ASPG
American Scientific Publishing Group

INTERNATIONAL JOURNAL OF NEUTROSOPHIC SCIENCE

Volume 6, 2020

Editor in Chief: Broumi Said & Florentin Smarandache

ISSN: 2690-6805

Table of Content

International Journal of Neutrosophic Science (IJNS)

Editorial Board

Wadei faris Mohammed Al-omeri, Department of Mathematics, Faculty of Science, Al-Balqa Applied University, Salt 19117, Jordan (wadeimoon1@hotmail.com, wadeialomeri@bau.edu.jo).

Abhijit Saha, Department of Mathematics, Techno College of Engineering Agartala Maheshkhola-799004, Tripura, India (abhijit84.math@gmail.com).

D.Nagarajan, Department of Mathematics, Hindustan Institute of Technology & Science, India (dnagarajan@hindustanuniv.ac.in).

Prem Kumar Singh, Amity Institute of Information Technology, Amity University-Sector 125 Noida-201313, Uttar Pradesh-India (premsingh.csjm@gmail.com).

Avishek Chakraborty, Department of Basic Science, University/College- Narula Institute of Technology Under MAKAUT, India (tirtha.avishek93@gmail.com).

Arindam Dey, Department of Computer Science and Engineering, Saroj Mohan Institute of Technology, Hooghly 712512, West Bengal, India (arindam84nit@gmail.com).

Muhammad Gulistan, Department of Mathematics & Statistics, Hazara University Mansehra, Khyber Pakhtunkhwa, Pakistan(**gulistanmath@hu.edu.pk**).

Mohsin Khalid, The University of Lahore, Pakistan(**mk4605107@gmail.com**).

Hoang Viet Long, Faculty of Information Technology, People's Police University of Technology and Logistics, Bac Ninh, Viet Nam **(longhv08@gmail.com).**

Kishore Kumar P.K, Department of Information Technology, Al Musanna College of Technology, Sultanate of Oman (**kishore2982@gmail.com**).

Mohamed Talea, Laboratory of Information processing, Faculty of Science Ben M'Sik, Morocco (taleamohamed@yahoo.fr)

Assia Bakali, Ecole Royale Navale, Casablanca, Morocco (assiabakali@yahoo.fr).

Tahir Mahmood, Department of Mathematics and Statistics, International Islamic University Islamabad, Pakistan (**tahirbakhat@iiu.edu.pk**).

Faruk KARAASLAN, Çankırı Karatekin University, Faculty of Sciences, Department of Mathematics, 18100, Çankırı, TURKEY(fkaraaslan@karatekin.edu)

Mohamed Abdel-Basset, Department of Computer Science, Zagazig University, Egypt (**analyst_mohamed@yahoo.com, analyst_mohamed@zu.edu.eg).**

Riad Khider AlHamido, Department of mathematics Faculty of Sciences - Al- Furat University, Syria (riad-hamido1983@hotmail.com).

Fahad Mohammed Alsharari, Mathematics Department, College of Science and Human Studies at Hotat Sudair, Majmaah University, Saudi Arabia (f.alsharari@mu.edu.sa).

Maikel Leyva Vázquez, Universidad Politécnica Salesiana, Ecuador (**mleyvaz@gmail.com).**

Ir. Victor Christianto, DDiv. (associate of NSIA), Satyabhakti Advanced School of Theology - Jakarta Chapter, Indonesia(**victorchristianto@gmail.com**).

Xiaohong Zhang, Shaanxi University of Science and Technology, China (zhangxiaohong@sust.edu.cn, zxhonghz@263.net).

Huda E. Khalid, Head of the Scientific Affairs and Cultural Relations, Presidency of Telafer University ,Iraq (hodaesmail@yahoo.com, dr.huda-ismael@uotelafer.edu.iq).

Ranjan Kumar, Department of Mathematics, National Institute of Technology, Adityapur, Jamshedpur, 831014, India. (**ranjank.nit52@gmail.com**).

Choonkil Park, Department of Mathematics, Hanyang University, Republic of Korea (baak@hanyang.ac.kr**).**

Shahzaib Ashraf, Department of Mathematic Abdul Wali Khan University, Mardan 23200, Pakistan (shahzaibashraf@awkum.edu.pk).

Madeleine Al-Tahan, Lebanese International University,Lebanon (madeline.tahan@liu.edu.lb).

Rafif alhabib, Department of mathmatical statistic, Faculty of Sciences, albaath University, Syria. (rafif.alhabib85@gmail.com)

Giorgio Nordo, MIFT - Department of Mathematical and Computer Science, Physical Sciences and Earth Sciences, Messina University, Italy,(giorgio.nordo@unime.it)

Angelo de Oliveira, Departamento Academico de Ciencia da Computacao - Universidade Federal de Rondonia, Brasil, (angelo@unir.br)

A.A.A. Agboola, Department of Mathematics, Federal University of Agriculture, Abeokuta, Nigeria,(agboolaaaa@funaab.edu.ng)

A. A. Salama, Department of Mathematics and Computer Science, Faculty of Science, Port Said University, Port Said, Egypt, (drsalama44@gmail.com)

Akbar Rezaei, Department of Mathematics, Payame Noor University, P.O..Box. 19395-3697, Tehran, IRAN, (rezaei@pnu.ac.ir)

Ahmed Hatip, Department of Mathematics, Gaziantep University, Turkey, (kollnaar5@gmail.com)

Metawee Songsaeng , Department of Mathematics, School of Science, University of Phayao, Phayao 56000, Thailand, (metawee.faith@gmail.com)

Aim and Scope

International Journal of Neutrosophic Science (IJNS) is a peer-review journal publishing high quality experimental and theoretical research in all areas of Neutrosophic and its Applications. IJNS is published quarterly. IJNS is devoted to the publication of peer-reviewed original research papers lying in the domain of neutrosophic sets and systems. Papers submitted for possible publication may concern with foundations, neutrosophic logic and mathematical structures in the neutrosophic setting. Besides providing emphasis on topics like artificial intelligence, pattern recognition, image processing, robotics, decision making, data analysis, data mining, applications of neutrosophic mathematical theories contributing to economics, finance, management, industries, electronics, and communications are promoted. Variants of neutrosophic sets including refined neutrosophic set (RNS). Articles evolving algorithms making computational work handy are welcome.

Topics of Interest

IJNS promotes research and reflects the most recent advances of neutrosophic Sciences in diverse disciplines, with emphasis on the following aspects, but certainly not limited to:

- Neutrosophic sets
- Neutrosophic algebra
- Neutrosophic topolog
- Neutrosophic graphs
- Neutrosophic probabilities
- Neutrosophic tools for decision making
- Neutrosophic theory for machine learning
- Neutrosophic statistics
- Neutrosophic numerical measures
- Classical neutrosophic numbers
- A neutrosophic hypothesis
- The neutrosophic level of significance
- The neutrosophic confidence interval
- The neutrosophic central limit theorem
- Neutrosophic theory in bioinformatics and medical analytics
- Neutrosophic tools for big data analytics
- Neutrosophic tools for deep learning
- Neutrosophic tools for data visualization
- Quadripartitioned single-valued neutrosophic sets
- Refined single-valued neutrosophic sets
- Applications of neutrosophic logic in image processing
- Neutrosophic logic for feature learning, classification, regression, and clustering

- Neutrosophic knowledge retrieval of medical images
- Neutrosophic set theory for large-scale image and multimedia processing
- Neutrosophic set theory for brain-machine interfaces and medical signal analysis
- Applications of neutrosophic theory in large-scale healthcare data
- Neutrosophic set-based multimodal sensor data
- Neutrosophic set-based array processing and analysis
- Wireless sensor networks Neutrosophic set-based Crowd-sourcing
- Neutrosophic set-based heterogeneous data mining
- Neutrosophic in Virtual Reality
- Neutrosophic and Plithogenic theories in Humanities and Social Sciences
- Neutrosophic and Plithogenic theories in decision making
- Neutrosophic in Astronomy and Space Sciences

Published and typeset in American Scientific Publishing Group (ASPG) is a USA academic publisher, established as LLC company on 2019 at New Orleans, Louisiana, USA. ASPG publishes online scholarly journals that are free of submission charges.

American Scientific Publishing Group (ASPG) LLC,
New Orleans, USA
Mailing Address: 625 Wright Ave, Gretna, LA 70056, USA
Phone: +1(504) 336-3385
Fax: +1-888-417-7465
e-mail: manager@americaspg.com
www.americaspg.com

Single Valued Neutrosophic Filters

[1]Giorgio Nordo, [2]Arif Mehmood, [3]Said Broumi
[1]MIFT - Department of Mathematical and Computer Science, Physical Sciences and Earth Sciences, Messina University, Italy.
[2]Department of Mathematics and Statistics, Riphah International University Sector I-14, Islamabad, Pakistan
[3]Laboratory of Information Processing, Faculty of Science Ben M'sick, University Hassan II, Casablanca, Morocco

giorgio.nordo@unime.it[1], mehdaniyal@gmail.com[2], broumisaid78@gmail.com[3]

Abstract

In this paper we give a comprehensive presentation of the notions of filter base, filter and ultrafilter on single valued neutrosophic set and we investigate some of their properties and relationships. More precisely, we discuss properties related to filter completion, the image of neutrosophic filter base by a neutrosophic induced mapping and the infimum and supremum of two neutrosophic filter bases.
Keywords: neutrosophic set, single valued neutrosphic set, neutrosophic induced mapping, single valued neutrosophic filter, neutrosophic completion, single valued neutrosophic ultrafilter.

1 Introduction

The notion of neutrosophic set was introduced in 1999 by Smarandache [20] as a generalization of both the notions of fuzzy set introduced by Zadeh in 1965 [22] and intuitionistic fuzzy set introduced by Atanassov in 1983 [2].

In 2012, Salama and Alblowi [17] introduced the notion of neutrosophic topological space which generalizes both fuzzy topological spaces given by Chang [8] and that of intuitionistic fuzzy topological spaces given by Coker [9]. Further contributions to Neutrosophic Sets Theory, which also involve many fields of theoretical and applied mathematics were recently given by numerous authors (see, for example, [6], [3], [4], [15], [1], [13], [14] and [16]). In particular, in [18] Salama and Alagamy introduced and studied the notion of neutrosophic filter and they gave some applications to neutrosophic topological space.

In General Topology, filter bases, filters and ultrafilters are widely known notions and very popular tools for proving many properties and characterizations (see, for example [5,7,10]).
Rather surprisingly, despite the fact that the class of single valued neutrosophic sets, is more versatile and has a particular aptitude for application purposes and resolution of practical real-world problems than that of neutrosophic sets, the authors of this article were not able to find any generalizations of such notions respect on single valued neutrosophic sets, in known scientific literature.

In this paper, we introduce the notions of filter base, filter and ultrafilter on single valued neutrosophic sets, and we prove some of their fundamental properties and relationships which may be useful for further studies and applications in the class of single valued neutrosophic topological spaces.

2 Preliminaries

In this section we present some basic definitions and results on neutrosophic sets and suitably exemplify them. Terms and undefined concepts are used as in [10] and [11].

The original definition of neutrosophic set, given in 1999 by Smarandache [20], refers to the interval $]0^-, 1^+[$ of the nonstandard real numbers and although it is consistent from a philosophical point of view, unfortunately, it is not suitable to be used for approaching real-world problems. For such a reason, in 2010, the same author, jointly with Wang, Zhang and Sunderraman [21], also introduced the notion of single valued

Doi :10.5281/zenodo.3830930

Received: February 27, 2020 Revised: March 31, 2020 Accepted: May 15, 2020

neutrosophic set which, referring instead to the $[0, 1]$ unit range of the usual $\mathbb{R}$ set of real numbers, can be usefully used in scientific and engineering applications.

Notation 2.1. *Let $\mathbb{U}$ be a set, $I = [0, 1]$ be the unit interval of the real numbers, for every $r \in I$, with $\underline{r}$ we denote the constant mapping $\underline{r} : \mathbb{U} \to I$ that, for every $u \in \mathbb{U}$ is defined by $\underline{r}(u) = r$.*
For every family $\{f_i\}_{i \in I}$ of mappings $f_i : \mathbb{U} \to I$, we denote by:

- $\bigwedge_{i \in I} f_i$ *the infimum mapping* $\bigwedge_{i \in I} f_i : \mathbb{U} \to I$ *that, for every* $u \in U$ *is defined by* $\left(\bigwedge_{i \in I} f_i\right)(u) = \bigwedge_{i \in I} f_i(u) = \inf\{f_i(u) : i \in I\}$*, and by*
- $\bigvee_{i \in I} f_i$ *the supremum mapping* $\bigvee_{i \in I} f_i : \mathbb{U} \to I$ *that, for every* $u \in U$ *is defined by* $\left(\bigvee_{i \in I} f_i\right)(u) = \bigvee_{i \in I} f_i(u) = \sup\{f_i(u) : i \in I\}$.

In particular, if f and g are two mappings from $\mathbb{U}$ to I, we denote their infimum (which is the minimum) by $f \wedge g$ and their supremum (which is the maximum) by $f \vee g$.

Definition 2.2. [21] Let $\mathbb{U}$ be an initial universe set and $A \subseteq \mathbb{U}$, a ***single valued neutrosophic set*** over $\mathbb{U}$ (***SVN-set*** for short), denoted by $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle$, is a set of the form

$$\widetilde{A} = \{(u, \mu_A(u), \sigma_A(u), \omega_A(u)) : u \in \mathbb{U}\}$$

where $\mu_A : \mathbb{U} \to I$, $\sigma_A : \mathbb{U} \to I$ and $\omega_A : \mathbb{U} \to I$ are the ***membership function***, the ***indeterminacy function*** and the ***nonmembership function*** of A, respectively. For every $u \in \mathbb{U}$, $\mu_A(u)$, $\sigma_A(u)$ and $\omega_A(u)$ are said the ***degree of membership***, the ***degree of indeterminacy*** and the ***degree of nonmembership*** of u, respectively.

Since $I = [0, 1]$, it clearly results $0 \leq \mu_A(u) + \sigma_A(u) + \omega_A(u) \leq 3$, for every $u \in \mathbb{U}$.

Notation 2.3. *The set of all the single valued neutrosophic sets over a universe $\mathbb{U}$ will be denoted by $\mathcal{SVN}(\mathbb{U})$.*

Definition 2.4. [20, 21] Let $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle$ and $\widetilde{B} = \langle \mathbb{U}, \mu_B, \sigma_B, \omega_B \rangle$ be two SVN-sets over the universe set $\mathbb{U}$, we say that $\widetilde{A}$ is a ***neutrosophic subset*** (or simply a subset) of $\widetilde{B}$ and we write $\widetilde{A} \Subset \widetilde{B}$ if, for every $u \in \mathbb{U}$, it results $\mu_A(u) \leq \mu_B(u)$, $\sigma_A(u) \leq \sigma_B(u)$ and $\omega_A(u) \geq \omega_B(u)$. We also say that $\widetilde{A}$ is contained in $\widetilde{B}$ or that $\widetilde{B}$ contains $\widetilde{A}$.

It is worth noting that the relation $\Subset$ satisfies the reflexive, antisymmetrical and transitive properties and so that $(\mathcal{SVN}(\mathbb{U}), \Subset)$ forms a partial ordered set (poset) but not a totally ordered set (loset) as shown in the following example.

Example 2.5. Let $\mathbb{U} = \{a, b, c\}$ be a finite universe set and $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle$, $\widetilde{B} = \langle \mathbb{U}, \mu_B, \sigma_B, \omega_B \rangle$ be two SVN-sets on $\mathcal{SVN}(\mathbb{U})$ respectively defined by the following tabular representations:

$\mathbb{U}$ \ $\widetilde{A}$	μ_A	σ_A	ω_A
a	0.5	0.3	0.2
b	0.6	0.2	0.3
c	0.4	0.2	0.7

$\mathbb{U}$ \ $\widetilde{B}$	μ_B	σ_B	ω_B
a	0.2	0.2	0.2
b	0.4	0.1	0.6
c	0.8	0.3	0.1

Then $\widetilde{A} \not\Subset \widetilde{B}$ because $\mu_A(a) = 0.5 > 0.2 = \mu_B(a)$ and $\widetilde{B} \not\Subset \widetilde{A}$ because $\omega_B(c) = 0.1 < 0.7 = \omega_A(c)$ and so the SVN-sets $\widetilde{A}$ and $\widetilde{B}$ are not comparable.

Definition 2.6. [20, 21] Let $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle$ and $\widetilde{B} = \langle \mathbb{U}, \mu_B, \sigma_B, \omega_B \rangle$ be two SVN-sets over the universe set $\mathbb{U}$, we say that $\widetilde{A}$ is a ***neutrosophically equal*** (or simply equal) to $\widetilde{B}$ and we write $\widetilde{A} \equiv \widetilde{B}$ if $\widetilde{A} \Subset \widetilde{B}$ and $\widetilde{B} \Subset \widetilde{A}$.

Definition 2.7. [21] The SVN-set $\langle \mathbb{U}, \underline{0}, \underline{0}, \underline{1} \rangle$ is said to be the ***neutrosophic empty set*** over $\mathbb{U}$ and it is denoted by $\widetilde{\emptyset}$, or more precisely by $\widetilde{\emptyset}_{\mathbb{U}}$ in case it is necessary to specify the corresponding universe set.

Definition 2.8. [21] The SVN-set $\langle \mathbb{U}, \underline{1}, \underline{1}, \underline{0} \rangle$ is said to be the ***neutrosophic absolute set*** over $\mathbb{U}$ and it is denoted by $\widetilde{\mathbb{U}}$.

Evidently, for every $A \in \mathcal{SVN}(\mathbb{U})$, it results $\widetilde{\emptyset} \Subset A \Subset \widetilde{\mathbb{U}}$.

Definition 2.9. [20, 21] Let $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle$ be a SVN-set over the universe set $\mathbb{U}$, the ***neutrosophic complement*** (or, simply, the complement) of $\widetilde{A}$, denoted by $\widetilde{A}^{\complement}$, is the SVN-set $\widetilde{A}^{\complement} = \langle \mathbb{U}, \omega_A, \underline{1} - \sigma_A, \mu_A \rangle$ that is $\widetilde{A}^{\complement} = \{(u, \omega_A(u), 1 - \sigma_A(u), \mu_A(u)) : u \in \mathbb{U}\}$.

It is a simple matter to verify that for every $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle \in \mathcal{SVN}(\mathbb{U})$, it results $\left(\widetilde{A}^{\complement}\right)^{\complement} \equiv \widetilde{A}$ and, in particular, that $\widetilde{\mathbb{U}}^{\complement} \equiv \widetilde{\emptyset}$ and $\widetilde{\emptyset}^{\complement} \equiv \widetilde{\mathbb{U}}$.

Remark 2.10. It is important to point out that, unlike in the crisp sets theory, the neutrosophic intersection of a SVN-set with its complement is not always the neutrosophic empty set, and the neutrosophic intersection of a SVN-set with its complement is not always the neutrosophic absolute set. In fact, if we consider the universe set $\mathbb{U} = \{a, b\}$ and the SVN-set on $\mathcal{SVN}(\mathbb{U})$ defined by the following tabular representations:

$\mathbb{U}$ \ $\widetilde{A}$	μ_A	σ_A	ω_A
a	0.2	0.6	0.8
b	1	0.5	0

we can easily verify that the neutrosophic intersection, $\widetilde{A} \Cap \widetilde{A}^{\complement}$ and the neutrosophic union, $\widetilde{A} \Cup \widetilde{A}^{\complement}$ are, respectively, given by the following tabular representations:

$\mathbb{U}$ \ $\widetilde{A} \Cap \widetilde{A}^{\complement}$	$\mu_{\widetilde{A} \Cap \widetilde{A}^{\complement}}$	$\sigma_{\widetilde{A} \Cap \widetilde{A}^{\complement}}$	$\omega_{\widetilde{A} \Cap \widetilde{A}^{\complement}}$
a	0.2	0.4	0.8
b	0	0.5	1

and

$\mathbb{U}$ \ $\widetilde{A} \Cup \widetilde{A}^{\complement}$	$\mu_{\widetilde{A} \Cup \widetilde{A}^{\complement}}$	$\sigma_{\widetilde{A} \Cup \widetilde{A}^{\complement}}$	$\omega_{\widetilde{A} \Cup \widetilde{A}^{\complement}}$
a	0.8	0.6	0.2
b	1	0.5	0

and so that $\widetilde{A} \Cap \widetilde{A}^{\complement} \not\equiv \widetilde{\emptyset}$ and $\widetilde{A} \Cup \widetilde{A}^{\complement} \not\equiv \widetilde{\mathbb{U}}$.

Proposition 2.11. [21] *For every pair $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle$ and $\widetilde{B} = \langle \mathbb{U}, \mu_B, \sigma_B, \omega_B \rangle$ of SVN-sets in $\mathcal{SVN}(\mathbb{U})$, we have that $\widetilde{A} \Subset \widetilde{B}$ iff $\widetilde{B}^{\complement} \Subset \widetilde{A}^{\complement}$.*

Definition 2.12. [18] Let $\left\{\widetilde{A}_i\right\}_{i \in I}$ be a family of SVN-sets $\widetilde{A}_i = \langle \mathbb{U}, \mu_{A_i}, \sigma_{A_i}, \omega_{A_i} \rangle$ over a common universe set $\mathbb{U}$, its ***neutrosophic union*** (or simply union), denoted by $\bigcup_{i \in I} \widetilde{A}_i$, is the neutrosophic set $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle$ where $\mu_A = \bigvee_{i \in I} \mu_{A_i}$, $\sigma_A = \bigvee_{i \in I} \sigma_{A_i}$, and $\omega_A = \bigwedge_{i \in I} \omega_{A_i}$.
In particular, the neutrosophic union of two single SVN-sets $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle$ and $\widetilde{B} = \langle \mathbb{U}, \mu_B, \sigma_B, \omega_B \rangle$, denoted by $\widetilde{A} \Cup \widetilde{B}$, is the neutrosophic set defined by $\langle \mathbb{U}, \mu_A \vee \mu_B, \sigma_A \vee \sigma_B, \omega_A \wedge \omega_B \rangle$.

Definition 2.13. [18] Let $\left\{\widetilde{A}_i\right\}_{i \in I}$ be a family of SVN-sets $\widetilde{A}_i = \langle \mathbb{U}, \mu_{A_i}, \sigma_{A_i}, \omega_{A_i} \rangle$ over a common universe set $\mathbb{U}$, its ***neutrosophic intersection*** (or simply intersection), denoted by $\bigcap_{i \in I} \widetilde{A}_i$, is the neutrosophic set $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle$ where $\mu_A = \bigwedge_{i \in I} \mu_{A_i}$, $\sigma_A = \bigwedge_{i \in I} \sigma_{A_i}$, and $\omega_A = \bigvee_{i \in I} \omega_{A_i}$. In particular, the neutrosophic intersection of two SVN-sets $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle$ and $\widetilde{B} = \langle \mathbb{U}, \mu_B, \sigma_B, \omega_B \rangle$, denoted by $\widetilde{A} \Cap \widetilde{B}$, is the neutrosophic set defined by $\langle \mathbb{U}, \mu_A \wedge \mu_B, \sigma_A \wedge \sigma_B, \omega_A \vee \omega_B \rangle$.

Definition 2.14. [21] Let $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle$ and $\widetilde{B} = \langle \mathbb{U}, \mu_B, \sigma_B, \omega_B \rangle$ be two SVN-sets over $\mathbb{U}$, we say that $\widetilde{A}$ and $\widetilde{B}$ are ***neutrosophically disjoint*** if $\widetilde{A} \Cap \widetilde{B} \equiv \widetilde{\emptyset}$. On the contrary, if $\widetilde{A} \Cap \widetilde{B} \not\equiv \widetilde{\emptyset}$ we say that $\widetilde{A}$ ***neutrosophically meets*** $\widetilde{B}$ (or that $\widetilde{A}$ and $\widetilde{B}$ neutrosophically meet each other).

Definition 2.15. [18] Let $\mathcal{A}, \mathcal{B} \subseteq \mathcal{SVN}(\mathbb{U})$ be two nonempty families of SVN-sets over $\mathbb{U}$, we say that $\mathcal{A}$ ***neutrosophically meets*** $\mathcal{B}$ (or that $\mathcal{A}$ and $\mathcal{B}$ neutrosophically meet each other) if every member of $\mathcal{A}$ neutrosophically meets any member of $\mathcal{B}$, that is if for every $\widetilde{A} \in \mathcal{A}$ and every $\widetilde{B} \in \mathcal{B}$ it results $\widetilde{A} \Cap \widetilde{B} \not\equiv \widetilde{\emptyset}$.
In particular, if $\widetilde{C} = \langle \mathbb{U}, \mu_C, \sigma_C, \omega_C \rangle$ is a SVN-set over $\mathbb{U}$ which neutrosophically meets each member of the family $\mathcal{A}$, we say that $\widetilde{C}$ neutrosophically meets $\mathcal{A}$.

The neutrosophic operators of union, intersection and complement satisfy many relations similar to those of crisp set theory, which are summarized in the following propositions.

Proposition 2.16. [21] *For every SVN-set* $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle \in \mathcal{SVN}(\mathbb{U})$*, we have:*

(1) $\widetilde{A} \Cup \widetilde{A} \equiv \widetilde{A}$

(2) $\widetilde{A} \Cup \widetilde{\emptyset} \equiv \widetilde{A}$

(3) $\widetilde{A} \Cup \widetilde{\mathbb{U}} \equiv \widetilde{\mathbb{U}}$

(4) $\widetilde{A} \Cap \widetilde{A} \equiv \widetilde{A}$

(5) $\widetilde{A} \Cap \widetilde{\emptyset} \equiv \widetilde{\emptyset}$

(6) $\widetilde{A} \Cap \widetilde{\mathbb{U}} \equiv A$

Proposition 2.17. [21] *For every pair* $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle$ *and* $\widetilde{B} = \langle \mathbb{U}, \mu_B, \sigma_B, \omega_B \rangle$ *of SVN-sets in* $\mathcal{SVN}(\mathbb{U})$*, we have:*

(1) $\widetilde{A} \Cup \widetilde{B} \equiv \widetilde{B} \Cup \widetilde{A}$

(2) $\widetilde{A} \Cap \widetilde{B} \equiv \widetilde{B} \Cap \widetilde{A}$

Proposition 2.18. [21] *For every triplet* $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle$, $\widetilde{B} = \langle \mathbb{U}, \mu_B, \sigma_B, \omega_B \rangle$ *and* $\widetilde{C} = \langle \mathbb{U}, \mu_C, \sigma_C, \omega_C \rangle$ *of SVN-sets in* $\mathcal{SVN}(\mathbb{U})$*, we have:*

(1) $\widetilde{A} \Cap \left(\widetilde{B} \Cap \widetilde{C}\right) \equiv \left(\widetilde{A} \Cap \widetilde{B}\right) \Cap \widetilde{C}$

(2) $\widetilde{A} \Cup \left(\widetilde{B} \Cup \widetilde{C}\right) \equiv \left(\widetilde{A} \Cup \widetilde{B}\right) \Cup \widetilde{C}$

Proposition 2.19. [21] *Let* $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle$, $\widetilde{B} = \langle \mathbb{U}, \mu_B, \sigma_B, \omega_B \rangle \in \mathcal{SVN}(\mathbb{U})$ *be two SVN-sets over a universe* $\mathbb{U}$*, then:*

(1) $\widetilde{A} \Subset \widetilde{B}$ *iff* $\widetilde{A} \Cap \widetilde{B} \equiv \widetilde{A}$

(2) $\widetilde{A} \Subset \widetilde{B}$ *iff* $\widetilde{A} \Cup \widetilde{B} \equiv \widetilde{B}$

Proposition 2.20. [21] *For every pair* $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle$ *and* $\widetilde{B} = \langle \mathbb{U}, \mu_B, \sigma_B, \omega_B \rangle$ *of SVN-sets in* $\mathcal{SVN}(\mathbb{U})$*, we have:*

(1) $\widetilde{A} \Cup \left(\widetilde{A} \Cap \widetilde{B}\right) \equiv \widetilde{A}$

(2) $\widetilde{A} \Cap \left(\widetilde{A} \Cup \widetilde{B}\right) \equiv \widetilde{A}$

Proposition 2.21. [21] *Let* $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle$, $\widetilde{B} = \langle \mathbb{U}, \mu_B, \sigma_B, \omega_B \rangle$, $\widetilde{C} = \langle \mathbb{U}, \mu_C, \sigma_C, \omega_C \rangle$ *and* $\widetilde{D} = \langle \mathbb{U}, \mu_D, \sigma_D, \omega_D \rangle$ *be SVN-sets in* $\mathcal{SVN}(\mathbb{U})$ *such that* $\widetilde{A} \Subset \widetilde{B}$ *and* $\widetilde{C} \Subset \widetilde{D}$*, then:*

(1) $\widetilde{A} \Cup \widetilde{C} \Subset \widetilde{B} \Cup \widetilde{D}$

(2) $\widetilde{A} \Cap \widetilde{C} \Subset \widetilde{B} \Cap \widetilde{D}$

Proposition 2.22. [21] *Let* $\left\{\widetilde{A}_i\right\}_{i \in I}$ *be a family of SVN-sets* $\widetilde{A}_i = \langle \mathbb{U}, \mu_{A_i}, \sigma_{A_i}, \omega_{A_i} \rangle$ *over a common universe set* $\mathbb{U}$*, then, for every* $i \in I$*, we have that* $\bigcap_{i \in I} \widetilde{A}_i \Subset \widetilde{A}_i \Subset \bigcup_{i \in I} \widetilde{A}_i$.

Proposition 2.23. [21] *Let respectively* $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle \in \mathcal{SVN}(\mathbb{U})$ *be a SVN-set and* $\left\{\widetilde{B}_i\right\}_{i \in I} \subseteq \mathcal{SVN}(\mathbb{U})$ *be a family of SVN-sets* $\widetilde{B}_i = \langle \mathbb{U}, \mu_{B_i}, \sigma_{B_i}, \omega_{B_i} \rangle$ *over a common universe set* $\mathbb{U}$*, then we have:*

(1) $\widetilde{A} \Cap \left(\bigcup_{i \in I} \widetilde{B}_i\right) \equiv \bigcup_{i \in I} \left(\widetilde{A} \Cap \widetilde{B}_i\right)$

(2) $\widetilde{A} \Cup \left(\bigcap_{i \in I} \widetilde{B}_i\right) \equiv \bigcap_{i \in I} \left(\widetilde{A} \Cup \widetilde{B}_i\right)$

Proposition 2.24. [21] *Let* $\left\{\widetilde{A}_i\right\}_{i\in I} \subseteq \mathcal{SVN}(\mathbb{U})$ *be a family of SVN-sets* $\widetilde{A}_i = \langle \mathbb{U}, \mu_{A_i}, \sigma_{A_i}, \omega_{A_i}\rangle$ *over a common universe set* $\mathbb{U}$*, it results:*

(1) $\left(\bigcup_{i\in I} \widetilde{A}_i\right)^{\complement} \equiv \bigcap_{i\in I} \widetilde{A}_i^{\complement}$

(2) $\left(\bigcap_{i\in I} \widetilde{A}_i\right)^{\complement} \equiv \bigcup_{i\in I} \widetilde{A}_i^{\complement}$

Definition 2.25. [12, 19] Let $f : \mathbb{U} \to \mathbb{V}$ be a mapping between two universe sets $\mathbb{U}$ and $\mathbb{V}$, and $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A\rangle$ be a SVN-set over $\mathbb{U}$. The ***neutrosophic image*** of $\widetilde{A}$ by f, denoted by $\widetilde{f}\left(\widetilde{A}\right)$, is the SVN-set over $\mathbb{V}$ defined by:

$$\widetilde{f}\left(\widetilde{A}\right) = \langle \mathbb{V}, f(\mu_A), f(\sigma_A), f(\omega_A)\rangle$$

where the mappings $f(\mu_A) : \mathbb{V} \to I$, $f(\sigma_A) : \mathbb{V} \to I$ and $f(\omega_A) : \mathbb{V} \to I$ are defined respectively by:

$$f(\mu_A)(v) = \begin{cases} \inf\limits_{u\in f^{-1}(\{v\})} \mu_A(u) & \text{if } f^{-1}(\{v\}) \neq \emptyset \\ 0 & \text{otherwise} \end{cases},$$

$$f(\sigma_A)(v) = \begin{cases} \inf\limits_{u\in f^{-1}(\{v\})} \sigma_A(u) & \text{if } f^{-1}(\{v\}) \neq \emptyset \\ 0 & \text{otherwise} \end{cases},$$

$$f(\omega_A)(v) = \begin{cases} \sup\limits_{u\in f^{-1}(\{v\})} \omega_A(u) & \text{if } f^{-1}(\{v\}) \neq \emptyset \\ 1 & \text{otherwise} \end{cases}$$

for every $v \in \mathbb{V}$.

Definition 2.26. [12, 19] Let $f : \mathbb{U} \to \mathbb{V}$ be a mapping between two universe sets $\mathbb{U}$ and $\mathbb{V}$, and $\widetilde{B} = \langle \mathbb{V}, \mu_B, \sigma_B, \omega_B\rangle$ be a SVN-set over $\mathbb{V}$. The ***neutrosophic inverse image*** of $\widetilde{B}$ by f, denoted by $\widetilde{f}^{-1}\left(\widetilde{B}\right)$, is the SVN-set over $\mathbb{U}$ defined by:

$$\widetilde{f}^{-1}\left(\widetilde{B}\right) = \left\langle \mathbb{U}, f^{-1}(\mu_B), f^{-1}(\sigma_B), f^{-1}(\omega_B)\right\rangle$$

where the mappings $f^{-1}(\mu_B) : \mathbb{U} \to I$, $f^{-1}(\sigma_B) : \mathbb{U} \to I$ and $f^{-1}(\omega_B) : \mathbb{U} \to I$ are defined respectively by:

$$f^{-1}(\mu_B)(u) = \mu_B(f(u)),$$

$$f^{-1}(\sigma_B)(u) = \sigma_B(f(u)),$$

$$f^{-1}(\omega_B)(u) = \omega_B(f(u))$$

for every $u \in \mathbb{U}$.

Remark 2.27. Let us note that the notation used to denote neutrosophic images and neutrosophic inverse images implicitly underlines the fact that they are not real images or counterimages since the mapping f is defined between the universe sets $\mathbb{U}$ and $\mathbb{V}$ while the definition refers to the sets $\mathcal{SVN}(\mathbb{U})$ and $\mathcal{SVN}(\mathbb{V})$ of all the SVN-sets of that respective universe sets. More properly, this means that we consider a mapping $\widetilde{f} : \mathcal{SVN}(\mathbb{U}) \to \mathcal{SVN}(\mathbb{V})$ induced by $f : \mathbb{U} \to \mathbb{V}$ over the corresponding sets of all SVN-sets.

Example 2.28. Let $\mathbb{U} = \{a, b, c\}$ and $\mathbb{V} = \{\alpha, \beta, \gamma, \delta\}$ be two finite universe sets, $f : \mathbb{U} \to \mathbb{V}$ be a mapping defined by $f(a) = f(c) = \beta$ and $f(b) = \alpha$. Let us consider a SVN-set $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A\rangle$ on $\mathcal{SVN}(\mathbb{U})$ and a SVN-set $\widetilde{B} = \langle \mathbb{U}, \mu_B, \sigma_B, \omega_B\rangle$ on $\mathcal{SVN}(\mathbb{V})$ respectively defined by the following tabular representations:

$\mathbb{U}$ \ $\widetilde{A}$	μ_A	σ_A	ω_A
a	0.5	0.3	0.2
b	0.6	0.2	0.3
c	0.4	0.2	0.7

$\mathbb{V}$ \ $\widetilde{B}$	μ_B	σ_B	ω_B
α	0.1	0.7	0.9
β	0.5	0.3	0.1
γ	0.8	0.4	0.2
δ	0.4	0.6	0.8

Then the neutrosophic image $\widetilde{f}\left(\widetilde{A}\right) = \langle \mathbb{V}, f(\mu_A), f(\sigma_A), f(\omega_A)\rangle$ of $\widetilde{A}$ by f and the neutrosophic inverse image $\widetilde{f}^{-1}\left(\widetilde{B}\right) = \left\langle \mathbb{U}, f^{-1}(\mu_B), f^{-1}(\sigma_B), f^{-1}(\omega_B)\right\rangle$ of $\widetilde{B}$ by f are given by:

$\mathbb{V}$ \ $\widetilde{f}\left(\widetilde{A}\right)$	$f(\mu_A)$	$f(\sigma_A)$	$f(\omega_A)$
α	0.6	0.2	0.3
β	0.4	0.2	0.7
γ	0	0	1
δ	0	0	1

and:

$\mathbb{U}$ \ $\widetilde{f}^{-1}\left(\widetilde{B}\right)$	$f^{-1}(\mu_B)$	$f^{-1}(\sigma_B)$	$f^{-1}(\omega_B)$
a	0.5	0.3	0.1
b	0.1	0.7	0.9
c	0.5	0.3	0.1

respectively.

