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American Scientific Publishing Group (ASPG) LLC,
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Mailing Address: 625 Wright Ave, Gretna, LA 70056, USA
Phone: +1(504) 336-3385
Fax: +1-888-417-7465
e-mail: manager@americaspg.com
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Aim and Scope

International Journal of Neutrosophic Science (IJNS) is a peer-review journal publishing high quality experimental and theoretical research in all areas of Neutrosophic and its Applications. IJNS is published quarterly. IJNS is devoted to the publication of peer-reviewed original research papers lying in the domain of neutrosophic sets and systems. Papers submitted for possible publication may concern with foundations, neutrosophic logic and mathematical structures in the neutrosophic setting. Besides providing emphasis on topics like artificial intelligence, pattern recognition, image processing, robotics, decision making, data analysis, data mining, applications of neutrosophic mathematical theories contributing to economics, finance, management, industries, electronics, and communications are promoted. Variants of neutrosophic sets including refined neutrosophic set (RNS). Articles evolving algorithms making computational work handy are welcome.

Topics of Interest

IJNS promotes research and reflects the most recent advances of neutrosophic Sciences in diverse disciplines, with emphasis on the following aspects, but certainly not limited to:

- Neutrosophic sets
- Neutrosophic algebra
- Neutrosophic topolog
- Neutrosophic graphs
- Neutrosophic probabilities
- Neutrosophic tools for decision making
- Neutrosophic theory for machine learning
- Neutrosophic statistics
- Neutrosophic numerical measures
- Classical neutrosophic numbers
- A neutrosophic hypothesis
- The neutrosophic level of significance
- The neutrosophic confidence interval
- The neutrosophic central limit theorem
- Neutrosophic theory in bioinformatics and medical analytics
- Neutrosophic tools for big data analytics
- Neutrosophic tools for deep learning
- Neutrosophic tools for data visualization
- Quadripartitioned single-valued neutrosophic sets
- Refined single-valued neutrosophic sets

- Applications of neutrosophic logic in image processing
- Neutrosophic logic for feature learning, classification, regression, and clustering
- Neutrosophic knowledge retrieval of medical images
- Neutrosophic set theory for large-scale image and multimedia processing
- Neutrosophic set theory for brain-machine interfaces and medical signal analysis
- Applications of neutrosophic theory in large-scale healthcare data
- Neutrosophic set-based multimodal sensor data
- Neutrosophic set-based array processing and analysis
- Wireless sensor networks Neutrosophic set-based Crowd-sourcing
- Neutrosophic set-based heterogeneous data mining
- Neutrosophic in Virtual Reality
- Neutrosophic and Plithogenic theories in Humanities and Social Sciences
- Neutrosophic and Plithogenic theories in decision making
- Neutrosophic in Astronomy and Space Sciences

A New Trend to Extensions of CI-algebras

[1] Florentin Smarandache, [2,*] Akbar Rezaei, [3] Hee Sik Kim

[1] Department of Mathematics & Sciences, University of New Mexico, Gallup, NM 87301, USA.
E-mail: smarand@unm.edu
[2] Department of Mathematics, Payame Noor University, P.O. Box. 19395-3697, Tehran, Iran.
E-mail: rezaei@pnu.ac.ir
[3] Department of Mathematics, Hanyang University, Seoul 04763, Korea.
E-mail: heekim@hanyang.ac.kr
* Correspondence: rezaei@pnu.ac.ir

Abstract

In this paper, as an extension of CI-algebras, we discuss the new notions of Neutro-CI-algebras and Anti-CI-algebras. First, some examples are given to show that these definitions are different. We prove that any proper CI-algebra is a Neutro-BE-algebra or Anti-BE-algebra. Also, we show that any NeutroSelf-distributive and AntiCommutative CI-algebras are not BE-algebras.

Keywords: CI-algebra, Neutro-CI-algebra, Anti-CI-algebra, Self-distributive, NeutroSelf-distributive, AntiSelf-distributive, Commutative, NeutroCommuative, AntiCommutative.

1. Introduction

H.S. Kim et al. introduced the notion of BE-algebras as a generalization of dual BCK-algebras [1]. A. Walendziak defined the notion of commutative BE-algebras and discussed some of their properties [11]. A. Rezaei et al. investigated the relationship between Hilbert algebras and BE-algebras [5]. B.L. Meng introduced the notion of CI-algebras as a generalization of BE-algebras and dual BCI/BCK-algebras, and studied some relations with BE-algebras [2]. Then he defined the notion of atoms in CI-algebras and singular CI-algebras and investigated their properties [3]. Filters and upper sets were studied in detail by B. Piekart et al. [4].

Recently, in 2019-2020 F. Smarandache [8, 9, 10] constructed for the first time the neutrosophic triple corresponding to the Algebraic Structures as (Algebraic Structure, NeutroAlgebraic Structure, AntiAlgebraic Structure), where a (classical) Algebraic Structure is an algebraic structure dealing only with (classical) Operations) (that are totally well-defined) and (classical) Axioms (that are totally true). A NeutroAlgebraic Structure is an algebraic structure that has at least one NeutroOperation or NeutroAxiom, and no AntiOperation and no AntiAxiom, while an AntiAlgebraic Structure is an algebraic structure that has at least one AntiOperation or one AntiAxiom. Moreover, some left (right)-quasi neutrosophic triplet structures in BE-algebras were studied by X. Zhang et al. [12].

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The aim of this paper is to characterize these definitions to CI-algebras. Also, the notions of NeutroSelf-distributive / AntiSelf-distributive and NeutroCommutative / AntiCommutative in CI-algebras are studied. Finally, as an alternative to the definition of CI-algebra, Neutro-CI-algebra and Anti-CI-algebra are defined.

2. Preliminaries

In this section we recall some basic notions and results regarding CI-algebras and BE-algebras. CI-algebras were introduced in [2] as a generalization of BE-algebras (see [1]) and properties of them have recently been studied in [3] and [4].

Definition 2.1. ([2]) A *CI-algebra* is an algebra $(X, \to, 1)$ of type $(2, 0)$ (i.e. X is a non-empty set, $\to$ is a binary operation and 1 is a constant element) satisfying the following axioms, for all $x, y, z \in X$:

(CI1) $(\forall x \in X)(x \to x = 1)$;

(CI2) $(\forall x \in X)(1 \to x = x)$;

(CI3) $(\forall x, y, z \in X, with\, x \neq y)(x \to (y \to z) = y \to (x \to z))$.

We introduce a binary relation $\leq$ on X by $x \leq y$ if and only if $x \to y = 1$. A CI-algebra $(X, \to, 1)$ is said to be a *BE-algebra* ([1]) if

(BE) $(\forall x \in X)(x \to 1 = 1)$.

By (CI1) $\leq$ is only reflexive.

In what follows, let X be a CI-algebra unless otherwise specified. A CI-algebra X is proper if it is not a BE-algebra.

For example, the set $X = \{1, a\}$, with the following Cayley table is a proper CI-algebra, since $a \to 1 = a \neq 1$.

Table 1

$\to$	1	a
1	1	a
a	a	1

Theorem 2.2. Let $(X, \to, 1)$ be a CI-algebra. The binary operation $\to$ is associative if and only if $x \to 1 = x$, for all $x \in X$.

Proof. Assume that $\to$ is associative. Using (CI2) and associativity, we have

$$x = 1 \to x = (x \to x) \to x = x \to (x \to x) = x \to 1.$$

Conversely, suppose that $x \to 1 = x$, for all $x \in X$. Let $x, y, z \in X$, then by applying assumption and three times (CI3), we get

$(x \to y) \to z = (x \to y) \to (z \to 1) = z \to ((x \to y) \to 1) = z \to (x \to y) = z \to (x \to (y \to 1)) = x \to (\to (y \to 1)) = x \to (y \to (z \to 1)) = x \to (y \to z)$.

Thus, $(x \to y) \to z = x \to (y \to z)$.

Also, if $\to$ is associative relation, then CI-algebra $(X, \to, 1)$ is an Abelian group with identity 1, since

$$x \to y = x \to (y \to 1) = y \to (x \to 1) = y \to x.$$

Example 2.3. (i) Let $\mathbb{R}$ be the set of all real numbers and $\to$ be the binary operation on $\mathbb{R}$ defined by $x \to y = y \div x$, where $\div$ is the binary operation of division. Then $(\mathbb{R} - \{0\}, \to, 1)$ is a CI-algebra, but it is not a BE-algebra.

(CI1) holds, since for every $0 \neq x \in \mathbb{R}$, $x \to x = x \div x = 1$;

(CI2) valid, since for all $x \in X$, $1 \to x = x$;

(CI3) holds. Let $x, y, z \in X$. Then we have

$x \to (y \to z) = x \to (z \div y) = (z \div y) \div x = (z \div x) \div y = y \to (z \div x) = y \to (x \to z)$.

(BE) is not valid, since $5 \to 1 = 1 \div 5 = \frac{1}{5} \neq 1$. Thus, $(\mathbb{R} - \{0\}, \to, 1)$ is a proper CI-algebra.

(ii) Consider the real interval [0,1] and let $\to$ be the binary operation on [0,1] defined by $x \to y = 1 - x + xy$. Then $([0,1], \to, 1)$ is not a CI-algebra (so, is not a BE-algebra), since (CI1) and (CI3) are not valid. Note that (BE) holds, since $x \to 1 = 1 - x + x.1 = 1 - x + x = 1$.

Proposition 2.4. ([2]) Let $\boldsymbol{X}$ be a CI-algebra. Then for all $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X}$,

(i) $\boldsymbol{y} \to ((\boldsymbol{y} \to \boldsymbol{x}) \to \boldsymbol{x}) = \mathbf{1}$;

(ii) $(\boldsymbol{x} \to \mathbf{1}) \to (\boldsymbol{y} \to \mathbf{1}) = (\boldsymbol{x} \to \boldsymbol{y}) \to \mathbf{1}$.

Definition 2.5. ([1, 2]) A CI/BE-algebra $\boldsymbol{X}$ is said to be *self-distributive* if for any $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in \boldsymbol{X}$,

$$\boldsymbol{x} \to (\boldsymbol{y} \to \boldsymbol{z}) = (\boldsymbol{x} \to \boldsymbol{y}) \to (\boldsymbol{x} \to \boldsymbol{z}).$$

Example 2.6. Consider the CI-algebra given in Example 2.3 (i). It is not self-distributive. Let $\boldsymbol{x} \coloneqq \mathbf{5}, \boldsymbol{y} \coloneqq \mathbf{4}$ and $\boldsymbol{z} \coloneqq \mathbf{7}$. Then we have $\mathbf{5} \to (\mathbf{4} \to \mathbf{7}) = \mathbf{5} \to \frac{\mathbf{7}}{\mathbf{4}} = \frac{\mathbf{7}}{\mathbf{20}} \neq (\mathbf{5} \to \mathbf{4}) \to (\mathbf{5} \to \mathbf{7}) = \frac{\mathbf{4}}{\mathbf{5}} \to \frac{\mathbf{7}}{\mathbf{5}} = \frac{\mathbf{7}}{\mathbf{4}}$.

Proposition 2.7. ([2]) Every self-distributive CI-algebra $\boldsymbol{X}$ is a BE-algebra.

Note that if $\boldsymbol{X}$ is self-distributive, then $\leq$ is transitive ([6]).

Definition 2.8. ([2, 5, 11]) A CI/BE-algebra X is said to be commutative if for any $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X}$,

$$\boldsymbol{x} \to (\boldsymbol{x} \to \boldsymbol{y}) = \boldsymbol{y} \to (\boldsymbol{y} \to \boldsymbol{x}).$$

Example 2.9. ([6]) (i) Let $\mathbb{N}$ be the set of all natural numbers and $\to$ be the binary operation on $\mathbb{N}$ defined by

$$\boldsymbol{x} \to \boldsymbol{y} = \begin{cases} \boldsymbol{y} & \boldsymbol{if} \;\; \boldsymbol{x} = \mathbf{1}; \\ \mathbf{1} & \boldsymbol{otherwise}. \end{cases}$$

Then $(\mathbb{N}, \to, \mathbf{1})$ is a non-commutative BE-algebra.

(ii) Let $\mathbb{N}_{\mathbf{0}} = \mathbb{N} \cup \{\mathbf{0}\}$ and let $\to$ be the binary operation on $\mathbb{N}_{\mathbf{0}}$ defined by

$$\boldsymbol{x} \to \boldsymbol{y} = \begin{cases} \mathbf{0} & \boldsymbol{if} \; \boldsymbol{x} \geq \boldsymbol{y}; \\ \boldsymbol{y} - \boldsymbol{x} & \boldsymbol{otherwise}. \end{cases}$$

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Then $(\mathbb{N}_0, \to, \mathbf{0})$ is a commutative BE-algebra ([6]), but it is not self-distributive, since

$$\mathbf{5 \to (6 \to 7) = 5 \to 1 = 0 \neq (5 \to 6) \to (5 \to 7) = 1 \to 2 = 1.}$$

Proposition 2.10. ([2]) Every commutative CI-algebra $\boldsymbol{X}$ is a BE-algebra.

Note that if $\boldsymbol{X}$ is commutative, then $\leq$ is anti-symmetric ([6]). Hence, if $\boldsymbol{X}$ is a commutative and self-distributive CI-algebra, then $\leq$ is a partially ordered set ([6]).

3. On NeutroSelf-distributive and AntiSelf-distributive CI-algebras

Definition 3.1. A CI-algebra $\boldsymbol{X}$ is said to be *NeutroSelf-distributive* if

$$(\exists \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in \boldsymbol{X})(\boldsymbol{x} \to (\boldsymbol{y} \to \boldsymbol{z}) = (\boldsymbol{x} \to \boldsymbol{y}) \to (\boldsymbol{x} \to \boldsymbol{z})) \text{ and } (\exists \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in \boldsymbol{X})(\boldsymbol{x} \to (\boldsymbol{y} \to \boldsymbol{z}) \neq (\boldsymbol{x} \to \boldsymbol{y}) \to (\boldsymbol{x} \to \boldsymbol{z})).$$

Example 3.2. Consider the non-self-distributive CI-algebra given in Example 2.3 (i). If $\boldsymbol{x} \coloneqq \mathbf{1}$, then for all $\boldsymbol{y}, \boldsymbol{z} \in \mathbb{R} - \{\mathbf{0}\}$, we have $\boldsymbol{x} \to (\boldsymbol{y} \to \boldsymbol{z}) = (\boldsymbol{x} \to \boldsymbol{y}) \to (\boldsymbol{x} \to \boldsymbol{z})$. If $\boldsymbol{x} \neq \mathbf{1}$, then for all $\boldsymbol{y}, \boldsymbol{z} \in \mathbb{R} - \{\mathbf{0}\}$, we have $\boldsymbol{x} \to (\boldsymbol{y} \to \boldsymbol{z}) \neq (\boldsymbol{x} \to \boldsymbol{y}) \to (\boldsymbol{x} \to \boldsymbol{z})$. Hence $(\mathbb{R} - \{\mathbf{0}\}, \to, \mathbf{1})$ is a NeutroSelf-distributive CI-algebra.

Definition 3.3. A CI-algebra $\boldsymbol{X}$ is said to be *AntiSelf-distributive* if

$$(\forall \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in \boldsymbol{X}, \boldsymbol{with}\ \boldsymbol{x} \neq \mathbf{1})(\boldsymbol{x} \to (\boldsymbol{y} \to \boldsymbol{z}) \neq (\boldsymbol{x} \to \boldsymbol{y}) \to (\boldsymbol{x} \to \boldsymbol{z})).$$

Example 3.4. Consider the CI-algebra given in Example 2.3 (i). Then it is an AntiSelf-distributive CI-algebra, since for all $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in \mathbb{R} - \{\mathbf{0}\}$ and $\boldsymbol{x} \neq \mathbf{1}$, we can see that

$$\boldsymbol{x} \to (\boldsymbol{y} \to \boldsymbol{z}) = (\boldsymbol{z} \div \boldsymbol{y}) \div \boldsymbol{x} = \frac{\boldsymbol{z}}{\boldsymbol{yx}} \neq (\boldsymbol{x} \to \boldsymbol{y}) \to (\boldsymbol{x} \to \boldsymbol{z}) = (\boldsymbol{z} \div \boldsymbol{x}) \div (\boldsymbol{y} \div \boldsymbol{x}) = \frac{\boldsymbol{z}}{\boldsymbol{y}}.$$

Theorem 3.5. Let $\boldsymbol{X}$ be an AntiSelf-distributive CI-algebra. Then $\boldsymbol{X}$ is not a BE-algebra.

Proof. Assume that $\boldsymbol{X}$ is an AntiSelf-distributive CI-algebra and $\mathbf{1} \neq \boldsymbol{x} \in \boldsymbol{X}$. Take $\boldsymbol{y} = \boldsymbol{z} = \mathbf{1}$ and using AntiSelf-distributivity and applying (CI1) two times, we have

$$\boldsymbol{x} \to \mathbf{1} = \boldsymbol{x} \to (\mathbf{1} \to \mathbf{1}) \neq (\boldsymbol{x} \to \mathbf{1}) \to (\boldsymbol{x} \to \mathbf{1}) = \mathbf{1}.$$

Thus, $(\forall \boldsymbol{x} \in \boldsymbol{X}, \boldsymbol{with}\ \boldsymbol{x} \neq \mathbf{1})(\boldsymbol{x} \to \mathbf{1} \neq \mathbf{1})$, and so $\boldsymbol{X}$ is not a BE-algebra.

Corollary 3.6. There is no AntiSelf-distributive BE-algebra.

Proposition 3.7. Let $\boldsymbol{X}$ be an AntiSelf-distributive CI-algebra. Then

$$(\forall \boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X}, \boldsymbol{with}\ \boldsymbol{x} \neq \mathbf{1})(\boldsymbol{x} \to (\boldsymbol{x} \to \boldsymbol{y}) \neq \boldsymbol{x} \to \boldsymbol{y}).$$

Proof. Let $\boldsymbol{X}$ be a CI-algebra and $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X}$. Using AntiSelf-distributivity and (CI2), we get

$$\boldsymbol{x} \to (\boldsymbol{x} \to \boldsymbol{y}) \neq (\boldsymbol{x} \to \boldsymbol{x}) \to (\boldsymbol{x} \to \boldsymbol{y}) = \mathbf{1} \to (\boldsymbol{x} \to \boldsymbol{y}) = \boldsymbol{x} \to \boldsymbol{y}.$$

Thus, $\boldsymbol{x} \to (\boldsymbol{x} \to \boldsymbol{y}) \neq \boldsymbol{x} \to \boldsymbol{y}$.

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Proposition 3.8. Let X be an AntiSelf-distributive CI-algebra, and $x \leq y$. Then $z \to x \nleq z \to y$, for all $1 \neq z \in X$.

Proof. Suppose that X is an AntiSelf-distributive CI-algebra, $x \leq y$ and $1 \neq z \in X$. Then $x \to y = 1$. Applying AntiSelf-distributivity and (BE), we get

$$(z \to x) \to (z \to y) \neq z \to (x \to y) = z \to 1 \neq 1.$$

Thus, $z \to x \nleq z \to y$, for all $1 \neq z \in X$.

Proposition 3.9. Let X be an AntiSelf-distributive CI-algebra. Then $\leq$ is not transitive.

Proof. Suppose that X is an AntiSelf-distributive CI-algebra, $x \leq y$ and $y \leq z$. Then $x \to y = 1$ and $y \to z = 1$. Using (CI2) and AntiSelf-distributivity, we have

$$x \to z = 1 \to (x \to z) = (x \to y) \to (x \to z) \neq x \to (y \to z) = x \to 1 \neq 1.$$

Thus, $x \nleq z$.

4. On NeutroCommutative and AntiCommutative CI-algebras

Definition 4.1. A CI/BE-algebra X is said to be *NeutroCommutative* if

$$(\exists x, y \in X \text{ with } x \neq y)(x \to (x \to y) = y \to (y \to x)) \text{ and } (\exists x, y \in X)(x \to (x \to y) \neq y \to (y \to x)).$$

Example 4.2. (i) Consider the non-commutative BE-algebra given in Example 2.9 (i). If $x, y \in \mathbb{N} - \{1\}$, then $x \to (x \to y) = y \to (y \to x)$. If $x = 1$ and $y \neq 1$, then $x \to (x \to y) = y \neq y \to (y \to x) = 1$.

(ii) Consider the CI-algebra given in Example 2.3 (i). Then it is not a NeutroCommutative CI-algebra, since, for all $x, y \in \mathbb{R} - \{0\}$, we have $x \to (x \to y) \neq y \to (y \to x)$, only if $x = y = 1$, then $x \to (x \to y) = y \to (y \to x)$. Thus, there is not $x \neq y$ such that $x \to (x \to y) = y \to (y \to x)$.

Definition 4.3. A CI/BE-algebra X is said to be *AntiCommutative* if

$$(\forall x, y \in X \text{ with } x \neq y)((x \to y) \to y \neq (y \to x) \to x).$$

Example 4.4. Consider the CI-algebra given in Example 2.3 (i). Then it is an AntiCommutative CI-algebra.

Proposition 4.5. Let X be an AntiCommutative CI-algebra. Then X is not a BE-algebra.

Proof. By contrary, let X be a BE-algebra. Then for all $x \in X$, $x \to 1 = 1$. Hence $(x \to 1) \to 1 = 1 \to 1 = 1$, by assumption and (CI1). Now, applying AntiCommutaitivity and (CI2) we get

$$1 = (x \to 1) \to 1 \neq (1 \to x) \to x = x \to x = 1.$$

Thus, $1 \neq 1$ which is a contradiction.

Corollary 4.6. There is no AntiCommutative BE-algebra.

Proposition 4.7. Let X be an AntiCommutative CI-algebra. If $x \leq y$, then $y \neq (y \to x) \to x$.

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Proof. Assume that $\boldsymbol{X}$ be an AntiCommutative CI-algebra and $\boldsymbol{x} \leq \boldsymbol{y}$. Then $\boldsymbol{x} \to \boldsymbol{y} = \mathbf{1}$. Using (CI2) and AntiCommutaitivity, we have

$$\boldsymbol{y} = \mathbf{1} \to \boldsymbol{y} = (\boldsymbol{x} \to \boldsymbol{y}) \to \boldsymbol{y} \neq (\boldsymbol{y} \to \boldsymbol{x}) \to \boldsymbol{x}.$$

Proposition 4.8. Let $\boldsymbol{X}$ be an AntiCommutative CI-algebra. Then $\leq$ is not an anti-symmetric relation on $\boldsymbol{X}$.

Proof. Assume that $\boldsymbol{X}$ be an AntiCommutative CI-algebra. Let $\boldsymbol{x} \leq \boldsymbol{y}$ and $\boldsymbol{y} \leq \boldsymbol{x}$. Then $\boldsymbol{x} \to \boldsymbol{y} = \mathbf{1}$ and $\boldsymbol{y} \to \boldsymbol{x} = \mathbf{1}$. Applying (CI2) and AntiCommutativity, we get

$$\boldsymbol{x} = \mathbf{1} \to \boldsymbol{x} = (\boldsymbol{y} \to \boldsymbol{x}) \to \boldsymbol{x} \neq (\boldsymbol{x} \to \boldsymbol{y}) \to \boldsymbol{y} = \mathbf{1} \to \boldsymbol{y} = \boldsymbol{y}.$$

Corollary 4.9. If $\boldsymbol{X}$ is an AntiSelf-distributive or an AntiCommutative CI-algebra, then X endowed with the induced relation $\leq$ is not a partially ordered set.

Proof. By Propositions 3.9 and 4.8, we get the desired result.

If $\boldsymbol{X}$ is not a partially ordered set, then $\boldsymbol{X}$ is either totally ordered set, or totally unordered set (i.e. for any two distinct elements $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X}$, neither $\boldsymbol{x} \leq \boldsymbol{y}$ nor $\boldsymbol{y} \leq \boldsymbol{x}$).

We have the neutrosophic triplet for the order relationship $\leq$ in a similar way as for CI-algebras:

(totally ordered, partially ordered and partially unordered, totally unordered) or (Ordered, NeutroOrdered, AntiOrdered).

Corollary 4.10. If $\boldsymbol{X}$ is an AntiSelf-distributive or an AntiCommutative CI-algebra, then $\boldsymbol{x} \to (\boldsymbol{y} \to \boldsymbol{x}) \neq \mathbf{1}$, for all $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X}$.

Proof. Using Corollaries 3.6 and 4.5, $\boldsymbol{X}$ is not a BE-algebra, and so applying (CI3) and (CI1) we get, for all $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X}$

$$\boldsymbol{x} \to (\boldsymbol{y} \to \boldsymbol{x}) = \boldsymbol{y} \to (\boldsymbol{x} \to \boldsymbol{x}) = \boldsymbol{y} \to \mathbf{1} \neq \mathbf{1}.$$

5. On Neutro-CI-algebras and Anti-CI-algebras

The Neutro-BE-algebra and the Anti-BE-algebra as an alternative of a BE-algebra was defined in 2020 by A. Rezaei and F. Smarandache. Now, we can define Neutro-CI-algebra and Anti-CI-algebra (for detail see [7]).

Definition 5.1. (Neutro-sophications)

The Neutro-sophication of the Law (degree of well-defined, degree of indeterminacy, degree of outer-defined)

(NL) $(\exists \boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X})(\boldsymbol{x} \to \boldsymbol{y} \in \boldsymbol{X})$ and $(\exists \boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X})(\boldsymbol{x} \to \boldsymbol{y} = \boldsymbol{indeterminate}$ or $\boldsymbol{x} \to \boldsymbol{y} \notin \boldsymbol{X})$.

The Neutro-sophication of the Axioms (degree of truth, degree of indeterminacy, degree of falsehood)

(NCI1) $(\exists \boldsymbol{x} \in \boldsymbol{X})(\boldsymbol{x} \to \boldsymbol{x} = \mathbf{1})$ and $(\exists \boldsymbol{x} \in \boldsymbol{X})(\boldsymbol{x} \to \boldsymbol{x} =$ *indeterminate* or $\boldsymbol{x} \to \boldsymbol{x} \neq \mathbf{1})$;

(NCI2) $(\exists \boldsymbol{x} \in \boldsymbol{X})(\mathbf{1} \to \boldsymbol{x} = \boldsymbol{x})$ **and** $(\exists \boldsymbol{x} \in \boldsymbol{X})(\mathbf{1} \to \boldsymbol{x} =$ *indeterminate* or $1 \to \boldsymbol{x} \neq \boldsymbol{x})$;

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(NCI3) $(\exists \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in \boldsymbol{X}, \boldsymbol{with}\ \boldsymbol{x} \neq \boldsymbol{y})(\boldsymbol{x} \rightarrow (\boldsymbol{y} \rightarrow \boldsymbol{z}) = \boldsymbol{y} \rightarrow (\boldsymbol{x} \rightarrow \boldsymbol{z}))$ and $(\exists \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in \boldsymbol{X}, \boldsymbol{with}\ \boldsymbol{x} \neq \boldsymbol{y})(\boldsymbol{x} \rightarrow (\boldsymbol{y} \rightarrow \boldsymbol{z}) = \boldsymbol{indeterminate}\ \ \boldsymbol{or}\ \boldsymbol{x} \rightarrow (\boldsymbol{y} \rightarrow \boldsymbol{z}) \neq \boldsymbol{y} \rightarrow (\boldsymbol{x} \rightarrow \boldsymbol{z}))$.

Definition 5.2. (Anti-sophications)

The Anti-sophication of the Law (totally outer-defined)

(AL) $(\forall \boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X})(\boldsymbol{x} \rightarrow \boldsymbol{y} \notin \boldsymbol{X})$.

The Anti-sophication of the Axioms (totally false)

(ACI1) $(\forall \boldsymbol{x} \in \boldsymbol{X})(\boldsymbol{x} \rightarrow \boldsymbol{x} \neq \mathbf{1})$;

(ACI2) $(\forall \boldsymbol{x} \in \boldsymbol{X})(\mathbf{1} \rightarrow \boldsymbol{x} \neq \boldsymbol{x})$;

(ACI3) $(\forall \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in \boldsymbol{X}, \mathbf{with}\ \boldsymbol{x} \neq \boldsymbol{y})(\boldsymbol{x} \rightarrow (\boldsymbol{y} \rightarrow \boldsymbol{z}) \neq \boldsymbol{y} \rightarrow (\boldsymbol{x} \rightarrow \boldsymbol{z}))$.

Definition 5.3. A *Neutro-CI-algebra* is an alternative of CI-algebra that has at least a (NL) or at least one (NCIt), $\boldsymbol{t} \in \{\mathbf{1}, \mathbf{2}, \mathbf{3}\}$, with no anti-law and no anti-axiom.

Definition 5.4. An *Anti-CI-algebra* is an alternative of CI-algebra that has at least an (AL) or at least one (NCIt), $\boldsymbol{t} \in \{\mathbf{1}, \mathbf{2}, \mathbf{3}\}$.

A Neutro-BE-algebra ([7]) is a Neutro-CI-algebra or has (NBE), where

(NBE) $(\exists \boldsymbol{x} \in \boldsymbol{X})(\boldsymbol{x} \rightarrow \mathbf{1} = \mathbf{1})$ and $(\exists \boldsymbol{x} \in \boldsymbol{X})(\boldsymbol{x} \rightarrow \mathbf{1} = \boldsymbol{indeterminate}$ or $\boldsymbol{x} \rightarrow \mathbf{1} \neq \mathbf{1})$.

An Anti-BE-algebra ([7]) is an Anti-CI-algebra or has (ABE), where

(ABE) $(\forall \boldsymbol{x} \in \boldsymbol{X})(\boldsymbol{x} \rightarrow \mathbf{1} \neq \mathbf{1})$.

Note that any proper CI-algebra may be a Neutro-BE-algebra or Anti-BE-algebra.

Proposition 5.5. Every NeutroSelf-distributive CI-algebra is a Neutro-CI-algebra.

Proposition 5.6. Every AntiSelf-distributive CI-algebra is an Anti-BE-algebra.

Proposition 5.7. Every AntiCommutative CI-algebra is an Anti-BE-algebra.

6. Conclusions

In this paper, Neutro-CI-algebras and Anti-CI-algebras are introduced and discussed based on the definition of CI-algebras. By some examples we showed that these notions were different. Some of their properties were provided. We proved that any proper CI-algebra is a Neutro-BE-algebra or Anti-BE-algebra. Further, for every classical CI-algebra, it was shown that, if it is AntiSelf-distributive or AntiCommutative, then it is an Anti-BE-algebra.

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A SVTrN-number approach of multi-objective optimisation on the basis of simple ratio analysis based on MCDM method

[1]D. Ajay, [2]J. Aldring, [3]S. Abirami, [3]D. Jeni Seles Martina
[1]Assistant Professor, Department of Mathematics, Sacred Heart Collge, Tirupattur District, India.
[2,3] Department of Mathematics, Sacred Heart Collge, Tirupattur District, India.
*Correspondence email: dajaypravin@gmail.com[1]

Abstract

An essential process in the study of human work behaviour or human resource management is the personnel selection/recruitment and a major challenge of an organization is to determine the most potential personnel. From this perspective, this paper tries to formulate MOOSRA approach in the neutrosophic environment to select potential personnel in organizations. The method proves to reduce complexity in computation and is comprehensible. An illustrative example is shown for better understanding of the proposed method.

Keywords: MCDM, Personnel Selection, Neutrosophic Set, SVTrN-number, MOOSRA Method

1 Introduction

The quality of an organization can be estimated by being able to select efficient personnel for the organization. Hence it is the essential process of human resource management. Sometimes the numerous and conflict personnel criteria may result in confusing the decision maker to select suitable persons. In view of rectifying such situations, the classical ideas or methods need to be enhanced with our recent trends. The study of literature reveals that many researchers used fuzzy set theory as an important tool for solving the multi critria decision making approach such as ELECTRE[1] , LINMAP (Linear Programming Techniques for Multidimensional Analysis of Preference)[2] , VIKOR model[3] , Analytic Hierarchic Process (AHP)[4,5] , TOPSIS[6,7] , WASPAS[8] , COPRAS[9] , and MOORA[10] . These methods can be employed in any of the fields which requires a decision making situation of fuzzy nature.

Fuzzy sets theories have been developed and generalised but found to be not able to deal with every situation of uncertainty that we come across in real problems. For example, Atanassov's intuitionistic fuzzy sets[11] deal with the degree of membership, non-membership and indeterminacy, and furthermore these functions are entirely mutually dependent. Therefore, this necessitated the emergence of new theories. F. Smarandache introduced a new theory which accounts for the truth, indeterminacy and falsity membership functions simultaneously and the corresponding set is called a neutrosophic set[12] . This notion laid a new path to research on uncertainty and many theories evolved from it in the last two decades and a good number of researchers have applied it to decision making science[16–18] .

Subas (2015) described single valued triangular neutrosophic (SVTrN) numbers as a special type of single valued neutrosophic numbers. From the available literature we can see that single valued triangular neutrosophic (SVTrN) numbers have been extended to different forms[19] . This SVTrN-numbers are used in many decision-making problems[20,21] and several complexities and dynamic issues arise in decision-making applications. Further, this paper makes an attempt to develop an new decision approach called neutrosophic MOOSRA method based on SVTrN-numbers which have broad application fields. It goes without saying that the personnel selection depends on many criteria and having to choose from many alternatives the decision makers' job of obtaining the best candidate becomes tedious. As these conflicting criteria and alternatives have the nature of full, partially full and null possibilities, employing neutrosophic sets in this context is justified.

The organisation of the rest of the paper: in section 2 the basic concepts of fuzzy sets are explained, section 3 provides detailed explanation of neutrosophic MOOSRA method. An illustration presented in section 4 validates the proposed theory and finally the conclusion based on the study is presented.

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2 Basic Concepts

This section recalls some of the fundamental concepts of fuzzy sets[13] and neutrosophic notion

Definition 2.1. [13] If x is a particular element of universe of discourse X, then a fuzzy set A is defined by a fuzzy membership function (μ_A) which takes the membership values in the unit closed interval of zero and one. i.e. $\mu_A : X \longrightarrow [0,1]$

Definition 2.2. A fuzzy decision matrix of the order $m \times n$ is defined as $A = [\mu_{ij}]_{m\times n}$, where μ_{ij} is the fuzzy membership values of the element in A which is generated by decision makers according to the alternatives on criteria.

Definition 2.3. [12] Let B be a neutrosophic set in the universal discourse or non empty set X and any object x in B has the form $B = \{< x, \mu_B(x), \sigma_B(x), \gamma_B(x) >; x \in X\}$, where $\mu_B(x), \sigma_B(x)$ and $\gamma_B(x)$ represent the degree of truth, the degree of indeterminacy and the degree of falsity membership functions respectively which take their values in the unit closed interval of zero and one, and it satisfies the following relation $0 \leq \mu_B(x) + \sigma_B(x) + \gamma_B(x) \leq 3$.

Definition 2.4. [14] A triangular single valued neutrosophic number $B = \langle (l_1, m_1, u_1) ; \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle$ is a special type of neutrosophic set on real number set R, whose degree of truthness, indeterminacy and falsity membership functions are defined as

$$\mu_B(x) = \begin{cases} \frac{(x-l_1)T_B}{m_1-l_1} & (l_1 \leq x \leq m_1), \\ T_B & (x = m_1), \\ \frac{(u_1-x)T_B}{u_1-m_1} & (m_1 \leq x \leq u_1), \\ 0 & otherwise. \end{cases}$$

$$\sigma_B(x) = \begin{cases} \frac{(m_1-x+I_B(x-l_1))}{m_1-l_1} & (l_1 \leq x \leq m_1), \\ I_B & (x = m_1), \\ \frac{(x-u_1+I_B(u_1-x))}{u_1-m_1} & (m_1 \leq x \leq u_1), \\ 1 & otherwise. \end{cases}$$

$$\gamma_B(x) = \begin{cases} \frac{(m_1-x+F_B(x-l_1))}{m_1-l_1} & (l_1 \leq x \leq m_1), \\ I_B & (x = m_1), \\ \frac{(x-u_1+F_B(u_1-x))}{u_1-m_1} & (m_1 \leq x \leq u_1), \\ 1 & otherwise. \end{cases}$$

respectively, where $T_B, I_B, F_B \in [0,1]$ and $l_1, m_1, u_1 \in R$.

3 Neutrosophic MOOSRA Methodology

MOOSRA was first introduced by Das et al.[15] and further many more researchers utilized it in selection process. Formation of a decision matrix is the first step in method which involves four parameters namely, alternatives available for selection, criteria upon which the selection has to be made, weights attributed to individual alternative and calculation of performance by alternatives over the fixed set of criteria. The similar process is adopted here for the neutrosophic MOOSRA method, for which the detailed procedure is described below.

Step 1 Form neutrosophic triangular scale

Step 2 Set the consistency for pairwise comparison matrix

Step 3 Create random consistency index for various criterion

PHASE I Acquire the expert's information in neutrosophic environment

Phase I starts with the concept of neutrosophic matrix in which the output of each alternative in relation to each criteria is evaluated.

Step 4 Create the pairwise comparison decision matrix of each criteria by decision maker's judgments as mentioned

$$C^M = \begin{bmatrix} B_{11}^M & \cdots & B_{1z}^M \\ \vdots & \ddots & \vdots \\ B_{y1}^M & \cdots & B_{yz}^M \end{bmatrix}$$

Step 5 Find crisp values of aggregated pairwise comparison matrix of criteria

The formula for finding the aggregate pairwise decision matrix is

$$B_{uv} = \left\langle \left(\frac{1}{M} \sum_{i=1}^{M} l_{uv}^i, \frac{1}{M} \sum_{i=1}^{M} m_{uv}^i, \frac{1}{M} \sum_{i=1}^{M} u_{uv}^i \right); \frac{1}{M} \sum_{i=1}^{M} T_{uv}^i, \frac{1}{M} \sum_{i=1}^{M} I_{uv}^i, \frac{1}{M} \sum_{i=1}^{M} F_{uv}^i \right\rangle$$

where, M - number of decision makers, l_{uv}^M, m_{uv}^M, u_{uv}^M are lower, middle, upper bound of neutrosophic numbers, and T_{uv}^M, I_{uv}^M, F_{uv}^M are truth, indeterminacy, falsity membership values respectively.

By using score function of B_{uv} convert neutrosophic scales to crisp values,

$$s(B_{uv}) = \left| \frac{(l_{uv} * m_{uv} * u_{uv})^{(T_{uv}+I_{uv}+F_{uv})}}{9} \right| \tag{1}$$

where l, m, u denote lower, middle, upper scale of triangular neutrosophic numbers respectively.

PHASE II

Step 6 Calculate weight of criteria (w_u^y)

Compute average value of row by $w_u = \dfrac{\sum_{i=1}^{z} s(B_{uv})}{z}$; $u = 1, 2, 3, \ldots, y$; $v = 1, 2, 3, \ldots, z$

Step 7 The given equation measures normalization of crisp values

$$w_u^y = \frac{w_u}{\sum_{u=1}^{y} w_u} \tag{2}$$

PHASE III Evaluate expert judgement using consistency rate

Step 8 Calculate weighted columns sum i.e. Multiply the weights of criteria with each value of pairwise comparison matrix

Step 9 Next, weighted sum values is divided by the weights of each criteria

Step 10 Compute λ_{max} values by finding the means of previous step

Step 11 Compute consistency index by the formula

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

where, n is the number of criteria.

