)} \{A_T(x)\} & \text{if } f^{-1}(y) \text{ is non-empty,} \\ [0, 0] & \text{otherwise,} \end{cases}$$

$$f(A)_I(y) = \begin{cases} \text{rinf}_{x \in f^{-1}(y)} \{A_I(x)\} & \text{if } f^{-1}(y) \text{ is non-empty,} \\ [1, 1] & \text{otherwise,} \end{cases}$$

$$f(A)_F(y) = \begin{cases} \text{rsup}_{x \in f^{-1}(y)} \{A_F(x)\} & \text{if } f^{-1}(y) \text{ is non-empty,} \\ [0, 0] & \text{otherwise,} \end{cases}$$

$$f(\lambda)_T(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \{\lambda_T(x)\} & \text{if } f^{-1}(y) \text{ is non-empty,} \\ 1 & \text{otherwise,} \end{cases}$$

$$f(\lambda)_I(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \{\lambda_I(x)\} & \text{if } f^{-1}(y) \text{ is non-empty,} \\ 0 & \text{otherwise,} \end{cases}$$

$$f(\lambda)_F(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \{\lambda_F(x)\} & \text{if } f^{-1}(y) \text{ is non-empty,} \\ 1 & \text{otherwise.} \end{cases}$$

Example 3.2. Let $X = \{0_X, 1_X, 2_X\}$ be a UP-algebra with a fixed element 0_X and a binary operation \circ defined by the following Cayley table:

\circ	0_X	1_X	2_X
0_X	0_X	1_X	2_X
1_X	0_X	0_X	1_X
2_X	0_X	0_X	0_X

and let $Y = \{0_Y, 1_Y, 2_Y\}$ be a UP-algebra with a fixed element 0_Y and a binary operation \bullet defined by the following Cayley table:

\bullet	0_Y	1_Y	2_Y
0_Y	0_Y	1_Y	2_Y
1_Y	0_Y	0_Y	2_Y
2_Y	0_Y	0_Y	0_Y

We define a function $f : X \rightarrow Y$ as follows:

$$f(0_X) = 0_Y, f(1_X) = 1_Y, \text{ and } f(2_X) = 1_Y.$$

We define a NCS $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ in X with the tabular representation as follows:

X	$\mathbf{A}(x)$	$\Lambda(x)$
0_X	$([0.4, 0.7], [0.5, 0.7], [0.2, 0.4])$	$(0.1, 0.3, 0.4)$
1_X	$([0.1, 0.2], [0.1, 0.5], [0.4, 0.5])$	$(0.3, 0.8, 0.4)$
2_X	$([0.8, 0.9], [0.7, 0.8], [0.1, 0.6])$	$(0.1, 0.5, 0.7)$

Then $f(\mathcal{A}) = (f(A)_{T,I,F}, f(\lambda)_{T,I,F})$ in Y with the tabular representation as follows:

Y	$\mathbf{A}(x)$	$\Lambda(x)$
0_Y	$([0.4, 0.7], [0.5, 0.7], [0.2, 0.4])$	$(0.1, 0.3, 0.4)$
1_Y	$([0.8, 0.9], [0.1, 0.5], [0.4, 0.6])$	$(0.1, 0.8, 0.4)$
2_Y	$([0, 0], [1, 1], [0, 0])$	$(1, 0, 1)$

Hence, $f(\mathcal{A}) = (f(A)_{T,I,F}, f(\lambda)_{T,I,F})$ is a NCS in Y .

Definition 3.3. Let f be a function from a non-empty set X into a non-empty set Y and $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ be a NCS in Y . Then the inverse image of \mathcal{A} is defined as a NCS $f^{-1}(\mathcal{A}) = (f^{-1}(A)_{T,I,F}, f^{-1}(\lambda)_{T,I,F})$ in X , where

$$(\text{for all } x \in X)(f^{-1}(A)_{T,I,F}(x) = A_{T,I,F}(f(x))), \quad (3.1)$$

$$(\text{for all } x \in X)(f^{-1}(\lambda)_{T,I,F}(x) = \lambda_{T,I,F}(f(x))). \quad (3.2)$$

Example 3.4. In Example 3.2 we have $(X, \circ, 0_X)$ and $(Y, \bullet, 0_Y)$ are two UP-algebras. We define a function $f : X \rightarrow Y$ as follows:

$$f(0_X) = 0_Y, f(1_X) = 1_Y, \text{ and } f(2_X) = 1_Y.$$

We define a NCS $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ in Y with the tabular representation as follows:

Y	$\mathbf{A}(x)$	$\Lambda(x)$
0_Y	$([0.3, 0.7], [0.3, 0.5], [0.1, 0.4])$	$(0.5, 0.4, 0.7)$
1_Y	$([0.6, 0.7], [0.1, 0.3], [0.4, 0.5])$	$(0.2, 0.7, 0.8)$
2_Y	$([0.5, 0.9], [0.3, 0.5], [0.5, 0.8])$	$(0.3, 0.5, 0.4)$

Then $f^{-1}(\mathcal{A}) = (f^{-1}(A)_{T,I,F}, f^{-1}(\lambda)_{T,I,F})$ in X with the tabular representation as follows:

X	$\mathbf{A}(x)$	$\Lambda(x)$
0_X	$([0.3, 0.7], [0.3, 0.5], [0.1, 0.4])$	$(0.5, 0.4, 0.7)$
1_X	$([0.6, 0.7], [0.1, 0.3], [0.4, 0.5])$	$(0.2, 0.7, 0.8)$
2_X	$([0.6, 0.7], [0.1, 0.3], [0.4, 0.5])$	$(0.2, 0.7, 0.8)$

Hence, $f^{-1}(\mathcal{A}) = (f^{-1}(A)_{T,I,F}, f^{-1}(\lambda)_{T,I,F})$ is a NCS in X .

Definition 3.5. A NCS

$\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ in X is said to be *order preserving* if

$$(\text{for all } x, y \in X) \left(x \leq y \Rightarrow \left\{ \begin{array}{l} A_T(x) \preceq A_T(y), A_I(x) \succeq A_I(y), A_F(x) \preceq A_F(y), \\ \lambda_T(x) \geq \lambda_T(y), \lambda_I(x) \leq \lambda_I(y), \lambda_F(x) \geq \lambda_F(y) \end{array} \right\} \right). \quad (3.3)$$

Lemma 3.6. Every neutrosophic cubic UP-filter (resp., neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal) of X is order preserving.

Proof. Assume that $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ is a neutrosophic cubic UP-filter of X . Let $x, y \in X$ be such that $x \leq y$ in X . Then $x \circ y = 0$. Thus

$$A_T(y) \succeq \text{rmin}\{A_T(x \circ y), A_T(x)\} = \text{rmin}\{A_T(0), A_T(x)\} = A_T(x), \quad ((2.59), (2.55), (2.36))$$

$$A_I(y) \preceq \text{rmax}\{A_I(x \circ y), A_I(x)\} = \text{rmin}\{A_I(0), A_I(x)\} = A_I(x), \quad ((2.59), (2.55), (2.37))$$

$$A_F(y) \succeq \text{rmin}\{A_F(x \circ y), A_F(x)\} = \text{rmin}\{A_F(0), A_F(x)\} = A_F(x), \quad ((2.59), (2.55), (2.36))$$

$$\lambda_T(y) \leq \max\{\lambda_T(x \circ y), \lambda_T(x)\} = \max\{\lambda_T(0), \lambda_T(x)\} = \lambda_T(x), \quad ((2.60), (2.56))$$

$$\lambda_I(y) \geq \min\{\lambda_I(x \circ y), \lambda_I(x)\} = \min\{\lambda_I(0), \lambda_I(x)\} = \lambda_I(x), \quad ((2.60), (2.56))$$

$$\lambda_F(y) \leq \max\{\lambda_F(x \circ y), \lambda_F(x)\} = \max\{\lambda_F(0), \lambda_F(x)\} = \lambda_F(x). \quad ((2.60), (2.56))$$

Hence, \mathcal{A} is order preserving. \square

Theorem 3.7. Let $(X, \circ, 0_X)$ and $(Y, \bullet, 0_Y)$ be two UP-algebras, $f: X \rightarrow Y$ be a UP-homomorphism, and $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ be a NCS in Y . Then the followings hold:

- (1) If \mathcal{A} is a neutrosophic cubic UP-subalgebra of Y , then the inverse image $f^{-1}(\mathcal{A})$ of \mathcal{A} under f is a neutrosophic cubic UP-subalgebra of X .
- (2) If \mathcal{A} is a neutrosophic cubic near UP-filter of Y which is order preserving, then the inverse image $f^{-1}(\mathcal{A})$ of \mathcal{A} under f is a neutrosophic cubic near UP-filter of X .
- (3) If \mathcal{A} is a neutrosophic cubic UP-filter of Y , then the inverse image $f^{-1}(\mathcal{A})$ of \mathcal{A} under f is a neutrosophic cubic UP-filter of X .
- (4) If \mathcal{A} is a neutrosophic cubic UP-ideal of Y , then the inverse image $f^{-1}(\mathcal{A})$ of \mathcal{A} under f is a neutrosophic cubic UP-ideal of X .
- (5) If \mathcal{A} is a neutrosophic cubic strong UP-ideal of Y , then the inverse image $f^{-1}(\mathcal{A})$ of \mathcal{A} under f is a neutrosophic cubic strong UP-ideal of X .

Proof. (1) Assume that \mathcal{A} is a neutrosophic cubic UP-subalgebra of Y . Then for all $x, y \in X$,

$$f^{-1}(A)_T(x \circ y) = A_T(f(x \circ y)) \quad ((3.1))$$

$$= A_T(f(x) \bullet f(y))$$

$$\succeq \text{rmin}\{A_T(f(x)), A_T(f(y))\} \quad ((2.53))$$

$$= \text{rmin}\{f^{-1}(A)_T(x), f^{-1}(A)_T(y)\}, \quad ((3.1))$$

$$f^{-1}(A)_I(x \circ y) = A_I(f(x \circ y)) \quad ((3.1))$$

$$= A_I(f(x) \bullet f(y))$$

$$\preceq \text{rmax}\{A_I(f(x)), A_I(f(y))\} \quad ((2.53))$$

$$= \text{rmax}\{f^{-1}(A)_I(x), f^{-1}(A)_I(y)\}, \quad ((3.1))$$

$$f^{-1}(A)_F(x \circ y) = A_F(f(x \circ y)) \quad ((3.1))$$

$$= A_F(f(x) \bullet f(y))$$

$$\succeq \text{rmin}\{A_F(f(x)), A_F(f(y))\} \quad ((2.53))$$

$$= \text{rmin}\{f^{-1}(A)_F(x), f^{-1}(A)_F(y)\}, \quad ((3.1))$$

$$f^{-1}(\lambda)_T(x \circ y) = \lambda_T(f(x \circ y)) \quad ((3.2))$$

$$= \lambda_T(f(x) \bullet f(y))$$

$$\leq \max\{\lambda_T(f(x)), \lambda_T(f(y))\} \quad ((2.54))$$

$$= \max\{f^{-1}(\lambda)_T(x), f^{-1}(\lambda)_T(y)\}, \quad ((3.2))$$

$$f^{-1}(\lambda)_I(x \circ y) = \lambda_I(f(x \circ y)) \quad ((3.2))$$

$$= \lambda_I(f(x) \bullet f(y))$$

$$\geq \min\{\lambda_I(f(x)), \lambda_I(f(y))\} \quad ((2.54))$$

$$= \min\{f^{-1}(\lambda)_I(x), f^{-1}(\lambda)_I(y)\}, \quad ((3.2))$$

$$f^{-1}(\lambda)_F(x \circ y) = \lambda_F(f(x \circ y)) \quad (3.2)$$

$$= \lambda_F(f(x) \bullet f(y)) \\ \leq \max\{\lambda_F(f(x)), \lambda_F(f(y))\} \quad (2.54)$$

$$= \max\{f^{-1}(\lambda)_F(x), f^{-1}(\lambda)_F(y)\}. \quad (3.2)$$

Hence, $f^{-1}(\mathcal{A})$ is a neutrosophic cubic UP-subalgebra of X .