Proposition 2.29. [19] *Let $f : \mathbb{U} \to \mathbb{V}$ be a mapping between two universe sets $\mathbb{U}$ and $\mathbb{V}$, $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A\rangle$ be a SVN-set over $\mathbb{U}$ and $\widetilde{B} = \langle \mathbb{V}, \mu_B, \sigma_B, \omega_B\rangle$ be a SVN-set over $\mathbb{V}$, then the following hold:*

(1) $\widetilde{f}\left(\widetilde{\emptyset}_{\mathbb{U}}\right) \Cap \widetilde{\emptyset}_{\mathbb{V}}$

(2) $\widetilde{f}^{-1}\left(\widetilde{\emptyset}_{\mathbb{V}}\right) \equiv \widetilde{\emptyset}_{\mathbb{U}}$

(3) $\widetilde{f}^{-1}\left(\widetilde{\mathbb{V}}\right) \equiv \widetilde{\mathbb{U}}$

(4) $\widetilde{A} \Subset \widetilde{f}^{-1}\left(\widetilde{f}\left(\widetilde{A}\right)\right)$ *and the identity holds if $\widetilde{f}$ is injective*

(5) $\widetilde{f}\left(\widetilde{f}^{-1}\left(\widetilde{B}\right)\right) \Subset \widetilde{B}$ *and the identity holds if $\widetilde{f}$ is surjective*

(6) $\widetilde{f}^{-1}\left(\widetilde{B}^{\complement}\right) \equiv \widetilde{f}^{-1}\left(\widetilde{B}\right)^{\complement}$

Proposition 2.30. [19] *Let $f : \mathbb{U} \to \mathbb{V}$ be a mapping between two universe sets $\mathbb{U}$ and $\mathbb{V}$, $\widetilde{A}_i = \langle \mathbb{U}, \mu_{A_i}, \sigma_{A_i}, \omega_{A_i}\rangle$ (with $i = 1, 2$) be SVN-sets over $\mathbb{U}$ and $\widetilde{B}_i = \langle \mathbb{V}, \mu_{B_i}, \sigma_{B_i}, \omega_{B_i}\rangle$ (with $i = 1, 2$) be SVN-sets over $\mathbb{V}$, then the following hold:*

(1) *if $\widetilde{A}_1 \Subset \widetilde{A}_2$ then $\widetilde{f}\left(\widetilde{A}_1\right) \Subset \widetilde{f}\left(\widetilde{A}_2\right)$*

(2) *if $\widetilde{B}_1 \Subset \widetilde{B}_2$ then $\widetilde{f}^{-1}\left(\widetilde{B}_1\right) \Subset \widetilde{f}^{-1}\left(\widetilde{B}_2\right)$*

Proposition 2.31. [19] *Let $f : \mathbb{U} \to \mathbb{V}$ be a mapping between two universe sets $\mathbb{U}$ and $\mathbb{V}$, $\left\{\widetilde{A}_i\right\}_{i\in I}$ be a family of SVN-sets $\widetilde{A}_i = \langle \mathbb{U}, \mu_{A_i}, \sigma_{A_i}, \omega_{A_i}\rangle$ over $\mathbb{U}$ and $\left\{\widetilde{B}_i\right\}_{i\in I}$ be a family of SVN-sets $\widetilde{B}_i = \langle \mathbb{V}, \mu_{B_i}, \sigma_{B_i}, \omega_{B_i}\rangle$ over $\mathbb{V}$, then the following hold:*

(1) $\widetilde{f}\left(\Cup_{i\in I} \widetilde{A}_i\right) \equiv \Cup_{i\in I} \widetilde{f}\left(\widetilde{A}_i\right)$

(2) $\widetilde{f}\left(\Cap_{i\in I} \widetilde{A}_i\right) \Subset \Cap_{i\in I} \widetilde{f}\left(\widetilde{A}_i\right)$ *and the identity holds if $\widetilde{f}$ is injective*

(3) $\widetilde{f}^{-1}\left(\Cup_{i\in I} \widetilde{A}_i\right) \equiv \Cup_{i\in I} \widetilde{f}^{-1}\left(\widetilde{A}_i\right)$

(4) $\widetilde{f}^{-1}\left(\Cap_{i\in I} \widetilde{A}_i\right) \equiv \Cap_{i\in I} \widetilde{f}^{-1}\left(\widetilde{A}_i\right)$

3 Single Valued Neutrosophic Filters

In [18], A.A. Salama and H. Alagamy introduced the notion of filter on neutrosophic set. That study, unfortunately, is rather incomplete and certainly not exhaustive because it does not cover the neutrosophic equivalent of many fundamental notions and properties such as those concerning the lower or upper bound of two filter bases, the proof of the ultrafilter's existence, etc. For this reason, together with the fact that – as already mentioned – the class of the single valued neutrosophic sets lends itself with greater ductility to resolution of real life problems and applications, we give here a comprehensive presentation about theory of filters on SVN-sets which includes new notions and properties that are not present in that article.

Definition 3.1. Let $\mathcal{A} \subseteq \mathcal{SVN}(\mathbb{U})$ be a nonempty family of SVN-sets over the universe set $\mathbb{U}$, we say that $\mathcal{F}$ is a ***single valued neutrosophic filter subbase*** (***SVN-filter subbase*** for short, or simply a filter subbase) on $\mathcal{SVN}(\mathbb{U})$ if it has the finite intersection property, i.e. if for every $\widetilde{A_1}, \dots \widetilde{A_n} \in \mathcal{A}$ (with $\widetilde{A}_i = \langle \mathbb{U}, \mu_{A_i}, \sigma_{A_i}, \omega_{A_i} \rangle$ for every $i = 1, \dots n$, and $n \in \mathbb{N}^*$), it results $\bigcap_{i=1}^{n} \widetilde{A}_i \not\equiv \widetilde{\emptyset}$.

Definition 3.2. A nonempty family $\mathcal{F} \subseteq \mathcal{SVN}(\mathbb{U})$ of SVN-sets over the universe set $\mathbb{U}$ is ***single valued neutrosophic filter base*** (***SVN-filter base*** for short, or simply a filter base) on $\mathcal{SVN}(\mathbb{U})$ if the following two conditions hold:

(i) $\widetilde{\emptyset} \notin \mathcal{F}$

(ii) for every $\widetilde{F}, \widetilde{G} \in \mathcal{F}$ there exists some $\widetilde{H} \in \mathcal{F}$ such that $\widetilde{H} \Subset \widetilde{F} \Cap \widetilde{G}$.

Definition 3.3. A nonempty family $\mathcal{F} \subseteq \mathcal{SVN}(\mathbb{U})$ of SVN-sets over the universe set $\mathbb{U}$ is ***single valued neutrosophic filter*** (***SVN-filter*** for short or simply a filter) on $\mathcal{SVN}(\mathbb{U})$ if:

(i) $\mathcal{F}$ is a SVN-filter base, and

(ii) for every $\widetilde{F} = \langle \mathbb{U}, \mu_F, \sigma_F, \omega_F \rangle \in \mathcal{F}$ and every $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle \in \mathcal{SVN}(\mathbb{U})$ such that $\widetilde{F} \Subset \widetilde{A}$ it follows that $\widetilde{A} \in \mathcal{F}$.

An equivalent definition of SVN-filter is given by the following proposition.

Proposition 3.4. *A nonempty family $\mathcal{F} \subseteq \mathcal{SVN}(\mathbb{U})$ of SVN-sets over the universe set $\mathbb{U}$ is a SVN-filter on $\mathcal{SVN}(\mathbb{U})$ if and only if the following three conditions hold:*

(i) $\widetilde{\emptyset} \notin \mathcal{F}$

(ii) *for every $\widetilde{F}, \widetilde{G} \in \mathcal{F}$, it follows that $\widetilde{F} \Cap \widetilde{G} \in \mathcal{F}$*

(iii) *for every $\widetilde{F} \in \mathcal{F}$ and every $\widetilde{A} \in \mathcal{SVN}(\mathbb{U})$ such that $\widetilde{F} \Subset \widetilde{A}$ we have that $\widetilde{A} \in \mathcal{F}$.*

Proof. Let $\mathcal{F}$ be a SVN-filter over $\mathbb{U}$. Evidently, conditions (i) and (iii) are satisfied. Let $\widetilde{F}, \widetilde{G} \in \mathcal{F}$. By condition (ii) of the definition of SVN-filter base, there exists some $\widetilde{H} \in \mathcal{F}$ such that $\widetilde{H} \Subset \widetilde{F} \Cap \widetilde{G}$ and so, by the peculiar condition of SVN-filter, it also follows that $\widetilde{F} \Cap \widetilde{G} \in \mathcal{F}$.

Conversely, if $\mathcal{F}$ is a nonempty family of SVN-sets over the universe set $\mathbb{U}$ satisfying conditions (i), (ii) and (iii), it is enough to note that (ii) implies condition (ii) of Definition 3.2 and so that $\mathcal{F}$ is SVN-filter over $\mathbb{U}$. □

It is a simple routine to show that the condition (ii) of the Proposition 3.4 can be generalized to any finite neutrosophic intersection as it is pointed out in the following corollary.

Corollary 3.5. *A nonempty family $\mathcal{F} \subseteq \mathcal{SVN}(\mathbb{U})$ of SVN-sets over the universe set $\mathbb{U}$ is a SVN-filter on $\mathcal{SVN}(\mathbb{U})$ if and only if the following three conditions hold:*

(i) $\widetilde{\emptyset} \notin \mathcal{F}$

(ii) *for every $\widetilde{F}_1, \dots \widetilde{F}_n \in \mathcal{F}$, it follows that $\bigcap_{i \in I} \widetilde{F}_i \in \mathcal{F}$*

(iii) *for every $\widetilde{F} \in \mathcal{F}$ and every $\widetilde{A} \in \mathcal{SVN}(\mathbb{U})$ such that $\widetilde{F} \Subset \widetilde{A}$ we have that $\widetilde{A} \in \mathcal{F}$.*

Remark 3.6. Evidently every SVN-filter is a SVN-filter base and every SVN-filter base is a SVN-filter subbase. It is also trivial to note that every SVN-filter on $\mathcal{SVN}(\mathbb{U})$ contains $\mathbb{U}$.

Example 3.7. Let $\mathbb{U} = \{a, b, c\}$ be a finite universe set and let $\mathcal{F} = \left\{ \widetilde{F}, \widetilde{G}, \widetilde{H}, \widetilde{\mathbb{U}} \right\} \subseteq \mathcal{SVN}(\mathbb{U})$ be a set of the SVN-sets $\widetilde{F} = \langle \mathbb{U}, \mu_F, \sigma_F, \omega_F \rangle$, $\widetilde{G} = \langle \mathbb{U}, \mu_G, \sigma_G, \omega_G \rangle$, $\widetilde{H} = \langle \mathbb{U}, \mu_H, \sigma_H, \omega_H \rangle$ and $\widetilde{\mathbb{U}} = \langle \mathbb{U}, \underline{1}, \underline{1}, \underline{0} \rangle$ respectively defined by the following tabular representations:

$\mathbb{U}$ \ $\widetilde{F}$	μ_F	σ_F	ω_F
a	0.4	0.3	0.2
b	0.8	0.2	0.1
c	0.6	0.5	0.4

$\mathbb{U}$ \ $\widetilde{G}$	μ_G	σ_G	ω_G
a	0.7	0.1	0.3
b	0.9	0.2	0.2
c	0.2	0.6	0.5

$\mathbb{U}$ \ $\widetilde{H}$	μ_H	σ_H	ω_H
a	0	0.4	0.8
b	0.5	0.3	0.6
c	0.1	0.8	0.5

$\mathbb{U}$ \ $\widetilde{\mathbb{U}}$	$\mu_{\mathbb{U}}$	$\sigma_{\mathbb{U}}$	$\omega_{\mathbb{U}}$
a	1	1	0
b	1	1	0
c	1	1	0

It is easy to check that $\mathcal{F}$ is a SVN-filter base on $\mathcal{SVN}(\mathbb{U})$. However, by using Proposition 3.4, we have that $\mathcal{F}$ is not a SVN-filter since, for example, the neutrosophic intersection $\widetilde{W} \equiv \widetilde{F} \Cap \widetilde{G}$ of $\widetilde{F}, \widetilde{G} \in \mathcal{F}$ computated as shown in the following tabular representation:

$\mathbb{U}$ \ $\widetilde{W}$	μ_W	σ_W	ω_W
a	0.4	0.3	0.3
b	0.8	0.2	0.2
c	0.2	0.6	0.5

is a SVN-set over $\mathbb{U}$ which does not belong to the family $\mathcal{F}$.

Notation 3.8. *The set of all the single valued neutrosophic filters over the universe set $\mathbb{U}$ will be denoted by $\mathfrak{F}(\mathbb{U})$.*

Definition 3.9. Let $\mathcal{F}$ and $\mathcal{G}$ be two SVN-filter bases on $\mathcal{SVN}(\mathbb{U})$, we say that $\mathcal{G}$ is ***finer*** than $\mathcal{F}$ if $\mathcal{F} \subseteq \mathcal{G}$. We also say that $\mathcal{F}$ is ***coarser*** than $\mathcal{G}$.

Let us note that the set $\mathfrak{F}(\mathbb{U})$ equipped with the finess relation $\subseteq$ forms a poset although it is not a loset.

Proposition 3.10. *Let $\mathcal{S}$ be a SVN-filter subbase on $\mathcal{SVN}(\mathbb{U})$ and let*

$$\mathcal{S}^* = \left\{ \bigcap_{i=1}^{n} \widetilde{A_i} : \widetilde{A_i} = \langle \mathbb{U}, \mu_{A_i}, \sigma_{A_i}, \omega_{A_i} \rangle \in \mathcal{S},\ n \in \mathbb{N}^* \right\}$$

be the set of all finite neutrosophic intersections of $\mathcal{S}$. Then $\mathcal{S}^*$ is a SVN-filter base on $\mathcal{SVN}(\mathbb{U})$ containing $\mathcal{S}$, i.e. $\mathcal{S} \subseteq \mathcal{S}^*$.

Proof. Let $\mathcal{S}$ be a SVN-filter subbase over $\mathbb{U}$. Since, by Definition 3.1, $\mathcal{S}$ has the finite intersection property, it is evident that $\mathcal{S}^*$ satisfies the condition (i) of Definition 3.2. Furthermore, for every $\widetilde{A}, \widetilde{B} \in \mathcal{S}^*$, there exist some $m, n \in \mathbb{N}^*$, $\widetilde{A_i} = \langle \mathbb{U}, \mu_{A_i}, \sigma_{A_i}, \omega_{A_i} \rangle \in \mathcal{S}$ (with $i = 1, \dots m$) and $\widetilde{B_j} = \left\langle \mathbb{U}, \mu_{B_j}, \sigma_{B_j}, \omega_{B_j} \right\rangle \in \mathcal{S}$ (with $j = 1, \dots n$) such that $\widetilde{A} = \bigcap_{i=1}^{m} \widetilde{A_i}$ and $\widetilde{B} = \bigcap_{j=1}^{n} \widetilde{B_i}$ and so, by definition of $\mathcal{S}^*$, it is clear that $\widetilde{A} \Cap \widetilde{B} \in \mathcal{S}^*$. Thus, $\mathcal{S}^*$ also satisfies condition (ii) of Definition 3.2 and it is a SVN-filter base. Finally, for every $\widetilde{A} \in \mathcal{S}$, since by Proposition 2.16(4) it results $\widetilde{A} \Cap \widetilde{A} = \widetilde{A}$, we have that $\widetilde{A} \in \mathcal{S}^*$ and hence that $\mathcal{S} \subseteq \mathcal{S}^*$. $\square$

Definition 3.11. Let $\mathcal{S}$ be a SVN-filter subbase on $\mathcal{SVN}(\mathbb{U})$, the SVN-filter base $\mathcal{S}^*$ defined in the proposition above is called the ***neutrosophic filter base generated*** by its ***neutrosophic filter subbase*** $\mathcal{S}$.

Proposition 3.12. *Let $\mathcal{F}$ be a SVN-filter base on $\mathcal{SVN}(\mathbb{U})$ and let*

$$\langle \mathcal{F} \rangle = \left\{ \widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle \in \mathcal{SVN}(\mathbb{U}) :\ \exists \widetilde{F} = \langle \mathbb{U}, \mu_F, \sigma_F, \omega_F \rangle \in \mathcal{F},\ \widetilde{F} \Subset \widetilde{A} \right\}$$

be the set of all neutrosophic supersets of members of $\mathcal{F}$, then $\langle \mathcal{F} \rangle$ is a SVN-filter on $\mathcal{SVN}(\mathbb{U})$ containing $\mathcal{F}$, i.e. $\mathcal{F} \subseteq \langle \mathcal{F} \rangle$.

Proof. Let $\mathcal{F}$ be a SVN-filter base over $\mathbb{U}$. Evidently, every member of $\langle\mathcal{F}\rangle$ is nonempty and for every $\widetilde{A},\widetilde{B}\in\langle\mathcal{F}\rangle$, there exist some $\widetilde{F},\widetilde{G}\in\mathcal{F}$ such that $\widetilde{F}\Subset\widetilde{A}$ and $\widetilde{G}\Subset\widetilde{B}$. So, by Proposition 2.21 (2), it follows that $\widetilde{F}\Cap\widetilde{G}\Subset\widetilde{A}\Cap\widetilde{B}$ and, by condition (ii) of Definition 3.2, we have that there exists some $\widetilde{H}\in\mathcal{F}$ such that $\widetilde{H}\Subset\widetilde{F}\Cap\widetilde{G}$ which implies that $\widetilde{H}\Subset\widetilde{A}\Cap\widetilde{B}$ and hence that $\widetilde{A}\Cap\widetilde{B}\in\langle\mathcal{F}\rangle$. Furthermore, for every $\widetilde{A}\in\langle\mathcal{F}\rangle$ and $\widetilde{B}\in\mathcal{SVN}(\mathbb{U})$ such that $\widetilde{A}\Subset\widetilde{B}$, we have that there exists some $\widetilde{F}\in\mathcal{F}$ such that $\widetilde{F}\Subset\widetilde{A}$ and, consequently, $\widetilde{F}\Subset\widetilde{B}$ which means that $\widetilde{B}\in\langle\mathcal{F}\rangle$. Thus, $\langle\mathcal{F}\rangle$ satisfies all the conditions of Proposition 3.4 and so it is a SVN-filter on $\mathcal{SVN}(\mathbb{U})$. Furthemore, we have that $\mathcal{F}\subseteq\langle\mathcal{F}\rangle$ since it is clear that for every $\widetilde{A}\in\mathcal{F}$, it results $\widetilde{A}\Subset\widetilde{A}$ and hence $\widetilde{A}\in\langle\mathcal{F}\rangle$. □

Definition 3.13. Let $\mathcal{F}$ be a SVN-filter base on $\mathcal{SVN}(\mathbb{U})$, the SVN-filter $\langle\mathcal{F}\rangle$ defined in the proposition above is called the ***neutrosophic filter completion*** of $\mathcal{F}$. Additionally, we say that $\mathcal{F}$ is a ***neutrosophic filter base*** for the SVN-filter $\langle\mathcal{F}\rangle$.

Proposition 3.14. *If $\mathcal{F}$ and $\mathcal{G}$ are two SVN-filter bases on $\mathcal{SVN}(\mathbb{U})$ such that $\mathcal{F}\subseteq\mathcal{G}$ then $\langle\mathcal{F}\rangle\subseteq\langle\mathcal{G}\rangle$.*

Proof. In fact, for every $\widetilde{A}\in\langle\mathcal{F}\rangle$ we have that there exists some $\widetilde{F}\in\mathcal{F}$ such that $\widetilde{F}\Subset\widetilde{A}$ and since $\mathcal{F}\subseteq\mathcal{G}$ it also follows that $\widetilde{F}\in\mathcal{G}$ and hence that $\widetilde{A}\in\langle\mathcal{G}\rangle$. □

Definition 3.15. Let $\mathcal{F}$ and $\mathcal{G}$ be two SVN-filter bases on $\mathcal{SVN}(\mathbb{U})$. We say that $\mathcal{F}$ and $\mathcal{G}$ are ***equivalent*** if they are both neutrosophic filter base for the same SVN-filter, that is if $\langle\mathcal{F}\rangle=\langle\mathcal{G}\rangle$.

Remark 3.16. It is a simple matter to verify that:

(1) if $\mathcal{S}$ is a SVN-filter base on $\mathcal{SVN}(\mathbb{U})$ then $\mathcal{S}^*=\mathcal{S}$,

(2) if $\mathcal{F}$ is a SVN-filter on $\mathcal{SVN}(\mathbb{U})$ then $\mathcal{F}^*=\mathcal{F}$ and $\langle\mathcal{F}\rangle=\mathcal{F}$.

Proposition 3.17. *If $\mathcal{S}$ is a SVN-filter subbase on $\mathcal{SVN}(\mathbb{U})$ then $\langle\mathcal{S}^*\rangle$ is the coarsest SVN-filter on $\mathcal{SVN}(\mathbb{U})$ containing $\mathcal{S}$, i.e. such that:*

(i) *$\mathcal{S}\subseteq\langle\mathcal{S}^*\rangle$, and*

(ii) *for every SVN-filter $\mathcal{H}$ on $\mathcal{SVN}(\mathbb{U})$ such that $\mathcal{S}\subseteq\mathcal{H}$ it follows that $\langle\mathcal{S}^*\rangle\subseteq\mathcal{H}$.*

Proof. Let $\mathcal{S}$ be a SVN-filter subbase over $\mathbb{U}$. Condition (i) is trivially verified since by Propositions 3.10 and 3.12, we immediately have that $\mathcal{S}\subseteq\mathcal{S}^*\subseteq\langle\mathcal{S}^*\rangle$. Now, suppose that $\mathcal{H}$ is a SVN-filter on $\mathcal{SVN}(\mathbb{U})$ such that $\mathcal{S}\subseteq\mathcal{H}$ and let $\widetilde{A}\in\langle\mathcal{S}^*\rangle$. Then, for some $n\in\mathbb{N}^*$, there exist $\widetilde{B}_1,\dots\widetilde{B}_n\in\mathcal{S}$ such that $\bigcap_{i=1}^{n}\widetilde{B}_i\Subset\widetilde{A}$. Since $\mathcal{S}\subseteq\mathcal{H}$, it follows that every $\widetilde{B}_i\in\mathcal{H}$ (for $i=1,\dots n$) and, by Corollary 3.5, we obtain that $\bigcap_{i=1}^{n}\widetilde{B}_i\in\mathcal{H}$ and therefore that $\widetilde{A}\in\mathcal{H}$ which proves condition (ii) and concludes our proof. □

Definition 3.18. Let $\mathcal{S}$ be a SVN-filter subbase on $\mathcal{SVN}(\mathbb{U})$, the SVN-filter $\langle\mathcal{S}^*\rangle$ defined in the proposition above is called the ***neutrosophic filter generated*** by its neutrosophic filter subbase $\mathcal{S}$.

In particular, if $\widetilde{A}=\langle\mathbb{U},\mu_A,\sigma_A,\omega_A\rangle$ is a nonempty SVN-set over the universe set $\mathbb{U}$, the SVN-filter $\langle\mathcal{S}^*\rangle$ generated by the singleton $\mathcal{S}=\left\{\widetilde{A}\right\}$, being the coarser (smallest) SVN-set containing $\mathcal{S}$, coincides with the family of all single valued neutrosophic superset of $\widetilde{A}$, is denoted simply with $\left\langle\widetilde{A}\right\rangle$ and is called the ***SVN-principal filter*** generated by $\widetilde{A}$.

Proposition 3.19. *If $\mathcal{F}$ is a finite SVN-filter base on $\mathcal{SVN}(\mathbb{U})$, then the neutrosophic filter completion $\langle\mathcal{F}\rangle$ is a SVN-principal filter over $\mathbb{U}$.*

Proof. Let $\mathcal{F}=\left\{\widetilde{F}_1,\dots\widetilde{F}_n\right\}$ (with $\widetilde{F}_i=\langle\mathbb{U},\mu_{F_i},\sigma_{F_i},\omega_{F_i}\rangle$, $i=1,\dots n$) be a finite SVN-filter base and let $\widetilde{G}\equiv\bigcap_{i=1}^{n}\widetilde{F}_i$. We will show that $\mathcal{G}=\left\{\widetilde{G}\right\}$ is an equivalent SVN-filter base for the SVN-filter $\langle\mathcal{F}\rangle$. In fact, since $\mathcal{F}$ is a SVN-filter base, by Remark 3.16 (1), we have that $\widetilde{G}\in\mathcal{F}^*=\mathcal{F}$. Thus $\mathcal{G}\subseteq\mathcal{F}$ and by Proposition 3.14 it follows that $\langle\mathcal{G}\rangle\subseteq\langle\mathcal{F}\rangle$. On the other hand, for every $\widetilde{A}\in\langle\mathcal{F}\rangle$, we have that there exists some $j=1,\dots n$ such that $\widetilde{F}_j\Subset\widetilde{A}$ and since by Proposition 2.22 we know that $\widetilde{G}=\bigcap_{i=1}^{n}\widetilde{F}_i\Subset\widetilde{F}_j$, it follows that $\widetilde{G}\Subset\widetilde{A}$ and so that $\widetilde{A}\in\langle\mathcal{G}\rangle$. This proves that $\langle\mathcal{F}\rangle\subseteq\langle\mathcal{G}\rangle$ and consequently that $\langle\mathcal{F}\rangle=\langle\mathcal{G}\rangle=\left\langle\widetilde{G}\right\rangle$, i.e. that $\langle\mathcal{F}\rangle$ is a SVN-principal filter generated by $\widetilde{G}$. □

Proposition 3.20. *Let $\mathcal{F}$ and $\mathcal{G}$ be two SVN-filter bases on $\mathcal{SVN}(\mathbb{U})$ and let*

$$\mathcal{F} \wedge \mathcal{G} = \left\{ \widetilde{F} \Cup \widetilde{G} : \ \widetilde{F} = \langle \mathbb{U}, \mu_F, \sigma_F, \omega_F \rangle \in \mathcal{F}, \, \widetilde{G} = \langle \mathbb{U}, \mu_G, \sigma_G, \omega_G \rangle \in \mathcal{G} \right\}$$

be the set of all neutrosophic unions of the members of $\mathcal{F}$ and $\mathcal{G}$, then $\mathcal{F} \wedge \mathcal{G}$ is a SVN-filter base on $\mathcal{SVN}(\mathbb{U})$. Additionally, if $\mathcal{F}$ and $\mathcal{G}$ are SVN-filter over $\mathbb{U}$ then $\mathcal{F} \wedge \mathcal{G}$ is a SVN-filter on $\mathcal{SVN}(\mathbb{U})$ which is coarser than both $\mathcal{F}$ and $\mathcal{G}$, i.e. $\mathcal{F} \wedge \mathcal{G} \subseteq \mathcal{F}$ and $\mathcal{F} \wedge \mathcal{G} \subseteq \mathcal{G}$.

Proof. If $\mathcal{F}$ and $\mathcal{G}$ are two SVN-filter bases on $\mathcal{SVN}(\mathbb{U})$, for every $\widetilde{F} \in \mathcal{F}$ and $\widetilde{G} \in \mathcal{G}$, it is evident that $\widetilde{F} \Cup \widetilde{G} \not\equiv \widetilde{\emptyset}$ and so that $\mathcal{F} \wedge \mathcal{G}$ verifies the condition (i) of Definition 3.2. Moreover, for every $\widetilde{A}_1, \widetilde{A}_2 \in \mathcal{F} \wedge \mathcal{G}$, we have that there exist some $\widetilde{F}_1, \widetilde{F}_2 \in \mathcal{F}$ and $\widetilde{G}_1, \widetilde{G}_2 \in \mathcal{G}$ such that $\widetilde{A}_1 \equiv \widetilde{F}_1 \Cup \widetilde{G}_1$ and $\widetilde{A}_2 \equiv \widetilde{F}_2 \Cup \widetilde{G}_2$. Since $\mathcal{F}$ and $\mathcal{G}$ are SVN-filter bases, there exist $\widetilde{F}_3 \in \mathcal{F}$ and $\widetilde{G}_3 \in \mathcal{G}$ such that $\widetilde{F}_3 \Subset \widetilde{F}_1 \Cap \widetilde{F}_2$ and $\widetilde{G}_3 \Subset \widetilde{G}_1 \Cap \widetilde{G}_2$. So, $\widetilde{F}_3 \Cup \widetilde{G}_3 \in \mathcal{F} \wedge \mathcal{G}$ and, by using Proposition 2.23, it results $\widetilde{F}_3 \Cup \widetilde{G}_3 \Subset \left(\widetilde{F}_1 \Cap \widetilde{F}_2\right) \Cup \left(\widetilde{G}_1 \Cap \widetilde{G}_2\right) \equiv \left(\widetilde{F}_1 \Cup \left(\widetilde{G}_1 \Cap \widetilde{G}_2\right)\right) \Cap \left(\widetilde{F}_2 \Cup \left(\widetilde{G}_1 \Cap \widetilde{G}_2\right)\right) \Subset \left(\widetilde{F}_1 \Cup \widetilde{G}_1\right) \Cap \left(\widetilde{F}_2 \Cup \widetilde{G}_2\right) \equiv \widetilde{A}_1 \Cap \widetilde{A}_2$ and this means that $\mathcal{F} \wedge \mathcal{G}$ also verifies condition (ii) of Definition 3.2 and hence that it is a SVN-filter base on $\mathcal{SVN}(\mathbb{U})$.
Now, suppose that $\mathcal{F}$ and $\mathcal{G}$ are are SVN-filters and let $\widetilde{F} \in \mathcal{F}$, $\widetilde{G} \in \mathcal{G}$ and $\widetilde{A} \in \mathcal{SVN}(\mathbb{U})$ such that $\widetilde{F} \Cup \widetilde{G} \Subset \widetilde{A}$. Since $\mathcal{F}$ is a SVN-filter and $\widetilde{F} \Subset \widetilde{F} \Cup \widetilde{G} \Subset \widetilde{A}$, we have that $\widetilde{A} \in \mathcal{F}$. Analogously, by the fact that $\mathcal{G}$ is a SVN-filter and $\widetilde{G} \Subset \widetilde{F} \Cup \widetilde{G} \Subset \widetilde{A}$, we have that $\widetilde{A} \in \mathcal{G}$. Thus $\widetilde{A} = \widetilde{A} \Cup \widetilde{A} \in \mathcal{F} \wedge \mathcal{G}$ and this proves that $\mathcal{F} \wedge \mathcal{G}$ is a SVN-filter over $\mathbb{U}$.
In such a situation, for every $\widetilde{F} \Cup \widetilde{G} \in \mathcal{F} \wedge \mathcal{G}$, with $\widetilde{F} \in \mathcal{F}$ and $\widetilde{G} \in \mathcal{G}$, being $\widetilde{F} \Subset \widetilde{F} \Cup \widetilde{G}$, we have that $\widetilde{F} \Cup \widetilde{G} \in \mathcal{F}$ and so that $\mathcal{F} \wedge \mathcal{G} \subseteq \mathcal{F}$. In a similar way, one can also proves that $\mathcal{F} \wedge \mathcal{G} \subseteq \mathcal{G}$. □

Proposition 3.21. *Let $\mathcal{F}$ and $\mathcal{G}$ be two SVN-filter bases on $\mathcal{SVN}(\mathbb{U})$ such that $\mathcal{F}$ nuetrosophically meets $\mathcal{G}$ and let*

$$\mathcal{F} \vee \mathcal{G} = \left\{ \widetilde{F} \Cap \widetilde{G} : \ \widetilde{F} = \langle \mathbb{U}, \mu_F, \sigma_F, \omega_F \rangle \in \mathcal{F}, \, \widetilde{G} = \langle \mathbb{U}, \mu_G, \sigma_G, \omega_G \rangle \in \mathcal{G} \right\}$$

be the set of all neutrosophic intersections of the members of $\mathcal{F}$ and $\mathcal{G}$, then $\mathcal{F} \vee \mathcal{G}$ is a SVN-filter base on $\mathcal{SVN}(\mathbb{U})$.
Additionally, if $\mathcal{F}$ and $\mathcal{G}$ are SVN-filters over $\mathbb{U}$ then $\mathcal{F} \vee \mathcal{G}$ is a SVN-filter on $\mathcal{SVN}(\mathbb{U})$ which is finer than both $\mathcal{F}$ and $\mathcal{G}$, i.e. $\mathcal{F} \subseteq \mathcal{F} \vee \mathcal{G}$ and $\mathcal{G} \subseteq \mathcal{F} \vee \mathcal{G}$.