Step 12 Compute consistency rate by the following equation

$$CR = \frac{CI}{RI} \tag{3}$$

Here, CR - consistency rate, CI - consistency index, RI - random index for consistency matrix

PHASE IV MOOSRA Method

Step 13 Form decision judgment values of each alternative on criteria interms of triangular neutrosophic pairwise comparison decision matrix

Step 14 Ultilize the score function to calculate crisp values of aggregated comparison decision matrix

Step 15 Next, find the normalization of the decision matrix by

$$B^*_{uv} = \frac{B_{uv}}{\sqrt{\sum_{u=1}^{y} B^2_{uv}}}$$

Step 16 Compute the sum of beneficial and non-beneficial criteria of weighted normalized values of matrix and denote as Y^+ and Y^- respectively.

$$Y^+ = \sum_{j=1}^{g} w_v x^*_{ij}$$

$$Y^- = \sum_{j=g+1}^{n} w_v x^*_{ij}$$

Step 17 Overall performances of each alternatives is obtained by MOOSRA method using the following equation

$$y^*_i = \frac{\sum_{j=1}^{g} x^*_{ij}}{\sum_{j=g+1}^{n} x^*_{ij}}$$

Step 18 Ranking of the alternatives is obtained according to the overall performance score of each alternative y^*_i

4 Illustration of the Proposed method

An illustrative example is chosen based on the above framework for explaining the research work and the importance of the proposed model. The case study is about how to select an organization's employees. The following criteria are used in the case study namely Boldness (C_1), Good in work (C_2), Team management (C_3), Uniqueness (C_4), Good character (C_5), Commitment (C_6) and five alternatives have been taken. Here C_1, C_2, C_4, C_5 are beneficial criteria and C_3, C_6 are non-beneficial criteria and also four decision makers are provided to form decision matrix.

The beginning frame work has been taken in the following table.1-3.

Table 1: Neutrosophic triangular scale

Satty scale	Significance level	Neutrosophic triangular scale
1	Even	$\bar{1}$ =$<< 1, 1, 1 >$; 0.50, 0.50, 0.50$>$
3	A little	$\bar{3}$ =$<< 2, 3, 4 >$; 0.30, 0.75, 0.70$>$
5	Powerful	$\bar{5}$ = $<< 4, 5, 6 >$; 0.80, 0.15, 0.20$>$
7	Completely powerful	$\bar{7}$ = $<< 6, 7, 8 >$; 0.90, 0.10, 0.10$>$
9	Absolute	$\bar{9}$ = $<< 9, 9, 0 >$; 1.00, 0.00, 0.00$>$
2	Values lying between two close scales	$\bar{2}$ = $<< 1, 2, 3 >$; 0.40, 0.60, 0.65$>$
4		$\bar{4}$ = $<< 3, 4, 5 >$; 0.35, 0.60, 0.40$>$
6		$\bar{6}$ = $<< 5, 6, 7 >$; 0.70, 0.25, 0.30$>$
8		$\bar{8}$ = $<< 7, 8, 9 >$; 0.85, 0.10, 0.15$>$

Table 2: The consistency rate for pair-wise comparison matrix

N	4*4	5*5	N>4
CR$\leq$	0.58	0.90	1.12

Table 3: Random Consistency Table

Size of matrix	1	2	3	4	5	6	7	8	9	10
Random Consistency	0.00	0.00	0.58	0.0	1.12	1.24	1.32	1.41	1.45	1.49

Next, the decision makers provide pairwise comparison decision matrix of each criteria and their judgments values are listed in the appendix A from table.9 to table.12

Using score function formula 1 , we convert neutrosophic values to crisp values

$$\begin{aligned} C_1C_2 \implies & << 4,5,6 >; 0.80, 0.15, 0.20 >, \\ & << 2,3,4 >; 0.30, 0.75, 0.70 >, \\ & << 2,3,4 >; 0.30, 0.75, 0.70 >, \\ & << 4,5,6 >; 0.80, 0.15, 0.20 > . \end{aligned}$$

$$\begin{aligned} C_1C_2 \implies & << 12,16,20 >; 2.2, 1.8, 1.8 >, \\ & << 3,4,5 >; 0.55, 0.45, 0.45 >, \\ & \Rightarrow (60)^{0.161111111} = 1.93410, \end{aligned}$$

Similarly, we get all the other values presented in the below table.

Table 4: Aggregated crisp value of pairwise comparison decision matrix

criteria	C_1	C_2	C_3	C_4	C_5	C_6
C_1	1	1.93410	1.91656	1.99676	1.98939	1.93410
C_2	0.51703	1	1.81218	1.64382	2.06426	2.13217
C_3	0.52176	0.55182	1	1.97710	1.93410	1.64382
C_4	0.50081	0.60833	0.50579	1	1.83468	2.04143
C_5	0.50266	0.48443	0.51703	0.54505	1	1.58543
C_6	0.51703	0.46900	0.60833	0.48985	0.63074	1

PHASE 2

Weights of each criteria is calculated by step.6

w_1 = 10.77091, w_2 = 9.16946, w_3 = 7.6286, w_4 = 6.49104, w_5 = 4.6346, w_6 = 3.71495

Using equation.2 the normalization of crisp value is calculated and the values are listed as follows:

$$\begin{aligned} \sum_{i=1}^{6} w_i &= w_1 + w_2 + w_3 + w_4 + w_5 + w_6 \\ &= 10.77091 + 9.16946 + 7.6286 + 6.49104 + 4.6346 + 3.71495 \\ \sum_{i=1}^{6} w_i &= 42.40956 \end{aligned}$$

$$w_1 = \frac{10.77091}{42.40956} \Rightarrow w_1 = 0.25397$$

similarly, w_2 = 0.21621, w_3 = 0.17987, w_4 = 0.15305, w_5 = 0.10928, w_6 = 0.08759

PHASE 3 Compute weighted column sum

The weighted criteria are multiplied by each column in the pairwise comparison decision matrix and adding these values we can get weighted columns sum and the values are given as

w_1 = 1.70927, w_2 = 1.33740, w_3 = 1.08962, w_4 = 0.88204, $w_5 = 0.65696$, $w_6 = 0.57362$

Therefore, λ_{max} = 6.21137

Consistency index

The formula for finding the consistency index is CI = $\frac{\lambda_{max}-n}{n-1}$ where, n is the number of criteria

The value of CI = 0.04227.

Consistency ratio

Consistency Rate (CR) = $\frac{ConsistencyIndex}{RandomIndex}$ = $\frac{0.04227}{1.24}$ = 0.03408

PHASE 4 Judgment values of each alternative on criteria is formed interms of triangular neutrosophic decision matrix from four decision makers and the values are represented in the appendix B from tables 13 to 15.

Using score function formula 1, we can convert the neutrosophic values to crisp values

$A_1C_1 \implies << 4,5,6 >; 0.80, 0.15, 0.20 >, << 4,5,6 >; 0.80, 0.15, 0.20 >,$

$<< 6,7,8 >; 0.90, 0.10, 0.10 >, << 5,6,7 >; 0.70, 0.25, 0.30 >$.

$\Rightarrow << 4.75, 5.75, 6.75 >; 0.8, 0.1625, 0.2 >$ = $(184.359375)^{0.129166666}$ = 1.96177

similarly all the other values are obtained and presented below in table.5

Table 5: The aggregated pairwise comparision decision matrix

A/C	C_1	C_2	C_3	C_4	C_5	C_6
A_1	1.96177	2.03598	1.93410	2.03931	1.43878	1.90053
A_2	2.08883	2.02514	1.87745	1.79506	1.99676	2.04893
A_3	2.03699	2.03931	1.91059	2.03948	1.55864	2.10585
A_4	2.02744	1.83493	1.84362	2.05460	2.12452	2.08595
A_5	1.98352	2.03931	1.68484	1.77208	2.01160	1.85179

Utiling the formula of normalization, we can get table.6

Table 6: The normalization decision matrix

A/C	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.43428	0.45604	0.46698	0.46906	0.34847	0.42470
A_2	0.46240	0.45361	0.45330	0.41288	0.48362	0.45786
A_3	0.45093	0.45679	0.46130	0.46909	0.37750	0.47058
A_4	0.44881	0.41101	0.44513	0.47257	0.51456	0.46614
A_5	0.43909	0.45679	0.40680	0.40759	0.48721	0.41381

Let $Y^{+}=\sum_{v=1}^{g} w_v\ B_{uv}^{*}$ be a beneficial criteria and $Y^{-}=\sum_{v=1}^{z}\ w_v\ B_{uv}^{*}$ be a non-beneficial criteria.

Table 7: The weighted normalized decision matrix

A/C	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.110294	0.098600	0.083995	0.071789	0.038080	0.037199
A_2	0.117435	0.098075	0.081535	0.063191	0.052849	0.040103
A_3	0.114522	0.098762	0.082974	0.071794	0.041253	0.041218
A_4	0.113984	0.08864	0.080065	0.072326	0.056231	0.040829
A_5	0.111515	0.098762	0.073171	0.062381	0.053242	0.036245

Table 8: Over all performances of the alternatives

	$\sum_{j=1}^{g} x_{ij}^*$	$\sum_{j=g+1}^{n} x_{ij}^*$	Y_i^*	Ranking
A_1	0.318763	0.121194	2.63018	4
A_2	0.33155	0.121638	2.72571	3
A_3	0.326331	0.124192	2.62763	5
A_4	0.331405	0.120894	2.741285	2
A_5	0.32590	0.109416	2.97854	1

5 Conclusion

To test the efficiency of the proposed neutrosophic MOOSRA method, a decision matrix is derived in consultation with experts and is utilized to finding the potential personnel selection in an organization. The application of the proposed technique implies that A5 is the best alternative or the requirement of the organization. Hence we propose that the decision makers can utilize neutrosophic MOOSRA method as the method is shown to be effective in personnel selection and reduces computational complexity and provides desired results in neutrosophic environment.

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Appendix A

PHASE-I

Table 9: The pairwise comparison neutrosophic decision matrix of each criteria

		C_1	C_2	C_3	C_4	C_5	C_6
DM1	C_1	<<1,1,1>; 0.50,0.50,0.50>	<<4,5,6>; 0.80,0.15,0.20>	<<6,7,8>; 0.90,0.10,0.10>	<<3,4,5>; 0.35,0.60,0.40>	<<2,3,4>; 0.30,0.75,0.70>	<<5,6,7>; 0.70,0.25,0.30>
	C_2	1/<<4,5,6>; 0.80,0.15,0.20>	<<1,1,1>; 0.50,0.50,0.50>	<<3,4,5>; 0.35,0.60,0.40>	<<5,6,7>; 0.70,0.25,0.30>	<<6,7,8>; 0.90,0.10,0.10>	<<2,3,4>; 0.30,0.75,0.70>
	C_3	1/<<6,7,8>; 0.90,0.10,0.10>	1/<<3,4,5>; 0.35,0.60,0.40>	<<1,1,1>; 0.50,0.50,0.50>	<<2,3,4>; 0.30,0.75,0.70>	<<5,6,7>; 0.70,0.25,0.30>	<<4,5,6>; 0.80,0.15,0.20>
	C_4	1/<<3,4,5>; 0.35,0.60,0.40>	1/<<5,6,7>; 0.70,0.25,0.30>	1/<<2,3,4>; 0.30,0.75,0.70>	<<1,1,1>; 0.50,0.50,0.50>	<<4,5,6>; 0.80,0.15,0.20>	<<7,8,9>; 0.85,0.10,0.15>
	C_5	1/<<2,3,4>; 0.30,0.75,0.70>	1/<<6,7,8>; 0.90,0.10,0.10>	1/<<5,6,7>; 0.70,0.25,0.30>	1/<<4,5,6>; 0.80,0.15,0.20>	<<1,1,1>; 0.50,0.50,0.50>	<<3,4,5>; 0.35,0.60,0.40>
	C_6	1/<<5,6,7>; 0.70,0.25,0.30>	1/<<2,3,4>; 0.30,0.75,0.70>	1/<<4,5,6>; 0.80,0.15,0.20>	1/<<7,8,9> 0.85,0.10,0.15>	1/ <<3,4,5> 0.35,0.60,0.40>	<<1,1,1>; 0.50,0.50,0.50>

Table 10: The pairwise comparison neutrosophic decision matrix of each criteria

		C_1	C_2	C_3	C_4	C_5	C_6
DM2	C_1	<<1,1,1>; 0.50,0.50,0.50>	<<2,3,4>; 0.30,0.75,0.70>	<<1,1,1>; 0.50,0.50,0.50>	<<3,4,5>; 0.35,0.60,0.40>	<<6,7,8>; 0.90,0.10,0.10>	<<1,2,3>; 0.40,0.60,0.65>
	C_2	1/<<2,3,4>; 0.30,0.75,0.70>	<<1,1,1>; 0.50,0.50,0.50>	<<4,5,6>; 0.80,0.15,0.20>	<<1,2,3>; 0.40,0.60,0.65>	<<3,4,5>; 0.35,0.60,0.40>	<<5,6,7>; 0.70,0.25,0.30>
	C_3	1/<<1,1,1>; 0.50,0.50,0.50>	1/<<4,5,6>; 0.80,0.15,0.20>	<<1,1,1>; 0.50,0.50,0.50>	<<4,5,6>; 0.80,0.15,0.20>	<<1,2,3>; 0.40,0.60,0.65>	<<2,3,4>; 0.30,0.75,0.70>
	C_4	1/<<3,4,5>; 0.35,0.60,0.40>	1/<<1,2,3>; 0.40,0.60,0.65>	1/<<4,5,6>; 0.80,0.15,0.20>	<<1,1,1>; 0.50,0.50,0.50>	<<4,5,6>; 0.80,0.15,0.20>	<<3,4,5>; 0.35,0.60,0.40>
	C_5	1/<<6,7,8>; 0.90,0.10,0.10>	1/<<3,4,5>; 0.35,0.60,0.40>	1/<<1,2,3>; 0.40,0.60,0.65>	1/<<4,5,6>; 0.80,0.15,0.20>	<<1,1,1>; 0.50,0.50,0.50>	<<2,3,4>; 0.30,0.75,0.70>
	C_6	1/<<1,2,3>; 0.40,0.60,0.65>	1/<<5,6,7>; 0.70,0.25,0.30>	1/<<2,3,4>; 0.30,0.75,0.70>	1/<<3,4,5> 0.35,0.60,0.40>	1/<<2,3,4> 0.30,0.75,0.70>	<<1,1,1>; 0.50,0.50,0.50>

Table 11: The pairwise comparison neutrosophic decision matrix of each criteria

		C_1	C_2	C_3	C_4	C_5	C_6
DM3	C_1	<<1,1,1>; 0.50,0.50,0.50>	<<2,3,4>; 0.30,0.75,0.70>	<<4,5,6 >; 0.80,0.15,0.20>	<<6,7,8>; 0.90,0.10,0.10>	<<3,4,5>; 0.35,0.60,0.40>	<<1,2,3>; 0.40,0.60,0.65>
	C_2	1/<<2,3,4>; 0.30,0.75,0.70>	<<1,1,1>; 0.50,0.50,0.50>	<<3,4,5>; 0.35,0.60,0.40>	<<1,2,3>; 0.40,0.60,0.65>	<<5,6,7>; 0.70,0.25,0.30>	<<7,8,9>; 0.85,0.10,0.15>
	C_3	1/<<4,5,6>; 0.80,0.15,0.20>	1/<<3,4,5>; 0.35,0.60,0.40>	<<1,1,1>; 0.50,0.50,0.50>	<<5,6,7>; 0.70,0.25,0.30>	<<2,3,4>; 0.30,0.75,0.70>	<<1,2,3>; 0.40,0.60,0.65>
	C_4	1/<<6,7,8>; 0.90,0.10,0.10>	1/<<1,2,3>; 0.40,0.60,0.65>	1/<<5,6,7>; 0.70,0.25,0.30>	<<1,1,1>; 0.50,0.50,0.50>	<< 4, 5, 6 >; 0.80,0.15,0.20>	<<3,4,5>; 0.35,0.60,0.40>
	C_5	1/<<3,4,5>; 0.35,0.60,0.40>	1/<<5,6,7>; 0.70,0.25,0.30>	1/<<2,3,4>; 0.30,0.75,0.70>	1/<<4,5,6>; 0.80,0.15,0.20>	<<1,1,1>; 0.50,0.50,0.50>	<<1,2,3>; 0.40,0.60,0.65>
	C_6	1/<<1,2,3>; 0.40,0.60,0.65>;	1/<<7,8,9>; 0.85,0.10,0.15>;	1/<<1,2,3>; 0.40,0.60,0.65>;	1/<<3,4,5> 0.35,0.60,0.40>;	1/<<1,2,3> 0.40,0.60,0.65>;	<<1,1,1>; 0.50,0.50,0.50>;

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Table 12: The pairwise comparison neutrosophic decision matrix of each criteria

		C_1	C_2	C_3	C_4	C_5	C_6
DM4	C_1	<<1,1,1>; 0.50,0.50,0.50>	<<4,5,6>; 0.80,0.15,0.20>	<<2,3,4 >; 0.30,0.75,0.70>	<<6,7,8>; 0.90,0.10,0.10>	<<3,4,5>; 0.35,0.60,0.40>	<<5,6,7>; 0.70,0.25,0.30>
	C_2	1/<<4,5,6>; 0.80,0.15,0.20>	<<1,1,1>; 0.50,0.50,0.50>	<<1,2,3>; 0.40,0.60,0.65>	<<1,1,1>; 0.50,0.50,0.50>	<<2,3,4>; 0.30,0.75,0.70>	<<6,7,8>; 0.90,0.10,0.10>
	C_3	1/<<2,3,4>; 0.30,0.75,0.70>	1/<<1,2,3>; 0.40,0.60,0.65>	<<1,1,1>; 0.50,0.50,0.50>	<<3,4,5>; 0.35,0.60,0.40>	<<4,5,6>; 0.80,0.15,0.20>	<<1,1,1>; 0.50,0.50,0.50>
	C_4	1/<<6,7,8>; 0.90,0.10,0.10>	1/<<1,2,3>; 0.40,0.60,0.65>	1/<<5,6,7>; 0.70,0.25,0.30>	<<1,1,1>; 0.50,0.50,0.50>	<<1,2,3>; 0.40,0.60,0.65>	<<2,3,4>; 0.30,0.75,0.70>
	C_5	1/<<3,4,5>; 0.35,0.60,0.40>	1/<<2,3,4>; 0.30,0.75,0.70>	1/<<4,5,6>; 0.80,0.15,0.20>	1/<<1,2,3>; 0.40,0.60,0.65>	<<1,1,1>; 0.50,0.50,0.50>	<<1,1,1>; 0.50,0.50,0.50>
	C_6	1/<<5,6,7>; 0.70,0.25,0.30>;	1/<<6,7,8>; 0.90,0.10,0.10>;	1/<<1,1,1>; 0.50,0.50,0.50>;	1/<<2,3,4> 0.30,0.75,0.70>;	1/<< 1, 1, 1 > 0.50,0.50,0.50>;	<<1,1,1>; 0.50,0.50,0.50>;

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Appendix B

PHASE 4 Decision judgments values of each alternatives on criteria is formed interms of neutrosophic decision matrix from multiple decision makers

Table 13: The judgments values of first and second decision makers

		C_1	C_2	C_3	C_4	C_5	C_6
DM1	A_1	<<4,5,6>; 0.80,0.15,0.20>	<<6,7,8>; 0.90,0.10,0.10>	<<2,3,4>; 0.30,0.75,0.70>	<<4,5,6>; 0.80,0.15,0.20>	<<2,3,4>; 0.30,0.75,0.70>	<<4,5,6>; 0.80,0.15,0.20>
	A_2	<<7,8,9>; 0.85,0.10,0.15>	<<7,8,9>; 0.85,0.10,0.15>	<<4,5,6>; 0.80,0.15,0.20>	<<5,6,7>; 0.70,0.25,0.30>	<<6,7,8>; 0.90,0.10,0.10>	<<6,7,8>; 0.90,0.10,0.10>
	A_3	<<7,8,9>; 0.85,0.10,0.15>	<<2,3,4>; 0.30,0.75,0.70>	<<4,5,6>; 0.80,0.15,0.20>	<<7,8,9>; 0.85,0.10,0.15>	<<4,5,6>; 0.80,0.15,0.20>	<<6,7,8>; 0.90,0.10,0.10>
	A_4	<<6,7,8>; 0.90,0.10,0.10>	<<4,5,6>; 0.80,0.15,0.20>	<<4,56>; 0.80,0.15,0.20>	<<4,5,6>; 0.80,0.15,0.20>	<<6,7,8>; 0.90,0.10,0.10>	<<7,8,9> 0.85,0.10,0.15>
	A_5	<<6,7,8>; 0.90,0.10,0.10>	<<2,3,4>; 0.30,0.75,0.70>	<<2,3,4>; 0.30,0.75,0.70>	<<4,5,6>; 0.80,0.15,0.20>	<<7,8,9>; 0.85,0.10,0.15>	<<4,5,6> 0.80,0.15,0.20>

		C_1	C_2	C_3	C_4	C_5	C_6
DM2	A_1	<<4,5,6>; 0.80,0.15,0.20>	<<6,7,8>; 0.90,0.10,0.10>	<<2,3,4>; 0.30,0.75,0.70>	<<2,3,4>; 0.30,0.75,0.70>	<<1,1,1>; 0.50,0.50,0.50>	<<4,5,6>; 0.80,0.15,0.20>
	A_2	<<6,7,8>; 0.90,0.10,0.10>	<<1,2,3>; 0.40,0.60,0.65>	<<1,1,1>; 0.50,0.50,0.50>	<<2,3,4>; 0.30,0.75,0.70>	<<6,7,8>; 0.90,0.10,0.10>	<<3,4,5>; 0.35,0.60,0.40>
	A_3	<<3,4,5>; 0.35,0.60,0.40>	<<6,7,8>; 0.90,0.10,0.10>	<<6,7,8>; 0.90,0.10,0.10>	<<5,6,7>; 0.70,0.25,0.30>	<<1,1,1>; 0.50,0.50,0.50>	<<7,8,9>; 0.85,0.10,0.15>
	A_4	<<6,7,8>; 0.90,0.10,0.10>	<<4,5,6>; 0.80,0.15,0.20>	<<4,56>; 0.80,0.15,0.20>	<<6,7,8>; 0.90,0.10,0.10>	<<6,7,8>; 0.90,0.10,0.10>	<<6,7,8> 0.90,0.10,0.10>
	A_5	<<4,5,6>; 0.80,0.15,0.20>	<<6,7,8>; 0.90,0.10,0.10>	<<2,3,4>; 0.30,0.75,0.70>	<<4,5,6>; 0.80,0.15,0.20>	<<4,5,6>; 0.80,0.15,0.20>	<<4,5,6> 0.80,0.15,0.20>

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Table 14: The judgments values of third decision maker

		C_1	C_2	C_3	C_4	C_5	C_6
DM3	A_1	<<6,7,8>; 0.90,0.10,0.10>	<<6,7,8>; 0.90,0.10,0.10>	<<4,5,6>; 0.80,0.15,0.20>	<<2,3,4>; 0.30,0.75,0.70>	<<1,1,1>; 0.50,0.50,0.50>	<<6,7,8>; 0.90,0.10,0.10>
	A_2	<<7,8,9>; 0.85,0.10,0.15>	<<6,7,8>; 0.90,0.10,0.10>	<<2,3,4>; 0.30,0.75,0.70>	<<1,1,1>; 0.50,0.50,0.50>	<<3,4,5>; 0.35,0.60,0.40>	<<5,6,7>; 0.70,0.25,0.30>
	A_3	<<4,5,6>; 0.80,0.15,0.20>	<<2,3,4>; 0.30,0.75,0.70>	<<6,7,8>; 0.90,0.10,0.10>	<<3,4,5>; 0.35,0.60,0.40>	<<1,1,1>; 0.50,0.50,0.50>	<<1,2,3>; 0.40,0.60,0.65>
	A_4	<<6,7,8>; 0.90,0.10,0.10>	<<1,1,1>; 0.50,0.50,0.50>	<<4,5,6>; 0.80,0.15,0.20>	<<5,6,7>; 0.70,0.25,0.30>	<<2,3,4>; 0.30,0.75,0.70>	<<1,2,3> 0.40,0.60,0.65>
	A_5	<<2,3,4>; 0.30,0.75,0.70>	<<4,5,6>; 0.80,0.15,0.20>	<<3,4,5>; 0.35,0.60,0.40>	<<1,1,1>; 0.50,0.50,0.50>	<<7,8,9>; 0.85,0.10,0.15>	<<1,2,3> 0.40,0.60,0.65>

Table 15: The judgments values of fourth decision maker

		C_1	C_2	C_3	C_4	C_5	C_6
DM4	A_1	<<5,6,7>; 0.70,0.25,0.30>	<<6,7,8>; 0.90,0.10,0.10>	<<4,5,6>; 0.80,0.15,0.20>	<<6,7,8>; 0.90,0.10,0.10>	<<2,3,4>; 0.30,0.75,0.70>	<<4,5,6>; 0.80,0.15,0.20>
	A_2	<<6,7,8>; 0.90,0.10,0.10>	<<4,5,6>; 0.80,0.15,0.20>	<<6,7,8>; 0.90,0.10,0.10>	<<2,3,4>; 0.30,0.75,0.70>	<<3,4,5>; 0.35,0.60,0.40>	<<5,6,7>; 0.70,0.25,0.30>
	A_3	<<2,3,4>; 0.30,0.75,0.70>	<<4,5,6>; 0.80,0.15,0.20>	<<1,1,1>; 0.50,0.50,0.50>	<<3,4,5>; 0.35,0.60,0.40>	<<2,3,4>; 0.30,0.75,0.70>	<<2,3,4>; 0.30,0.75,0.70>
	A_4	<<3,4,5>; 0.35,0.60,0.40>	<<4,5,6>; 0.80,0.15,0.20>	<<4,5,6>; 0.80,0.15,0.20>	<<2,3,4>; 0.30,0.75,0.70>	<<5,6,7>; 0.70,0.25,0.30>	<<6,7,8> 0.90,0.10,0.10>
	A_5	<<4,5,6>; 0.80,0.15,0.20>	<<2,3,4>; 0.30,0.75,0.70>	<<1,1,1>; 0.50,0.50,0.50>	<<2,3,4>; 0.30,0.75,0.70>	<<4,5,6>; 0.80,0.15,0.20>	<<2,3,4> 0.30,0.75,0.70>

On Refined Neutrosophic Algebraic Hyperstructures I

[1] A.A.A. Agboola , [2]M.A. Ibrahim, [3] E.O. Adeleke, [4]S.A. Akinleye
[1,2,3,4]Department of Mathematics, Federal University of Agriculture, Abeokuta, Nigeria.
agboolaaaa@funaab.edu.ng[1], muritalaibrahim40@gmail.com[2], yemi376@yahoo.com[3], sa_akinleye@yahoo.com[4]

Abstract

Given any algebraic hyperstructure $(X, *, \circ)$, the objective of this paper is to generate a refined neutrosophic algebraic hyperstructure $(X(I_1, I_2), *', \circ')$ from X, I_1 and I_2 and study refined neutrosophic Krasner hyperrings in particular.
Keywords: refined neutrosophic group, refined neutrosophic ring, refined neutrosophic hyperring.

1 Introduction

The concept of refined neutrosophic algebraic structures was introduced by Agboola in.[1] Adeleke et.al in[24,25] introduced and studied refined neutrosophic rings and ideals. In the present paper, we introduce the concept of refined neutrosophic algebraic hyperstructure and we study refined neutrosophic Krasner hyperrings in particular. In this section, we present introduction and some necessary definitions for completeness.

Definition 1.1 ([22]). A hypergroup $(X, \circ)$ is called a canonical hypergroup if the following conditions are satisfied:

(i) $(X, \circ)$ is commutative.

(ii) $(X, \circ)$ has a scalar identity that is $\forall x \in X$, there exists $e \in X$ such that

$$x \circ e = e \circ x = x.$$

(iii) Every element of $(X, \circ)$ has a unique inverse that is $\forall x \in X$, there exists a unique $x^{-1} \in X$ such that

$$e \in x \circ x^{-1} \cap x^{-1} \circ x.$$

(iv) $(X, \circ)$ is reversible that is if $x \in y \circ z$, then there exist $y^{-1}, z^{-1} \in X$ such that $z \in y^{-1} \circ x$ and $y \in x \circ z^{-1}$.

Definition 1.2 ([22]). An algebraic hyperstructure $(X, +, .)$ where $+$ is a hyperoperation and . is the usual multiplication operation is called a Krasner hyperring if the following conditions are satisfied:

(i) $(X, +)$ is a canonical hypergroup with identity 0.

(ii) $(X, .)$ is a semigroup with 0 as a bilateral absorbing element that is

$$x.0 = 0.x = 0 \quad \forall x \in X.$$

(iii) . is distributive over $+$.

A Krasner hyperring $(X, +, .)$ is said to be commutative with unit element if $(X, .)$ is a commutative semigroup with unit element.

Definition 1.3 ([22]). (i) Let $(X, +, .)$ be a Krasner hyperring and let A be a nonempty subset of X. A is called a subhyperring of X if $(A, +, .)$ is a hyperring in its own right.

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(ii) A is called a normal subhyperring of X if and only if $x + A - x \subseteq A$ for all $x \in X$.

(iii) A is called a hyperideal of X if for all $x \in X$, $a \in A$, we have $a.x, x.a \in A$.

Definition 1.4 ([22]). Let $(X_1, +, .)$ and $(X_2, +', .')$ be two Krasner hyperrings. The mapping $\phi : X_1 \rightarrow X_2$ is called a good(strong) homomorphism if $\forall x, y \in X_1$, we have the following:

(i) $\phi(x + y) = \phi(x) +' \phi(y)$ and

(ii) $\phi(x.y) = \phi(x).'\phi(y)$.

The kernel of ϕ denoted by $Ker\phi = \{x \in X_1 : \phi(x) = 0\}$.

For full details about algebraic hyperstructures and applications, the readers should see [9–12,15–21,23,30–38,40,41].

The concept of neutrosophic logic was introduced by Florentin Smarandache in 1995. Neutrosophic logic is an extension and generalization of fuzzy logic and intuitionistic fuzzy logic of Lofti Zadeh and Atanassov respectively. In neutrosophic logic, each proposition is approximated to have the percentage of truth in the subset T, percentage of indeterminacy in the subset I and the percentage of falsity in the subset F where $T, I, F \subseteq]^{-}0, 1^{+}[$. For simplicity and practical applications, T, I, F are taken as single-valued numbers from the interval $[0, 1]$ with $0 \leq T+I+F \leq 3$. In neutrosophy, if $< A >$ is an item or a concept, then only the triads $(< A >, < neutA >, < antiA >)$ that makes sense in the real world. The neutrosophication of a concept into a (T, I, F)-concept, or more generally into a refined $(T_1, T_2, \cdots, I_1, I_2, \cdots, F_1, F_2, \cdots)$-concept is possible if the triad $(< concept >, < neutconcept >, < anticoncept >$, or more generally its corresponding refined concept triad, makes sense in the real world. If the indeterminacy factor I is refined into two indeterminacies I_1 and I_2 in the sense of Smarandache in,[27] then we can take $I_1 =$ contradiction (true (T) and false (F)), $I_2 =$ ignorance (true (T) or false (F)) so that logically we have

$$I_1^2 = I_1, I_2^2 = I_2, I_1 I_2 = I_2 I_1 = I_1.$$

For full details about neutrosophy, neutrosphic logic and neutrosophic algebraic structures, the readers should see [26–28,39]

Definition 1.5 ([24]). Let $(R, +, .)$ be any ring. The abstract system $(R(I_1, I_2), +, .)$ is called a refined neutrosophic ring generated by R, I_1, I_2. $(R(I_1, I_2), +, .)$ is called a commutative refined neutrosophic ring if for all $x, y \in R(I_1, I_2)$, we have $xy = yx$. If there exists an element $e = (1, 0, 0) \in R(I_1, I_2)$ such that $ex = xe = x$ for all $x \in R(I_1, I_2)$, then we say that $(R(I_1, I_2), +, .)$ is a refined neutrosophic ring with unity.

Definition 1.6 ([24]). Let $(R(I_1, I_2), +, .)$ be a refined neutrosophic ring and let $J(I_1, I_2)$ be a nonempty subset of $R(I_1, I_2)$. $J(I_1, I_2)$ is called a refined neutrosophic subring of $R(I_1, I_2)$ if $(J(I_1, I_2), +, .)$ is itself a refined neutrosophic ring. It is essential that $J(I_1, I_2)$ contains a proper subset which is a ring. Otherwise, $J(I_1, I_2)$ will be called a pseudo refined neutrosophic subring of $R(I_1, I_2)$.

Definition 1.7 ([25]). Let $R(I_1, I_2)$ be a refined neutrosophic ring and let J be a nonempty subset of $R(I_1, I_2)$. J is called a refined neutrosophic ideal of $R(I_1, I_2)$ if the following conditions hold:

(i) J is a refined neutrosophic subring of $R(I_1, I_2)$.

(ii) For every $x \in J$ and $r \in R(I_1, I_2)$, we have $xr, rx \in J$.

If J is a pseudo refined neutrosophic subring of $R(I_1, I_2)$ and for every $x \in J$ and $r \in R(I_1, I_2)$, we have $xr \in J$, then J is called a pseudo refined neutrosophic ideal of $R(I_1, I_2)$.

Definition 1.8 ([25]). Let J be a refined neutrosophic ideal of the refined neutrosophic ring $R(I_1, I_2)$. The set $R(I_1, I_2)/J$ is defined by

$$R(I_1, I_2)/J = \{r + J : r \in R(I_1, I_2)\}.$$

If $\bar{x} = r_1 + J$ and $\bar{y} = r_2 + J$ are two arbitrary elements of $R(I_1, I_2)/J$ and $\oplus, \odot$ are two binary operations on $R(I_1, I_2)/J$ defined by

$$\begin{aligned} \bar{x} \oplus \bar{y} &= (x + y) + J, \\ \bar{x} \odot \bar{y} &= (xy) + J. \end{aligned}$$

It can be shown that $(R(I_1, I_2)/J, \oplus, \odot)$ is a refined neutrosophic ring with the additive identity J.
$(R(I_1, I_2)/J, \oplus, \odot)$ is called a refined quotient neutrosophic ring.

Definition 1.9 ([25]). Let $(R(I_1, I_2), +, .)$ and and $(S(I_1, I_2), +, .)$ be two refined neutrosophic rings. The mapping $\phi : (R(I_1, I_2), +, .) \rightarrow (S(I_1, I_2), +, .)$ is called a refined neutrosophic ring homomorphism if the following conditions hold:

(i) $\phi(x + y) = \phi(x) + \phi(y)$.

(ii) $\phi(x.y) = \phi(x).\phi(y)$.

(iii) $\phi(I_k) = I_k \;\; \forall x, y \in R(I_1, I_2)$ and $k = 1, 2$.

The image of ϕ denoted by $Im\phi$ is defined by the set

$$Im\phi = \{y \in S(I_1, I_2) : y = \phi(x) \text{ for some } x \in R(I_1, I_2)\}.$$

The kernel of ϕ denoted by $Ker\phi$ is defined by the set

$$Ker\phi = \{x \in R(I_1, I_2) : \phi(x) = (0, 0, 0)\}.$$

Epimorphism, monomorphism, isomorphism, endomorphism and automorphism of ϕ are similarly defined as in the classical cases. For full details about neutrosophic and refined neutrosophic algebraic structures and algebraic hyperstructures, the readers should see [1–8, 13, 14, 24, 25, 29]

2 Development of Refined Neutrosophic Algebraic Hyperstructures

In this section, we present the development of refined neutrosophic algebraic hyperstructures. Refined neutrosophic Krasner hyperrings are studied in particular and some of their basic properties are presented.

Suppose that $(X, *, \circ)$ is any given algebraic hyperstructure with $*$ as a hyperoperation on X and $\circ$ is an operation on X. Let $X(I_1, I_2) =< X, I_1, I_2 >$ be a refined neutrosophic set generated by X, I_1, I_2 defined by

$$X(I_1, I_2) = \{(x, yI_1, zI_2) : x, y, z \in X\}.$$

Suppose that

$$*' : X(I_1, I_2) \times X(I_1, I_2) \rightarrow 2^{X(I_1,I_2)} \quad \text{and} \quad \circ' : X(I_1, I_2) \times X(I_1, I_2) \rightarrow X(I_1, I_2)$$

are respectively a hyperoperation and an operation on $X(I_1, I_2)$ and such that $\forall (a, bI_1, cI_2), (d, eI_1, fI_2) \in X(I_1, I_2)$ with $a, b, c, d, e, f \in X$, we define

$$\begin{aligned} (a, bI_1, cI_2)*'(d, eI_1, fI_2) \;\; &= \;\; \{((x, yI_1, zI_2) : x \in a * d, y \in (a * e) \\ &\quad\; *(b * d) * (b * e) * (b * f) * (c * e), \\ &\quad\; z \in (a * f) * (c * d) * (c * f)\} \qquad (1) \\ (a, bI_1, cI_2)\circ'(d, eI_1, fI_2) \;\; &= \;\; ((x, yI_1, zI_2) \;\; \text{such that} \\ &\quad\; x = a \circ d, y = (a \circ e) \circ (b \circ d) \\ &\quad\; \circ(b \circ e) \circ (b \circ f) \circ (c \circ e), \\ &\quad\; z = (a \circ f) \circ (c \circ d) \circ (c \circ f). \qquad (2) \end{aligned}$$

With the composition of the elements of $X(I_1, I_2)$ according to equations (1) and (2), we call the algebraic structure $(X(I_1, I_2), *', \circ')$ a refined neutrosophic algebraic hyperstructure. $(X(I_1, I_2), *', \circ')$ is named according to the algebraic laws satisfied by $*'$ and $\circ'$.

If $(X(I_1, I_2), *', \circ')$ and $(Y(I_1, I_2), *'', \circ'')$ are two given refined neutrosophic algebraic hyperstructures, then the mapping $\phi : (X(I_1, I_2), *', \circ') \rightarrow (Y(I_1, I_2), *'', \circ'')$ is called a refined neutrosophic good(strong) homomorphism if
$\forall (a, bI_1, cI_2), (d, eI_1, fI_2) \in X(I_1, I_2)$, the following conditions hold:

(i) $\phi((a, bI_1, cI_2)*'(d, eI_1, fI_2)) = \phi((a, bI_1, cI_2))*''\phi((d, eI_1, fI_2)))$,

(ii) $\phi((a, bI_1, cI_2)\circ'(d, eI_1, fI_2)) = \phi((a, bI_1, cI_2))\circ''\phi((d, eI_1, fI_2)))$,

(iii) $\phi(I_i) = I_i$ for $i = 1, 2$.

In all that follows in the sequel, all hyperrings $(X, +, .)$ will be Krasner hyperrings.

Definition 2.1. Let $(X, +, .)$ be any hyperring and let $X(I_1, I_2) =< X, I_1, I_2 >$ be a set generated by X, I_1 and I_2. The triple $(X(I_1, I_2), +, .)$ is called a refined neutrosophic hyperring. It is said to be commutative if for all $x, y \in X(I_1, I_2)$, we have $x.y = y.x$.