(2) Assume that \mathcal{A} is a neutrosophic cubic near UP-filter of Y which is order preserving. By Theorem 2.6 (2) and (UP-3), we have for all $x \in X$,

$$\begin{aligned} f^{-1}(A)_T(0_X) &= A_T(f(0_X)) \succeq A_T(f(x)) = f^{-1}(A)_T(x), \\ f^{-1}(A)_I(0_X) &= A_I(f(0_X)) \preceq A_I(f(x)) = f^{-1}(A)_I(x), \\ f^{-1}(A)_F(0_X) &= A_F(f(0_X)) \succeq A_F(f(x)) = f^{-1}(A)_F(x), \\ f^{-1}(\lambda)_T(0_X) &= \lambda_T(f(0_X)) \leq \lambda_T(f(x)) = f^{-1}(\lambda)_T(x), \\ f^{-1}(\lambda)_I(0_X) &= \lambda_I(f(0_X)) \geq \lambda_I(f(x)) = f^{-1}(\lambda)_I(x), \\ f^{-1}(\lambda)_F(0_X) &= \lambda_F(f(0_X)) \leq \lambda_F(f(x)) = f^{-1}(\lambda)_F(x). \end{aligned}$$

Let $x, y \in X$. Then

$$f^{-1}(A)_T(x \circ y) = A_T(f(x \circ y)) = A_T(f(x) \bullet f(y)) \succeq A_T(f(y)) = f^{-1}(A)_T(y), \quad (2.57), (3.1)$$

$$f^{-1}(A)_I(x \circ y) = A_I(f(x \circ y)) = A_I(f(x) \bullet f(y)) \preceq A_I(f(y)) = f^{-1}(A)_I(y), \quad (2.57), (3.1)$$

$$f^{-1}(A)_F(x \circ y) = A_F(f(x \circ y)) = A_F(f(x) \bullet f(y)) \succeq A_F(f(y)) = f^{-1}(A)_F(y), \quad (2.57), (3.1)$$

$$f^{-1}(\lambda)_T(x \circ y) = \lambda_T(f(x \circ y)) = \lambda_T(f(x) \bullet f(y)) \leq \lambda_T(f(y)) = f^{-1}(\lambda)_T(y), \quad (2.58), (3.2)$$

$$f^{-1}(\lambda)_I(x \circ y) = \lambda_I(f(x \circ y)) = \lambda_I(f(x) \bullet f(y)) \geq \lambda_I(f(y)) = f^{-1}(\lambda)_I(y), \quad (2.58), (3.2)$$

$$f^{-1}(\lambda)_F(x \circ y) = \lambda_F(f(x \circ y)) = \lambda_F(f(x) \bullet f(y)) \leq \lambda_F(f(y)) = f^{-1}(\lambda)_F(y). \quad (2.58), (3.2)$$

Hence, $f^{-1}(\mathcal{A})$ is a neutrosophic cubic near UP-filter of X .

(3) Assume that \mathcal{A} is a neutrosophic cubic UP-filter of Y . Then \mathcal{A} is a neutrosophic cubic near UP-filter of Y . By Lemma 3.6 and the proof of (2), we have $f^{-1}(\mathcal{A})$ satisfies the assertions (2.55) and (2.56). Let $x, y \in X$. Then

$$f^{-1}(A)_T(y) = A_T(f(y)) \quad (3.1)$$

$$\succeq \text{rmin}\{A_T(f(x) \bullet f(y)), A_T(f(x))\} \quad (2.59)$$

$$= \text{rmin}\{A_T(f(x \circ y)), A_T(f(x))\}$$

$$= \text{rmin}\{f^{-1}(A)_T(x \circ y), f^{-1}(A)_T(x)\}, \quad (3.1)$$

$$f^{-1}(A)_I(y) = A_I(f(y)) \quad (3.1)$$

$$\preceq \text{rmax}\{A_I(f(x) \bullet f(y)), A_I(f(x))\} \quad (2.59)$$

$$= \text{rmax}\{A_I(f(x \circ y)), A_I(f(x))\}$$

$$= \text{rmax}\{f^{-1}(A)_I(x \circ y), f^{-1}(A)_I(x)\}, \quad (3.1)$$

$$f^{-1}(A)_F(y) = A_F(f(y)) \quad (3.1)$$

$$\succeq \text{rmin}\{A_F(f(x) \bullet f(y)), A_F(f(x))\} \quad (2.59)$$

$$= \text{rmin}\{A_F(f(x \circ y)), A_F(f(x))\}$$

$$= \text{rmin}\{f^{-1}(A)_F(x \circ y), f^{-1}(A)_F(x)\}, \quad (3.1)$$

$$f^{-1}(\lambda)_T(y) = \lambda_T(f(y)) \quad (3.2)$$

$$\leq \max\{\lambda_T(f(x) \bullet f(y)), \lambda_T(f(x))\} \quad (2.60)$$

$$= \max\{\lambda_T(f(x \circ y)), \lambda_T(f(x))\}$$

$$= \max\{f^{-1}(\lambda)_T(x \circ y), f^{-1}(\lambda)_T(x)\}, \quad (3.2)$$

$$f^{-1}(\lambda)_I(y) = \lambda_I(f(y)) \quad (3.2)$$

$$\geq \min\{\lambda_I(f(x) \bullet f(y)), \lambda_I(f(x))\} \quad (2.60)$$

$$= \min\{\lambda_I(f(x \circ y)), \lambda_I(f(x))\}$$

$$= \min\{f^{-1}(\lambda)_I(x \circ y), f^{-1}(\lambda)_I(x)\}, \quad (3.2)$$

$$f^{-1}(\lambda)_F(y) = \lambda_F(f(y)) \quad (3.2)$$

$$\leq \max\{\lambda_F(f(x) \bullet f(y)), \lambda_F(f(x))\} \quad (2.60)$$

$$= \max\{\lambda_F(f(x \circ y)), \lambda_F(f(x))\}$$

$$= \max\{f^{-1}(\lambda)_F(x \circ y), f^{-1}(\lambda)_F(x)\}. \quad (3.2)$$

Hence, $f^{-1}(\mathcal{A})$ is a neutrosophic cubic UP-filter of X .

(4) Assume that \mathcal{A} is a neutrosophic cubic UP-ideal of Y . Then \mathcal{A} is a neutrosophic cubic UP-filter of Y . By the proof of (3), we have $f^{-1}(\mathcal{A})$ satisfies the assertions (2.55) and (2.56). Let $x, y, z \in X$. Then

$$f^{-1}(A)_T(x \circ z) = A_T(f(x \circ z)) \quad (3.1)$$

$$= A_T(f(x) \bullet f(z))$$

$$\succeq \text{rmin}\{A_T(f(x) \bullet (f(y) \bullet f(z))), A_T(f(y))\} \quad (2.61)$$

$$= \text{rmin}\{A_T(f(x) \bullet (f(y \circ z))), A_T(f(y))\}$$

$$= \text{rmin}\{A_T(f(x \circ (y \circ z))), A_T(f(y))\}$$

$$= \text{rmin}\{f^{-1}(A)_T(x \circ (y \circ z)), f^{-1}(A)_T(y)\}, \quad (3.1)$$

$$f^{-1}(A)_I(x \circ z) = A_I(f(x \circ z)) \quad (3.1)$$

$$= A_I(f(x) \bullet f(z))$$

$$\preceq \text{rmax}\{A_I(f(x) \bullet (f(y) \bullet f(z))), A_I(f(y))\} \quad (2.61)$$

$$= \text{rmax}\{A_I(f(x) \bullet (f(y \circ z))), A_I(f(y))\}$$

$$= \text{rmax}\{A_I(f(x \circ (y \circ z))), A_I(f(y))\}$$

$$= \text{rmax}\{f^{-1}(A)_I(x \circ (y \circ z)), f^{-1}(A)_I(y)\}, \quad (3.1)$$

$$f^{-1}(A)_F(x \circ z) = A_F(f(x \circ z)) \quad (3.1)$$

$$= A_F(f(x) \bullet f(z))$$

$$\succeq \text{rmin}\{A_F(f(x) \bullet (f(y) \bullet f(z))), A_F(f(y))\} \quad (2.61)$$

$$= \text{rmin}\{A_F(f(x) \bullet (f(y \circ z))), A_F(f(y))\}$$

$$= \text{rmin}\{A_F(f(x \circ (y \circ z))), A_F(f(y))\}$$

$$= \text{rmin}\{f^{-1}(A)_F(x \circ (y \circ z)), f^{-1}(A)_F(y)\}, \quad (3.1)$$

$$f^{-1}(\lambda)_T(x \circ z) = \lambda_T(f(x \circ z)) \quad (3.2)$$

$$= \lambda_T(f(x) \bullet f(z))$$

$$\leq \max\{\lambda_T(f(x) \bullet (f(y) \bullet f(z))), \lambda_T(f(y))\} \quad (2.62)$$

$$= \max\{\lambda_T(f(x) \bullet (f(y \circ z))), \lambda_T(f(y))\}$$

$$= \max\{\lambda_T(f(x \circ (y \circ z))), \lambda_T(f(y))\}$$

$$= \max\{f^{-1}(\lambda)_T(x \circ (y \circ z)), f^{-1}(\lambda)_T(y)\}, \quad (3.2)$$

$$f^{-1}(\lambda)_I(x \circ z) = \lambda_I(f(x \circ z)) \quad (3.2)$$

$$= \lambda_I(f(x) \bullet f(z))$$

$$\geq \min\{\lambda_I(f(x) \bullet (f(y) \bullet f(z))), \lambda_I(f(y))\} \quad (2.62)$$

$$= \min\{\lambda_I(f(x) \bullet (f(y \circ z))), \lambda_I(f(y))\}$$

$$= \min\{\lambda_I(f(x \circ (y \circ z))), \lambda_I(f(y))\}$$

$$= \min\{f^{-1}(\lambda)_I(x \circ (y \circ z)), f^{-1}(\lambda)_I(y)\}, \quad (3.2)$$

$$f^{-1}(\lambda)_F(x \circ z) = \lambda_F(f(x \circ z)) \quad (3.2)$$

$$= \lambda_F(f(x) \bullet f(z))$$

$$\leq \max\{\lambda_F(f(x) \bullet (f(y) \bullet f(z))), \lambda_F(f(y))\} \quad (2.62)$$

$$= \max\{\lambda_F(f(x) \bullet (f(y \circ z))), \lambda_F(f(y))\}$$

$$= \max\{\lambda_F(f(x \circ (y \circ z))), \lambda_F(f(y))\}$$

$$= \max\{f^{-1}(\lambda)_F(x \circ (y \circ z)), f^{-1}(\lambda)_F(y)\}. \quad (3.2)$$

Hence, $f^{-1}(\mathcal{A})$ is a neutrosophic cubic UP-ideal of X .