Proof. Since $\mathcal{F}$ neutrosophically meets $\mathcal{G}$, it is clear that $\widetilde{\emptyset} \notin \mathcal{F} \vee \mathcal{G}$, i.e. that $\mathcal{F} \vee \mathcal{G}$ verifies the condition (i) of Definition 3.2. Moreover, for every $\widetilde{A}_1, \widetilde{A}_2 \in \mathcal{F} \vee \mathcal{G}$, we have that there exist some $\widetilde{F}_1, \widetilde{F}_2 \in \mathcal{F}$ and $\widetilde{G}_1, \widetilde{G}_2 \in \mathcal{G}$ such that $\widetilde{A}_1 \equiv \widetilde{F}_1 \Cap \widetilde{G}_1$ and $\widetilde{A}_2 \equiv \widetilde{F}_2 \Cap \widetilde{G}_2$. Since $\mathcal{F}$ and $\mathcal{G}$ are SVN-filter bases, there exist $\widetilde{F}_3 \in \mathcal{F}$ and $\widetilde{G}_3 \in \mathcal{G}$ such that $\widetilde{F}_3 \Subset \widetilde{F}_1 \Cap \widetilde{F}_2$ and $\widetilde{G}_3 \Subset \widetilde{G}_1 \Cap \widetilde{G}_2$. So, $\widetilde{F}_3 \Cap \widetilde{G}_3 \in \mathcal{F} \vee \mathcal{G}$ and it results $\widetilde{F}_3 \Cap \widetilde{G}_3 \Subset \left(\widetilde{F}_1 \Cap \widetilde{F}_2\right) \Cap \left(\widetilde{G}_1 \Cap \widetilde{G}_2\right) \equiv \left(\widetilde{F}_1 \Cap \widetilde{G}_1\right) \Cap \left(\widetilde{F}_2 \Cap \widetilde{G}_2\right) \equiv \widetilde{A}_1 \Cap \widetilde{A}_2$ and this means that $\mathcal{F} \vee \mathcal{G}$ also verifies the condition (ii) of Definition 3.2 is verified and hence that it is a SVN-filter base on $\mathcal{SVN}(\mathbb{U})$.
Now, suppose that $\mathcal{F}$ and $\mathcal{G}$ are SVN-filters and let $\widetilde{F} \in \mathcal{F}$, $\widetilde{G} \in \mathcal{G}$ and $\widetilde{A} \in \mathcal{SVN}(\mathbb{U})$ such that $\widetilde{F} \Cap \widetilde{G} \Subset \widetilde{A}$. Since $\widetilde{F} \equiv \widetilde{F} \Cup \left(\widetilde{F} \Cap \widetilde{G}\right) \Subset \widetilde{F} \Cup \widetilde{A}$ and $\mathcal{F}$ is a SVN-filter, we have that $\widetilde{F} \Cup \widetilde{A} \in \mathcal{F}$. In a similar way, since $\widetilde{G} \Subset \widetilde{G} \Cup \widetilde{A}$ and $\mathcal{G}$ is a SVN-filter, it follows that $\widetilde{G} \Cup \widetilde{A} \in \mathcal{G}$ and hence that $\left(\widetilde{F} \Cup \widetilde{A}\right) \Cap \left(\widetilde{G} \Cup \widetilde{A}\right) \in \mathcal{F} \vee \mathcal{G}$. By Proposition 2.23(2) and Proposition 2.19(2) we have that $\left(\widetilde{F} \Cup \widetilde{A}\right) \Cap \left(\widetilde{G} \Cup \widetilde{A}\right) \equiv \left(\widetilde{F} \Cap \widetilde{G}\right) \Cup \widetilde{A} = \widetilde{A}$ and so that $\widetilde{A} \in \mathcal{F} \vee \mathcal{G}$ which proves that $\mathcal{F} \vee \mathcal{G}$ is a SVN-filter over $\mathbb{U}$.
In such a situation, for every $\widetilde{F} \in \mathcal{F}$ and for any fixed $\widetilde{G} \in \mathcal{G}$, we have that $\widetilde{F} \Cap \widetilde{G} \Subset \widetilde{F}$ with $\widetilde{F} \Cap \widetilde{G} \in \mathcal{F} \vee \mathcal{G}$ and so that also $\widetilde{F} \in \mathcal{F} \vee \mathcal{G}$ which proves that $\mathcal{F} \subseteq \mathcal{F} \vee \mathcal{G}$. In a similar way, one can also proves that $\mathcal{G} \subseteq \mathcal{F} \vee \mathcal{G}$. □

Proposition 3.22. *Let $\mathcal{F}$ be a SVN-filter base on $\mathcal{SVN}(\mathbb{U})$ and $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle$ be a SVN-set over $\mathbb{U}$ which neutrosophically meets $\mathcal{F}$, then the set $\mathcal{F} \vee \widetilde{A} = \left\{ \widetilde{F} \Cap \widetilde{A} : \ \widetilde{F} \in \mathcal{F} \right\}$ of all neutrosophic intersections of $\widetilde{A}$ with the members of $\mathcal{F}$ is a SVN-filter base over $\mathbb{U}$. Additionally, if $\mathcal{F}$ is a SVN-filter then $\mathcal{F} \vee \widetilde{A}$ is a SVN-filter on $\mathcal{SVN}(\mathbb{U})$ which is finer than $\mathcal{F}$, i.e. $\mathcal{F} \subseteq \mathcal{F} \vee \widetilde{A}$.*

Proof. Since by hypothesis $\mathcal{F}$ neutrosophically meets $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle$ it is evident that $\widetilde{\emptyset} \notin \mathcal{F} \vee \widetilde{A}$. Moreover, for every $\widetilde{G}_1, \widetilde{G}_2 \in \mathcal{F} \vee \widetilde{A}$ there exist $\widetilde{F}_1, \widetilde{F}_2 \in \mathcal{F}$ such that $\widetilde{G}_1 = \widetilde{F}_1 \Cap \widetilde{A}$ and $\widetilde{G}_2 = \widetilde{F}_2 \Cap \widetilde{A}$. Since $\mathcal{F}$ is a SVN-filter base, there exists some $\widetilde{F}_3 \in \mathcal{F}$ such that such that $\widetilde{F}_3 \Subset \widetilde{F}_1 \Cap \widetilde{F}_2$. So, let $\widetilde{G}_3 = \widetilde{F}_3 \Cap \widetilde{A}$, we note that $\widetilde{G}_3 \in \mathcal{F} \vee \widetilde{A}$ and it results $\widetilde{G}_3 \Subset \widetilde{G}_1 \Cap \widetilde{G}_2$ which proves that $\mathcal{F} \vee \widetilde{A}$ is a SVN-filter base on $\mathcal{SVN}(\mathbb{U})$.
Now, suppose that $\mathcal{F}$ is a SVN-filter over $\mathbb{U}$ and let $\widetilde{F} \in \mathcal{F}$ and $\widetilde{B} \in \mathcal{SVN}(\mathbb{U})$ such that $\widetilde{F} \Cap \widetilde{A} \Subset \widetilde{B}$, with

$\widetilde{B} \Subset \widetilde{A}$. Since $\widetilde{F} \equiv \widetilde{F} \Cup \left(\widetilde{F} \Cap \widetilde{A}\right) \Subset \widetilde{F} \Cup \widetilde{B}$ and $\mathcal{F}$ is a SVN-filter, we have that $\widetilde{F} \Cup \widetilde{B} \in \mathcal{F}$ and hence that $\left(\widetilde{F} \Cup \widetilde{B}\right) \Cap \widetilde{A} \in \mathcal{F} \vee \widetilde{A}$. Moreover, by Proposition 2.23(2) and Proposition 2.19(2), $\widetilde{B} \Subset \widetilde{A}$ and $\widetilde{F} \Cap \widetilde{A} \Subset \widetilde{B}$ imply that $\left(\widetilde{F} \Cup \widetilde{B}\right) \Cap \widetilde{A} \equiv \left(\widetilde{F} \Cap \widetilde{A}\right) \Cup \left(\widetilde{B} \Cap \widetilde{A}\right) \equiv \left(\widetilde{F} \Cap \widetilde{A}\right) \Cup \widetilde{B} \equiv \widetilde{B}$ and hence that $\widetilde{B} \in \mathcal{F} \vee \widetilde{A}$ which proves that $\mathcal{F} \vee \widetilde{A}$ is a SVN-filter over $\mathbb{U}$.
In such a situation, for every $\widetilde{F} \in \mathcal{F}$, we have that $\widetilde{F} \Cap \widetilde{A} \in \mathcal{F} \vee \widetilde{A}$ and since $\widetilde{F} \Cap \widetilde{A} \Subset \widetilde{F}$ and $\mathcal{F} \vee \widetilde{A}$ is a SVN-filter, it follows that also $\widetilde{F} \in \mathcal{F} \vee \widetilde{A}$ and hence that $\mathcal{F} \subseteq \mathcal{F} \vee \widetilde{A}$. □

Proposition 3.23. *Let $f : \mathbb{U} \to \mathbb{V}$ be a mapping between two universe sets $\mathbb{U}$ and $\mathbb{V}$ and let $\mathcal{F}$ be a SVN-filter base on $\mathcal{SVN}(\mathbb{U})$. Then, the family $\widetilde{f}(\mathcal{F}) = \left\{\widetilde{f}\left(\widetilde{F}\right) : \widetilde{F} \in \mathcal{F}\right\}$ of all neutrosophic images on $\mathcal{SVN}(\mathbb{U})$ by the mapping f is a neutrosophic filter base on $\mathcal{SVN}(\mathbb{V})$.*

Proof. Let us consider a mapping $f : \mathbb{U} \to \mathbb{V}$ and a SVN-filter base $\mathcal{F}$ over $\mathbb{U}$. Evidently, for every $\widetilde{A} \in \mathcal{F}$, being $\widetilde{A} \not\equiv \widetilde{\emptyset}_{\mathbb{U}}$, by Proposition 2.29(1), we also have that $\widetilde{f}\left(\widetilde{F}\right) \not\equiv \widetilde{\emptyset}_{\mathbb{V}}$ and this means that $\widetilde{f}(\mathcal{F})$ satisfies the condition (i) of Definition 3.2. Moreover, for every $\widetilde{G}_1, \widetilde{G}_2 \in \widetilde{f}(\mathcal{F})$, there are some $\widetilde{F}_1, \widetilde{F}_2 \in \mathcal{F}$ such that $\widetilde{G}_1 \equiv \widetilde{f}\left(\widetilde{F}_1\right)$ and $\widetilde{G}_2 \equiv \widetilde{f}\left(\widetilde{F}_2\right)$. Since $\mathcal{F}$ is a SVN-filter base, there exists some $\widetilde{F}_3 \in \mathcal{F}$ such that $\widetilde{F}_3 \Subset \widetilde{F}_1 \Cap \widetilde{F}_2$. Hence, said $\widetilde{G}_3 \equiv \widetilde{f}\left(\widetilde{F}_3\right)$, we have that $\widetilde{G}_3 \in \widetilde{f}(\mathcal{F})$, while by Proposition 2.30(1) and Proposition 2.31(2), we obtain that $\widetilde{G}_3 \equiv \widetilde{f}\left(\widetilde{F}_3\right) \Subset \widetilde{f}\left(\widetilde{F}_1 \Cap \widetilde{F}_2\right) \Subset \widetilde{f}\left(\widetilde{F}_1\right) \Cap \widetilde{f}\left(\widetilde{F}_2\right) \equiv \widetilde{G}_1 \Cap \widetilde{G}_2$. This shows that $\widetilde{f}(\mathcal{F})$ satisfies also the condition (ii) of Definition 3.2 and concludes our proof. □

4 Single Valued Neutrosophic Ultrafilters

In this section we consider the class of SVN-ultrafilters, first proving that it is not empty and then establishing some characterizations and properties that we think may be useful for further investigations.

Definition 4.1. Let $\mathcal{U}$ be a SVN-filter base on $\mathcal{SVN}(\mathbb{U})$, we say that it is a ***SVN-ultrafilter*** if it is maximal in the partial ordered set $(\mathfrak{F}(\mathbb{U}), \subseteq)$ of all SVN-filters over $\mathbb{U}$, that is if there is no SVN-filter on $\mathcal{SVN}(\mathbb{U})$ strictly finer than $\mathcal{U}$, or, equivalently, if any other SVN-filter containing $\mathcal{U}$ coincides with $\mathcal{U}$.

Proposition 4.2. *Every SVN-filter base on $\mathcal{SVN}(\mathbb{U})$ is contained in some SVN-ultrafilter over $\mathbb{U}$.*

Proof. Let $\mathcal{F}$ be a SVN-filter base over $\mathbb{U}$ and let us consider the set $\mathfrak{S} = \{\mathcal{G} \in \mathfrak{F}(\mathbb{U}) : \mathcal{F} \subseteq \mathcal{G}\}$ of all SVN-filters on $\mathcal{SVN}(\mathbb{U})$ containing $\mathcal{F}$. Obviously $\mathfrak{S}$ is a nonempty subset of the poset $(\mathfrak{F}(\mathbb{U}), \subseteq)$ because $\langle\mathcal{F}\rangle \in \mathfrak{S}$. Now, let $\mathfrak{C}$ be a nonempty chain of $\mathfrak{S}$ and define $\mathcal{M} = \bigcup_{\mathcal{G}\in\mathfrak{C}} \mathcal{G}$ as the union of all SVN-filters of $\mathfrak{C}$. Now, we have that $\mathcal{M}$ is a SVN-filter over $\mathbb{U}$ because it satisfies all the conditions of Definitions 3.3 and 3.2, that is:

(i) For every $\widetilde{G} \in \mathcal{M}$, we have that there exists some $\mathcal{G} \in \mathfrak{C}$ such that $\widetilde{G} \in \mathcal{G}$ and since $\mathcal{G}$ is a SVN-filter, we immediately have that $\widetilde{G} \not\equiv \widetilde{\emptyset}$.

(ii) For every $\widetilde{G}_1, \widetilde{G}_2 \in \mathcal{M}$, there are some $\mathcal{G}_1, \mathcal{G}_2 \in \mathfrak{C}$ such that $\widetilde{G}_1 \in \mathcal{G}_1$ and $\widetilde{G}_2 \in \mathcal{G}_2$. Since $\mathfrak{C}$ is a chain, i.e. a totally ordered subset of $(\mathfrak{S}, \subseteq)$, $\mathcal{G}_1$ and $\mathcal{G}_2$ must be comparable and, without loss of generality, we can suppose that $\mathcal{G}_1 \subseteq \mathcal{G}_2$. Thus, $\widetilde{G}_1, \widetilde{G}_2 \in \mathcal{G}_2$ and since $\mathcal{G}_2$ is a SVN-filter, there exists some $\widetilde{G}_3 \in \mathcal{G}_2$ such that $\widetilde{G}_3 \Subset \widetilde{G}_1 \Cap \widetilde{G}_2$ with $\widetilde{G}_3 \in \mathcal{G}_2 \subseteq \bigcup_{\mathcal{G}\in\mathfrak{C}} \mathcal{G} = \mathcal{M}$.

(iii) For every $\widetilde{G} \in \mathcal{M}$, and $\widetilde{A} \in \mathcal{SVN}(\mathbb{U})$ such that $\widetilde{G} \Subset \widetilde{A}$, there exists some $\mathcal{G} \in \mathfrak{C}$ such that $\widetilde{G} \in \mathcal{G}$ and since $\mathcal{G}$ is a SVN-filter, we immediately have that also $\widetilde{A} \in \mathcal{G}$ and hence that $\widetilde{A} \in \mathcal{G} \subseteq \bigcup_{\mathcal{G}\in\mathfrak{C}} \mathcal{G} = \mathcal{M}$.

Moreover, being $\mathcal{F} \subseteq \mathcal{G}$, for every $\mathcal{G} \in \mathfrak{C}$, we have that $\mathcal{F} \subseteq \bigcup_{\mathcal{G}\in\mathfrak{C}} \mathcal{G} = \mathcal{M}$ and so that $\mathcal{M} \in \mathfrak{S}$ is an upper bound for $\mathfrak{C}$. Hence, by the Zorn's Lemma (see [11]), it follows that $(\mathfrak{S}, \subseteq)$ has a maximal element, that is a SVN-ultrafilter $\mathcal{U}$ containing $\mathcal{F}$. □

Proposition 4.3. *Let $\mathcal{U}$ be a SVN-filter base on $\mathcal{SVN}(\mathbb{U})$. Then $\mathcal{U}$ is a SVN-ultrafilter over $\mathbb{U}$ if and only if for every $\widetilde{A} = \langle\mathbb{U}, \mu_A, \sigma_A, \omega_A\rangle \in \mathcal{SVN}(\mathbb{U})$ which neutrosophically meets $\mathcal{U}$ we have that $\widetilde{A} \in \mathcal{U}$.*

Proof. Suppose that $\mathcal{U}$ is a SVN-ultrafilter and consider a SVN-set $\widetilde{A} = \langle\mathbb{U}, \mu_A, \sigma_A, \omega_A\rangle \in \mathcal{SVN}(\mathbb{U})$ which neutrosophically meets $\mathcal{U}$, By Proposition 3.22, we have that $\mathcal{U} \vee \widetilde{A}$ is a SVN-filter such that $\mathcal{U} \subseteq \mathcal{U} \vee \widetilde{A}$ but, being $\mathcal{U}$ a SVN-ultrafilter, it must necessarily follow that $\mathcal{U} \vee \widetilde{A} = \mathcal{U}$ and so that $\widetilde{A} \in \mathcal{U}$.
Conversely, suppose that every SVN-set which neutrosophically meets the SVN-filter base $\mathcal{U}$ belongs to $\mathcal{U}$. In

order to prove first that $\mathcal{U}$ is a SVN-filter over $\mathbb{U}$, let $\widetilde{U} \in \mathcal{U}$ and $\widetilde{A} \in \mathcal{SVN}(\mathbb{U})$ such that $\widetilde{U} \Subset \widetilde{A}$. We claim that $\widetilde{A}$ meets $\mathcal{U}$. In fact, for each $\widetilde{V} \in \mathcal{U}$, since $\mathcal{U}$ is a SVN-filter base, we have that there exist some $\widetilde{W} \in \mathcal{U}$ such that $\widetilde{W} \Subset \widetilde{U} \Cap \widetilde{V} \Subset \widetilde{U} \Subset \widetilde{A}$ and, by Proposition 2.21(2), we have that $\widetilde{W} \Cap \widetilde{V} \Subset \widetilde{A} \Cap \widetilde{V}$. On the other hand, being also $\widetilde{W} \Subset \widetilde{U} \Cap \widetilde{V} \Subset \widetilde{V}$, by Proposition 2.19(1), we have $\widetilde{W} \Cap \widetilde{V} \equiv \widetilde{W}$ and hence that $\widetilde{W} \Subset \widetilde{A} \Cap \widetilde{V}$, with $\widetilde{W} \in \mathcal{U}$. Thus, by Definition 3.2, it follows that $\widetilde{A} \Cap \widetilde{V} \not\equiv \widetilde{\emptyset}$ and, by hypothesis, we obtain that $\widetilde{A} \in \mathcal{U}$, which proves that $\mathcal{U}$ is a SVN-filter on $\mathcal{SVN}(\mathbb{U})$.
Moreover, in order to prove that $\mathcal{U}$ is SVN-ultrafilter over $\mathbb{U}$, suppose, by contradiction, that there is some SVN-filter $\mathcal{M}$ over $\mathbb{U}$ such that $\mathcal{U} \subset \mathcal{M}$ and so that there exists some $\widetilde{M} \in \mathcal{M}$ such that $\widetilde{M} \notin \mathcal{U}$. Hence, by hypothesis, we have that $\widetilde{M}$ does not meet $\mathcal{U}$, i.e. that there exists some $\widetilde{U} \in \mathcal{U}$ such that $\widetilde{U} \Cap \widetilde{M} \equiv \widetilde{\emptyset}$ but, being $\widetilde{M} \in \mathcal{M}$ and $\widetilde{U} \in \mathcal{U} \subset \mathcal{M}$, this contradicts the fact that $\mathcal{M}$ is a SVN-filter and concludes our proof. □

Corollary 4.4. *If $\mathcal{U}$ is a SVN-ultraffilter on $\mathcal{SVN}(\mathbb{U})$, then for every $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle \in \mathcal{SVN}(\mathbb{U})$ it results $\widetilde{A} \in \mathcal{U}$ or $\widetilde{A}^{\complement} \in \mathcal{U}$.*

Proof. Let $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle \in \mathcal{SVN}(\mathbb{U})$ an suppose, by contradiction, that $\widetilde{A} \notin \mathcal{U}$ and $\widetilde{A}^{\complement} \notin \mathcal{U}$. By Proposition 4.3, we should have that neither $\widetilde{A}$ does not neutrosophically meet $\mathcal{U}$ nor $\widetilde{A}^{\complement}$ does not neutrosophically meet $\mathcal{U}$, i.e. that there are some $\widetilde{U}_1, \widetilde{U}_2 \in \mathcal{U}$ such that $\widetilde{U}_1 \Cap \widetilde{A} \equiv \widetilde{\emptyset}$ and $\widetilde{U}_2 \Cap \widetilde{A}^{\complement} \equiv \widetilde{\emptyset}$. Since $\mathcal{U}$ is a SVN-filter base, it follows that there exists some $\widetilde{U}_3 \in \mathcal{U}$ such that $\widetilde{U}_3 \Subset \widetilde{U}_1 \Cap \widetilde{U}_2$. Hence, by Proposition 2.22, we have that $\widetilde{U}_3 \Subset \widetilde{U}_1$ and $\widetilde{U}_3 \Subset \widetilde{U}_2$ and, by Proposition 2.21(2), it also follows that $\widetilde{U}_3 \Cap \widetilde{A} \Subset \widetilde{U}_1 \Cap \widetilde{A}$ and $\widetilde{U}_3 \Cap \widetilde{A}^{\complement} \Subset \widetilde{U}_2 \Cap \widetilde{A}^{\complement}$. Hence, $\widetilde{U}_3 \Cap \widetilde{A} \equiv \widetilde{\emptyset}$ and $\widetilde{U}_3 \Cap \widetilde{A}^{\complement} \equiv \widetilde{\emptyset}$. Thus, by Propositions 2.16 (4), 2.20 and 2.23(1),we have that $\widetilde{U}_3 \equiv \widetilde{U}_3 \Cap \widetilde{U}_3 \equiv \left(\widetilde{U}_3 \Cup \left(\widetilde{U}_3 \Cap \widetilde{A}\right)\right) \Cap \left(\widetilde{U}_3 \Cup \left(\widetilde{U}_3 \Cap \widetilde{A}^{\complement}\right)\right) \equiv \widetilde{U}_3 \Cap \left(\left(\widetilde{U}_3 \Cap \widetilde{A}\right) \Cup \left(\widetilde{U}_3 \Cap \widetilde{A}^{\complement}\right)\right) \equiv \widetilde{U}_3 \Cap \left(\widetilde{\emptyset} \Cup \widetilde{\emptyset}\right) \equiv \widetilde{\emptyset}$ which is a contradiction to the fact that $\widetilde{U}_3 \in \mathcal{U}$ and $\mathcal{U}$ is a SVN-filter. □

Remark 4.5. In the classical filter's theory on crisp sets, the condition of Corollary 4.4 is a ultrafilters characterization, but in the case of filters on single valued neutrosophic sets the converse does not hold. This is due to the fact that, as pointed out in Remark 2.10, in general, the neutrosophic intersection of a SVN-set with its neutrosophic complement is not the neutrosophic empty set, and it can be confirmed by the following example.

Example 4.6. Let $\mathbb{U} = \{a, b, c\}$ be a finite universe set and consider the SVN-principal filter $\mathcal{F} = \left\langle \widetilde{A} \right\rangle = \left\{\widetilde{A}, \widetilde{B}, \widetilde{C}, \widetilde{\mathbb{U}}\right\}$ generated by $\widetilde{A}$, where the SVN-sets $\widetilde{A} = \langle \mathbb{U}, \mu_A, \sigma_A, \omega_A \rangle$, $\widetilde{B} = \langle \mathbb{U}, \mu_B, \sigma_B, \omega_B \rangle$, $\widetilde{C} = \langle \mathbb{U}, \mu_C, \sigma_C, \omega_C \rangle$ and $\widetilde{\mathbb{U}} = \langle \mathbb{U}, \underline{1}, \underline{1}, \underline{0} \rangle$ are respectively defined by the following tabular representations:

$\mathbb{U}$ \ $\widetilde{A}$	μ_A	σ_A	ω_A
a	0.8	0.4	0
b	0	0.1	0.9
b	0	0	1

$\mathbb{U}$ \ $\widetilde{B}$	μ_B	σ_B	ω_B
a	0.9	0.5	0
b	0.8	0.6	0.1
c	0	0.2	0.3

$\mathbb{U}$ \ $\widetilde{C}$	μ_C	σ_C	ω_C
a	1	0.5	0
b	0	0.2	0.8
c	0.7	0.6	0.5

$\mathbb{U}$ \ $\widetilde{\mathbb{U}}$	$\mu_{\mathbb{U}}$	$\sigma_{\mathbb{U}}$	$\omega_{\mathbb{U}}$
a	1	1	0
b	1	1	0
c	1	1	0

After observing that $\widetilde{A} \Subset \widetilde{B}$ and $\widetilde{A} \Subset \widetilde{C}$, one can easily check that $\left\langle \widetilde{A} \right\rangle$ is a SVN-ultrafilter on $\mathcal{SVN}(\mathbb{U})$. However, if we consider the SVN-set $\widetilde{Z} = \langle \mathbb{U}, \mu_Z, \sigma_Z, \omega_Z \rangle$ defined by the following tabular representation:

$\mathbb{U}$ \ $\widetilde{Z}$	μ_Z	σ_Z	ω_Z
a	0	0	1
b	0.7	0.3	0.5
c	0.8	0.4	0.6

it is a trivial matter to verify that neither $\widetilde{Z}$ nor its complement $\widetilde{Z}^{\complement}$ belong to $\left\langle \widetilde{A} \right\rangle$.

5 Conclusions and Perspectives

In this paper we have introduced the notions of SVN-filter base, SVN-filter and SVN-ultrafilter on the set $\mathcal{SVN}(\mathbb{U})$ of all the single valued neutrosophic sets and we have investigated some of their fundamental properties and relationships concerning, in particular, the SVN-filter and SVN-filter base generated by some neutrosophic filter subbase, the neutrosophic filter completion, the principal SVN-filters, the infimum and the supremum of two SVN-filter bases, the image of a SVN-filter base by a neutrosophic induced mapping between two universe sets, the existence of SVN-ultrafilters and some their characterizations.
We expect to continue the research on these topics by investigating supplementary features and properties related to SVN-filters and we hope that this comprehensive study will stimulate further developments in the theory of Neutrosophic Sets and will provide useful tools to demonstrate in an elegant and concise way new properties for the class of single valued neutrosophic topological spaces.

Acknowledgements The authors would like to express their sincere gratitude to the Anonymous Referees for their valuable suggestions and comments which were precious in improving of the paper.

References

[1] Al-Omeri W., Jafari S. On Generalized Closed Sets and Generalized Pre-Closed Sets in Neutrosophic Topological Spaces. *Mathematics* **7** (1), pp. 1-12. 2019.

[2] Atanassov K.T. Intuitionistic Fuzzy Sets. *Fuzzy Sets and Systems* **20** (1), pp. 87-96, 1986.

[3] Bera T., Mahapatra N.K. Introduction to neutrosophic soft topological spaces, *Opsearch* (March, 2017), DOI: 10.1007/s12597-017-0308-7.

[4] Bera T., Mahapatra N.K. On neutrosophic soft topological space. *Neutrosophic Sets and Systems* **19** (1), pp. 3-15, 2018.

[5] Bourbaki N. .*General Topology*. Springer Science and Business Media, 2020.

[6] Broumi S., Talea M., Bakali A., Smarandache F. Single valued neutrosophic graphs. *Journal of New theory* **10**, pp. 86-101, 2016.

[7] Cartan H. Théorie des filtres. *Comptes Rendus de l'Académie des Sciences* **205**, pp. 595-598, 1937.

[8] Chang C.L. Fuzzy Topological Spaces. *Journal of Mathematical Analysis and Applications* **24**, pp. 182-190, 1968.

[9] Coker D. An Introduction to Intuitionistic Fuzzy Topological Spaces. *Fuzzy Sets and Systems* **88**, pp. 81-89, 1997.

[10] Engelking R. *General Topology*. Berlin: Heldermann Verlag, 1989.

[11] Gemignani M.C. *Elementary Topology*. New York: Dover Publications Inc., 1990.

[12] Latreche A., Barkat O., Milles S., Ismail F. Single valued neutrosophic mappings defined by single valued neutrosophic relations with applications. *Neutrosophic Sets and Systems* **1**, pp. 203-220, 2020.

[13] Mehmood A., Nordo G., Zamir M., Park C., Nazia H., Nadeem F., Shamona J. Soft b-separation axioms in neutrosophic soft topological structures. *Annals of Fuzzy Mathematics and Informatics* **18**, pp. 93–105, 2019.

[14] Mehmood A., Nadeem F., Nordo G., Zamir M., Park C., Kalsoom H., Jabeen S., Khan M.Y. Generalized neutrosophic separation axioms in neutrosophic soft topological spaces. *Neutrosophic Sets and Systems* **32** (1), pp. 38–51, 2020.

[15] Parimala M., Karthika M., Smarandache F., Broumi S., On $\alpha\omega$-closed sets and itsconnectedness in terms of neutrosophic topological spaces, *International Journal of Neutrosophic Science* **2** (2), pp. 82-88, 2020.

[16] Saber Y., Alsharari F., Smarandache F. On Single-Valued Neutrosophic Ideals in Şostak Sense. *Symmetry* **12** (2):193, 2020.

[17] Salama A.A., Alblowi S.A. Neutrosophic Set and Neutrosophic Topological Spaces. *ISOR Journal of Mathematics* **3** (4), pp. 31-35, 2012.

[18] Salama A.A., Alagamy H. Neutrosophic Filters. *International Journal of Computer Science Engineering and Information Technologt Research* **3** (1), pp. 307-312, 2013.

[19] Salama A.A., Smarandache F., Kromov V. Neutrosophic Closed Set and Neutrosophic Continuous Functions, *Neutrosophic Sets and Systems* **4**, pp- 4-8, 2014.

[20] Smarandache F. A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic. Rehoboth: American Research Press, 1999.

[21] Wang H., Smarandache F., Zhang Y.Q., Sunderraman R. Single Valued Neutrosophic Sets. *Technical Sciences and Applied Mathematics*, pp. 10-14, 2012.

[22] Zadeh L.A. Fuzzy Sets, *Information and Control* **8** (3), pp. 338-353, 1965.

Classical Logic as a subclass of Neutrosophic Logic

[1]Angelo de Oliveira, [2]Marina Nogueira Carvalho de Oliveira
[1]Departamento Acadêmico de Ciência da Computação - Universidade Federal de Rondônia - Brasil
[2]Undergraduate Student in Civil Engineering - Universidade Tecnológica Federal do Paraná - Brasil
angelo@unir.br[1], marina.nogueira.co@gmail.com.br[2]

Abstract

It is customary in mathematics that almost all new developments maintain compatibility with what is already proved and accepted. Following this way, neutrosophic logic has the classical logic as subset. However, in mathematics, all the affirmations must be proved first to be accepted, so the claim that the neutrosophic logic encompass classical logic must be also proved. Thus, this paper show that the main properties of the classical logic hold when translated to neutrosophic form at propositional level.

Keywords: Neutrosophic Logic; Classical Logic.

1 Introduction

Neutrosophic Logic (NL) is a recent logic born in the middle of the nineties by Florentin Smarandache,[1] it is characterized by having another component added to the common (standard) components of any logic, namely, truth and falsity. This added component is called indeterminacy.

Formally, a propositional neutrosophic logic variable x is represented by the triple

$$x = (t, i, f)$$

where t is the degree of truth, i is the degree of indeterminacy and f is the degree of falsity.

In this way, neutrosophic logic clearly embodies indeterminacy as component that must be treated in its framework, while other logics this element is not explicit (though sometimes taken in account).

In a general way, the definition of the components representing the truth, indeterminacy and falsity are sets. Thus, the neutrosophic logic can be considered as (and indeed it is) a multivalued logic. Now it is time to present the formal definition.

Definition 1.1. Let T, I, F be sets (real standard or non-standard) in the non-standard unity interval $]^{-}0, 1^{+}[$, with

$$\begin{aligned} supT &= t\_sup, infT = t\_inf \\ supI &= i\_sup, infI = i\_inf \\ supF &= f\_sup, infF = f\_inf \end{aligned}$$

and

$$\begin{aligned} n\_sup &= t\_sup + i\_sup + f\_sup \\ n\_inf &= t\_inf + i\_inf + f\_inf \end{aligned}$$

Doi :10.5281/zenodo.3830094

Received: February 14, 2020 Revised: April15, 2020 Accepted: May 10, 2020

then the triplet $X = (T, I, F)$ represents the values of the truth (T), indeterminacy (I) and falsity (F) with respect of the variable X. The sets T, I and F are called neutrosophic components.

Additionally, the following restrictions must hold:

$$n\_sup = t\_sup + i\_sup + f\_sup \leq 3^+$$
$$n\_inf = t\_inf + i\_inf + f\_inf \geq^- 0$$

The use of real non-standard mathematics is just one of many topics that makes *neutrosophic logic* different from others types of logics. Here, should also be noted that the use of non-standard values allows the *neutrosophic logic* to distinguish between *absolute truth* (truth in all possible worlds, represented by 1^+) and *relative truth* (truth in one or just a few worlds, represented by 1). The same applies for *absolute falsity* (represented by $^-0$) and *relative falsity* (represented by 0).

The definition 1.1 is so general that the set (or subsets) does not necessarily have to be intervals, they can be any type of sets (discrete, continuous, open or closed or half-open/half-closed intervals, intersections or unions of the previous sets, etc.) in accordance with the given proposition. Also, a subset may have one element only in special cases of this logic.

The degree of generality is such that every time that someone wish to apply the neutrosophic logic to any specific matter, the operators and their meanings must be first defined. Besides, the most common case of neutrosophic logic use is real intervals or points in the real unitary interval.

Not being viable to list all possible nuances, it is time show how the neutrosophic logic encompass the classical logic.

2 Neutrosophic Logic and Classical Logic

Classical logic (or crisp logic) is nowadays a branch of mathematical logic which roots dated before Christ birth, being definitely developed from centuries XIX and XX from the works of Boole, Frege, Russell and Whitehead, just to name a few.

What constitutes a main characteristic of the classical logic is its bivalence, just two absolute values are considered, V (verum) for truth and F (falsum) for falsity, that can be conveniently substituted by 1 and 0 (or any two different signs). Thus, in the framework of the classical logic, indeterminacy cannot be considered (here is not being considered variants of the classical logic developed from the beginning of the XX century, such as modal logic, temporal logic and alike).

Now, let $V = (1, 0, 0)$ represent the truth value in classical logic, now represented in neutrosophic form. Also, let $F = (0, 0, 1)$ represent the falsity value. With these two values, it is easy to see how one can be the negation of the other, being enough to define the negation operator as:

$$\neg x = \neg(t, 0, f) = (f, 0, t)$$

thus, $\neg \underbrace{(1,0,0)}_{V} = \underbrace{(0,0,1)}_{F}$ and $\neg \underbrace{(0,0,1)}_{F} = \underbrace{(1,0,0)}_{V}$.

Likewise, if the operators $\vee$ (disjunction) and $\wedge$ (conjunction) are defined as:

$$(t_1, 0, f_1) \vee (t_2, 0, f_2) = \{\max\{t_1, t_2\}, 0, \min\{f_1, f_2\})$$
$$(t_1, 0, f_1) \wedge (t_2, 0, f_2) = \{\min\{t_1, t_2\}, 0, \max\{f_1, f_2\})$$

then, those operators have the same behavior of their classical counterparts, being given by the following table:

Table 1: Disjunction and conjunction operators in NL with behavior of the classical ones.

p_1	p_2	$\vee$	$\wedge$
$(1,0,0)$	$(1,0,0)$	$(1,0,0)$	$(1,0,0)$
$(1,0,0)$	$(0,0,1)$	$(1,0,0)$	$(0,0,1)$
$(0,0,1)$	$(1,0,0)$	$(1,0,0)$	$(0,0,1)$
$(0,0,1)$	$(0,0,1)$	$(0,0,1)$	$(0,0,1)$

In classical logic, the set operators $CS = \{\neg, \vee, \wedge\}$ is said to be complete, that is, all the other operators in classical logic can be defined based in these operators. However, someone must be careful if he/she wishes that the operators and properties in classical logic holds in NL. The properties that remain true with the already given definitions of negation, conjunction and disjunction are next given.

Theorem 2.1 (Double Negation). *Let $x = (t, 0, f)$ be a neutrosophic logical variable representing a classical logic variable. The double negation propriety remains true, that is*

$$\neg\neg x = x$$

Proof.

$$\neg\neg x = \neg(\neg x) = \neg(\neg(t, 0, f)) = \neg(f, 0, t) = (t, 0, f)$$

□

Theorem 2.2 (Commutativity). *Let $x = (t_1, 0, f_1)$ and $y = (t_2, 0, f_2)$ be neutrosophic logic variables representing classical logic variables. The operators $\vee$ and $\wedge$ are commutative.*

Proof.

$$\begin{aligned} x \vee y = (t_1, 0, f_1) \vee (t_2, 0, f_2) = (\max\{t_1, t_2\}, 0, \min\{f_1, f_2\}) = \\ (\max\{t_2, t_1\}, 0, \min\{f_2, f_1\}) = (t_2, 0, f_2) \vee (t_1, 0, f_1) = y \vee x \end{aligned}$$

$$\begin{aligned} x \wedge y = (t_1, 0, f_1) \wedge (t_2, 0, f_2) = (\min\{t_1, t_2\}, 0, \max\{f_1, f_2\}) = \\ (\min\{t_2, t_1\}, 0, \max\{f_2, f_1\}) = (t_2, 0, f_2) \wedge (t_1, 0, f_1) = y \wedge x \end{aligned}$$

□

Theorem 2.3 (Associativity). *Let $x = (t_1, 0, f_1)$, $y = (t_2, 0, f_2)$ and $z = (t_3, 0, f_3)$ be neutrosophic logic variables representing classical logic variables. The operators $\vee$ and $\wedge$ are associative.*

Proof.

$$\begin{aligned} x \vee (y \vee z) = (t_1, 0, f_1) \vee ((t_2, 0, f_2) \vee (t_3, 0, f_3)) = (t_1, 0, f_1) \vee (\max\{t_2, t_3\}, 0, \min\{f_2, f_3)\}) = \\ (\max\{t_1, \max\{t_2, t_3\}\}, 0, \min\{f_1, \min\{f_2, f_3\}\}) = (\{\max\{t_1, t_2, t_3\}, 0, \min\{f_1, f_2, f_3\}) = \\ (\max\{\max\{t_1, t_2\}, t_3\}, 0, \min\{\min\{f_1, f_2\}, f_3\}) = (\max\{t_1, t_2\}, 0, \min\{f_1, f_2\}) \vee (t_3, 0, f_3) = \\ ((t_1, 0, f_1) \vee (t_2, 0, f_3)) \vee (t_3, 0, f_3) = (x \vee y) \vee z \end{aligned}$$

$$\begin{aligned} x \wedge (y \wedge z) = (t_1, 0, f_1) \wedge ((t_2, 0, f_2) \wedge (t_3, 0, f_3)) = (t_1, 0, f_1) \wedge (\min\{t_2, t_3\}, 0, \max\{f_2, f_3)\}) = \\ (\min\{t_1, \min\{t_2, t_3\}\}, 0, \max\{f_1, \max\{f_2, f_3\}\}) = (\{\min\{t_1, t_2, t_3\}, 0, \max\{f_1, f_2, f_3\}) = \\ (\min\{\min\{t_1, t_2\}, t_3\}, 0, \max\{\max\{f_1, f_2\}, f_3\}) = (\min\{t_1, t_2\}, 0, \max\{f_1, f_2\}) \wedge (t_3, 0, f_3) = \\ ((t_1, 0, f_1) \wedge (t_2, 0, f_3)) \wedge (t_3, 0, f_3) = (x \wedge y) \wedge z \end{aligned}$$

□

Theorem 2.4 (Distributivity). *Let $x = (t_1, 0, f_1)$, $y = (t_2, 0, f_2)$ and $z = (t_3, 0, f_3)$ be neutrosophic logic variables representing classical logic variables. The operators $\vee$ and $\wedge$ are distributive.*

Proof. At first, this property does not seem to be valid owing to the fact that when it is translated into the definitions of the operators in NL, the left and right side yields different formulas.

For $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$:

$$\begin{aligned} x \vee (y \wedge v) = x \vee ((t_2, 0, f_2) \wedge (t_3, 0, f_3)) = x \vee (\min\{t_2, t_3\}, 0, \max\{f_2, f_3\}) = \\ (t_1, 0, f_1) \vee (\min\{t_2, t_3\}, 0, \max\{f_2, f_3\}) = \\ (\max\{t_1, \min\{t_2, t_3\}\}, 0, \min\{f_1, \max\{f_2, f_3\}\}) \end{aligned}$$

and

$$\begin{aligned} (x \vee y) \wedge (x \vee z) = ((t_1, 0, f_1) \vee (t_2, 0, f_2)) \wedge ((t_1, 0, f_1) \vee (t_3, 0, f_3)) = \\ (\max\{t_1, t_2\}, 0, \min\{f_1, f_2\}) \wedge (\max\{t_1, t_3\}, 0, \min\{f_1, f_3\}) = \\ (\min\{\max\{t_1, t_2\}, \max\{t_1, t_3\}\}, 0, \max\{\min\{f_1, f_2\}, \min\{f_1, f_3\}\}) \end{aligned}$$

For $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$:

$$\begin{aligned} x \wedge (y \vee z) = x \wedge ((t_2, 0, f_2) \vee (t_3, 0, f_3)) = x \wedge (\max\{t_2, t_3\}, 0, \min\{f_2, f_3\}) = \\ (t_1, 0, f_1) \wedge (\max\{t_2, t_3\}, 0, \min\{f_2, f_3\}) = \\ (\min\{t_1, \max\{t_2, t_3\}\}, 0, \max\{f_1, \min\{f_2, f_3\}\}) \end{aligned}$$

and

$$\begin{aligned} (x \wedge y) \vee (x \wedge z) = ((t_1, 0, f_1) \wedge (t_2, 0, f_2)) \vee ((t_1, 0, f_1) \wedge (t_3, 0, f_3)) = \\ (\min\{t_1, t_2\}, 0, \max\{f_1, f_2\}) \vee (\min\{t_1, t_3\}, 0, \max\{f_1, f_3\}) = \\ (\max\{\min\{t_1, t_2\}, \min\{t_1, t_3\}\}, 0, \min\{\max\{f_1, f_2\}, \max\{f_1, f_3\}\}) \end{aligned}$$

But now, checking the truth tables for $x \vee (y \wedge z)$ (Table 2) and $(x \vee y) \wedge (x \vee z)$ (Table 3), can be seen that both of the sides yields the same result.