For all $(a, bI_1, cI_2), (d, eI_1, fI_2) \in X(I_1, I_2)$, we define the composition of elements with respect to the hyperoperation $+$ and ordinary operation . in $X(I_1, I_2)$ as follows.

$$\begin{aligned}
(a, bI_1, cI_2) + (d, eI_1, fI_2) &= \{(x, yI_1, zI_2) : x \in a + d, y \in b + e, \\
&\quad z \in c + f\}. \qquad (3)\\
(a, bI_1, cI_2).(d, eI_1, fI_2) &= (x, yI_1, zI_2) \text{ such that } x = ad, \\
&\quad y \in (ae + bd + be + bf + ce), \\
&\quad z \in (af + cd + cf). \qquad (4)
\end{aligned}$$

Definition 2.2. (i) Let $(X(I_1, I_2), +, .)$ be a refined neutrosophic hyperring and let $A(I_1, I_2)$ be a nonempty subset of $X(I_1, I_2)$. Then $A(I_1, I_2)$ is called a refined neutrosophic subhyperring of $X(I_1, I_2)$ if $(A(I_1, I_2), +, .)$ is a refined neutrosophic hyperring in its own right. $-A(I_1, I_2)$ is defined as

$$-A(I_1, I_2) = \{-a : a \in A(I_1, I_2)\}.$$

(iii) $A(I_1, I_2)$ is called a refined neutrosophic hyperideal of $X(I_1, I_2)$ if for all $x \in X(I_1, I_2), a \in A(I_1, I_2)$, we have $a.x, x.a \in A(I_1, I_2)$.

Definition 2.3. Let $A(I_1, I_2)$ and $B(I_1, I_2)$ be two refined neutrosophic hyperideals of a refined neutrosophic hyperring $(X(I_1, I_2), +, .)$.

(i) The sum $A(I_1, I_2) + B(I_1, I_2)$ is defined by

$$A(I_1, I_2) + B(I_1, I_2) = \{x : x \in a + b \text{ for some } a \in A(I_1, I_2), b \in B(I_1, I_2)\}.$$

(ii) The product $A(I_1, I_2)B(I_1, I_2)$ is defined by

$$A(I_1, I_2)B(I_1, I_2) = \left\{x : x \in \sum_i^n a_i b_i, a_i \in A(I_1, I_2), b_i \in B(I_1, I_2)\}, n \in \mathbb{Z}^+\right\}.$$

Definition 2.4. Let $A_1(I_1, I_2), A_2(I_1, I_2), \cdots, A_n(I_1, I_2)$ be refined neutrosophic hyperideals of a refined neutrosophic hyperring $(X(I_1, I_2), +, .)$.

$$\mathbb{A}(I_1, I_2) = A_1(I_1, I_2) \oplus A_2(I_1, I_2) \oplus \cdots \oplus A_n(I_1, I_2) = \oplus \sum_{i=1}^n A_i(I_1, I_2)$$

is called a direct sum if for each element x in $\mathbb{A}(I_1, I_2)$, we have

$$x \in \sum_i^n a_i \text{ (uniquely), with } a_i \in A_i(I_1, I_2).$$

Definition 2.5. Let $(X_1(I_1, I_2), +, .)$ and $X_2(I_1, I_2), +', .')$ be two refined neutrosophic hyperrings. The mapping $\phi : X_1(I_1, I_2) \rightarrow X_2(I_1, I_2)$ is called a refined neutrosophic good(strong) homomorphism if $\forall x, y \in X_1(I_1, I_2)$, we have the following:

(i) $\phi(x + y) = \phi(x) +' \phi(y)$ and

(ii) $\phi(x.y) = \phi(x).'\phi(y)$.

(iii) $\phi(I_i) = I_i$ for all $i = 1, 2$.

The kernel of ϕ denoted by $Ker\phi = \{x \in X_1(I_1, I_2) : \phi(x) = 0\}$. The image of ϕ denoted by $Im\phi = \{y \in X_2(I_1, I_2) : y = \phi(x)$ for some $x \in X_1(I_1, I_2)\}$.

Example 2.6. Let $(\mathbb{R}(I_1, I_2), +, .)$ be a refined neutrosophic ring of real numbers and let $J(I_1, I_2)$ be a refined neutrosophic ideal of $\mathbb{R}(I_1, I_2)$. Let $\mathbb{R}(I_1, I_2)/J(I_1, I_2)$ be a refined neutrosophic quotient ring defined by

$$\mathbb{R}(I_1, I_2)/J(I_1, I_2) = \{x + J(I_1, I_2) : x \in \mathbb{R}(I_1, I_2)\}.$$

Let $\oplus$ and $\odot$ be additive hyperoperation and ordinary multiplicative operation respectively defined on $\mathbb{R}(I_1, I_2)/J(I_1, I_2)$ as follows:

$$\begin{aligned}(x + J(I_1, I_2)) \oplus (y + J(I_1, I_2)) &= \{z + J(I_1, I_2) : z \in (x + y)J(I_1, I_2)\},\\ (x + J(I_1, I_2)) \odot (y + J(I_1, I_2)) &= xy + J(I_1, I_2).\end{aligned}$$

Then $(\mathbb{R}(I_1, I_2)/J(I_1, I_2), \oplus, \odot)$ is a refined neutrosophic hyperring.

Theorem 2.7. *Any refined neutrosophic hyperring is a hyperring.*

Proof. Suppose that $(R(I_1, I_2), +, .)$ is a refined neutrosophic hyperring. It is clear that:

(1) $(R(I_1, I_2), +)$ is a canonical hypergroup.

(2) $(R(I_1, I_2), .)$ is a semigroup and $0 = (0, 0I_1, 0I_2)$ is a bilateral absorbing element.

(3) We establish the distributive laws. Let $x = (a, bI_1, cI_2), y = (d, eI_1, fI_2), z = (g, hI_1, iI_2)$ be arbitrary elements in $R(I_1, I_2)$ with $a, b, c, d, e, f, g, h, i \in R$.

$$\begin{aligned}x(y + z) &= \{(p, qI_1, rI_2) : p \in a(d + g), q \in (a(e + h) + b(d + g) + b(e + h)\\ &\quad +b(f + i) + c(e + h)), r \in (a(f + i) + c(d + g) + c(f + i))\}, \text{and}\\ xy + xz &= \{(u, vI_1, wI_2) : u \in a(d + g), v \in (a(e + h) + b(d + g) + b(e + h)\\ &\quad +b(f + i) + c(e + h)), w \in (a(f + i) + c(d + g) + c(f + i))\}.\end{aligned}$$

This shows that $x(y+z) = xy+xz$. Similarly, it can be shown that $(y+z)x = yx+zx$ and $\therefore .$ is distributive over $+$. The proof is complete. □

Lemma 2.8. *Let $(X(I_1, I_2), +, .)$ be a refined neutrosophic hyperring. Then for all $x, y, z, w \in X(I_1, I_2)$, we have*

(i) $-(-x) = x$.

(ii) $-(x + y) = -x - y$.

(iii) $(x + y)(z + w) = xz + xw + yz + yw$.

Lemma 2.9. *Let $A(I_1, I_2)$ be a nonempty subset of a refined neutrosophic hyperring $(X(I_1, I_2), +, .)$. $A(I_1, I_2)$ is a refined neutrosophic hyperideal of $X(I_1, I_2)$ if and only if:*

(i) $x, y \in A(I_1, I_2)$ implies that $x - y \subseteq A(I_1, I_2)$,and

(ii) $x \in A(I_1, I_2)$ and $r \in X(I_1, I_2)$ imply that $xr, rx \in A(I_1, I_2)$.

Theorem 2.10. *Let $A(I_1, I_2)$ and $B(I_1, I_2)$ be refined neutrosophic hyperideals of a refined neutrosophic hyperring $(X(I_1, I_2), +, .)$ and let $\{A(I_1, I_2)_i\}_{i=1}^{n}$ be a family of refined neutrosophic hyperideals of $(X(I_1, I_2), +, .)$. Then:*

(i) $\bigcap_{i=1}^{n} A(I_1, I_2)_i$ is a refined neutrosophic hyperideal of $X(I_1, I_2)$.

(ii) $\oplus \sum_{i=1}^{n} A_i(I_1, I_2)$ is a refined neutrosophic hyperideal of $X(I_1, I_2)$.

(iii) $A(I_1, I_2) + B(I_1, I_2)$ is a refined neutrosophic hyperideal of $X(I_1, I_2)$.

(iv) $A(I_1, I_2) + B(I_1, I_2)$ is a refined neutrosophic hyperideal of $X(I_1, I_2)$.

Lemma 2.11. *Let $A(I_1, I_2)$ be a refined neutrosophic hyperideal of a refined neutrosophic hyperring $(X(I_1, I_2), +, .)$. Then*

(i) $A(I_1, I_2) + A(I_1, I_2) = A(I_1, I_2)$.

(ii) $x + A(I_1, I_2) = A(I_1, I_2)$ for all $x \in A(I_1, I_2)$.

Proof. (i) Let $y = (p, qI_1, rI_2)$. Then

$$\begin{aligned} y &\in A(I_1, I_2) + A(I_1, I_2) \\ &\Leftrightarrow y \in u + u \quad \text{for some} \quad u = (a, bI_1, cI_2) \in A(I_1, I_2) \\ &\Leftrightarrow y \in A(I_1, I_2) \quad \text{since} \quad u + u \subseteq A(I_1, I_2) \\ &\therefore \quad A(I_1, I_2) + A(I_1, I_2) = A(I_1, I_2). \end{aligned}$$

(ii) Let $y = (p, qI_1, rI_2)$. Then

$$\begin{aligned} y &\in x + A(I_1, I_2) \\ &\Leftrightarrow y \in x + u \quad \text{for some} \quad u = (a, bI_1, cI_2) \in A(I_1, I_2) \\ &\Leftrightarrow y \in A(I_1, I_2) \quad \text{since} \quad x + u \subseteq A(I_1, I_2) \end{aligned}$$

$\therefore\ x + A(I_1, I_2) = A(I_1, I_2)$. □

Theorem 2.12. *Let $A(I_1, I_2)$ be a refined neutrosophic hyperideal of a refined neutrosophic hyperring $(X(I_1, I_2), +, .)$. Then*

(i) $(A(I_1, I_2) + x) + (A(I_1, I_2) + y) = A(I_1, I_2) + x + y$ *for all* $x, y \in X(I_1, I_2)$.

(ii) $A(I_1, I_2) + x = A(I_1, I_2) + y$ *for all* $y \in A(I_1, I_2) + x$.

(iii) $A(I_1, I_2) + x + y = A(I_1, I_2) + z$ *if* $z \in x + y$ *for all* $x, y, z \in X(I_1, I_2)$.

Proof. (i) Let $v = (p, qI_1, rI_2)$. Then

$$\begin{aligned} v &\in (A(I_1, I_2) + x) + (A(I_1, I_2) + y) \\ &\Leftrightarrow v \in (u + u) + x + y \quad \text{for some} \quad u = (a, bI_1, cI_2) \in A(I_1, I_2) \\ &\Leftrightarrow v \in A(I_1, I_2) + x + y \quad \text{since} \quad u + u \subseteq A(I_1, I_2) \\ &\therefore \quad (A(I_1, I_2) + x) + (A(I_1, I_2) + y) = A(I_1, I_2) + x + y. \end{aligned}$$

(ii) Suppose that $y \in A(I_1, I_2) + x$. Let $v = (p, qI_1, rI_2)$. Then

$$\begin{aligned} v &\in A(I_1, I_2) + x \\ &\Leftrightarrow v \in A(I_1, I_2) + A(I_1, I_2) + x \\ &\Leftrightarrow v \in A(I_1, I_2) + y \\ &\therefore \quad A(I_1, I_2) + x = A(I_1, I_2) + y. \end{aligned}$$

(iii) Suppose that $z \in x + y$ for all $x, y, z \in X(I_1, I_2)$. Let $v = (p, qI_1, rI_2)$. Then

$$\begin{aligned} v &\in A(I_1, I_2) + x + y \\ &\Leftrightarrow v \in u + (x + y) \quad \text{for some} \quad u = (a, bI_1, cI_2) \in A(I_1, I_2) \\ &\Leftrightarrow v \in u + z \\ &\Leftrightarrow v \in A(I_1, I_2) + z. \end{aligned}$$

$\therefore A(I_1, I_2) + x + y = A(I_1, I_2) + z$. □

Theorem 2.13. *Let $A(I_1, I_2)$ be a refined neutrosophic hyperideal of a refined neutrosophic hyperring $(X(I_1, I_2), +, .)$. Then $A(I_1, I_2)$ is normal in $X(I_1, I_2)$ only if A is normal in X.*

Proof. Suppose that A is normal in X. Let $u = (x, yI_1, zI_2) \in X(I_1, I_2)$. Then there exists $v = (a, bI_1, cI_2) \in A(I_1, I_2)$ such that

$$\begin{aligned} u + A(I_1, I_2) - u &= u + v - u \\ &= (x, yI_1, zI_2) + (a, bI_1, cI_2) - (x, yI_1, zI_2) \\ &= \{(p, qI_1, rI_2) : p \in x + a - x, q \in y + b - y, \\ &\quad r \in z + c - z\} \\ &= \{(p, qI_1, rI_2) : p \in x + A - x \subseteq A, \\ &\quad q \in y + A - y \subseteq A, r \in z + A - z \subseteq A\} \\ &= \{(p, qI_1, rI_2) : p \in A, q \in A, r \in A\} \\ &\subseteq A(I_1, I_2) \end{aligned}$$

which shows that $A(I_1, I_2)$ is normal in $X(I_1, I_2)$. □

Example 2.14. Let $X_1(I_1, I_2)$ and $X_2(I_1, I_2)$ be two refined neutrosophic hyperrings and let $X(I_1, I_2) = X_1(I_1, I_2) \oplus X_2(I_1, I_2)$ be their direct sum. Let $\phi_i : X(I_1, I_2) \rightarrow X_i(I_1, I_2)$ with $i = 1, 2$ be the projections defined by

$$\phi_1(x, y) = x, \phi_2(x, y) = y \quad \forall\, (x, y) \in X(I_1, I_2).$$

Then ϕ_i are refined neutrosophic good(strong) homomorphisms.

Theorem 2.15. *Let $\phi : X_1(I_1, I_2) \rightarrow X_2(I_1, I_2)$ be a refined neutrosophic good(strong) hyperring homomorphism from $(X_1(I_1, I_2), +, .)$ into $X_2(I_1, I_2), +', .')$. Then*

(i) $\phi((0, 0I_1, 0I_2)) = (0, 0I_1, 0I_2)$.

(ii) $\phi(-x) = -\phi(x)$ *for all* $x \in X_1(I_1, I_2)$.

(iii) $Ker\phi$ *is not a refined neutrosophic hyperideal of* $X_1(I_1, I_2)$.

Proof. (i) and (ii) are clear.

(iii) It should be noted that only the element $(0, 0I_1, 0I_2) \in X_1(I_1, I_2)$ is the only element in $Ker\phi$. Since $\phi(I_i) = I_i$ for all $i = 1, 2$, it follows that no other element of $X_1(I_1, I_2)$ can be in the kernel of ϕ . Hence, $Ker\phi$ is not closed under $-$ and consequently, it cannot be a refined neutrosophic hyperideal of $X_1(I_1, I_2)$.

However, it can be shown that $Ker\phi$ is a subhyperring of $X_1(I_1, I_2)$ but not a refined neutrosophic subhyperring of $X_1(I_1, I_2)$. □

3 Conclusion

We have in this paper introduced the new concept of refined neutrosophic algebraic hyperstructures. We have studied refined neutrosophic Krasner hyperrings in particular. Their refined neutrosophic hypersubstructures were studied and their basic properties were presented. It was established that every refined neutrosophic Krasner hyperring is a Krasner hyperring. It was also established that the kernel of a refined neutrosophic Krasner hyperring homomorphism cannot be be a refined neutrosophic hyperideal but it can be a subhyperring. In our next paper to be titled Refined Neutrosophic Algebraic Hyperstructures II, we will study refined neutrosophic multiplicative hyperrings, their hypersubstructures and their properties.

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Dominating Energy in Neutrosophic graphs

M. Mullai[1]*, Said Broumi[2]
[1]Department of Mathematics(DDE), Alagappa University, Karaikudi, Tamilnadu, India.
[2]Laboratory of Information processing, Faculty of Sciences , Ben M'SiK, University Hassan II,B.P 7955, Sidi Othman, Casablanca, Morocco.
[1]mullaim@alagappauniversity.ac.in, [2]broumisaid78@gmail.com

Abstract

Dominating energy of graphs plays a vital role in the field of application in energy. Results by applying neutrosophic graph theory is more efficient than other existing methods. So, dominating energy of neutrosophic graphs will also give more accurate results than other exixting methods in the field of energy. This article introduces dominating energy of neutrosophic graphs. Dominating energy of a neutrosophic graph, dominating neutrosophic adjacency matrix, eigen values for the dominating energy of a neutrosophic graphs and complement of neutrosophic graphs are defined with examples. Also, dominating energy in union and join operations of neutrosophic graphs are developed and some theorems in dominating energy of a neutrosophic graphs are derived here.

Keywords: Neutrosophic graph, dominating energy of neutrosophic graph, eigen values for dominating energy of a neutrosophic graph.

1.Introduction

Fuzzy set plays a vital role in the area of interdisciplinary research. Fuzzy graph relation was introduce by Zadeh[20] and it has many real world applications. Rosenfield[12] used fuzzy relations on fuzzy sets and derived the structure of fuzzy graphs.

Recently, intuitionistic fuzzy set area takes important rule from normal mathematics to computer sciences, information sciences and communications systems. Spectrum of graphs is used in statistical physics problems and in combinatorial optimization problems. Spectrum of a graph also plays an important role in pattern recognition, virus propagation in computer networks and in secure datas in databases. The spectrum of a graph is used in the field of of energy.

Let d_i be the degree of i^{th} vertex of G, i=1,2,.....,n. The spectrum of graph G consisting of $\lambda_1,\lambda_2,\ldots\ldots,\lambda_n$ is the spectrum of its adjacency matrix[3]. The Laplacian spectrum of the graph G consisting of $\mu_1,\mu_2,\ldots\ldots,\mu_n$ is the spectrum of its Laplacian matrix.

The following relations are satisfied by ordinary and laplacian graph eigen values.

$$\sum_{i=1}^{n} \lambda_i = 0, \sum_{i=1}^{n} \lambda_i^2 = 2m, \ \sum_{i=1}^{n} \mu_i = 2m,$$

$$\sum_{i=1}^{n} \mu_i^2 = 2m + \sum_{i=1}^{n} d_i^2$$

The study of domination in graphs was Started in 1960,. C.F.De jaenisch[2] tried to find the minimum number of queens required to cover a n×n chess board in 1862,. The independent domination number in graphs was established by Cockayne[1]. Domination in graphs has many applications in several fields. A.Somasundaram and

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S.Somasundaram[17] introduced domination in fuzzy graph in terms of effective edges. Domination using strong arcs was introduced by A Nagoorgani and V.T.Chandrasekaran[4]. R.Parvathi and G.Thamizhenthi[6] developed dominating sets, dominating number, independent set, total dominating and total dominating number in intuitionistic fuzzy graphs. In[19] Vijayragavan et.al developed the dominating energy in products of intuitionistic fuzzy graph. Many authors introduced various concepts and their applications of neutrosophic theory in [3,5,7,9,10,11,14,15,16,18,22,23,24,25,26,27,28]

Domination in Neutrosophic graphs are more convenient than fuzzy and intuitionistic fuzzy graphs, which is useful in the field of traffic and communication systems, because the neutrosophic set is a generalization of fuzzy and intuitionistic fuzzy sets. Also neutrosophic concept plays an important role in real world applications when uncertainty and inderminacy occur. The results obtained by using neutrosophics sets are more accurate than fuzzy and intuitionistic fuzzy sets. Dominations in neutrosophic graphs was introduced by M.Mullai [21].

The energy of a graph is used in quantum theory by relating edge of a graph with electron energy of a class of molecule and many applications in the field of energy. Similarly energy of fuzzy graphs and intuitionistic fuzzy graphs are applied in many fields. Dominating energy is more efficient in the field of energy. Compared to dominating energy of fuzzy graphs and intuitionistic fuzzy graphs, dominating energy of neutrosophic graphs is more efficient by giving accurate results in various real life applications. Before analyzing these concepts, dominating energy of neutrosophic graph and dominating energy of different operations of neutrosophic graph are defined with examples and some theorems in dominating energy of neutrosophic graph are established and various results are discussed in this article.

2 Preliminaries

This part includes some basic definitions and results in domination theory of graphs that is very helpful to the proposed research work.

Definition 2.1. [5]An intuitionistic fuzzy graph is defined as $G=(V,E,\mu,\gamma)$, where V is the set of vertices and E is the set of edges, μ is a fuzzy membership function defined on V×V and γ is a fuzzy non membership function. Define $\mu(v_i,v_j)$ by μ_{ij} and $\gamma(v_i,v_j)$ by γ_{ij} such that

1.$0\leq \mu_{ij}+\gamma_{ij}\leq 1$

2.$0\leq \mu_{ij},\gamma_{ij},\pi_{ij}\leq 1$, where $\pi_{ij}=1-\mu_{ij}-\gamma_{ij}$.

Hence, $(V\times V,\mu,\gamma)$ is an intuitionistic fuzzy graph.

Definition 2.2. [8] An intuitionistic fuzzy graph is of the form $G=(V,E)$, where

(i)$V=\{v_1,v_2,\dots.v_n\}$ such that $\mu: V\to[0,1]$, $\gamma: V\to[0,1]$ denote the degree of membership and nonmembership of the element $v\in V$ respectively and $0\leq\mu_1(v_i)+\gamma_1(v_i)\leq 1$ for every $v_i\in V_i$ $(i=1,2,3,\dots)$

(ii)$E\subseteq V\times V$ where $\mu_2: V\times V\to[0,1]$ and $\gamma_2: V\times V\to[0,1]$ are suchthat

$$\mu_2(v_i,v_j)\leq\mu_1(v_i)\wedge\mu_1(v_j)$$

$\gamma_2(v_i,v_j)\leq\gamma_1(v_i)\wedge\gamma_1(v_j)$ and

$0\leq\mu_2(v_i,v_j)+\gamma_2(v_i,v_j)\leq 1$.

Definition 2.3. [8] An arc (v_i,v_j) of an intuitionistic fuzzy graph G is called a strong arc if

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$\mu_2(v_i, v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j)$ and $\gamma_2(v_i, v_j) \leq \gamma_1(v_i) \wedge \gamma_1(v_j)$.

Definition 2.4. [8] Let $G = (V, E, \mu, \gamma, \mu_1, \gamma_1)$ be a dominating intuitionistic fuzzy graph. A dominating intuitionistic fuzzy adjacency matrix $D(G) = [d_{ij}]$, where

$$d_{ij} = \begin{cases} (\mu_{ij}, \gamma_{ij}) & \text{if} \quad (v_i, v_j) \in E \\ (1,1) & \text{if} \quad i = j \quad \text{and} \quad v_i \in D \\ (0,0) & \quad \text{otherwise} \end{cases}$$

This dominating intuitionistic fuzzy graph adjacency matrix $D(G)$ can be written as $D(G) = (\mu_D(G), \gamma_D(G))$ where

$$\mu_D(G) = \begin{cases} \mu_{ij} & \text{if} \quad (v_i, v_j) \in E \\ 1 & \text{if} \quad i = j \quad \text{and} \quad v_i \in D \\ 0 & \quad \text{otherwise} \end{cases}$$

and

$$\gamma_D(G) = \begin{cases} \gamma_{ij} & \text{if} \quad (v_i, v_j) \in E \\ 1 & \text{if} \quad i = j \quad \text{and} \quad v_i \in D \\ 0 & \quad \text{otherwise} \end{cases}$$

Definition 2.5. [8] The eigen values of dominating intuitionistic fuzzy adjacency matrix $D(G)$ is defined as (X, Y) where X is the set of eigen values of $\mu_D(G)$ and Y is the set of eigen values of $\gamma_D(G)$. The energy of a dominating intuitionistic fuzzy graph $G = (V, E, \mu, \gamma, \mu_1, \gamma_1)$ is defined $(\sum_{\lambda_i \in X} |\lambda_i|, \sum_{\delta_i \in Y} |\delta_i|)$ where $\sum_{\lambda_i \in X} |\lambda_i|$ is the sum of the absolute values of the eigen values of $\mu_D(G)$ and it is denoted by the energy of the membership matrix $E(\mu_D(G))$ and $\sum_{\delta_i \in Y} |\delta_i|$ is the sum of the absolute values of the eigen values of $\gamma_D(G)$ and it is denoted by the energy of the membership matrix $E(\gamma_D(G))$.

Definition 2.6. [19] Let $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$ be two intuitionistic fuzzy graphs with $V_1 \cap V_2 = \emptyset$ and $G = G_1 \cup G_2 = \langle V_1 \cup V_2, E_1 \cup E_2 \rangle$ be the union of G_1 and G_2. Then the union of intuitionistic fuzzy graphs G_1 and G_2 is an intuitionistic fuzzy graph defined by

$$(\mu_1 \cup \mu_{1'})(v) = \begin{cases} \mu_1(v) & \text{if } v \in v_1 - v_2 \\ \mu_{1'}(v) & \text{if } v \in v_2 - v_1 \end{cases}$$

$$(\gamma_1 \cup \gamma_{1'})(v) = \begin{cases} \gamma_1(v) & \text{if} \quad v \in v_1 - v_2 \\ \gamma_{1'}(v) & \text{if} \quad v \in v_2 - v_1 \end{cases} \text{and}$$

$$(\mu_2 \cup \mu_{2'})(v_i, v_j) = \begin{cases} \mu_2(e_{ij}) & \text{if} \quad e_{ij} \in E_1 - E_2 \\ \mu_{2'}(e_{ij}) & \text{if} \quad e_{ij} \in E_2 - E_1 \end{cases}$$

where (μ_1, γ_1) and $(\mu_{1'}, \gamma_{1'})$ refer the vertex membership and non-membership of G_1 and G_2 respectively, (μ_2, γ_2) and $(\mu_{2'}, \gamma_{2'})$ refer the edge membership and non-membership of G_1 and G_2 respectively.

Definition 2.7. [19]The join of two intutionistic fuzzy graphs

$G = G_1 + G_2 = \langle V_1 \cup V_2, E_1 \cup E_2 \rangle$ defined by

$(\mu_1 + \mu_{1'})(v) = (\mu_1 \cup \mu_{1'})$ if $v \in V_1 \cup V_2$

$(\gamma_1 + \gamma_{1'})(v) = (\gamma_1 \cup \gamma_{1'})(v)$ if $v \in V_1 \cup V_2$

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$(\mu_2 + \mu_{2'})(v_i v_j) = (\mu_2 \cup \mu_{2'})(v_i v_j)$ if $v_i v_j \in E_1 \cup E_2$

Definition 2.8. [19] The α-product of two intuitionistic fuzzy graphs $G_1 = (V_1, E_1)$ and

$G_2 = (V_2, E_2)$ denoted by $G_1 \odot G_2$ is an intuitionistic fuzzy graph

$G = (V, E, \langle \mu_r, v_r \rangle, \langle \mu_{rs}, v_{rs} \rangle)$ where

1.$V = v_i u_p$ for all $v_i \in V_1$ and $u_p \in V_2$, $(V_1 \cap V_2) = \emptyset$, i=1,2,3,...m, p=1,2,3,....n

2.$E = \langle v_i u_p, v_j u_q \rangle$ such that either one of the following is true:

(i)$(v_i, v_j) \in E_1$ and $(u_p, u_q) \notin E_2$

(ii)$(u_p, u_q) \in E_2$ and $(v_i, v_j) \notin E_1$

3.$\langle \mu_r, v_r \rangle$ denote the degrees of membership and non-membership of

vertices of G,and is given by $\langle \mu_r, v_r \rangle = \langle \min(\mu_i, \mu_p), \max(v_i, v_p) \rangle$ for all

$v_r \in V$, r=1,2,3,...m,n

4.$\langle \mu_{rs}, v_{rs} \rangle$ denote the degrees of membership and non-membership of edges of G, and is given by

$$\langle \mu_{rs}, v_{rs} \rangle = \begin{cases} \langle \min(\mu_i, \mu_j, \mu_p), \max(v_i, v_j, v_p) \rangle & \text{if} \quad (v_i, v_j) \notin E_1 \text{and} \quad (u_p, u_q) \in E_2 \\ \langle \min(\mu_p, \mu_j, \mu_{ij}), \max(v_p, v_q, v_{ij}) \rangle & \text{if} \quad (v_i, v_j) \in E_1 \text{and} \quad (u_p, u_q) \notin E_2 \\ \langle 0,0 \rangle & \text{if} \quad (v_i, v_j) \in E_1 \text{and} \quad (u_p, u_q) \in E_2 \end{cases}$$

Definition 2.9. [19] The β-product of two intuitionistic fuzzy graphs $G_1 = (V_1, E_1)$ and

$G_2 = (V_2, E_2)$ denoted by $G_1 * G_2$ is an intuitionistic fuzzy graph

$G = (V, E, \langle \mu_r, v_r \rangle, \langle \mu_{rs}, v_{rs} \rangle)$ where

1.$V = v_i u_p$ for all $v_i \in V_1$ and $u_p \in V_2$,$V_1 \cap V_2) = \emptyset$, i=1,2,3,...m, p=1,2,3,....n

2.$E = \langle v_i u_p, v_j u_q \rangle$ such that either one of the following is true:

(i)$(v_i, v_j) \in E_1$ when $p \neq q, i \neq j$

(ii)$(u_p, u_q) \in E_2$ when $i \neq j, p \neq q$

3.$\langle \mu_r, v_r \rangle$ denote the degrees of membership and non-membership of vertices of G, and is given by $\langle \mu_r, v_r \rangle = \langle \min(\mu_i, \mu_p), \max(v_i, v_p) \rangle$ for all

$v_r \in V$, r=1,2,3,...m,n

4.$\langle \mu_{rs}, v_{rs} \rangle$ denote the degrees of membership and non-membership of edges of G, and is given by

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$$\langle \mu_{rs}, v_{rs} \rangle = \begin{cases} \langle \min(\mu_i, \mu_j, \mu_{pq}), \max(v_i, v_j, v_{pq}) \rangle & \text{if } i \neq j, (v_i, v_j) \notin E_1 \text{and } (u_p, u_q) \in E_2 \\ \langle \min(\mu_p, \mu_q, \mu_{ij}), \max(v_p, v_q, v_{ij}) \rangle & \text{if } p \neq q, (u_p, u_q) \notin E_2 \text{and } (v_i, v_j) \in E_1 \\ \langle \min(\mu_{ij}, \mu_{pq}), \max(v_{ij}, v_{pq}) \rangle & \text{if } i \neq j, p \neq q, (v_i, v_j) \in E_1 \text{and } (u_p, u_q) \in E_2 \\ \langle 0,0 \rangle & \text{otherwise} \end{cases}$$

Definition 2.10. [19] The γ-product of two intuitionistic fuzzy graphs $G_1 = (V_1, E_1)$ and

$G_2 = (V_2, E_2)$ denoted by $G_1 \boxdot G_2$ is an intuitionistic fuzzy graphs

$G = (V, E, \langle \mu_r, v_r \rangle, \langle \mu_{rs}, v_{rs} \rangle)$ where

1.$V = v_i u_p$ for all $v_i \in V_1$ and $u_p \in V_2$, $V_1 \cap V_2 = \emptyset$, i=1,2,3,...m, p=1,2,3,....n

2.$E = \langle v_i u_p, v_j u_q \rangle$ such that either $(v_i, v_j) \in E_1$ or $(u_p, u_q) \in E_2$

3.$\langle \mu_r, v_r \rangle$ denote the degrees of membership and non-membership of vertices of G,and is given by $\langle \mu_r, v_r \rangle = \langle \min(\mu_i, \mu_p), \max(v_i, v_p) \rangle$ for all

$v_r \in V$, r=1,2,3,...m,n

4.$\langle \mu_{rs}, v_{rs} \rangle$ denote the degrees of membership and non-membership of edges of G ,and is given by

$$\langle \mu_{rs}, v_{rs} \rangle = \begin{cases} \langle \min(\mu_i, \mu_j, \mu_{pq}), \min(v_i, v_j, v_{pq}) \rangle & \text{if } (v_i, v_j) \notin E_1 \text{and } (u_p, u_q) \in E_2 \\ \langle \min(\mu_p, \mu_q, \mu_{ij}), \min(v_p, v_q, v_{ij}) \rangle & \text{if } (u_p, u_q) \notin E_2 \text{and } (v_i, v_j) \in E_1 \\ \langle \min(\mu_{ij}, \mu_{pq}), \max(v_{ij}, v_{pq}) \rangle & \text{if } (v_i, v_j) \in E_1 \text{and } (u_p, u_q) \in E_2 \\ \langle 0,0 \rangle & \text{otherwise} \end{cases}$$

Definition 2.11. [13] A single valued neutrosophic graph with underlying set V is defined to be a pair $G = (A, B)$ where,

(i) The functions $T_A V \to [0,1], I_A: V \to [0,1]$ and $F_A: V \to [0,1]$ denote the degree of truth-membership, degree of indeterminacy membership, and degree of falsity-membership of the element $v_i \in V$, respectively and

$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$ for all $v_i \in V$ (i=1,2,..n)

(ii)The functions $T_B: E \subseteq V \times V \to [0,1], I_B: E \subseteq V \times V \to [0,1]$ and $F_B: E \subseteq V \times V \to [0,1]$ are defined by

$T_B(\{v_i, v_j\}) \leq \min(T_A(v_i), T_A(v_j))$,

$I_B(\{v_i, v_j\}) \geq \max(I_A(v_i), I_A(v_j))$and

$F_B(\{v_i, v_j\}) \geq \max(F_A(v_i), F_A(v_j))$

denotes the degree of truth-membership,degree of indeterminacy-membership and degree of falsity-membership of the edge $(v_i, v_j) \in E$ respectively, where

$0 \leq T_B(\{v_i, v_j\}) + I_B(\{v_i, v_j\}) + F_B(\{v_i, v_j\}) \leq 3$ for all $\{v_i, v_j\} \in E$ (i,j=1,2,...n)

Definition 2.12. [13]Let G be the neutrosophic graph.Let x,y ∈V. x dominates y in G if

$\mu_1(x, y) = \min\{\mu(x), \mu(y)\}, \gamma_1(x, y) = \min\{\gamma(x), \gamma(y)\}$ and $\sigma_1(x, y) = \min\{\sigma(x), \sigma(y)\}$.

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A subset D^N of V is called a dominating set in G if for every vertex $v \notin D^n$, there exists $u \in D^N$ such that u dominates v.

Definition 2.13. [21]A dominating set D^N of neutrosophic graph is said to be minimal dominating set if no paper subset of D^N is a dominating set.

Definition 2.14. [21]Minimum cardinality of a dominating set in a neutrosophic graph G is called the domination number of G and is denoted by $\gamma^N(G)(or)\gamma^N$.

3 Dominating energy in neutrosophic graphs

Dominating energy of a neutrosophic graph using various operations and some theorems on these operations are established here. Let $G = (V, E, \mu, \gamma, \sigma, \mu_1, \gamma_1, \sigma_1)$ be a dominating neutrosophic graph. Define a dominating neutrosophic adjacency matrix $D^N(G) = [d_{ij}]$, where

$$d_{ij} = \begin{cases} (\mu_{ij}, \gamma_{ij}, \sigma_{ij}) & \text{if } (v_i, v_j) \in E \\ (1,1) & \text{if } i = j \text{ and } v_i \in D^N \\ (0,0) & \text{otherwise} \end{cases}$$

This dominating neutrosophic adjacency matrix $D^N(G)$ can be written as $D^N(G) = \left(\mu_{D^N}(G), \gamma_{D^N}(G), \sigma_{D^N}(G)\right)$, where

$$\mu_{D^N}(G) = \begin{cases} \mu_{ij} & \text{if } (v_i, v_j) \in E \\ 1 & \text{if } i = j \text{ and } v_i \in D^N \\ 0 & \text{otherwise} \end{cases}, \quad \gamma_{D^N}(G) = \begin{cases} \gamma_{ij} & \text{if } (v_i, v_j) \in E \\ 1 & \text{if } i = j \text{ and } v_i \in D^N \\ 0 & otherwise \end{cases}$$

and

$$\sigma_{D^N}(G) = \begin{cases} \sigma_{ij} & \text{if } (v_i, v_j) \in E \\ 1 & \text{if } i = j \text{ and } v_i \in D^N \\ 0 & \text{otherwise} \end{cases}$$

For example, consider the neutrosophic graph G = (V,E), where $V = \{v_1, v_2, v_3, v_4\}$, $E = \{(v_1v_2), (v_2v_3), (v_3v_4), (v_4v_1)\}$ as in Fig.1

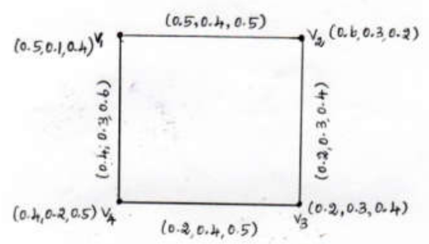

Fig.1

Then, the above dominating neutrosophic graph can be written as $G = (V, E, \mu, \gamma, \sigma, \mu_1, \gamma_1, \sigma_1)$, where $V = \{v_1, v_2, v_3, v_4\}$ and $\mu_1, \gamma_1, \sigma_1$ are given by $\mu_1: V \to [0,1], \gamma_1: V \to [0,1]$ and $\sigma_1: V \to [0,1]$, where

$$\mu_1(v_1) = \min[\mu(v_1, v_2), \mu(v_1, v_4)] = \min[0.5, 0.4] = 0.4$$

$$\mu_1(v_2) = \min[\mu(v_2,v_1),\mu(v_2,v_3)] = \min[0.5,0.2] = 0.2$$

$$\mu_1(v_3) = \min[\mu(v_3,v_2),\mu(v_3,v_4)] = \min[0.2,0.2] = 0.2$$

$$\mu_1(v_4) = \min[\mu(v_4,v_1),\mu(v_4,v_3)] = \min[0.4,0.2] = 0.2$$

$$\gamma_1(v_1) = \max[\gamma(v_1,v_2),\gamma(v_1,v_4)] = \max[0.4,0.3] = 0.4$$

$$\gamma_1(v_2) = \max[\gamma(v_2,v_1),\gamma(v_2,v_3)] = \max[0.4,0.3] = 0.4$$

$$\gamma_1(v_3) = \max[\gamma(v_3,v_2),\gamma(v_3,v_4)] = \max[0.3,0.4] = 0.4$$

$$\gamma_1(v_4) = \max[\gamma(v_4,v_1),\gamma(v_4,v_3)] = \max[0.3,0.4] = 0.4$$

$$\sigma_1(v_1) = \max[\sigma(v_1,v_2),\sigma(v_1,v_4)] = \max[0.5,0.6] = 0.6$$

$$\sigma_1(v_2) = \max[\sigma(v_2,v_1),\sigma(v_2,v_3)] = \max[0.5,0.4] = 0.6$$

$$\sigma_1(v_3) = \max[\sigma(v_3,v_2),\sigma(v_3,v_4)] = \max[0.4,0.5] = 0.5$$

$$\sigma_1(v_4) = \max[\sigma(v_4,v_1),\sigma(v_4,v_3)] = \max[0.6,0.5] = 0.6$$

Here, v_2 dominates v_3 because

$$\mu(v_2v_3) \le \mu_1(v_2) \wedge \mu_1(v_3) 0.2 \le 0.2 \wedge 0.29$$

$$\gamma(v_2v_3) \le \gamma(v_2) \wedge \gamma(v_3) 0.3 \le 0.4 \wedge 0.4$$

$$\sigma(v_2v_3) \le \sigma(v_2) \wedge \sigma(v_3) 0.4 \le 0.5 \wedge 0.5$$

Here, v_3 dominates v_4 because

$$\mu(v_3v_4) \le \mu_1(v_3) \wedge \mu_1(v_4) 0.2 \le 0.2 \wedge 0.2$$

$$\gamma(v_3v_4) \le \gamma_1(v_3) \wedge \gamma_1(v_4) 0.4 \le 0.4 \wedge 0.4$$

$$\sigma(v_3v_4) \le \sigma(v_3) \wedge \sigma(v_4) 0.5 \le 0.5 \wedge 0.6$$

$$V = \{v_1,v_2,v_3,v_4\}, D^N = \{v_2,v_3\} \text{ and } V - D^N = \{v_1,v_4\}$$

$|D^N|$=2=sum of dominating elements

$$D^N(G) = \begin{bmatrix} (0,0,0) & (0.5,0.4,0.5) & (0,0,0) & (0.4,0.2,0.6) \\ (0.5,0.4,0.5) & (1,1,1) & (0.2,0.3,0.4) & (0,0,0) \\ (0,0,0) & (0.2,0.3,0.4) & (1,1,1) & (0.4,0.3) \\ (0.2,0.3) & (0.3,0.1) & (0.2,0.4,0.5) & (0,0,0) \end{bmatrix} \text{where}$$

$$\mu_{D^N}(G) = \begin{bmatrix} 1 & 0.5 & 0 & 0.4 \\ 0.5 & 1 & 0.2 & 0 \\ 0 & 0.2 & 1 & 0.2 \\ 0.4 & 0 & 0.2 & 0 \end{bmatrix} \quad \gamma_{D^N}(G) = \begin{bmatrix} 0 & 0.4 & 0 & 0.2 \\ 0.4 & 1 & 0.3 & 0 \\ 0 & 0.3 & 1 & 0.4 \\ 0.3 & 0 & 0.4 & 0 \end{bmatrix} \text{and } \sigma_{D^N}(G) = \begin{bmatrix} 0 & 0.5 & 0 & 0.6 \\ 0.5 & 1 & 0.4 & 0 \\ 0 & 0.4 & 1 & 0.5 \\ 0.6 & 0 & 0.5 & 0 \end{bmatrix}$$

3.1 Dominating energy in operations on neutrosophic graph

Dominating energy in complement of neutrosophic graph:

The complement of neutrosophic graph $G = (V, E)$ is neutrosophic graph, $\overline{G} = \langle \overline{V}, \overline{E} \rangle$, where $\overline{\mu}_{1i} = \mu_{1i}, \overline{\gamma}_{1i} = \gamma_{1i}$ and $\overline{\sigma}_{1i} = \sigma_{ij}$, for all i=1,2,.....n $\overline{\mu}_{2ij} = \mu_{1i}\mu_{1j} - \mu_{ij}$, $\overline{\gamma}_{2ij} = \gamma_{1i}\gamma_{1j} - \gamma_{2ij}$ and $\overline{\sigma}_{2ij} = \sigma_{1i}\sigma_{1j} - \sigma_{2ij}$, for all i,j=1,2,....n

First we find the dominating energy of neutrosophic graph $G(V, E)$.