(5) Assume that \mathcal{A} is a neutrosophic cubic strong UP-ideal of Y . Then \mathcal{A} is a neutrosophic cubic UP-ideal of Y . By the proof of (4), we have $f^{-1}(\mathcal{A})$ satisfies the assertions (2.55) and (2.56). Let $x, y, z \in X$. Then

$$f^{-1}(A)_T(x) = A_T(f(x)) \quad (3.1)$$

$$\succeq \text{rmin}\{A_T((f(z) \bullet f(y)) \bullet (f(z) \bullet f(x))), A_T(f(y))\} \quad (2.63)$$

$$= \text{rmin}\{A_T(f(z \circ y) \bullet f(z \circ x)), A_T(f(y))\}$$

$$= \text{rmin}\{A_T(f((z \circ y) \circ (z \circ x))), A_T(f(y))\}$$

$$= \text{rmin}\{f^{-1}(A)_T((z \circ y) \circ (z \circ x)), f^{-1}(A)_T(y)\}, \quad (3.1)$$

$$f^{-1}(A)_I(x) = A_I(f(x)) \quad (3.1)$$

$$\preceq \text{rmax}\{A_I((f(z) \bullet f(y)) \bullet (f(z) \bullet f(x))), A_I(f(y))\} \quad (2.63)$$

$$= \text{rmax}\{A_I(f(z \circ y) \bullet f(z \circ x)), A_I(f(y))\}$$

$$= \text{rmax}\{A_I(f((z \circ y) \circ (z \circ x))), A_I(f(y))\}$$

$$= \text{rmax}\{f^{-1}(A)_I((z \circ y) \circ (z \circ x)), f^{-1}(A)_I(y)\}, \quad (3.1)$$

$$f^{-1}(A)_F(x) = A_F(f(x)) \quad (3.1)$$

$$\succeq \text{rmin}\{A_F((f(z) \bullet f(y)) \bullet (f(z) \bullet f(x))), A_F(f(y))\} \quad (2.63)$$

$$= \text{rmin}\{A_F(f(z \circ y) \bullet f(z \circ x)), A_F(f(y))\}$$

$$= \text{rmin}\{A_F(f((z \circ y) \circ (z \circ x))), A_F(f(y))\}$$

$$= \text{rmin}\{f^{-1}(A)_F((z \circ y) \circ (z \circ x)), f^{-1}(A)_F(y)\}, \quad (3.1)$$

$$f^{-1}(\lambda)_T(x) = \lambda_T(f(x)) \quad (3.2)$$

$$\leq \max\{\lambda_T((f(z) \bullet f(y)) \bullet (f(z) \bullet f(x))), \lambda_T(f(y))\} \quad (2.64)$$

$$= \max\{\lambda_T(f(z \circ y) \bullet f(z \circ x)), \lambda_T(f(y))\}$$

$$= \max\{\lambda_T(f((z \circ y) \circ (z \circ x))), \lambda_T(f(y))\}$$

$$= \max\{f^{-1}(\lambda)_T((z \circ y) \circ (z \circ x)), f^{-1}(\lambda)_T(y)\}, \quad (3.2)$$

$$f^{-1}(\lambda)_I(x) = \lambda_I(f(x)) \quad (3.2)$$

$$\geq \min\{\lambda_I((f(z) \bullet f(y)) \bullet (f(z) \bullet f(x))), \lambda_I(f(y))\} \quad (2.64)$$

$$= \min\{\lambda_I(f(z \circ y) \bullet f(z \circ x)), \lambda_I(f(y))\}$$

$$= \min\{\lambda_I(f((z \circ y) \circ (z \circ x))), \lambda_I(f(y))\}$$

$$= \min\{f^{-1}(\lambda)_I((z \circ y) \circ (z \circ x)), f^{-1}(\lambda)_I(y)\}, \quad (3.2)$$

$$f^{-1}(\lambda)_F(x) = \lambda_F(f(x)) \quad (3.2)$$

$$\leq \max\{\lambda_F((f(z) \bullet f(y)) \bullet (f(z) \bullet f(x))), \lambda_F(f(y))\} \quad (2.64)$$

$$= \max\{\lambda_F(f(z \circ y) \bullet f(z \circ x)), \lambda_F(f(y))\}$$

$$= \max\{\lambda_F(f((z \circ y) \circ (z \circ x))), \lambda_F(f(y))\}$$

$$= \max\{f^{-1}(\lambda)_F((z \circ y) \circ (z \circ x)), f^{-1}(\lambda)_F(y)\}. \quad (3.2)$$

Hence, $f^{-1}(\mathcal{A})$ is a neutrosophic cubic strong UP-ideal of X . \square

Definition 3.8. A NCS $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ in X has *NCS-property* if for any non-empty subset A of X , there exist elements $\alpha_T, \alpha_I, \alpha_F, \beta_T, \beta_I, \beta_F \in A$ (instead of $\alpha_T, \alpha_I, \alpha_F, \beta_T, \beta_I, \beta_F \in A$) such that

$$A_T(\alpha_T) = \text{rsup}_{s \in A}\{A_T(s)\}, A_I(\alpha_I) = \text{rinf}_{s \in A}\{A_I(s)\}, A_F(\alpha_F) = \text{rsup}_{s \in A}\{A_F(s)\},$$

$$\lambda_T(\beta_T) = \text{inf}_{s \in A}\{\lambda_T(s)\}, \lambda_I(\beta_I) = \text{sup}_{s \in A}\{\lambda_I(s)\}, \lambda_F(\beta_F) = \text{inf}_{s \in A}\{\lambda_F(s)\}.$$

Definition 3.9. Let X and Y be any two non-empty sets and let $f : X \rightarrow Y$ be any function. A NCS $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ in X is said to be *f-invariant* if

$$(\text{for all } x, y \in X)(f(x) = f(y) \Rightarrow A_{T,I,F}(x) = A_{T,I,F}(y), \lambda_{T,I,F}(x) = \lambda_{T,I,F}(y)). \quad (3.4)$$

Lemma 3.10. Let $(X, \circ, 0_X)$ and $(Y, \bullet, 0_Y)$ be two UP-algebras and let $f : X \rightarrow Y$ be a UP-epimorphism. Let $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ be an *f-invariant* NCS in X with *NCS-property*. For any $x, y \in Y$, there exist

elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$ and $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ such that

$$\begin{aligned} f(A)_T(x) &= A_T(\alpha_T), f(A)_I(x) = A_I(\alpha_I), f(A)_F(x) = A_F(\alpha_F), \\ f(\lambda)_T(x) &= \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F), \\ f(A)_T(y) &= A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F), \\ f(\lambda)_T(y) &= \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F), \\ f(A)_T(x \bullet y) &= A_T(\alpha_T \circ \beta_T), f(A)_I(x \bullet y) = A_I(\alpha_I \circ \beta_I), f(A)_F(x \bullet y) = A_F(\alpha_F \circ \beta_F), \\ f(\lambda)_T(x \bullet y) &= \lambda_T(\gamma_T \circ \phi_T), f(\lambda)_I(x \bullet y) = \lambda_I(\gamma_I \circ \phi_I), f(\lambda)_F(x \bullet y) = \lambda_F(\gamma_F \circ \phi_F). \end{aligned}$$

Proof. Let $x, y \in Y$. Since f is surjective, we have $f^{-1}(x)$, $f^{-1}(y)$, and $f^{-1}(x \circ y)$ are non-empty subsets of X . Since \mathcal{A} has NCS-property, there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$, $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$, and $a_{T,I,F}, b_{T,I,F} \in f^{-1}(x \bullet y)$ such that

$$\begin{aligned} f(A)_T(x) &= \text{rsup}_{s \in f^{-1}(x)} \{A_T(s)\} = A_T(\alpha_T), \\ f(A)_I(x) &= \text{rinf}_{s \in f^{-1}(x)} \{A_I(s)\} = A_I(\alpha_I), \\ f(A)_F(x) &= \text{rsup}_{s \in f^{-1}(x)} \{A_F(s)\} = A_F(\alpha_F), \\ f(\lambda)_T(x) &= \text{inf}_{s \in f^{-1}(x)} \{\lambda_T(s)\} = \lambda_T(\gamma_T), \\ f(\lambda)_I(x) &= \text{sup}_{s \in f^{-1}(x)} \{\lambda_I(s)\} = \lambda_I(\gamma_I), \\ f(\lambda)_F(x) &= \text{inf}_{s \in f^{-1}(x)} \{\lambda_F(s)\} = \lambda_F(\gamma_F), \\ f(A)_T(y) &= \text{rsup}_{s \in f^{-1}(y)} \{A_T(s)\} = A_T(\beta_T), \\ f(A)_I(y) &= \text{rinf}_{s \in f^{-1}(y)} \{A_I(s)\} = A_I(\beta_I), \\ f(A)_F(y) &= \text{rsup}_{s \in f^{-1}(y)} \{A_F(s)\} = A_F(\beta_F), \\ f(\lambda)_T(y) &= \text{inf}_{s \in f^{-1}(y)} \{\lambda_T(s)\} = \lambda_T(\phi_T), \\ f(\lambda)_I(y) &= \text{sup}_{s \in f^{-1}(y)} \{\lambda_I(s)\} = \lambda_I(\phi_I), \\ f(\lambda)_F(y) &= \text{inf}_{s \in f^{-1}(y)} \{\lambda_F(s)\} = \lambda_F(\phi_F), \end{aligned}$$

and

$$\begin{aligned} f(A)_T(x \bullet y) &= \text{rsup}_{s \in f^{-1}(x \bullet y)} \{A_T(s)\} = A_T(a_T), \\ f(A)_I(x \bullet y) &= \text{rinf}_{s \in f^{-1}(x \bullet y)} \{A_I(s)\} = A_I(a_I), \\ f(A)_F(x \bullet y) &= \text{rsup}_{s \in f^{-1}(x \bullet y)} \{A_F(s)\} = A_F(a_F), \\ f(\lambda)_T(x \bullet y) &= \text{inf}_{s \in f^{-1}(x \bullet y)} \{\lambda_T(s)\} = \lambda_T(b_T), \\ f(\lambda)_I(x \bullet y) &= \text{sup}_{s \in f^{-1}(x \bullet y)} \{\lambda_I(s)\} = \lambda_I(b_I), \\ f(\lambda)_F(x \bullet y) &= \text{inf}_{s \in f^{-1}(x \bullet y)} \{\lambda_F(s)\} = \lambda_F(b_F). \end{aligned}$$

Since

$$\begin{aligned} f(a_T) &= x \bullet y = f(\alpha_T) \bullet f(\beta_T) = f(\alpha_T \circ \beta_T), \\ f(a_I) &= x \bullet y = f(\alpha_I) \bullet f(\beta_I) = f(\alpha_I \circ \beta_I), \\ f(a_F) &= x \bullet y = f(\alpha_F) \bullet f(\beta_F) = f(\alpha_F \circ \beta_F), \\ f(b_T) &= x \bullet y = f(\gamma_T) \bullet f(\phi_T) = f(\gamma_T \circ \phi_T), \\ f(b_I) &= x \bullet y = f(\gamma_I) \bullet f(\phi_I) = f(\gamma_I \circ \phi_I), \\ f(b_F) &= x \bullet y = f(\gamma_F) \bullet f(\phi_F) = f(\gamma_F \circ \phi_F), \end{aligned}$$

and \mathcal{A} is f -invariant, it follows that

$$\begin{aligned} f(A)_T(x \bullet y) &= A_T(a_T) = A_T(\alpha_T \circ \beta_T), \\ f(A)_I(x \bullet y) &= A_I(a_I) = A_I(\alpha_I \circ \beta_I), \\ f(A)_F(x \bullet y) &= A_F(a_F) = A_F(\alpha_F \circ \beta_F), \\ f(\lambda)_T(x \bullet y) &= \lambda_T(b_T) = \lambda_T(\gamma_T \circ \phi_T), \\ f(\lambda)_I(x \bullet y) &= \lambda_I(b_I) = \lambda_I(\gamma_I \circ \phi_I), \\ f(\lambda)_F(x \bullet y) &= \lambda_F(b_F) = \lambda_F(\gamma_F \circ \phi_F). \end{aligned}$$

The proof is completed. □

Theorem 3.11. Let $(X, \circ, 0_X)$ and $(Y, \bullet, 0_Y)$ be two UP-algebras, $f: X \rightarrow Y$ be a UP-epimorphism, and $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ be a NCS in X . Then the followings hold:

- (1) If \mathcal{A} is an f -invariant neutrosophic cubic UP-subalgebra of X with NCS-property, then the image $f(\mathcal{A})$ of \mathcal{A} under f is a neutrosophic cubic UP-subalgebra of Y .
- (2) If \mathcal{A} is an f -invariant neutrosophic cubic near UP-filter of X with NCS-property, then the image $f(\mathcal{A})$ of \mathcal{A} under f is a neutrosophic cubic near UP-filter of Y .
- (3) If \mathcal{A} is an f -invariant neutrosophic cubic UP-filter of X with NCS-property, then the image $f(\mathcal{A})$ of \mathcal{A} under f is a neutrosophic cubic UP-filter of Y .
- (4) If \mathcal{A} is an f -invariant neutrosophic cubic UP-ideal of X with NCS-property, then the image $f(\mathcal{A})$ of \mathcal{A} under f is a neutrosophic cubic UP-ideal of Y .
- (5) If \mathcal{A} is an f -invariant neutrosophic cubic strong UP-ideal of X with NCS-property, then the image $f(\mathcal{A})$ of \mathcal{A} under f is a neutrosophic cubic strong UP-ideal of Y .