Table 2: Truth table for $x \vee (y \wedge z)$.

x	y	z	$\max\{t_1, \min\{t_2, t_3\}\}$	$\min\{f_1, \max\{f_2, f_3\}\}$	Result
(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	1	0	(1, 0, 0)
(1, 0, 0)	(1, 0, 0)	(0, 0, 1)	1	0	(1, 0, 0)
(1, 0, 0)	(0, 0, 1)	(1, 0, 0)	1	0	(1, 0, 0)
(1, 0, 0)	(0, 0, 1)	(0, 0, 1)	1	0	(1, 0, 0)
(0, 0, 1)	(1, 0, 0)	(1, 0, 0)	1	0	(1, 0, 0)
(0, 0, 1)	(1, 0, 0)	(0, 0, 1)	0	1	(0, 0, 1)
(0, 0, 1)	(0, 0, 1)	(1, 0, 0)	0	1	(0, 0, 1)
(0, 0, 1)	(0, 0, 1)	(0, 0, 1)	0	1	(0, 0, 1)

Table 3: Truth table for $(x \vee y) \wedge (x \vee z)$.

x	y	z	$\min\{\max\{t_1, t_2\}, \max\{t_1, t_3\}\}$	$\max\{\min\{f1, f2\}, \min\{f_1, f_3\}\}$	Result
(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	1	0	(1, 0, 0)
(1, 0, 0)	(1, 0, 0)	(0, 0, 1)	1	0	(1, 0, 0)
(1, 0, 0)	(0, 0, 1)	(1, 0, 0)	1	0	(1, 0, 0)
(1, 0, 0)	(0, 0, 1)	(0, 0, 1)	1	0	(1, 0, 0)
(0, 0, 1)	(1, 0, 0)	(1, 0, 0)	1	0	(1, 0, 0)
(0, 0, 1)	(1, 0, 0)	(0, 0, 1)	0	1	(0, 0, 1)
(0, 0, 1)	(0, 0, 1)	(1, 0, 0)	0	1	(0, 0, 1)
(0, 0, 1)	(0, 0, 1)	(0, 0, 1)	0	1	(0, 0, 1)

Similarly, the same occurs for $x \wedge (y \vee z)$ (Table 4) and $(x \wedge y) \vee (x \wedge z)$ (Table 5).

Table 4: Truth table for $x \wedge (y \vee z)$.

x	y	z	$\min\{t_1, \max\{t_2, t_3\}\}$	$\max\{f_1, \min\{f_2, f_3\}\}$	Result
(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	1	0	(1, 0, 0)
(1, 0, 0)	(1, 0, 0)	(0, 0, 1)	1	0	(1, 0, 0)
(1, 0, 0)	(0, 0, 1)	(1, 0, 0)	1	0	(1, 0, 0)
(1, 0, 0)	(0, 0, 1)	(0, 0, 1)	0	1	(0, 0, 1)
(0, 0, 1)	(1, 0, 0)	(1, 0, 0)	0	1	(0, 0, 1)
(0, 0, 1)	(1, 0, 0)	(0, 0, 1)	0	1	(0, 0, 1)
(0, 0, 1)	(0, 0, 1)	(1, 0, 0)	0	1	(0, 0, 1)
(0, 0, 1)	(0, 0, 1)	(0, 0, 1)	0	1	(0, 0, 1)

Table 5: Truth table for $(x \wedge y) \vee (x \wedge z)$.

x	y	z	$\min\{\max\{t_1, t_2\}, \max\{t_1, t_3\}\}$	$\max\{\min\{f_1, f_2\}, \min\{f_1, f_3\}\}$	Result
(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	1	0	(1, 0, 0)
(1, 0, 0)	(1, 0, 0)	(0, 0, 1)	1	0	(1, 0, 0)
(1, 0, 0)	(0, 0, 1)	(1, 0, 0)	1	0	(1, 0, 0)
(1, 0, 0)	(0, 0, 1)	(0, 0, 1)	0	1	(0, 0, 1)
(0, 0, 1)	(1, 0, 0)	(1, 0, 0)	0	1	(0, 0, 1)
(0, 0, 1)	(1, 0, 0)	(0, 0, 1)	0	1	(0, 0, 1)
(0, 0, 1)	(0, 0, 1)	(1, 0, 0)	0	1	(0, 0, 1)
(0, 0, 1)	(0, 0, 1)	(0, 0, 1)	0	1	(0, 0, 1)

□

Theorem 2.5 (De Morgan's Laws). *Let $x = (t_1, 0, f_1)$ and $y = (t_2, 0, f_2)$ be neutrosophic logic variables representing classical logic variables. Then, it is true that $\neg(x \vee y) = \neg x \wedge \neg y$ and $\neg(x \wedge y) = \neg x \vee \neg y$, that is, the De Morgan's Laws remain valid.*

Proof.

$$\begin{aligned}\neg(x \vee y) &= \neg((t_1, 0, f_1) \vee (t_2, 0, f_2)) = \neg(\max\{t_1, t_2\}, 0, \min\{f_1, f_2\}) \\ &= (\min\{f_1, f_2\}, 0, \max\{t_1, t_2\}) = (f_1, 0, t_1) \wedge (f_2, 0, t_2) \\ &= \neg(t_1, 0, f_1) \wedge \neg(t_2, 0, f_2) = \neg x \wedge \neg y\end{aligned}$$

$$\begin{aligned}\neg(x \wedge y) &= \neg((t_1, 0, f_1) \wedge (t_2, 0, f_2)) = \neg(\min\{t_1, t_2\}, 0, \max\{f_1, f_2\}) \\ &= (\max\{f_1, f_2\}, 0, \min\{t_1, t_2\}) = (f_1, 0, t_1) \vee (f_2, 0, t_2) \\ &= \neg(t_1, 0, f_1) \vee \neg(t_2, 0, f_2) = \neg x \vee \neg y\end{aligned}$$

□

Definition 2.6 (Tautology). A tautology in NL representing a classical logic variable is an expression that is always equal to $(1, 0, 0)$, independent of the values of its components. A tautology may be represented by τ.

Definition 2.7 (Contradiction). A contradiction in NL representing a classical logic variable is an expression that is always equal to $(0, 0, 1)$, independent of the values of its components. A contradiction may be represented by γ.

Theorem 2.8 (Law of Noncontradiction). *Let $x = (t_1, 0, f_1)$ be a neutrosophic logic variable representing a classical logic variable. The Law of Noncontradicition, that is, $\neg(x \wedge \neg x)$, remains valid, being a tautology.*

Proof.

$$\begin{aligned}\neg(x \wedge \neg x) &= \neg((t_1, 0, f_1) \wedge \neg(t_1, 0, f_1)) = \neg((t_1, 0, f_1) \wedge (f_1, 0, t_1)) \\ &= \neg(min\{t_1, f_1\}, 0, \max\{f_1, t_1\}) = (\max\{f_1, t_1\}, 0, min\{t_1, f_1\})\end{aligned}$$

The result above is always equal to $(1, 0, 0)$. To comprehend this, consider that x is $(1, 0, 0)$ or $(0, 0, 1)$, exclusively. So, $t_1 = 1$ and $f_1 = 0$, or $t_1 = 0$ and $f_1 = 1$, such $\max\{f_1, t_1\}$ is always equal to 1 and $min\{t_1, f_1\}$ equal to 0. The following table indicates this as well.

Table 6: Table for the noncontradiction law.

x	$\neg x$	$\max\{f_1, t_1\}$	$min\{t_1, f_1\}$	Result
(1, 0, 0)	(0, 0, 1)	1	0	(1, 0, 0)
(0, 0, 1)	(1, 0, 0)	1	0	(1, 0, 0)

Thus, $\neg(x \wedge \neg x) = \tau$. □

Remark 2.9. The Law of Noncontradiction can be summarized in a few words: *it's false that one proposition can be true and false simultaneously*.

Theorem 2.10 (Law of Excluded Middle)**.** *Let $x = (t_1, 0, f_1)$ be a neutrosophic logic variable representing a classical logic variable. The Law of Excluded Middle (*tertium non datur*), that is, $x \vee \neg x$, remains valid, being a tautology.*

Proof.

$$(x \vee \neg x) = (t_1, 0, f_1) \vee \neg(t_1, 0, f_1) = (t_1, 0, f_1) \vee (f_1, 0, t_1)$$
$$= (max\{t_1, f_1\}, 0, \min\{f_1, t_1\})$$

The result above is always equal to $(1, 0, 0)$. To comprehend this, consider that x is $(1, 0, 0)$ or $(0, 0, 1)$, exclusively. So, $t_1 = 1$ and $f_1 = 0$, or $t_1 = 0$ and $f_1 = 1$, such $\max\{t_1, f_1\}$ is always equal to 1 and $min\{f_1, t_1\}$ equal to 0. The following table reveals this as well.

Table 7: Table for the law excluded middle.

x	$\neg x$	$\max\{t_1, f_1\}$	$min\{f_1, t_1\}$	Result
(1, 0, 0)	(0, 0, 1)	1	0	(1, 0, 0)
(0, 0, 1)	(1, 0, 0)	1	0	(1, 0, 0)

Thereby, $x \wedge \neg x = \tau$. □

Remark 2.11. The Law of Excluded Middle can be summarized in a few words: *one proposition can be true or false, not being accepted another value*.

Theorem 2.12 (Identity Laws)**.** *Let $x = (t_1, 0, f_1)$ be a neutrosophic logic variable representing a classical logic variable. It is true that: (i) $x \vee \gamma = x$, (ii) $x \wedge \gamma = \gamma$, (iii) $x \vee \tau = \tau$ and (iv) $x \wedge \tau = x$.*

Proof.
(i) $x \vee \gamma = (t_1, 0, f_1) \vee (0, 0, 1) = (\max\{t_1, 0\}, 0, \min\{f_1, 1\})$. As $t_1 \geq 0$, then $\max\{t_1, 0\} = t_1$ and as $f_1 \leq 1$, then $\min\{f_1, 1\} = f_1$. So, $(\max\{t_1, 0\}, 0, \min\{f_1, 1\}) = (t_1, 0, f_1) = x$.

(ii) $x \wedge \gamma = (t_1, 0, f_1) \wedge (0, 0, 1) = (\min\{t_1, 0\}, 0, \max\{f_1, 1\})$. As $t_1 \geq 0$, then $\min\{t_1, 0\} = 0$ and as $f_1 \leq 1$, then $\max\{f_1, 1\} = 1$. So, $(\min\{t_1, 0\}, 0, \max\{f_1, 1\}) = (0, 0, 1) = \gamma$.

(iii) $x \vee \tau = (t_1, 0, f_1) \vee (1, 0, 0) = (\max\{t_1, 1\}, 0, \min\{f_1, 0\})$. As $t_1 \leq 1$, then $\max\{t_1, 1\} = 1$ and as $f_1 \geq 0$, then $\min\{f_1, 0\} = 0$. So, $(\max\{t_1, 1\}, 0, \min\{f_1, 0\}) = (1, 0, 0) = \tau$.

(iv) $x \wedge \tau = (t_1, 0, f_1) \wedge (1, 0, 0) = (\min\{t_1, 1\}, 0, \max\{f_1, 0\})$. As $t_1 \leq 1$, then $\min\{t_1, 1\} = t_1$ and as $f_1 \geq 0$, then $\max\{f_1, 0\} = f_1$. So, $(\min\{t_1, 1\}, 0, \max\{f_1, 0\}) = (t_1, 0, f_1) = x$. □

Theorem 2.13 (Idempotent Laws)**.** *Let $x = (t_1, 0, f_1)$ be a neutrosophic logic variable representing a classical logic variable. It is true that: (i) $x \wedge x = x$ and (ii) $x \vee x = x$.*

Proof.
(i) $x \wedge x = (t_1, 0, f_1) \wedge (t_1, 0, f_1) = (\min\{t_1, t_1\}, 0, \max\{f_1, f_1\}) = (t_1, 0, f_1) = x$.
(ii) $x \vee x = (t_1, 0, f_1) \vee (t_1, 0, f_1) = (\max\{t_1, t_1\}, 0, \min\{f_1, f_1\}) = (t_1, 0, f_1) = x$. □

Now, let's turn the attention to the operator $\rightarrow$ (conditional) and $\leftrightarrow$ (biconditional).

The conditional operator has, in the classical form (now represented in neutrosophic form), the following table:

Table 8: Table for the conditional operator.

x	y	$x \rightarrow y$
(1, 0, 0)	(1, 0, 0)	(1, 0, 0)
(1, 0, 0)	(0, 0, 1)	(0, 0, 1)
(0, 0, 1)	(1, 0, 0)	(1, 0, 0)
(0, 0, 1)	(0, 0, 1)	(1, 0, 0)

Theorem 2.14. *The conditional operator $\rightarrow$ is equivalent to $\neg(x \wedge \neg y)$, that in turn is equivalent to $\neg x \vee y$.*

Proof. Let $x = (t_1, 0, f_1)$ and $y = (t_2, 0, f_2)$ be neutrosophic logic variables representing classical logic variables, then:

$$\neg(p \wedge \neg q) = \neg((t_1, 0, f_1) \wedge (f_2, 0, t_2)) = \neg(\min\{t_1, f_2\}, 0, \max\{f_1, t_2\}) = (\max\{f_1, t_2\}, 0, \min\{t_1, f_2\})$$
$$\neg p \vee q = (f_1, 0, t_1) \vee (t_2, 0, f_2) = (\max\{f_1, t_2\}, 0, \min\{t_1, f_2\})$$

So, $\neg(p \wedge \neg q)$ and $\neg p \vee q$ are equals. Now, let's examine the truth table bellow:

Table 9: Table showing that $z \to y = \neg(p \wedge \neg q) = \neg p \vee q$.

x	y	$(\max\{f_1, t_2\}, 0, \min\{t_1, f_2\})$
(1, 0, 0)	(1, 0, 0)	(1, 0, 0)
(1, 0, 0)	(0, 0, 1)	(0, 0, 1)
(0, 0, 1)	(1, 0, 0)	(1, 0, 0)
(0, 0, 1)	(0, 0, 1)	(1, 0, 0)

From the truth table, it turns out that $\neg(p \wedge \neg q) = \neg p \vee q$ and equivalent to $x \to y$. □

The biconditional operator has, in the classical form (now represented in neutrosophic form), the following table:

Table 10: Table for the biconditional operator.

x	y	$x \leftrightarrow y$
(1, 0, 0)	(1, 0, 0)	(1, 0, 0)
(1, 0, 0)	(0, 0, 1)	(0, 0, 1)
(0, 0, 1)	(1, 0, 0)	(0, 0, 1)
(0, 0, 1)	(0, 0, 1)	(1, 0, 0)

From the table, it is notable that the result is true iff the two variables have the same value.

Theorem 2.15. *The biconditional operator $\leftrightarrow$ is equivalent to $(x \to y) \wedge (y \to x)$.*

Proof. Let $x = (t_1, 0, f_1)$ and $y = (t_2, 0, f_2)$ be neutrosophic logic variables representing classical logic variables, then:

$$x \to y = \neg x \vee y = (f_1, 0, t_1) \vee (t_2, 0, f_2) = (\max\{f_1, t_2\}, 0, \min\{t_1, f_2\})$$
$$y \to z = \neg y \vee x = (f_2, 0, t_2) \vee (t_1, 0, f_1) = (\max\{f_2, t_1\}, 0, \min\{t_2, f_1\})$$
$$(x \to y) \wedge (y \to x) = (\min\{\max\{f_1, t_2\}, \max\{f_2, t_1\}\}, 0, \max\{\min\{t_1, f_2\}, \min\{t_2, f_1\}\})$$

Let's examine the truth table:

Table 11: Table showing that $x \leftrightarrow y = (x \to y) \wedge y(\to x)$.

x	y	$x \to y$	$y \to x$	$(x \to y) \wedge (y \to x)$
1 0 0	1 0 0	1 0 0	1 0 0	1 0 0
1 0 0	0 0 1	0 0 1	1 0 0	0 0 1
0 0 1	1 0 0	1 0 0	0 0 1	0 0 1
0 0 1	0 0 1	1 0 0	1 0 0	1 0 1

From the truth table, it turns out that $x \leftrightarrow y$ is equivalent to $(x \to y) \wedge y(\to x)$. □

Furthermore, other operators could be defined, for example, the operators $\oplus$ (XOR), $\downarrow$ (NOR) and $\uparrow$ (NAND).

Definition 2.16 (Exclusive OR - XOR)**.** The operator $\oplus$ (Exclusive OR - XOR) has its behavior given by the following truth table:

Table 12: Table for the Exclusive OR operator.

x	y	$x \oplus y$
1 0 0	1 0 0	0 0 1
1 0 0	0 0 1	1 0 0
0 0 1	1 0 0	1 0 0
0 0 1	0 0 1	0 0 1

A rapid view on the table truth of the operator XOR is enough to perceive that XOR is the negation of biconditional operator, so $\neg(x \leftrightarrow y) = x \oplus y$. To observe this, consider the following.

Theorem 2.17. *The negation of $x \to y$, that is, $\neg(x \to y)$ is equivalent to $x \wedge \neg y$.*

Proof.

$$\neg(x \to y) = \neg(\neg x \vee y) = \neg(\neg x) \wedge \neg y = x \wedge \neg y$$

□

Now, given that $x \leftrightarrow x = (x \to y) \wedge (y \to x)$, then $\neg(x \leftrightarrow y) = \neg(x \to y) \vee \neg(y \to x) = (x \wedge \neg y) \vee (y \wedge \neg x)$. So, letting $x = (t_1, 0, f_1)$ and $y = (t_2, 0, f_2)$:

$$\begin{aligned}(x \wedge \neg y) \vee (y \wedge \neg x) &= ((t_1, 0, f_1) \wedge (f_2, 0, t_2)) \vee ((t_2, 0, f_2), \wedge (f_1, 0, t_1)) = \\ &= (\min\{t_1, f_2\}, 0, \max\{f_1, t_2\} \vee (\min\{t_2, f_1\}, 0, \max\{f_2, t_1\}) \\ &= (\max\{\min\{t_1, f_2\}, \min\{t_2, f_1\}\}, 0, \min\{\max\{f_1, t_2\}, \max\{f_2, t_1\}\})\end{aligned}$$

Now, examining the truth table, one can be sure that $x \oplus y$ is equal to $\neg(x \leftrightarrow y)$.

Table 13: Table showing that $x \oplus y$ has the same value as $\neg(x \leftrightarrow y)$.

x	$\neg y$	y	$\neg x$	$x \wedge \neg y$	$y \wedge \neg x$	$(x \wedge \neg y) \vee (y \wedge \neg x)$
(1, 0, 0)	(0, 0, 1)	(1, 0, 0)	(0, 0, 1)	(0, 0, 1)	(0, 0, 1)	(0, 0, 1)
(1, 0, 0)	(1, 0, 0)	(0, 0, 1)	(0, 0, 1)	(1, 0, 0)	(0, 0, 1)	(1, 0, 0)
(0, 0, 1)	(0, 0, 1)	(1, 0, 0)	(1, 0, 0)	(0, 0, 1)	(1, 0, 0)	(1, 0, 0)
(0, 0, 1)	(1, 0, 0)	(0, 0, 1)	(1, 0, 0)	(0, 0, 1)	(0, 0, 1)	(0, 0, 1)

Definition 2.18 (Operators NOR and NAND)**.** The operators $\downarrow$ (NOR) and $\uparrow$ (NAND) has behavior given by the following truth table:

Table 14: Table for the operators $\downarrow$ (NOR) and $\uparrow$ (NAND).

x	y	$\downarrow$	$\uparrow$
1 0 0	1 0 0	0 0 1	0 0 1
1 0 0	0 0 1	0 0 1	1 0 0
0 0 1	1 0 0	0 0 1	1 0 0
0 0 1	0 0 1	1 0 0	1 0 0

As the acronym suggests, NOR and NAND are the negations of OR and AND, that is, $\downarrow = \neg\vee$ and $\uparrow = \neg\wedge$, being enough to look at the truth table of OR and AND and negate the entries of the respective columns.

So far was showed that the main connectives and properties of the classical logic remains true when translated to neutrosophic form. However, for any logic be useful, it must supply rules of inference, and this is the scope of the following lines.

Theorem 2.19 (Modus Ponens)**.** *Let $x = (t_1, 0, f_1)$ and $y = (t_2, 0, f_2)$ be neutrosophic logic variables representing classical logic variables. If $x \to y$ is true and x is true, then could be inferred that y is true. This is represented by*

$$x \to y, x \vdash y$$

This rule is named as modus ponens *(mode that assert).*

Proof. Its enough to see the truth table of the conditional operator (Table 8). In the first line of the truth table, $x \to y$ is true iff x and y are true, so, given that $x \to y$ and x are true, one must infer that y is also true. This is the only possibility. □

Theorem 2.20 (Modus Tollens)**.** *Let $x = (t_1, 0, f_1)$ and $y = (t_2, 0, f_2)$ be neutrosophic logic variables representing classical logic variables. If $x \to y$ is true and $\neg y$ is true, then could be inferred that $\neg x$ is true. This is represented by*

$$x \to y, \neg y \vdash \neg x$$

This rule is named as modus tollens *(mode that denies).*

Proof. It is know that $x \to y = \neg x \vee y$ (Theorem 2.15), so:

$$\begin{aligned} \neg x \vee y &= \neg x \vee (\neg\neg y) \text{ (double negation of y - Theorem 2.1)} \\ &= \neg(\neg y) \vee \neg x \text{ (commutativity of the operator } \vee \text{ - Theorem 2.2)} \\ &= \neg y \to \neg x \text{ (backing to conditional form)} \end{aligned}$$

Another way to discern this is looking at the truth table of the conditional operator (Table 8). In the fourth line of the truth table, $x \to y$ is true iff x and y are false, so, given that $x \to y$ is true and $\neg x$ is true, can be inferred that $\neg y$ is also true. This is the only possibility. □

Theorem 2.21 (And Elimination)**.** *Let $x = (t_1, 0, f_1)$ and $y = (t_2, 0, f_2)$ be neutrosophic variables representing classical logic variables. If $x \wedge y$ is true, then can be inferred that x (or y) is also true. This can be represented by:*

$$x \wedge y \vdash x \qquad \textit{or} \qquad x \wedge y \vdash y$$

Proof. It is enough to recall that the result given by the conjunction operator is true iff both of its operands are true (first line/last column of the Table 1). □

Theorem 2.22 (And Introduction)**.** *Let $x = (t_1, 0, f_1)$ and $y = (t_2, 0, f_2)$ be neutrosophic variables representing classical logic variables. If x and y are both true, then can be inferred that $x \wedge y$ is also true. This can be represented by:*

$$x, y \vdash x \wedge y$$

Proof. Its enough to recall that the result given by the conjunction operator is true iff both of its operands are true (first line/last column of the Table 1). □

Theorem 2.23 (Or Introduction)**.** *Let $x = (t_1, 0, f_1)$ and $y = (t_2, 0, f_2)$ be neutrosophic variables representing classical logic variables. If x is true, then $x \vee y$ is true, independent of the logical value of y. This can be represented by:*

$$x \vdash x \vee y$$

Proof. Its enough to recall that the result given by the disjunction operator is false iff both of its operands are false. However, as the first component is true, the result of the operation is true (first line/third column of the Table 1). Note that the theorem remains true if the roles of x and y are swapped, that is: $y \vdash x \vee y$. □

Theorem 2.24 (Unit Resolution)**.** *Let $x = (t_1, 0, f_1)$ and $y = (t_2, 0, f_2)$ be neutrosophic variables representing classical logic variables. If $x \vee y$ is true and one of their elements is false, then the other element must be true. This can be represented by:*

$$x \vee y, \neg y \vdash x \qquad \textit{or} \qquad x \vee y, \neg x \vdash y$$

Proof. Its enough to recall the result given by the disjunction operator in the second and third line/third column of the Table 1. □

Theorem 2.25 (Resolution)**.** *Let $x = (t_1, 0, f_1)$ and $y = (t_2, 0, f_2)$ and $z = (t_3, 0, f_3)$ be neutrosophic logic variables representing classical logic variables. The conditional operator is transitive, that is*

$$x \to y, y \to z \vdash x \to z$$

Proof. Firstly, take as hypothesis that $x \to y$ is true and x is true, so under the rule of *modus ponens* (Theorem 2.19), it is possible to infer that y is true, that is

$$x \to y, x \vdash y \tag{1}$$

Now, take as hypothesis that $y \to z$ is true. Noting that y is true by (1), it is possible to conclude that z is true, that is

$$y \to z, y \vdash z \tag{2}$$

Finally, given that x (by assumption) is true and z is true by (2), can be concluded that $x \to z$ is true (recall the first line from the Table 8). So,

$$x \to y, y \to z \vdash x \to z \tag{3}$$

□

The rules of inference given are not the only ones, there are several other. However, it must be sufficient to demonstrate that the main characters of the classical logical remain true when translated to neutrosophic form.

Is something missing? Yes. For example, the view of the classical logic as a formal system (*propositional calculi*), not to mention first, second and higher-order logics, have not been addressed.

3 Conclusion

This short paper has shown that neutrosophic logic, a recent type of logic, encompass the classical logic, at least at the propositional level. Then, it could be said that the classical logic is a type of neutrosophic logic, or that CL is a subset of NL.

Given the limited scope, it is not reasonable to show all aspects of the classical logic translated in neutrosophic form. However, this could be done in another time if the necessity arises.

Now, which future directions of research can be pursued? Given that the neutrosophic branch is in a wide scope, there are a bunch of paths, but of possible interest is to compare neutrosophic logic with other logics, mainly the class of the paraconsistent annotated Logics.

The paraconsistent annotated logics are a family of non-classical logics initially employed in logical programming by Subrahmanian.[2] Due to the obtained applications, a study of the foundations of the underlying logic of the investigated programming languages became convenient. It was verified that it was a paraconsistent logic, in some cases, also contained characteristics of paracomplete and non-alethic logic. To be concise and short, the class of paraconsistent annotated logics has been applied to several fields, from digital circuits,[3,4] decision making,[5] neural networks,[6] just to name a few.

Finally, it is worth follow this way? The answer to this question cannot be given here, but consider that comparing the strengths of neutrosophic logic and other logics, one or another can be enhanced, and as lateral effect, the general knowledgment grows with this.

Acknowledgments: The first author is in profound debit with Wandercy Savedra Gomes, a very understanding old friend, that kindly made the revision of the english language for this work.

References

1. Florentin Smarandache. *A Unifying Field in Logics, Neutrosophic Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics*. InfoLearnQuest, 2007. Disponible at: http://fs.unm.edu/eBook-Neutrosophics6.pdf.

2. N. C. A da Costa, J. M. Abe, and V. S. Subrahmanian. Remarks on annotated logic. *Zeitschrift fur Mathematische Logik und Grundlagen der Mathematik*, (37):561–570, 1991.

3. Joao Inácio da Silva Filho. Implementação de circuitos lógicos fundamentados em uma classe de lógicas paraconsistentes anotadas. Master's thesis, Universidade de São Paulo, 1997.

4. Joao Inácio da Silva Filho. *Métodos de aplicações da lógica paraconsistente anotada de anotação com dois valores-LPA2v com construção de algoritmo e implementação de circuitos eletrônicos*. PhD thesis, Universidade de São Paulo, 1999.

5. Fábio Romeu de Carvalho and Jair Minoro Abe. *A Paraconsistent Decision-Making Method*, volume 87 of *Smart Innovation, Systems and Technologies*. Springer, 2018.

6. Sheila Souza and Jair Minoro Abe. *Paraconsistent Artificial Neural Networks and Aspects of Pattern Recognition*, volume 94 of *Intelligent Systems Reference Library*, chapter 9, pages 207–231. Springer, 2015.

Neutrosophic Crisp Semi Separation Axioms In Neutrosophic Crisp Topological Spaces

Riad K. Al-Hamido[1,*] Luai Salha[2] Taleb Gharibah[3]

[1*]Department of Mathematics, College of Science, AlFurat University, Deir-ez-Zor, Syria.
[1*]E-mail: riad-hamido1983@hotmail.com ; Tel.: (+963988619783)
[2,3]Department of Mathematics, College of Science, AlBaath University, Homs, Syria.

Abstract

The main goal of this paper is to propose a new type of separation axioms via neutrosophic crisp semi open sets and neutrosophic crisp points in neutrosophic crisp topological spaces, namely neutrosophic crisp semi separation axioms. Finally, we examine the relationship between them in details. And also includes the study of the connections between these neutrosophic crisp semi separation axioms and the existing neutrosophic crisp separation axioms. Moreover, many examples are presented, to illustrate the concepts introduced in this paper. and investigate their fundamental properties, relationships and characterizations.

Keywords: Neutrosophic crisp semi separation axiom, neutrosophic crisp separation axiom, neutrosophic crisp point.

1. Introduction

After F.Samarandache established the concept of neutrosophy in 1980 the neutrosophy in 1980 as a new logic which generalizes the fuzzy logic, many of the pure mathematical concepts, especially in topology, were found according to this new logic. One of the most important topological developments according to this logic is finding out and defining the neutrosophic crisp topological space [1] in 2014 by A. Salama and et al. Since the elements of the neutrosophic crisp sets[1] are neutrosophic crisp points, A. Salama defined the concept of neutrosophic crisp points [1,2] in 2014. Recently, the neutrosophic crisp set theory may have applications in image processing [3],[4], the field of geographic information systems[5] and possible applications to database[6]. Also, neutrosophic sets [7] have applications in the medical field [8], [9], [10], [11]. We can't use neutrosophic crisp points were defined in [1,2] for defining the neutrosophic crisp separation axioms and this encouraged A. Al-nafey, R. Al-Hamido and F. Smarandache to think of presenting another new concept of the neutrosophic crisp points [12] in 2018, which enabled them to define separation axioms in the neutrosophic crisp space for the first time in [12] . Moreover, neutrosophic crisp semi open sets were first defined and investigated by A. Salama [5] in 2015. Since the separation

DOI: 10.5281/zenodo.3831081

Received: February 14, 2020 Revised: April 28, 2020 Accepted: May 14, 2020

axioms are considered one of the very important useful topics and one of the newly studied in topology, we thought of developing and generalizing the neutrosophic crisp separation axioms to neutrosophic crisp semi separation axioms. Finally, we study the relations between them on the one hand and between the separation neutrosophic crisps in [12] on the other hand. Many researchers studied topology, and they had many contributions to neutrosophic toplogy as [13], [14], [15], [16] and [17] and in neutrosophic bitopology in [18], [19], [20] and [21], and in neutrosophic algebra in [22], [23], [24], [25] and [26].

In this paper, Section 2 focuses on the related definitions. Section 3 presents new separation axioms in the neutrosophic crisp topological spaces, the relationship among these new separation axioms and the neutrosophic crisp separation axioms is determined.

2. Preliminaries

Throughout the paper, (χ,Ț) means neutrosophic crisp topological space (N_cTS).

N_c. OS (N_c. CS) means a neutrosophic crisp open(closed) sets and N_cS. OS means a neutrosophic crisp semi open set in N_cTS.
Now, we recall some definitions which are useful in this paper.

Definition 2.1. [1] Let $X \neq \emptyset$ be a fixed set. A neutrosophic crisp set (N_c. S) U is an object with the $U=< U_1, U_2, U_3>$ shape ; U_1, U_2 *and* U_3 are subsets of X.

Definition 2.2. [1]

$\emptyset_N$ can be defined in four ways, as below :

1. $\emptyset_N =< \emptyset, \ \emptyset, \ \emptyset >$.
2. $\emptyset_N =< \emptyset, \ X, \ \emptyset >$.
3. $\emptyset_N =< \emptyset, \ X, \ X >$.
4. $\emptyset_N =< \emptyset, \ \emptyset, \ \emptyset >$.

X_N can be defined in four ways, as below :

1. $X_N =< X, \ \emptyset, \ \emptyset >$.
2. $X_N =< X, \ X, \ \emptyset >$.
3. $\emptyset_N =< X, \ \emptyset, \ X >$.
4. $X_N =< X, \ X, \ X >$.

Definition 2.3. [1]

Let $\chi \neq \emptyset$ be a fixed set, and $U =< U_1, U_2, U_3 >, V =< V_1, V_2, V_3 >$ are two neutrosophic crisp sets, then:

$U \cup V$ can be defined as two ways, as below :

1. $U \cup V =< U_1 \cup V_1, U_2 \cup V_2, U_3 \cap V_3 >$.
2. $U \cup V =< U_1 \cup V_1, U_2 \cap V_2, U_3 \cap V_3 >$.

$U \cap V$ can be defined as two ways, as below :

1. $U \cap V =< U_1 \cap V_1, U_2 \cap V_2, U_3 \cup V_3 >$.
2. $U \cap V =< U_1 \cap V_1, U_2 \cup V_2, U_3 \cup V_3 >$.

Definition 2.4. [1]

A neutrosophic crisp topology (NCT) on a non-empty set χ is a family Ț of neutrosophic crisp subsets in χ may be satisying the following axioms:

1. X_N and $\emptyset_N$ belong to Ț.

DOI: 10.5281/zenodo.3831081

2. T is closed under finite intersection.
3. T is closed under arbitrary union.

The pair $(\chi,\mathrm{T})$is neutrosophic crisp topological space (NCTS) in Ţ. Moreover, the elements in Ţ are said to be neutrosophic crisp open sets (NCOS). A neutrosophic crisp set F is closed (NCCS) if and only if its complement F^c is neutrosophic crisp open set.

Definition 2.5. [12]

Let χ be a non-empty set. And x,y,z$\in\chi$, then:

a. $x_{N_1}=<\{x\},\emptyset,\emptyset>$ is called a neutrosophic crisp point (NCP_{N_1}) in χ .

b. $y_{N_2}=<\emptyset,\{y\},\emptyset>$ is called a neutrosophic crisp point (NCP_{N_2}) in χ.

c. $z_{N_3}=<\emptyset,\emptyset,\{z\}>$ is called a neutrosophic crisp point (NCP_{N_3}) in χ .

The set of all neutrosophic crisp points (NCP_{N_1}, NCP_{N_2}, NCP_{N_3}) is denoted by NCP_N.

Definition 2.6. [12]

Let (χ,T) be an N_cTS. Then χ is called:

a. $N_1\mathrm{T}_o$-space if for every $x_{N_1}\neq y_{N_1}\in\chi$ there exists N_c.OS M in χ containing one of them but not the other.

b. $N_2\mathrm{T}_o$-space if $\forall\ x_{N_2}\neq y_{N_2}\in\mathcal{X}$ there exists N_c.OS M in χ containing one of them but not the other.

c. $N_3\mathrm{T}_o$-space if $\forall\ x_{N_3}\neq y_{N_3}\in\chi$ there exists N_c.OS M in χ containing one of them but not the other.

d. $N_1\mathrm{T}_1$-space if for every $x_{N_1}\neq y_{N_1}\in\chi$ there exists N_c.OS M_1, M_2 in χ such that $x_{N_1}\in M_1$, $y_{N_1}\notin M_1$ and $x_{N_1}\notin M_2$, $y_{N_1}\in M_2$.

e. $N_2\mathrm{T}_1$-space if $\forall\ x_{N_2}\neq y_{N_2}\in\chi$ there exist N_c.OS M_1, M_2 in χ such that $x_{N_2}\in G_1$, $y_{N_2}\notin G_1$ and $x_{N_2}\notin G_2$, $y_{N_2}\in G_2$.

f. $N_3\mathrm{T}_1$-space if $\forall\ x_{N_3}\neq y_{N_3}\in\chi$ there exist N_c.OS M_1, M_2 in χ such that $x_{N_3}\in M_1$, $y_{N_3}\notin M_1$ and $x_{N_3}\notin M_2$, $y_{N_3}\in M_2$.

g. $N_1\mathrm{T}_2$-space if for every $x_{N_3}\neq y_{N_3}\in\chi$ there exist N_c.OS M_1, M_2 in χ such that $x_{N_1}\in M_1$, $y_{N_1}\notin M_1$ and $x_{N_1}\notin M_2$, $y_{N_1}\in M_2$. with $M_1\cap M_2=\emptyset$.

h. $N_2\mathrm{T}_2$-space if $\forall\ x_{N_2}\neq y_{N_2}\in\chi$ there exist N_c.OS M_1, M_2 in χ such that $x_{N_2}\in M_1$, $y_{N_2}\notin M_1$ and $x_{N_2}\notin M_2$, $y_{N_2}\in M_2$ with $M_1\cap M_2=\emptyset$.

i. $N_3\mathrm{T}_2$-space if $\forall\ x_{N_3}\neq y_{N_3}\in\chi$ there exist N_c.OS M_1, M_2 in χ such that $x_{N_3}\in M_1$, $y_{N_3}\notin M_1$ and $x_{N_3}\notin M_2$, $y_{N_3}\in M_2$ with $M_1\cap M_2=\emptyset$.