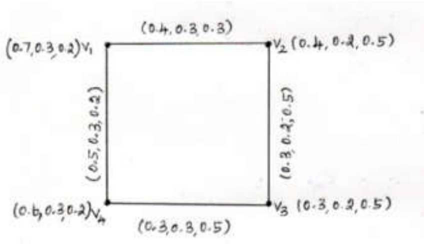

Figure 2: $G = (V, E)$

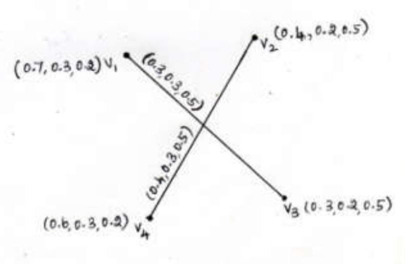

Figure 3: $\overline{G} = \langle \overline{V}, \overline{E} \rangle$

Consider a dominating neutrosophic graph $G = (V, E, \mu, \gamma, \sigma, \mu_1, \gamma_1, \sigma_1)$, where $V = \{v_1, v_2, v_3, v_4\}$ and $\mu_1, \gamma_1, \sigma_1$ are given by $\mu_1 : V \to [0,1], \gamma_1 : V \to [0,1]$ and $\sigma_1 : V \to [0,1]$ where

$$\mu_1(v_1) = \min[\mu(v_1v_2), \mu(v_1v_4)] = \min[0.4,0.5] = 0.4$$

$$\mu_1(v_2) = \min[\mu(v_2v_1), \mu(v_2v_3)] = \min[0.4,0.3] = 0.3$$

$$\mu_1(v_3) = \min[\mu(v_3v_2), \mu(v_3v_4)] = \min[0.3,0.2] = 0.2$$

$$\mu_1(v_4) = \min[\mu(v_4v_1), \mu(v_4v_3)] = \min[0.2,0.5] = 0.2$$

$$\gamma_1(v_1) = \max[\gamma(v_1v_2), \gamma(v_1v_4)] = \max[0.3,0.3] = 0.3$$

$$\gamma_1(v_2) = \max[\gamma(v_2v_1), \gamma(v_2v_3)] = \max[0.3,0.2] = 0.3$$

$$\gamma_1(v_3) = \max[\gamma(v_3v_2), \gamma(v_3v_4)] = \max[0.2,0.3] = 0.3$$

$$\gamma_1(v_4) = \max[\gamma(v_4v_1), \gamma(v_4v_3)] = \max[0.3,0.3] = 0.3$$

$$\sigma_1(v_1) = \max[\sigma(v_1v_2), \sigma(v_1v_4)] = \max[0.3,0.2] = 0.3$$

$$\sigma_1(v_2) = \max[\sigma(v_2v_1), \sigma(v_2v_3)] = \max[0.5,0.5] = 0.5$$

$$\sigma_1(v_3) = \max[\sigma(v_3v_2), \sigma(v_3v_4)] = \max[0.5,0.5] = 0.5$$

$$\sigma_1(v_4) = \max[\sigma(v_4v_1), \sigma(v_4v_3)] = \max[0.5,0.2] = 0.5$$

Here, v_3 dominates v_4 because

$$\mu(v_3v_4) \leq \mu_1(v_3) \wedge \mu_1(v_4) 0.2 \leq 0.2 \wedge 0.2$$

$$\gamma(v_3v_4) \leq \gamma(v_3) \wedge \gamma(v_4) 0.3 \leq 0.3 \wedge 0.3$$

$$\sigma(v_3v_4) \leq \sigma(v_3) \wedge \sigma(v_4) 0.5 \leq 0.5 \wedge 0.5$$

$V = \{v_1, v_2, v_3, v_4\}, D^N = \{v_3\}$ and $V - D^N = \{v_1, v_2, v_4\}$

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$|D^N|$=1=sum of dominating elements

$D^N(G) = \begin{bmatrix} (0,0,0) & (0.4,0.3,0.3) & (0,0,0) & (0.2,0.3,0.5) \\ (0.4,0.3,0.3) & (0,0,0) & (0.3,0.2,0.5) & (0,0,0) \\ (0,0,0) & (0.3,0.2,0.5) & (1,1,1) & (0.2,0.3,0.5) \\ (0.5,0.3,0.2) & (0,0,0) & (0.2,0.3,0.5) & (0,0,0) \end{bmatrix}$,where $\mu_{D^N}(G) = \begin{bmatrix} 0 & 0.4 & 0 & 0.2 \\ 0.4 & 0 & 0.3 & 0 \\ 0 & 0.3 & 1 & 0.2 \\ 0.5 & 0 & 0.2 & 0 \end{bmatrix}$, $\gamma_{D^N}(G) = \begin{bmatrix} 0 & 0.3 & 0 & 0.3 \\ 0.3 & 0 & 0.2 & 0 \\ 0 & 0.2 & 1 & 0.3 \\ 0.3 & 0 & 0.3 & 0 \end{bmatrix}$ and $\sigma_{D^N}(G) = \begin{bmatrix} 0 & 0.3 & 0 & 0.5 \\ 0.3 & 0 & 0.5 & 0 \\ 0 & 0.5 & 1 & 0.5 \\ 0.2 & 0 & 0.5 & 0 \end{bmatrix}$

Eigen values of $\mu_{D^N}(G) = \{-0.5549, 0.4064, 1.1431, 0,0054\}$=spectrum of $\mu_{D^N}(G)$

Eigen values of $\gamma_{D^N}(G) = \{-0.4692, 0.3414, 1.1327, -0.0049\}$=spectrum of $\gamma_{D^N}(G)$

Eigen values of $\sigma_{D^N}(G) = \{1.3981, -0.6218, 0.1940, 0.0296\}$=spectrum of $\sigma_{D^N}(G)$

Dominating energy of neutrosophic graph

$G = (V, E) = \left[\sum_{\lambda_i \in X} |\lambda_i|, \sum_{\delta_i \in Y} |\delta_i|, \sum_{\rho_i \in Z} |\rho_i|\right]$=[2.1098,1.9482,2.2435]

Now we find the dominating energy of neutrosophic graph $G(V, E)$.

Consider a dominating neutrosophic graph $G = (V, E, \mu, \gamma, \sigma, \mu_1, \gamma_1, \sigma_1)$ where $V = \{v_1, v_2, v_3, v_4\}$ and $\mu_1, \gamma_1, \sigma_1$ are given by $\mu_1 : V \to [0,1], \gamma_1 : V \to [0,1]$ and $\sigma_1 : V \to [0,1]$ where

$$\mu_1(v_1) = \min[\mu(v_1 v_3)] = \min[0.3] = 0.3$$

$$\mu_1(v_2) = \min[\mu(v_2 v_4)] = \min[0.4] = 0.4$$

$$\mu_1(v_3) = \min[\mu(v_3 v_1)] = \min[0.3] = 0.3$$

$$\mu_1(v_4) = \min[\mu(v_4 v_2] = \min[0.4] = 0.4$$

$$\gamma_1(v_1) = \max[\gamma(v_1 v_3)] = \max[0.3] = 0.3$$

$$\gamma_1(v_2) = \max[\gamma(v_2 v_4)] = \max[0.3] = 0.3$$

$$\gamma_1(v_3) = \max[\gamma(v_3 v_1)] = \max[0.3] = 0.3$$

$$\gamma_1(v_4) = \max[\gamma(v_4 v_2)] = \max[0.3] = 0.3$$

$$\sigma_1(v_1) = \max[\sigma(v_1 v_3)] = \max[0.5] = 0.5$$

$$\sigma_1(v_2) = \max[\sigma(v_2 v_4)] = \max[0.5] = 0.5$$

$$\sigma_1(v_3) = \max[\sigma(v_3 v_1)] = \max[0.5] = 0.5$$

$$\sigma_1(v_4) = \max[\sigma(v_4 v_2)] = \max[0.5] = 0.5$$

Here, v_1 dominates v_3 because

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$$\mu(v_1v_3) \leq \mu_1(v_1) \wedge \mu_1(v_3) 0.3 \leq 0.3 \wedge 0.3$$

$$\gamma(v_1v_3) \leq \gamma(v_1) \wedge \gamma(v_3) 0.3 \leq 0.3 \wedge 0.3$$

$$\sigma(v_1v_3) \leq \sigma(v_1) \wedge \sigma(v_3) 0.5 \leq 0.5 \wedge 0.5$$

Here, v_2 dominates v_4 because

$$\mu(v_2v_4) \leq \mu_1(v_2) \wedge \mu_1(v_4) 0.4 \leq 0.4 \wedge 0.4$$

$$\gamma(v_2v_4) \leq \gamma(v_2) \wedge \gamma(v_4) 0.3 \leq 0.3 \wedge 0.3$$

$$\sigma(v_2v_4) \leq \sigma(v_2) \wedge \sigma(v_4) 0.5 \leq 0.5 \wedge 0.5$$

$V = \{v_1, v_2, v_3, v_4\}, D^N = \{v_1, v_2\}$ and $V - D^N = \{v_3, v_4\}$

$|D^N|$=2=sum of dominating elements

$$D^N(G) = \begin{bmatrix} (1,1,1) & (0,0,0) & (0.3,0.3,0.5) & (0,0,0) \\ (0,0,0) & (1,1,1) & (0,0,0) & (0.4,0.3,0.5) \\ (0.3,0.3,0.5) & (0,0,0) & (0,0,0) & (0,0,0) \\ (0,0,0) & (0.4,0.3,0.5) & (0,0,0) & (0,0,0) \end{bmatrix} \text{ where}$$

$$\mu_{D^N}(G) = \begin{bmatrix} 1 & 0 & 0.3 & 0 \\ 0 & 1 & 0 & 0.4 \\ 0.3 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 \end{bmatrix}, \gamma_{D^N}(G) = \begin{bmatrix} 1 & 0 & 0.3 & 0 \\ 0 & 1 & 0 & 0.3 \\ 0.3 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 \end{bmatrix} \text{ and } \sigma_{D^N}(G) = \begin{bmatrix} 1 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0.5 \\ 0.5 & 1 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \end{bmatrix}$$

Eigen values of $\mu_{D^N}(G) = \{1.0830, -0.0830, 1.4031, -0.1403\}$=spectrum of $\mu_{D^N}(G)$

Eigen values of $\gamma_{D^N}(G) = \{1.0830, -0.0830, 1.0830, -0.0830\}$ =spectrum of $\gamma_{D^N}(G)$

Eigen values of $\sigma_{D^N}(G) = \{1.2071, -0.2071, 1.2071, -0.2071\}$ =spectrum of $\sigma_{D^N}(G)$

Dominating energy of complement of neutrosophic graph

$G = (V, E) = \left[\sum_{\lambda_i \in X} |\lambda_i|, \sum_{\delta_i \in Y} |\delta_i|, \sum_{\rho_i \in Z} |\rho_i|\right]$=[2.7094,2.332,2.8284]

3.2 Dominating energy in union of neutrosophic graph

Let $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$ be two neutrosophic Graphs with $V_1 \cap V_2 = \emptyset$ and $G = G_1 \cup G_2 = \langle V_1 \cup V_2, E_1 \cup E_2 \rangle$ be the union of G_1 and G_2. Then the union of neutrosophic graphs G_1 and G_2 is neutrosophic graph defined by

$$(\mu_1 \cup \mu_{1'})(v) = \begin{cases} \mu_1(v) & \text{if } v \in v_1 - v_2 \\ \mu_{1'}(v) & \text{if } v \in v_2 - v_1 \end{cases}, \quad (\gamma_1 \cup \gamma_{1'})(v) = \begin{cases} \gamma_1(v) & \text{if } v \in v_1 - v_2 \\ \gamma_{1'}(v) & \text{if } v \in v_2 - v_1 \end{cases}$$

$$(\sigma_1 \cup \sigma_{1'})(v) = \begin{cases} \sigma_1(v) & \text{if } v \in v_1 - v_2 \\ \sigma_{1'}(v) & \text{if } v \in v_2 - v_1 \end{cases} \text{ and } (\mu_2 \cup \mu_{2'})(v_i, v_j) = \begin{cases} \mu_2(e_{ij}) & \text{if } e_{ij} \in E_1 - E_2 \\ \mu_{2'}(e_{ij}) & \text{if } e_{ij} \in E_2 - E_1 \end{cases}$$

where $(\mu_1, \gamma_1, \sigma_1)$ and $(\mu_{1'}, \gamma_{1'}, \sigma_{1'})$ refer the vertex truth-membership, indeterminacy-membership and falsity-membership of G_1 and G_2 respectively,$(\mu_2, \gamma_2, \sigma_2)$ and $(\mu_{2'}, \gamma_{2'}, \sigma_{2'})$ refer the edge truth-membership, indeterminacy-membership and falsity-membership of G_1 and G_2 respectively.

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First we find the dominating energy of neutrosophic graph $G_1(V, E)$

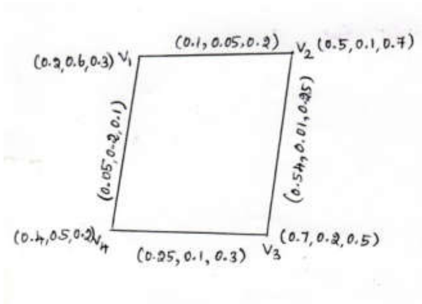

Fig. 4: G_1

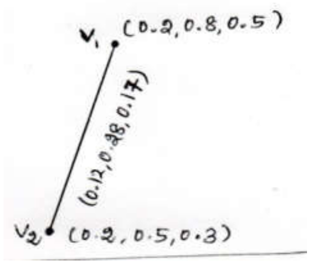

Fig.5: G_2

Consider a dominating neutrosophic graph $G = (V, E, \mu, \gamma, \sigma, \mu_1, \gamma_1, \sigma_1)$, where $V = \{v_1, v_2, v_3, v_4\}$ and $\mu_1, \gamma_1, \sigma_1$ are given by $\mu_1: V \to [0,1], \gamma_1: V \to [0,1]$ and $\sigma_1: V \to [0,1]$ where

$$\mu_1(v_1) = \min[\mu(v_1v_2), \mu(v_1v_4)] = \min[0.1,0.05] = 0.05$$

$$\mu_1(v_2) = \min[\mu(v_2v_1), \mu(v_2v_3)] = \min[0.54,0.1] = 0.1$$

$$\mu_1(v_3) = \min[\mu(v_3v_2), \mu(v_3v_4)] = \min[0.54,0.25] = 0.25$$

$$\mu_1(v_4) = \min[\mu(v_4v_1), \mu(v_4v_3)] = \min[0.05,0.25] = 0.05$$

$$\gamma_1(v_1) = \max[\gamma(v_1v_2), \gamma(v_1v_4)] = \max[0.05,0.2] = 0.2$$

$$\gamma_1(v_2) = \max[\gamma(v_2v_1), \gamma(v_2v_3)] = \max[0.05,0.01] = 0.05$$

$$\gamma_1(v_3) = \max[\gamma(v_3v_2), \gamma(v_3v_4)] = \max[0.01,0.1] = 0.1$$

$$\gamma_1(v_4) = \max[\gamma(v_4v_1), \gamma(v_4v_3)] = \max[0.2,0.1] = 0.2$$

$$\sigma_1(v_1) = \max[\sigma(v_1v_2), \sigma(v_1v_4)] = \max[0.2,0.1] = 0.2$$

$$\sigma_1(v_2) = \max[\sigma(v_2v_1), \sigma(v_2v_3)] = \max[0.2,0.25] = 0.25$$

$$\sigma_1(v_3) = \max[\sigma(v_3v_2), \sigma(v_3v_4)] = \max[0.25,0.3] = 0.3$$

$$\sigma_1(v_4) = \max[\sigma(v_4v_1), \sigma(v_4v_3)] = \max[0.1,0.3] = 0.3$$

Here, v_1 dominates v_4 because

$$\mu(v_1v_4) \le \mu_1(v_1) \wedge \mu_1(v_4) 0.05 \le 0.05 \wedge 0.05$$

$$\gamma(v_1v_4) \le \gamma(v_1) \wedge \gamma(v_4) 0.2 \le 0.2 \wedge 0.2$$

$$\sigma(v_1v_4) \le \sigma(v_1) \wedge \sigma(v_4) 0.05 \le 0.2 \wedge 0.3$$

$V = \{v_1, v_2, v_3, v_4\}, D^N = \{v_1\}$ and $V - D^N = \{v_2, v_3, v_4\}$

$|D^N|$=1=sum of dominating elements

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$$D^N(G_1) = \begin{bmatrix} (1,1,1) & (0.1,0.05,0.2) & (0,0,0) & (0.05,0.5,0.2) \\ (0.1,0.05,0.2) & (0,0,0) & (0.54,0.01,0.25) & (0,0,0) \\ (0,0,0) & (0.54,0.01,0.25) & (0,0,0) & (0.25,0.1,0.3) \\ (0.05,0.5,0.2) & (0,0,0) & (0.25,0.1,0.3) & (0,0,0) \end{bmatrix}$$ where

$$\mu_{D^N}(G_1) = \begin{bmatrix} 1 & 0.1 & 0 & 0.05 \\ 0.1 & 0 & 0.54 & 0 \\ 0 & 0.54 & 0 & 0.25 \\ 0.05 & 0 & 0.25 & 0 \end{bmatrix} \gamma_{D^N}(G_1) = \begin{bmatrix} 1 & 0.05 & 0 & 0.5 \\ 0.05 & 0 & 0.01 & 0 \\ 0 & 0.01 & 0 & 0.1 \\ 0.5 & 0 & 0.1 & 0 \end{bmatrix}$$ and

$$\sigma_{D^N}(G_1) = \begin{bmatrix} 1 & 0.2 & 0 & 0.2 \\ 0.2 & 0 & 0.25 & 0 \\ 0 & 0.25 & 0 & 0.3 \\ 0.2 & 0 & 0.3 & 0 \end{bmatrix}$$

Eigen values of $\mu_{D^N}(G_1) = \{1.0186, -0.5989, 0.5803, 0\}$=spectrum of $\mu_{D^N}(G_1)$

Eigen values of $\gamma_{D^N}(G_1) = \{1.2101, -0.2442, 0.0341, 0\}$=spectrum of $\gamma_{D^N}(G_1)$

Eigen values of $\sigma_{D^N}(G_1) = \{1.0846, -0.4194, 0.3354, -0.0006\}$=spectrum of $\sigma_{D^N}(G_1)$

Dominating energy of neutrosophic graph

$G = (V, E) = \left[\sum_{\lambda_i \in X} |\lambda_i|, \sum_{\delta_i \in Y} |\delta_i|, \sum_{\rho_i \in Z} |\rho_i|\right]$=[2.1978,1.4884,1.84]

Also we find the dominating energy of neutrosophic graph $G_2(V, E)$:

Let $V = \{v_1, v_2\}$

$$\mu_1(v_1) = \min[\mu(v_1 v_2] = \max[0.12] = 0.12$$

$$\mu_1(v_2) = \min[\mu(v_2 v_1)] = \max[0.12] = 0.12$$

$$\gamma_1(v_1) = \max[\gamma(v_1 v_2)] = \min[0.28] = 0.28$$

$$\gamma_1(v_2) = \max[\gamma(v_2 v_1)] = \min[0.28] = 0.28$$

$$\sigma_1(v_1) = \max[\sigma(v_1 v_2)] = \min[0.17] = 0.17$$

$$\sigma_1(v_2) = \max[\sigma(v_2 v_1)] = \min[0.17] = 0.17$$

Here, v_1 dominates v_2 because

$$\mu(v_1 v_2) \leq \mu_1(v_1) \wedge \mu_1(v_2) 0.12 \leq 0.12 \wedge 0.12$$

$$\gamma(v_1 v_2) \leq \gamma_1(v_1) \wedge \gamma_1(v_2) 0.28 \leq 0.28 \wedge 0.28$$

$$\sigma(v_1 v_2) \leq \sigma_1(v_1) \wedge \sigma_1(v_2) 0.17 \leq 0.17 \wedge 0.17$$

Here, $V = \{v_1, v_2\}$ and $D^N = \{v_1\}$;$V - D^N = \{v_2\}$

$|D^N|$=1=sum of dominating elements.

$$D^N(G_2) = \begin{bmatrix} (1,1,1) & (0.12,0.28,0.17) \\ (0.12,0.28,0.17) & (0,0,0) \end{bmatrix}$$, where

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$\mu_{D^N}(G_2) = \begin{bmatrix} 1 & 0.12 \\ 0.12 & 0 \end{bmatrix}$ $\gamma_{D^N}(G_2) = \begin{bmatrix} 1 & 0.28 \\ 0.28 & 0 \end{bmatrix}$ and $\sigma_{D^N}(G_2) = \begin{bmatrix} 1 & 0.17 \\ 0.17 & 0 \end{bmatrix}$

Eigen values of$\mu_{D^N}(G_2) = \{1.0142, -0.0141\}$=spectrum of $\mu_{D^N}(G_2)$

Eigen values of $\gamma_{D^N}(G_2) = \{1.07306, -0.0736\}$=spectrum of $\gamma_{D^N}(G_2)$

Eigen values of $\sigma_{D^N}(G_2) = \{1.0281, -0.0281\}$=spectrum of $\sigma_{D^N}(G_2)$

Dominating energy of neutrosophic graph

$G = (V, E) = \left[\sum_{\lambda_i \in X} |\lambda_i|, \sum_{\delta_i \in Y} |\delta_i|, \sum_{\rho_i \in Z} |\rho_i|\right]$=[0.0183,1.1466,1.0562]

Now we find the dominating energy of union of neutrosophic graph $G_1 \cup G_2$

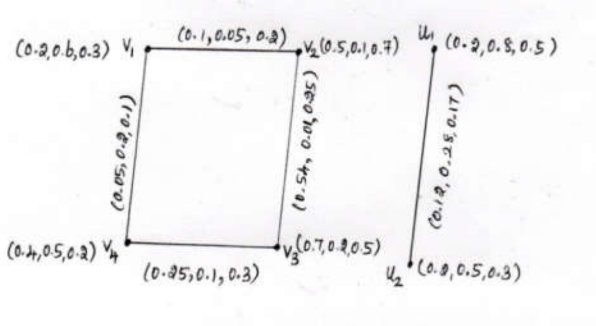

Fig.6: $G_1 \cup D_2$

$$\mu_1(v_1) = \min[\mu(v_1v_2), \mu(v_1v_4)] = \min[0.1,0.05] = 0.05$$

$$\mu_1(v_2) = \min[\mu(v_2v_1), \mu(v_2v_3)] = \min[0.54,0.1] = 0.1$$

$$\mu_1(v_3) = \min[\mu(v_3v_2), \mu(v_3v_4)] = \min[0.54,0.25] = 0.25$$

$$\mu_1(v_4) = \min[\mu(v_4v_1), \mu(v_4v_3)] = \min[0.05,0.25] = 0.05$$

$$\mu_1(u_1) = \min[\mu(u_1u_2] = \min[0.12] = 0.12$$

$$\mu_1(u_2) = \min[\mu(v_2u_1)] = \min[0.12] = 0.12$$

$$\gamma_1(v_1) = \max[\gamma(v_1v_2), \gamma(v_1v_4)] = \max[0.05,0.2] = 0.2$$

$$\gamma_1(v_2) = \max[\gamma(v_2v_1), \gamma(v_2v_3)] = \max[0.05,0.01] = 0.05$$

$$\gamma_1(v_3) = \max[\gamma(v_3v_2), \gamma(v_3v_4)] = \max[0.01,0.1] = 0.1$$

$$\gamma_1(v_4) = \max[\gamma(v_4v_1), \gamma(v_4v_3)] = \max[0.2,0.1] = 0.2$$

$$\gamma_1(u_1) = \max[\gamma(u_1u_2)] = \max[0.28] = 0.28$$

$$\gamma_1(u_2) = \max[\gamma(u_2u_1)] = \max[0.28] = 0.28$$

$$\sigma_1(v_1) = \max[\sigma(v_1v_2), \sigma(v_1v_4)] = \max[0.2,0.1] = 0.2$$

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$$\sigma_1(v_2) = \max[\sigma(v_2v_1), \sigma(v_2v_3)] = \max[0.2, 0.25] = 0.25$$

$$\sigma_1(v_3) = \max[\sigma(v_3v_2), \sigma(v_3v_4)] = \max[0.25, 0.3] = 0.3$$

$$\sigma_1(v_4) = \max[\sigma(v_4v_1), \sigma(v_4v_3)] = \max[0.1, 0.3] = 0.3$$

$$\sigma_1(u_1) = \max[\sigma(u_1u_2)] = \max[0.17] = 0.17$$

$$\sigma_1(u_2) = \max[\sigma(u_2u_1)] = \max[0.17] = 0.17$$

Here, v_1 dominates v_4 because

$$\mu(v_1v_4) \leq \mu_1(v_1) \wedge \mu_1(v_4) 0.05 \leq 0.05 \wedge 0.05$$

$$\gamma(v_1v_4) \leq \gamma(v_1) \wedge \gamma(v_4) 0.2 \leq 0.2 \wedge 0.2$$

$$\sigma(v_1v_4) \leq \sigma(v_1) \wedge \sigma(v_4) 0.05 \leq 0.2 \wedge 0.3$$

Here, u_1 dominates u_2 because

$$\mu(u_1v_2) \leq \mu_1(u_1) \wedge \mu_1(u_2) 0.12 \leq 0.12 \wedge 0.12$$

$$\gamma(u_1u_2) \leq \gamma_1(u_1) \wedge \gamma_1(u_2) 0.28 \leq 0.28 \wedge 0.28$$

$$\sigma(u_1u_2) \leq \sigma_1(u_1) \wedge \sigma_1(u_2) 0.17 \leq 0.17 \wedge 0.17$$

$V = \{v_1, v_2, v_3, v_4, u_1, u_2\}$, $D^N = \{v_1, u_1\}$ and $V - D^N = \{v_2, v_3, v_4, u_2\}$

$|D^N|$=2=sum of dominating elements

$$D^N(G_1 \cup G_2) = \begin{bmatrix} (1,1,1) & (0.1,0.05,0.2) & (0,0,0) & (0.05,0.2,0.1) & (0,0,0) & (0,0,0) \\ (0.1,0.05,0.2) & (0,0,0) & (0.54,0.01,0.25) & (0,0,0) & (0,0,0) & (0,0,0) \\ (0,0,0,) & (0.54,0.01,0.25) & (0,0,0) & (0.25,0.1,0.3) & (0,0,0) & (0,0,0) \\ (0.05,0.2,0.1) & (0,0,0) & (0.25,0.1,0.3) & (0,0,0) & (0,0,0) & (0.12,0.28,0.17) \\ (0,0,0,) & (0,0,0) & (0,0,0) & (0,0,0) & (1,1,1) & (0,0,0) \\ (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) & (0.12,0.28,0.17) & (0,0,0) \end{bmatrix}$$

where

$$\mu_{D^N}(G_1 \cup G_2) = \begin{bmatrix} 1 & 0.1 & 0 & 0.05 & 0 & 0 \\ 0.1 & 0 & 0.54 & 0 & 0 & 0 \\ 0 & 0.54 & 0 & 0.25 & 0 & 0 \\ 0.05 & 0 & 0.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.12 \\ 0 & 0 & 0 & 0 & 0.12 & 0 \end{bmatrix}, (G_1 \cup G_2) = \begin{bmatrix} 1 & 0.05 & 0 & 0.2 & 0 & 0 \\ 0.05 & 0 & 0.01 & 0 & 0 & 0 \\ 0 & 0.54 & 0 & 0.25 & 0 & 0 \\ 0.05 & 0 & 0.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.28 \\ 0 & 0 & 0 & 0 & 0.28 & 0 \end{bmatrix}$$

and

$$\sigma_{D^N}(G_1 \cup G_2) = \begin{bmatrix} 1 & 0.2 & 0 & 0.1 & 0 & 0 \\ 0.2 & 0 & 0.25 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0.3 & 0 & 0 \\ 0.1 & 0 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.17 \\ 0 & 0 & 0 & 0 & 0.17 & 0 \end{bmatrix}$$

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Eigen values of $\mu_{D^N}(G_1 \cup G_2) = \{1.0186, -0.5989, 0.5803, 0, 1.0141, -0.0141\}$=spectrum of $\mu_{D^N}(G_1 \cup G_2)$

Eigen values of $\gamma_{D^N}(G_1 \cup G_2) = \{1.0143, 0.0173, -0.2718, 0.2401, 1.0730, -0.0730\}$=spectrum of $\gamma_{D^N}(G_1 \cup G_2)$

Eigen values of $\sigma_{D^N}(G_1 \cup G_2) = \{1.0537, -0.4059, 0.3601, -0.0079, 1.028, -0.0281\}$=spectrum of $\sigma_{D^N}(G_1 \cup G_2)$

Dominating energy of union of neutrosophic graph

$G = (V, E) = \left[\sum_{\lambda_i \in X} |\lambda_i|, \sum_{\delta_i \in Y} |\delta_i|, \sum_{\rho_i \in Z} |\rho_i|\right]$=[3.226,2.6895,2.8897]

3.3 Dominating energy in join of neutrosophic graph

The join of two neutrosophic graph

$G = G_1 + G_2 = \langle V_1 \cup V_2, E_1 \cup E_2 \rangle$ defined by

$(\mu_1 + \mu_{1'})(v) = (\mu_1 \cup \mu_{1'})(v)$ if $v \in V_1 \cup V_2$

$(\gamma_1 + \gamma_{1'})(v) = (\gamma_1 \cup \gamma_{1'})(v)$ if $v \in V_1 \cup V_2$

$(\sigma_1 + \sigma_{1'})(v) = (\sigma_1 \cup \sigma_{1'})(v)$ if $v \in V_1 \cup V_2$

$(\mu_2 + \mu_{2'})(v_i v_j) = (\mu_2 \cup \mu_{2'})(v_i v_j)$ if $v_i v_j \in E_1 \cup E_2$

Now we find the dominating energy of join of neutrosophic graph $G(V, E)$:

Consider a dominating neutrosophic graph $G = (V, E, \mu, \gamma, \sigma, \mu_1, \gamma_1, \sigma_1)$ where $V = \{v_1, v_2, v_3, v_4\}$ and $\mu_1, \gamma_1, \sigma_1$ are given by $\mu_1: V \to [0,1], \gamma_1: V \to [0,1]$ and $\sigma_1: V \to [0,1]$ where

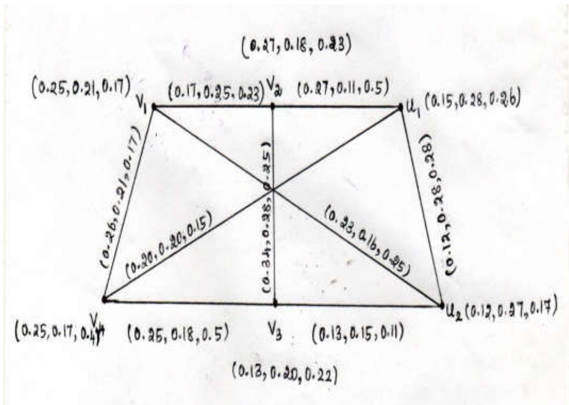

Fig.7: $G = G_1 + G_2$

$\mu_1(v_1) = \min[\mu(v_1v_2), \mu(v_1u_2), \mu(v_1v_4)] = \min[0.17, 0.23, 0.26] = 0.17$

$\mu_1(v_2) = \min[\mu(v_2v_1), \mu(v_2u_1), \mu(v_2v_3)] = \min[0.17, 0.27, 0.34] = 0.17$

$\mu_1(v_3) = \min[\mu(v_3v_2), \mu(v_3v_4), \mu(v_3u_2)] = \min[0.34, 0.24, 0.13] = 0.13$

$\mu_1(v_4) = \min[\mu(v_4v_1), \mu(v_4u_1), \mu(v_4v_3)] = \min[0.26, 0.20, 0.25]$

$\mu_1(u_1) = \min[\mu(u_1v_2), \mu(u_1v_4), \mu(u_1u_2)] = \min[0.27, 0.20, 0.12] = 0.12$

$$\mu_1(u_2) = \min[\mu(u_2u_1), \mu(u_2v_1), \mu(u_2v_3)] = \min[0.12,0.23,0.13] = 0.12$$

$$\gamma_1(v_1) = \max[\gamma(v_1v_2), \gamma(v_1u_2), \gamma(v_1v_4)] = \max[0.25,0.16,0.21] = 0.25$$

$$\gamma_1(v_2) = \max[\gamma(v_2v_1), \gamma(v_2u_1), \gamma(v_2v_3)] = \max[0.25,0.11,0.28] = 0.28$$

$$\gamma_1(v_3) = \max[\gamma(v_3v_2), \gamma(v_3v_4, \gamma(v_3u_2)] = \max[0.28,0.18,0.15] = 0.28$$

$$\gamma_1(v_4) = \max[\gamma(v_4v_1), \gamma(v_4u_1), \gamma(v_4v_3)] = \max[0.21,0.20,0.18] = 0.21$$

$$\gamma_1(u_1) = \max[\gamma(u_1v_2), \gamma(u_1v_4), \gamma(u_1u_2)] = \max[0.11,0.20,0.28] = 0.28$$

$$\gamma_1(u_2) = \max[\gamma(u_2u_1), \gamma(u_2v_1), \gamma(u_2v_3)] = \max[0.28,0.16,0.15] = 0.28$$

$$\sigma_1(v_1) = \max[\sigma(v_1v_2), \sigma(v_1u_2), \sigma(v_1v_4)] = \max[0.23,0.25,0.17] = 0.25$$

$$\sigma_1(v_2) = \max[\sigma(v_2v_1), \sigma(v_2u_1), \sigma(v_2v_3)] = \max[0.23,0.5,0.25] = 0.25$$

$$\sigma_1(v_3) = \max[\sigma(v_3v_2), \sigma(v_3v_4, \sigma(v_3u_2)] = \max[0.25,0.5,0.11] = 0.25$$

$$\sigma_1(v_4) = \max[\sigma(v_4v_1), \sigma(v_4u_1), \sigma(v_4v_3)] = \max[0.17,0.15,0.5] = 0.17$$

$$\sigma_1(u_1) = \max[\sigma(u_1v_2), \sigma(u_1v_4), \sigma(u_1u_2)] = \max[0.5,0.15,0.28] = 0.28$$

$$\sigma_1(u_2) = \max[\sigma(u_2u_1), \sigma(u_2v_1), \sigma(u_2v_3)] = \min[0.28,0.25,0.11] = 0.28$$

Here, v_1 dominates v_2 because

$$\mu(v_1v_2) \le \mu_1(v_1) \wedge \mu_1(v_2) 0.17 \le 0.17 \wedge 0.17 \gamma(v_1v_2) \le \gamma_1(v_1) \wedge \gamma_1(v_2) 0.25 \le 0.25 \wedge 0.25$$

$V = \{v_1, v_2, v_3, v_4, u_1, u_2\}$, $D^N = \{v_1, u_1\}$ and $V - D^N = \{v_2, v_3, v_4, u_2\}$

$|D^N|$=2=sum of dominating elements

$$D^N(G) = \begin{bmatrix} (1,1,1) & (0.17,0.25,0.23) & (0,0,0) & (0.26,0.21,0.17) & (0,0,0) & (0,0,0) \\ (0.17,0.25,0.23) & (0,0,0) & (0.36,0.28,0.25) & (0,0,0) & (0.27,0.11,0.5) & (0,0,0) \\ (0,0,0,) & (0.36,0.28,0.25) & (0,0,0) & (0.25,0.18,0.5) & (0,0,0) & (0.13,0.15,0.11) \\ (0.26,0.21,0.17) & (0,0,0) & (0.25,0.18,0.5) & (0,0,0) & (0.20,0.20,0.15) & (0,0,0) \\ (0,0,0,) & (0.27,0.11,0.5) & (0,0,0) & (0.20,0.20,0.15) & (1,1,1) & (0.12,0.28,0.28) \\ (0.17,0.25,0.23) & (0,0,0) & (0.13,0.15,0.11) & (0,0,0) & (0.12,0.28,0.28) & (0,0,0) \end{bmatrix}$$

where

$$\mu_{D^N}(G) = \begin{bmatrix} 1 & 0.17 & 0 & 0.28 & 0 & 0 \\ 0.17 & 0 & 0.36 & 0 & 0.27 & 0 \\ 0 & 0.36 & 0 & 0.25 & 0 & 0.13 \\ 0.26 & 0 & 0.25 & 0 & 0.20 & 0 \\ 0 & 0.27 & 0 & 0.20 & 1 & 0.12 \\ 0.17 & 0 & 0.13 & 0 & 0.28 & 0 \end{bmatrix}, \gamma_{D^N}(G) = \begin{bmatrix} 1 & 0.25 & 0 & 0.21 & 0 & 0 \\ 0.25 & 0 & 0.28 & 0 & 0.11 & 0 \\ 0 & 0.28 & 0 & 0.18 & 0 & 0.5 \\ 0.21 & 0 & 0.18 & 0 & 0.20 & 0 \\ 0 & 0.11 & 0 & 0.20 & 1 & 0.28 \\ 0.25 & 0 & 0.15 & 0 & 0.28 & 0 \end{bmatrix}$$

and

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$$\sigma_{D^N}(G) = \begin{bmatrix} 1 & 0.23 & 0 & 0.17 & 0 & 0 \\ 0.23 & 0 & 0.25 & 0 & 0.5 & 0 \\ 0 & 0.25 & 0 & 0.5 & 0 & 0.11 \\ 0.17 & 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0.15 & 1 & 0.28 \\ 0.23 & 0 & 0.11 & 0 & 0.28 & 0 \end{bmatrix}$$

Eigen values of $\mu_{D^N}(G) = \{1.2241, 1.0118, -0.5402, 0.3163, -0.0061 + i0.0054, -0.0061 - i0.0054\}$ = spectrum of $\mu_{D^N}(G)$

Eigen values of $\gamma_{D^N}(G) = \{1.2108, 1.0184, -0.5190, 0.2687, 0.0255, -0.004\}$=spectrum of $\gamma_{D^N}(G)$

Eigen values of $\sigma_{D^N}(G) = \{1.4069, 0.9828, -0.7028, 0.4109, -0.1051, 0.0072\}$=spectrum of $\sigma_{D^N}(G)$

Dominating Energy of join of neutrosophic graph

$G = (V, E) = \left[\sum_{\lambda_i \in X} |\lambda_i|, \sum_{\delta_i \in Y} |\delta_i|, \sum_{\rho_i \in Z} |\rho_i|\right]$=[0.0121+i0.0108,3.0469,3.6157]

Let $G = (V, E, \mu, \gamma, \sigma, \mu_1, \gamma_1, \sigma_1)$ be a dominating neutrosophic graph with vertex set $V = \{v_1, v_2, \ldots. v_n\}$, edge set E. Let

$D^N = \{u_1, u_2, \ldots. u_n\}$ be a dominating set. If $\lambda_1, \lambda_2, \ldots. \lambda_n$ are the eigen values of dominating matrix $\mu_{D^N}(G)$ then

(i) $\sum_{i=1}^{n} \lambda_i = |D^N|$, (ii)$\sum_{i=1}^{n} \lambda_i^2 = \sum_{i=1}^{n} \mu_{ii}^2 + 2\sum_{1 \le i < j \le n} \mu_{ij}\mu_{ji}$

if $\delta_1, \delta_2, \ldots. \delta_n$ are the eigen values of dominating matrix $\gamma_{D^N}(G)$ then

(iii)$\sum_{i=1}^{n} \delta_i = |D^N|$, (iv)$\sum_{i=1}^{n} \delta_i^2 = \sum_{i=1}^{n} \gamma_{ii}^2 + \sum_{1 \le i < j \le n} \gamma_{ij}\gamma_{ji}$

and if $\rho_1, \rho_2, \ldots. \rho_n$ are the eigen values of dominating matrix $\sigma_{D^N}(G)$ then (v)$\sum_{i=1}^{n} \rho_i = |D^N|$, (vi)$\sum_{i=1}^{n} \rho_i^2 = \sum_{i=1}^{n} \sigma_{ii}^2 + \sum_{1 \le i < j \le n} \sigma_{ij}\sigma_{ji}$

Proof:

(i) We know that the sum of the eigen values of $\mu_{D^N}(G)$ is equal to the trace of $\mu_{D^N}(G)$ $\sum_{i=1}^{n} \lambda_i = \sum_{i=1}^{n} \mu_{ii} = |D^N|$.