Proof. (1) Assume that $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ is an f -invariant neutrosophic cubic UP-subalgebra of X with NCS-property. Let $x, y \in Y$. Since f is surjective, we have $f^{-1}(x), f^{-1}(y)$, and $f^{-1}(x \bullet y)$ are non-empty. By Lemma 3.10, there exist elements $\alpha_T, \gamma_T, \beta_T, \phi_T \in f^{-1}(x)$ and $\alpha_I, \gamma_I, \beta_I, \phi_I \in f^{-1}(y)$ such that

$$\begin{aligned} f(A)_T(x) &= A_T(\alpha_T), f(A)_I(x) = A_I(\alpha_I), f(A)_F(x) = A_F(\alpha_F), \\ f(\lambda)_T(x) &= \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F), \\ f(A)_T(y) &= A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F), \\ f(\lambda)_T(y) &= \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F), \\ f(A)_T(x \bullet y) &= A_T(\alpha_T \circ \beta_T), f(A)_I(x \bullet y) = A_I(\alpha_I \circ \beta_I), f(A)_F(x \bullet y) = A_F(\alpha_F \circ \beta_F), \\ f(\lambda)_T(x \bullet y) &= \lambda_T(\gamma_T \circ \phi_T), f(\lambda)_I(x \bullet y) = \lambda_I(\gamma_I \circ \phi_I), f(\lambda)_F(x \bullet y) = \lambda_F(\gamma_F \circ \phi_F). \end{aligned}$$

Then

$$f(A)_T(x \bullet y) = A_T(\alpha_T \circ \beta_T) \succeq \text{rmin}\{A_T(\alpha_T), A_T(\beta_T)\} = \text{rmin}\{f(A)_T(x), f(A)_T(y)\}, \quad (2.53)$$

$$f(A)_I(x \bullet y) = A_I(\alpha_I \circ \beta_I) \preceq \text{rmax}\{A_I(\alpha_I), A_I(\beta_I)\} = \text{rmax}\{f(A)_I(x), f(A)_I(y)\}, \quad (2.53)$$

$$f(A)_F(x \bullet y) = A_F(\alpha_F \circ \beta_F) \succeq \text{rmin}\{A_F(\alpha_F), A_F(\beta_F)\} = \text{rmin}\{f(A)_F(x), f(A)_F(y)\}, \quad (2.53)$$

$$f(\lambda)_T(x \bullet y) = \lambda_T(\gamma_T \circ \phi_T) \leq \max\{\lambda_T(\gamma_T), \lambda_T(\phi_T)\} = \max\{f(\lambda)_T(x), f(\lambda)_T(y)\}, \quad (2.54)$$

$$f(\lambda)_I(x \bullet y) = \lambda_I(\gamma_I \circ \phi_I) \geq \min\{\lambda_I(\gamma_I), \lambda_I(\phi_I)\} = \min\{f(\lambda)_I(x), f(\lambda)_I(y)\}, \quad (2.54)$$

$$f(\lambda)_F(x \bullet y) = \lambda_F(\gamma_F \circ \phi_F) \leq \max\{\lambda_F(\gamma_F), \lambda_F(\phi_F)\} = \max\{f(\lambda)_F(x), f(\lambda)_F(y)\}. \quad (2.54)$$

Hence, $f(\mathcal{A})$ is a neutrosophic cubic UP-subalgebra of Y .

(2) Assume that $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ is an f -invariant neutrosophic cubic near UP-filter of X with NCS-property. By Theorem 2.6(1), we have $0_X \in f^{-1}(0_Y)$ and so $f^{-1}(0_Y)$ is non-empty. Thus

$$\left(\begin{aligned} f(A)_T(0_Y) &= \text{rsup}_{s \in f^{-1}(0_Y)} \{A_T(s)\} \succeq A_T(0_X) \\ f(A)_I(0_Y) &= \text{rinf}_{s \in f^{-1}(0_Y)} \{A_I(s)\} \preceq A_I(0_X) \\ f(A)_F(0_Y) &= \text{rsup}_{s \in f^{-1}(0_Y)} \{A_F(s)\} \succeq A_F(0_X) \\ f(\lambda)_T(0_Y) &= \text{inf}_{s \in f^{-1}(0_Y)} \{\lambda_T(s)\} \leq \lambda_T(0_X) \\ f(\lambda)_I(0_Y) &= \text{sup}_{s \in f^{-1}(0_Y)} \{\lambda_I(s)\} \geq \lambda_I(0_X) \\ f(\lambda)_F(0_Y) &= \text{inf}_{s \in f^{-1}(0_Y)} \{\lambda_F(s)\} \leq \lambda_F(0_X) \end{aligned} \right). \quad (3.5)$$

Let $y \in Y$. Since f is surjective, we have $f^{-1}(y)$ is non-empty. By (2.55) and (2.56), we have $A_T(0_X) \succeq A_T(s), A_I(0_X) \preceq A_I(s), A_F(0_X) \succeq A_F(s), \lambda_T(0_X) \leq \lambda_T(s), \lambda_I(0_X) \geq \lambda_I(s), \lambda_F(0_X) \leq \lambda_F(s)$ for all $s \in f^{-1}(y)$. Then $A_T(0_X)$ is an upper bound of $\{A_T(s)\}_{s \in f^{-1}(y)}$, $A_I(0_X)$ is a lower bound of $\{A_I(s)\}_{s \in f^{-1}(y)}$, $A_F(0_X)$ is an upper bound of $\{A_F(s)\}_{s \in f^{-1}(y)}$, $\lambda_T(0_X)$ is a lower bound of $\{\lambda_T(s)\}_{s \in f^{-1}(y)}$, $\lambda_I(0_X)$ is an

upper bound of $\{\lambda_I(s)\}_{s \in f^{-1}(y)}$, and $\lambda_F(0_X)$ is a lower bound of $\{\lambda_F(s)\}_{s \in f^{-1}(y)}$. By (3.5), we have

$$\begin{aligned} f(A)_T(0_Y) &\succeq A_T(0_X) \succeq \text{rsup}_{s \in f^{-1}(y)} \{A_T(s)\} = f(A)_T(y), \\ f(A)_I(0_Y) &\preceq A_I(0_X) \preceq \text{rinf}_{s \in f^{-1}(y)} \{A_I(s)\} = f(A)_I(y), \\ f(A)_F(0_Y) &\succeq A_F(0_X) \succeq \text{rsup}_{s \in f^{-1}(y)} \{A_F(s)\} = f(A)_F(y), \\ f(\lambda)_T(0_Y) &\leq \lambda_T(0_X) \leq \text{inf}_{s \in f^{-1}(y)} \{\lambda_T(s)\} = f(\lambda)_T(y), \\ f(\lambda)_I(0_Y) &\geq \lambda_I(0_X) \geq \text{sup}_{s \in f^{-1}(y)} \{\lambda_I(s)\} = f(\lambda)_I(y), \\ f(\lambda)_F(0_Y) &\leq \lambda_F(0_X) \leq \text{inf}_{s \in f^{-1}(y)} \{\lambda_F(s)\} = f(\lambda)_F(y). \end{aligned}$$

Let $x, y \in Y$. By Lemma 3.10, there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$ and $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ such that

$$\begin{aligned} f(A)_T(x) &= A_T(\alpha_T), f(A)_I(x) = A_I(\alpha_I), f(A)_F(x) = A_F(\alpha_F), \\ f(\lambda)_T(x) &= \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F), \\ f(A)_T(y) &= A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F), \\ f(\lambda)_T(y) &= \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F), \\ f(A)_T(x \bullet y) &= A_T(\alpha_T \circ \beta_T), f(A)_I(x \bullet y) = A_I(\alpha_I \circ \beta_I), f(A)_F(x \bullet y) = A_F(\alpha_F \circ \beta_F), \\ f(\lambda)_T(x \bullet y) &= \lambda_T(\gamma_T \circ \phi_T), f(\lambda)_I(x \bullet y) = \lambda_I(\gamma_I \circ \phi_I), f(\lambda)_F(x \bullet y) = \lambda_F(\gamma_F \circ \phi_F). \end{aligned}$$

Then

$$\begin{aligned} f(A)_T(x \bullet y) &= A_T(\alpha_T \circ \beta_T) \succeq A_T(\beta_T) = f(A)_T(y), & (2.57) \\ f(A)_I(x \bullet y) &= A_I(\alpha_I \circ \beta_I) \preceq A_I(\beta_I) = f(A)_I(y), & (2.57) \\ f(A)_F(x \bullet y) &= A_F(\alpha_F \circ \beta_F) \succeq A_F(\beta_F) = f(A)_F(y), & (2.57) \\ f(\lambda)_T(x \bullet y) &= \lambda_T(\gamma_T \circ \phi_T) \leq \lambda_T(\phi_T) = f(\lambda)_T(y), & (2.58) \\ f(\lambda)_I(x \bullet y) &= \lambda_I(\gamma_I \circ \phi_I) \geq \lambda_I(\phi_I) = f(\lambda)_I(y), & (2.58) \\ f(\lambda)_F(x \bullet y) &= \lambda_F(\gamma_F \circ \phi_F) \leq \lambda_F(\phi_F) = f(\lambda)_F(y). & (2.58) \end{aligned}$$

Hence, $f(\mathcal{A})$ is a neutrosophic cubic near UP-filter of Y .

(3) Assume that $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ is an f -invariant neutrosophic cubic UP-filter of X with NCS-property. Then \mathcal{A} is a neutrosophic cubic near UP-filter of X . By the proof of (2), we have $f(\mathcal{A})$ satisfies the assertions (2.55) and (2.56). Let $x, y \in Y$. By Lemma 3.10, there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$ and $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ such that

$$\begin{aligned} f(A)_T(x) &= A_T(\alpha_T), f(A)_I(x) = A_I(\alpha_I), f(A)_F(x) = A_F(\alpha_F), \\ f(\lambda)_T(x) &= \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F), \\ f(A)_T(y) &= A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F), \\ f(\lambda)_T(y) &= \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F), \\ f(A)_T(x \bullet y) &= A_T(\alpha_T \circ \beta_T), f(A)_I(x \bullet y) = A_I(\alpha_I \circ \beta_I), f(A)_F(x \bullet y) = A_F(\alpha_F \circ \beta_F), \\ f(\lambda)_T(x \bullet y) &= \lambda_T(\gamma_T \circ \phi_T), f(\lambda)_I(x \bullet y) = \lambda_I(\gamma_I \circ \phi_I), f(\lambda)_F(x \bullet y) = \lambda_F(\gamma_F \circ \phi_F). \end{aligned}$$

Then

$$\begin{aligned} f(A)_T(y) &= A_T(\beta_T) \succeq \text{rmin}\{A_T(\alpha_T \circ \beta_T), A_T(\alpha_T)\} = \text{rmin}\{f(A)_T(x \bullet y), f(A)_T(x)\}, & (2.59) \\ f(A)_I(y) &= A_I(\beta_I) \preceq \text{rmax}\{A_I(\alpha_I \circ \beta_I), A_I(\alpha_I)\} = \text{rmax}\{f(A)_I(x \bullet y), f(A)_I(x)\}, & (2.59) \\ f(A)_F(y) &= A_F(\beta_F) \succeq \text{rmin}\{A_F(\alpha_F \circ \beta_F), A_F(\alpha_F)\} = \text{rmin}\{f(A)_F(x \bullet y), f(A)_F(x)\}, & (2.59) \\ f(\lambda)_T(y) &= \lambda_T(\phi_T) \leq \text{max}\{\lambda_T(\gamma_T \circ \phi_T), \lambda_T(\gamma_T)\} = \text{max}\{f(\lambda)_T(x \bullet y), f(\lambda)_T(x)\}, & (2.60) \\ f(\lambda)_I(y) &= \lambda_I(\phi_I) \geq \text{min}\{\lambda_I(\gamma_I \circ \phi_I), \lambda_I(\gamma_I)\} = \text{min}\{f(\lambda)_I(x \bullet y), f(\lambda)_I(x)\}, & (2.60) \\ f(\lambda)_F(y) &= \lambda_F(\phi_F) \leq \text{max}\{\lambda_F(\gamma_F \circ \phi_F), \lambda_F(\gamma_F)\} = \text{max}\{f(\lambda)_F(x \bullet y), f(\lambda)_F(x)\}. & (2.60) \end{aligned}$$

Hence, $f(\mathcal{A})$ is a neutrosophic cubic UP-filter of Y .