3. Separation axioms in a neutrosophic crisp topological space

Definition 3.1.

Let (χ,T) be an NcTS. Then χ is called:

a. N_1semiŢo-space if for every $x_{N_1}\neq y_{N_1}\in\chi$ there exists NcS.OS M in χ containing one of them but not the other.

b. N_2semiŢo-space if $\forall\ x_{N_2}\neq y_{N_2}\in\mathcal{X}$ there exists NcS.OS M in χ containing one of them but not the other.

c. N_3semiŢo-space if $\forall\ x_{N_3}\neq y_{N_3}\in\chi$ there exists NcS.OS M in χ containing one of them but not the other.

Definition 3.2.

Let (χ,T) be an N_cTS. Then χ is called:

a. N_1semiT_1-space if for every $x_{N_1}\neq y_{N_1}\in\chi$ there exist N_cS.OS M_1, M_2 in χ such that $x_{N_1}\in M_1$, $y_{N_1}\notin M_1$ and $x_{N_1}\notin M_2$, $y_{N_1}\in M_2$.

b. N_2semiT_1-space if $\forall\ x_{N_2}\neq y_{N_2}\in\chi$ there exist N_cS.OS M_1, M_2 in χ such that $x_{N_2}\in G_1$, $y_{N_2}\notin G_1$ and $x_{N_2}\notin G_2$, $y_{N_2}\in G_2$

c. N_3semiT_1-space if $\forall\ x_{N_3}\neq y_{N_3}\in\chi$ there exist N_cS.OS M_1, M_2 in χ such that $x_{N_3}\in M_1$, $y_{N_3}\notin M_1$ and $x_{N_3}\notin M_2$, $y_{N_3}\in M_2$

Definition 3.3.

Let (χ,T) be an N_cTS. Then χ is called:

a. N_1semiT_2-space if for every $x_{N_1}\neq y_{N_1}\in\chi$ there exists N_cS.OS M_1, M_2 in χ such that $x_{N_1}\in M_1$, $y_{N_1}\notin M_1$ and $x_{N_1}\notin M_2$, $y_{N_1}\in M_2$. with $M_1\cap M_2=\emptyset$.

DOI: 10.5281/zenodo.3831081

b. N_2semiT_2-space if $\forall$ $x_{N_2} \neq y_{N_2} \in \chi$ there exists N_cS.OS M_1, M_2 in χ such that $x_{N_2} \in M_1$, $y_{N_2} \notin M_1$ and $x_{N_2} \notin M_2$, $y_{N_2} \in M_2$ with $M_1 \cap M_2 = \emptyset$.

c. N_3semiT_2-space if $\forall$ $x_{N_3} \neq y_{N_3} \in \chi$ there exists N_cS.OS M_1, M_2 in χ such that $x_{N_3} \in M_1$, $y_{N_3} \notin M_1$ and $x_{N_3} \notin M_2$, $y_{N_3} \in M_2$ with $M_1 \cap M_2 = \emptyset$.

Theorem 3.4.

Let (χ,T) be an N_cTS, then :

1. Every $\mathrm{N}_1\mathrm{T}_0$-space is N_1semiT_0-space.
2. Every $\mathrm{N}_2\mathrm{T}_0$-space is N_2semiT_0-space.
3. Every $\mathrm{N}_3\mathrm{T}_0$-space is N_3semiT_0-space.

Proof:

1. Suppose that (χ,T) is an $\mathrm{N}_1\mathrm{T}_0$-space , therefore for every two $x_{N_1} \neq y_{N_1}$, there exists an N_c.OS M in χ containing one of them to which the other does not belong. So there exists an N_cS.OS M in χ containing one of them to which the other does not belong, therefore X is N_1semiT_0-space.
2. Similar to Proof 1.
3. Similar to Proof 1.

Remark 3.5.

The converse of theorem 3.4 is not true., as it is shown in the following examples.

Example 3.6.

Let $\chi = \{a,b,c\}, \mathrm{T} = \{\emptyset_N, X_N, A\}, A = \{<\{a\},\emptyset,\emptyset>\}$.
$N_cS.OS = \mathrm{T} \cup \{C = \{<\{a,b\},\emptyset,\emptyset>\}, B = \{<\{a,c\},\emptyset,\emptyset>\}$.
Let $x_{N_1} = \{<\{b\},\emptyset,\emptyset>\} \neq y_{N_1} = \{<\{c\},\emptyset,\emptyset>\} \in \chi$ there is no a N_c.OS M in χ containing one of them but not the other. Therefore (χ,T) is not $\mathrm{N}_1\mathrm{T}_0$-space.
Then (χ,T) $\mathrm{N}_1 semi\mathrm{T}_0$-space, But (χ,T) is not $\mathrm{N}_1\mathrm{T}_0$-space.

Example 3.7.

Let $\chi = \{a,b,c\}, \mathrm{T} = \{\emptyset_N, X_N, A\}, A = \{<\emptyset,\{a\},\emptyset>\}$.
$N_cS.OS = \mathrm{T} \cup \{C = \{<\emptyset,\{a,b\},\emptyset>\}, B = \{<\emptyset,\{a,c\},\emptyset>\}$.
Let $x_{N_2} = \{<\emptyset,\{b\},\emptyset>\} \neq y_{N_2} = \{<\emptyset,\{c\},\emptyset>\} \in \chi$ there is no a N_c.OS M in χ containing one of them but not the other. Therefore (χ,T) is not $\mathrm{N}_2\mathrm{T}_0$-space.
Then (χ,T) $\mathrm{N}_2 semi\mathrm{T}_0$-space, But (χ,T) is not $\mathrm{N}_2\mathrm{T}_0$-space.

Example 3.8.

Let $\chi = \{a,b,c\}, \mathrm{T} = \{\emptyset_N, X_N, A\}, A = \{<\emptyset,\{a\},\emptyset>\}$.
$N_cS.OS = \mathrm{T} \cup \{C = \{<\emptyset,\emptyset,\{a,b\}>\}, B = \{<\emptyset,\emptyset,\{a,c\},>\}$.
Let $x_{N_3} = \{<\emptyset,\emptyset,\{b\}>\} \neq y_{N_3} = \{<\emptyset,\emptyset,\{c\}>\} \in \chi$ there is no a N_c.OS M in χ containing one of them but not the other. Therefore (χ,T) is not $\mathrm{N}_3\mathrm{T}_0$-space.
Then (χ,T) $\mathrm{N}_3 semi\mathrm{T}_0$-space, But (χ,T) is not $\mathrm{N}_3\mathrm{T}_0$-space.

Theorem 3.9.

Let (χ,T) be an N_cTS, then :

1. Every $\mathrm{N}_1\mathrm{T}_1$-space is N_1semiT_1-space.
2. Every $\mathrm{N}_2\mathrm{T}_1$-space is N_2semiT_1-space.
3. Every $\mathrm{N}_3\mathrm{T}_1$-space is N_3semiT_1-space.

Proof:

1. Suppose that (χ,T) is an $\mathrm{N}_1\mathrm{T}_1$-space , therefore for every two $x_{N_1} \neq y_{N_1}$, there exist an N_c.OS M_1, M_2 in χ such that $x_{N_1} \in M_1$, $y_{N_1} \notin M_1$ and $x_{N_1} \notin M_2$, $y_{N_1} \in M_2$. So there exists an N_cS.OS M_1, M_2 in χ such that $x_{N_1} \in M_1$, $y_{N_1} \notin M_1$ and $x_{N_1} \notin M_2$, $y_{N_1} \in M_2$.
   Therefore X is N_1semiT_1-space.
2. Similar to Proof 1.
3. Similar to Proof 1.

Remark 3.10.

The converse of a theorem 3.9 is not true, as it is shown in the following example.

Example 3.11.

Let $\chi = \{a,b,c\}, \mathrm{T} = \{\emptyset_N, X_N, A, B, C\}, A = \{<\{a\},\emptyset,\emptyset>\}, B = \{<\{b\},\emptyset,\emptyset>\}, C = \{<\{a,b\},\emptyset,\emptyset>\}$.

DOI: 10.5281/zenodo.3831081

$N_cS.OS = Ţ \cup \{G = \{< \{a, c\}, \emptyset, \emptyset >\}, H = \{< \{b, c\}, \emptyset, \emptyset >\}$.
Let $x_{N_1} = \{< \{b\}, \emptyset, \emptyset >\} \neq y_{N_1} = \{< \{c\}, \emptyset, \emptyset >\} \in \chi$ there is no N_c.OS M_1, M_2 in χ such that $x_{N_1} \in M_1$, $y_{N_1} \notin M_1$ and $x_{N_1} \notin M_2$, $y_{N_1} \in M_2$. Therefore $(\chi,Ţ)$ is not $N_1Ţ_1$-space.

Then $(\chi,Ţ)$ $N_1semiŢ_1$-space, But $(\chi,Ţ)$ is not $N_1Ţ_1$-space.
Also $(\chi,Ţ)$ $N_1semiŢ_2$-space, But $(\chi,Ţ)$ is not $N_1Ţ_2$-space.

Example 3.12.
Let $\chi = \{a, b, c\}, Ţ = \{\emptyset_N, X_N, A, B, C\}, A = \{< \emptyset, \{a\}, \emptyset >\}, B = \{< \emptyset, \{b\}, \emptyset >\}, C = \{< \emptyset, \{a, b\}, \emptyset >\}$.
$N_cS.OS = Ţ \cup \{G = \{< \emptyset, \{a, c\}, \emptyset >\}, H = \{< \emptyset, \{b, c\}, \emptyset >\}$.
Let $x_{N_1} = \{< \emptyset, \{b\}, \emptyset >\} \neq y_{N_1} = \{\emptyset, < \{c\}, \emptyset >\} \in \chi$ there are no a N_c.OS M_1, M_2 in χ such that $x_{N_2} \in M_1$, $y_{N_2} \notin M_1$ and $x_{N_2} \notin M_2$, $y_{N_2} \in M_2$. Therefore $(\chi,Ţ)$ is not $N_2Ţ_1$-space.

Then $(\chi,Ţ)$ $N_2semiŢ_1$-space, But $(\chi,Ţ)$ is not $N_2Ţ_1$-space.

Also $(\chi,Ţ)$ $N_2semiŢ_2$-space, But $(\chi,Ţ)$ is not $N_2Ţ_2$-space.

Example 3.13.
Let $\chi = \{a, b, c\}, Ţ = \{\emptyset_N, X_N, A, B, C\}, A = \{< \emptyset, \emptyset, \{a\} >\}, B = \{< \emptyset, \emptyset, \{b\} >\}, C = \{< \emptyset, \emptyset, \{a, b\} >\}$.
$N_cS.OS = Ţ \cup \{G = \{< \emptyset, \emptyset, \{a, c\} >\}, H = \{< \emptyset, \emptyset, \{b, c\} >\}$.
Let $x_{N_3} = \{< \emptyset, \emptyset, \{b\} >\} \neq y_{N_3} = \{< \emptyset, \emptyset, \{c\} >\} \in \chi$ there are no N_c.OS M_1, M_2 in χ such that $x_{N_3} \in M_1$, $y_{N_3} \notin M_1$ and $x_{N_3} \notin M_2$, $y_{N_3} \in M_2$. Therefore $(\chi,Ţ)$ is not $N_3Ţ_1$-space.

Then $(\chi,Ţ)$ $N_3semiŢ_1$-space, But $(\chi,Ţ)$ is not $N_3Ţ_1$-space.
Also $(\chi,Ţ)$ $N_3semiŢ_2$-space, But $(\chi,Ţ)$ is not $N_3Ţ_2$-space.

Theorem 3.14.

Let $(\chi,Ţ)$ be an N_cTS, then :

1. Every $N_1Ţ_2$-space is N_1semi$Ţ_2$-space.
2. Every $N_2Ţ_2$-space is N_2semi$Ţ_2$-space.
3. Every $N_3Ţ_2$-space is N_3semi$Ţ_2$-space.

Proof:

1. Suppose that $(\chi,Ţ)$ is an $N_1Ţ_2$-space , therefore for every two $x_{N_1} \neq y_{N_1}$, there exists an N_c.OS M_1, M_2 in χ such that $x_{N_1} \in M_1$, $y_{N_1} \notin M_1$ and $x_{N_1} \notin M_2$, $y_{N_1} \in M_2$. with $M_1 \cap M_2 = \emptyset$. So there exists N_cS.OS M_1, M_2 in χ such that $x_{N_1} \in M_1$, $y_{N_1} \notin M_1$ and $x_{N_1} \notin M_2$, $y_{N_1} \in M_2$. with $M_1 \cap M_2 = \emptyset$.
   Therefore X is N_1semi$Ţ_2$-space.
2. Similar to Proof 1.
3. Similar to Proof 1.

Remark 3.15.
The converse of the Theorem 3.14 is not true., as it is shown in the following example.
Example 3.16.
Let $\chi = \{a, b, c\}, Ţ = \{\emptyset_N, X_N, A, B, C\}, A = \{< \{a\}, \emptyset, \emptyset >\}, B = \{< \{b\}, \emptyset, \emptyset >\}, C = \{< \{a, b\}, \emptyset, \emptyset >\}$.
$N_cS.OS = Ţ \cup \{G = \{< \{a, c\}, \emptyset, \emptyset >\}, H = \{< \{b, c\}, \emptyset, \emptyset >\}$.
Let $x_{N_1} = \{< \{b\}, \emptyset, \emptyset >\} \neq y_{N_1} = \{< \{c\}, \emptyset, \emptyset >\} \in \chi$ there are no a N_c.OS M_1, M_2 in χ such that $x_{N_1} \in M_1$, $y_{N_1} \notin M_1$ and $x_{N_1} \notin M_2$, $y_{N_1} \in M_2$. Therefore $(\chi,Ţ)$ is not $N_1Ţ_1$-space.

Then $(\chi,Ţ)$ $N_1semiŢ_1$-space, but $(\chi,Ţ)$ is not $N_1Ţ_1$-space.
Also $(\chi,Ţ)$ $N_1semiŢ_2$-space, but $(\chi,Ţ)$ is not $N_1Ţ_2$-space.

Example 3.17.
Let $\chi = \{a, b, c\}, Ţ = \{\emptyset_N, X_N, A, B, C\}, A = \{< \emptyset, \{a\}, \emptyset >\}, B = \{< \emptyset, \{b\}, \emptyset >\}, C = \{< \emptyset, \{a, b\}, \emptyset >\}$.
$N_cS.OS = Ţ \cup \{G = \{< \emptyset, \{a, c\}, \emptyset >\}, H = \{< \emptyset, \{b, c\}, \emptyset >\}$.

DOI: 10.5281/zenodo.3831081

Let $x_{N_1} = \{<\emptyset,\{b\},\emptyset>\} \neq y_{N_1} = \{\emptyset,<\{c\},\emptyset>\} \in \chi$ there are no N_c.OS M_1, M_2 in χ such that $x_{N_2} \in M_1$, $y_{N_2} \notin M_1$ and $x_{N_2} \notin M_2$, $y_{N_2} \in M_2$. Therefore (χ,T) is not N_2T_1-space.

Then (χ,T) $N_2 semi T_1$-space, but (χ,T) is not N_2T_1-space.

Also (χ,T) $N_2 semi T_2$-space, but (χ,T) is not N_2T_2-space.

Example 3.18.
Let $\chi = \{a,b,c\}, T = \{\emptyset_N, X_N, A, B, C\}, A = \{<\emptyset,\emptyset,\{a\}>\}, B = \{<\emptyset,\emptyset,\{b\}>\}, C = \{<\emptyset,\emptyset,\{a,b\}>\}$.
$N_cS.OS = T \cup \{G = \{<\emptyset,\emptyset,\{a,c\}>\}, H = \{<\emptyset,\emptyset,\{b,c\}>\}$.
Let $x_{N_3} = \{<\emptyset,\emptyset,\{b\}>\} \neq y_{N_3} = \{<\emptyset,\emptyset,\{c\}>\} \in \chi$ there are no N_c.OS M_1, M_2 in χ such that $x_{N_3} \in M_1$, $y_{N_3} \notin M_1$ and $x_{N_3} \notin M_2$, $y_{N_3} \in M_2$. Therefore (χ,T) is not N_3T_1-space.

Then (χ,T) $N_3 semi T_1$-space, But (χ,T) is not N_3T_1-space.
Also (χ,T) $N_3 semi T_2$-space, But (χ,T) is not N_3T_2-space.

Theorem 3.19.

Let (χ,T) be an N_cTS, then :

1. N_1semiT_2-space $\Rightarrow$ N_1semiT_1-space $\Rightarrow$ N_1semiT_0-space.
2. N_2semiT_2-space $\Rightarrow$ N_2semiT_1-space $\Rightarrow$ N_2semiT_0-space.
3. N_3semiT_2-space $\Rightarrow$ N_3semiT_1-space $\Rightarrow$ N_3semiT_0-space.

The converse of the Theorem 3.19 is not true.

Example 3.20.
Let $X = \{a,b,c\}, S^{NC} = \{\emptyset_N, X_N, A\}, A = \{<\{a\},\emptyset,\emptyset>\}$.
$N_cS.OS = \{A = \{<\{a\},\emptyset,\emptyset>\}, C = \{<\{a,b\},\emptyset,\emptyset>\}, B = \{<\{a,c\},\emptyset,\emptyset>\}$.
Then (χ,T) $N_1 semi T_0$-space, but (χ,T) is not $N_1 semi T_1$-space. So (χ,T) $N_1 semi T_0$-space, But (χ,T) is not $N_1 semi T_2$-space.

Remark. 3.21.
Relations among the different types of neutrosophic crisp separation axioms which were studied in this paper, appear in the following diagram.

$$
\begin{array}{ccccc}
\text{Ni-}T_2\text{-space} & \underset{\nLeftarrow}{\Rightarrow} & \text{Ni-}T_1\text{-space} & \underset{\nLeftarrow}{\Rightarrow} & \text{Ni-}T_o\text{-space.} \\
\Downarrow \; \nUparrow & & \Downarrow \; \nUparrow & & \Downarrow \; \nUparrow \\
\text{Ni-}ST_2\text{-space} & \underset{\nLeftarrow}{\Rightarrow} & \text{Ni-}ST_1\text{-space} & \underset{\nLeftarrow}{\Rightarrow} & \text{Ni-}ST_o\text{-space.}
\end{array}
$$

i=0,1,2.

Definition 3.22.
An N_cTS (χ,T) is called:
a. $N semi T_0$-space if $(\mathcal{X},\mathcal{T})$ is $N_1 semi T_0$-space and $N_2 semi T_0$-space and $N_3 semi T_0$-space.
b. $N semi T_1$-space if $(\mathcal{X},\mathcal{T})$ is $N_1 semi T_1$-space and $N_2 semi T_1$-space and $N_3 semi T_1$-space.
c. $N semi T_2$-space if $(\mathcal{X},\mathcal{T})$ is $N_1 semi T_2$-space and $N_2 semi T_2$-space and $N_3 semi T_2$-space.

Remark 3.23.
For an N_cTS (χ,T)
1. Every $N semi T_0$-space if $(\mathcal{X},\mathcal{T})$ is $N_1 semi T_0$-space.

DOI: 10.5281/zenodo.3831081

2. Every $\mathrm{N}semi\mathrm{T}_0$-space if $(\mathcal{X},\mathcal{T})$ is $\mathrm{N}_2semi\mathrm{T}_0$-space.
3. Every $\mathrm{N}semi\mathrm{T}_0$-space if $(\mathcal{X},\mathcal{T})$ is $\mathrm{N}_3semi\mathrm{T}_0$-space.
The converse is not true as it is shown in the below example.
Example 3.24.
Assume that $\mathrm{T} = \{x, y\}$, $\mathrm{T}_1 = \{\mathcal{X}_\mathrm{N}, \emptyset_\mathrm{N}, A\}$,$\mathrm{T}_2 = \{\mathcal{X}_\mathrm{N}, \emptyset_\mathrm{N}, B\}$, $\mathrm{T}_3 = \{\mathcal{X}_\mathrm{N}, \emptyset_\mathrm{N}, G\}$.
A =<{x},∅,∅>
B =<∅,{y},∅>
G =<∅,∅,{x}>
Then: (χ,T) is $\mathrm{N}_1semi\mathrm{T}_0$-space but not $\mathrm{N}semi\mathrm{T}_0$-space.
(χ,T) is $\mathrm{N}_2semi\mathrm{T}_0$-space but not $\mathrm{N}semi\mathrm{T}_0$-space.
(χ,T) is $\mathrm{N}_3semi\mathrm{T}_0$-space but not $\mathrm{N}semi\mathrm{T}_0$-space.
Remark 3.25.
For an $\mathrm{N_cTS}$ (χ,T)
1. Every $\mathrm{N}semi\mathrm{T}_1$-space if $(\mathcal{X},\mathcal{T})$ is $\mathrm{N}_1semi\mathrm{T}_1$-space.
2. Every $\mathrm{N}semi\mathrm{T}_1$-space if $(\mathcal{X},\mathcal{T})$ is $\mathrm{N}_2semi\mathrm{T}_1$-space.
3. Every $\mathrm{N}semi\mathrm{T}_1$-space if $(\mathcal{X},\mathcal{T})$ is $\mathrm{N}_3semi\mathrm{T}_1$-space.
The converse is not true as it is shown in the following example.
Example 3.26.
Assume that = {x, y} ,$\mathcal{T}_1 = \{\mathcal{X}_\mathrm{N} , \emptyset_\mathrm{N} , A, B\}$, $\mathcal{T}_2 = \{\mathcal{X}_\mathrm{N}, \emptyset_\mathrm{N}, G , F\}$.
A =<{x},{y},∅>
B =<{y},{x},∅>
G =<∅,∅,{x}>
F =<∅,∅,{y}>
Then: $(\chi,\mathrm{T}_1)$is $\mathrm{N}_1semi\mathrm{T}_1$-space but not $\mathrm{N}semi\mathrm{T}_1$.
$(\chi,\mathrm{T}_1)$is $\mathrm{N}_2semi\mathrm{T}_1$-space but not $\mathrm{N}semi\mathrm{T}_1$.
$(\chi,\mathrm{T}_2)$is $\mathrm{N}_3semi\mathrm{T}_1$-space but not $\mathrm{N}semi\mathrm{T}_1$.
Remark 3.27.
For an $\mathrm{N_cTS}$ (χ,T)
1. Every $\mathrm{N}semi\mathrm{T}_2$-space if $(\mathcal{X},\mathcal{T})$ is $\mathrm{N}_1semi\mathrm{T}_2$-space.
2. Every $\mathrm{N}semi\mathrm{T}_2$-space if $(\mathcal{X},\mathcal{T})$ is $\mathrm{N}_2semi\mathrm{T}_2$-space.
3. Every $\mathrm{N}semi\mathrm{T}_2$-space if $(\mathcal{X},\mathcal{T})$ is $\mathrm{N}_3semi\mathrm{T}_2$-space.
The converse is not true as it is shown in the example.
Remark 3.28.
For an neutrosophic crisp topological space $(\mathcal{X},\mathcal{T})$
1. Every $\mathrm{N}semi\mathrm{T}_1$-space but not $\mathrm{N}semi\mathrm{T}_0$-space.
2. Every $\mathrm{N}semi\mathrm{T}_2$-space but not $\mathrm{N}semi\mathrm{T}_1$-space.
The converse is not true as it is shown in the following example :
Example 3.29.
Assume that = {x, y} , $\mathcal{T} = \{\mathcal{X}_\mathrm{N} , \emptyset_\mathrm{N} , A, B, G\}$.

A =<{x},∅,∅>
B =<∅,{y},∅>
G =<∅,∅,{x}>
Then: $(\mathcal{X},\mathcal{T})$ is $\mathrm{N}semi\mathrm{T}_0$-space but not $\mathrm{N}semi\mathrm{T}_1$-space.

Conclusion

In this paper, we have defined a new type of neutrosophic crisp separation axioms by using neutrosophic crisp semi open sets and certain point in the neutrosophic crisp topological spaces. Moreover,we study the connections between neutrosophic crisp semi separation axioms and the existing neutrosophic crisp separation axioms. And many examples are presented, to illustrate the concepts introduced in this paper. Also, investigate their fundamental properties and characterizations.

References

[1] A. A. Salama, F. Smarandache and Kroumov, “Neutrosophic crisp Sets and Neutrosophic crisp Topological Spaces” Neutrosophic Sets and Systems Vlo.2, pp.25-30, 2014.
[2] A. A. Salama, "Neutrosophic Crisp Points and Neutrosophic Crisp Ideals". Neutrosophic Sets and Systems 1, pp.50-54, 2013.
[3] A. A Salama, I. M Hanafy, Hewayda Elghawalby Dabash M.S, “Neutrosophic Crisp Closed Region and Neutrosophic Crisp Continuous Functions”, New Trends in Neutrosophic Theory and Applications.
[4] A. A Salama, Hewayda Elghawalby,M.S. Dabash, A. M. NASR , “Retrac Neutrosophic Crisp System For Gray Scale Image”, Asian Journal of Mathematics and Computer Research, Vol. 24, pp.104-117-22, 2018.
[5] A. A Salama, “Basic Structure of Some Classes of Neutrosophic Crisp Nearly Open Sets & Possible Application to GIS Topology”, Neutrosophic Sets and Systems, Vol. 7, pp.18-22, 2015.
[6] A. A Salama, F. Smarandache, “Neutrosophic Crisp Set Theory”, Neutrosophic Sets and Systems,Vol. 5, pp.1-9, 2014.
[7] F. Smarandache, “Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics” University of New Mexico, Gallup, NM 87301, USA 2002.
[8] M. Abdel-Basset, M. Mai, E. Mohamed, C. Francisco,H. Z. Abd El-Nasser, "Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases" Artificial Intelligence in Medicine Vol. 101 , 101735, 2019.
[9] M. Abdel-Basset, E. Mohamed, G. Abduallah and F. Smarandache"A novel model for evaluation Hospital medical care systems based on plithogenic sets" Artificial intelligence in medicine 100 (2019), 101710.
[10] M. Abdel-Basset, G. Gunasekaran Mohamed, G. Abduallah. C. Victor, "A Novel Intelligent Medical Decision Support Model Based on Soft Computing and IoT" IEEE Internet of Things Journal, Vol. 7 ,2019.
[11] M. Abdel-Basset, G. Abduallah, G. Gunasekaran, L. Hoang Viet,"A novel group decision making model based on neutrosophic sets for heart disease diagnosis" Multimedia Tools and Applications, pp.1-26, 2019.
[12] A. B.AL-Nafeen R.K. Al-Hamido, F.Smarandache, "Separation Axioms In Neutrosophic Crisp Topological Spaces", Neutrosophic Sets and Systems, vol. 25, 25-32, 2019.
[13] F. Smarandache, S. Pramanik, "New Neutrosophic Sets via Neutrosophic Topological Spaces". In Neutrosophic Operational Research; Pons Editions: Brussels, Belgium, Vol. 1, pp.189–209, 2017.
[14] W. Al-Omeri, "Neutrosophic crisp sets via neutrosophic crisp topological spaces". Neutrosophic Sets Syst, Vol. 13, pp.96–104, 2016.
[15] W. Al-Omeri, S. Jafari "On Generalized Closed Sets and Generalized Pre-Closed Sets in Neutrosophic Topological Spaces", Mathematics, Vol. 7, pp.1-12, 2019. doi:doi.org/10.3390/math7010001.
[16] R.K. Al-Hamido, Q. H. Imran, K. A. Alghurabi, T. Gharibah, “On Neutrosophic Crisp Semi Alpha Closed Sets”, Neutrosophic Sets and Systems”, vol. 21, pp.28-35, 2018.
[17] Q. H. Imran, F. Smarandache, R.K. Al-Hamido, R. Dhavasselan, “On Neutrosophic Semi Alpha open Sets”, Neutrosophic Sets and Systems, vol. 18, pp.37-42, 2017.
[18] R. K.Al-Hamido, “A study of multi-Topological Spaces”, PhD Theses, AlBaath university , Syria, 2019.
[19] R. K.Al-Hamido,“Neutrosophic Crisp Supra Bi-Topological Spaces”, International Journal of Neutrosophic Science, Vol. 1, pp.66-73, 2018.
[20] R.K. Al-Hamido, “Neutrosophic Crisp Bi-Topological Spaces”, Neutrosophic Sets and Systems, vol. 21, pp.66-73, 2018.

DOI: 10.5281/zenodo.3831081

[21] R.K. Al-Hamido, T. Gharibah, S. Jafari, F.Smarandache, “On Neutrosophic Crisp Topology via N-Topology”, Neutrosophic Sets and Systems, vol. 21, pp.96-109, 2018.

[22] V. Christianto, F. Smarandache, M. Aslam, “How we can extend the standard deviation notion with neutrosophic interval and quadruple neutrosophic numbers”, International Journal of Neutrosophic Science, vol. 2, No. 2, pp.72-76 , 2020.

[23] E. O. Adeleke, A. A. A. Agboola, F. Smarandache, “Refined Neutrosophic Rings I ”, International Journal of Neutrosophic Science, vol. 2 , No. 2, pp.77-81 , 2020.

[24] E. O. Adeleke, A. A. A. Agboola, F. Smarandache, “Refined Neutrosophic Rings II”, International Journal of Neutrosophic Science, vol 2 , No. 2, pp.89-94 , 2020.

[25] M. A. Ibrahim, A. A. A. Agboola, E. O. Adeleke, S. A. Akinleye, “Introduction to Neutrosophic Subtraction Algebra and Neutrosophic Subtraction Semigroup”, International Journal of Neutrosophic Science, vol. 2 , No. 1, pp.47-62 , 2020.

[26] A. Hatip, “The Special Functions”, International Journal of Neutrosophic Science, vol. 4 , No. 2, pp.104-116, 2020.

DOI: 10.5281/zenodo.3831081

Introduction to NeutroGroups

Agboola[1] A.A.A.
Department of Mathematics, Federal University of Agriculture, Abeokuta, Nigeria.
agboolaaaa@funaab.edu.ng

Abstract

The objective of this paper is to formally present the concept of NeutroGroups by considering three NeutroAxioms (NeutroAssociativity, existence of NeutroNeutral element and existence of NeutroInverse element). Several interesting results and examples of NeutroGroups, NeutroSubgroups, NeutroCyclicGroups, NeutroQuotientGroups and NeutroGroupHomomorphisms are presented. It is shown that generally, Lagrange's theorem and 1st isomorphism theorem of the classical groups do not hold in the class of NeutroGroups.

Keywords: Neutrosophy, NeutroGroup, NeutroSubgroup, NeutroCyclicGroup, NeutroQuotientGroup and NeutroGroupHomomorphism.

1 Introduction

In 2019, Florentin Smarandache[2] introduced new fields of research in neutrosophy which he called NeutroStructures and AntiStructures respectively. The concepts of NeutroAlgebras and AntiAlgebras were recently introduced by Smarandache in.[3] Smarandache in[4] revisited the notions of NeutroAlgebras and AntiAlgebras where he studied Partial Algebras, Universal Algebras, Effect Algebras and Boole's Partial Algebras and showed that NeutroAlgebras are generalization of Partial Algebras. In,[1] Agboola et al examined NeutroAlgebras and AntiAlgebras viz-a-viz the classical number systems $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$. The mention of NeutroGroup by Smarandache in[2] motivated us to write the present paper. The concept of NeutroGroup is formally presented in this paper by considering three NeutroAxioms (NeutroAssociativity, existence of NeutroNeutral element and existence of NeutroInverse element). We study NeutroSubgroups, NeutroCyclicGroups, NeutroQuotientGroups and NeutroGroupHomomorphisms. We present several interesting results and examples. It is shown that generally, Lagrange's theorem and 1st isomorphism theorem of the classical groups do not hold in the class of NeutroGroups.

For more details about NeutroAlgebras, AntiAlgebras, NeutroAlgebraic Structures and AntiAlgebraic Structures, the readers should see.[1–4]

2 Formal Presentation of NeutroGroup and Properties

In this section, we formally present the concept of a NeutroGroup by considering three NeutroAxioms (NeutroAssociativity, existence of NeutroNeutral element and existence of NeutroInverse element) and we present its basic properties.

Definition 2.1. Let G be a nonempty set and let $* : G \times G \to G$ be a binary operation on G. The couple $(G, *)$ is called a NeutroGroup if the following conditions are satisfied:

(i) $*$ is NeutroAssociative that is there exits at least one triplet $(a, b, c) \in G$ such that

$$a * (b * c) = (a * b) * c \tag{1}$$

and there exists at least one triplet $(x, y, z) \in G$ such that

$$x * (y * z) \neq (x * y) * z. \tag{2}$$

[1]Correspondence:aaaola2003@yahoo.com

Doi :10.5281/zenodo.3840761

Received: February 26, 2020 Revised: March 17, 2020 Accepted: May19, 2020

(ii) There exists a NeutroNeutral element in G that is there exists at least an element $a \in G$ that has a single neutral element that is we have $e \in G$ such that

$$a * e = e * a = a \tag{3}$$

and for $b \in G$ there does not exist $e \in G$ such that

$$b * e = e * b = b \tag{4}$$

or there exist $e_1, e_2 \in G$ such that

$$\begin{aligned} b * e_1 &= e_1 * b = b \quad \text{or} &(5)\\ b * e_2 &= e_2 * b = b \quad \text{with} \quad e_1 \neq e_2 &(6)\end{aligned}$$

(iii) There exists a NeutroInverse element that is there exists an element $a \in G$ that has an inverse $b \in G$ with respect to a unit element $e \in G$ that is

$$a * b = b * a = e \tag{7}$$

or there exists at least one element $b \in G$ that has two or more inverses $c, d \in G$ with respect to some unit element $u \in G$ that is

$$\begin{aligned} b * c &= c * b = u &(8)\\ b * d &= d * b = u. &(9)\end{aligned}$$

In addition, if $*$ is NeutroCommutative that is there exists at least a duplet $(a, b) \in G$ such that

$$a * b = b * a \tag{10}$$

and there exists at least a duplet $(c, d) \in G$ such that

$$c * d \neq d * c, \tag{11}$$

then $(G, *)$ is called a NeutroCommutativeGroup or a NeutroAbelianGroup.

If only condition (i) is satisfied, then $(G, *)$ is called a NeutroSemiGroup and if only conditions (i) and (ii) are satisfied, then $(G, *)$ is called a NeutroMonoid.

Definition 2.2. Let $(G, *)$ be a NeutroGroup. G is said to be finite of order n if the cardinality of G is n that is $o(G) = n$. Otherwise, G is called an infinite NeutroGroup and we write $o(G) = \infty$.

Example 2.3. Let $\mathbb{U} = \{a, b, c, d, e, f\}$ be a universe of discourse and let $G = \{a, b, c, d\}$ be a subset of $\mathbb{U}$. Let $*$ be a binary operation defined on G as shown in the Cayley table below:

$*$	a	b	c	d
a	b	c	d	a
b	c	d	a	c
c	d	a	b	d
d	a	b	c	a

.

It is clear from the table that:

$$\begin{aligned} a * (b * c) &= (a * b) * c = d,\\ b * (d * c) &= a, \text{ but } (b * d) * c = b.\end{aligned}$$

This shows that $*$ is NeutroAssociative and hence $(G, *)$ is a NeutroSemiGroup.

Next, let N_x and I_x represent the neutral element and the inverse element respectively with respect to any element $x \in G$. Then

$$\begin{aligned} N_a &= d,\\ I_a &= c,\\ N_b, N_c, N_d & \quad \text{do not exist},\\ I_b, I_c, I_d & \quad \text{do not exist}.\end{aligned}$$

This in addition to $(G, *)$ being a NeutroSemiGroup implies that $(G, *)$ is a NeutroGroup.

It is also clear from the table that $*$ is NeutroCommutative. Hence, $(G, *)$ is a NeutroAbelianGroup.

Example 2.4. Let $G = \mathbb{Z}_{10}$ and let $*$ be a binary operation on G defined by $x * y = x + 2y$ for all $x, y \in G$ where $+$ is addition modulo 10. Then $(G, *)$ is a NeutroAbeliaGroup. To see this, for $x, y, z \in G$, we have

$$x * (y * z) = x + 2y + 4z, \tag{12}$$
$$(x * y) * z = x + 2y + 2z. \tag{13}$$

Equating (16) and (17) we obtain $z = 0, 5$. Hence only the triplets $(x, y, 0), (x, y, 5)$ can verify associativity of $*$ and not any other triplet $(x, y, z) \in G$. Hence, $*$ is NeutroAssociative and therefore, $(G, *)$ is a NeutroSemigroup.

Next, let $e \in G$ such that $x * e = x + 2e = x$ and $e * x = e + 2x$. Then, $x + 2e = e + 2x$ from which we obtain $e = x$. But then, only $5 * 5 = 5$ in G. This shows that G has a NeutroNeutral element. It can also be shown that G has a NeutroInverse element. Hence,$(G, *)$ is a NeutroGroup.