(ii) Similarly, the sum of the squares of the eigen values of $\mu_{D^N}(G)$ is equal to the trace of $(\mu_{D^N}(G))^2$.

$\sum_{i=1}^{n} \lambda_i^2 = \text{traceof}(\mu_{D^N}(G))^2$=$\mu_{11}\mu_{11} + \mu_{12}\mu_{21} + \mu_{13}\mu_{31} + \ldots.. + \mu_{1n}\mu_{n1} + \mu_{21}\mu_{12} + \mu_{22}\mu_{22} + \mu_{23}\mu_{32} + \ldots.. + \mu_{2n}\mu_{n2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + + \mu_{n1}\mu_{1n} + \mu_{n2}\mu_{2n} + \mu_{n3}\mu_{3n} + \ldots.. + \mu_{nn}\mu_{nn}$

$$\sum_{i=1}^{n} \lambda_i^2 = \sum_{i=1}^{n} \mu_{ii}^2 + 2\sum_{1 \le i < j \le n} \mu_{ij}\mu_{ji}$$

(iii) We know that the sum of the eigen values of $\gamma_{D^N}(G)$ is equal to the trace of $\gamma_{D^N}(G)$ $\sum_{i=1}^{n} \delta_i = \sum_{i=1}^{n} \gamma_{ii} = |D^N|$.

(iv)Similarly, the sum of the squares of the eigen values of $\gamma_{D^N}(G)$ is equal to the trace of $(\gamma_{D^N}(G))^2$.

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$\sum_{i=1}^{n} \delta_i^2$=trace of $(\gamma_{D^N}(G))^2 = \gamma_{11}\gamma_{11} + \gamma_{12}\gamma_{21} + \gamma_{13}\gamma_{31} + \ldots + \gamma_{1n}\gamma_{n1} + \gamma_{21}\gamma_{12} + \gamma_{22}\gamma_{22} + \gamma_{23}\gamma_{32} + \ldots + \gamma_{2n}\gamma_{n2} + \ldots\ldots + + \gamma_{n1}\gamma_{1n} + \gamma_{n2}\gamma_{2n} + \gamma_{n3}\gamma_{3n} + \ldots + \gamma_{nn}\gamma_{nn}$

$$\sum_{i=1}^{n} \delta_i^2 = \sum_{i=1}^{n} \gamma_{ii}^2 + \sum_{1\le i<j\le n} \gamma_{ij}\gamma_{ji}$$

(v) We know that the sum of the eigen values of $\sigma_{D^N}(G)$ is equal to the trace of $\sigma_{D^N}(G)\sum_{i=1}^{n} \rho_i = \sum_{i=1}^{n} \sigma_{ii} = |D^N|$.

(vi)Similarly the sum of the squares of the eigen values of $\sigma_{D^N}(G)$ is equal to the trace of $(\sigma_{D^N}(G))^2$.

$\sum_{i=1}^{n} \rho_i^2$=trace of $(\sigma_{D^N}(G))^2 = \sigma_{11}\sigma_{11} + \sigma_{12}\sigma_{21} + \sigma_{13}\sigma_{31} + \ldots + \sigma_{1n}\sigma_{n1} + \sigma_{21}\sigma_{12} + \sigma_{22}\sigma_{22} + \sigma_{23}\sigma_{32} + \ldots + \sigma_{2n}\sigma_{n2} + \ldots\ldots + \sigma_{n1}\sigma_{1n} + \sigma_{n2}\sigma_{2n} + \sigma_{n3}\sigma_{3n} + \ldots + \sigma_{nn}\sigma_{nn}$

$\sum_{i=1}^{n} \rho_i^2 = \sum_{i=1}^{n} \sigma_{ii}^2 + \sum_{1\le i<j\le n} \sigma_{ij}\sigma_{ji}$ Let $G = (V, E, \mu, \gamma, \sigma, \mu_1, \gamma_1, \sigma_1)$ be a dominating neutrosophic graph with n vertices and m edges. If D^N is the dominating set then

(i) $\sqrt{\sum_{i=1}^{n} \mu_{ii}^2 + 2\sum_{1\le i<j\le n} \mu_{ij}\mu_{ji} + n(n-1)|A|^{\frac{2}{n}}} \le E(\mu_{D^N}G) \le \sqrt{n\left[\sum_{i=1}^{n} \mu_{ii}^2 + 2\sum_{1\le i<j\le n} \mu_{ij}\mu_{ji}\right]}$ where $|A|$ is the determinant of $\mu_{D^N}(G)$.

(ii) $\sqrt{\sum_{i=1}^{n} \gamma_{ii}^2 + 2\sum_{1\le i<j\le n} \gamma_{ij}\gamma_{ji} + n(n-1)|B|^{\frac{2}{n}}} \le E(\gamma_{D^N}G) \le \sqrt{n\left[\sum_{i=1}^{n} \gamma_{ii}^2 + 2\sum_{1\le i<j\le n} \gamma_{ij}\gamma_{ji}\right]}$ Where $|B|$ is the determinant of $\gamma_{D^N}(G)$.

(iii) $\sqrt{\sum_{i=1}^{n} \sigma_{ii}^2 + 2\sum_{1\le i<j\le n} \sigma_{ij}\sigma_{ji} + n(n-1)|C|^{\frac{2}{n}}} \le E(\sigma_{D^N}G) \le \sqrt{n\left[\sum_{i=1}^{n} \sigma_{ii}^2 + 2\sum_{1\le i<j\le n} \sigma_{ij}\sigma_{ji}\right]}$ Where $|C|$ is the determinant of $\sigma_{D^N}(G)$.

Proof: Cauchy Schwarz inequality is $[\sum_{i=1}^{n} a_i b_i]^2 \le [\sum_{i=1}^{n} a_i^2][\sum_{i=1}^{n} b_i^2]$

Upper bound:

If $a_i = 1, b_i = |\lambda_i|$ then $[\sum_{i=1}^{n} |\lambda_i|]^2 \le [\sum_{i=1}^{n} 1][\sum_{i=1}^{n} \lambda_i^2]$

$(E(\mu_{D^N}(G)))^2 \le n\left[\sum_{i=1}^{n} \mu_{ii}^2 + 2\sum_{1\le i<j\le n} \mu_{ij}\mu_{ji}\right]$

$E(\mu_{D^N}(G)) \le \sqrt{n\left[\sum_{i=1}^{n} \mu_{ii}^2 + 2\sum_{1\le i<j\le n} \mu_{ij}\mu_{ji}\right]}$..............(1)

Lower bound:

$$(E(\mu_{D^N}(G)))^2 = \left[\sum_{i=i}^{n} |\lambda_i|\right]^2 = \sum_{i=1}^{n} |\lambda_i|^2 + 2\sum_{1\le i<j\le n} |\lambda_i||\lambda_j$$
$$= \left[\sum_{i=1}^{n} \mu_{ii}^2 + 2\sum_{1\le i<j\le n} \mu_{ij}\mu_{ji}\right] + 2\frac{n(n-1)}{2} AM_{1\le i<j\le n}\{|\lambda_i||\lambda_j|\}$$

But, $AM_{1\le i<j\le n}\{|\lambda_i||\lambda_j|\} \ge GM_{1\le i<j\le n}\{|\lambda_i||\lambda_j|\}$

Therefore, $E(\mu_{D^N}(G)) \ge \sqrt{\sum_{i=1}^{n} \mu_{ii}^2 + 2\sum_{1\le i<j\le n} \mu_{ij}\mu_{ji} + n(n-1)GM_{1\le i<j\le n}\{|\lambda_i||\lambda_j|\}}$

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$$\mathrm{GM}_{1\le i<j\le n}\{|\lambda_i||\lambda_j|\} = \left[\prod_{1\le i<j\le n} |\lambda_i||\lambda_j|\right]^{\frac{2}{n(n-1)}}$$

$= [\prod_{i=1}^{n} |\lambda_i|^{n-1}]^{\frac{2}{n(n+1)}} = [\prod_{i=1}^{n} |\lambda_i|]^{\frac{2}{n}} = |A|^{\frac{2}{n}} E(\mu_{D^N}(G)) \ge$
$\sqrt{\sum_{i=1}^{n} \mu_{ii}^2 + 2\sum_{1\le i<j\le n} \mu_{ij}\mu_{ji} + n(n-1)|A|^{\frac{2}{n}}}$............(2)

From (1) and (2)

$$\sqrt{\sum_{i=1}^{n} \mu_{ii}^2 + 2\sum_{1\le i<j\le n} \mu_{ij}\mu_{ji} + n(n-1)|A|^{\frac{2}{n}}} \le E(\mu_{D^N}G) \le \sqrt{n\left[\sum_{i=1}^{n} \mu_{ii}^2 + 2\sum_{1\le i<j\le n} \mu_{ij}\mu_{ji}\right]}$$

Similarly, we can prove

$$\sqrt{\sum_{i=1}^{n} \gamma_{ii}^2 + 2\sum_{1\le i<j\le n} \gamma_{ij}\gamma_{ji} + n(n-1)|B|^{\frac{2}{n}}} \le E(\gamma_{D^N}G) \le \sqrt{n\left[\sum_{i=1}^{n} \gamma_{ii}^2 + 2\sum_{1\le i<j\le n} \gamma_{ij}\gamma_{ji}\right]} \text{ and}$$

$$\sqrt{\sum_{i=1}^{n} \sigma_{ii}^2 + 2\sum_{1\le i<j\le n} \sigma_{ij}\sigma_{ji} + n(n-1)|C|^{\frac{2}{n}}} \le E(\sigma_{D^N}G) \le \sqrt{n\left[\sum_{i=1}^{n} \sigma_{ii}^2 + 2\sum_{1\le i<j\le n} \sigma_{ij}\sigma_{ji}\right]}$$

Let $G = (V, E, \mu, \gamma, \sigma)$ be an neutrosophic graph and let $A(G) = (\mu(G), \gamma(G), \sigma(G))$ be an neutrosophic graph adjacency matrix of G. Let $G_1 = (V, E, \mu, \gamma, \sigma, \mu_1, \gamma_1, \sigma_1)$ be the dominating neutrosophic graph of G and let $D(G) = (\mu_{D^N}(G), \gamma_{D^N}(G), \sigma_{D^N}(G))$ be the dominating neutrosophic adjacency matrix of G_1. Then

(i)$(E(\mu_{D^N}(G)))^2 \le n[\sum_{i=1}^{n} \mu_{ii}^2 + (E(\mu(G)))^2](ii)(E(\gamma_{D^N}(G)))^2 \le n[\sum_{i=1}^{n} \gamma_{ii}^2 + (E(\gamma(G)))^2](iii)(E(\sigma_{D^N}(G)))^2 \le n[\sum_{i=1}^{n} \sigma_{ii}^2 + (E(\sigma(G)))^2]$

Proof:

$$(E(\mu(G)))^2 \ge 2\sum_{1\le i<j\le n} \mu_{ij}\mu_{ji} + n(n-1)|A|^{\frac{2}{n}} \ge 2\sum_{1\le i<j\le n} \mu_{ij}\mu_{ji} (\text{i.e}) 2\sum_{1\le i<j\le n} \mu_{ij}\mu_{ji} \le (E(\mu(G)))^2$$

Now $(E(\mu_{D^N}(G)))^2 \le n\sum_{i=1}^{n} \mu_{ii}^2 + 2n\sum_{1\le i<j\le n} \mu_{ij}\mu_{ji} (E(\mu_{D^N}(G)))^2 \le n\sum_{i=1}^{n} \mu_{ii}^2 + (E(\mu(G)))^2$

Similarly, we can prove

$$(E(\gamma(G)))^2 \ge 2\sum_{1\le i<j\le n} \gamma_{ij}\mu_{ji} + n(n-1)|A|^{\frac{2}{n}} \ge 2\sum_{1\le i<j\le n} \gamma_{ij}\gamma_{ji} (\text{i.e}) 2\sum_{1\le i<j\le n} \gamma_{ij}\gamma_{ji} \le (E(\gamma(G)))^2$$

Now, $(E(\gamma_{D^N}(G)))^2 \le n\sum_{i=1}^{n} \gamma_{ii}^2 + 2n\sum_{1\le i<j\le n} \gamma_{ij}\gamma_{ji} (E(\gamma_{D^N}(G)))^2 \le n\sum_{i=1}^{n} \gamma_{ii}^2 + (E(\gamma(G)))^2$

and

$$(E(\sigma(G)))^2 \ge 2\sum_{1\le i<j\le n} \sigma_{ij}\mu_{ji} + n(n-1)|A|^{\frac{2}{n}} \ge 2\sum_{1\le i<j\le n} \sigma_{ij}\sigma_{ji} (\text{i.e}) 2\sum_{1\le i<j\le n} \sigma_{ij}\sigma_{ji} \le (E(\sigma(G)))^2$$

Now, $(E(\sigma_{D^N}(G)))^2 \le n\sum_{i=1}^{n} \sigma_{ii}^2 + 2n\sum_{1\le i<j\le n} \sigma_{ij}\sigma_{ji} (E(\sigma_{D^N}(G)))^2 \le n\sum_{i=1}^{n} \sigma_{ii}^2 + (E(\sigma(G)))^2$

4 Conclusion

The dominating energy of neutrosophic graph is introduced in this proposed research. Dominating energy of a neutrosophic graph, dominating neutrosophic adjacency matrix, eigen values for the dominating energy of neutrosophic graph and complement neutrosophic graphs are defined with examples. Also dominating energy in union and join operations of neutrosophic graph are developed with suitable examples and some theorems in dominating energy of neutrosophic graph are established. These results will be applied in various real life situations in future.

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On Some Special Substructures of Refined Neutrosophic Rings

Mohammad Abobala

Tishreen University, Faculty of Science, Lattakia, Syria

e-mail: mohammadabobala777@gmail.com

Abstract

The objective of this article is to define and study the concepts of refined neurosophic AH-ideal and AHS-ideal in refined neutrosophic rings.We investigate the elementary properties of these concepts.

Keywords: Refined neutrosophic ring, Refined neutrosophic AH-ideal, Refined neutrosophicAHS-Ideal, Refined AHS-homomorphism.

1.Introduction

Neutrosophy as a new branch of philosophy can be applied into the algebraic systems, which leads to a better comprehension and evolution of these systems.The notion of neutrosophic groups and rings was defined by Kandasamy and Smarandache in [10], and studied widely in [4, 5, 7,8]. Studies were carried out on neutrosophic rings and neutrosophic hyperring. See[1, 3, 4,6].

Refined neutrosophic rings were defined and studied carefully in [2, 3], where special substructures such as refined neutrosophic subrings and refined neutrosophic ideals are defined. Many interesting results were proved. In [1] concepts as AH-ideal and AHS-ideal were defined and studied as interesting substructures of neutrosophic ring. Some related concepts such as weak principal, maximal, and prime AH-ideals were introduced. These concepts have many properties which are similar to classical case of rings.In this paper, we try to define concepts such as AH-ideal and AHS-ideal in refined neutrosophic ring with some related concepts such as weak prime, principal and maximal refined neutrosophic AH-ideals. Also, we introduce the notion of refined AHS-homomorphism in similar way to AHS-homomorphism defined in [1].

Motivation

This paper is the continuation of the work began in the paper entitled "On Some Special Substructures of Neutrosophic Rings and Their properties".

2. Preliminaries

Definition 2.1:[5]

Let (R,+,×) be a ring then R(I)={a+bI; a,b∈ $R\}$ is called the neutrosophic ring where I is a neutrosophic element with the condition $I^2 = I$.

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Remark 2.2: [2]

The element I can be splitinto two indeterminacies I_1 , I_2 with conditions:

$$I_1^{\,2} = I_1 \,, I_2^{\,2} = I_2 \,, I_1 I_2 = I_2 I_1 = I_1.$$

Definition 2.3: [2]

If X is a set then $X(I_1, I_2) = \{(a, bI_1, cI_2); a, b, c \in X\}$ is called the refined neutrosophic set generated by X, I_1, I_2.

Definition 2.4: [2]

Let (R,+,×) be a ring, ($R(I_1, I_2)$, +,×) is called a refined neutrosophic ring generated by R,I_1, I_2.

Definition 2.5: [2]

Let ($R(I_1, I_2)$, +,×) be a refined neutrosophic ring; it is called commutative if

$$x \times y = y \times x, \forall x\,, y \in R(I_1, I_2).$$

Theorem 2.6: [2]

Let ($R(I_1, I_2)$, +,×) be a refined neutrosophic ring then it is a ring.

Definition 2.7: [3]

Let ($R(I_1, I_2)$, +,×) be a refined neutrosophic ring and J be a nonempty subset of $R(I_1, I_2)$ then J is called a neutrosophic refined ideal if:

(a) J is a refined neutrosophic subring of $R(I_1, I_2)$.

(b) For every $x \in J$ and $r \in R(I_1, I_2)$ then $x \times r \in R(I_1, I_2)$.

Definition 2.8:[1]

Let R(I) be a neutrosophic ring and $P = P_0 + P_1 I = \{a_0 + a_1 I;\; a_0 \in P_0, a_1 \in P_1\}$.

(a)We say that P is an AH-ideal if P_0, P_1 are ideals in the ring R.

(b)We say that P is an AHS-ideal if $P_0 = P_1$.

Definition 2.9:[1]

Let R(I), T(J) be two neutrosophic rings and the map $f: R(I) \to T(J)$; we say that f is a neutrosophic AHS-homomrphism :

Restriction of the map f on R is a ring homomorphism from R to T, i.e $f_R: R \to T$ is homomorphism and

$f(a + b\mathrm{I}) = f_R(a) + f_R(b)\, J$.

We say that R(I), T(J) are AHS-isomomrphic neutrosophic rings if there is a neutrosophic AHS-homomorphism

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$f: R(I) \to T(J)$ which is a bijective map; i.e (R $\cong T$), we say that f is a neutrosophic AHS-isomorphism.

Definition 2.10:[1]

Let R(I) be a neutrosophic ring and $P = P_0 + P_1 I$ be an AH-ideal, we define the AH-factor as:

$R(I)/P = R/P_0 + R/P_1\, I$.

Theorem 2.11:[1]

Let R(I) be a neutrosophic ring and $P = P_0 + P_1 I$ be an AH-ideal then $R(I)/P$ is a ring.

Theorem 2.12:[1]

Let R(I), T(J) be two neutrosophic rings and f: R(I)→T(J) is a neutrosophic ring AHS-homomorphism, let $P = P_0 + P_1 I$ be an AH-ideal of R(I) and $Q = Q_0 + Q_1 J$ be an AH-ideal of T(J), we have:

(a) f(P) is an AH-ideal of f(R(I)).

(b) $f^{-1}(Q)$ is an AH-ideal of R(I).

(c) If P ia an AHS-ideal of R(I),f(P) is an AHS-ideal of f(R(I)).

(d) $AH - kerf = kerf_R + kerf_R I$ is an AHS-ideal;f_R is the restriction of f on the ring R.

(e) The AH-factor $R(I)/kerf\ \ is\ AHS - isomorphic\ to\ f(R(I))$.

Definition 2.13: [1]

Let R(I) be a neutrosophic commutative ring and $P = P_0 + P_1 I$ be an AH-ideal, we say that:

(a) P is a weak prime AH-ideal if P_0, P_1 are prime ideals in R.

(b) P is a weak maximal AH-ideal if P_0, P_1 are maximal ideals in R.

(c) P is a weak principal AH-ideal if P_0, P_1 are principal ideals in R.

3. Main concepts and discussion

Definition 3.1:

Let (R(I_1, I_2), +,×) be a refined neutrosophic ring, and P_0, P_1, P_2 be three ideals in the ring R then the set

$P = (P_0, P_1 I_1, P_2 I_2) = \{(a, bI_1, cI_2); a \in P_0, b \in P_1, c \in P_2\}$is called a refined neutrosophic AH-ideal.

If $P_0 = P_1 = P_2$ then P is called a refined neutrosophic AHS-ideal.

Definition 3.2:

Let (R , +,×), (T, +, ×) be two rings and $f_R: R \to T$ is a homomorphism :

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The map $f: \mathrm{R}(I_1, I_2) \to \mathrm{T}(I_1, I_2); f(x, yI_1, zI_2) = (f_R(x), f_R(y)I_1, f_R(z)I_2)$ is called a refined AHS-homomorphism.

It is easy to see that for all $x, y \in \mathrm{R}(I_1, I_2)$, we have $f(x+y) = f(x) + f(y), f(x \times y) = f(x) \times f(y)$.

Example 3.3:

Suppose that $\mathrm{R} = (Z_6, +, \times), T = (Z_{10}, +, \times)$ are two rings and $f_R: R \to T\,; f(a) = 5a$ is homomorphism, the related refined AHS-homomorphism can be defined:

$f: \mathrm{R}(I_1, I_2) \to \mathrm{T}(I_1, I_2); f(x, yI_1, zI_2) = (5x, 5yI_1, 5zI_2)$.

The previous example shows that refined AH-homomorphism is not a refined neutrosophic homomorphism in general because:

$f(I_1) \neq I_1$

Definition 3.4:

(a) Let $f: \mathrm{R}(I_1, I_2) \to \mathrm{T}(I_1, I_2)$ be a refined AHS-homomorphism, we define refined AH-Kernel of f by:

$\mathrm{AH} - Kerf = \{(a, bI_1, cI_2); a, b, c \in Kerf_R\} = (Kerf_R, Kerf_R I_1, Kerf_R I_2)$.

(b) Let $S = (S_0, S_1I_1, S_2I_2)$ be a subset of $\mathrm{R}(I_1, I_2)$, then: $f(S) = (f_R(S_0), f_R(S_1)I_1, f_R(S_2)I_2) = \{(f_R(a_0), f_R(a_1)I_1, f_R(a_2)I_2); a_i \in S_i\}$.

(c) Let $S = (S_0, S_1I_1, S_2I_2)$ be a subset of $\mathrm{T}(I_1, I_2)$, then:

$$f^{-1}(S) = (f_T^{-1}(S_0), f_T^{-1}(S_1)I_1, f_T^{-1}(S_2)I_2).$$

Definition 3.5:

Let $f: \mathrm{R}(I_1, I_2) \to \mathrm{T}(I_1, I_2)$ be a refined AHS-homomorphism we say that f is a refined AHS-isomorphism if it is a bijective map, $\mathrm{R}(I_1, I_2), \mathrm{T}(I_1, I_2)$ are called AHS-isomorphic refined neutrosophic rings.

It is easy to see that restriction f_R will be an isomorphism between R, T .

Theorem 3.6:

Let $f: \mathrm{R}(I_1, I_2) \to \mathrm{T}(I_1, I_2)$ be a refined AHS-homomorphism we have:

(a) $AH - Ker\,f$ is a refinedneutrosophic AHS-ideal of $\mathrm{R}(I_1, I_2)$.

(b) If P is a refined neutrosophic AH-ideal of $\mathrm{R}(I_1, I_2)$, $f(P)$ is a refined neutrosophic AH-ideal of $\mathrm{T}(I_1, I_2)$.

(c) If P is a refined neutrosophic AHS-ideal of $\mathrm{R}(I_1, I_2)$, $f(P)$ is a refined neutrosophic AHS-ideal of $\mathrm{T}(I_1, I_2)$.

Proof:

(a) Since $Kerf_R$ is an ideal of R, $\mathrm{AH} - kerf = (Kerf_R\,, Kerf_R I_1\,, Kerf_R I_2)$ is a refined neutrosophic AHS-ideal of $\mathrm{R}(I_1, I_2)$.

(b)Suppose that $P = (P_0, P_1I_1, P_2I_2)$ is a refined neutrosophic AH-ideal of $\mathrm{R}(I_1, I_2)$. Since $f_R(P_i)$ is an ideal of T, $f(P) = (f_R(P_0), f_R(P_1)I_1, f_R(P_2)I_2)$ is a refined neutrosophic AH-ideal.

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(c) The proof is similar to (b).

Definition 3.7:

Let ($R(I_1, I_2), +,\times$) be a refined neutrosophic ring and P= $(P_0, P_1 I_1, P_2 I_2)$ be a refined neutrosophic AH-ideal then:

(a) We say that P is a weak prime refined neutrosophic AH-ideal if $P_i; i \in \{0,1,2\}$ are prime ideals in R.

(b) We say that P is a weak maximal refined neutrosophic AH-ideal if $P_i; i \in \{0,1,2\}$ are maximal ideals in R.

(c) We say that P is a weak principal refined neutrosophic AH-ideal if $P_i; i \in \{0,1,2\}$ are principal ideals in R.

(d) We define the refined neutrosophic AH-factor as:

$\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2)/\mathrm{P} = (\mathrm{R}/\mathrm{P}_0 , \mathrm{R}/\mathrm{P}_1\mathrm{I}_1 , \mathrm{R}/\mathrm{P}_2\mathrm{I}_2) = \{([x_0 + \mathrm{P}_0], [\mathrm{x}_1 + \mathrm{P}_1]\mathrm{I}_1, [\mathrm{x}_2 + \mathrm{P}_2]\mathrm{I}_2);\ x_0, x_1, x_2 \in R\}$.

Theorem 3.8:

Let $f: \mathrm{R}(I_1, I_2) \to \mathrm{T}(I_1, I_2)$ be a refined AHS-homomorphism and P= $(P_0, P_1 I_1, P_2 I_2)$ be a refined neutroosphic AH-ideal of $\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2)$, let Q=$(Q_0, Q_1 I_1, Q_2 I_2) \neq \mathrm{T}(I_1, I_2)$ be a refined neutrosophic AH-ideal of $\mathrm{T}(I_1, I_2)$, assume that $Kerf_R \leq P_i \neq R$ then:

(a) P is a weak prime refined neutrosophic AH-ideal of $\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2)$ if and only if $f(P)$ is a weak primerefinedneutrosophic AH-ideal in $f(\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2))$.

(b) P is a weak maximal AH-ideal of $\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2)$ if and only if $f(P)$ is a weak maximal in $f(\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2))$.

(c) Q is a weak prime AH-ideal of $\mathrm{T}(\mathrm{I}_1, \mathrm{I}_2)$ if and only if f^{-1}(Q) is a weak prime in $\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2)$.

(d) Q is a weak maximal AH-ideal of $\mathrm{T}(\mathrm{I}_1, \mathrm{I}_2)$ if and only if f^{-1}(Q) is a weak maximal in $\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2)$.

Proof:

The proof is similar to the Theorem 3.15 in [1].

It is easy to see that conditions (a), (b) are still true if P is an AHS-ideal and conditions (c), (d) are still true if Q is an AHS-ideal.

Theorem 3.9:

The refined neutrosophic AH-factor $\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2)/\mathrm{P}$ is a ring with respect to the following operations:

Let $x = (x_0 + P_0 , (x_1 + P_1)I_1 , (x_2 + P_2)\, I_2)$, $y = (y_0 + P_0, (y_1 + P_1)I_1, (y_2 + P_2)\, I_2)$,be two arbitrary elements in $\mathrm{R}(I_1, I_2)$ then:

$x + y = ([x_0 + y_0] + P_0, ([x_1 + y_1] + P_1)I_1, ([x_2 + y_2] + P_2)\, I_2)$,

$x \times y = ([x_0 \times y_0] + P_0, ([x_1 \times y_1] + P_1)I_1, ([x_2 \times y_2] + P_2)\, I_2)$.

Proof:

The proofis similar to the Theorem 3.9 in [1].

Example 3.10:

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Let R = $(Z_6, +,\times)$, T= $(Z_{10}, +,\times)$ be two rings, and $f$be the refined neutrosophic AHS-homomorphism defined in Example 3.3, we have the following:

(a) $P_0 = \{0,2,4\}, P_1 = \{0,3\}$are two ideals in Z_6 thus P=$(P_0, P_0 I_1, P_1 I_2)$ is a refined neutrosophic AH-ideal of $\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2)$.

$(b) f(P) = (f(P_0), f(P_0)I_1, f(P_1)I_2)$= { (0,0, 0), (0,0,5$I_2$)} is a refined neutrosophic AH-ideal in $\mathrm{T}(\mathrm{I}_1, \mathrm{I}_2)$.

(c) $Q_0 = \{0,2,4,6,8\}$ is a maximal ideal in Z_{10} and $f_T^{-1}(Q_0) = \{0,2,4\}$, so $Q = (Q_0, Q_0 I_1, Q_0 I_2)$ is a weak maximal refined neutrosophic AHS-ideal in $\mathrm{T}(\mathrm{I}_1, \mathrm{I}_2)$, we have $f^{-1}(Q)$ =({0,2,4}, {0,2,4}I_1, {0,2,4}I_2) is a weak maximal refined neutrosophic AHS-ideal in $\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2)$.

Example 3.11:

(a) In the ring (Z,+, ×), P= <3>, Q=<2> are two prime and maximal ideals, thus M=(P ,QI_1, QI_2) = {(3a, 2bI_1, 2cI_2); a, b, c∈ Z } is a weak maximal/prime refined neutrosophic AH-ideal of (Z(I_1, I_2), +,×).

(b)The map $f_Z : Z \to Z_6; f(a) = a \; mod \; 6$ is a homomorphism so the related refined neutrosophic AHS-homomorphism is

f: $\mathrm{Z}(\mathrm{I}_1, \mathrm{I}_2) \to \mathrm{Z}_6(\mathrm{I}_1, \mathrm{I}_2); f(a, bI_1, cI_2) = (a \; mod6, (b \; mod6)I_1, (c \; mod6)I_2), AH - kerf$ = (6Z,6ZI_1, 6ZI_2) $\leq M$ since 6Z≤ P, Q.

(c) $f(M) = (\{0,3\}, \{0,2,4\}I_1, \{0,2,4\}I_2)$ is a weak maximal/prime refined neutrosophic AH-ideal of $\mathrm{Z}_6(\mathrm{I}_1, \mathrm{I}_2)$.

Definition 3.12:

A refined neutrosophic ring $\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2)$is called weak principal refined neutrosophic AH-ring if every refined neutrosophic AH-ideal is weak principal.

Theorem 3.13:

Let R be a principal ideal ring then $\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2)$ is weak principal refined neutrosophic AH-ring.

Proof:

Let P= $(P_0, P_1 I_1, P_2 I_2)$ be a refined neutrosophic AH-ideal of $\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2)$. Since P_i are ideals in R and then principal this implies that P is a weak refined neutrosophic AH-ideal; thus $\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2)$ must be weak principal refined neutrosophic AH-ring.

Example 3.14:

The ring (Z,+, ×) is principal ideals ring; thus $\mathrm{Z}(\mathrm{I}_1, \mathrm{I}_2)$ is weak principal refined neutrosophic AH-ring.

Definition 3.15:

Let $(\mathrm{R}(I_1, I_2), +,\times)$ be a refined neutrosophic ring and P= $(P_0, P_1 I_1, P_2 I_2)$, Q = $(Q_0, Q_1 I_1, Q_2 I_2)$ be two refined neutrosophic AH-ideals of R(I_1, I_2), then we define:

(a) $P \cap Q = (P_0 \cap Q_0, [P_1 \cap Q_1]I_1, [P_2 \cap Q_2]I_2)$.

$$(b) P + Q = (P_0 + Q_0, [P_1 + Q_1]I_1, [P_2 + Q_2]I_2).$$

(c) $P \times Q = (P_0 \times Q_0, [P_1 \times Q_1]I_1, [P_2 \times Q_2]I_2)$.

Theorem 3.16:

Let (R$(I_1, I_2), +, \times$) be a refined neutrosophic ring and P= (P_0, P_1I_1, P_2I_2), Q = (Q_0, Q_1I_1, Q_2I_2) be two refined neutrosophic AH-ideals of R(I_1, I_2), then:

$P \cap Q, P + Q, P \times Q$are refined neutrosophic AH-ideals of R(I_1, I_2).

Proof:

As a result of Theorem 2.5 in[1], we have $P_i + Q_i, P_i \cap Q_i, P_i \times Q_i$are ideals of R, thus the proof is complete.

Remark 3.17:

Theorem 3.16 is still true if P and Q are refined neutrosophic AHS-ideals.

Example 3.18:

Let $R(I_1, I_2) = Z_8(I_1, I_2)$ and Q = {0, 4}, S = {0,2,4,6} be two principal ideals in R, then:

(a) P = (S, QI_1, SI_2) is a refined neutrosophic AH-ideal of $R(I_1, I_2)$, the related refined neutrosophic AH-factor is:

$R(I_1, I_2)/P$= $(R/S\,, R/Q\; I_1, R/S\; I_2)$ = ({S, 1+S}, {Q, 1+Q, 2+Q, 3+Q}I_1, {S, 1+S}I_2).

To clarify addition and multiplication on $R(I_1, I_2)/P$ we take:

$x = (1 + S, (1 + Q)I_1$, SI_2), $y = (S, (2 + Q)I_1, (1 + S)I_2)$, we have:

$x + y =$ ([1 + 0] + S, ([1 + 2] + Q) I_1, ([0 + 1] + S)I_2)= (1+S, (3+Q)I_1, $(1 + S)I_2$).

$x \times y =$ ([1 × 0] + S, ([1 × 2] + Q) I_1, ([0 × 1] + S)I_2)= (S, (2+Q) I_1, SI_2).

Conclusion

In this article we defined concepts of refined neutrosophic AH-ideal/ AHS-ideal in a refined neutrosophic ring.We studied some of elementary properties of these concepts.Also, notions as weak maximal, prime and principal refined neutrosophic AH-ideal and refined AHS-homomorphisms were introduced and checked.

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On neutrosophic nano $\alpha g^{\#}\psi$-closed sets in neutrosophic nano topological spaces

T.Nandhini[1], M.Vigneshwaran[2]
Research Scholar[1], Assistant Professor[2], Department of Mathematics,
Kongunadu Arts and Science College, Coimbatore-641 029.
Email: nandhinimaths11@gmail.com and vignesh.mat@gmail.com

Abstract

The object of the present paper is to introduce neutrosophic nano $\alpha g^{\#}\psi$-closed sets in neutrosophic nano topological spaces and characterize some of its basic properties in neutrosophic nano topological spaces.
Keywords: Neutrosophic set, Neutrosophic topology, neutrosophic nano topology, neutrosophic nano $\alpha g^{\#}\psi$-closed set.

1 Introduction

The first successful attempt towards containing non-probabilistic uncertainty, i.e. uncertainty which is not incite by randomness of an event, into mathematical modeling was made in 1965 by Zadeh[17] through his significant theory on fuzzy sets. A fuzzy set is a set where each element of the universe belongs to it but with some value or degree of belongingness which lies between 0 and 1 and such values are called membership value of an element in that set. Later on Chang[3] was the first to introduce the concept of fuzzy topology.