(4) Assume that $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ is an f -invariant neutrosophic cubic UP-ideal of X with NCS-property. Then \mathcal{A} is a neutrosophic cubic UP-filter of X . By the proof of (3), we have $f(\mathcal{A})$ satisfies the

assertions (2.55) and (2.56). Let $x, y, z \in Y$. By Lemma 3.10, there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$, $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ and $\psi_{T,I,F}, \omega_{T,I,F} \in f^{-1}(z)$ such that

$$\begin{aligned} f(A)_T(y) &= A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F), \\ f(\lambda)_T(y) &= \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F), \\ f(A)_T(x \bullet z) &= A_T(\alpha_T \circ \psi_T), f(A)_I(x \bullet z) = A_I(\alpha_I \circ \psi_I), f(A)_F(x \bullet z) = A_F(\alpha_F \circ \psi_F), \\ f(\lambda)_T(x \bullet z) &= \lambda_T(\gamma_T \circ \omega_T), f(\lambda)_I(x \bullet z) = \lambda_I(\gamma_I \circ \omega_I), f(\lambda)_F(x \bullet z) = \lambda_F(\gamma_F \circ \omega_F), \\ f(A)_T(x \bullet (y \bullet z)) &= A_T(\alpha_T \circ (\beta_T \circ \psi_T)), \\ f(A)_I(x \bullet (y \bullet z)) &= A_I(\alpha_I \circ (\beta_I \circ \psi_I)), \\ f(A)_F(x \bullet (y \bullet z)) &= A_F(\alpha_F \circ (\beta_F \circ \psi_F)), \\ f(\lambda)_T(x \bullet (y \bullet z)) &= \lambda_T(\gamma_T \circ (\phi_T \circ \omega_T)), \\ f(\lambda)_I(x \bullet (y \bullet z)) &= \lambda_I(\gamma_I \circ (\phi_I \circ \omega_I)), \\ f(\lambda)_F(x \bullet (y \bullet z)) &= \lambda_F(\gamma_F \circ (\phi_F \circ \omega_F)). \end{aligned}$$

Then

$$\begin{aligned} f(A)_T(x \bullet z) &= A_T(\alpha_T \circ \psi_T) \\ &\succeq \text{rmin}\{A_T(\alpha_T \circ (\beta_T \circ \psi_T)), A_T(\beta_T)\} \\ &= \text{rmin}\{f(A)_T(x \bullet (y \bullet z)), f(A)_T(y)\}, \end{aligned} \quad (2.61)$$

$$\begin{aligned} f(A)_I(x \bullet z) &= A_I(\alpha_I \circ \psi_I) \\ &\preceq \text{rmax}\{A_I(\alpha_I \circ (\beta_I \circ \psi_I)), A_I(\beta_I)\} \\ &= \text{rmax}\{f(A)_I(x \bullet (y \bullet z)), f(A)_I(y)\}, \end{aligned} \quad (2.61)$$

$$\begin{aligned} f(A)_F(x \bullet z) &= A_F(\alpha_F \circ \psi_F) \\ &\succeq \text{rmin}\{A_F(\alpha_F \circ (\beta_F \circ \psi_F)), A_F(\beta_F)\} \\ &= \text{rmin}\{f(A)_F(x \bullet (y \bullet z)), f(A)_F(y)\}, \end{aligned} \quad (2.61)$$

$$\begin{aligned} f(\lambda)_T(x \bullet z) &= \lambda_T(\gamma_T \circ \omega_T) \\ &\leq \text{max}\{\lambda_T(\gamma_T \circ (\phi_T \circ \omega_T)), \lambda_T(\phi_T)\} \\ &= \text{max}\{f(\lambda)_T(x \bullet (y \bullet z)), f(\lambda)_T(y)\}, \end{aligned} \quad (2.62)$$

$$\begin{aligned} f(\lambda)_I(x \bullet z) &= \lambda_I(\gamma_I \circ \omega_I) \\ &\geq \text{min}\{\lambda_I(\gamma_I \circ (\phi_I \circ \omega_I)), \lambda_I(\phi_I)\} \\ &= \text{min}\{f(\lambda)_I(x \bullet (y \bullet z)), f(\lambda)_I(y)\}, \end{aligned} \quad (2.62)$$

$$\begin{aligned} f(\lambda)_F(x \bullet z) &= \lambda_F(\gamma_F \circ \omega_F) \\ &\leq \text{max}\{\lambda_F(\gamma_F \circ (\phi_F \circ \omega_F)), \lambda_F(\phi_F)\} \\ &= \text{max}\{f(\lambda)_F(x \bullet (y \bullet z)), f(\lambda)_F(y)\}. \end{aligned} \quad (2.62)$$

Hence, $f(\mathcal{A})$ is a neutrosophic cubic UP-ideal of Y .

(5) Assume that $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ is an f -invariant neutrosophic cubic strong UP-ideal of X with NCS-property. Then \mathcal{A} is a neutrosophic cubic UP-ideal of X . By the proof of (4), we have $f(\mathcal{A})$ satisfies the assertions (2.55) and (2.56). Let $x, y, z \in Y$. By Lemma 3.10, there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$, $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ and $\psi_{T,I,F}, \omega_{T,I,F} \in f^{-1}(z)$ such that

$$\begin{aligned} f(A)_T(x) &= A_T(\alpha_T), f(A)_I(x) = A_I(\alpha_I), f(A)_F(x) = A_F(\alpha_F), \\ f(\lambda)_T(x) &= \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F), \\ f(A)_T(y) &= A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F), \\ f(\lambda)_T(y) &= \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F), \\ f(A)_T((z \bullet y) \bullet (z \bullet x)) &= A_T((\psi_T \circ \beta_T) \circ (\psi_T \circ \alpha_T)), \\ f(A)_I((z \bullet y) \bullet (z \bullet x)) &= A_I((\psi_I \circ \beta_I) \circ (\psi_I \circ \alpha_I)), \\ f(A)_F((z \bullet y) \bullet (z \bullet x)) &= A_F((\psi_F \circ \beta_F) \circ (\psi_F \circ \alpha_F)), \\ f(\lambda)_T((z \bullet y) \bullet (z \bullet x)) &= \lambda_T((\omega_T \circ \phi_T) \circ (\omega_T \circ \gamma_T)), \\ f(\lambda)_I((z \bullet y) \bullet (z \bullet x)) &= \lambda_I((\omega_I \circ \phi_I) \circ (\omega_I \circ \gamma_I)), \\ f(\lambda)_F((z \bullet y) \bullet (z \bullet x)) &= \lambda_F((\omega_F \circ \phi_F) \circ (\omega_F \circ \gamma_F)). \end{aligned}$$

Then

$$f(A)_T(x) = A_T(\alpha_T) \succeq \text{rmin}\{A_T((\psi_T \circ \beta_T) \circ (\psi_T \circ \alpha_T)), A_T(\beta_T)\} \quad ((2.63))$$

$$= \text{rmin}\{f(A)_T((z \bullet y) \bullet (z \bullet x)), f(A)_T(y)\},$$

$$f(A)_I(x) = A_I(\alpha_I) \preceq \text{rmax}\{A_I((\psi_I \circ \beta_I) \circ (\psi_I \circ \alpha_I)), A_I(\beta_I)\} \quad ((2.63))$$

$$= \text{rmax}\{f(A)_I((z \bullet y) \bullet (z \bullet x)), f(A)_I(y)\},$$

$$f(A)_F(x) = A_F(\alpha_F) \succeq \text{rmin}\{A_F((\psi_F \circ \beta_F) \circ (\psi_F \circ \alpha_F)), A_F(\beta_F)\} \quad ((2.63))$$

$$= \text{rmin}\{f(A)_F((z \bullet y) \bullet (z \bullet x)), f(A)_F(y)\},$$

$$f(\lambda)_T(x) = \lambda_T(\gamma_T) \leq \max\{\lambda_T((\omega_T \circ \phi_T) \circ (\omega_T \circ \gamma_T)), \lambda_T(\phi_T)\} \quad ((2.64))$$

$$= \max\{f(\lambda)_T((z \bullet y) \bullet (z \bullet x)), f(\lambda)_T(y)\},$$

$$f(\lambda)_I(x) = \lambda_I(\gamma_I) \geq \min\{\lambda_I((\omega_I \circ \phi_I) \circ (\omega_I \circ \gamma_I)), \lambda_I(\phi_I)\} \quad ((2.64))$$

$$= \min\{f(\lambda)_I((z \bullet y) \bullet (z \bullet x)), f(\lambda)_I(y)\},$$

$$f(\lambda)_F(x) = \lambda_F(\gamma_F) \leq \max\{\lambda_F((\omega_F \circ \phi_F) \circ (\omega_F \circ \gamma_F)), \lambda_F(\phi_F)\} \quad ((2.64))$$

$$= \max\{f(\lambda)_F((z \bullet y) \bullet (z \bullet x)), f(\lambda)_F(y)\}.$$

Hence, $f(\mathcal{A})$ is a neutrosophic cubic strong UP-ideal of Y . \square

4 Conclusions and future work

In this paper, we have studied the image and inverse image of a neutrosophic cubic UP-subalgebra (resp., neutrosophic cubic near UP-filter, neutrosophic cubic UP-filter, neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal) of a UP-algebra under some UP-homomorphisms. The results of the study, in the case of inverse image, we noticed that only a neutrosophic cubic near UP-filter required order preserving condition. In the case of image, we noticed that all concepts of NCSs required f -invariant and NCS-property assertions and UP-epimorphism.

In our future study, we will apply this concept/results to other types of NCSs in a UP-algebra. Also, we will study the P-intersection, P-union, R-intersection, R-union of neutrosophic cubic UP-subalgebras, neutrosophic cubic near UP-filters, neutrosophic cubic UP-filters, neutrosophic cubic UP-ideals, and neutrosophic cubic strong UP-ideals of a UP-algebra.

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On single valued neutrosophic sets and neutrosophic \mathbb{N} -structures: Applications on algebraic structures (hyperstructures)

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Abstract

In this paper, we find a relationship between SVNS and neutrosophic \mathbb{N} -structures and study it. Moreover, we apply our results to algebraic structures (hyperstructures) and prove that the results on neutrosophic \mathbb{N} -substructure (subhyperstructure) of a given algebraic structure (hyperstructure) can be deduced from single valued neutrosophic algebraic structure (hyperstructure) and vice versa.

Keywords: Neutrosophic \mathbb{N} -structures, SVNS, (α, β, γ) -level set, neutrosophic \mathbb{N} -ideals, neutrosophic \mathbb{N} -substructures (subhypersructures)

1 Introduction

Neutrosophy,^[19] a new branch of science that deals with indeterminacy, was launched by Smarandache in 1998. The theory of neutrosophy considers every notion or idea $\langle A \rangle$ together with its opposite or negation $\langle antiA \rangle$ and with their spectrum of neutralities $\langle neutA \rangle$ in between them (i.e. notions or ideas supporting neither $\langle A \rangle$ nor $\langle antiA \rangle$). The $\langle neutA \rangle$ and $\langle antiA \rangle$ ideas together are referred to as $\langle nonA \rangle$. Smarandach^[20] defined neutrosophic sets as a generalization of the fuzzy sets introduced by Zadeh^[22] in 1965 and as a generalization of intuitionistic fuzzy sets introduced by Atanassov^[8] in 1986. Fuzzy sets allow gradual membership of an element in a set by assigning each element a degree of membership between 0 and 1 that are both inclusive. Whereas intuitionistic fuzzy sets allow gradual membership as well as gradual non-membership of an element in a set by assigning each element a degree of membership and a degree of non-membership in a way that their sum is a real number in the unit interval $[0, 1]$. Single valued neutrosophic sets (SVNS)^[24] generalize these two concepts so that each element has a truth value, indeterminacy value, and a falsity value and each of these values is a number in the unit interval $[0, 1]$. Sometimes we have negative information. As an example, "The rate increase in a certain bank depends on employees' performance. It increases by 3% annually if the employee's performance is outstanding (convincing many business women/men to invest their money in the bank), by 2% annually if the employee's performance is very good, by 1% annually if the employee's performance is good, and no increase if the employee's performance is average. Let's say that Sam convinces annually around twenty business women/men to invest their money in the bank, so he got the 3% annual increase as a result of his excellent job. And there is an employee "Bella" who comes always late to her work, leaves early, complains about the bank in public and as a result, she leads to the loss of some possible investors in the bank. So, in this case Bella is making the bank loses and as a result she does not deserve an annual increase but instead she should be given a decrease in her salary." In order to deal with such negative information, we need negative functions. So, by means of negative functions, neutrosophic \mathbb{N} -structures were introduced.^{[14][15]} They are similar to SVNS where each element has a truth value, indeterminacy value, and a falsity value but each of these values is a number in the interval $[-1, 0]$, i.e., the truth, indeterminacy, and the falsity functions are negative-valued functions. Neutrosophy has many applications in different fields of Science. Many researchers^{[3][5][7][14][17][21]} worked on the connection between neutrosophy and algebraic structures (hyperstructures). More precisely, the connection between SVNS and algebraic structures (hypersructures) and the connection between neutrosophic \mathbb{N} -structures and algebraic structures (hypersructures) grabbed the attention of algebraist researchers. For example, Al-Tahan^[5] studied single valued neutrosophic polygroups, Khan et al.^[15] discussed neutrosophic \mathbb{N} -subsemigroups, Park studied

neutrosophic ideals of subtraction algebras, and Al-Tahan and Davvaz^[7] studied neutrosophic \mathbb{N} -ideals of subtraction algebras.