Lastly,

$$x * y = x + 2y, \tag{14}$$
$$y * x = y + 2x. \tag{15}$$

Equating (18) and (19), we obtain $x = y$. Hence only the duplet $(x, x) \in G$ can verify commutativity of $*$ and not any other duplet $(x, y) \in G$. Hence, $*$ is NeutroCommutative and thus $(G, *)$ is a NeutroAbelianGroup.

Remark 2.5. General NeutroGroup is a particular case of general NeutroAlgebra which is an algebra which has at least one NeutroOperation or one NeutroAxiom (axiom that is true for some elements, indeterminate for other elements, and false for the other elements). Therefore, a NeutroGroup is a group that has either one NeutroOperation (partially well-defined, partially indeterminate, and partially outer-defined), or atleast one NeutroAxiom (NeutroAssociativity, NeutroElement, or NeutroInverse).

It is possible to define NeutroGroup in another way by considering only one NeutroAxiom or by considering two NeutroAxioms.

Theorem 2.6. *Let $(G_i, *), i = 1, 2, \cdots, n$ be a family of NeutroGroups. Then*

(i) $G = \bigcap_{i=1}^{n} G_i$ is a NeutroGroup.

(ii) $G = \prod_{i=1}^{n} G_i$ is a NeutroGroup.

Proof. Obvious. □

Definition 2.7. Let $(G, *)$ be a NeutroGroup. A nonempty subset H of G is called a NeutroSubgroup of G if $(H, *)$ is also a NeutroGroup.

The only trivial NeutroSubgroup of G is G.

Example 2.8. Let $(G, *)$ be the NeutroGroup of **Example 2.3** and let $H = \{a, c, d\}$. The compositions of elements of H are given in the Cayley table below.

$*$	a	c	d
a	b	d	a
c	d	b	d
d	a	c	a

It is clear from the table that:

$$\begin{aligned} a * (c * d) &= (a * c) * d = a, \\ d * (c * a) &= a, \text{ but } (d * c) * a = d \neq a. \\ N_a &= d, \\ I_a &= c, \\ N_c, N_d & \quad \text{do not exist,} \\ I_c, I_d & \quad \text{do not exist.} \\ a * d &= d * a = a, \text{ but } c * d = d, d * c = c \neq d. \end{aligned}$$

All these show that $(H, *)$ is a NeutroAbelianGroup. Since $H \subset G$, it follows that H is a NeutroSubgroup of G.

It should be observed that the order of H is not a divisor of the order of G. Hence, Lagrange's theorem does not hold.

Theorem 2.9. *Let $(G, *)$ be a NeutroGroup and let $(H_i, *), i = 1, 2, \cdots, n$ be a family of NeutroSubgroups of G. Then*

(i) $H = \bigcap_{i=1}^{n} H_i$ *is a NeutroSubgroup of* G.

(ii) $H = \prod_{i=1}^{n} H_i$ *is a NeutroSubgroup of* G.

Proof. Obvious. □

Definition 2.10. Let H be a NeutroSubgroup of the NeutroGroup $(G, *)$ and let $x \in G$.

(i) xH the left coset of H in G is defined by

$$xH = \{xh : h \in H\}. \tag{16}$$

(i) Hx the right coset of H in G is defined by

$$Hx = \{hx : h \in H\}. \tag{17}$$

Example 2.11. Let $(G, *)$ be the NeutroGroup of **Example 2.3** and let H be the NeutroSubgroup of **Example 2.8**. H_l the set of distinct left cosets of H in G is given by

$$H_l = \{\{a, b, d\}, \{a, c\}, \{b, d\}\}$$

and H_r the set of distinct right cosets of H in G is given by

$$H_r = \{\{a, b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, d\}\}.$$

It should be observed that H_l and H_r do not partition G. This is different from what is obtainable in the classical groups. However, the order of H_l is 3 which is not a divisor of the order of G and therefore, $[G : H]$ the index of H in G is 3, that is $[G : H] = 3$. Also, the order of H_r is 4 which is a divisor of G. Hence, $[G : H] = 4$.

Example 2.12. Let $\mathbb{U} = \{a, b, c, d\}$ be a universe of discourse and let $G = \{a, b, c\}$ be a NeutroGroup given in the Cayley table below:

$*$	a	b	c
a	a	c	b
b	c	a	c
c	a	c	d

Let $H = \{a, b\}$ be a NeutroSubgroup of G given in the Cayley table below:

$*$	a	b
a	a	c
b	c	a

Then, the sets of distinct left and right cosets of H in G are respectively obtained as:

$$\begin{aligned} H_l &= \{\{a, c\}\}, \\ H_r &= \{\{a, \}, \{c\}, \{b, c\}\}. \end{aligned}$$

In this example, the order of H_l the set of distinct left cosets of H in G is 1 which is not a divisor of the order of G and therefore, $[G : H] = 1$. However, the order of H_r the set of distinct right cosets of H in G is 3 which is a divisor of the order of G and therefore, $[G : H] = 3$. This is also different from what is obtainable in the classical groups.

Consequent on **Examples 2.8, 2.11 and 2.12**, we state the following theorem:

Theorem 2.13. *Let H be a NeutroSubgroup of the finite NeutroGroup $(G, *)$. Then generally:*

(i) $o(H)$ *is not a divisor of* $o(G)$.

(ii) There is no 1-1 correspondence between any two left(right) cosets of H in G.

(iii) *There is no 1-1 correspondence between any left(right) coset of H in G and H.*

(iv) *If $N_x = e$ that is $xe = ex = x$ for any $x \in G$, then $eH \neq H$, $He \neq H$ and $\{e\}$ is not a NeutroSubgroup of G.*

(v) $o(G) \neq [G : H]o(H)$.

(vi) *The set of distinct left(right) cosets of H in G is not a partition of G.*

Definition 2.14. Let $(G, *)$ be a NeuroGroup. Since $*$ is associative for at least one triplet $(x, x, x) \in G$, the powers of x are defined as follows:

$$\begin{array}{rcl} x^1 & = & x \\ x^2 & = & xx \\ x^3 & = & xxx \\ \vdots & & \vdots \\ x^n & = & xxx\cdots x \quad n \text{ factors } \ \forall n \in \mathbb{N}. \end{array}$$

Theorem 2.15. *Let $(G, *)$ be a NeutroGroup and let $x \in G$. Then for any $m, n \in \mathbb{N}$, we have:*

(i) $x^m x^n = x^{m+n}$.

(ii) $(x^m)^n = x^{mn}$.

Definition 2.16. Let $(G, *)$ be a NeutroGroup. G is said to be cyclic if G can be generated by an element $x \in G$ that is

$$G =< x >= \{x^n : n \in \mathbb{N}\}. \tag{18}$$

Example 2.17. Let $(G, *)$ be the NeutroGroup given in **Example 2.3** and consider the following:

$$\begin{array}{rcl} a^1 & = & a, a^2 = b, a^3 = c, a^4 = d. \\ b^1 & = & b, b^2 = d, b^3 = c, b^4 = a. \\ c^1 & = & c, c^2 = b, c^3 = a, c^4 = d. \\ d^1 & = & d, d^2 = a, d^3 = a, d^4 = a. \\ \therefore \ G & = & < a >=< b >=< c >, G \neq< d > . \end{array}$$

These show that G is cyclic with the generators a, b, c. The element $d \in G$ does not generate G.

Definition 2.18. Let H be a NeutroSubgroup of the NeutroGroup $(G, *)$. The sets $(G/H)_l$ and $(G/H)_r$ are defined by:

$$(G/H)_l = \{xH : x \in G\} \tag{19}$$

$$(G/H)_r = \{Hx : x \in G\}. \tag{20}$$

Let $xH, yH \in (G/H)_l$ and let $\odot_l$ be a binary operation defined on $(G/H)_l$ by

$$xH \odot_l yH = x * yH \quad \forall\, x, y \in G. \tag{21}$$

Also, let $xH, yH \in (G/H)_r$ and let $\odot_r$ be a binary operation defined on $(G/H)_r$ by

$$Hx \odot_r Hy = Hx * y \quad \forall\, x, y \in G. \tag{22}$$

It can be shown that the couples $((G/H)_l, \odot_l)$ and $((G/H)_r, \odot_r)$ are NeutroGroups.

Example 2.19. Let G and H be as given in **Example 2.12** and consider

$$(G/H)_l = \{aH, bH, cH\} = \{aH\} = \{\{a, c\}\}.$$

Then $((G/H)_l, \odot_l)$ is a NeutroGroup.

Definition 2.20. Let $(G, *)$ and $(H, \circ)$ be any two NeutroGroups. The mapping $\phi : G \to H$ is called a NeutroGroupHomomorphism if ϕ preserves the binary operations $*$ and $\circ$ that is if for all $x, y \in G$, we have

$$\phi(x * y) = \phi(x) \circ \phi(y). \tag{23}$$

The kernel of ϕ denoted by $Ker\phi$ is defined as

$$Ker\phi = \{x : \phi(x) = e_H\} \tag{24}$$

where $e_H \in H$ is such that $N_h = e_H$ for at least one $h \in H$.

The image of ϕ denoted by $Im\phi$ is defined as

$$Im\phi = \{y \in H : y = \phi(x) \text{ for some } h \in H\}. \tag{25}$$

If in addition ϕ is a bijection, then ϕ is called a NeutroGroupIsomorphism and we write $G \cong H$. NeutroGroupEpimorphism, NeutroGroupMonomorphism, NeutroGroupEndomorphism, and NeutroGroupAutomorphism are similarly defined.

Example 2.21. Let $(G, *)$ be a NeutroGroup of **Example** 2.12 and let $\psi : G \times G \to G$ be a projection given by

$$\psi((x, y)) = x \quad \forall x, y \in G.$$

Then ψ is a NeutroGroupHomomorphism. The $Ker\psi = \{(a, a), (a, b), a, c)\}$ which is a NeutroSubgroup of $G \times G$ as shown in the Cayley table below.

$*$	(a,a)	(a,b)	(a,c)
(a,a)	(a,a)	(a,c)	(a,b)
(a,b)	(a,c)	(a,a)	(a,c)
(a,c)	(a,a)	(a,c)	(a,d)

and $Im\psi = \{a, b, c\} = G$.

Consequent on **Example** 2.21, we state the following theorem:

Theorem 2.22. *Let $(G, *)$ and $(H, \circ)$ be NeutroGroups and let $N_x = e_G$ such that $e_G * x = x * e_G = x$ for at least one $x \in G$ and let $N_y = e_H$ such that $e_H * y = y * e_H = y$ for at least one $y \in H$. Suppose that $\phi : G \to H$ is a NeutroGroupHomomorphism. Then:*

(i) $\phi(e_G) = e_H$.

(ii) *$Ker\phi$ is a NeutroSubgroup of G.*

(iii) *$Im\phi$ is a NeutroSubgroup of H.*

(iv) *ϕ is injective if and only if $Ker\phi = \{e_G\}$.*

Example 2.23. Considering **Example** 2.19, let $\phi : G \to G/H$ be a mapping defined by $\phi(x) = xH$ for all $x \in G$. Then, $\phi(a) = \phi(b) = \phi(c) = aH = \{a, c\}$ from which we have that ϕ is a NeutroGroupHomomorphism.

$$Ker\phi = \{x \in G : \phi(x) = e_{G/H}\} = \{x \in G : xH = e_{G/H} = e_{\{\{a,c\}\}}\} \neq H.$$

Consequent on **Example** 2.23, we state the following theorem:

Theorem 2.24. *Let H be a NeutroSubgroup of a NeutroGroup $(G, *)$. The mapping $\psi : G \to G/H$ defined by*

$$\psi(x) = xH \quad \forall\, x \in G$$

is a NeutroGroupHomomorphism and the $Ker\psi \neq H$.

Theorem 2.25. *Let $\phi : G \to H$ be a NeutroGroupHomomorpism and let $K = Ker\phi$. Then the mapping $\psi : G/K \to Im\phi$ defined by*

$$\psi(xK) = \phi(x) \quad \forall\, x \in G$$

is a NeutroGroupEpimorphism and not a NeutroGroupIsomorphism.

Proof. That ψ is a well defined surjective mapping is clear. Let $xK, yK \in G/K$ be arbitrary. Then

$$\begin{aligned}
\psi(xKyK) &= \psi(xyK) \\
&= \phi(xy) \\
&= \phi(x)\phi(y) \\
&= \psi(xK)\psi(yK). \\
Ker\psi &= \{xK \in G/K : \psi(xK) = e_{\phi(x)}\} \\
&= \{xK \in G/K : \phi(x) = e_{\phi(x)}\} \\
&\neq \{e_{G/K}\}.
\end{aligned}$$

This shows that ψ is a surjective NeutroGroupHomomorphism and therefore it is a NeutroGroupEpimorphism. Since ψ is not injective, it follows that $G/K \not\cong Im\phi$ which is different from what is obtainable in the classical groups. □

Theorem 2.26. *NeutroGroupIsomorphism of Neutrogroups is an equivalence relation.*

Proof. The same as the classical groups. □

3 Conclusion

We have formally presented the concept of NeutroGroup in this paper by considering three NeutroAxioms (NeutroAssociativity, existence of NeutroNeutral element and existence of NeutroInverse element). We presented and studied several interesting results and examples on NeutroSubgroups, NeutroCyclicGroups, NeutroQuotientGroups and NeutroGroupHomomorphisms. We have shown that generally, Lagrange's theorem and 1st isomorphism theorem of the classical groups do not hold in the class of NeutroGroups. Further studies of NeutroGroups will be presented in our future papers. Other NeutroAlgebraicStructures such as NeutroRings, NeutroModules, NeutroVectorSpaces etc are opened to studies for Neutrosophic Researchers.

4 Appreciation

The author is very grateful to all anonymous reviewers for valuable comments and suggestions which we have found very useful in the improvement of the paper.

References

[1] Agboola A.A.A., Ibrahim M.A. and Adeleke E.O.,"Elementary Examination of NeutroAlgebras and AntiAlgebras viz-a-viz the Classical Number Systems", International Journal of Neutrosophic Science,Volume 4 , Issue 1, pp. 16-19 , 2020.

[2] Smarandache, F., "Introduction to NeutroAlgebraic Structures, in Advances of Standard and Nonstandard Neutrosophic Theories," Pons Publishing House Brussels, Belgium, Ch. 6, pp. 240-265, 2019.

[3] Smarandache, F., "Introduction to NeutroAlgebraic Structures and AntiAlgebraic Structures (revisited)", Neutrosophic Sets and Systems, vol. 31, pp. 1-16, 2020. DOI: 10.5281/zenodo.3638232.

[4] Smarandache, F., "NeutroAlgebra is a Generalization of Partial Algebra", International Journal of Neutrosophic Science, vol. 2 (1), pp. 08-17, 2020.

Neutrosophic Event-Based Queueing Model

Mohamed Bisher Zeina

Dept. of Mathematical Statistics, Faculty of Sciences, University of Aleppo, Aleppo, Syria
bisherzeina@alepuniv.edu.sy

Abstract

In this paper we have defined the concept of neutrosophic queueing systems and defined its neutrosophic performance measures. An important application of neutrosophic logic in queueing systems we face in real life were discussed, that is the neutrosophic events accuring times, because of its wide applications in networking and simulating communication systems specialy when probability distribution is not known, and because it's more realistic to consider and to not ignore the imprecise events times. Event-based table of a neutrosophic queueing system was presented and its neutrosophic performance measures were derived, i.e. neutrosophic mean waiting time in queue, neutrosophic mean waiting time in system, neutrosophic expected number of customers in queue and neutrosophic expected number of customers in system. Neutrosophic Little's Formulas (NLF) were also defined which is a main tool in queueing systems problems to make it easier finding performance measures from each other.

Keywords: Neutrosophic Logic; Queueing Theory; Performance Measures; Little's Formula

1.Introduction

Neutrosophic Logic that was presented by F.Smarandache in 1995 as generalization of intuitionistic fuzzy logic has lots of applications in many branches of science specially in probability, statistics and operation research [1,2,3,4,5,6,7]. In these three mentioned branches of science we face lots of imprecise, ambiguity, fluctuation and incomplete data [3,4,5,6,7]. Before the foundation of neutrosophic logic we couldn't deal with these cases [8,9,10,11,12]. Queueing theory or waiting lines theory is one of the most important applications of probability theory and operations research presented by Erlang in 1909, which is a modelling of systems where customers have to wait unitl they get required services [13]. Queueing theory has many applications in traffic flow, scheduling, designing communication networks and facility design [13,14,15].

In queueing theory, efficiency of service providing systems can be determined depending on the time that customer must wait unitl he takes his desired service and the cost that is resulting from system's design [14,15].

Modeling a queuenig system depends on lots of characteristics including: interarrival times, departures times, servicing times, number of servers, etc. but these are not always determined exactly and most times are inaccurate [3,10,14] .

In classical cirsp qeues researcher often use rates, midpoints or most probable values to represent in queue's parameters [16,17]. The extension of classical queueing theory to neutrosophic queueing theory means that the elements of the queue are imprecise, i.e. times of arrivals, timer of taking services and departures are not known exactly. In this paper, the definition of neutrosophic performance measures of a general queue depending on event-

DOI: 10.5281/zenodo.3840771
Received: March 09, 2020 Revised: April 18, 2020 Accepted: May 14, 2020

based table and NLF was introduced, then a solved example was presented to show the power of neutrosophic crisp sets [3,5,18] in solving problems better and more accurate than in classical crisp logic.

2. Neutrosophic queues

2.1 Definitions and notions

Suppose that the queueing system consists of one waiting line. Arriving customer takes his service from only one server. If the server is busy then arriving customer waits in the queue until the server is empty. The server can serve one customer at time according to first comes first served policy. The customer departs after he takes his service.

Let by definition:

- $NA^{(n)}$ denotes the neutrosophic time of customer (n) arrival.
- $NS^{(n)}$ denotes the neutrosophic service time of customer (n).
- $NT^{(n)}$ denotes the neutrosophic interarrival time of customer (n) and customer (n+1) so:
$$NT^{(n)} = NA^{(n+1)} - NA^{(n)}$$
- $NU^{(n)}$ denotes the neutrosophic time that customer (n) starts taking the service so:
$$NU^{(n+1)} = \max\left(ND^{(n)}, NA^{(n+1)}\right)$$
- $ND^{(n)}$ denotes the neutrosophic time of customer (n) departure that is:
$$ND^{(n)} = NU^{(n)} + NS^{(n)}$$
- $NW_q^{(n)}$ denotes waiting time in queue of customer (n) so:
$$NW_q^{(n)} = NU^{(n)} - NA^{(n)}$$
- $NW^{(n)}$ denotes waiting time in system of customer (n) so:
$$NW^{(n)} = NW_q^{(n)} + NS^{(n)}$$
- NW_q denotes the neutrosophic mean waiting time in queue.
- NW denotes the neutrosophic mean waiting time in system.
- NL_q denotes the neutrosophic expected number of customers in queue.
- NL denotes the neutrosophic expected number of customers in system.

NLF will be then (based on classical little's formula in [14,15]):

$$NL = \lambda * NW$$
$$NL_q = \lambda * NW_q$$

Where λ is the arrival rate, that is total number of arrivals divided by the time between last departure and starting time.

Among this paper, neutrosophic statistical numbers presented in [19] will be used, which is an effective method to describe indeterminacy in data specially data that it is not useful to be presented in the form (T, I, F) like parameters of distributions, statistical data, statistical data tables, frequency tables, time series, etc [4, 5, 10 ,19, 20].

2.2 Example

Suppose that we have the following event-based table of a queueing system representing arriving customers to an ATM (all times are in minutes):

Customer	$NA^{(n)}$	$NS^{(n)}$
1	0	[1,2]
2	[3,4]	4

DOI: 10.5281/zenodo.3840771

3	6	2
4	8	1
5	12	[1,2]
6	[13,14]	2

Table 1: Input data

Lets calculate the performance measures of this ATM system according to Neutrosophic queues then according to cirsp queues:

Neutrosophic Solution:

For customer 1: we start when the system is empty so $NA^{(1)} = 0$ that is the arrival time of ther first customer.

Since the system is empty then the arrival customer will be served without any waiting so $NU^{(1)} = 0$.

The service time for this customer is $NS^{(1)} = [1,2]$ so the departure time will be $ND^{(1)} = NU^{(1)} + NS^{(1)} = 0 + [1,2] = [1,2]$ that is the time of start serving the customer plus the serving time.

The waiting time in queue is the difference between starting serving the customer $NU^{(1)}$and arriving time of this customer $NA^{(1)}$ so : $NW_q^{(1)} = NU^{(1)} - NA^{(1)} = 0 - 0 = 0$.

Waiting time in system is sum of waiting time in queue and serving time that is : $NW^{(1)} = NW_q^{(1)} + NS^{(1)} = 0 + [1,2] = [1,2]$.

Interarrival time between first and next customer is $NT^{(1)} = NA^{(2)} - NA^{(1)} = [3,4] - 0 = [3,4]$.

For customer 2: $NA^{(2)} = [3,4]$ that is the arrival time of the second customer.

The service starting time of this customer (customer 2) will be its arriving time if the server is empty or it will be immediately after the departure time of the previous customer if the server is busy, so it's $NU^{(2)} = \max\left(ND^{(1)}, NA^{(2)}\right) = \max([1,2],[3,4]) = [3,4]$.

The service time for this customer is $NS^{(2)} = 4$ so the departure time will be $D^{(2)} = NU^{(2)} + NS^{(2)} = [3,4] + 4 = [7,8]$.

The waiting time in queue is the difference between starting serving the customer $NU^{(2)}$and arriving time of this customer $NA^{(2)}$ so : $NW_q^{(2)} = NU^{(2)} - NA^{(2)} = [3,4] - [3,4] = 0$.

Waiting time in system is sum of waiting time in queue and serving time that is : $NW^{(2)} = NW_q^{(2)} + NS^{(2)} = 0 + 4 = 4$.

Interarrival time between second and third customer is $NT^{(2)} = NA^{(3)} - NA^{(2)} = 6 - [3,4] = [2,3]$.

DOI: 10.5281/zenodo.3840771

<u>For customer 3:</u> $NA^{(3)} = 6$.

The service starting time of this customer will be $NU^{(3)} = \max\left(ND^{(2)}, NA^{(3)}\right) = \max([7,8],6) = [7,8]$.

The service time for this customer is $NS^{(3)} = 2$ so the departure time will be $D^{(3)} = NU^{(3)} + NS^{(3)} = [7,8] + 2 = [9,10]$.

The waiting time in queue is: $NW_q^{(3)} = NU^{(3)} - NA^{(3)} = [7,8] - 6 = [1,2]$.

Waiting time in system is : $NW^{(3)} = NW_q^{(3)} + NS^{(3)} = [1,2] + 2 = [3,4]$.

Interarrival time between third and forth customer is $NT^{(3)} = NA^{(4)} - NA^{(3)} = 8 - 6 = 2$.

And so on.. We can form the following table:

Customer	$\boldsymbol{NA^{(n)}}$	$\boldsymbol{NS^{(n)}}$	$\boldsymbol{NT^{(n)}}$	$\boldsymbol{NU^{(n)}}$	$\boldsymbol{ND^{(n)}}$	$\boldsymbol{NW_q^{(n)}}$	$\boldsymbol{NW^{(n)}}$
1	0	[1,2]	[3,4]	0	[1,2]	0	[1,2]
2	[3,4]	4	[2,3]	[3,4]	[7,8]	0	4
3	6	2	2	[7,8]	[9,10]	[1,2]	[3,4]
4	8	1	4	[9,10]	[10,11]	[1,2]	[2,3]
5	12	[1,2]	[1,2]	12	[13,14]	0	[1,2]
6	[13,14]	2	-	[13,14]	[15,16]	0	2

Table 2: Neutrosophic event-based queue table

Now we can calculate the performance measures as follows:

$$NW_q = \frac{\sum_{k=1}^{6}\left[NW_q^{(n)}\right]_k}{6} = \frac{0 + 0 + [1,2] + [1,2] + 0 + 0}{6} = \frac{[2,4]}{6} = [0.33,0.67]\ minutes$$

Which means that customer will wait in the queue between 0.33 and 0.67 mins or between 20 and 40 secs before starting being served.

$$NW = \frac{\sum_{k=1}^{6}\left[NW^{(n)}\right]_k}{6} = \frac{[1,2] + 4 + [3,4] + [2,3] + [1,2] + 2}{6} = \frac{[13,17]}{6} = [2.17,2.833]\ minutes$$

Which means that customer will stay in the system between 2.17 and 2.833 that is between 2 mins and 10 secs and 2 mins and 50 secs from the time he arrives until he departs from the system.

We have also $\lambda = \frac{6}{16} = 0.375$ that is the arrival rate because we totally have 6 customers arrived during 16 minutes.

DOI: 10.5281/zenodo.3840771

Now using Little's formula we have:

$NL = \lambda * NW = 0.375 * [2.17, 2.833] = [0.81375, 1.062375]$ *customers*

That means expected number of customers in the system will be between 0.81375 and 1.062375 customers.

$NL_q = \lambda * NW_q = 0.375 * [0.33, 0.67] = [0.12375, 0.25125]$ *customers*

That means expected number of custoemrs in queue will range between 0.12375 and 0.25125 customers.

Crisp Solution:

To solve this kind of problems using crisp queues we may take midpoints of intervals, so:

Customer	$\boldsymbol{A^{(n)}}$	$\boldsymbol{S^{(n)}}$
1	0	1.5
2	3.5	4
3	6	2
4	8	1
5	12	1.5
6	13.5	2

Table 3: Input data converted into crisp data

For customer 1: Since the system is empty then the arrival customer will be served without any waiting so $U^{(1)} = 0$.

The service time for this customer is approximatley $S^{(1)} = 1.5$ so the departure time will be approximately $D^{(1)} = U^{(1)} + S^{(1)} = 0 + 1.5 = 1.5$ that is the apprpximated time of start serving the customer plus the approximated serving time.

The waiting time in queue is the difference between starting serving the customer $U^{(1)}$and arriving time of this customer $A^{(1)}$ so : $W_q^{(1)} = U^{(1)} - A^{(1)} = 0 - 0 = 0$.

Waiting time in system is sum of waiting time in queue and serving time that is : $W^{(1)} = W_q^{(1)} + S^{(1)} = 0 + 1.5 = 1.5$.

Interarrival time between first and next customer is approximately $T^{(1)} = A^{(2)} - A^{(1)} = 3.5 - 0 = 3.5$

For customer 2: $A^{(2)} = 3.5$ that is the approximated arrival time of the second customer.

DOI: 10.5281/zenodo.3840771

The service starting time of this customer will be its arriving time if the server is empty or it will be immediately after the departure time of the previous customer if the server is busy, so it's $U^{(2)} = \max\left(D^{(1)}, A^{(2)}\right) = \max(1.5,3.5) = 3.5$

The service time for this customer is $S^{(2)} = 4$ so the departure time will be $D^{(2)} = U^{(2)} + S^{(2)} = 3.5 + 4 = 7.5$.

The waiting time in queue is the difference between starting serving the customer $U^{(2)}$and arriving time of this customer $A^{(2)}$ so : $W_q^{(2)} = U^{(2)} - A^{(2)} = 3.5 - 3.5 = 0$.

Waiting time in system is sum of waiting time in queue and serving time that is : $W^{(2)} = W_q^{(2)} + S^{(2)} = 0 + 4 = 4$.

Interarrival time between second and third customer is $T^{(2)} = A^{(3)} - A^{(2)} = 6 - 3.5 = 2.5$

And so on.. We can form the following table:

Customer	$\boldsymbol{A^{(n)}}$	$\boldsymbol{S^{(n)}}$	$\boldsymbol{T^{(n)}}$	$\boldsymbol{U^{(n)}}$	$\boldsymbol{D^{(n)}}$	$\boldsymbol{W_q^{(n)}}$	$\boldsymbol{W^{(n)}}$
1	0	1.5	3.5	0	1.5	0	1.5
2	3.5	4	2.5	3.5	7.5	0	4
3	6	2	2	7.5	9.5	1.5	3.5
4	8	1	4	9.5	10.5	1.5	2.5
5	12	1.5	1.5	12	13.5	0	1.5
6	13.5	2	-	13.5	15.5	0	2

Table 4: Crisp event-based queue table

Now performance measures can be calculated as follows:

$$W_q = \frac{\sum_{k=1}^{6}\left[W_q^{(n)}\right]_k}{6} = \frac{0 + 0 + 1.5 + 1.5 + 0 + 0}{6} = \frac{3}{6} = 0.5\ minutes \in [0.33,0.67] = NW_q$$

$$W = \frac{\sum_{k=1}^{6}\left[W^{(n)}\right]_k}{6} = \frac{1.5 + 4 + 3.5 + 2.5 + 1.5 + 2}{6} = \frac{15}{6} = 2.5\ minutes \in [2.17,2.833] = NW$$

We had before $\lambda = 0.375$, now using crisp Little's formula:

$L = \lambda * W = 0.375 * 2.5 = 0.9375\ customers \in [0.81375,1.062375] = NL$

$L_q = \lambda * W_q = 0.375 * 0.5 = 0.1875\ customers \in [0.12375,0.25125] = NL_q$

DOI: 10.5281/zenodo.3840771

We notice that neutrosophic solutions are more accurate and realistic than crisp solutions.

5. Conclusions

In this article, we discussed basics of neutrosophic queueing theory and have shown the power of neutrosophic logic in modelling queues with imprecise and incomplete inputs in even-based tables of queues. We solved an example which contains indetermined times of arrivals, departures, services and services starting times and calculated the neutrosophic mean waiting time in queue, neutrosophic mean waiting time in system, neutrosophic expected number of customers in queue and neutrosophic expected number of customers in system then compared it to crisp solutions.

We found that neutrosophic solutions can be considered as an extension to crisp solutions, also neutrosophic solutions are more accurate than crisp solutions.

In future work author looking forward to study more complex applications of neutrosophic logic to more genereal queueing problems like bulk queues, balk queues, networks of queues and impatient customers behaviour in queues like jockeying and reneging.

Acknowledgment: "The author is highly thankful to the Editor-in-Chief and the reviewers for their valuable comments and suggestions that improve the quality of paper".

Funding: "This research received no external funding"

Conflicts of Interest: "The authors declare no conflict of interest."

References

[1] Smarandache.F., "Introduction to Neutrosophic Measure, Neutrosophic Integral, and Neutrosophic Probability", Sitech-Education Publisher, Craiova – Columbus, 2013

[2] Atanassov .k, "Intuitionistic fuzzy sets," In V. Sgurev, ed., ITKRS Session, Sofia, June 1983, Central Sci. and Techn. Library, Bulg. Academy of Sciences, 1984.

[3] Salama, A. A. and Smarandache, F. "Neutrosophic crisp probability theory & decision making process." Critical Review: A Publication of Society for Mathematics of Uncertainty, vol. 12, p. 34-48, 2016.

[4] Alhabib, R., A. A. Salama, "Using Moving Averages To Pave The Neutrosophic Time Series", International Journal of Neutrosophic Science, Vol. 3, 2020

[5] Alhabib, R., Ranna, M., Farah, H. and Salama, A. " Some neutrosophic probability distributions," Neutrosophic Sets and Systems, 22, pp.30-38, 2018.

[6] Sapan Kumar Das; S.A. Edalatpanah, "A new ranking function of triangular neutrosophic number and its application in integer programming", International Journal of Neutrosophic Science, *Vol. 4, No. 2, 02020*

[7] Christianto.V. ,Boyd. R.N. , Smarandach.F. , "Three possible applications of Neutrosophic Logic in Fundamental and Applied Sciences," International Journal of Neutrosophic Science, Volume 1 , Issue 2, PP: 90-95 , 2020.

[8] Smarandache. F, "Neutrosophic set a generalization of the intuitionistic fuzzy sets", Inter. J. Pure Appl. Math., 24, 287 – 297, 2005

[9] Smarandache, F., "A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability", American Research Press, Rehoboth, NM, 1999

[10] Smarandache, F." Introduction to Neutrosophic statistics," Sitech & Education Publishing, 2014

[11] Hatip.A., "The Special Neutrosophic Functions" , International Journal of Neutrosophic Science,Vol. 4, 2020

DOI: 10.5281/zenodo.3840771

[12] A. Chakraborty, A "New Score Function of Pentagonal Neutrosophic Number and its Application in Networking Problem", International Journal of Neutrosophic Science, Volume 1 , Issue 1, pp. 40-51 , 2020
[13] Erlang, Agner Krarup, "The theory of probabilities and telephone conversations" . NYT Tidsskrift for Matematik B. 20: 33–39, 1909.
[14] F Shortle J.; M Thompson J.; Gross D.; M Harris C., "Fundamentals of Queueing Theory", 5th ed.; Wiley: United States of America, 2018
[15] Nick T. Thomopoulos; "Fundamentals of Queuing Systems", 1st ed.; Springer: United States of America, 2012
[16] Balter. M.H., "Performance Modeling and Design of Computer Systems: Queueing Theory in Action", Cambridge, 2013
[17] Robertazzi T.G., "Computer Networks and Systems: Queueing Theory and Performance Evaluation", 2nd Edition, Springer, 1994
[18] Salama.A.A, Smarandache. F, Neutrosophic Crisp Set Theory. Education Publishing, Columbus, 2015.
[19] Patro.S.K, Smarandache. F., "The Neutrosophic Statistical Distribution, More Problems, More Solutions", Neutrosophic Sets and Systems, Vol. 12, 2016
[20] Kawther Fawzi Hamza Alhasan, F. Smarandache, "Neutrosophic Weibull distribution and Neutrosophic Family Weibull Distribution", Neutrosophic Sets and Systems, vol. 28, pp. 191-199, 2019

DOI: 10.5281/zenodo.3840771

Plithogenic Subjective Hyper-Super-Soft Matrices with New Definitions & Local, Global, Universal Subjective Ranking Model

Shazia Rana [1,2,*], Muhammad Saeed [1], Midha Qayyum [2], Florentin Smarandache [3]

[1] Dept. Math, University of Management and Technology, Johar Town Campus, Lahore, 54000, Pakistan.; *shaziaranams@gmail.com

[2] COMSATS University Islamabad, Lahore Campus, Department of Mathematics, Lahore, 54000, Pakistan.; mqayyum17@gmail.com

[3] Dept. Math and Sciences, University of New Mexico, Gallup, NM 87301, USA; smarand@unm.edu

Correspondence: shaziaranams@gmail.com

Abstract

In this paper, we initially introduce a novel type of matrix representation of Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hypersoft Set named as **Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hypersoft Matrix,** which is generated by multiple parallel sheets of matrices. Furthermore, these parallel sheets are representing parallel universes or parallel realities (a combination of attributes and sub-attributes w.r.t. subjects). We represent cross-sectional cuts of these hyper-soft matrices as parallel sheets (images of the expanded universe). Later, we utilize these Hypersoft matrices to formulate **Plithogenic Subjective Crisp/Fuzzy/Intuitionistic/Neutrosophic Hyper-Super-Soft Matrix.** These matrices are framed by the generalization of Whole Hyper-Soft Set to Subjective Whole Hyper-Soft Set and then their representation in such **hyper-super-soft-matrix (**parallel sheets of matrices) whose elements are matrices. The Hypersoft matrices and hyper-super-soft matrices are tensors of rank three and four, respectively, having three and four indices of variations. Later we provide an application of these Plithogenic Hyper super soft matrices in the form of **Local, Global, Universal Subjective Ranking Model.** The specialty of this model is that it offers precise classification of the universe from micro-universe to macro-universe levels by observing them through several angles of visions in many environments having several ambiguities and hesitation levels. This model provides optimal and neutral values of universes and can compact the expanded universe to a single point in such a way that the compacted universe reflects the cumulative effect of the whole universe. It further offers a transparent ranking by giving a percentage authenticity measure of the ranking. Finally, we provide an application of the model as a numerical example.

Keywords: Plithogenic Hyper-Super-Soft matrices, Sheets of matrices, Expanded Universe, Compacted Universe, Subjective, Local, Global, Universal Ranking,

1.Introduction

DOI: 10.5281/zenodo.3841624

Received: February 25, 2020 Revised: March 28, 2020 Accepted: May 20, 2020

The theory of fuzziness in mathematics was initially introduced by Zadeh [**7**] in 1965 named as fuzzy set theory (FST). As in crisp Set, a member either completely belongs to a set $\boldsymbol{A}$ or completely doesn't belong to the set $\boldsymbol{A}$ which means we assignee membership to set $\boldsymbol{A}$ either $\mathbf{1}$ or $\mathbf{0}$ while in a fuzzy set, a doubt of belongingness is addressed as a natural trait of the human mind in decision making. The complete membership reduced according to the doubt of belongingness. We may say a fuzzy set is a set where each element of the universe of discourse $\boldsymbol{X}$ has some degree of membership in the unit closed interval $[\mathbf{0}, \mathbf{1}]$ in given set $\boldsymbol{A}$, where $\boldsymbol{A}$ is a subset of universal Set $\boldsymbol{X}$ with respect to an attribute say $\boldsymbol{M}$ with an imposed condition that the sum of membership and non-membership is one unlike crisp Set where the membership is not partial but completely one or completely zero. In fuzzy Set, elements are represented with one variable quantity, i.e., degree of membership noted as $\boldsymbol{\mu_A(x)} \in [\mathbf{0}, \mathbf{1}] \forall \boldsymbol{x} \in \boldsymbol{X}$ and to express the degree of non-membership a notation $\boldsymbol{v_A(x)} \in [\mathbf{0}, \mathbf{1}] \forall \boldsymbol{x} \in \boldsymbol{X}$ was used. $\{\boldsymbol{x}: \boldsymbol{\mu_A(x)}\}$ is the general representation of a fuzzy Set.