Further generalization of this fuzzy set was introduced by Atanassov[1,2] in 1986, which is known as Intuitionistic fuzzy sets. In intuitionistic fuzzy set, instead of one membership value there is also a non-membership value devoted to each element. Further there is a restriction that the sum of these two values is less or equal to unity. In Intuitionistic fuzzy set the degree of non-belongingness is not independent but it is dependent on the degree of belongingness. Fuzzy set theory can be considered as a special case of an Intuitionistic fuzzy set where the degree of non-belongingness of an element is exactly equal to 1 minus the degree of belongingness. Along with these IFS are also studied extensively in the topological framework introduced by Coker[4].

Neutrosophic logic was introduced by Smarandache[15] in 1995. It's a logic during which each proposition is calculated to possess a degree of truth, a degree of indeterminacy and a degree of falsity. In 2012, Salama et.al[16] introduced the neutrosophic topological spaces as sort of a generalization concerning intuitionistic fuzzy topological space and a neutrosophic set without the degree concerning membership, the degree of indeterminacy and therefore the degree regarding non-membership over each element.

The neutrosophic concept have wide range of real time applications for the fields of [7,10,12,13] Information Systems, Computer Science, Artificial Intelligence, Applied Mathematics, decision making. Mechanics, Electrical Electronic, Medicine and Management Science etc,.

Rough set theory is introduced by Pawlak[11] as a replacement mathematical tool for representing reasoning and deciding handling vagueness and uncertainty. This theory provides the approximation of sets by means of equivalence relations and is taken into account together of the primary non-statistical approaches in data analysis. A rough set are often described by a pair of definable sets called lower and upper approximations. The lower approximation is that the greatest definable set contained within the given set of objects while the upper approximation is that the smallest definable set that contains the given set. Rough set concept are often defined quite generally by means of topological operations, interior and closure, called approximations.

In 2013, a new topology called Nano topology was introduced by Lellis Thivagar[5] which is an extension of rough set theory. He also introduced Nano topological spaces which were defined in terms of approximations and boundary region of a subset of a universe using an equivalence relation on it. The elements of a

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Nano topological space are called the Nano open sets and its complements are called the Nano closed sets. Nano means something very small. Nano topology thus literally means the study of very small surface. The fundamental ideas in Nano topology are those of approximations and indiscernibility relation.

Now Lellis Thivagar et.al[6] explored a new concept of neutrosophic nano topology. In that paper he discussed about neutrosophic nano interior and neutrosophic nano closure.

In this article, we introduce neutrosophic nano $\alpha g^{\#}\psi(N_{N-\alpha g^{\#}\psi})$-closed sets and study some basic properties in neutrosophic nano topological spaces.

2 Preliminaries

Definition 2.1 (14)**.** A neutrosophic set $\mathcal{S}$ is an object of the following form

$$A = \{\langle s, \mathcal{P}_A(s), \mathcal{Q}_A(s), \mathcal{R}_A(s) : s \in \mathcal{S}\rangle\}$$

where $\mathcal{P}_A(s)$, $\mathcal{Q}_A(s)$ and $\mathcal{R}_A(s)$ denote the degree of membership, the degree of indeterminacy and the degree of nonmembership for each element $s \in \mathcal{S}$ to the set A, respectively.

Definition 2.2 (14)**.** Let A and B be Neutrosophic sets of the form
$A = \{\langle s, \mathcal{P}_A(s), \mathcal{Q}_A(s), \mathcal{R}_A(s) : s \in \mathcal{S}\rangle\}$ and
$B = \{\langle s, \mathcal{P}_B(s), \mathcal{Q}_B(s), \mathcal{R}_B(s) : s \in \mathcal{S}\rangle\}$. Then
(i) $A \subseteq B$ if and only if $\mathcal{P}_A(s) \leq \mathcal{P}_B(s)$, $\mathcal{Q}_A(s) \leq \mathcal{Q}_B(s)$ and $\mathcal{R}_A(s) \geq \mathcal{R}_B(s)$;
(ii) $\overline{A} = \{\langle x, \mathcal{R}_A(s), \mathcal{Q}_A(s), \mathcal{P}_A(s) : s \in \mathcal{S}\rangle\}$;
(iii) $A \cup B$=$\{\langle s, \mathcal{P}_A(s) \vee \mathcal{P}_B(s), \mathcal{Q}_A(s) \wedge \mathcal{Q}_B(s), \mathcal{R}_A(s) \wedge \mathcal{R}_B(s)\rangle : s \in \mathcal{S}\}$;
(iv) $A \cap B$=$\{\langle s, \mathcal{P}_A(s) \wedge \mathcal{P}_B(s), \mathcal{Q}_A(s) \vee \mathcal{Q}_B(s), \mathcal{R}_A(s) \vee \mathcal{R}_B(s)\rangle : s \in \mathcal{S}\}$.

Definition 2.3 (16)**.** A neutrosophic topology on a non-empty set X is a family τ of neutrosophic sets in X satisfying the following axioms:

i. $0_N, 1_N \in \tau$,

ii. $\mathcal{G}_1 \cap \mathcal{G}_2 \in \tau$ for any $\mathcal{G}_1, \mathcal{G}_2 \in \tau$,

iii. $\cup \mathcal{G}_i \in \tau$ for arbitrary family $\{\mathcal{G}_i | i \in j\} \subseteq \tau$.

Definition 2.4 (6)**.** Let $\mathcal{U}$ be a universe and $\mathcal{R}$ be an equivalence relation on $\mathcal{U}$ and Let $\mathcal{S}$ be a neutrosophic subset of $\mathcal{U}$. Then the neutrosophic nano topology is defined by $\tau_N(\mathcal{S}) = \{0_N, 1_N, \overline{N}(\mathcal{S}), \underline{N}(\mathcal{S}), B_N(\mathcal{S})\}$, where

i. $\underline{N}(\mathcal{S})$=$\{\langle y, \mathcal{M}_{\underline{\mathcal{R}}(y)}, \mathcal{I}_{\underline{\mathcal{R}}(y)}, \mathcal{N}_{\underline{\mathcal{R}}(y)}\rangle / z \in [y]\mathcal{R}, y \in \mathcal{U}\}$.

ii. $\overline{N}(\mathcal{S})$ =$\{\left\langle y, \mathcal{M}_{\overline{\mathcal{R}}(y)}, \mathcal{I}_{\overline{\mathcal{R}}(y)}, \mathcal{N}_{\overline{\mathcal{R}}(y)}\right\rangle / z \in [y]\mathcal{R}, y \in \mathcal{U}\}$.

iii. $B_N(S)$=$\underline{N}(S) - \overline{N}(S)$.
where $\mathcal{M}_{\underline{\mathcal{R}}(y)} = \wedge_{z\in[y]\mathcal{R}}\mathcal{M}_S(z)$, $\mathcal{I}_{\underline{\mathcal{R}}(y)} = \wedge_{z\in[y]\mathcal{R}}\mathcal{I}_S(z)$, $\mathcal{N}_{\underline{\mathcal{R}}(y)} = \vee_{z\in[y]\mathcal{R}}\mathcal{N}_S(z)$, $\mathcal{M}_{\overline{\mathcal{R}}(y)} = \vee_{z\in[y]\mathcal{R}}\mathcal{M}_S(z)$, $\mathcal{I}_{\overline{\mathcal{R}}(y)} = \vee_{z\in[y]\mathcal{R}}\mathcal{I}_S(z)$, $\mathcal{N}_{\overline{\mathcal{R}}(y)} = \wedge_{z\in[y]\mathcal{R}}\mathcal{N}_S(z)$.

Definition 2.5 (6)**.** Let A be a neutrosophic set in a neutrosophic nano topological space (X, τ). Then

i. $\mathcal{N}_N int(A) = \bigcup\{\mathcal{G}|\mathcal{G}$ is a neutrosophic nano open set in (X, τ) and $\mathcal{G} \subseteq A$ is called the neutrosophic nano interior of A.

ii. $\mathcal{N}_N cl(A) = \bigcap\{\mathcal{H}|\mathcal{H}$ is a neutrosophic nano closed set in (X, τ) and $\mathcal{H} \supseteq A$ is called the neutrosophic nano closure of A.

Definition 2.6 (9)**.** A neutrosophic set A in a neutrosophic nano topological space (X, τ) is called,

i. a neutrosophic nano semi-open set if $A \subseteq \mathcal{N}_N cl(\mathcal{N}_N int(A))$.

ii. a neutrosophic nano α-open set if $A \subseteq \mathcal{N}_N int(\mathcal{N}_N cl(\mathcal{N}_N int(A)))$.

iii. a neutrosophic nano pre-open set if $A \subseteq \mathcal{N}_N int(\mathcal{N}_N cl(A))$.

iv. a neutrosophic nano regular-open set if $A = \mathcal{N}_N int(\mathcal{N}_N cl(A))$.

Definition 2.7. A subset A of a space (X, τ) is called

i. a neutrosophic $\alpha g^{\#}\psi$-closed set[8] if $\mathcal{N}\alpha cl(A) \subseteq \mathcal{U}$ whenever $A \subseteq \mathcal{U}$ and $\mathcal{U}$ is neutrosophic $g^{\#}\psi$-open in (X, τ)

ii. a neutrosophic nano semi-generalized closed set[9] if $\mathcal{N}_N scl(A) \subseteq \mathcal{U}$ whenever $A \subseteq \mathcal{U}$ and $\mathcal{U}$ is neutrosophic nano semi-open in (X, τ),

iii. a neutrosophic nano ψ-closed set[9] if $\mathcal{N}_N scl(A) \subseteq \mathcal{U}$ whenever $A \subseteq \mathcal{U}$ and $\mathcal{U}$ is neutrosophic nano sg-open in (X, τ),

3 Basic Properties of $N_{N-\alpha g^{\#}\psi}$-closed sets

Definition 3.1. A subset A of (X, τ) is called

i. a neutrosophic nano $g^{\#}\psi$-closed set if $\mathcal{N}_N\psi cl(A) \subseteq \mathcal{U}$ whenever $A \subseteq \mathcal{U}$ and $\mathcal{U}$ is neutrosophic nano ψ-open in (X, τ).

ii. a $N_{N-\alpha g^{\#}\psi}$-closed set if $\mathcal{N}_N\alpha cl(A) \subseteq \mathcal{U}$ whenever $A \subseteq \mathcal{U}$ and $\mathcal{U}$ is neutrosophic nano $g^{\#}\psi$-open in (X, τ).

Theorem 3.2. *Every neutrosophic nano closed set is $N_{N-\alpha g^{\#}\psi}$-closed set.*

Proof: Let $A \subseteq \mathcal{U}$ and $\mathcal{U}$ is neutrosophic nano $g^{\#}\psi$-open in (X, τ). Since A is neutrosophic nano closed set, then $\mathcal{N}_N cl(A) = A$. But $\mathcal{N}_N\alpha cl(A) \subseteq \mathcal{N}_N cl(A)$, then $\mathcal{N}_N\alpha cl(A) \subseteq \mathcal{U}$. Hence A is $N_{N-\alpha g^{\#}\psi}$-closed.

Theorem 3.3. *Every neutrosophic nano regular-closed set is $N_{N-\alpha g^{\#}\psi}$-closed set.*

Proof: Let $A \subseteq \mathcal{U}$ and $\mathcal{U}$ is neutrosophic nano $g^{\#}\psi$-open in (X, τ). Since A is neutrosophic nano regular-closed set, then $\mathcal{N}_N rcl(A) = A$. But $\mathcal{N}_N\alpha cl(A) \subseteq \mathcal{N}_N rcl(A)$, then $\mathcal{N}_N\alpha cl(A) \subseteq \mathcal{U}$. Hence A is $N_{N-\alpha g^{\#}\psi}$-closed.

Theorem 3.4. *Every neutrosophic nano α-closed set is $N_{N-\alpha g^{\#}\psi}$-closed set.*

Proof: Let $A \subseteq \mathcal{U}$ and $\mathcal{U}$ is neutrosophic nano $g^{\#}\psi$-open in (X, τ). Since A is neutrosophic nano α-closed set, then $\mathcal{N}_N\alpha cl(A) = A$. But $\mathcal{N}_N\alpha cl(A) \subseteq \mathcal{U}$. Hence A is $N_{N-\alpha g^{\#}\psi}$-closed.

Theorem 3.5. *Every $N_{N-\alpha g^{\#}\psi}$-closed set is neutrosophic nano sg-closed set.*

Proof: Let $A \subseteq \mathcal{U}$ and $\mathcal{U}$ is neutrosophic nano semi-open in (X, τ). Since every neutrosophic nano semi-open set is neutrosophic nano $g^{\#}\psi$-open, $\mathcal{U}$ is neutrosophic nano $g^{\#}\psi$-open. Since A is $N_{N-\alpha g^{\#}\psi}$-closed, $\mathcal{N}_N\alpha cl(A) \subseteq \mathcal{U}$. But $\mathcal{N}_N scl(A) \subseteq \mathcal{N}_N\alpha cl(A)$, then $\mathcal{N}_N scl(A) \subseteq \mathcal{U}$. Therefore, A is neutrosophic nano sg-closed.

Theorem 3.6. *Every $N_{N-\alpha g^{\#}\psi}$-closed set is neutrosophic nano ψ-closed set.*

Proof: Let $A \subseteq \mathcal{U}$ and $\mathcal{U}$ is neutrosophic nano sg-open in (X, τ). Since every neutrosophic nano sg-open set is neutrosophic nano $g^{\#}\psi$-open, $\mathcal{U}$ is neutrosophic nano $g^{\#}\psi$-open. Since A is $N_{N-\alpha g^{\#}\psi}$-closed, $\mathcal{N}_N\alpha cl(A) \subseteq \mathcal{U}$. But $\mathcal{N}_N scl(A) \subseteq \mathcal{N}_N\alpha cl(A)$, then $\mathcal{N}_N scl(A) \subseteq \mathcal{U}$. Therefore, A is neutrosophic nano ψ-closed.

Theorem 3.7. *Every $N_{N-\alpha g^{\#}\psi}$-closed set is neutrosophic nano $g^{\#}\psi$-closed set.*

Proof: Let $A \subseteq \mathcal{U}$ and $\mathcal{U}$ is neutrosophic nano ψ-open in (X, τ). Since every neutrosophic nano ψ-open set is neutrosophic nano $g^{\#}\psi$-open, $\mathcal{U}$ is neutrosophic nano $g^{\#}\psi$-open. Since A is $N_{N-\alpha g^{\#}\psi}$-closed, $\mathcal{N}_N\alpha cl(A) \subseteq \mathcal{U}$. But $\mathcal{N}_N\psi cl(A) \subseteq \mathcal{N}_N\alpha cl(A)$, then $\mathcal{N}_N\psi cl(A) \subseteq \mathcal{U}$. Therefore, A is neutrosophic nano $g^{\#}\psi$-closed.

Remark 3.8. The reverse implication of the above theorems is not true as shown in the following example.

Example 3.9. Assume $\mathcal{U} = \{p, q, r\}$ be the universe set and the equivalence relation is $\mathcal{U}/\mathcal{R} = \{\{p, r\}, \{r\}\}$. Let
$\mathcal{A} = \{\langle p, (0.4, 0.4, 0.3)\rangle, \langle q, (0.3, 0.4, 0.2)\rangle, \langle r, (0.4, 0.3, 0.4)\rangle\}$
be a neutrosophic nano subset of $\mathcal{U}$. Then
$\underline{N}(\mathcal{A}) = \{\langle p, (0.3, 0.4, 0.3)\rangle, \langle q, (0.3, 0.4, 0.3)\rangle, \langle r, (0.4, 0.3, 0.4)\rangle\}$

$\overline{N}(\mathcal{A}) = \{\langle p,(0.4,0.4,0.2)\rangle, \langle q,(0.4,0.4,0.2)\rangle, \langle r,(0.4,0.3,0.4)\rangle\}$
$\mathcal{B}(\mathcal{A}) = \{\langle p,(0.2,0.4,0.4)\rangle, \langle q,(0.2,0.4,0.4)\rangle, \langle r,(0.4,0.3,0.4)\rangle\}$
N_r-closed set=
$D_1 = \{\langle p,(0.3,0.4,0.3)\rangle, \langle q,(0.3,0.4,0.3)\rangle, \langle r,(0.4,0.3,0.4)\rangle\}$
neutrosophic nano α-closed set=
$D_2 = \{\langle p,(0.3,0.4,0.3)\rangle, \langle q,(0.3,0.4,0.3)\rangle, \langle r,(0.4,0.3,0.4)\rangle\}$
neutrosophic nano sg-closed set=
$D_3 = \{\langle p,(0.2,0.3,0.4)\rangle, \langle q,(0.2,0.3,0.4)\rangle, \langle r,(0.3,0.2,0.4)\rangle\}$
neutrosophic nano ψ-closed set=
$D_4 = \{\langle p,(0.2,0.2,0.2)\rangle, \langle q,(0.2,0.2,0.2)\rangle, \langle r,(0.3,0.2,0.4)\rangle\}$
neutrosophic nano $g^{\#}\psi$-closed set=
$D_5 = \{\langle p,(0.2,0.1,0.3)\rangle, \langle q,(0.2,0.2,0.3)\rangle, \langle r,(0.3,0.2,0.4)\rangle\}$
$N_{N-\alpha g^{\#}\psi}$-closed set=$D_6 = \{\langle p,(0.2,0.1,0.4)\rangle, \langle q,(0.2,0.2,0.4)\rangle, \langle r,(0.3,0.2,0.4)\rangle\}$
Let $\tau = \{0_N, \underline{N}(\mathcal{A}), \overline{N}(\mathcal{A}), \mathcal{B}(\mathcal{A}), 1_N\}$. Here $(D_5)^c$ is neutrosophic nano $g^{\#}\psi$-open set, $N\alpha cl(D_6) \subseteq (D_5)^c$. Then D_6 is $N_{N-\alpha g^{\#}\psi}$-closed set in $(X,\mathcal{T})$ but not neutrosophic closed set, N_r-closed set and neutrosophic nano α-closed set.
Here D_3, D_4 and D_5 are neutrosophic nano sg-closed set, neutrosophic nano ψ-closed set and neutrosophic nano $g^{\#}\psi$-closed set respectively. But not $N_{N-\alpha g^{\#}\psi}$-closed set because $N\alpha cl(D_3) \nsubseteq (D_5)^c$, $N\alpha cl(D_4) \nsubseteq (D_5)^c$ and $N\alpha cl(D_5) \nsubseteq (D_5)^c$.

Theorem 3.10. *Intersection of two $N_{N-\alpha g^{\#}\psi}$-closed sets in (X,τ) is again $N_{N-\alpha g^{\#}\psi}$-closed set.*

Proof: Let A and B be the subsets of $N_{N-\alpha g^{\#}\psi}$-closed sets, $A \subseteq \mathcal{U}$ and $\mathcal{N}_N\alpha cl(A) \subseteq \mathcal{U}$, $B \subseteq \mathcal{U}$ and $\mathcal{N}_N\alpha cl(B) \subseteq \mathcal{U}$, $\mathcal{U}$ is a neutrosophic nano $g^{\#}\psi$-open. Therefore, $A \cap B \subseteq A$ and $\alpha cl(A \cap B) \subseteq \alpha cl(A)$, $A \cap B \subseteq B$ and $\alpha cl(A \cap B) \subseteq \mathcal{N}_N\alpha cl(B)$. Hence $\mathcal{N}_N\alpha cl(A \cap B) \subseteq \mathcal{U}$ and $\mathcal{U}$ is a neutrosophic nano $g^{\#}\psi$-open. Thus $A \cap B$ is $N_{N-\alpha g^{\#}\psi}$-closed set.

Theorem 3.11. *Union of two $N_{N-\alpha g^{\#}\psi}$-closed sets in (X,τ) is again $N_{N-\alpha g^{\#}\psi}$-closed set.*

Proof: Let A and B be the subsets of $N_{N-\alpha g^{\#}\psi}$-closed sets, $A \subseteq \mathcal{U}$ and $\mathcal{N}_N\alpha cl(A) \subseteq \mathcal{U}$, $B \subseteq \mathcal{U}$ and $\mathcal{N}_N\alpha cl(B) \subseteq \mathcal{U}$, $\mathcal{U}$is a neutrosophic nano $g^{\#}\psi$-open. Therefore, $A \cup B \subseteq \mathcal{U}$ and $\mathcal{N}_N\alpha cl(A \cup B) = \mathcal{N}_N\alpha cl(A) \cup \mathcal{N}_N\alpha cl(B) \subseteq \mathcal{U}$. That is $\mathcal{N}_N\alpha cl(A \cup B) \subseteq \mathcal{U}$. Therefore, $A \cup B$ is $N_{N-\alpha g^{\#}\psi}$-closed set.

Theorem 3.12. *If a set A is $N_{N-\alpha g^{\#}\psi}$-closed in (X,τ) iff $\mathcal{N}_N\alpha cl(A) - A$ contains no non-empty neutrosophic nano $g^{\#}\psi$-closed set.*

Proof: Necessity: Let $\mathcal{F}$ be a neutrosophic nano $g^{\#}\psi$-closed in (X,τ) such that $\mathcal{F} \subseteq \mathcal{N}_N\alpha cl(A) - A$. Then $\mathcal{F} \subseteq X - A$. This implies $A \subseteq X - \mathcal{F}$. Now $X - \mathcal{F}$ is neutrosophic nano $g^{\#}\psi$-open set of (X,τ) such that $A \subseteq X - \mathcal{F}$. Since A is $N_{N-\alpha g^{\#}\psi}$-closed set then $\mathcal{N}_N\alpha cl(A) \subseteq X - \mathcal{F}$. Thus $\mathcal{F} \subseteq X - \mathcal{N}_N\alpha cl(A)$. Now $\mathcal{F} \subseteq \mathcal{N}_N\alpha cl(A) \cap (X - \mathcal{N}_N\alpha cl(A)) = 0_N$.
Sufficiency: Assume that $\mathcal{N}_N\alpha cl(A) - A$ contains no non-empty neutrosophic nano $g^{\#}\psi$-closed set. Let $A \subseteq \mathcal{U}, \mathcal{U}$ is $N_{g^{\#}\psi}$-open set. Suppose that $\mathcal{N}_N\alpha cl(A)$ is not contained in $\mathcal{U}$ then $\mathcal{N}_N\alpha cl(A) \cap \mathcal{U}^c$ is a non-empty neutrosophic nano $g^{\#}\psi$-closed set of $\mathcal{N}_N\alpha cl(A) - A$, which is a contradiction. Therefore, $\mathcal{N}_N\alpha cl(A) \subseteq \mathcal{U}$ and hence A is $N_{N-\alpha g^{\#}\psi}$-closed.

Theorem 3.13. *If a subset A is $N_{N-\alpha g^{\#}\psi}$-closed and $A \subseteq B \subseteq N_{N\alpha cl(A)}$, then B is $N_{N-\alpha g^{\#}\psi}$-closed set.*

Proof: Let $B \subseteq \mathcal{U}, \mathcal{U}$ is a neutrosophic nano $g^{\#}\psi$-open, then $A \subseteq B$ and $A \subseteq \mathcal{U}$. Since A is $N_{N-\alpha g^{\#}\psi}$-closed, $\mathcal{N}_N\alpha cl(A) \subseteq \mathcal{U}$ but $B \subseteq \mathcal{N}_N\alpha cl(A)$ this implies that $\mathcal{N}_N\alpha cl(B) \subseteq \mathcal{N}_N\alpha cl(A)$. Therefore, $\mathcal{N}_N\alpha cl(B) \subseteq \mathcal{N}_N\alpha cl(A) \subseteq \mathcal{U}$. Thus $\mathcal{N}_N\alpha cl(B) \subseteq \mathcal{U}$ and $\mathcal{U}$ is neutrosophic nano $g^{\#}\psi$-open. Hence B is $N_{N-\alpha g^{\#}\psi}$-closed.

Conclusion

In this article the new concept of $N_{N-\alpha g^{\#}\psi}$-closed sets is introduced in neutrosophic nano topological spaces. Furthermore, the work was extended as its basic properties.

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Classical Homomorphisms Between Refined Neutrosophic Rings and Neutrosophic Rings

Mohammad Abobala

Faculty of Science, Tishreen University, Lattakia, Syria.
E-mail: mohammadabobala777@gmail.com

Abstract

The aim of this paper is to study homomorphisms between refined neutrosophic rings and neutrosophic rings. We prove that every neutrosophic ring $R(I)$ is a homomorphic image of the refined neutrosophic ring $R(I_1, I_2)$. Furthermore, we prove the following interesting result:

Theorem. Let R be a ring with $Char(R) = 2$, then $R(I_1, I_2)/K \cong R(I)$, where K is a subring of $R(I_1, I_2)$ with the property $K \cong R$.

Keywords: Neutrosophic Ring, Refined Neutrosophic Ring, Homomorphic Image.

1. Introduction

Neutrosophy as a generalization of classical logic introduced by Smarandache plays an important role in algebraic studies, many neutrosophic algebraic structures were defined and studied such as neutrosophic groups, and neutrosophic fields. See[4, 5, 6, 8]. Neutrosophic ring was defined in [8] as a new kind of rings. It has been studied carefully in [4, 5], many related concepts such as neutrosophic subring, neutrosophic ideal, and neutrosophic homomorphism were introduced and checked. Also, concepts of refined neutrosophic ring, refined neutrosophic subring, refined neutrosophic ideal were defined and studied in [2, 3]. Concepts such as AH-ideal, AHS-ideal, and AHS-homomorphisms are introduced in [1]. In [3], Adeleke, Agboola and Smarandache proved the following result:

Let $Z_n(I_1, I_2)$ be a refined neutrosophic ring with addition and multiplication modulo n, then $O(Z_n(I_1, I_2)) = n^3$.

In general it is easy to check that the order of finite refined neutrosophic ring $R(I_1, I_2) = n^3$, where n is the order of the finite ring R. The previous result motivates us to check if there is a classical ring homomorphism between $R(I_1, I_2)$, $R(I)$ and to investigate the kernel of a homomorphism under the assumption that if there is a classical homomorphism between $R(I_1, I_2)$, $R(I)$. Since neutrosophic ring $R(I)$ and refined neutrosophic ring $R(I_1, I_2)$ are rings by classical meaning, we can use concepts such as classical ring homomorphism/isomorphism to obtain some properties of these rings.

All homomorphisms and isomorphisms through this paper are considered by classical meaning not neutrosophical meaning defined in [2, 3 ,5].

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2. Preliminaries

In this section we recall some basic notions and results regarding to neutrosophic rings.

Definition 2.1.([5]) Let $(R,+,\times)$ be a ring.$R(I) = \{a + bI: a, b \in R\}$is called the neutrosophic ring, where I is a neutrosophic element with condition $I^2 = I$.

Remark 2.2.([2]) The element I can be split into two indeterminacies I_1 , I_2 with conditions:

$I_1{}^2 = I_1, I_2{}^2 = I_2, I_1 I_2 = I_2 I_1 = I_1$.

Definition 2.3.([2]) If X is a set then $X(I_1, I_2) = \{(a, bI_1, cI_2); a , b , c \in X\}$ is called the refined neutrosophic set generated by X, I_1, I_2.

Definition 2.4.([2]) Let $(R,+,\times)$ be a ring. $(R(I_1, I_2) , + ,\times)$ is called a refined neutrosophic ring generated by R,I_1, I_2.

Theorem 2.5.([2]) Let $(R(I_1, I_2) , + ,\times)$ be a refined neutrosophic ring. Then it is a ring.

Definition 2.6.([1]) Let R be a ring and $R(I)$ be the corresponding neutrosophic ring and

$P = P_0 + P_1 I = \{a_0 + a_1 I ; a_0 \in P_0 , a_1 \in P_1\}$, where $P_0, P_1$are two subsets of R. Then P is an

a) AH-ideal if P_0, P_1 are ideals in the ring R,

b) AHS-ideal if $P_0 = P_1$.

3. Main discussion

Theorem 3.1: Let $(R ,+,\times)$ be a ring and $R(I)$, $R(I_1, I_2)$ the corresponding neutrosophic ring and refined neutrosophic ring respectively. Then

a) There is a ring homomorphism $f: R(I_1, I_2) \to R(I)$ defined by $f(a, bI_1, cI_2) = a + (b + c)I$.

b) The additive group $(Ker(f) , +)$ is isomorphic to the additive group $(R, +)$.

Proof.(a) Let $a, b, c \in R$. Define $f: R(I_1, I_2) \to R(I)$ by $f(a, bI_1, cI_2) = a + (b + c)I$. Then f is well defined, since if $(a, bI_1, cI_2) = (x, yI_1, zI_2)$, then $a = x , b = y , c = z$. Thus, $a + (b + c)I = x + (y + z)I$.

Also, f is a ring homomorphism. For this, suppose that $m = (a, bI_1, cI_2), n = (x, yI_1, zI_2) \in R(I_1, I_2)$, we have:

$m + n = (a + x, (b + y)I_1, (c + z)I_2)$, and

$m \times n = (a \times x , (a \times y + b \times y + b \times z + b \times x + c \times y)I_1, (a \times z + c \times x + c \times z)I_2)$.

So, $f(m + n) = [a + x] + [b + y + c + z]I = f(m) + f(n)$ and

$f(m \times n) = [a \times x] + [a \times y + b \times y + b \times z + b \times x + c \times y + a \times z + c \times x + c \times z]I =$

$[a + (b + c)I] \times [x + (y + z)I] = f(m) \times f(n)$.

It is clear that f is a surjective map.

(b) By the definition $Ker(f) = \{(a, bI_1, cI_2) \in R(I_1, I_2): a + (b + c)I = 0 = 0 + 0I\}$. Hence $a = 0 , b + c = 0$,this means $b = -c$. Thus, $Ker(f) = \{(0, bI_1, -bI_2): b \in R\}$.

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It is easy to check that $\varphi : Ker(f) \to R$ defined by $\varphi(0, bI_1, -bI_2) = b$ is a group isomorphism between $(Ker(f)\ , +)$ and $(R, +)$.

As a simple result from the Lagrange theorem we have:

If R is finite ring, then $\frac{O(R(I_1,I_2))}{O(R)} = o(R(I))$.

Theorem 3.2. Let R be a ring, where $Char(R) = 2$. Thenthere is a subring of $R(I_1, I_1)$say $K$with the property $K \cong R$ and $R(I_1, I_2)/K \cong R(I)$.

Proof. Applying Isomorphism Theorem, we get $R(I_1, I_2)/Ker\ f \cong R(I)$, and φ (defined in Theorem 3.1) is a group isomorphism between $(Ker(f), +)$ and $(R, +)$. Now, we prove that φ is a ring isomorphism under the condition $Char(R) = 2$.

Suppose that $m = (0, bI_1, -bI_2)$ and $n = (0, cI_1, -cI_2)$are two arbitrary elements in $Ker(f)$, we have:

$$m \times n = (0, (b \times c - b \times c - b \times c)I_1, (b \times c)I_2) = (0, [-b \times c]I_1, [b \times c]I_2).$$

So,$f(m \times n) = -b \times c = b \times c = f(m) \times f(n)$.

$[-\mathrm{b} \times \mathrm{c} = \mathrm{b} \times \mathrm{c}$,since $Char(R) = 2]$.

By the previous aspect we find that $K = (Ker(f)\ , +, \times) \cong (R\ , +\ , \times)$.Thus, $R(I_1, I_2)/K \cong R(I)$.

Theorem 3.3. Let R be a ring.Then there is a subring of $R(I)$say $K$with the property $K \cong R$and$R(I)\ /\ K \cong R$.

Proof. Define $f: R(I) \to R$ by $f(a + bI) = a$. It is easy to check that f is a surjective ring homomorphism. Thus, $R(I)/Ker(f) \cong R$.

Also, $Ker\ (f) = \{a + bI \in R(I): a = 0\} = \{bI: b \in R\} = RI$, since $K = RI$ is a subring of $R(I)$ and the map

$\varphi: K \to R$ defined by $\varphi(bI) = b$ is a ring isomorphism, we get the proof.

Example 3.4. Let $R = Z_2$. Then R is a ring with respect to addition and multiplication modulo2, we have $Char(R) = 2$.

$R(I_1, I_2) = \{(a, bI_1, cI_2): a, b, c \in Z_2\}$ is the corresponding refined neutrosophic ring.

$R(I) = \{a + bI: a, b \in Z_2\}$ is the corresponding neutrosophic ring.

Let f be the ring homomorphism defined in Theorem 3.1, we have:

$Ker(f) = \{(0, bI_1, -bI_2): b \in Z_2\} = \{(0,0,0), (0, I_1, I_2)\} \cong R$and

$R(I_1, I_2)/K = \{K\ , (1,0,0) + K\ , (0, I_1, 0) + K\ , (1,0, I_2) + K\ \} \cong R(I)$.

So, $O(R(I_1, I_2)) = 8, O\big(R(I)\big) = 4, O(R) = 2$. Therefore, $O(R(I_1, I_2))/O(R) = O\big(R(I)\big) = 4$.

4. Conclusion

In this paper we have studied the relationship between refined neutrosophic rings and neutrosophic rings by using classical methods in Ring Theory. In particular have shown:

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a) Every neutrosophic ring $R(I)$ is a homomorphic image of refined neutrosophic ring $R(I_1, I_2)$.
b) If $Char(R) = 2$, then there is a subring of $R(I_1, I_2)$ say $K$with property $K \cong R$ andR$(I_1, I_2)/K \cong R(I)$.

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On Refined Neutrosophic Quotient Groups

A.A.A. Agboola[1]
Department of Mathematics, Federal University of Agriculture, Abeokuta, Nigeria.
agboolaaaa@funaab.edu.ng

Abstract

This paper is devoted to the study of refined neutrosophic quotient groups. It is shown that the classical isomorphism theorems of groups do not hold for the refined neutrosophic groups. Moreover, the existence of classical morphisms between refined neutrosophic groups $G(I_1, I_2)$ and neutrosophic groups $G(I)$ is established.

Keywords: Neutrosophy, neutrosophic group, neutrosophic subgroup, refined neutrosophic group, refined neutrosophic subgroup, refined neutrosophic group homomorphism.

1 Introduction and Preliminaries

Agboola in [1] introduced the concept of refined neutrosophic algebraic structures and studied refined neutrosophic groups in particular. Since the introduction of refined neutrosophic algebraic structures, many neutrosophic researchers have established and studied more refined neutrosophic algebraic structures. Adeleke et al. in [5] studied refined neutrosophic rings and refined neutrosophic subrings and presented their fundamental properties. Also in [6], Adeleke et al. studied refined neutrosophic ideals and refined neutrosophic homomorphisms and presented their basic properties. The present paper is devoted to the study of refined neutrosophic quotient groups. More properties of refined neutrosophic groups will be presented and it will be shown that the classical isomorphism theorems of groups do not hold in the refined neutrosophic groups. The existence of classical morphisms between refined neutrosophic groups $G(I_1, I_2)$ and neutrosophic groups $G(I)$ will be established. For more details about neutrosophy, refined neutrosophic logic, neutrosophic groups and refined neutrosophic groups, the readers should see [2,4,7,13].

Definition 1.1. [1] Let $(X(I_1, I_2), +, .)$ be any refined neutrosophic algebraic structure where $+$ and $.$ are ordinary addition and multiplication respectively. I_1 and I_2 are the split components of the indeterminacy factor I that is $I = \alpha_1 I_1 + \alpha_2 I_2$ with $\alpha_i \in \mathbb{R}, \mathbb{C}, i = 1, 2$. Also, I_1 and I_2 are taken to have the properties $I_1^2 = I_1$, $I_2^2 = I_2$ and $I_1 I_2 = I_2 I_1 = I_1$ or I_2. For the purposes of this paper, we will take $I_1 I_2 = I_2 I_1 = I_1$.

For any two elements $(a, bI_1, cI_2), (d, eI_1, fI_2) \in X(I_1, I_2)$, we define

$$\begin{aligned}
(a, bI_1, cI_2) + (d, eI_1, fI_2) &= (a + d, (b + e)I_1, (c + f)I_2), && (1)\\
(a, bI_1, cI_2).(d, eI_1, fI_2) &= (ad, (ae + bd + be + bf + ce)I_1, \\
&\quad (af + cd + cf)I_2). && (2)
\end{aligned}$$

Definition 1.2. [1] Let $(G, *)$ be any group. The couple $(G(I_1, I_2), *)$ is called a refined neutrosophic group generated by G, I_1 and I_2. $(G(I_1, I_2), *)$ is said to be commutative if for all $x, y \in G(I_1, I_2)$, we have $x * y = y * x$. Otherwise, we call $(G(I_1, I_2), *)$ a non-commutative refined neutrosophic group.

Example 1.3. [1] Let $\mathbb{Z}_2(I_1, I_2) = \{(0, 0, 0), (1, 0, 0), (0, I_1, 0), (0, 0, I_2),$
$(0, I_1, I_2), (1, I_1, 0), (1, 0, I_2), (1, I_1, I_2)\}$. Then $(\mathbb{Z}_2(I_1, I_2), +)$ is a commutative refined neutrosophic group of integers modulo 2. Generally for a positive integer $n \geq 2$, $(\mathbb{Z}_n(I_1, I_2), +)$ is a finite commutative refined neutrosophic group of integers modulo n.

[1]Correspondence:aaaola2003@yahoo.com

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Theorem 1.4. [7]

(1) *Every refined neutrosophic group is a semigroup but not a group.*

(2) *Every refined neutrosophic group contains a group.*

Corollary 1.5. [7] *Every refined neutrosophic group $(G(I_1, I_2), +)$ is a group.*

Definition 1.6. [1] Let $(G(I_1, I_2), *)$ be a refined neutrosophic group and let $A(I_1, I_2)$ be a nonempty subset of $G(I_1, I_2)$. $A(I_1, I_2)$ is called a refined neutrosophic subgroup of $G(I_1, I_2)$ if $(A(I_1, I_2), *)$ is a refined neutrosophic group. It is essential that $A(I_1, I_2)$ contains a proper subset which is a group. Otherwise, $A(I_1, I_2)$ will be called a pseudo refined neutrosophic subgroup of $G(I_1, I_2)$.

Example 1.7. [1]

(1) Let $G(I_1, I_2) = (\mathbb{Z}(I_1, I_2), +)$ and let $A(I_1, I_2) = (3\mathbb{Z}(I_1, I_2), +)$. Then $A(I_1, I_2)$ is a refined neutrosophic subgroup of $G(I_1, I_2)$.

(2) Let $G(I_1, I_2) = (\mathbb{Z}_6(I_1, I_2), +)$ and let

$$\begin{aligned} A(I_1, I_2) &= \{(0,0,0), (0, I_1, 0), (0, 0, I_2), (0, I_1, I_2), \\ &\quad (0, 2I_1, 0), (0, 0, 2I_2), (0, 2I_1, 2I_2), \\ &\quad (0, 3I_1, 0), (0, 0, 3I_2), (0, 3_1, 3I_2), \\ &\quad (0, 4I_1, 0), (0, 0, 4I_2), (0, 4I_1, 4I_2), \\ &\quad (0, 5I_1, 0), (0, 0, 5I_2), (0, 5I_1, 5I_2)\}. \end{aligned}$$

Then $A(I_1, I_2)$ is a pseudo refined neutrosophic subgroup of $G(I_1, I_2)$.