A question arises here:

“Is there a certain relationship between SVNS and neutrosophic \mathbb{N} -structures?”

Another question arises now:

“What would be the effect of such a relationship between SVNS and neutrosophic \mathbb{N} -structures on the study of both: single valued neutrosophic algebraic structures (hypertstructures) and neutrosophic \mathbb{N} -substructures (subhypertstructures)?”

This article answers the above two questions and it is constructed as follows: after an Introduction, in Section 2, we find a relationship between SVNS and neutrosophic \mathbb{N} -structures. In Section 3, we discuss the effect of such a relationship between SVNS and neutrosophic \mathbb{N} -structures on the study of both: single valued neutrosophic algebraic structures (hypertstructures) and neutrosophic \mathbb{N} -substructures (subhypertstructures) and we deal with some examples of algebraic structures (hypertstructures).

2 Relationship between SVNS and neutrosophic \mathbb{N} -structures

In this section, we find a relationship between SVNS and neutrosophic \mathbb{N} -structures and study it. Moreover, we illustrate our results by some examples.

Definition 2.1. ^[24] Let X be a space of points (objects), with a generic element in X denoted by x . A single valued neutrosophic set (SVNS) A on X is characterized by truth-membership T_A , indeterminacy-membership function I_A and falsity-membership function F_A . For each point $x \in X$, $T_A(x), I_A(x), F_A(x) \in [0, 1]$.

Definition 2.2. ^{[14][15]} Let X be a non-empty set. A neutrosophic \mathbb{N} -structure over X is defined as follows:

$$S_N = \left\{ \frac{x}{(T_N(x), I_N(x), F_N(x))} : x \in X \right\}$$

where T_N, I_N, F_N are \mathbb{N} -functions on X (i.e. functions on X with codomain $[-1, 0]$) which are called the negative truth membership function, the negative indeterminacy membership function and the negative falsity membership function, respectively, on X .

Definition 2.3. Let X be a non-empty set, $\alpha, \beta, \gamma \in [0, 1]$, and A a SVNS over X . Then the (α, β, γ) -level set of A is defined as follows:

$$L_{(\alpha, \beta, \gamma)} = \{x \in X : T_A(x) \geq \alpha, I_A(x) \geq \beta, F_A(x) \leq \gamma\}.$$

Definition 2.4. Let X be a non-empty set, $\alpha, \beta, \gamma \in [-1, 0]$, and S_N a neutrosophic \mathbb{N} -structure over X . Then the (α, β, γ) -level set of S_N is defined as follows:

$$\bar{L}_{(\alpha, \beta, \gamma)} = \{x \in X : T_N(x) \leq \alpha, I_N(x) \geq \beta, F_N(x) \leq \gamma\}.$$

Definition 2.5. ^[24] Let X be a non-empty set and A, B be single valued neutrosophic sets over X defined as follows.

$$A = \left\{ \frac{x}{(T_A(x), I_A(x), F_A(x))} : x \in X \right\}, B = \left\{ \frac{x}{(T_B(x), I_B(x), F_B(x))} : x \in X \right\}$$

Then

- A is called a single valued neutrosophic subset of B and denoted as $A \subseteq B$ if $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$, and $F_A(x) \geq F_B(x)$ for all $x \in X$.
If $A \subseteq B$ and $B \subseteq A$ then $A = B$.
- The union of A and B is defined to be the SVNS over X :

$$A \cup B = \left\{ \frac{x}{(T_{A \cup B}(x), I_{A \cup B}(x), F_{A \cup B}(x))} : x \in X \right\}.$$

Where $T_{A \cup B}(x) = T_A(x) \vee T_B(x)$, $I_{A \cup B}(x) = I_A(x) \vee I_B(x)$, and $F_{A \cup B}(x) = F_A(x) \wedge F_B(x)$ for all $x \in X$.

- The intersection of A and B is defined to be the SVN over X :

$$S_{A \cap B} = \left\{ \frac{x}{(T_{A \cap B}(x), I_{A \cap B}(x), F_{A \cap B}(x))} : x \in X \right\}.$$

Where $T_{A \cap B}(x) = T_A(x) \wedge T_B(x)$, $I_{A \cap B}(x) = I_A(x) \wedge I_B(x)$, and $F_{A \cap B}(x) = F_A(x) \vee F_B(x)$ for all $x \in X$.

Definition 2.6. [15] Let X be a non-empty set and S_N, S_M be neutrosophic \mathbb{N} -structures over X defined as follows.

$$S_N = \left\{ \frac{x}{(T_N(x), I_N(x), F_N(x))} : x \in X \right\}, S_M = \left\{ \frac{x}{(T_M(x), I_M(x), F_M(x))} : x \in X \right\}$$

Then

- S_N is called a neutrosophic \mathbb{N} -substructure of S_M and denoted as $S_N \subseteq S_M$ if $T_N(x) \geq T_M(x)$, $I_N(x) \leq I_M(x)$, and $F_N(x) \geq F_M(x)$ for all $x \in X$.
If $S_N \subseteq S_M$ and $S_M \subseteq S_N$ then $S_N = S_M$.

- The union of S_N and S_M is defined to be the \mathbb{N} -structure over X :

$$S_{N \cup M} = \left\{ \frac{x}{(T_{N \cup M}(x), I_{N \cup M}(x), F_{N \cup M}(x))} : x \in X \right\}.$$

Where $T_{N \cup M}(x) = T_N(x) \wedge T_M(x)$, $I_{N \cup M}(x) = I_N(x) \vee I_M(x)$, and $F_{N \cup M}(x) = F_N(x) \wedge F_M(x)$ for all $x \in X$.

- The intersection of S_N and S_M is defined to be the \mathbb{N} -structure over X :

$$S_{N \cap M} = \left\{ \frac{x}{(T_{N \cap M}(x), I_{N \cap M}(x), F_{N \cap M}(x))} : x \in X \right\}.$$

Where $T_{N \cap M}(x) = T_N(x) \vee T_M(x)$, $I_{N \cap M}(x) = I_N(x) \wedge I_M(x)$, and $F_{N \cap M}(x) = F_N(x) \vee F_M(x)$ for all $x \in X$.

For more details about operations on SVN and operations on neutrosophic \mathbb{N} -structures, we refer to the papers [14][15][24]

Proposition 2.7. Let X be a non-empty set, A, S_N be defined as follows:

$$A = \left\{ \frac{x}{(T_A(x), I_A(x), F_A(x))} : x \in X \right\}, S_N = \left\{ \frac{x}{(-T_A(x), I_A(x) - 1, F_A(x) - 1)} : x \in X \right\}.$$

Then A is a SVN over X if and only if S_N is a neutrosophic \mathbb{N} -structure of X .

Proof. Let A be a SVN of X . Then for every $x \in X$, $0 \leq T_A(x), I_A(x), F_A(x) \leq 1$. The latter implies that $-1 \leq -T_A(x), I_A(x) - 1, F_A(x) - 1 \leq 0$. Thus, S_N is a neutrosophic \mathbb{N} -structure of X . Similarly, if S_N is a neutrosophic \mathbb{N} -structure of X then A is a SVN of X . \square

Example 2.8. Let $X = \{0, 1, 2\}$ and $A = \left\{ \frac{0}{(0.1, 0.9, 0.3)}, \frac{1}{(0.7, 0.3, 0.5)}, \frac{2}{(0.8, 0.5, 0.3)} \right\}$ be a SVN over X . Then $S_N = \left\{ \frac{0}{(-0.1, -0.1, -0.7)}, \frac{1}{(-0.7, -0.7, -0.5)}, \frac{2}{(-0.8, -0.5, -0.7)} \right\}$ is a neutrosophic \mathbb{N} -structure of X .

Theorem 2.9. Let A be a SVN of X and $0 \leq \alpha, \beta, \gamma \leq 1$. Then $L_{\alpha, \beta, \gamma} = \bar{L}_{-\alpha, \beta-1, \gamma-1}$.

Proof. We have $L_{\alpha, \beta, \gamma} = \{x \in X : T_A(x) \geq \alpha, I_A(x) \geq \beta, F_A(x) \leq \gamma\}$ and $\bar{L}_{-\alpha, \beta-1, \gamma-1} = \{x \in X : T_N(x) \leq -\alpha, I_N(x) \geq \beta - 1, F_N(x) \leq \gamma - 1\}$. Having $T_A(x) \geq \alpha, I_A(x) \geq \beta$, and $F_A(x) \leq \gamma$ equivalent to $T_N(x) = -T_A(x) \leq -\alpha, I_N(x) = I_A(x) - 1 \geq \beta - 1$, and $F_N(x) = F_A(x) - 1 \leq \gamma - 1$ respectively implies that $L_{\alpha, \beta, \gamma} = \bar{L}_{-\alpha, \beta-1, \gamma-1}$. \square

Proposition 2.10. Let X be a non-empty set, S_N, A be defined as follows:

$$S_N = \left\{ \frac{x}{(T_N(x), I_N(x), F_N(x))} : x \in X \right\}, A = \left\{ \frac{x}{(-T_N(x), I_N(x) + 1, F_N(x) + 1)} : x \in X \right\}.$$

Then A is a SVN of X if and only if S_N is a neutrosophic \mathbb{N} -structure of X .

Proof. Let A be a SVN of X . Then for every $x \in X$, $0 \leq -T_N(x), I_N(x) + 1, F_N(x) + 1 \leq 1$. The latter implies that $-1 \leq T_N(x), I_N(x), F_N(x) \leq 0$. Thus, S_N is a neutrosophic \mathbb{N} -structure of X . Similarly, if S_N is a neutrosophic \mathbb{N} -structure of X then A is a SVN of X . \square

Example 2.11. Let $X = \{0, 1, 2\}$ and $S_N = \left\{ \frac{0}{(-0.1, -0.9, -0.3)}, \frac{1}{(-0.7, -0.3, -0.5)}, \frac{2}{(0, -1, -0.3)} \right\}$ be a neutrosophic \mathbb{N} -structure over X . Then $A = \left\{ \frac{0}{(0.1, 0.1, 0.7)}, \frac{1}{(0.7, 0.7, 0.5)}, \frac{2}{(0, 0, 0.7)} \right\}$ a SVN over X .

Theorem 2.12. Let A be a SVN of X and $-1 \leq \alpha, \beta, \gamma \leq 0$. Then $L_{-\alpha, 1+\beta, 1+\gamma} = \bar{L}_{\alpha, \beta, \gamma}$

Proof. The proof is similar to that of Theorem 2.9 \square

3 Applications to algebraic structures (hyperstructures)

In this section, we apply the relationship we found in Section 2 between SVN and neutrosophic \mathbb{N} -structures on some algebraic structure (hypersructures) and we present our main theorems in Subsection 3.4.