Further generalization of fuzzy Set was made by Atanassov [2-3] in 1986, which is known as the Intuitionistic fuzzy set theory (IFS). IFS addresses the doubt on assigning the membership to an element in the previously discussed fuzzy Set. This expanded doubt is the natural trait of the human mind, known as the hesitation factor of IFS. By introducing the hesitation factor in IFS, the sum of opposite membership values was modified. In IFS, the sum of membership, non-membership, and hesitation is one. The degree of hesitation was generally expressed by the notation"$\boldsymbol{\iota}$"now the modified condition is $\boldsymbol{\mu_A(x)} + \boldsymbol{v_A(x)} + \boldsymbol{\iota_A(x)} = \mathbf{1} \forall \boldsymbol{x} \in \boldsymbol{X}$. The elements of IFS expressed by using two variable quantities."$\boldsymbol{\mu_A(x)}$"and "$\boldsymbol{v_A(x)}$"$\{\boldsymbol{x}: (\boldsymbol{\mu_A(x)}, \boldsymbol{v_A(x)})\}$. Later Intuitionistic fuzzy set theory (IFS) was further modernized by Smarandache [1, 15, 17] in 1995, where he assigned an independent degree to the doubt which was aroused in IFS. He represented membership by $\boldsymbol{T(x)}$, the truth of an event and non-membership by $\boldsymbol{F(x)}$ falsity or opposite to truth and the Indeterminacy $\boldsymbol{I(x)}$ is the neutrality between the truth and its opposite. These three factors are considered as independent factors and represented in a unit cube in the non-standard unit interval $]\mathbf{0}^{-}, \mathbf{1}^{+}[$. Smarandache represented the elements of Standard Neutrosophic Set by using three independent quantities i.e.

$\{\boldsymbol{x}: (\boldsymbol{T(x)}, \boldsymbol{I(x)}, \boldsymbol{F(x)})\}$ with condition $\mathbf{0} \leq (\boldsymbol{T(x)} + \boldsymbol{I(x)} + \boldsymbol{F(x)}) \leq \mathbf{3}$

Soft Set was introduced by Molodtsov [5] in 1999, where he exhibited them as a parametrized family of a subset of the universal set. Soft Set further expanded by Deli, Broumi, Çağman [23][24] in 2015.

Later, Smarandache [16] in 2018 generalized the Soft Set to Hypersoft Set and Plithogenic Hypersoft Set by expanding the function of one attribute into a multi attributes/sub-attribute function. He assigned a combined membership $\boldsymbol{\mu_{A_1 \times A_2 \times \ldots \times A_N}(x)}$, non-membership $\boldsymbol{v_{A_1 \times A_2 \times \ldots \times A_N}(x)}$ and indeteriminacy $\boldsymbol{\iota_{A_1 \times A_2 \times \ldots \times A_N}(x)}$ $\forall \boldsymbol{x} \in \boldsymbol{X}$ with condition $\boldsymbol{A_i} \cap \boldsymbol{A_j} = \boldsymbol{\phi}$ and introduced hybrids of Crisp/ Fuzzy/ Intuitionistic Fuzzy and Neutrosophic hypersoft set and addressed many open problems of development of new literature and MADM techniques.

We have answered some of the open concerns raised by Smarandache [16] in his article "Extension of Soft set to Hypersoft Set and then to Plithogenic Hypersoft Set" published in 2018, in our previous article, titled "Plithogenic Fuzzy Whole Hyper Soft Set, Construction of Local Operators and their Application in Frequency Matrix Multi-

DOI: 10.5281/zenodo.3841624

Attribute Decision Making Technique"[14]. With the help of this Matrix, some local operators for the Plithogenic Fuzzy Hypersoft Set (PFHSS) originated, and their numerical examples constructed. Once these local operators applied to (PFHSS) they gave birth to a new type of Soft Set termed as Plithogenic Fuzzy Whole Hypersoft set (PFWHSS).

Now in this article on its first stage, we represent Plithogenic/Fuzzy/Intuitionistic/Neutrosophic Hyper Soft Set in different and advanced forms of Matrix termed as Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hypersoft Matrix (PC/F/I/NHS-Matrix). This special type of Matrix is generated by parallel sheets of matrices representing parallel universes or parallel realities. Then by hybridization of this novel Hyper-soft Matrix and Plithogenic Fuzzy Whole Hyper Soft Set, we generate a more generalized and expanded version of Soft Set and name these new Soft Set, "Plithogenic Fuzzy/Intuitionistic/Neutrosophic Subjective Hyper- Soft Set." To represent these sets, we further generate a more expanded form of Hyper-Soft-Matrix naming as Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Subjective Hyper-Super-Soft Matrix (PC/F/I/NHSSM) which is a more advance form of Matrix. These matrices are a hybridization of Super-matrices Introduced by Smarandache [8][9] and Hyper matrices [11].

* Now the question arises why we are using hyper-soft and hyper-super-soft matrices for the expression of Plithogenic Hyper-Soft Set and Plithogenic Subjective Hyper-Soft Set? The answer is very simply and truly convincing that is, the Plithogenic universe is so vast and expanded interiorly (like having Fuzzy, Intuitionistic Fuzzy, Picture Fuzzy, Neutrosophic, Pythogorian, Universes with memberships non-memberships and indeterminacies) and exteriorly (dealing with many attributes, sub-attributes and might be sub-sub-attributes regarding many subjects. The expression for such a vastly expanded universe is not possible by using ordinary algebra or matrices. For the repercussion of an expanded universe with vast indeterminate data, we need some new theories like super or hyper-super algebra or hyper-soft and hyper-super-soft matrices.

* This article is an initial draft or effort for initiating and opening a new dimension of expression by using hyper or hyper-super matrices. It is an expanding field, so in this paper, we introduce names and general expressions of these matrices in many environments. A detailed constructed example is presented in Crisp and Fuzzy environments to keep the length of the article within the required limits of the journal. Later, the detailed constructed examples in other suitable environments would be displayed in the form of upcoming articles. For further motivation see [20-26].

In this paper, we discuss some applications of these matrices in the Crisp and fuzzy environment. These Plithogenic Hyper Soft Matrices and Plithogenic Subjective Hyper Super Soft Matrices are tensors of rank three and four, respectively, having three and four indices of variations. The first index represents rows. The second index represents columns. The third index represents parallel sheets of matrices. The fourth index represents several packets of parallel sheets.

In the second stage, we will utilize these special advanced matrices in the development of a ranking model named "Plithogenic Fuzzy/Intuitionistic/Neutrosophic Local, Global, Universal Subjective Ranking model. This subjective

DOI: 10.5281/zenodo.3841624

ranking model will provide several types of subjective ranking. Initially, it gives Local Subjective Ranking, then expands it to Global Subjective Ranking than further extends it to Universal Subjective ranking.

This model offers decision making in different levels of fuzziness according to the state of mind of decision-makers. We may vary the level of fuzziness by choosing a suitable environment like Fuzzy, Intuitionistic Neutrosophic, or all environments combined in one environment, etc.

This model offers a transparent decision by analyzing the universe through several angles of visions. The choice of aggregation operator provides a different angle of vision, either optimist pessimist or neutral.

In this model, Local Subjective Ranking is associated with a particular angle of vision. Global Subjective Ranking combines several angles of vision to offer a transparent decision while the Universal, Subjective Ranking is through compacting the expanded universe to have an outer look of the universe.

At the final stage, we provide an application of this subjective ranking model with the help of a numerical example.

We aim to establish such data handling structures that can reduce the complexity and long calculations in decision-making techniques. Decisions will often be made understeer and clear uncertainties (i.e., with incomplete and uncertain knowledge). With the help of our "Subjective ranking model," problems of incomplete and uncertain knowledge of Artificial Intelligence can be solved to a greater extent.

We are giving such a Mathematical structure/pattern that would possibly deal with the expanded data and will shrink its size in such a manner that, in a glimpse, the outcome of it will be obvious to the observer.

2. Preliminaries

In this section, we present some basic definitions of soft sets, fuzzy soft sets, hypersoft Set, crisp hypersoft Set, fuzzy hypersoft Set, plithogenic hypersoft Set, plithogenic crisp hypersoft sets and plithogenic fuzzy hypersoft Set, hyper Matrix and super-matrix.

Definition 2.1 [5] (Soft Set)

"Let $\boldsymbol{U}$ be the initial universe of discourse, and $\boldsymbol{E}$ be a set of parameters or attributes with respect to $\boldsymbol{U}$ let $\boldsymbol{P(U)}$ denote the power set of $\boldsymbol{U}$, and $\boldsymbol{A \subseteq E}$ is a set of attributes. Then pair $(\boldsymbol{F, A})$, where $\boldsymbol{F: A \longrightarrow P(U)}$ is called Soft Set over $\boldsymbol{U}$. For $\boldsymbol{e \in A}$, $\boldsymbol{F(e)}$ may be considered as Set of $\boldsymbol{e}$ elements or $\boldsymbol{e}$ approximate elements $(\boldsymbol{F, A}) = \{(\boldsymbol{F(e) \in P(U)}: \boldsymbol{e \in E}, \boldsymbol{F(e) = \varphi} \textbf{ if } \boldsymbol{e \notin A}\}$ "

Definition 2.2 [6] (Soft Subset)

"For two soft sets $(\boldsymbol{F, A})$ and $(\boldsymbol{G, B})$ over a universe $\boldsymbol{U}$, we say that $(\boldsymbol{F, A})$ is a soft subset of $(\boldsymbol{G, B})$ if (i) $\boldsymbol{A \subseteq B}$, and (ii) $\boldsymbol{\forall e \in A}$, $\boldsymbol{F(e) \subseteq G(e)}$."

Definition 2.3 [7] (Fuzzy Set)

"Let $\boldsymbol{U}$ be the universe. A fuzzy set $\boldsymbol{X}$ over $\boldsymbol{U}$ is a set defined by a membership function $\boldsymbol{\mu_X}$ representing a mapping $\boldsymbol{\mu_X} : \boldsymbol{U} \longrightarrow [\mathbf{0}, \mathbf{1}]$. The vale of $\boldsymbol{\mu_X}\ (\boldsymbol{x})$ for the fuzzy set $\boldsymbol{X}$ is called the membership value of the grade of membership of $\boldsymbol{x} \in \boldsymbol{U}.$ The membership value represents the degree of belonging to fuzzy set $\boldsymbol{X}$. "A fuzzy set $\boldsymbol{X}$ on $\boldsymbol{U}$ can be presented as follows. $\boldsymbol{X} = \{(\boldsymbol{\mu_X}(\boldsymbol{x})/\boldsymbol{x}) : \boldsymbol{x} \in \boldsymbol{U}, \boldsymbol{\mu_X}(\boldsymbol{x}) \in [\mathbf{0}, \mathbf{1}]\}$ "

Definition 2.4 [4] (Fuzzy Soft Set)

"Let $\boldsymbol{U}$ be the initial universe of discourse, $\boldsymbol{F}(\boldsymbol{U})$ be all fuzzy sets over $\boldsymbol{U}$. $\boldsymbol{E}$ be the Set of all parameters or attributes with respect to $\boldsymbol{U}$ and $\boldsymbol{A} \subseteq \boldsymbol{E}$ is a set of attributes. A fuzzy soft set $\boldsymbol{\Gamma_A}$ on the universe $\boldsymbol{U}$ is defined by the Set of ordered pairs as follows, $\boldsymbol{\Gamma_A} = \{\boldsymbol{x}, \boldsymbol{\gamma_A}(\boldsymbol{x}) : \boldsymbol{x} \in \boldsymbol{E}, \boldsymbol{\gamma_A}(\boldsymbol{x}) \in \boldsymbol{F}(\boldsymbol{U})\}$, where $\boldsymbol{\gamma_A} : \boldsymbol{E} \longrightarrow \boldsymbol{F}(\boldsymbol{U})$ such that $\boldsymbol{\gamma_A}(\boldsymbol{x}) = \boldsymbol{\phi}$ if $\boldsymbol{x} \notin \boldsymbol{A}$, $\boldsymbol{\gamma_A}(\boldsymbol{x}) = \left\{\boldsymbol{\mu_{\gamma_A(x)}}(\boldsymbol{u})/\boldsymbol{u} : \boldsymbol{u} \in \boldsymbol{U}, \boldsymbol{\mu_{\gamma_A(x)}}(\boldsymbol{u}) \in [\mathbf{0}, \mathbf{1}]\right\}$ "

Definition 2.5 [16] (Hypersoft set)

"Let $\boldsymbol{U}$ be the initial universe of discourse $\boldsymbol{P}(\boldsymbol{U})$ the power set of $\boldsymbol{U}$. Let $\boldsymbol{a_1}, \boldsymbol{a_2}, \ldots, \boldsymbol{a_n}$ for $\boldsymbol{n} \geq \mathbf{1}$ be $\boldsymbol{n}$ distinct attributes, whose corresponding attributes values are respectively the sets $\boldsymbol{A_1}, \boldsymbol{A_2}, \ldots, \boldsymbol{A_n}$ with $\boldsymbol{A_i} \cap \boldsymbol{A_j} = \boldsymbol{\varphi}$ for $\boldsymbol{i} \neq \boldsymbol{j}$ and $\boldsymbol{i}, \boldsymbol{j} \in \{\mathbf{1}, \mathbf{2}, \ldots, \boldsymbol{n}\}$. Then the pair $(\boldsymbol{F}, \boldsymbol{A_1} \times \boldsymbol{A} \times \ldots \times \boldsymbol{A_n})$ where, $\boldsymbol{F} : \boldsymbol{A_1} \times \boldsymbol{A} \times \ldots \times \boldsymbol{A_n} \rightarrow \boldsymbol{P}(\boldsymbol{U})$, is called a Hypersoft set over $\boldsymbol{U}$"

Definition 2.6 [16] (Crisp Universe of Discourse)

"A Universe of Discourse $\boldsymbol{U_C}$ is called Crisp if $\forall \boldsymbol{x} \in \boldsymbol{U_c}\ \boldsymbol{x} \in \mathbf{100}\%$ to $\boldsymbol{U_C}$ or membership of $\boldsymbol{x}\ \boldsymbol{T}(\boldsymbol{x})$ with respect to $\boldsymbol{A}$ in $\boldsymbol{M}$ is $\mathbf{1}$ denoted as $\boldsymbol{x}(\mathbf{1})$"

Definition 2.7 [16] (Fuzzy Universe of Discourse)

"A Universe of Discourse $\boldsymbol{U_F}$ is called Fuzzy if $\forall \boldsymbol{x} \in \boldsymbol{U_C}\ \ \boldsymbol{x}$ partially belongs to $\boldsymbol{U_F}$ or membership of $\boldsymbol{x}\ \boldsymbol{T}(\boldsymbol{x}) \subseteq [\mathbf{0}, \mathbf{1}]$ where $\boldsymbol{T}(\boldsymbol{x})$ may be a subset, an interval, a hesitant set, a single value set, denoted as $\boldsymbol{x}(\boldsymbol{T_X})$"

Definition 2.8 [16] (Crisp Hypersoft set)

"Let $\boldsymbol{U_c}$ be the initial universe of discourse $\boldsymbol{P}(\boldsymbol{U_c})$ the power set of $\boldsymbol{U}$. let $\boldsymbol{a_1}, \boldsymbol{a_2}, \ldots, \boldsymbol{a_n}$ for $\boldsymbol{n} \geq \mathbf{1}$ be $\boldsymbol{n}$ distinct attributes, whose corresponding attributes values are respectively the sets $\boldsymbol{A_1}, \boldsymbol{A_2}, \ldots, \boldsymbol{A_n}$ with $\boldsymbol{A_i} \cap \boldsymbol{A_j} = \boldsymbol{\varphi}$ for $\boldsymbol{i} \neq \boldsymbol{j}$ and $\boldsymbol{i}, \boldsymbol{j} \in \{\mathbf{1}, \mathbf{2}, \ldots, \boldsymbol{n}\}$ Then the pair $(\boldsymbol{F_c},\ \boldsymbol{A_1} \times \boldsymbol{A} \times \ldots \times \boldsymbol{A_n})$ where, $\boldsymbol{F_c} : \boldsymbol{A_1} \times \boldsymbol{A} \times \ldots \times \boldsymbol{A_n} \rightarrow \boldsymbol{P}(\boldsymbol{U_c})$, is called Crisp Hypersoft set over $\boldsymbol{U_c}$."

Definition 2.9 [16] (Fuzzy Hypersoft set)

"Let $\boldsymbol{U_F}$ be the initial universe of discourse $\boldsymbol{P}(\boldsymbol{U_F})$ the power set of $\boldsymbol{U_F}$. Let $\boldsymbol{a_1}, \boldsymbol{a_2}, \ldots, \boldsymbol{a_n}$ for $\boldsymbol{n} \geq \mathbf{1}$ be $\boldsymbol{n}$ distinct attributes, whose corresponding attributes values are respectively the sets $\boldsymbol{A_1}, \boldsymbol{A_2}, \ldots, \boldsymbol{A_n}$ with $\boldsymbol{A_i} \cap \boldsymbol{A_j} = \boldsymbol{\varphi}$ for $\boldsymbol{i} \neq \boldsymbol{j}$ and $\boldsymbol{i}, \boldsymbol{j} \in \{\mathbf{1}, \mathbf{2}, \ldots, \boldsymbol{n}\}$. Then the pair $(\boldsymbol{F_F}, \boldsymbol{A_1}, \boldsymbol{A_2}, \ldots, \boldsymbol{A_n})$ where, $\boldsymbol{F_F} : \boldsymbol{A_1} \times \boldsymbol{A} \times \ldots \times \boldsymbol{A_n} \rightarrow \boldsymbol{P}(\boldsymbol{U_F})$, is called Fuzzy Hypersoft set over $\boldsymbol{U_c}$, Now instead of assigning combined membership $\boldsymbol{\mu_{A_1 \times A_2 \times \ldots \times A_N}}(\boldsymbol{x})\ \forall \boldsymbol{x} \in \boldsymbol{X}$ for Hyper Soft sets if each attribute $\boldsymbol{A_j}$ is assigned an individual membership $\boldsymbol{\mu_{A_j}}(\boldsymbol{x})$, non-membership $\boldsymbol{\upsilon_{A_j}}(\boldsymbol{x})$ and Indeteriminacy $\boldsymbol{\iota_{A_j}}(\boldsymbol{x}) \forall \boldsymbol{x} \in \boldsymbol{X}\ \ \boldsymbol{j} = \mathbf{1}, \mathbf{2}, \ldots, \boldsymbol{n}$ in Crisp, Fuzzy, Intuitionistic and Neutrosophic Hypersoft set then these generalized Crisp, Fuzzy, Intuitionistic and Neutrosophic Hypersoft sets are called Plithogenic Crisp, Fuzzy, Intuitionistic Fuzzy and Neutrosophic Hypersoft sets"

DOI: 10.5281/zenodo.3841624

Definition 2.10 [8][9] (Super-matrix)

"A Square or rectangular arrangements of numbers in rows and columns are matrices we shall call them as simple matrices while, super-matrix is one whose elements are themselves matrices with elements that can be either scalars or other matrices. $\boldsymbol{a} = \begin{bmatrix} \boldsymbol{a_{11}} & \boldsymbol{a_{12}} \\ \boldsymbol{a_{21}} & \boldsymbol{a_{22}} \end{bmatrix}$, where $\boldsymbol{a_{11}} = \begin{bmatrix} \mathbf{2} & \mathbf{-4} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}, \boldsymbol{a_{12}} = \begin{bmatrix} \mathbf{0} & \mathbf{40} \\ \mathbf{21} & \mathbf{-12} \end{bmatrix}, \boldsymbol{a_{21}} = \begin{bmatrix} \mathbf{3} & \mathbf{-1} \\ \mathbf{5} & \mathbf{7} \\ \mathbf{-2} & \mathbf{9} \end{bmatrix}$, $\boldsymbol{a_{22}} = \begin{bmatrix} \mathbf{4} & \mathbf{12} \\ \mathbf{-17} & \mathbf{6} \\ \mathbf{3} & \mathbf{7} \end{bmatrix}$ $\boldsymbol{a}$ is a super-matrix".

Note: The elements of super-matrices are called sub-matrices i.e. $\boldsymbol{a_{11}}, \boldsymbol{a_{12}}, \boldsymbol{a_{21}}, \boldsymbol{a_{22}}$ are sub-matrices of the super-matrix $\boldsymbol{a}$.

in this example, the order of super-matrix $\boldsymbol{a}$ is $\mathbf{2 \times 2}$ and order of sub-matrices $\boldsymbol{a_{11}}$ is $\mathbf{2 \times 2}$, $\boldsymbol{a_{12}}$ is $\mathbf{2 \times 2}$ $\boldsymbol{a_{21}}$ is $\mathbf{3 \times 2}$ and order of sub-matrix $\boldsymbol{a_{22}}$ is $\mathbf{3 \times 2}$, we can see that the order of super-matrix doesn't tell us about the order of its sub-matrices.

Definition 2.11 [11] (Hyper-matrix)

"For $\boldsymbol{n_1, \ldots, n_d} \in \boldsymbol{N}$, a function $\boldsymbol{f}: (\boldsymbol{n_1}) \times \ldots \times (\boldsymbol{n_d}) \rightarrow \boldsymbol{F}$ is a hyper-matrix, also called an order-d hyper-matrix or d-hyper-matrix. We often just write $\boldsymbol{a_{k_1 \ldots k_d}}$ to denote the value $\boldsymbol{f(k_1 \ldots k_d)}$ of $\boldsymbol{f}$ at $(\boldsymbol{k_1 \ldots k_d})$ and think of $\boldsymbol{f}$ (renamed as $\boldsymbol{A}$) as specified by a d-dimensional table of values, writing $\boldsymbol{A} = [\boldsymbol{a_{k_1 \ldots k_d}}]_{k_1 \ldots k_d}^{n_1, \ldots, n_d}$, A 3-hypermatrix may be conveniently written down on a (2-dimensional) piece of paper as a list of usual matrices, called slices". For example

$$\boldsymbol{A} = [\boldsymbol{a_{ijk}}] = \begin{bmatrix} \boldsymbol{a_{111}} & \boldsymbol{a_{121}} & \boldsymbol{a_{131}} & \cdot & \boldsymbol{a_{112}} & \boldsymbol{a_{122}} & \boldsymbol{a_{132}} \\ \boldsymbol{a_{211}} & \boldsymbol{a_{221}} & \boldsymbol{a_{231}} & \cdot & \boldsymbol{a_{212}} & \boldsymbol{a_{222}} & \boldsymbol{a_{232}} \\ \boldsymbol{a_{311}} & \boldsymbol{a_{321}} & \boldsymbol{a_{331}} & \cdot & \boldsymbol{a_{312}} & \boldsymbol{a_{322}} & \boldsymbol{a_{332}} \end{bmatrix}$$

3. Plithogenic Hyper-Soft Matrices and Plithogenic Subjective Hyper-Super-Soft Matrices

In this section, we develop some literature about the Plithogenic Hypersoft Set in the following order.

1. Introduction to some basic new concepts and definitions relevant to the hypersoft Set.
2. Generalization of plithogenic whole hypersoft Set to plithogenic subjective hypersoft Set in Crisp, Fuzzy, Intuitionistic, and Neutrosophic environments.
3. Generation of new types of Plithogenic Hypersoft matrix and plithogenic Subjective Hyper-Super-Soft matrix in Crisp, Fuzzy, Intuitionistic, and Neutrosophic environments to represent these new type of sets.
4. Development of expanded and compacted versions of plithogenic Subjective Hyper-Super-Soft Matrix in plithogenic environment.
5. Utilization of new types of matrices for the development of a Subjective ranking model. The specialty of this model is that it offers the classification of non-physical phenomena and Plithogenic

Definition 3.1 (Plithogenic Crisp/ Fuzzy,/Intuitionistic/ Neutrosophic/ Whole Hypersoft set)

If Plithogenic Crisp/Fuzzy/Intuitionistic Fuzzy/Neutrosophic Hyper Soft set expressed by both individual memberships $\boldsymbol{\mu_{A_j^k}(x_i)}$ non-membership $\boldsymbol{\upsilon_{A_j^k}(x_i)}$ and Indeterminacy $\boldsymbol{\iota_{A_j^k}(x_i)}$ $\forall \boldsymbol{x_i} \in \boldsymbol{X}$ $\boldsymbol{j = 1, 2, \ldots, N}$, $\boldsymbol{i = 1, 2, \ldots, M}$

and $\boldsymbol{k = 1, 2, \ldots, L}$ for each given attribute and Combined memberships $\boldsymbol{\Omega_{A_\alpha}(x_i)}$, non-membership $\boldsymbol{\Phi_{A_\alpha}(x_i)}$ and indeteriminacy $\boldsymbol{\Psi_{A_\alpha}(x_i)}$ $\forall \boldsymbol{x_i \in X}$ for the given $\boldsymbol{\alpha}$ Combination of attributes/sub-attributes and subjects ($\boldsymbol{\alpha}$ universe), then These Plithogenic Crisp/Fuzzy/Intuitionistic Fuzzy/Neutrosophic Hyper Soft set are called Plithogenic Crisp/Fuzzy/ Intuitionistic Fuzzy/Neutrosophic Whole Hypersoft set.

Definition 3.2 (Plithogenic Crisp/Fuzzy,/Intuitionistic/Neutrosophic Subjective Hypersoft Set)

If Plithogenic Crisp/Fuzzy/Intuitionistic/ Neutrosophic Hypersoft Set is reflected through subjects in such a way that it exhibits both an interior and exterior view of the universe, then, These Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hyper-Soft-Set titled, Plithogenic Crisp/Fuzzy/ Intuitionistic/ Neutrosophic Subjective Hypersoft set. The interior view of the universe is reflected through individual memberships $\boldsymbol{\mu_{A_j^k}(x_i)}$ non-memberships $\boldsymbol{\upsilon_{A_j^k}(x_i)}$ and Indeterminacies $\boldsymbol{\iota_{A_j^k}(x_i)}$ $\forall \boldsymbol{x_i \in X}$ $\boldsymbol{j = 1, 2, \ldots, N}$, $\boldsymbol{i = 1, 2, \ldots, M}$ and $\boldsymbol{k = 1, 2, \ldots, L}$ of subjects and the exterior view is reflected through summative memberships $\boldsymbol{\Omega_{A_j}^k(X)}$, non-memberships $\boldsymbol{\Phi_{A_j}^k(X)}$ and indeteriminacy $\boldsymbol{\Psi_{A_j}^k(X)}$ $\forall \boldsymbol{x_i \in X}$ united specifically for all attribute/sub-attributes and exhibited w.r.t subjects.

Note: In Plithogenic Crisp/Fuzzy/ Intuitionistic Fuzzy/ Neutrosophic Subjective Hypersoft set the united membership $\boldsymbol{\Omega_{A_\alpha}(x_i)}$, non-membership $\boldsymbol{\Phi_{A_\alpha}(x_i)}$ and indeteriminacy $\boldsymbol{\Psi_{A_\alpha}(x_i)}$ $\forall \boldsymbol{x \in X}$ are dependent on individual membership $\boldsymbol{\mu_{A_j^k}(x_i)}$, non-membership $\boldsymbol{\upsilon_{A_j^k}(x_i)}$ and Indeterminacy $\boldsymbol{\iota_{A_j^k}(x_i)}$.

To achieve a universe, reflected through its subjects (matter for), some terminologies are introduced and described here.

Definition 3.3 (Local Subjective Operators): The aggregation operators, used to cumulate the memberships $\boldsymbol{\mu_{A_j^k}(x_i)}$ non-memberships $\boldsymbol{\upsilon_{A_j^k}(x_i)}$ and Indeterminacies $\boldsymbol{\iota_{A_j^k}(x_i)}$ Of subjects to achieve a universe reflected through its subjects are termed as local subjective operators.

Definition 3.4 (Local Subjective Ranking): Ranking of subjects under consideration by using a particular aggregation operator is called Local Subjective Ranking. This is the case of the classification of the micro-universe.

Definition 3.5 (Global Subjective Ranking): Ranking of subjects under consideration by using multiple aggregation operators and then obtaining a combined ordering in the form of final ranking is called Global Ranking. This is the case of combining several angles of visions.

Definition 3.6 (Universal Ranking): Ranking of Universes by cumulating effects of attributes and subjects with the help of some local operators is called "Universal Ranking." In Universal Ranking, We converge the whole universe to a single numeric value. Where the combined effect of all attributes and subjects of the universe would be represented by a single numeric value and then writing these converged numeric values in descending order for universal ranking purpose.

Definition 3.7 (Hyper-Soft-Matrix): Let $\boldsymbol{U}$ be the initial universe of discourse $\boldsymbol{P(U)}$ be the powerset of $\boldsymbol{U}$. Let $\boldsymbol{A_1^k, A_2^k, \ldots, A_n^k}$ for $\boldsymbol{n \geq 1}$ be $\boldsymbol{n}$ distinct attributes, $\boldsymbol{k = 1, 2, \ldots L}$ representing attributes values, a function $\boldsymbol{F}: \boldsymbol{A_1} \times \boldsymbol{A} \times \ldots \times \boldsymbol{A_n} \rightarrow \boldsymbol{P(U)}$ is a hypersoft matrix, also called an order-$\boldsymbol{M} \times \boldsymbol{N} \times \boldsymbol{L}$ hypersoft matrix or d-hypersoft Matrix ($\boldsymbol{d = 3}$), i.e., a matrix representing a hypersoft set is a hyper-soft matrix (HS-Matrix). As we know, all simple $\boldsymbol{M} \times \boldsymbol{N}$ Matrices on real vector space are tensors of rank 2, so the new Hyper-Soft Matrix having three indices of variations are tensors of rank 3. The Hyper-Soft Matrix is obtained by the generalization of ordinary matrices, known as tensors of rank two. $\boldsymbol{B = [b_{ijk}]}$ is an example of Hyper-Soft Matrix where index $\boldsymbol{i}$ give variation on rows $\boldsymbol{j}$ gives a variation on columns, and $\boldsymbol{k}$ gives variation on clusters of rows and columns.

DOI: 10.5281/zenodo.3841624

The example and detailed illustration of the Hyper-Soft Matrix is presented in the form of the Plithogenic Crisp/Fuzzy Hypersoft Matrix.

The detailed descriptions and applications of the Plithogenic Intuitionistic and Neutrosophic Hyper-Soft Matrix would be exhibited in upcoming versions of research articles.

Definition 3.8 (Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hypersoft Matrix): Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hypersoft Set, when represented in the matrix form are called Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hyper Soft Matrices, i.e., if $\boldsymbol{B}$ is a hypersoft matrix then $\boldsymbol{B} = [\boldsymbol{b_{ijk}}]$, as $\boldsymbol{b_{ijk}}$ are elements of Matrix and would be expressed in Crisp, Fuzzy, Intuitionistic, Neutrosophic environments with memberships $\boldsymbol{\mu_{A_j^k}(x_i)}$ non-memberships $\boldsymbol{v_{A_j^k}(x_i)}$ and Indeterminacy $\boldsymbol{\iota_{A_j^k}(x_i)}$ $\forall \boldsymbol{x_i} \in \boldsymbol{X}$ $\boldsymbol{j = 1, 2, \ldots, N}$, $\boldsymbol{i = 1, 2, \ldots, M}$ and $\boldsymbol{k = 1, 2, \ldots, L}$.

A general form of Plithogenic Hyper-Soft Matrix in fuzzy environment is expressed as

$$\boldsymbol{B} = \left[\boldsymbol{\mu_{A_j^k}(x_i)}\right]$$

A general form of Plithogenic Hyper-Soft Matrix in Intuitionistic environment is expressed as

$$\boldsymbol{B} = \left[(\boldsymbol{\mu_{A_j^k}(x_i)}, \boldsymbol{v_{A_j^k}(x_i)}\right]$$

A general form of Plithogenic Hyper-Soft Matrix in Neutrosophic environment is expressed as

$$\boldsymbol{B} = \left[(\boldsymbol{\mu_{A_j^k}(x_i)}, \boldsymbol{\iota_{A_j^k}(x_i)}, \boldsymbol{v_{A_j^k}(x_i)}\right]$$

These matrices described as,

Let $\boldsymbol{U(X)}$ be the universe of discourse in fuzzy or crisp environment and

$$\boldsymbol{G}: \boldsymbol{A_1^k} \times \boldsymbol{A_2^k} \times \ldots \times \boldsymbol{A_N^k} \rightarrow \boldsymbol{P(U)}$$

The Plithogenic Crisp/Fuzzy Hypersoft Set having individual memberships $\boldsymbol{\mu_{A_j^k}(x_i)}$ $\forall \boldsymbol{x_i} \in \boldsymbol{X}$ $\boldsymbol{j = 1, 2, \ldots, N}$ vary with respect to each attribute $\boldsymbol{A_j}$ and sub-attribute $\boldsymbol{A_j^k}$ ($\boldsymbol{k = 1, 2, 3, \ldots, L}$) where $\boldsymbol{k}$ represent Numeric values of attributes called sub-attributes and $\boldsymbol{i = 1, 2, \ldots, M}$ are the number of subjects under consideration.

In fact in Plithogenic Crisp/Fuzzy Hyper Soft Matrix we have three types of variations introduced on memberships $\boldsymbol{\mu_{A_j^k}(x_i)}$. First variation of $\boldsymbol{i}$ is with respect to subjects representing $\boldsymbol{M}$ rows of $\boldsymbol{M \times N}$ Matrix. The second variation on $\boldsymbol{j}$ is with respect to attributes representing $\boldsymbol{N}$ columns of $\boldsymbol{M \times N}$ Matrix. A third Variation on $\boldsymbol{k}$ is for sub-attributes and represented in the form of $\boldsymbol{L}$ layers or $\boldsymbol{L}$ level sheets of $\boldsymbol{M \times N}$ Matrix. These level cuts of Hyper Matrix are categorized in three types:

1 Vertical front to back and interior level cuts are $\boldsymbol{M \times N}$ Matrix with $\boldsymbol{L}$ level sheets

2 Vertical left to right and interior level cuts are $\boldsymbol{L \times M}$ Matrix with $\boldsymbol{N}$ level sheets

3 Horizontal upper lower and interior level cuts are $\boldsymbol{L \times N}$ Matrix with $\boldsymbol{M}$ level sheets

Definition 3.9 (Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Cubical Hypersoft Matrix):

DOI: 10.5281/zenodo.3841624

If In Plithogenic Hyper-Soft Matrix $= \boldsymbol{M} = \boldsymbol{N}$. That is, the number of horizontal rows , vertical columns and parallel level cuts are equal, then it is termed a Cubicle Hypersoft Matrix.

Definition 3.10 (Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Square Rectangular Hypersoft Matrix):

If In plithogenic hypersoft matrix $\boldsymbol{L} \neq \boldsymbol{M} = \boldsymbol{N}$ we shall have an equal number of horizontal rows vertical columns in all sheets, and the number of sheets is different from the number of rows and columns then this hypersoft Matrix is called Square Rectangular Hypersoft Matrices. For example is for the first numeric value of sub-attribute $\boldsymbol{k} = \boldsymbol{1}$ we get the first sheet in the form of Matrix $[\boldsymbol{b}_{\boldsymbol{ij1}}]$ and for second level of sub-attributes, we get the second sheet of Matrix $[\boldsymbol{b}_{\boldsymbol{ij2}}]$. This procedure will continue until the $\boldsymbol{L}$th sheet obtained by taking $\boldsymbol{k} = \boldsymbol{L}$. These all sheets will form a hyper matrix $[\boldsymbol{b}_{\boldsymbol{ijk}}]$.

$\boldsymbol{L}$ −Hypermatrix for $\boldsymbol{L}$ numeric values of attributes forming $\boldsymbol{L}$ sheets of $\boldsymbol{M} \times \boldsymbol{N}$ Matrix, and if we take $\boldsymbol{L} = \boldsymbol{3}$ we will get $\boldsymbol{3}$ −Hypersoft Matrix in the form of three $\boldsymbol{M} \times \boldsymbol{N}$ parallel sheets.

If the environment of the plithogenic hypersoft Set is Crisp/Fuzzy/intuitionistic/neutrosophic, then the plithogenic hypersoft Matrix is called "Plithogenic Crisp/fuzzy/Intuitionistic/Neutrosophic hypersoft matrix."