Definition 1.8. [1] Let $A(I_1, I_2)$ and $B(I_1, I_2)$ be any two refined neutrosophic subgroups (pseudo refined neutrosophic subgroups) of a refined neutrosophic group $(G(I_1, I_2), +)$. We define the sum $A(I_1, I_2) + B(I_1, I_2)$ by the set

$$A(I_1, I_2) + B(I_1, I_2) = \{(a_1, a_2I_1, a_3I_2) + (b_1, b_2I_1, b_3I_2) : a_1, a_2, a_3 \in A(I_1, I_2), b_1, b_2, b_3 \in B(I_1, I_2)\} \quad (3)$$

which is a refined neutrosophic subgroup (pseudo refined neutrosophic subgroup) of $G(I_1, I_2)$.

Definition 1.9. [1] Let $(G(I_1, I_2), *)$ and and $(H(I_1, I_2), *')$ be two refined neutrosophic groups. The mapping $\phi : (G(I_1, I_2), *) \to (H(I_1, I_2), *')$ is called a neutrosophic homomorphism if the following conditions hold:

(1) $\phi(x * y) = \phi(x) *' \phi(y)$.

(2) $\phi(I_k) = I_k \;\; \forall x, y \in G(I_1, I_2)$ and $k = 1, 2$.

The image of ϕ is defined by the set

$$Im\phi = \{y \in H(I_1, I_2) : y = \phi(x) \text{ for some } x \in G(I_1, I_2)\}. \quad (4)$$

If $G(I_1, I_2)$ and $H(I_1, I_2)$ are additive refined neutrosophic groups, then the kernel of the neutrosophic homomorphism $\phi : (G(I_1, I_2), +) \to (H(I_1, I_2), +)$ is defined by the set

$$Ker\phi = \{x \in G(I_1, I_2) : \phi(x) = (0, 0I_1, 0I_2)\}. \quad (5)$$

Epimorphism, monomorphism, isomorphism, endomorphism and automorphism of ϕ have the same definitions as those of the classical cases.

Theorem 1.10. [7] *Let $\phi : (G(I_1, I_2), *) \to (H(I_1, I_2), *')$ be a refined neutrosophic group homomorphism. Then $Im\phi$ is a neutrosophic subgroup of $H(I_1, I_2)$.*

Theorem 1.11. [7] *Let $\phi : (G(I_1, I_2), +) \to (H(I_1, I_2), +)$ be a refined neutrosophic group homomorphism. Then $Ker\phi$ is a subgroup of G and not a neutrosophic subgroup of $G(I_1, I_2)$.*

Example 1.12. [1] Let $\phi : \mathbb{Z}_2(I_1, I_2) \times \mathbb{Z}_2(I_1, I_2) \to \mathbb{Z}_2(I_1, I_2)$ be a neutrosophic group homomorphism defined by $\phi(x, y) = x$ for all $x, y \in \mathbb{Z}_2(I_1, I_2)$. Then

$$\begin{aligned} Im\phi &= \{(0,0,0), (1,0,0), (0, I_1, 0), (0, 0, I_2), \\ &\quad (0, I_1, I_2), (1, I_1, 0), (1, 0, I_2), (1, I_1, I_2)\}. \\ Ker\phi &= \{((0,0,0), (0,0,0)), ((0,0,0), (1,0,0)), ((0,0,0), (0, I_1, 0)), \\ &\quad ((0,0,0), (0, I_1, I_2)), ((0,0,0), (0,0,I_2)), ((0,0,0), (1, I_1, 0)), \\ &\quad ((0,0,0), (1, 0, I_2)), ((0,0,0), (1, I_1, I_2))\}. \end{aligned}$$

2 Main Results

Definition 2.1. Let $H(I_1, I_2)$ be a refined neutrosophic subgroup of a refined neutrosophic group $(G(I_1, I_2), .)$ and let $x = (a, bI_1, cI_2) \in G(I_1, I_2)$.

(i) The set

$$xH(I_1, I_2) = \{xh : h \in H(I_1, I_2)\} \tag{6}$$

is called a refined left coset of $H(I_1, I_2)$ in $G(I_1, I_2)$.

(ii) The set

$$Hx(I_1, I_2) = \{hx : h \in H(I_1, I_2)\} \tag{7}$$

is called a refined right coset of $H(I_1, I_2)$ in $G(I_1, I_2)$.

If $''.''$ is commutative, there is no distinction between the refined left and refined right cosets of $H(I_1, I_2)$ in $G(I_1, I_2)$.

Definition 2.2. Let $H(I_1, I_2)$ be a refined neutrosophic subgroup of a refined neutrosophic group $(G(I_1, I_2), +)$ and let $x = (a, bI_1, cI_2) \in G(I_1, I_2)$.

(i) The set

$$x + H(I_1, I_2) = \{x + h : h \in H(I_1, I_2)\} \tag{8}$$

is called a refined left coset of $H(I_1, I_2)$ in $G(I_1, I_2)$.

(ii) The set

$$H(I_1, I_2) + x = \{h + x : h \in H(I_1, I_2)\} \tag{9}$$

is called a refined right coset of $H(I_1, I_2)$ in $G(I_1, I_2)$.

Definition 2.3. Let $H(I_1, I_2)$ be a refined neutrosophic subgroup of a refined neutrosophic group $(G(I_1, I_2), .)$. $H(I_1, I_2)$ is said to be normal in $G(I_1, I_2)$ if for all $x = (a, bI_1, cI_2) \in G(I_1, I_2)$, we have

$$xH(I_1, I_2) = H(I_1, I_2)x. \tag{10}$$

Definition 2.4. Let $H(I_1, I_2)$ be a normal refined neutrosophic subgroup of a refined neutrosophic group $(G(I_1, I_2), .)$. The quotient $G(I_1, I_2)/H(I_1, I_2)$ is defined by

$$G(I_1, I_2)/H(I_1, I_2) = \{xH(I_1, I_2) : x \in G(I_1, I_2)\}. \tag{11}$$

Definition 2.5. Let $H(I_1, I_2)$ be a normal refined neutrosophic subgroup of a refined neutrosophic group $(G(I_1, I_2), +)$. The quotient $G(I_1, I_2)/H(I_1, I_2)$ is defined by

$$G(I_1, I_2)/H(I_1, I_2) = \{x + H(I_1, I_2) : x \in G(I_1, I_2)\}. \tag{12}$$

Example 2.6. Let $G = \{e, a, b, c\}$ be the Klein-4 group and let $(G(I_1, I_2), .)$ be a refined neutrosophic group given by

$$\begin{aligned} G(I_1, I_2) \;\; = \;\; & \{(e, 0I_1, 0I_2), (a, 0I_1, 0I_2), (b, 0I_1, 0I_2), (c, 0I_1, 0I_2), (e, eI_1, eI_2), \\ & (e, aI_1, aI_2), (e, bI_1, bI_2), (e, cI_1, cI_2), (a, eI_1, eI_2), (a, aI_1, aI_2), \\ & (a, bI_1, bI_2), (a, cI_1, cI_2), (b, eI_1, eI_2), (b, aI_1, aI_2), (b, bI_1, bI_2), \\ & (b, cI_1, cI_2), (c, eI_1, eI_2), (c, aI_1, aI_2), (c, bI_1, bI_2), \cdots, (c, cI_1, cI_2)\} \end{aligned}$$

and let $H(I_1, I_2)$ be a subset of $G(I_1, I_2)$ given by

$$\begin{aligned} H(I_1, I_2) \;\; = \;\; & \{(e, 0I_1, 0I_2), (a, 0I_1, 0I_2), (e, eI_1, eI_2), (a, aI_1, aI_2), (e, aI_1, aI_2), \\ & (a, eI_1, eI_2), (e, aI_1, eI_2), (a, eI_1, aI_2)\}. \end{aligned}$$

The order of $G(I_1, I_2)$ is $4^3 = 64$ and it is clear that $H(I_1, I_2)$ is a refined neutrosophic subgroup of $G(I_1, I_2)$ of order 8. Since 8 is a divisor of 64, it follows that Lagrange's theorem holds in this case. It should be noted that Lagrange's theorem generally does not hold in refined neutrosophic groups. However, Lagrange's theorem holds in any refined neutrosophic group $G(I_1, I_2)$ whenever G is isomorphic to Klein-4 group.

Example 2.7. Let $G(I_1, I_2)$ and $H(I_1, I_2)$ be as defined in **Example 2.6**. Simple computations show that $xH(I_1, I_2)$ the set of all left cosets of $H(I_1, I_2)$ in $G(I_1, I_2)$ for all $x \in G(I_1, I_2)$ is a partition of $G(I_1, I_2)$.

Generally, the left(right) cosets of a refined neutrosophic subgroup in a refined neutrosophic group does not partition the refined neutrosophic group.

Theorem 2.8. *Let $H(I_1, I_2)$ be a refined neutrosophic subgroup of a refined neutrosophic group $(G(I_1, I_2), .)$ and let $G(I_1, I_2)/H(I_1, I_2)$ be a set defined by*

$$G(I_1, I_2)/H(I_1, I_2) = \{xH(I_1, I_2) : x \in G(I_1, I_2)\}. \tag{13}$$

Let $\odot$ be a binary operation defined on $G(I_1, I_2)/H(I_1, I_2)$ by

$$\begin{aligned} xH(I_1, I_2) \odot yH(I_1, I_2) &= xyH(I_1, I_2) \;\; \forall xH(I_1, I_2), \\ &\quad yH(I_1, I_2) \in G(I_1, I_2)/H(I_1, I_2). \end{aligned} \tag{14}$$

Then

(i) $(G(I_1, I_2)/H(I_1, I_2), \odot)$ is a refined neutrosophic monoid.

(ii) $(G(I_1, I_2)/H(I_1, I_2), \odot)$ is not a refined neutrosophic group but contains a proper subset which is a group.

Proof. (i) It is clear that $\odot$ is well defined. Let $xH(I_1, I_2), yH(I_1, I_2), zH(I_1, I_2)$ be arbitrary elements in $G(I_1, I_2)/H(I_1, I_2)$ with $x, y, z \in G(I_1, I_2)$. Now,

$$\begin{aligned} xH(I_1, I_2) \odot (yH(I_1, I_2) \odot zH(I_1, I_2)) &= xH(I_1, I_2) \odot (yzH(I_1, I_2)) \\ &= x(yz)H(I_1, I_2) \\ &= (xy)zH(I_1, I_2) \\ &= (xH(I_1, I_2) \odot yH(I_1, I_2)) \odot zH(I_1, I_2)). \end{aligned}$$

For all $xH(I_1, I_2) \in G(I_1, I_2)/H(I_1, I_2)$, there exists $eH(I_1, I_2) \in G(I_1, I_2)/H(I_1, I_2)$ with $e \in G(I_1, I_2)$ such that

$$\begin{aligned} xH(I_1, I_2) \odot eH(I_1, I_2) &= eH(I_1, I_2) \odot xH(I_1, I_2) \\ &= exH(I_1, I_2) \\ &= xH(I_1, I_2) \end{aligned}$$

$\therefore (G(I_1, I_2)/H(I_1, I_2), \odot)$ is a refined neutrosophic monoid.

(ii) For all $xH(I_1, I_2) \in G(I_1, I_2)/H(I_1, I_2)$ there does not exist any $yH(I_1, I_2) \in G(I_1, I_2)/H(I_1, I_2)$ such that

$$xH(I_1, I_2) \odot yH(I_1, I_2) = yH(I_1, I_2) \odot xH(I_1, I_2) = eH(I_1, I_2)$$

this shows that $x^{-1}H(I_1, I_2)$ does not exist and therefore, $(G(I_1, I_2)/H(I_1, I_2), \odot)$ is not a refined neutrosophic group. However, it contains a proper subset which is a group. □

Example 2.9. Let $G(I_1, I_2)$ and $H(I_1, I_2)$ be as defined in **Example 2.6**. It can be shown that $(G(I_1, I_2)/H(I_1, I_2), \odot)$ is not a refined neutrosophic group but a refined neutrosophic monoid.

Theorem 2.10. *Let $H(I_1, I_2)$ be a refined neutrosophic subgroup of a refined neutrosophic group $(G(I_1, I_2), .)$. Then $(G(I_1, I_2)/H(I_1, I_2), .)$ is a commutative refined neutrosophic monoid if and only if G is an abelian group.*

Proof. That $(G(I_1, I_2)/H(I_1, I_2), \odot)$ is a refined neutrosophic monoid follows from **Theorem 2.8**. Suppose that G is an abelian group. Then $((G(I_1, I_2), .)$ is a commutative refined neutrosophic monoid. Suppose that $xH(I_1, I_2), yH(I_1, I_2) \in G(I_1, I_2)/H(I_1, I_2)$ are arbitrary with $x, y \in G(I_1, I_2)$. Then

$$\begin{aligned} xH(I_1, I_2).yH(I_1, I_2) &= x.yH(I_1, I_2) \\ &= y.xH(I_1, I_2) \\ &= yH(I_1, I_2).xH(I_1, I_2). \end{aligned}$$

Conversely, suppose that $(G(I_1, I_2)/H(I_1, I_2), \odot)$ is a commutative refined neutrosophic monoid. Then $G(I_1, I_2)$ is a commutative refined neutrosophic group and consequently, G is an abelian group. □

Theorem 2.11. *Let $H(I_1, I_2)$ be a refined neutrosophic subgroup of a refined neutrosophic group $(G(I_1, I_2), +)$ and let $G(I_1, I_2)/H(I_1, I_2)$ be a set defined by*

$$G(I_1, I_2)/H(I_1, I_2) = \{x + H(I_1, I_2) : x \in G(I_1, I_2)\}. \tag{15}$$

Let $\oplus$ be a binary operation defined on $G(I_1, I_2)/H(I_1, I_2)$ by

$$\begin{aligned}(x + H(I_1, I_2)) \oplus (y + H(I_1, I_2)) &= (x + y) + H(I_1, I_2) \ \forall x + H(I_1, I_2), \\ &\quad y + H(I_1, I_2) \in G(I_1, I_2)/H(I_1, I_2).\end{aligned} \tag{16}$$

Then $(G(I_1, I_2)/H(I_1, I_2), \oplus)$ is a refined neutrosophic group.

Proof. It is clear that $\oplus$ is well defined. Let $x + H(I_1, I_2), y + H(I_1, I_2), z + H(I_1, I_2)$ be arbitrary elements in $G(I_1, I_2)/H(I_1, I_2)$ with $x, y, z \in G(I_1, I_2)$. Now,

$$\begin{aligned}(x + H(I_1, I_2)) \oplus ((y + H(I_1, I_2)) \oplus (z + H(I_1, I_2))) &= (x + H(I_1, I_2)) \oplus ((y + z) + H(I_1, I_2))) \\ &= (x + (y + z)) + H(I_1, I_2) \\ &= ((x + y) + z) + H(I_1, I_2) \\ &= ((x + H(I_1, I_2)) \oplus (y + H(I_1, I_2))) \\ &\quad \oplus (z + H(I_1, I_2))).\end{aligned}$$

Next, for all $x+H(I_1, I_2) \in G(I_1, I_2)/H(I_1, I_2)$, there exists $(0, 0I_1, 0I_2)+H(I_1, I_2) \in G(I_1, I_2)/H(I_1, I_2)$ such that

$$\begin{aligned}x + H(I_1, I_2) \oplus (0, 0I_1, 0I_2) + H(I_1, I_2) &= (0, 0I_1, 0I_2) + H(I_1, I_2) \oplus x + H(I_1, I_2) \\ &= (x + (0, 0I_1, 0I_2)) + H(I_1, I_2) \\ &= x + H(I_1, I_2).\end{aligned}$$

$$\therefore \ (0, 0I_1, 0I_2) + H(I_1, I_2) = H(I_1, I_2) \ \text{ is the additive identity element.}$$

Lastly, for all $x + H(I_1, I_2) \in G(I_1, I_2)/H(I_1, I_2)$, there exists $(-x) + H(I_1, I_2) \in G(I_1, I_2)/H(I_1, I_2)$ with $-x \in G(I_1, I_2)$ such that

$$\begin{aligned}(x + H(I_1, I_2)) \oplus ((-x) + H(I_1, I_2)) &= ((-x) + H(I_1, I_2)) \oplus (x + H(I_1, I_2)) \\ &= (x + (-x)) + H(I_1, I_2) \\ &= H(I_1, I_2).\end{aligned}$$

$$\therefore \ (-x) + H(I_1, I_2) \ \text{ is the additive inverse of } \ x + H(I_1, I_2).$$

Hence, $(G(I_1, I_2)/H(I_1, I_2), \oplus)$ is a refined neutrosophic group. □

Example 2.12. Let $G(I_1, I_2) = (\mathbb{Z}(I_1, I_2), +)$ be a refined neutrosophic group of integers and let $H(I_1, I_2) = 2\mathbb{Z}(I_1, I_2)$ be its refined neutrosophic subgroup. By definition,

$$\begin{aligned}G(I_1, I_2) &= \{(a, bI_1, cI_2) : a, b, c \in \mathbb{Z}\} \\ H(I_1, I_2) &= \{(x, yI_1, zI_2) : x, y, z \in 2\mathbb{Z}\}\end{aligned}$$

Simple computations show that

$$\begin{aligned}G(I_1, I_2)/H(I_1, I_2) = \{&(0, 0I_1, 0I_2) + H(I_1, I_2), (1, 0I_1, 0I_2) + H(I_1, I_2), \\ &(0, I_1, 0I_2) + H(I_1, I_2), (0, 0I_1, I_2) + H(I_1, I_2), (1, I_1, 0I_2) + H(I_1, I_2), \\ &(1, 0I_1, I_2) + H(I_1, I_2), (0, I_1, I_2) + H(I_1, I_2), (1, I_1, I_2) + H(I_1, I_2)\}.\end{aligned}$$

It can easily be shown that $(G(I_1, I_2)/H(I_1, I_2), +)$ is a refined neutrosophic group.

Theorem 2.13. *Let $H(I_1, I_2)$ be a refined neutrosophic subgroup of a refined neutrosophic group $(G(I_1, I_2), .)$. Then the mapping $\phi : G(I_1, I_2) \to G(I_1, I_2)/H(I_1, I_2)$ defined by*

$$\phi(x) = xH(I_1, I_2) \ \forall x \in G(I_1, I_2)$$

is not a neutrosophic homomorphism.

Proof. Let $x, y \in G(I_1, I_2)$. Then

$$\begin{aligned} \phi(xy) &= xyH(I_1, I_2) \\ &= xH(I_1, I_2)yH(I_1, I_2) \\ &= \phi(x)\phi(y). \end{aligned}$$

But then,

$$\begin{aligned} \phi(I_i) &= I_iH(I_1, I_2), i = 1, 2 \\ &\neq I_i. \end{aligned}$$

These show that ϕ is not a neutrosophic homomorphism. □

Remark 2.14. Since ϕ in **Theorem 2.13** is not a neutrosophic homomorphism, it follows that its kernel, $Ker\phi$ does not exist and the quotient $G(I_1, I_2)/Ker\phi$ cannot be found. Hence,

$$G(I_1, I_2)/Ker\phi \ncong \phi(G(I_1, I_2)).$$

This shows that 1st isomorphism theorem for classical groups does not hold in the refined neutrosophic groups. 2nd and 3rd isomorphism theorems for classical groups equally do not hold in the refined neutrosophic groups.

Theorem 2.15. *Let $H(I_1, I_2)$ be a refined neutrosophic subgroup of a refined neutrosophic group $(G(I_1, I_2), +)$. Then the mapping $\phi : G(I_1, I_2) \to G(I_1, I_2)/H(I_1, I_2)$ defined by*

$$\phi(x) = x + H(I_1, I_2) \;\; \forall x \in G(I_1, I_2)$$

is not a neutrosophic homomorphism.

Theorem 2.16. *Let $(G(I_1, I_2), +)$ be a refined neutrosophic group and let $(G(I), +)$ be a neutrosophic group where $I = xI_1 + yI_2$ with $x, y \in \mathbb{R}, \mathbb{C}$. Let $\psi : G(I_1, I_2) \to G(I)$ be a mapping defined by*

$$\psi((a, xI_1, yI_2)) = (a, (x + y)I) \;\; \forall (a, xI_1, yI_2) \in (G(I_1, I_2) \;\; \textit{with} \;\; a, x, y \in G.$$

Then ψ is a group homomorphism.

Proof. Obviously, ψ well defined. Suppose that $(a, xI_1, yI_2), (b, uI_1, vI_2) \in (G(I_1, I_2)$. Then

$$\begin{aligned} \psi((a, xI_1, yI_2) + (b, uI_1, vI_2)) &= \psi((a + b, (x + u)I_1, (y + v)I_2)) \\ &= (a + b, (x + u + y + v)I) \\ &= (a, (x + u)I) + (b, (y + v)I) \\ &= \psi((a, xI_1, yI_2)) + \psi((b, uI_1, vI_2)). \end{aligned}$$

This shows that ψ is a group homomorphism. □

Remark 2.17. From **Theorem 2.11** it follows that

$$\begin{aligned} Ker\psi &= \{(a, xI_1, yI_2) : \psi((a, xI_1, yI_2)) = (0, 0I)\} \\ &= \{(a, xI_1, yI_2) : (a, (x + y)I) = (0, 0I)\} \\ &= \{(0, xI_1, -xI_2) : x \in G\} \end{aligned}$$

which is a subgroup of $G(I_1, I_2)$. The mapping $\phi : Ker\psi \to G$ defined by $\phi((0, xI_1, -xI_2)) = x$ is a group isomorphism that is $Ker\phi \cong G$.

3 Conclusion

In this paper, we have studied refined neutrosophic quotient groups. We have shown that the classical isomorphism theorems of groups do not hold in the refined neutrosophic groups. Also, we have established existence of classical morphisms between refined neutrosophic groups $G(I_1, I_2)$ and neutrosophic groups $G(I)$.

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n- Refined Neutrosophic Rings

[1]Florentin Smarandache and [2]Mohammad Abobala
[1]Department of mathematics and science, University of New mexico, Gallup, NM 87301, USA
[2]Faculty of science, Tishreen University, Lattakia, Syria

e-mail: smarand@unm.edu

e-mail: mohammadabobala777@gmail.com

Abstract

The aim of this paper is to introduce the concept of n-refined neutrosophic ring as a generalization of refined neutrosophic ring. Also, wepresent concept of n-refined polynomial ring. We study some basic concepts related to these rings such as AH-subrings, AH-ideals, AH-factors, and AH-homomorphisms.

Keywords: n-Refined neutrosophic ring, AH-ideal, AHS-ideal, AH-homomorphism, n-Refined neutrosophic polynomial ring.

1.Introduction

Neutrosophy as a new branch of philosophy founded by F.Smarandache became a useful tool in algebraic studies.Many neutrosophic algebraic structures were defined and studied such as neutrosophic groups, neutrosophic rings, and neutrosophic vector spaces. (See [1,2,3,4,5,6]).Refined neutrosophic theory was introduced by Smarandache in 2013 when he extended the neutrosophic set / logic / probability to refined [n-valued] neutrosophic set / logic / probability respectively, i.e. the truth value T is refined/split into types of sub-truths such as (T_1, T_2, …), similarly indeterminacy I is refined/split into types of sub-indeterminacies (I_1, I_2, …) and the falsehood F is refined/split into sub-falsehood (F_1, F_2,..)[10]. In [9], Smarandache proposed a way to split the Indeterminacy element I into n sub-indeterminacies $I_1, I_2, \dots, I_n$. This idea is very interesting and helps to define new generalizations of refined neutrosophic algebraic structures.

For our purpose we define multiplication operation between indeterminacies $I_1, I_2, \dots, I_n$ as follows:

$I_m I_s = I_{\min(m,s)}$. For examples if $n = 4$ we get

$I_4 I_2 = I_2, I_1 I_2 = I_1, I_2 I_3 = I_2$. If $n = 6$ we get $I_2 I_4 = I_2, I_1 I_4 = I_1, I_4 I_5 = I_4$. If $n = 2$ we get $I_1 I_2 = I_1$ (2-refined neutrosophic ring).

AH-subtructures were firstly defined in [1]. AH-ideal in a neutrosophic ring R(I) has the form P+QI, where P,Q are ideals in the ring R. We can understand these substructures as two sections, each one is ideal (in rings). These ideals are interesting since they have properties which are similar to classical ideals and they lead us to study the concept of AHS-homomorphisms which are ring homomorphisms but not neutrosophic homomorphisms. In this article we aim to define these ideals in n-refined neutrosophic rings too.

2. Preliminaries

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Definition 2.1: [7]

Let (R,+,×) be a ring, $R(I) = \{a + bI : a, b \in R\}$ is called the neutrosophic ring where I is a neutrosophic element with condition $I^2 = I$.

Remark 2.2: [4]

The element I can be split into two indeterminacies I_1, I_2 with conditions:

$I_1^2 = I_1, I_2^2 = I_2, I_1 I_2 = I_2 I_1 = I_1$.

Definition 2.3: [4]

If X is a set then $X(I_1, I_2) = \{(a, bI_1, cI_2): a, b, c \in X\}$ is called the refined neutrosophic set generated by X , I_1, I_2.

Definition 2.4: [4]

Let (R,+,×) be a ring, $(R(I_1, I_2), +, \times)$ is called a 2-refined neutrosophic ring generated by R ,I_1, I_2.

Theorem 2.5: [4]

Let $(R(I_1, I_2), +, \times)$ be a 2-refined neutrosophic ring then it is a ring.

In the following we remind the reader about some AH-substructures.

Definition 2.6: [2]

Let $(R(I_1, I_2), +, .)$ be a refined neutrosophic ring and P_0, P_1, P_2 be ideals in the ring R then the set $P = (P_0, P_1 I_1, P_2 I_2) = \{(a, bI_1, cI_2): a \in P_0, b \in P_1, c \in P_2\}$ is called a refined neutrosophic AH-ideal.

If $P_0 = P_1 = P_2$ then P is called a refined neutrosophic AHS-ideal.

Definition 2.7: [1]

Let R be a ring and R(I) be the related neutrosophic ring and

$P = P_0 + P_1 I = \{a_0 + a_1 I ;\ a_0 \in P_0, a_1 \in P_1\};\ P_0, P_1$ are two subsets of R.

(a) We say that P is an AH-ideal if P_0, P_1 are ideals in the ring R.

(b) We say that P is an AHS-ideal if $P_0 = P_1$.

Definition 2.8: [2]

(a) Let $f: R(I_1, I_2) \to T(I_1, I_2)$ be an AHS-homomorphism we define AH-Kernel of f by : $AH - Kerf = \{ (a, bI_1, cI_2); a, b, c \in Kerf_R \} = (Kerf_R, Kerf_R I_1, Kerf_R I_2)$

(b) let $S=(S_0, S_1 I_1, S_2 I_2)$ be a subset of $R(I_1, I_2)$, then : $f(S) = (f_R(S_0), f_R(S_1) I_1, f_R(S_2) I_2) = \{(f_R(a_0), f_R(a_1) I_1, f_R(a_2) I_2); a_i \in S_i\}$.

(c) let $S=(S_0, S_1 I_1, S_2 I_2)$ be a subset of $T(I_1, I_2)$. Then

$f^{-1}(S) = (f_T^{-1}(S_0), f_T^{-1}(S_1) I_1, f_T^{-1}(S_2) I_2)$.

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Definition 2.9: [2]

Let f: R(I_1, I_2) → T(I_1, I_2) be an AHS-homomorphism we say that f is an AHS-isomorphism if it is a bijective map and R(I_1, I_2) , T(I_1, I_2) are called AHS-isomorphic refined neutrosophic rings.

It is easy to see that f_R will be an isomorphism between R , T .

Theorem 2.10 :

Let f: R(I_1, I_2) → T(I_1, I_2) be an AHS-homomorphism then we have :

(a) AH-Kerf is an AHS-ideal of R(I_1, I_2).

(b) If P is a refined neutrosophic AH-ideal of R(I_1, I_2) , f(P) is a refined neutrosophic AH-ideal of T(I_1, I_2).

(c) If P is a refined neutrosophic AHS-ideal of R(I_1, I_2) , f(P) is a refined neutrosophic AHS-ideal of T(I_1, I_2).

3. n-Refined neutrosophic rings

Definition 1.3:

Let (R,+,×) be a ring and $I_k; 1 \leq k \leq n$ be n indeterminacies. We define R_n(I)={$a_0 + a_1 I + \cdots + a_n I_n;\ a_i \in R$} to be n-refined neutrosophic ring. If n=2 we get a ring which is isomorphic to 2-refined neutrosophic ring $R(I_1, I_2)$.

Additionand multiplication on R_n(I)are defined as:

$\sum_{i=0}^{n} x_i I_i + \sum_{i=0}^{n} y_i I_i = \sum_{i=0}^{n} (x_i + y_i) I_i$, $\sum_{i=0}^{n} x_i I_i \times \sum_{i=0}^{n} y_i I_i = \sum_{i,j=0}^{n} (x_i \times y_j) I_i I_j$.

Where × is the multiplication defined on the ring R.

It is easy to see that R_n(I) is a ring in the classical concept and contains a proper ring R.

Definition 2.3:

Let R_n(I) be an n-refined neutrosophic ring, it is said to be commutative if $xy = yx$ for each x , y ∈ R_n(I), if there is 1∈ R_n(I) such $1.x = x.1 = x$, then it is called an n-refined neutrosophic ring with unity.

Theorem 3.3:

Let R_n(I) be an n-refinedneutrosophic ring. Then

(a) R is commutative if and only if R_n(I) is commutative,

(b) R has unity if and only if R_n(I) has unity,

(c) R_n(I) = $\sum_{i=0}^{n} RI_i$ = {$\sum_{i=0}^{n} x_i I_i : x_i \in R$}.

Proof:

(a) Holds directly from the definition of multiplication on R_n(I).

(b) If 1 is a unity of R then for each $a_0 + a_1 I + \cdots + a_n I^n \in R_n$(I) we have

1·$(a_0 + a_1 I + \cdots + a_n I_n) = (a_0 + a_1 I + \cdots + a_n I_n)$·1=$a_0 + a_1 I + \cdots + a_n I_n$ so 1 is the unity of R_n(I).

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(c) It is obvious that $\sum_{i=0}^{n} RI_i \leq R(I)$. Conversely assume that $a_0 + a_1 I + \cdots + a_n I_n \in R_n$(I) then by the definition we have that $a_0 + a_1 I + \cdots + a_n I_n \in \sum_{i=0}^{n} RI_i$.Thus the proof is complete.

Definition 4.3:

(a) Let R_n(I) be an n-refined neutrosophic ring and P = $\sum_{i=0}^{n} P_i I_i$ = { $a_0 + a_1 I + \cdots + a_n I_n$: $a_i \in P_i$} where P_i is a subset of R, we define P to be an AH-subring if P_i is a subring of R for all i, AHS-subring is defined by the condition $P_i = P_j$ for all i , j.

(b)P is an AH-ideal if P_i is an two sides ideal of R for all i, the AHS-ideal is defined by the condition $P_i = P_j$ for all i , j.

(c) The AH-ideal P is said to be null if $P_i = R$ or $P_i = \{0\}$ for all i.

Theorem 5.3:

Let R_n(I) be an n-refined neutrosophic ring and P is an AH-ideal,(P,+) is an abelian neutrosophic group with $k \leq n$ and r.p$\in P$ for all p $\in P$ and r $\in R$.

Proof :

Since P_i is abelian subgroup of $(R, +)$ and $r.x \in P_i$ for all $r \in R$, $x \in P_i$, the proof holds.

Remark 6.3:

We can define the right AH-ideal by the condition that P_i is a right ideal of R, the left AH-ideal can be defined as the same.

Definition 7.3:

Let R_n(I) be an n-refined neutrosophic ring and P = $\sum_{i=0}^{n} P_i I_i$, Q = $\sum_{i=0}^{n} Q_i I_i$be two AH-ideals then we define:

P+Q = $\sum_{i=0}^{n}(P_i + Q_i)I_i$, P∩Q = $\sum_{i=0}^{n}(P_i \cap Q_i)I_i$.

Theorem 8.3:

Let R_n(I) be an n-refined neutrosophic ring and P = $\sum_{i=0}^{n} P_i I_i$, Q = $\sum_{i=0}^{n} Q_i I_i$ be two AH- ideals then P+Q, P∩Q are AH-ideals. If P, Q are AHS-ideals then P+Q, P∩Q are AHS-ideals.

Proof :

Since $P_i + Q_i$, $P_i \cap Q_i$ are ideals of R then P+Q, P∩Q are AH-ideals of R_n(I).

Definition 9.3:

Let R_n(I) be an n-refined neutrosophic ring and P = $\sum_{i=0}^{n} P_i I_i$be an AH-ideal then the AH- radical of P can be defined as $H - rad(P) = \sum_{i=0}^{n}(\sqrt{P_i})I_i$.

Theorem 10.3:

The AH-radical of an AH-ideal is an AH-ideal.

Proof :

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Since $\sqrt{P_i}$ is an ideal of R then $AH - Rad(P)$ is an AH-ideal of R_n(I).

Definition 11.3:

Let R_n(I) be an n-refined neutrosophic ring and P = $\sum_{i=0}^{n} P_i I_i$ be an AH-ideal, we define AH-factor R(I)/P = $\sum_{i=0}^{n}(R/P_i)I_i = \sum_{i=0}^{n}(x_i + P_i)I_i\,; x_i \in R$.

Theorem 12.3:

Let R_n(I) be an n-refined neutrosophic ring and P = $\sum_{i=0}^{n} P_i I_i$ be an AH-ideal:

R_n(I)/P is aring with the following two binary operations

$\sum_{i=0}^{n}(x_i + P_i)I_i + \sum_{i=0}^{n}(y_i + P_i)I_i = \sum_{i=0}^{n}(x_i + y_i + P_i)I_i$,

$\sum_{i=0}^{n}(x_i + P_i)I_i \times \sum_{i=0}^{n}(y_i + P_i)I_i = \sum_{i=0}^{n}(x_i \times y_i + P_i)I_i$.

Proof :

Proof is similar tothat of Theorem 3.9 in [1].

Definition 13.3:

(a) Let R_n(I), T_n(I) be two n-refined neutrosophic rings respectively, and $f_R: R \to T$ be a ring homomorphism. We define n-refined neutrosophic AHS-homomorphism as :

f: R_n(I)→ T_n(I); $f(\sum_{i=0}^{n} x_i I_i) = \sum_{i=0}^{n} f_R(x_i) I_i$.

(b)f is an n-refined neutrosophic AHS-isomorphism if it is a bijective n-refined neutrosophic AHS-homomorphism.

(c) AH-Ker f = $\sum_{i=0}^{n} Ker(f_R)I_i$= $\{\sum_{i=0}^{n} x_i I_i\,;\ x_i \in Ker\ f_R\}$.

Theorem 14.3:

Let R_n(I), T_n(I) be two n-refined neutrosophic rings respectively and f be an n-refined neutrosophic AHS-homomorphism f: R_n(I)→ T_n(I). Then

(a) If P = $\sum_{i=0}^{n} P_i I_i$is an AH- subring of R_n(I) then f(P) is an AH- subring of T_n(I),

(b) If P = $\sum_{i=0}^{n} P_i I_i$ is an AHS- subring of R_n(I) then f(P) is an AHS- subring of T_n(I),

(c) If P = $\sum_{i=0}^{n} P_i I_i$ is an AH-ideal of R_n(I) then f(P) is an AH-ideal of f(R_n(I)),

(d)P = $\sum_{i=0}^{n} P_i I_i$ is an AHS-ideal of R_n(I) then f(P) is an AHS-ideal of f(R_n(I)),

(e) R_n(I)$/AH - Ker(f)\ is\ AHS - isomorphic\ to\ f\big(R(I)\big)$,

(f) Inverse image of an AH-ideal P in T_n(I) is an AH-ideal in R(I).

Proof :

(a) Since $f(P_i)$ is a subring of T then f(P) is an AH- subring of T_n(I).

(b) Holds by a similar way to (a).

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(c) Since $f(P_i)$ is an ideal of f(R) then f(P) is an AH- ideal of f(R(I)).

(d) It is similar to (c).

(e) We have $R/Ker(f_R) \cong f(R)$, by definition of AH-factor and $AH - Ker(f)$ we find that $R(I)/P \cong f(R(I))$.

(f) It is similar to the classical case.

Definition15.3:

(a) Let R(I) be a commutative n-refined neutrosophic ring, and P = $\sum_{i=0}^{n} P_i I_i$ be an AH- ideal, we define P to be a weak prime AH-ideal if P_i is a prime ideal of R for all i .

(b) P is called a weak maximal AH-ideal if P_i is a maximal ideal of R for all i.

(c) P is called a weak principal AH-ideal if P_i is a principal ideal of R for all i.

Theorem16.3:

Let R_n(I), T_n(I) be two commutative n-refined neutrosophic rings with an n-refined neutrosophic AHS-homomorphism $f: R_n$(I)→ T_n(I):

(a) If P = $\sum_{i=0}^{n} P_i I_i$ is an AHS- ideal of R_n(I) and Ker(f_R) $\le P_i \neq R_n$(I):

(a) P is a weak prime AHS-ideal if and only if f(P) is a weak prime AHS-ideal in f(R_n(I)).

(b) P is a weak maximal AHS-ideal if and only if f(P) is a weak maximal AHS-ideal in f(R_n(I)).

(c) If Q = $\sum_{i=0}^{n} Q_i I_i$ is an AHS-ideal of T_n(I)then it is a weak prime AHS-ideal if and only if $f^{-1}(Q)$ is a weak prime inR_n(I).

(d)if Q = $\sum_{i=0}^{n} Q_i I_i$ is an AHS-ideal of T_n(I) then it is a weak maximal AHS-ideal if and only if $f^{-1}(Q)$ is a weak maximal in R_n(I).

Proof :

Proof is similar to that of Theorem 3.8 in [1].

Example 17.3:

Let $R = Z\ be$ the ring of integers, $T = Z_6$ be thering of integers modulo 6 with multiplication and addition modulo 6, we have:

(a)$f_R: R \to T; f(x) = x\ mod 6$ is a ring homomorphism, $ker(f_R) = 6Z$, the corresponding AHS-homomorphism between R_4(I), T_4(I) is:

$f: R_4(\mathrm{I}) \to\ T_4(\mathrm{I}); f(a + bI_1 + cI_2 + dI_3 + eI_4) = (a\ mod 6) + (b\ mod 6)I_1 + (c\ mod 6)I_2 + (d\ mod 6)I_3 + (e\ mod 6)I_4;\ a, b, c, d, e \in Z.$

(b) $P =< 2 >, Q =< 3 >$are two prime and maximal and principal ideals in R,

M= $P + PI_1 + QI_2 + QI_3 + PI_4 = \{(2a + 2bI_1 + 3cI_2 + 3dI_3 + 2eI_4; a, b, c, d, e \in Z\}$ is a weak prime/ maximal AH-ideal of R_4(I).

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(c) $Ker(f_R) = 6Z \leq P, Q, f_R(P) = \{0,2,4\}, f_R(Q) = \{0,3\}$,

$f(M) = f(P) + f(P)I_1 + f(Q)I_2 + f(Q)I_3 + f(P)I_4$which is a weak maximal/ prime/principal AH-ideal of T_4(I).

(d) $AH - Ker(f) = 6Z + 6ZI_1 + 6ZI_2 + 6ZI_3 + 6ZI_4$which is an AHS-ideal of $R_4(I)$.