3.1 Applications to semigroups

In, [15] Khan et al. discussed neutrosophic \mathbb{N} -structures and applied it to semigroups. In this subsection, we deduce some of their results by applying the relationship that we found in Section 2 between SVN and neutrosophic \mathbb{N} -structures.

A semigroup is a groupoid that satisfies the associative axiom. For example, the set of positive integers under standard addition, the set of negative integers under standard addition, the set of integers modulo a positive integer n under standard multiplication modulo n are semigroups.

Definition 3.1. Let (X, \circ) be a semigroup and A a SVN over X . Then A is single valued neutrosophic semigroup over X if for all $x, y \in X$, the following conditions hold:

- $T_A(x \circ y) \geq T_A(x) \wedge T_A(y)$;
- $I_A(x \circ y) \geq I_A(x) \wedge I_A(y)$;
- $F_A(x \circ y) \leq F_A(x) \vee F_A(y)$.

Definition 3.2. [15] Let (X, \circ) be a semigroup and S_N a neutrosophic \mathbb{N} -structure over X . Then S_N is neutrosophic \mathbb{N} -subsemigroup of X if for all $x, y \in X$, the following conditions hold:

- $T_N(x \circ y) \leq T_N(x) \vee T_N(y)$;
- $I_N(x \circ y) \geq I_N(x) \wedge I_N(y)$;
- $F_N(x \circ y) \leq F_N(x) \vee F_N(y)$.

Remark 3.3. Let a, b be any real numbers. Then

- $1 + (a \wedge b) = (1 + a) \wedge (1 + b)$;
- $1 + (a \vee b) = (1 + a) \vee (1 + b)$;
- if $c = a \wedge b$ then $-c = (-a) \vee (-b)$;
- if $d = a \vee b$ then $-d = (-a) \wedge (-b)$.

Theorem 3.4. Let (X, \circ) be a semigroup and S_N a neutrosophic \mathbb{N} -structure over X . Then S_N is neutrosophic \mathbb{N} -subsemigroup of X if and only if A is a single valued neutrosophic semigroup over X . Here,

$$S_N = \left\{ \frac{x}{(T_N(x), I_N(x), F_N(x))} : x \in X \right\}, A = \left\{ \frac{x}{(-T_N(x), I_N(x) + 1, F_N(x) + 1)} : x \in X \right\}.$$

Proof. Let A be a single valued neutrosophic semigroup over X and $x, y \in X$. Then $-T_N(x \circ y) \geq (-T_N(x)) \wedge (-T_N(y))$, $1 + I_N(x \circ y) \geq (1 + I_N(x)) \wedge (1 + I_N(y))$, and $1 + F_N(x \circ y) \leq (1 + F_N(x)) \vee (1 + F_N(y))$. The latter implies that $T_N(x \circ y) \leq T_N(x) \vee T_N(y)$, $I_N(x \circ y) \geq I_N(x) \wedge I_N(y)$, and $F_N(x \circ y) \leq F_N(x) \vee F_N(y)$. Thus, S_N is neutrosophic \mathbb{N} -subsemigroup of X . Similarly, we can prove that if S_N is neutrosophic \mathbb{N} -subsemigroup of X then A is a single valued neutrosophic semigroup over X . \square

Theorem 3.5. Let (X, \circ) be a semigroup and A a SVN over X . Then A is a single valued neutrosophic semigroup over X if and only if $L_{(\alpha, \beta, \gamma)}$ is either the empty set or a subsemigroup of X for all $0 \leq \alpha, \beta, \gamma \leq 1$.

Proof. The proof is similar to that of Theorem 5.1. \square

Theorem 3.6. Let (X, \circ) be a semigroup and A a SVN over X . Then A is single valued neutrosophic semigroup over X if and only if $\bar{L}_{(\alpha, \beta, \gamma)}$ is either the empty set or a subsemigroup of X for all $-1 \leq \alpha, \beta, \gamma \leq 0$.

Proof. Let $-1 \leq \alpha, \beta, \gamma \leq 0$. Then there exist $0 \leq \alpha', \beta', \gamma' \leq 1$ such that $\alpha' = -\alpha$, $\beta' = \beta + 1$, and $\gamma' = \gamma + 1$. Theorem 3.5 asserts that $L_{(\alpha', \beta', \gamma')}$ is either the empty set or a subsemigroup of X . The latter and Theorem 2.12 imply that $\bar{L}_{(\alpha, \beta, \gamma)} = L_{(\alpha', \beta', \gamma')}$ is either the empty set or a subsemigroup of X .

Let $0 \leq \alpha', \beta', \gamma' \leq 1$. Then there exist $-1 \leq \alpha, \beta, \gamma \leq 0$ such that $\alpha' = -\alpha$, $\beta' = \beta + 1$, and $\gamma' = \gamma + 1$. But having $\bar{L}_{(\alpha, \beta, \gamma)}$ is either the empty set or a subsemigroup of X and that $L_{(\alpha', \beta', \gamma')} = \bar{L}_{(\alpha, \beta, \gamma)}$ imply that $L_{(\alpha', \beta', \gamma')}$ is either the empty set or a subsemigroup of X for all $0 \leq \alpha', \beta', \gamma' \leq 1$. Thus, A is single valued neutrosophic semigroup over X by Theorem 3.5. \square

Theorem 3.7. Let (X, \circ) be a semigroup and S_N a neutrosophic \aleph -structure over X where,

$$S_N = \left\{ \frac{x}{(T_N(x), I_N(x), F_N(x))} : x \in X \right\}, A = \left\{ \frac{x}{(-T_N(x), I_N(x) + 1, F_N(x) + 1)} : x \in X \right\}.$$

Then the following statements are equivalent.

1. S_N is a neutrosophic \aleph -subsemigroup of X ;
2. A is a single valued neutrosophic semigroup over X ;
3. $\bar{L}_{(\alpha, \beta, \gamma)}$ is either the empty set or a subsemigroup of X for all $-1 \leq \alpha, \beta, \gamma \leq 0$;
4. $L_{(\alpha, \beta, \gamma)}$ is either the empty set or a subsemigroup of X for all $0 \leq \alpha, \beta, \gamma \leq 1$.

Proof. The proof follows from Theorem 3.4, Theorem 3.5 and Theorem 3.6. \square

Example 3.8. Let $(\mathbb{Z}^+, +)$ be the semigroup of positive integers under standard addition. Let

$$(T_N(x), I_N(x), F_N(x)) = \begin{cases} (-0.6, -0.4, -0.7) & \text{if } x \text{ is a multiple of 2;} \\ (-0.5, -0.5, -0.6) & \text{otherwise.} \end{cases}$$

Then $S_N = \left\{ \frac{x}{(T_N(x), I_N(x), F_N(x))} : x \in \mathbb{Z}^+ \right\}$ is a neutrosophic \aleph -subsemigroup of \mathbb{Z}^+ as $A = \left\{ \frac{x}{(T_A(x), I_A(x), F_A(x))} : x \in \mathbb{Z}^+ \right\}$ is a single valued neutrosophic semigroup over \mathbb{Z}^+ . Where

$$(T_A(x), I_A(x), F_A(x)) = \begin{cases} (0.6, 0.6, 0.3) & \text{if } x \text{ is a multiple of 2;} \\ (0.5, 0.5, 0.4) & \text{otherwise.} \end{cases}$$

3.2 Applications to polygroups


In [5] Al-Tahan defined single valued neutrosophic polygroups and studied their properties. In this subsection, we use the result in [5] with the relationship we found in Section 2 between SVN and neutrosophic \aleph -structures to prove some results on neutrosophic \aleph -subpolygroups.

Algebraic hyperstructures represent a natural generalization of classical algebraic structures and they were introduced by Marty [16] in 1934 at the eighth Congress of Scandinavian Mathematicians. Where he generalized the notion of a group to that of a hypergroup. He defined a hypergroup as a set equipped with associative and reproductive hyperoperation. In a group, the composition of two elements is an element whereas in a hypergroup, the composition of two elements is a set. Many researchers worked on hypertstructure theory and its applications. We refer to [12][10][2]. A certain subclasses of hypergroups were introduced such as polygroups. The latter were introduced by Comer [9] where he emphasized their importance in connections to graphs, relations, Boolean and cylindric algebras. For more details about polygroups and their applications, we refer to [11][6][4].

Definition 3.9. [11] Let P be a non-empty set. Then, a mapping $\circ : P \times P \rightarrow \mathcal{P}^*(P)$ is called a *binary hyperoperation* on P , where $\mathcal{P}^*(P)$ is the family of all non-empty subsets of P . The couple (P, \circ) is called a *hypergroupoid*.

In the above definition, if A and B are two non-empty subsets of P and $x \in P$, then we define:

$$A \circ B = \bigcup_{\substack{a \in A \\ b \in B}} a \circ b, \quad x \circ A = \{x\} \circ A \text{ and } A \circ x = A \circ \{x\}.$$

Definition 3.10.  A polygroup is a system $\langle P, \circ, e, {}^{-1} \rangle$, where $e \in P$, ${}^{-1} : P \rightarrow P$ is a unitary operation on P , “ \circ ” maps $P \times P$ into the non-empty subsets of P , and the following axioms hold for all $x, y, z \in P$:


1. $(x \circ y) \circ z = x \circ (y \circ z)$,
2. $e \circ x = x \circ e = \{x\}$,
3. $x \in y \circ z$ implies $y \in x \circ z^{-1}$ and $z \in y^{-1} \circ x$.

Let (P, \circ) be a polygroup and $K \subseteq P$. Then (K, \circ) is a subpolygroup of (P, \circ) if for all $a, b \in K$, we have that $a \circ b \subseteq K$ and $a^{-1} \in K$.

Example 3.11. Let $P = \{e, a, b\}$ and define the polygroup (P_1, \circ_1) by Table [1](#).

Table 1: The polygroup (P_1, \circ_1)

\circ_1	e	a	b
e	e	a	b
a	a	$\{e, b\}$	$\{a, b\}$
b	b	$\{a, b\}$	$\{e, a\}$

Definition 3.12.  Let (P, \circ) be a polygroup and A a SVN over X . Then A is single valued neutrosophic polygroup of P if for all $x, y \in P$, the following conditions hold:

- $T_A(x \circ y) \geq T_A(x) \wedge T_A(y)$;
- $I_A(x \circ y) \geq I_A(x) \wedge I_A(y)$;
- $F_A(x \circ y) \leq F_A(x) \vee F_A(y)$;
- $T_A(x^{-1}) \geq T_A(x)$, $I_A(x^{-1}) \geq I_A(x)$, $F_A(x^{-1}) \leq F_A(x)$.


Definition 3.13. Let (P, \circ) be a polygroup and S_N a neutrosophic \aleph -structure over P . Then S_N is neutrosophic \aleph -subpolygroup of P if for all $x, y \in P$, the following conditions hold:

- $T_N(x \circ y) \leq T_N(x) \vee T_N(y)$;
- $I_N(x \circ y) \geq I_N(x) \wedge I_N(y)$;
- $F_N(x \circ y) \leq F_N(x) \vee F_N(y)$;
- $T_N(x^{-1}) \leq T_N(x)$, $I_N(x^{-1}) \geq I_N(x)$, $F_N(x^{-1}) \leq F_N(x)$.