(e) $R_4(I)/AH - Kerf = R/6Z + R/6Z\,I_1 + R/6Z\,I_2 + R/6Z\,I_3 + R/6Z\,I_4$which is AHS-isomorphic to $f\big(R_4(I)\big) = T_4(I)$, since $R/6Z \cong T$.

Example 18.3:

Let $R = Z_8$ be a ring with addition and multiplication modulo 8.

(a) 3-refined neutrosophic ring related with R is Z_{8_3}(I)={a+$bI_1 + cI_2 + dI_3; a, b, c, d \in Z_8$}.

(b) P={0,4} is an ideal of R, $\sqrt{P}$={0,2,4,6}, $M = P + PI_1 + PI_2 + PI_3$ is an AHS-ideal of Z_{8_3}(I),

$AH - Rad(M) = \sqrt{P} + \sqrt{P}I_1 + \sqrt{P}I_2 + \sqrt{P}I_3$which is an AHS-ideal of Z_{8_3}(I).

Example 19.3:

Let R=Z_2 the ring of integers modulo 2, let $n = 3$. The corresponding 3-refined neutrosophic ring is

Z_{2_3}(I)=$\{0,1,I_1, I_2, I_3, 1 + I_1, 1 + I_2, 1 + I_3, I_1 + I_2, I_1 + I_3, I_1 + I_2 + I_3, I_2 + I_3, 1 + I_1 + I_2 + I_3, 1 + I_2 + I_3, 1 + I_1 + I_3, 1 + I_1 + I_2\}$.

4. n-Refined neutrosophic polynomial rings

Definition1.4:

Let R_n(I) be a commutative n-refined neutrosophic ring and $P: R_n$(I) $\to R_n$(I) is a function defined as $P(x) = \sum_{i=0}^{m} a_i x^i$ such $a_i \in R_n$(I), we call P a neutrosophic polynomial on R_n(I).

We denote byR_n(I)[x] to the ring of neutrosophic polynomials over R_n(I).

Since R_n(I) is a classical ring then R_n(I)[x] is a classical ring.

Theorem 2.4:

Let R(I) be a commutative n-refined neutrosophic ring. Then R_n(I)[x] = $\sum_{i=0}^{n}$ R$[x]I_i$.

Proof :

Let P(x) = $\sum_{i=0}^{n} P_i(x)I^i \in \sum_{i=0}^{n}$ R$[x]I^i$,by rearranging the previous sum we can write it as P(x) = $\sum_{i=0}^{m} a_i x^i \in R_n(I)[x]$.

Conversely, if P(x) = $\sum_{i=0}^{n} a_i x^i \in R_n(I)[x]$, then we can write it as

P(x) = $\sum_{i=0}^{n} P_i(x)I_i \in \sum_{i=0}^{n} R[x]I_i$, by the previous argument we find the proof.

Example 3.4:

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Let $Z_3(I)$ be a 3-refined neutrosophic ring and P(x) = I_1 + (2+I_1) x + (I_1+I_3)x^2 a polynomial over $Z_{3_n}(I)$, then we can write P(x) = 2x + I_1 (1 + x + x^2) + $I_2 x^2$.

It is obvious that R_n(I)≤ R_n(I)$[x]$.

Definition4.4:

Let P(x) = $\sum_{i=0}^{n} P_i(x) I^i$ a neutrosophic polynomial over R_n(I) we define the degree of P by deg P = max(deg P_i).

5. Conclusion

In this paper we have defined the n-refined neutrosophic ring and n-refined neutrosophic polynomial ring, we have introduced and studied AH-structures such as:

AH-ideal, AHS-ideal, AH-weak principal ideal, AH-weak prime ideal. Authors hope that other n-refined neutrosophic algebraic structures will be defined in future research.

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Application of the neutrosophic system to tax havens with a criminal approach

Luis Andrés Crespo Berti [1]
Level III main professor-researcher and research coordinator of the Faculty of Jurisprudence of the Autonomous Regional University of the Andes, extension Ibarra-Ecuador
Id- Web of science R-8803-2019

Correspondence: crespoberti@gmail.com

Abstract

The investigation questions transparency (legitimacy) on the one hand and tax evasion (illegality) at tax haven on the other, so this paper the highlights importance of tax havens, either to the detriment of economies suffering from significant capital flight or to the benefit of jurisdictions declared as paradises, whose economy had been favored by the inflow of capital and investment Foreign. Tax havens are mechanisms of defense of wealthy taxpayers who seek to escape with their wealth from state taxes with progressive tax systems for the financing of social protection, education, and security of their population. Much of the study had to be based on the analysis of the information collected to clarify its importance with the support of neutrosophic numbers for the determination of fuzzy sets for a better understanding of the phenomenon under study inserted in tax havens. Besides, the heuristic evaluation methodology was used, as a form of financial investment with neutrosophic representation, since it allowed the search for qualitative results that helped to emphasize investment problems in those States, territories or jurisdictions that do not have taxation, profits, or apply it at very low rates, with serious limitations in the exchange of information (bank secrecy) and a marked absence of transparency. As the main conclusion, it was argued that governments should continue their transparency campaigns to prevent the continued use of money of public origin as a consequence of illegal acts, without affecting the sovereignty of each country.

Keywords: Administration; circumvention; evasion; fiscal; international; neutrosophic number; tax havens; transparency; tribute

1. Introduction

It is understood as a regime of Fiscal Transparency International, the intention of one State to tax, through the tax administration, those enrichments to obtain the investors or taxpayers domiciled in a determine country, from their investments in the territory or jurisdictions of low taxation which the laws international taxation have identified as Tax Havens [1], [2].

In this order of ideas, the application of strong fiscal pressures by some governments to promote their economies led to the creation of tax havens, which allow businesses to that work financial advantages, competitive and prosecutors do not exist in the world. In this sense, it is estimated that a high percentage of international transactions have been processed in some way, in the intervention of one of these jurisdictions, with the consequent detriment in of many countries that legally have the right to it [3], [4], [5].

The multiplicity and complexity of economic relations, internationalization, and globalization each day more extended to all areas of life have determined that the countries incorporate fiscal measures to regulate situations caused by the coexistence of tax regimes in different States. For example, consider $ø = \wedge$ (the minimum operation, with n = 2). Then, for A, B: $X \to J$, define $A \wedge B: X \to J$ by $(A \wedge B)(x) = A(x) \wedge B(x)$, for $x \in X$. Besides, tax identities are

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maintained between operations in J also spread punctually, producing tax identities for fuzzy sets. For example, the commutative and associative laws for ∧ in J produce the same laws for fuzzy sets in X. Equality and order relationships in J extend to fuzzy sets. ^ J(x)→its define A ∧ B: X → J by (A ∧ B) (x) = A (x) ∧ B (x), para x ∈ X. A [6], [7], [8].

Thus, neutrosophy has based tax havens on the analysis of oppositional propositions, as dialectic does, but on analysis of neutralities in between them as well.

In this case, neutrosophic Logic viewed as a general framework for the unification of many existing logics, such as fuzzy logic especially fuzzy intuitionist logic, paraconsistent logic, and intuitionist logic. The essential idea of Neutrosophical Logic was to characterize each logical statement in a 3D-Neutrosophical Space, where each dimension of the space represents, respectively, the truth (T), the falsehood (F) and the indeterminacy (I) of the statement under consideration, where t, i, f are real standard or non-standard subsets of] -0, 1+ [but in the connection between them [9].

Therefore, instead of a logical value with two clear states 0 or 1, the neutrosophic approach considers a representation by a triplet (t, i, f) where these three absolute values represent the equivalent of probabilities for truth (t), indetermination or neutral state (i) and falsity (f) respectively. As it, belonging functions according to the vocabulary used in fuzzy logic. These three values are between 0 and 1. Thus, the two classical binary logic values 0 and 1 are represented respectively by (0,0,1) and (1,0,0). Now a simple probability p of having the value 1 and therefore (1-p) of having the value 0 is represented by (p, 0,1-p). In this particular case, the neutrosophic representation mainly brings a general formulation (just like for a binary value), and thus it also makes possible to represent this conception which it encompasses in its generality (also for fuzzy logic and its numerous varieties of which the so-called “intuitionistic” one). [10].

What brings us a consequence problem with a high degree of complexity, which among others range from tax voracity of the different branches of national tax and their sense expansive, until the trend of some citizens to tax avoidance or evasion simple, passing through the complexity of economic relations and of the legal institutions and the difficulty of control of States on international relations. Intending to avoid the evasions of a fiscal nature and to protect the collection by the income generated in jurisdictions of low taxation, and under which there are no agreements for the exchange of tax information or conventions to avoid double taxation since the countries that are considered within the assumptions of these jurisdictions remain at the margin of the instruments of an international character, the tax authorities should seek alternative mechanisms that allow for the monitoring of the operations to be carried out in those countries through rules of an internal nature. In this regard, the investigation was justified to the extent of providing a referential theoretical framework of a regime of Transparency International Prosecutor and an analysis of the current situation of the fiscal control mechanisms implemented by the Tax Administration in general, since this legislation is only in its beginnings, in terms of the relevance that has in the tax law. On the other hand, this research is of great utility, as studied and addressed some series elements that will allow us to understand and identify the mechanisms of administrative control, technical or other nature established by the international tax administration [11], [12].

The implementation of an efficient and effective fiscal control system by the international tax administration is not an easy task, so it presents a series of interests that, given the current situation in the world, does not have the necessary resources to carry out this work. Therefore, analyzing the fiscal control mechanisms implemented by the Tax Administration to the International Fiscal Transparency regime involves elements of value judgment on the correspondence or inconsistency of duty.

To explain the importance of tax havens, either to the detriment of economies that suffer significant capital flight or to the benefit of jurisdictions declared paradises, whose economy has been favored by the influx of capital and foreign investment. A brief analysis of the implications of tax havens and current regulations is made. Taxes are instruments whose role is fundamental as a counter-pressure to the necessary common services received in society when a more developed and advanced society is required plus taxes to satisfy collective needs that citizens individually will not be able to attend to. Tax havens are defense mechanisms for wealthy taxpayers who seek to escape state taxes with their wealth with progressive tax systems to finance the social protection, education, and security of their population.

For its elaboration, the type of documentary research was used, with a bibliographic design, at an argument level, which favored defining concepts and establishing criteria. Much of the study had to be based on the analysis of the information collected to clarify its importance with the support of neutrosophic numbers for the determination of fuzzy sets for a better understanding of the phenomenon under study inserted in tax havens [13], [14]. Besides, the heuristic evaluation methodology was used, as a form of financial investment with neutrosophic representation [15], since it allowed the search for qualitative results that helped to emphasize investment problems in those States, territories or jurisdictions that do not have taxation, profits, or apply it at very low rates, with serious limitations in the exchange of information (bank secrecy) and a marked absence of transparency.

2. Questions of research, definition and example

Community international financial development is reinforced through tributary responsible behavior.

Consequently, the objective of this study is to analyze the mechanisms of fiscal control, implemented by the Tax Administration to the regime of International Transparency that is related to the active participation of the sector financial suggested by ISO 26000 standards.

The use of Neutrosophy was proposed by Florentin Smarandache [16] for the treatment of neutrality, it is a branch of philosophy that studies the origin, nature, and scope of neutrality. This has formed the basis for a series of mathematical theories that generalize classical and fuzzy theories such as neutrosophic sets and neutrosophic logic as referred to in [14]. The original definition of true value in neutrosophic logic is shown in [17].

According to the analysis of the theoretical bases carried out, the use of neutrosophic statistics is required for the analysis of tributary responsible participation in the international community. Neutrosophic statistics are useful because they describe the statistical calculation for several different samples that contain indeterminacy, each of the same size. The use of single-valued neutrosophic sets [18] (SVNS) was proposed, which through them it is possible to use tax terms [16], to obtain greater interpretability of the results obtained with this type of data.

With the use of classical statistics, we know the data, formed by clear numbers, in neutrosophic statistics the data have some indetermination, the data can be ambiguous, vague, imprecise, incomplete, contradictory even unknown. Instead of sharp numbers used in classical statistics, sets (which approximate these sharp numbers respectively) are used in neutrosophic statistics [19, 20, 21, 22].

Additionally, in neutrosophic statistics, the sample size may not be known exactly (for example, the sample size may be between 90 and 100), this may happen because, for example, the statistician is not sure what approximately they refer to, which are the individuals in the sample whether or not they belong to the population of interest, or because the individuals in the sample only partially belong to the population of interest, while partially do not belong). Another approach would be to consider only partially the data provided by individuals in the sample whose membership in the population of interest is only partial.

For example of neutrosophic data employed by investors in tax havens are the following sets: {4, [2, 6], [7, 8], 10, 23, [20, 23)}and {5, [4, 7], {1, 2, 6}, (65, 70], (4, 5)}. See that some data are imprecise like [7, 8] and {1, 2, 6}, because the exact datum is not known. In the framework of a neutrosophic sample, we can also have an imprecise sample size, where the sample size can be stated in 90, 91-100.

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However, the neutrosophic statistic is appropriate for this analysis since results are obtained that require interpretability. In that sense, neutrosophy was used in this study, like it, consider ø = ∧ the minimum operation, with $n = 2$. Then for $A, B: X \rightarrow J$ define, $A \wedge B: X \rightarrow J$ by $(A \wedge B)(x) = A(x) \wedge B(x)$, for x ∊ X.

Thus so let X be a universe of discourse, a space of points (objects) and x denotes a generic element of X. A *Single Valued Neutrosophic Set* (SVNS) A in X is characterized by a truth-membership function $\boldsymbol{T}_{\boldsymbol{A}(\boldsymbol{x})}$, an indeterminacy-membership function $\boldsymbol{I}_{\boldsymbol{A}(\boldsymbol{x})}$, and a falsity-membership function $\boldsymbol{F}_{\boldsymbol{A}(\boldsymbol{x})}$. Where, $\boldsymbol{T}_{\boldsymbol{A}(\boldsymbol{x})}$, $\boldsymbol{I}_{\boldsymbol{A}(\boldsymbol{x})}$, $\boldsymbol{F}_{\boldsymbol{A}(\boldsymbol{x})}$: $X \rightarrow [0, 1]$ such that: $0 \leq \boldsymbol{T}_{\boldsymbol{A}(\boldsymbol{x})} + \boldsymbol{I}_{\boldsymbol{A}(\boldsymbol{x})} + \boldsymbol{F}_{\boldsymbol{A}(\boldsymbol{x})} \leq 3$. A *single valued neutrosophic number* (SVNN) is symbolized by <T, I, F> for convenience, where T, I, F∈ [0, 1] and 0≤ T+ I+ F≤3.

In a numerical calculation, each value has an effect, possibly very small, on the final result. This corresponds better to the case of reality, where there are many cross - influences. A particular quantitative approach is a probabilistic approach. Each entry or intermediate result is no more longer, either true or false, but a probability of being true [23].

Based on the foregoing gives rise to the following questions:

- What are the mechanisms with which account the Tax Administration, to control the investments made in the territory of tax haven exists?

- Does the Tax Administration with the human resource, infrastructure, technology, and equipment needed to achieve an efficient implementation of the regime of Fiscal Transparency International?

- What do functional units of the Tax Administration, are responsible for controlling the implementation of the regime?

- What is tax havens?

3. Object of transformation

It is a well-known fact that with the dynamism of modern life, the international mobility of companies and capital has highlighted the need for its consideration in the field of international tax jurisdictions, because, given the diversity of national tax regimes, it is logical to some extent, that the people and companies seek a reduction of the tax burden through various combinations, that even within the more orthodox legality, provide them with such a result. Today more than ever the tax incidence is one of the reasons why determinants of transfer or location of capital and enterprises.

The allocation of income and expenditure on the part of the companies that work in various tax jurisdictions can mean a formula to locate utilities in certain countries or to evade taxes in all of them, governmental interests are at stake and therefore shall endeavor at all costs avoid a decline in its collection, establishing formulas or generally accepted criteria aimed at reducing fraud and tax evasion [24].

The current trend toward the globalization of economies at the international level, particularly the growth, efficiency and integration of capital markets, the need to broaden the tax base and hence tax collection, has generated in the Latin American countries a change concerning the criteria of fiscal policy, on which have been structured their tax systems.

On the other hand, it is necessary to highlight the concept of power tributary in the legal theory, which is generally regarded as an element or attribute in the State.

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So the tributary power, or taxing powers of the States, refers to the ability of the State to impose mandates of conduct to the people, consisting of the contribution of economic resources to the public coffers of the State [25], [26]. In many countries as a criterion conferring the powers of taxation, the principle of territoriality, based on circumstances of an economic nature of the passive subject of tribute, such as the headquarters of the business or the source in which it originated the taxable wealth.

For which they shall be considered taxable in the country, all the income received by the subject, whatever its source, national or foreign, subordinated the taxing powers of the State through linking criteria subjective, such as the domicile, nationality, and residence, to establish a nexus or substantial connection between the State and the passive subject, to tax the totality of the income obtained by the latter, in or outside the country. Equally, a permanent establishment or fixed base of natural or legal persons, not resident or non-resident will be taxed by the global enrichments attributable to that permanent establishment or fixed base in the country.

It indicates the classic position of the Latin American countries contained in the model of the Andean Pact, which holds the criterion for the application of the source as an exclusive criterion, given the feature imminently from importers of capital from these countries, and the contemporary position, which consists in the use of the source as a criterion of priority and with discretion, without prejudice to the restrictions which can be agreed in the treaties for the avoidance of double taxation, to tax the alternative income from foreign sources.

The adoption of the principle of global income, to create taxes on taxable transactions that occurred wholly or partly outside the national territories; is justified by reasons of tax technical and economic policy, aimed among other, to the implementation of special rules of fiscal control, governing investments in countries classified as jurisdictions of low taxation, or also called tax havens, given the current incident of the removal of the national capital. Tax Havens are understood by those countries, territories, or jurisdictions where the authorities have removed or reduced to its minimum expression, taxes imposed on foreign investment [27], [28]. The characteristics of the tax havens are:

1- Many were colonies of European metropolis (primarily) and in a few cases the possession of the United States of America. 2 - Some have achieved its political independence, but continue linked to the respective metropolis, for economic reasons. 3 - Are mostly islands or archipelagoes of little territorial extension and others are located in territorial areas. 4 - Most lack strong natural resources to develop farming activities, mining, or oil. 5 - With a low population and can low level of economic development and life. 6 - domestic economic activity does not generate enough wealth to levy taxes and to be able to survive without the cooperation of foreign investment. 7- Many of them offer beauties and natural landscapes that converts them into tourist havens. 8- Foreign investors have facilities legal and tax advantages, either natural or legal persons [29].

These jurisdictions provide restrictions on the exchange of information with the tax authorities of other countries, such territories offer confidentiality to develop financial operations, so that the ownership of bank accounts and the shares of the companies domiciled among others, are information reserved for third parties.

On the other hand, they are used as escape routes to less favorable tax rates and to the laws of fiscal control that implement the countries, which leads to a detriment in the collection of the country that legally has the right to it.

Following this order of ideas, it is defined as that the principle of fiscal transparency requires that tax laws in a broad sense, including regulatory orders, circulars, lines, among others, guidelines will be structured in such a way that they are technically and legally presented. The maximum intelligibility possible and its provisions are so clear and precise as to avoid any doubt about the rights and duties of the taxpayers, both in these as well as officials in the tax administration and with the arbitrariness in the liquidation and collection of taxes. [30].

The good performance of these powers leads to shelter from the fiscal interests, a situation that is reflected in the levels of tax collection with that account of the National Treasury of each country.

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Following this order of ideas, this research has as its fundamental purpose, analyze the mechanisms of fiscal control implemented by the Tax Administration, for the regime of Fiscal Transparency International.

4. Justification and importance of research

It is understood as a regime of Fiscal Transparency International, the intention of the State to tax, through the tax administration, those enrichments to obtain form investors or taxpayers domiciled in a determine country, from their investments in the territory or jurisdictions of low taxation which the laws international taxation have identified as tax, havens.

The application of strong fiscal pressures by some governments to promote their economies led to the creation of tax havens, that allow businesses to that work here financial advantages, competitive and prosecutors do not exist in other parts of the world. In this sense, it is estimated that a high percentage of international transactions have been processed in some way, with the intervention of one of these jurisdictions, with the consequent detriment in the collection of the country that legally has the right to it [31], [32], [33].

The multiplicity and complexity of economic relations, internationalization, and globalization each day more extended to all areas of life have determined that the countries incorporate fiscal measures to regulate situations caused by the coexistence of tax regimes in different states. Which brings as a consequence problem with a high degree of complexity, which among others range from tax voracity of the different branches of national tax and their sense expansive, until the trend of some citizens to tax avoidance or evasion simple, passing through the complexity of economic relations and of the legal institutions and the difficulty of monitoring of the States on international relations [34], [35], [36].

Intending to avoid the evasions of a fiscal nature and to protect the collection by the income generated in jurisdictions of low taxation, and under which there are no agreements for the exchange of tax information or conventions to avoid double taxation since the countries that are considered within the assumptions of these jurisdictions remain at the margin of the instruments of an international character, the tax authorities should seek alternative mechanisms that allow for the monitoring of the operations to be carried out in those countries through rules of an internal nature. In this regard, the investigation was justified to the extent of providing a referential theoretical framework of a regime of Transparency International Prosecutor established, and an analysis of the current situation of the fiscal control mechanisms implemented by the Tax Administration, as it since in recent years, legislated fiscal rules have come to command greater attention in terms of the relevance that has in the tax law [37], [38], [39].

On the other hand, this research is of great utility, as studied and addressed a series element that allowed us to understand and identify the mechanisms of administrative control, technical or other nature established by the tax administration. Therefore, analyze the mechanisms of fiscal control implemented by the Tax Administration to the regime of Fiscal Transparency International, involves elements of value judgment on the correspondence or inconsistency between the duty be and be.

5. Theoretical bases

Principle of global income: state of the question:

The principles in the legal theory of tax havens are always of transparency as elements or attributes own from the States. The tributary power, or taxing powers of the State, refers to the ability of the State to impose mandates of conduct to the people, consisting of the contribution of economic resources to the public coffers of the State [30].

It is possible to identify the doctrinal level and most importantly, at the level of comparative legislation, two types of binding criteria that determine the scope of the powers of taxation, or of the Tributary Power or criteria of linkage between the taxpayer and the fact taxed.

In the definition of the scope of the tax rules, States resort to linking criteria subjective criteria, linking objectives, or the two types of criteria simultaneously. When they relate to the characteristics of the people or passive subjects of the imposition, they are considered linking criteria "subjective" when they relate to the characteristics of the taxable transactions, they are considered linking criteria "goals".

The subjective criteria or personal, consider personal aspects of the taxpayer who do not serve both to the fact taxed but fundamentally inherent characteristics to the personality of the taxpayer, within which they identify the criteria of domicile and residence.

Within the so-called objective criteria, it can be considered a single predominant criterion, which would be the criterion of territoriality or source, which attends to the place where it produces the fact taxed, which has a content much more economical than the two subjective criteria mentioned above [40].

The characteristics of relationship or connection, in respect of the State, which usually relate the linking criteria, whether subjective or objective is explained by the practical needs of the Tax Administration.

The use of the criterion of the source has been normally associated with capital importing countries, in both that the criterion of residence was originally conceived by the exporting countries capitals.

The classic position of the Latin American countries supports the application of the principle of the source, as sole discretion. However, the contemporary position consists in the use of the source as a criterion of priority, understood as the recognition of the right of a country to taxing such goods in the first term, without prejudice to the restrictions that may be agreed upon in the treaties for the avoidance of double taxation, and subsidiarity to tax income from foreign sources, recognizing credits for taxes paid abroad based on the principle of the source.

The philosophies with that developing and developed countries have been structured their respective tax systems, has generated two types of tax problems, i) multiple international taxation (above all, double taxation) and (ii) evasion and circumvention, on several or all tax jurisdictions. To confront these problems, countries apply unilateral administrative measures, and measures of cooperation between the countries, which has led to the conclusion of tax treaties. The work concerning tax treaties at the level of developed countries has been collected important background in the Organization for Economic Cooperation and Development (OECD). In Latin American countries the concern for these problems is located at the level of the Latin American Free Trade Association (ALALC) and members of the Cartagena Agreement (Andean Pact) [41], [42].

All countries that have a system of income tax or the capital, applied to some extent the principle of the source, which has a universal character. The developed countries, but also developing countries as applied simultaneously with the criterion of domicile or the nationality criterion for these countries govern what is called the criterion of the global income.

When a State assumes the connecting factor of domicile or residence, is said to be in the presence of the principle of the "global income", because enrichment shall be determined by the physical presence in a given country and will, of course, be taxed, in principle, enrichments that obtained in other countries that have adopted the connecting factors of Territoriality and nationality [43].

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According to the principle of the global income, natural persons residing, and legal persons domiciled taxed by the totality of its enrichment obtained within and outside the country. Also, the permanent establishments or fixed base of natural or legal persons, not resident or non-resident will be taxed by the global enrichments attributable to that permanent establishment. Administrations should, on the one hand, to adapt to the constant changes in the legislation, and on the other, ensure the improvement of existing systems and the sustainability of tax collection. To do this, the reform process must be understood as a dynamic process, which requires adjustments and amendments in line with the new realities. But the greatest effort should be directed at strengthening of fields as to the oversight and control of evasion [44], [45].

Incidence of the principle of world income Tax:

The current trend toward the globalization of economies at an international level, particularly the growth, efficiency and integration of capital markets, the need to broaden the tax base and hence tax collection, has generated in the Latin American countries a change about the criteria of fiscal policy, on which have been structured their tax systems.

Following this order of ideas, it is presumed that the existence of permanent establishments in situations such as:

1. The possession or existence of a local or fixed place of business.
2. The existence of any center of activity.
3. The possession of agencies or representations authorized to act by order and account of the taxpayer.
4. The implementation of activities relating to the extraction of natural resources.
5. The conduct of professional activities or artistic by itself or using representatives.
6. The possession of installations for the purchase of goods or the provision of services.
7. The provision of independent professional services, by person resident abroad.

It should be noted that the concept of a permanent establishment, is a concept developed in the models of agreements to avoid double taxation, of the Organization of United Nations (UN) and the Organization for Economic Cooperation and Development (OECD) [46], [47].

The model of tax treaties for the avoidance of double taxation prepared by the OECD, expresses in its article 7 the concept of the permanent establishment, noting:

Article 7: the profits of an enterprise of a Contracting State may only be subject to tax in that State if be that the company performs its activity in the other contracting State through a permanent establishment situated in him. Or If the company carries out its activities in such a manner, the benefits of the company may be subject to tax in the other Contracting State, but only to the extent that they are attributable to that permanent establishment [48].

Thus, the concept of global income, means that the companies or natural persons carrying out activities to the detriment of the country, pay tax on the enrichments that obtaining in that territory or abroad. In this regard, the final declaration of this tribute must consider not only the income received in the country but also those obtained through investment abroad, among which are those made in tax havens.

The implementation of the principle of world income tax system should be observed rationally, inscribed in the stage of open competition with other countries of the continent and the world in general, which implies higher care and attention in the structuring as well as the participation of guard to the taxpayer of tax measures that can break the balance that should exist in the relationship treasury-contributor.

The fight against the tax breach refers to the need to establish mechanisms to detect and implement effective sanctions that discourage, which should minimize the evasion and thus safeguard the principle of equity in the collection of taxes. The international community is concerned by the proliferation of financial activities in districts of low taxation, known as "tax havens", because it adversely affects the fair competition and erodes the tax bases of nations. Without wishing to block the economic exchange, States have established policies aimed at avoiding double taxation, as well as the tax evasion originated in jurisdictions of low taxation. Therefore, measures are taken on Fiscal Transparency International, transfer pricing, and a network of international treaties. The use of tax havens has an impact both on the taxpayer who uses it as the country where it is coming from the capital. Definitions:

Etymologically the word transparency at its meaning more approximate to the economic sphere-legal means Political Economy: characteristic of the markets of perfect competition, in which all buyers know the proposals of all sellers and also in the opposite direction [49], [50].

The principle of tax transparency requires that the tax laws in the broad sense, i.e. including the regulations orders, circulars, line, among others, guidelines (...) will be structured in such a way that submit technically and legally the maximum possible intelligibility and its provisions are just as clear and precise as to preclude any doubt on the rights and duties of the contributors, both in these same as officials in tax administration and with it the arbitrariness in the liquidation and collection of taxes [51].

Objectives:

The fundamental objectives of the International Regime of Fiscal Transparency are:

1. Avoid conducting commercial transactions, financial, or any other type of relations with entities domiciled in jurisdictions of low taxation, to minimize the avoidance and tax evasion indirect taxes such as income tax.
2. Incorporate by Providence Administrative, a list of countries considered as jurisdictions of low taxation.
3. Accumulate for shareholders or beneficiaries residing in its countries, profits on investments located in jurisdictions of low taxation.
4. Forcing residents to inform and report annually the investments held in jurisdictions of low taxation.
5. Consider not deductible those payments that are made to entities located in jurisdictions of low taxation, except proof of your cancellation to market value.
6. Set by way of the presumption that payments between natural or legal persons resident or domiciled in its countries and entities located in jurisdictions of low taxation, payments are made between related parties.
7. Improve the system of global income, allowing tax at the head of the contributors the income derived by the foreign entity that has not been taxed in the outside, or has been taxed with lower rates.

Purpose:

The rules on Fiscal Transparency International is based on the constitutional principles of each country by its tax law, as are the contributory capacity, equality, justice, progressivity and not confiscation, give satisfaction to the same, will require tax following the totality of the income earned by the natural and legal persons domiciled in each country and this implies to do this regardless of the place where the abovementioned income had been produced [52].

On the other hand, a high percentage of international transactions have been processed in some way with the intervention of jurisdictions of low taxation, "tax havens", with the consequent detriment of collection of the country that legally has the right to it, this reason justifies the adoption of legislation of specific character, anti-tax havens.

Another of the aims of the regime of Fiscal Transparency International is to ensure that legal persons to pay the taxes to which they are entitled, irrespective of the country where the payment is made, this leads to the situation that the claim of the rules of Transparency International Prosecutor, is to prevent tax avoidance that there would be, about the principle of global income through, the interposition of non-resident entities [53], [54].

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Avoid the reduction of the tax burden or in their case the deferral of its causation, product of the materialization of tax planning by the utilization of tax havens, is other of the purposes to which the regime of Fiscal Transparency International.

Taxable persons:

Are subject to the regime laid down taxpayers who possess investments made directly, indirectly or through another person, in branch offices, legal persons, movable or immovable property, actions, bank or investment accounts, and any form of participation in local authorities with or without legal personality, trusts, associations in participation, investment funds, as well as in any other legal figure similar, created or acting under the foreign law, located in jurisdictions of low taxation.

It follows that they are taxable persons subject to the regime, taxpayers natural or legal persons domiciled in the country that have investments in jurisdictions of low taxation made in: (i) branches; (ii) legal persons; (iii) movable or immovable property; (iv) actions; (v) bank accounts; (vi) investment accounts and; (vii) trusts, associations, or accounts in participation or other similar [55].

Within the framework of the provisions will depend on the time in which the taxpayer decides to carry out the distribution or distribution of yields, profits or dividends from the jurisdictions of low taxation, or when you have the control of the administration of the same, either directly or indirectly or through another person.

Likewise, it establishes the presumption in the absence of proof to the contrary, that the taxpayer influences the administration and control of investments.

Will not have the character of passive subjects of the regime of Fiscal Transparency International, the Republic, the States, and the districts of the investments made in the form directly or through its decentralized entities in jurisdictions of low taxation.

Taxable income:

1. Income from investments in jurisdictions of low taxation, when causing in proportion to the direct or indirect participation in the investment.
2. Those income, dividends, or profits not distributed in this concept.
3. It is presumed that the amounts received from the jurisdiction of low taxation are gross income or dividend derived from the investment made in that jurisdiction.

Costs and deductions admissible in the regime of Fiscal Transparency International:

Shall be deducted in full the costs and expenses related to the income arising in jurisdictions of low taxation, in proportion to the participation you have the taxpayer in such investments, always and when available accounting as a means of proof and submit an informative statement of such investments.
Income not subject to the regime of Transparency International Prosecutor:

Income from activities in jurisdictions of low taxation, when more than 50 percent of the total assets of these investments consists of fixed assets affections to the execution of such activities and are located in those jurisdictions [56].

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Such an exception shall not apply where the taxpayer obtains income from the temporary use or enjoyment of property, dividends, interest, profits from the disposition of property or royalties representing more than twenty percent of the total income from investments held by it in such jurisdictions.

The investment located in a jurisdiction of low taxation when:

1. The accounts or investments of any kind are in institutions located in that jurisdiction.
2. There is a home paragraph portal in that jurisdiction.
3. There is a business address or effective administration or main.
4. There is a permanent establishment in that jurisdiction.
5. It is constituted a company or has a physical presence in that jurisdiction.
6. Be Held, regulate, or refine any type of legal business following the laws of such jurisdiction.

It was also considered as investments made in a jurisdiction of low taxation: presumptive element

When a taxpayer opens accounts at financial institutions of the jurisdiction of low taxation, its name, or to benefit your spouse, cohabitee or cohabitee, ascendants, descendants, proxies, or if the power conferred to the latter, gives them the power to sign or sort transfers.

It is presumed, in the absence of proof to the contrary, that the transfers made by the taxpayer to deposit accounts, investment and savings, opened in a jurisdiction of low taxation, are transfers made to accounts whose ownership belongs to the same taxpayer.

The loss or gain on sale of shares in investment in the jurisdiction of low taxation or revenue derived from the liquidation or reduction of social capital in investments in these jurisdictions. The procedure is the same as indicated in the case of foreign income (global income).

About the accreditation of the tax paid in the jurisdictions of low taxation: the taxpayer subject to the regime will be able to credit the tax paid in the jurisdictions of low taxation by the procedure for accreditation of imposed from outside, in the case of the global income provided for in the new law.

Tax Havens: definition and types of tax havens

It is understood by tax haven, anyone a sovereign country that has an income tax system that essentially excludes the application of any type of tax on the income earned by natural persons or legal persons domiciled in their territory. In other simple words, Tax Havens are understood by those countries' territories or jurisdictions where the authorities have removed or reduced to its minimum expression, taxes imposed on foreign investment [57].

Operate in them is open to any person without having necessarily great wealth, allowing a decrease in its taxation increase, the performance of its investments, preserving the anonymity, and acquiring a wider vision of the economy and business. In this sense, the tax haven operates as a kind of a free zone or free port from taxation. The English term of a tax haven is “Tax Heaven”, which means port franc prosecutor, unlike the free port of trade [58], [59].

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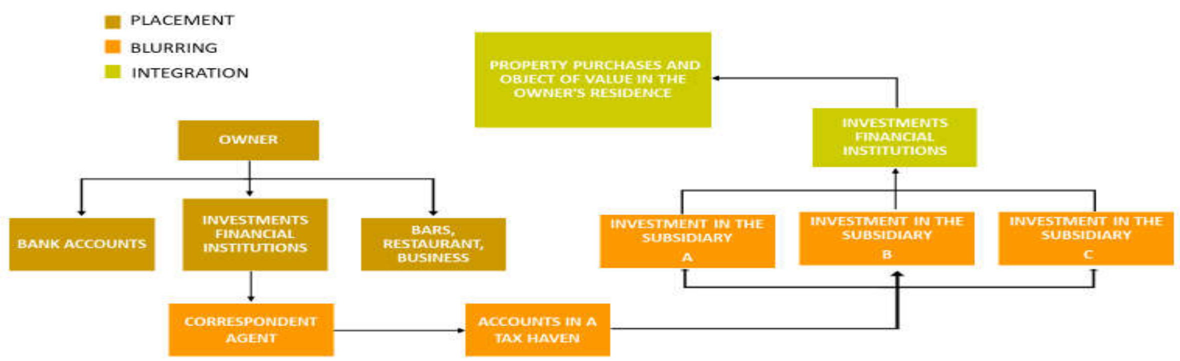

Chart 1. Process for money laundering through tax havens (source) [60].

This chart shows a process of money laundering in which the person has their assets, high investments in places where Banking Secrecy is relevant within the current regulations. In this way, the owner through the bank or a correspondent agent through a trust transfer ownership of property or rights to an investor or agent, so that it can trade with him through a third person [61].

Tax havens can be classified according to their graduation, by the greater or lesser degree of tax freedom that grant:

1. Countries or units with the absence of taxation.
2. Countries or dependencies that only taxed the acts and activities within its territory and not the Extraterritorial Income.
3. Countries that tax the acts and activities in its territory and the extraterritorial income at a very low rate; are also, with an important network of treaties to avoid double taxation.
4. Countries that privilege fiscally certain types of operations and societies [62].

Findings:

In general, the tax havens possess a series of characteristics and particular conditions, which leads to list them as elements of any jurisdiction typify qualified as a tax haven, being these features that are listed below:

- Absence or favorable level of taxation. As has been pointed out, there are four levels of jurisdictions based on the level of taxation that drives the denomination of privileged tax regimes.
- Absence of withholding taxes at the source on dividends or interest abroad. Companies incorporated in jurisdictions of low or zero taxation excluded from its tax legislation rules that establish any deduction at source.
- Secrecy and confidentiality financial and banking. It is a necessary and indispensable for the existence of tax havens.
- Banking regulation liberal. The owners of companies incorporated in tax havens can open accounts in any part of the world, besides, to counting with modern banking legislation, various financial services, and access to international credit systems.
- The network of communications and transport. Maintain excellent air services, telephone, postal, and couriers is indispensable to ensure that the operations are conducted with the corresponding speed.
- Proximity to major financial centers. Tax havens have born geographically in areas close to large financial centers.

- Legal and accounting services. In these jurisdictions, there is a multidisciplinary bureau, which will engage only in the specific provision of legal services, accounting, trust, maritime, and aviation, among others, to societies there constituted.
- Simplicity for the additions. Some jurisdictions incorporated societies with procedures that last very few minutes.
- Modern corporate laws. Presented as the dominant feature the absence of compulsory nominatives stockholding, in addition to corporate additions nearly instantaneous, and minimum requirements for the operation.
- Without double taxation conventions. Most of the tax havens do not have signed agreements of this kind since otherwise vulnerability confidentiality own of these territories.
- Political and economic stability. The Certainty, confidence, protection, and security of the assets in the jurisdiction chosen are elements that are considered in any capital investment.
- Absolute freedom would change. The entry and exit of foreign currency without restrictions and the invariance of change are conditions that offer these jurisdictions.
- Investment incentives. To advantages implied in these jurisdictions, are added rules that give favorable treatment to the location of industries, re-export and chartering, centers of marketing, administration, distribution centers, and transport.

7. Conclusion

As it can be seen in the manuscript, the government's strategy of alleviating the tax burden in a country or region is aimed at obtaining greater foreign investment, however, this can be considered as negative for the countries from which the resources that inhibit the collection of taxes for works necessary for the community. Financial globalization has brought with it the emergence of many offshore companies looking to maximize their profits. The various tax regimes encourage companies to take refuge in paradises that place a less financial weight on them. Due to its "secrecy" character, drug trafficking and various forms of corruption have a haven of refuge in their tax havens.

Governments should continue their transparency campaigns to prevent the continued use of the money of public origin because of illegal acts, without affecting the sovereignty of each country. One of the reasons why some countries do not undertake a mass struggle against these offshore uses is because, when inquiring about the company's accounts, they run the risk of investing or capital light.

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