Theorem 3.14. Let (P, \circ) be a polygroup and S_N a neutrosophic \aleph -structure over P . Then S_N is neutrosophic \aleph -subpolygroup of P if and only if A is a single valued neutrosophic polygroup over X . Here,

$$S_N = \left\{ \frac{x}{(T_N(x), I_N(x), F_N(x))} : x \in P \right\}, \quad A = \left\{ \frac{x}{(-T_N(x), I_N(x) + 1, F_N(x) + 1)} : x \in P \right\}.$$

Proof. Let A be a single valued neutrosophic polygroup over P and $x, y \in P$. Then $-T_N(x \circ y) \geq (-T_N(x)) \wedge (-T_N(y))$, $1 + I_N(x \circ y) \geq (1 + I_N(x)) \wedge (1 + I_N(y))$, and $1 + F_N(x \circ y) \leq (1 + F_N(x)) \vee (1 + F_N(y))$. The latter implies that $T_N(x \circ y) \leq T_N(x) \vee T_N(y)$, $I_N(x \circ y) \geq I_N(x) \wedge I_N(y)$, and $F_N(x \circ y) \leq F_N(x) \vee F_N(y)$. Moreover, having $-T_N(x^{-1}) \geq -T_N(x)$, $I_N(x^{-1}) - 1 \geq I_N(x) - 1$, $F_N(x^{-1}) - 1 \leq F_N(x) - 1$ implies that $T_N(x^{-1}) \leq T_N(x)$, $I_N(x^{-1}) \geq I_N(x)$, $F_N(x^{-1}) \leq F_N(x)$. Thus, S_N is neutrosophic \aleph -subpolygroup of P . Similarly, we can prove that if S_N is neutrosophic \aleph -subpolygroup of P then A is a single valued neutrosophic polygroup over P . \square

Theorem 3.15.  Let (P, \circ) be a polygroup and A a SVN over P . Then A is single valued neutrosophic polygroup over X if and only if $L_{(\alpha, \beta, \gamma)}$ is either the empty set or a subpolygroup of P for all $0 \leq \alpha, \beta, \gamma \leq 1$.

Theorem 3.16. Let (P, \circ) be a polygroup and A a SVN over X . Then A is single valued neutrosophic polygroup over X if and only if $\bar{L}_{(\alpha, \beta, \gamma)}$ is either the empty set or a subpolygroup of P for all $-1 \leq \alpha, \beta, \gamma \leq 0$.

Proof. The proof is similar to the proof of Theorem 3.6. □

Theorem 3.17. Let (P, \circ) be a polygroup and S_N a neutrosophic \aleph -structure over P where,

$$S_N = \left\{ \frac{x}{(T_N(x), I_N(x), F_N(x))} : x \in P \right\}, A = \left\{ \frac{x}{(-T_N(x), I_N(x) + 1, F_N(x) + 1)} : x \in P \right\}.$$

Then the following statements are equivalent.

1. S_N is a neutrosophic \aleph -subpolygroup of P ;
2. A is a single valued neutrosophic polygroup over P ;
3. $\bar{L}_{(\alpha, \beta, \gamma)}$ is either the empty set or a subpolygroup of X for all $-1 \leq \alpha, \beta, \gamma \leq 0$;
4. $L_{(\alpha, \beta, \gamma)}$ is either the empty set or a subpolygroup of X for all $0 \leq \alpha, \beta, \gamma \leq 1$.

Proof. The proof follows from Theorem 3.4, Theorem 3.14, and Theorem 3.15. □

Example 3.18. Let (P_1, \circ_1) be the polygroup defined in Example 3.11. Then

$$S_N = \left\{ \frac{e}{(-0.7, -0.4, -0.9)}, \frac{a}{(-0.6, -0.6, -0.8)}, \frac{b}{(-0.6, -0.6, -0.8)} \right\}$$

is a neutrosophic \aleph -subpolygroup of P_1 as $A = \left\{ \frac{e}{(0.7, 0.6, 0.1)}, \frac{a}{(0.6, 0.4, 0.2)}, \frac{b}{(0.6, 0.4, 0.2)} \right\}$ is a single valued neutrosophic polygroup over P_1 .

Remark 3.19. Theorem 3.17 implies that the results known for single valued neutrosophic polygroups in [5] hold also for neutrosophic \aleph -subpolygroups.

3.3 Applications to subtraction algebras

Park in [17] Al-Tahan and Davvaz in [7] defined neutrosophic ideals and \aleph -ideals of subtraction algebras respectively and studied their properties. In this subsection, we use the results in [17] with the relationship we found in Section 2 between SVN and neutrosophic \aleph -structures to some results on neutrosophic \aleph -ideals of subtraction algebras that were proved in [7].

Subtraction algebra was introduced by Shein in 1992 [18] and some results about it can be found in [13, 23].

Definition 3.20. [23] An algebra $(X, -)$ is called a subtraction algebra if the single binary operation “-” satisfies the following identities: for any $x, y, z \in X$,

1. $x - (y - x) = x$;
2. $x - (x - y) = y - (y - x)$;
3. $(x - y) - z = (x - z) - y$.

Definition 3.21. [13] A non-empty subset I of a subtraction algebra X is called an ideal of X if it satisfies the following conditions.

1. $a - x \in I$ for all $a \in I$ and $x \in X$;
2. for all $a, b \in I$, whenever $a \vee b$ exists in X then $a \vee b \in I$.

Example 3.22. Let $X_1 = \{0, 1, 2\}$ and define the subtraction algebra $(X_1, -_1)$ by Table 2.

Definition 3.23. [17] Let $(X, -)$ be a subtraction algebra and A a SVN over X . Then A is single valued neutrosophic ideal of X if for all $x, y \in X$, the following conditions hold:

- $T_A(x - y) \geq T_A(x)$;
- $I_A(x - y) \geq I_A(x)$;

Table 2: The subtraction algebra $(X_1, -_1)$

$-_1$	0	1	2
0	0	0	0
1	1	0	1
2	2	2	0

- $F_A(x - y) \leq F_A(x)$;
- if $x \vee y$ exists in X then $T_A(x \vee y) \geq T_A(x) \wedge T_A(y)$, $I_A(x \vee y) \geq I_A(x) \wedge I_A(y)$, and $F_A(x \vee y) \leq F_A(x) \vee F_A(y)$.

Definition 3.24. [7] Let (X, \circ) be a subtraction algebra and S_N a neutrosophic \aleph -structure over X . Then S_N is neutrosophic \aleph -ideal of X if for all $x, y \in X$, the following conditions hold:

- $T_N(x - y) \leq T_N(x)$;
- $I_N(x - y) \geq I_N(x)$;
- $F_N(x - y) \leq F_N(x)$;
- if $x \vee y$ exists in X then $T_N(x \vee y) \leq T_N(x) \vee T_N(y)$, $I_N(x \vee y) \geq I_N(x) \wedge I_N(y)$, and $F_N(x \vee y) \leq F_N(x) \vee F_N(y)$.

Theorem 3.25. Let $(X, -)$ be a subtraction algebra and S_N a neutrosophic \aleph -structure over X . Then S_N is neutrosophic \aleph -ideal of X if and only if A is a neutrosophic ideal of X . Here,

$$S_N = \left\{ \frac{x}{(T_N(x), I_N(x), F_N(x))} : x \in X \right\}, A = \left\{ \frac{x}{(-T_N(x), I_N(x) + 1, F_N(x) + 1)} : x \in X \right\}.$$

Proof. The proof is similar to the proof of Theorem 3.14. \square

Theorem 3.26. [7] Let (X, \circ) be a subtraction algebra and A a SVN over X . Then S_N is neutrosophic ideal of X if $L_{(\alpha, \beta, \gamma)}$ is either the empty set or ideal of X for all $0 \leq \alpha, \beta, \gamma \leq 1$.

Theorem 3.27. Let $(X, -)$ be a subtraction algebra and A a SVN over X . Then A is neutrosophic ideal of X if and only if $\bar{L}_{(\alpha, \beta, \gamma)}$ is either the empty set or an ideal of X for all $-1 \leq \alpha, \beta, \gamma \leq 0$.

Proof. The proof is similar to the proof of Theorem 3.6. \square

Theorem 3.28. Let $(X, -)$ be a subtraction algebra and S_N a neutrosophic \aleph -structure over X where,

$$S_N = \left\{ \frac{x}{(T_N(x), I_N(x), F_N(x))} : x \in X \right\}, A = \left\{ \frac{x}{(-T_N(x), I_N(x) + 1, F_N(x) + 1)} : x \in X \right\}.$$

. Then the following statements are equivalent.

1. S_N is a neutrosophic \aleph -subpolygroup of P ;
2. A is a single valued neutrosophic polygroup over P ;
3. $\bar{L}_{(\alpha, \beta, \gamma)}$ is either the empty set or a subpolygroup of X for all $-1 \leq \alpha, \beta, \gamma \leq 0$;
4. $L_{(\alpha, \beta, \gamma)}$ is either the empty set or a subpolygroup of X for all $0 \leq \alpha, \beta, \gamma \leq 1$.

Proof. The proof follows from Theorem 3.25, Theorem 3.26, and Theorem 3.27. \square

The authors proved in [7] the following theorem which can be deduced from Theorem 3.28.

Theorem 3.29. [7] Let $(X, -)$ be a subtraction algebra and S_N a neutrosophic \aleph -structure over X . Then S_N is neutrosophic \aleph -ideal of X if and only if $\bar{L}_{(\alpha, \beta, \gamma)}$ is either the empty set or ideal of X for all $-1 \leq \alpha, \beta, \gamma \leq 0$.

Example 3.30. Let $(X_1, -_1)$ be the subtraction algebra defined in Example 3.22. Then

$$S_N = \left\{ \frac{0}{(-0.7, -0.4, -0.9)}, \frac{1}{(-0.7, -0.4, -0.9)}, \frac{2}{(-0.6, -0.6, -0.8)} \right\}$$

is a neutrosophic \aleph -ideal of X_1 as $A = \left\{ \frac{0}{(0.7, 0.6, 0.1)}, \frac{1}{(0.7, 0.6, 0.1)}, \frac{2}{(0.6, 0.4, 0.2)} \right\}$ is a neutrosophic ideal of X_1 .

3.4 Generalization to any algebraic structure (hyperstructure)

We can deduce from the work presented in the previous subsections that neutrosophic substructures (subhyperstructures) and neutrosophic \aleph -substructures (subhyperstructures) are connected. The following two theorems generalize our work.

Theorem 3.31. *Let X be any algebraic structure (hyperstructure) and S_N a neutrosophic \aleph -structure over X . Then S_N is neutrosophic \aleph -substructure (subhyperstructure) of X if and only if A is a single valued neutrosophic algebraic structure (hyperstructure) over X . Here,*

$$S_N = \left\{ \frac{x}{(T_N(x), I_N(x), F_N(x))} : x \in X \right\}, A = \left\{ \frac{x}{(-T_N(x), I_N(x) + 1, F_N(x) + 1)} : x \in X \right\}.$$

Theorem 3.32. *Let X be any algebraic structure (hyperstructure) and S_N a neutrosophic \aleph -structure over X where,*

$$S_N = \left\{ \frac{x}{(T_N(x), I_N(x), F_N(x))} : x \in X \right\}, A = \left\{ \frac{x}{(-T_N(x), I_N(x) + 1, F_N(x) + 1)} : x \in X \right\}.$$

Then the following statements are equivalent.

1. S_N is a neutrosophic \aleph -substructure (subhyperstructure) of X ;
2. A is a single valued neutrosophic algebraic structure (hyperstructure) over P ;
3. $\bar{L}_{(\alpha, \beta, \gamma)}$ is either the empty set or a substructure (subhyperstructure) of X for all $-1 \leq \alpha, \beta, \gamma \leq 0$;
4. $L_{(\alpha, \beta, \gamma)}$ is either the empty set or a substructure (subhyperstructure) of X for all $0 \leq \alpha, \beta, \gamma \leq 1$.

Remark 3.33. Theorem 3.32 implies that if some results are known for single valued algebraic structures (hyperstructures) such as single valued neutrosophic groups, rings, hypergroups, hyperrings, etc., then these results hold also for neutrosophic \aleph -substructures (subhyperstructures) of these algebraic structures (hyperstructures).

4 Conclusion and discussion

SVNS and neutrosophic \aleph -structures grabbed the attention of neutrosophic researchers. In this paper, we found a relationship between the two concepts. And we used this relation to prove that there is a connection between neutrosophic substructures (subhyperstructures) and neutrosophic \aleph -substructures (subhyperstructures). Moreover, we presented examples on this connection by dealing with specific algebraic substructures (subhyperstructures) such as semigroups, polygroups, and subtraction algebras. As a result, we were able to deduce that by defining a new single valued neutrosophic structures (hyperstructures) over a given algebraic structure (hyperstructure) and working on it, we can immediately define neutrosophic \aleph -substructures (subhyperstructures) of the same algebraic structure (hyperstructure) and the results that we get for SVNS will be applicable for neutrosophic \aleph -structures.

For future work, it will be interesting to find more applications on SVNS and to project the relationship between SVNS and neutrosophic \aleph -structures we found in this paper on the new applications.

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