Plithogenic Crisp/Fuzzy Hypersoft Set when represented in a matrix form having memberships $\boldsymbol{\mu}_{\boldsymbol{A}_j^k}(\boldsymbol{x}_i)$, We name it as **Plithogenic Crisp/Fuzzy Hyper-Soft Matrix** is exibited as

Plithogenic Crisp/Fuzzy Hyper-Soft Matrix and level cuts,

$\boldsymbol{B} = \left[\mu_{A_j^k}(x_i)\right]$ by fixing $\boldsymbol{k}$ and giving variation to $\boldsymbol{i}$, $\boldsymbol{j}$ the expanded form of $\boldsymbol{B}$ is obtained as an $\boldsymbol{M} \times \boldsymbol{N}$ matrix, having **L** level cuts. These **L** Level cuts are termed as **Type-1 Level Cuts**.

$$\boldsymbol{B} = \begin{bmatrix} \begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_2^1}(x_1) & . & . & . & \mu_{A_N^1}(x_1) \\ \mu_{A_1^1}(x_2) & \mu_{A_2^1}(x_2) & . & . & . & \mu_{A_N^1}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_1^1}(x_M) & \mu_{A_2^1}(x_M) & . & . & . & \mu_{A_N^1}(x_M) \end{bmatrix} \\ \begin{bmatrix} \mu_{A_1^2}(x_1) & \mu_{A_2^2}(x_1) & . & . & . & \mu_{A_N^2}(x_1) \\ \mu_{A_1^2}(x_2) & \mu_{A_2^2}(x_2) & . & . & . & \mu_{A_N^2}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_1^2}(x_M) & \mu_{A_2^2}(x_M) & . & . & . & \mu_{A_N^2}(x_M) \end{bmatrix} \\ . \\ . \\ . \\ \begin{bmatrix} \mu_{A_1^L}(x_1) & \mu_{A_2^L}(x_1) & . & . & . & \mu_{A_N^k}(x_1) \\ \mu_{A_1^L}(x_2) & \mu_{A_2^L}(x_2) & . & . & . & \mu_{A_N^L}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_1^L}(x_M) & \mu_{A^L}(x_M) & . & . & . & \mu_{A_N^L}(x_M) \end{bmatrix} \end{bmatrix} \qquad (1)$$

DOI: 10.5281/zenodo.3841624

Similarly, on the other side, each column of the front sheet with its back columns will form $\boldsymbol{N}$ Level cuts of $\boldsymbol{M} \times \boldsymbol{L}$ matrix termed as **Type-2 Level Cuts**.

These left to right slices are level cuts of type 2, can and be expressed on a two-dimensional page by giving step by step variation to $\boldsymbol{j}$ and expressed as,

$$\boldsymbol{B} = \begin{bmatrix} \begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_1^2}(x_1) & . & . & . & \mu_{A_1^L}(x_1) \\ \mu_{A_1^1}(x_2) & \mu_{A_1^2}(x_2) & . & . & . & \mu_{A_1^L}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_1^1}(x_M) & \mu_{A_1^2}(x_M) & . & . & . & \mu_{A_1^L}(x_M) \end{bmatrix} \\ \begin{bmatrix} \mu_{A_2^1}(x_1) & \mu_{A_2^2}(x_1) & . & . & . & \mu_{A_2^L}(x_1) \\ \mu_{A_2^1}(x_2) & \mu_{A_2^2}(x_2) & . & . & . & \mu_{A_2^L}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_2^1}(x_M) & \mu_{A_2^2}(x_M) & . & . & . & \mu_{A_2^L}(x_M) \end{bmatrix} \\ . \\ . \\ . \\ \begin{bmatrix} \mu_{A_N^1}(x_1) & \mu_{A_N^2}(x_1) & . & . & . & \mu_{A_N^L}(x_1) \\ \mu_{A_N^1}(x_2) & \mu_{A_N^2}(x_2) & . & . & . & \mu_{A_N^L}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_N^1}(x_M) & \mu_{A_N^2}(x_M) & . & . & . & \mu_{A_N^L}(x_M) \end{bmatrix} \end{bmatrix} \quad (2)$$

Similarly, **Type-3 Level Cuts** are upper lower and central interior sheets. These sheets form $\boldsymbol{M}$ cuts of $\boldsymbol{N} \times \boldsymbol{L}$ matrix.

These top to bottom slices are level sheets of type 3 and can be expressed on a two-dimensional page by giving step by step variation to $\boldsymbol{i}$ in $\boldsymbol{B} = \left[\mu_{A_j^k}(x_i)\right]$ and described as,

$$\boldsymbol{B} = \begin{bmatrix} \begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_1^2}(x_1) & . & . & . & \mu_{A_1^L}(x_1) \\ \mu_{A_2^1}(x_1) & \mu_{A_2^2}(x_1) & . & . & . & \mu_{A_2^L}(x_1) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_N^1}(x_1) & \mu_{A_N^2}(x_1) & . & . & . & \mu_{A_N^L}(x_1) \end{bmatrix} \\ \begin{bmatrix} \mu_{A_1^1}(x_2) & \mu_{A_1^2}(x_2) & . & . & . & \mu_{A_1^L}(x_2) \\ \mu_{A_2^1}(x_2) & \mu_{A_2^2}(x_2) & . & . & . & \mu_{A_2^L}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_N^1}(x_2) & \mu_{A_N^2}(x_2) & . & . & . & \mu_{A_N^L}(x_2) \end{bmatrix} \\ . \\ . \\ . \\ \begin{bmatrix} \mu_{A_1^1}(x_M) & \mu_{A_1^2}(x_M) & . & . & . & \mu_{A_1^L}(x_M) \\ \mu_{A_2^1}(x_M) & \mu_{A_2^2}(x_M) & . & . & . & \mu_{A_2^L}(x_M) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_N^1}(x_M) & \mu_{A_N^2}(x_M) & . & . & . & \mu_{A_N^L}(x_M) \end{bmatrix} \end{bmatrix} \quad (3)$$

In $\boldsymbol{\mu_{A_j^k}(x_i)}$ $\boldsymbol{i}$ provides row-wise variations of subjects, $\boldsymbol{j}$ provides column-wise variations of attributes, and $\boldsymbol{k}$ provides variations of sub-attributes in sheets of matrices. These are hyper matrices $[\boldsymbol{a_{ijk}}]$ In the journal of order $\boldsymbol{M \times N \times L}$.

Definition 3.11 (Hyper-Super Matrix): Such a Hyper matrix whose elements are presented in matrix form is titled as Hyper-Super Matrix. These matrices are expressed by using more than two variation indices. These Hyper Super matrices have multiple sheets of ordinary matrices, i.e., = $\begin{bmatrix} [\boldsymbol{a_{ijk}}][\boldsymbol{b_{ijk}}] \\ [\boldsymbol{c_{ijk}}][\boldsymbol{d_{ijk}}] \end{bmatrix}$, B is an example of a hyper-super matrix. The elements of this Matrix ($[\boldsymbol{a_{ijk}}]$, $[\boldsymbol{b_{ijk}}]$, $[\boldsymbol{c_{ijk}}]$ and $[\boldsymbol{d_{ijk}}]$) are hyper-matrices.

* Tthe literature regarding to Hyper super matrices like operators properties application would be explored in upcoming articles)

A Hyper-Super Matrix, used to express a Plithogenic Subjective Hypersoft Set, is an example of Hyper-Super Matrix. The elements of this Matrix are matrices. These hyper-super matrices have multiple packets of parallel sheets and have more than three variations ($\boldsymbol{d > 3}$) .

The examples of HSS-Matrix are **Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Hyper-Super-Soft matrices**

A detailed illustration of the Plithogenic Crisp/fuzzy Hyper-Super-Soft Matrix is presented here, and the rest of the introduced matrices would be presented in upcoming articles.

3.12 (Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Subjective-Hyper-Super-Soft Matrices):

If the matrix representation form of PC/FWHSS is in such a way that the last column matrix of each sheet of the hyper Matrix represents the Combined effect of memberships for sub-attributes with respect to each subject under

DOI: 10.5281/zenodo.3841624

consideration, then these matrices are called Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Subjective Hyper-Super-Soft Matrices whose last column of cumulative memberships makes another matrix, we can see here the elements of hyper matrices are some matrices, so these matrices are basically hyper super matrices because these are such sheets of matrices whose elements are matrices. For example, if the combined memberships for Plithogenic Crisp/ Fuzzy Subjective Hyper Soft Set are denoted by $\boldsymbol{\Omega^t_{A^k}(x_i)}$ for $\boldsymbol{x_i}$ subject, combined attributes for $\boldsymbol{k}$th level of sub-attributes denoted by $\mathbf{A^k}$ and operator used to cumulate memberships for given attributes denoted by $\boldsymbol{t}$, ($\boldsymbol{t = 1, 2, 3, 4}$) one of the four local previously constructed operators (ref), then $\boldsymbol{B_{S_t}} = \left[[\boldsymbol{b_{ijk}}][\boldsymbol{b_{ikt}}]\right]$ Is Plithogenic Crisp/ Fuzzy Subjective Hyper super Soft Matrix, which will provide categorization of subjects from last $\boldsymbol{col}$ memberships values. As $\boldsymbol{b_{ijk}}$ the elements of Matrix representing crisp/ fuzzy memberships and $\boldsymbol{b_{ikt}}$ representing cumulative crisp/fuzzy memberships for some subject with respect to all given attributes can be expressed as $\boldsymbol{\Omega^t_{A^k}(x_i)}$.

More generalized form of $\boldsymbol{B_{S_t}}$ is. $\boldsymbol{B_{S_t}} = \left[\left[\boldsymbol{\mu_{A_j^k}(x_i)}\right]\left[\boldsymbol{\Omega^t_{A^k}(x_i)}\right]\right]$ and further expanded form with respect to $\boldsymbol{i}$ and $\boldsymbol{j}$ is,

$$\boldsymbol{B_{S_t}} = \begin{bmatrix} \begin{bmatrix} \mu_{A_1^k}(x_1) & \mu_{A_2^k}(x_1) & . & . & . & \mu_{A_N^k}(x_1) \\ \mu_{A_1^k}(x_2) & \mu_{A_2^k}(x_2) & . & . & . & \mu_{A_N^k}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_1^k}(x_M) & \mu_{A_2^k}(x_M) & . & . & . & \mu_{A_N^k}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^t_{A^k}(x_1) \\ \Omega^t_{A^k}(x_2) \\ . \\ . \\ . \\ \Omega^t_{A^k}(x_M) \end{bmatrix} \end{bmatrix} \tag{4}$$

In Plithogenic Crisp/ Fuzzy Subjective Hyper super Soft Matrix, $\boldsymbol{B_{S_t}}$ we give four type of variations

Example: Different cross-sectional cuts of $\boldsymbol{B_{S_t}}$ a super hyper matrix will form packets of multiple parallel sheets, i.e., multiple combinations of sheets, which is a more expanded universe.

In $\boldsymbol{B_{S_t}} = \left[\left[\boldsymbol{\mu_{A_j^k}(x_i)}\right]\left[\boldsymbol{\Omega^t_{A^k}(x_i)}\right]\right]$, Variations of $\boldsymbol{i}$ generates rows, variations of $\boldsymbol{j}$ generates columns, variations of $\boldsymbol{k}$ generates combinations of rows and columns as parallel layers of $\boldsymbol{M \times N}$ matrix and variation of $\boldsymbol{t = 1, 2, \ldots P}$ will provide $\boldsymbol{P}$ set of multiple $\boldsymbol{L}$ parallel layers of matrices.

These sets of combinations of sheets may represent parallel universes, on a two-dimensional sheet of paper the expanded form of general hyper super-matrix

$\boldsymbol{B_{S_t}} = \left[\left[\boldsymbol{\mu_{A_j^k}(x_i)}\right]\left[\boldsymbol{\Omega^t_{A^k}(x_i)}\right]\right]$ can be represented as

DOI: 10.5281/zenodo.3841624

$$
\begin{bmatrix}
\begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_2^1}(x_1) & \cdots & \mu_{A_N^1}(x_1) \\ \mu_{A_1^1}(x_2) & \mu_{A_2^1}(x_2) & \cdots & \mu_{A_N^1}(x_2) \\ \vdots & \vdots & & \vdots \\ \mu_{A_1^1}(x_M) & \mu_{A_2^1}(x_M) & \cdots & \mu_{A_N^1}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^1_{A^1}(x_1) \\ \Omega^t_{A^1}(x_2) \\ \vdots \\ \Omega^1_{A^1}(x_M) \end{bmatrix} \\
\begin{bmatrix} \mu_{A_1^2}(x_1) & \mu_{A_2^2}(x_1) & \cdots & \mu_{A_N^2}(x_1) \\ \mu_{A_1^2}(x_2) & \mu_{A_2^2}(x_2) & \cdots & \mu_{A_N^2}(x_2) \\ \vdots & \vdots & & \vdots \\ \mu_{A_1^2}(x_M) & \mu_{A_2^2}(x_M) & \cdots & \mu_{A_N^2}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^1_{A^2}(x_1) \\ \Omega^1_{A^2}(x_2) \\ \vdots \\ \Omega^1_{A^2}(x_M) \end{bmatrix} \\
\vdots \\
\begin{bmatrix} \mu_{A_1^L}(x_1) & \mu_{A_2^L}(x_1) & \cdots & \mu_{A_N^k}(x_1) \\ \mu_{A_1^L}(x_2) & \mu_{A_2^L}(x_2) & \cdots & \mu_{A_N^L}(x_2) \\ \vdots & \vdots & & \vdots \\ \mu_{A_1^L}(x_M) & \mu_{A^L}(x_M) & \cdots & \mu_{A_N^L}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^1_{A^L}(x_1) \\ \Omega^1_{A^L}(x_2) \\ \vdots \\ \Omega^1_{A^L}(x_M) \end{bmatrix} \\
\begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_2^1}(x_1) & \cdots & \mu_{A_N^1}(x_1) \\ \mu_{A_1^1}(x_2) & \mu_{A_2^1}(x_2) & \cdots & \mu_{A_N^1}(x_2) \\ \vdots & \vdots & & \vdots \\ \mu_{A_1^1}(x_M) & \mu_{A_2^1}(x_M) & \cdots & \mu_{A_N^1}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^2_{A^1}(x_1) \\ \Omega^2_{A^1}(x_2) \\ \vdots \\ \Omega^2_{A^1}(x_M) \end{bmatrix} \\
\begin{bmatrix} \mu_{A_1^2}(x_1) & \mu_{A_2^2}(x_1) & \cdots & \mu_{A_N^2}(x_1) \\ \mu_{A_1^2}(x_2) & \mu_{A_2^2}(x_2) & \cdots & \mu_{A_N^2}(x_2) \\ \vdots & \vdots & & \vdots \\ \mu_{A_1^2}(x_M) & \mu_{A_2^2}(x_M) & \cdots & \mu_{A_N^2}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^2_{A^2}(x_1) \\ \Omega^2_{A^2}(x_2) \\ \vdots \\ \Omega^2_{A^2}(x_M) \end{bmatrix} \\
\vdots \\
\begin{bmatrix} \mu_{A_1^L}(x_1) & \mu_{A_2^L}(x_1) & \cdots & \mu_{A_N^k}(x_1) \\ \mu_{A_1^L}(x_2) & \mu_{A_2^L}(x_2) & \cdots & \mu_{A_N^L}(x_2) \\ \vdots & \vdots & & \vdots \\ \mu_{A_1^L}(x_M) & \mu_{A^L}(x_M) & \cdots & \mu_{A_N^L}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^2_{A^L}(x_1) \\ \Omega^2_{A^L}(x_2) \\ \vdots \\ \Omega^2_{A^L}(x_M) \end{bmatrix} \\
\vdots \\
\begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_2^1}(x_1) & \cdots & \mu_{A_N^1}(x_1) \\ \mu_{A_1^1}(x_2) & \mu_{A_2^1}(x_2) & \cdots & \mu_{A_N^1}(x_2) \\ \vdots & \vdots & & \vdots \\ \mu_{A_1^1}(x_M) & \mu_{A_2^1}(x_M) & \cdots & \mu_{A_N^1}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^P_{A^1}(x_1) \\ \Omega^P_{A^1}(x_2) \\ \vdots \\ \Omega^P_{A^1}(x_M) \end{bmatrix} \\
\begin{bmatrix} \mu_{A_1^2}(x_1) & \mu_{A_2^2}(x_1) & \cdots & \mu_{A_N^2}(x_1) \\ \mu_{A_1^2}(x_2) & \mu_{A_2^2}(x_2) & \cdots & \mu_{A_N^2}(x_2) \\ \vdots & \vdots & & \vdots \\ \mu_{A_1^2}(x_M) & \mu_{A_2^2}(x_M) & \cdots & \mu_{A_N^2}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^P_{A^2}(x_1) \\ \Omega^P_{A^2}(x_2) \\ \vdots \\ \Omega^P_{A^2}(x_M) \end{bmatrix} \\
\vdots \\
\begin{bmatrix} \mu_{A_1^L}(x_1) & \mu_{A_2^L}(x_1) & \cdots & \mu_{A_N^k}(x_1) \\ \mu_{A_1^L}(x_2) & \mu_{A_2^L}(x_2) & \cdots & \mu_{A_N^L}(x_2) \\ \vdots & \vdots & & \vdots \\ \mu_{A_1^L}(x_M) & \mu_{A^L}(x_M) & \cdots & \mu_{A_N^L}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^P_{A^L}(x_1) \\ \Omega^P_{A^L}(x_2) \\ \vdots \\ \Omega^P_{A^L}(x_M) \end{bmatrix}
\end{bmatrix} \tag{5}
$$

DOI: 10.5281/zenodo.3841624

It is observed that the multiple sets of parallel sheets (universes) are achieved by giving step by step variation to the fourth index $\boldsymbol{t}$, which is used to represent several aggregation operators to homogenize attributes established as local operators.

It is interesting to note That if we see the hyper-super-soft-matrix on a two-dimensional screen, then columns of sheets of $\boldsymbol{M} \times \boldsymbol{N} \times \boldsymbol{L}$ Matrix will form a general $\boldsymbol{M} \times \boldsymbol{L}$ matrices.

Greater the numeric value of $\boldsymbol{\Omega}^{\boldsymbol{t}}_{\mathbf{A}^k}(\boldsymbol{x_i})$ better is the $\boldsymbol{x_i}$ subject under consideration. This shows that the whole effect of the Matrix can be visualized by observing the last column. $\left[\left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{\boldsymbol{t}}_{\mathbf{A}^k}(\boldsymbol{x_i})\right]\right]$ is representing a Hyper Super Soft Matrix and for subjective categorization with respect to different levels of sub-attributes is obtained by keeping in view a single sheet at a time, to achieve the purpose we shall use one of the sheets of this hyper super-matrix $\left[\left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{\boldsymbol{t}}_{\mathbf{A}^k}(\boldsymbol{x_i})\right]\right]$ by fixing $\boldsymbol{k}$ for a specific level of the attribute, and then we will vary " $\boldsymbol{t}$ " which is used to represent a particular aggregation operator, and this operator could be used to cumulate attributes effect at a certain level.

$\boldsymbol{B}_{\mathbf{S}_t} = \left[\left[\boldsymbol{\mu}_{A_j^1}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{\mathbf{1}}_{\mathbf{A}^\alpha}(\boldsymbol{x_i})\right]\right]$ will represent a single sheet of hyper super-matrix for $\boldsymbol{\alpha}$ universe at a fixed numeric value of attribute of first level $\boldsymbol{k} = \mathbf{1}$ and using particular operator $\boldsymbol{t} = \mathbf{1}$.

By fixing a combination of attributes, sub-attributes and subjects means considering one of the parallel sheets (one parallel universe) of the hyper-super-soft- Matrix by using Plithogenic Fuzzy Subjective Hyper Super Soft Matrices.

One may decide which subject is superior with respect to all attributes by analyzing the last column of the sheet (case of local Ranking), the subject which corresponds to the greater numeric value in the last column will be considered better, and then we will write them in descending order.

4. Application

Local, Global and Universal Subjective Ranking Model

In this section, we utilize the local operators constructed in an earlier published article ([26]) in formulation of Subjective Ranking model in plithogenic Crisp/Fuzzy environment.

1. The specialty of this model is that it offers the classification of subjects by observing them through several angles of vision.

2. Different angles of visions can be expressed by choosing a suitable local operator

3. This model offers classification in many environments where every environment has its ambiguity and uncertainty level.

4. This Subjective ranking model provides the ranking from micro-universe to the macro universe level.

5. Initially, this model provides the internal Ranking named "Local Subjective ranking" (classification of subjects of micro-universe). In Local Ranking, one perticular local operators is used for the classification purpose.

DOI: 10.5281/zenodo.3841624

6. At the next stage, this model offers an external classification of subjects named "Global Subjective Ranking." In Global Ranking, we combine several operators to formulate an expression that reflected through many angles of visions.

7. This model offers the third type of subjective Ranking named "Universal Subjective Ranking (case of macro universe Classification)

8. This model would provide extreme and neutral values of universes that would be helpful to find out the optimal and neutral behaviors of all types of universe micro to macro levels.

9. At the last stage, it provides a measure of the authority of classification by using the frequency matrix.

Initially, we consider the case of plithogenic Crisp/fuzzy Subjective-Hyper-Super-Soft matrices for classification of subjects. Later we may generalize these models in Plithogenic Intuitionistic and Plithogenic Neutrosophic environments. Describe the subjective ranking model in the given steps.

Step 1. Construction of Universe: Consider universe of discourse $\boldsymbol{U} = \{\boldsymbol{x_i}\}$ $\boldsymbol{i} = \mathbf{1, 2, 3, \ldots, P}$. Consider some attributes and subjects needed to be classified where attributes are $\boldsymbol{A_j^k}$ $\boldsymbol{j} = \mathbf{1, 2, 3, \ldots,} \boldsymbol{N}$ and $\boldsymbol{k} = \mathbf{1, 2, \ldots,} \boldsymbol{L}$ represents numeric values (levels) of $\boldsymbol{A_j}$, and subjects are $\boldsymbol{T} = \{\boldsymbol{x_i}\} \subset \boldsymbol{U}$ where $\boldsymbol{i}$ can take some values from $\mathbf{1}$ to $\boldsymbol{M}$ such that $\boldsymbol{M} \leq \boldsymbol{p}$ and define mappings $\boldsymbol{F}$ and $\boldsymbol{G}$ such that,

$$\boldsymbol{F}: \boldsymbol{A_1^k} \times \boldsymbol{A_2^k} \times \boldsymbol{A_3^k} \times \ldots \times \boldsymbol{A_N^k} \to \boldsymbol{P(U)} \qquad \text{For some fixed } \boldsymbol{k} \text{ (level} \mathbf{1})$$

$$\boldsymbol{G}: \boldsymbol{A_1^k} \times \boldsymbol{A_2^k} \times \boldsymbol{A_3^k} \times \ldots \times \boldsymbol{A_N^k} \to \boldsymbol{P(U)} \qquad \text{For some different fixed } \boldsymbol{k} \text{ (level } \mathbf{2})$$

Step 2. Plithogenic Crisp/Fuzzy Hyper Soft Matrix: Write the data or information (Memberships) in the form of Plithogenic Fuzzy Hyper Soft Matrix $\boldsymbol{B} = \left[\boldsymbol{\mu}_{\boldsymbol{A_j^k}}(\boldsymbol{x_i})\right]$

Step 3. Plithogenic Crisp/Fuzzy Subjective Hyper-super-Soft Matrix: By using previously constructed local operators formulate Plithogenic Fuzzy Hyper super Soft Matrix

$$\boldsymbol{B}_{\mathbf{S}_t} = \left[\left[\boldsymbol{\mu}_{\boldsymbol{A_j^k}}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{\boldsymbol{t}}_{\mathbf{A}^{k}}(\boldsymbol{x_i})\right]\right].$$

In formulating $\left[\boldsymbol{\Omega}^{\boldsymbol{t}}_{\mathbf{A}^{k}}(\boldsymbol{x_i})\right]$ we might be handling some favorable, some neutral and some non-favorable attributes, the non-favorable attributes can be handled by replacing their corresponding memberships with standard compliments i.e. $\boldsymbol{\mu}^{\boldsymbol{c}}_{\boldsymbol{A_j^k}}(\boldsymbol{x_i}) = \mathbf{1} - \boldsymbol{\mu}_{\boldsymbol{A_j^k}}(\boldsymbol{x_i})$, While for neutral attributes, the regular memberships can be used.The cumulative memberships $\boldsymbol{\Omega}^{\boldsymbol{t}}_{\mathbf{A}^{k}}(\boldsymbol{x_i})$ are obtained by uniting regular memberships for favorable and neutral attributes and compliments of memberships for non-favorable attributes by using local operators.

Step 4. Local Ranking: The Local Ranking would be obtained by observing cumulative memberships $\boldsymbol{\Omega}^{\boldsymbol{t}}_{\mathbf{A}^{\alpha}}(\boldsymbol{x_i})$ of last column of

$$\boldsymbol{B}_{\mathbf{S}_t} = \left[\left[\boldsymbol{\mu}_{\boldsymbol{A_j^k}}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{\boldsymbol{t}}_{\mathbf{A}^{k}}(\boldsymbol{x_i})\right]\right],$$

Higher the membership value better is the subject corresponding to that membership.

DOI: 10.5281/zenodo.3841624

At this stage, the classification of all sheets or one selected sheet would be provided according to the requirement. The process can be stopped here if transparent Ranking is achieved.

If there are some ties or ambiguities, would be removed in the next step, and more transparent Ranking can be provided with authenticity measurement.

Step 5. Global Ranking: Final Global Subjective ordering can be provided by using the Frequency matrix.

$$\boldsymbol{F_{ij}} = \begin{array}{c} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_M \end{array} \begin{bmatrix} f_{11} & f_{12} & \cdot & \cdot & \cdot & f_{1M} \\ f_{21} & f_{22} & \cdot & \cdot & \cdot & f_{2M} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ f_{M1} & f_{M2} & \cdot & \cdot & \cdot & f_{MM} \end{bmatrix} \tag{6}$$

In $\boldsymbol{F_{ij}}$ the elements of the first column will represent the frequency of obtaining the first position of given subjects. The elements of the second column will represent the frequency of obtaining the second position, and so on. To find out which subject would attain the first position we observe the entries of the first column of $\boldsymbol{F_{ij}}$ the subject corresponding to the highest value of the first column attains the first position, and then we delete the first position and the subject who have achieved that position. Afterward, for the selection of the second position, add the remaining frequencies of first positions in the frequencies of the second column and then look for the highest frequency in that column.

Once second position is decided to delete column and row of that position correspondingly and continue the procedure until the latest position is assigned

Step 6. Authenticity Measure of Global Ranking: We may check the authenticity in the last step by taking ratios.

percentage authenticity of j $_{th}$ position for i $_{th}$ subject

$$= \frac{\text{Highest frequency of } j_{th} \text{ position}}{\text{Total frequency of } j_{th} \text{position}} \times \mathbf{100} = \frac{\max_i (f_{ij})}{\sum_i (f_{ij})} \times \mathbf{100} \tag{7}$$

Step 7. Universal Ranking: If comparison of universes is required we may provide by combining cumulative memberships of last column for fixed $\boldsymbol{k}$ and $\boldsymbol{t}$ for each universe. The universe having the largest cumulative memberships is considered as a better universe and then writes these cumulative memberships in a descending order.

$$\boldsymbol{\Omega_{k1}} = \max_i \left[\boldsymbol{\Omega}_{\mathrm{A}^k}(\boldsymbol{x_i})\right] \tag{8}$$

$$\boldsymbol{\Omega_{k2}} = \max_i \left[\boldsymbol{\Omega}^2_{\mathrm{A}^k}(\boldsymbol{x_i})\right] \tag{9}$$

$$\boldsymbol{\Omega_{k3}} = \frac{\sum_i \left[\Omega^3_{\mathrm{A}^k}(x_i)\right]}{n} \tag{10}$$

5 Local operators for Construction of Plithogenic crisp/fuzzy Subjective Hyper Super Soft matrices.

These local operators for the plithogenic Crisp/Fuzzy hypersoft Set can be utilized to formulate Plithogenic Crisp/Fuzzy Subjective Hyper Super Soft Matrices.

DOI: 10.5281/zenodo.3841624

By using local disjunction, local conjunction and local averaging operators we developed a combined (whole) memberships $\boldsymbol{\Omega}^{t}_{\mathrm{A}^{\alpha}}(\boldsymbol{x_i})$ for Plithogenic Hyper-Soft-Set. By applying local aggregation operators on Crisp/fuzzy Subjective hyper soft matrices $\boldsymbol{B} = \left[\boldsymbol{\mu}_{A^k_j}(\boldsymbol{x_i})\right]$ the last column of cumulative memberships $\boldsymbol{\Omega}^{t}_{\mathrm{A}}(\boldsymbol{x_i})$ is obtained. To achieve the purpose we use three local operators, $\boldsymbol{t = 1}$ is used for $\mathbf{max}$, $\boldsymbol{t = 2}$ is used for $\mathbf{min}$, and $\boldsymbol{t = 3}$ is used for averaging operator described as,

$$\cup_{\boldsymbol{i}}\left(\boldsymbol{\mu}_{A^k_j}(\boldsymbol{x_i})\right) = \underset{j}{\boldsymbol{Max}}\left(\boldsymbol{\mu}_{A^k_j}(\boldsymbol{x_i})\right) = \boldsymbol{\Omega}^{1}_{\mathrm{A}}(\boldsymbol{x_i}) \tag{12}$$

$$\cap_{\boldsymbol{i}}\left(\boldsymbol{\mu}_{A^k_j}(\boldsymbol{x_i})\right) = \underset{j}{\boldsymbol{Min}}(\boldsymbol{\mu}_{A^k_j}(\boldsymbol{x_i})) = \boldsymbol{\Omega}^{2}_{\mathrm{A}}(\boldsymbol{x_i}) \tag{13}$$

$$\boldsymbol{\Gamma_i}\left(\boldsymbol{\mu}_{A^k_j}(\boldsymbol{x_i})\right) = \frac{\sum_j\left(\boldsymbol{\mu}_{A^k_j}(\boldsymbol{x_i})\right)}{\boldsymbol{M}} = \boldsymbol{\Omega}^{3}_{\mathrm{A}}(\boldsymbol{x_i}) \tag{14}$$

6 Application of Subjective ranking model in Crisp environment

Numerical Example:

To achieve the purpose, we will develop plithogenic Fuzzy hyper-soft matrix $\boldsymbol{B}$ for two Plithogenic crisp Hyper Soft Sets with $\boldsymbol{\alpha}$ Combination and $\boldsymbol{\beta}$ Combination of attributes, i.e., for $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ universes.

By choosing different numeric values of $\boldsymbol{k}$ consider $\boldsymbol{\alpha}$ Combination of attributes for $\boldsymbol{F}$ and $\boldsymbol{\beta}$ Combination of attributes for $\boldsymbol{G}$.

Step 1. Construction of Universe: Consider $\boldsymbol{U} =$ is a set of five candidates in the Mathematics department. $\boldsymbol{T} = \{\mathbf{Peter}, \mathbf{Aina}, \mathbf{kitty}\} \subset \boldsymbol{U}$ is the Set of three candidates, $\boldsymbol{T} = \{\mathbf{Peter}, \mathbf{Aina}, \mathbf{kitty}\}$Who have Applied for the selection of the post of Assistant professor are our subjects required to be classified with respect to following $\boldsymbol{A^k_j}$ attributes and sub-attributes.

$\boldsymbol{A^k_1} =$ Subject skill area with numeric values, $\boldsymbol{k = 1, 2}$

$\boldsymbol{A^1_1} =$ Pure Mathematics, $\boldsymbol{A^2_1} =$ Applied Mathematics

$\boldsymbol{A^k_2} =$ Qualification with numeric values, $\boldsymbol{k = 1, 2}$

$\boldsymbol{A^1_2} =$ Qualification like MS or Equivalent, $\boldsymbol{A^2_2} =$ Higher Qualification like Ph.D. or Equivalent

$\boldsymbol{A^k_3} =$ Teaching experience with numeric values, $\boldsymbol{k = 1, 2}$

$\boldsymbol{A^1_3} =$ Five years or less, $\boldsymbol{A^2_3} =$ More than Five years

$\boldsymbol{A^k_4} =$ Age , with numeric values $\boldsymbol{k = 1, 2}$

$\boldsymbol{A^1_4} =$ Age is less than forty years $\boldsymbol{A^2_4} =$ Age is more than forty years

Consider mapping $\boldsymbol{F}$ and $\boldsymbol{G}$ such that,

$\boldsymbol{F}: \boldsymbol{A^k_1} \times \boldsymbol{A^k_2} \times \boldsymbol{A^k_3} \times \ldots \times \boldsymbol{A^k_N} \to \boldsymbol{P(U)}$ (choosing some numeric values of $\boldsymbol{A^k_j}$ $\boldsymbol{k} \in (\boldsymbol{1, L})$

DOI: 10.5281/zenodo.3841624

$\boldsymbol{G}: \boldsymbol{A_1^k} \times \boldsymbol{A_2^k} \times \boldsymbol{A_3^k} \times \ldots \times \boldsymbol{A_N^k} \rightarrow \boldsymbol{P(U)}$ (choosing some different numeric values of $\boldsymbol{A_j^k}$ $\boldsymbol{k} \in (\mathbf{1}, \boldsymbol{L})$Let the these candidates are considered as subjects under consideration, and attributes are $\boldsymbol{A_j^k}$ $\boldsymbol{j} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}$ and $\boldsymbol{k} = \mathbf{1}, \mathbf{2}, \mathbf{3}$

we are looking for a single employ amongst the three candidates, $\boldsymbol{T} = \{\textbf{Peter}, \textbf{Aina}, \textbf{kitty}\} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}\}$, where $\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}$ represent $\boldsymbol{x_i}$ subjects under consideration, initially we consider the first level for $\boldsymbol{k} = \mathbf{1}$ i.e.

1. Subject skill area: Pure Mathematics $\boldsymbol{j} = \mathbf{1}$, $\boldsymbol{k} = \mathbf{1}$

2. Qualification: Qualification like MS or Equivalent $\boldsymbol{j} = \mathbf{2}$, $\boldsymbol{k} = \mathbf{1}$

3. Teaching experience: Five years or less $\boldsymbol{j} = \mathbf{3}$, $\boldsymbol{k} = \mathbf{1}$

4. Age: Age required is less than forty years $\boldsymbol{j} = \mathbf{4}$, $\boldsymbol{k} = \mathbf{1}$

Let the Function is $\boldsymbol{F}$ is defined as,

$\boldsymbol{F}(\boldsymbol{A_1^1}, \boldsymbol{A_2^1}, \boldsymbol{A_3^1}, \boldsymbol{A_4^1}) = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}\}$ let we name $\boldsymbol{A_1^1}$, $\boldsymbol{A_2^1}$, $\boldsymbol{A_3^1}$, $\boldsymbol{A_4^1}$ as $\boldsymbol{\alpha}$ Combination representing the first level for $\boldsymbol{k} = \mathbf{1}$

Let the Function is $\boldsymbol{G}$ is defined as,

$$\boldsymbol{G}: \boldsymbol{A_1^k} \times \boldsymbol{A_2^k} \times \boldsymbol{A_3^k} \times \boldsymbol{A_4^k} \rightarrow \boldsymbol{P(U)}$$

We are looking for another employ amongst these three candidates for the Category of applied mathematics with a different combination of attributes say $\boldsymbol{\beta}$ Combination for next level ($\boldsymbol{k} = \mathbf{2}$).

1. Subject skill area: Applied Mathematics $\boldsymbol{j} = \mathbf{1}$, $\boldsymbol{k} = \mathbf{2}$

2. Qualification: Higher Qualification like Ph.D. or equivalent $\boldsymbol{j} = \mathbf{2}$, $\boldsymbol{k} = \mathbf{2}$

3. Teaching experience: More than five years $\boldsymbol{j} = \mathbf{3}$, $\boldsymbol{k} = \mathbf{2}$

4. Age: Age is more than forty years $\boldsymbol{j} = \mathbf{3}$, $\boldsymbol{k} = \mathbf{2}$

$\boldsymbol{G}(\boldsymbol{A_1^2}, \boldsymbol{A_2^2}, \boldsymbol{A_3^2}, \boldsymbol{A_4^2}) = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}\}$ let we name $\boldsymbol{A_1^2}, \boldsymbol{A_2^2}, \boldsymbol{A_3^2}, \boldsymbol{A_4^2}$ as $\boldsymbol{\beta}$ Combination.with respect to. $\boldsymbol{T} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}\}$ Have memberships In PCHSS, PFHSS, which are assigned by decision-makers in crisp environment memberships, are considered as $\mathbf{1}$, or $\mathbf{0}$.

While in a fuzzy environment, memberships are assigned by decision-makers between $\mathbf{0}$ and $\mathbf{1}$ by using five-point scale linguistic chart.

Step 2. Plithogenic Crisp Hyper Soft Matrix:

With respect to $\boldsymbol{T} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}\}$ memberships In PCHSS are,

$$\boldsymbol{F}(\boldsymbol{A_1^1}, \boldsymbol{A_2^1}, \boldsymbol{A_3^1}, \boldsymbol{A_4^1}) = \boldsymbol{F}(\boldsymbol{\alpha}) = \{\boldsymbol{x_1}(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}), \boldsymbol{x_2}(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}), \boldsymbol{x_3}(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1})\}$$

$$\boldsymbol{G}(\boldsymbol{A_1^2}, \boldsymbol{A_2^2}, \boldsymbol{A_3^2}, \boldsymbol{A_4^2}) = \boldsymbol{F}(\boldsymbol{\beta}) = \{\boldsymbol{x_1}(\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}), \boldsymbol{x_2}(\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}), \boldsymbol{x_3}(\mathbf{0}, \mathbf{0}, \mathbf{0},)\}$$

DOI: 10.5281/zenodo.3841624

The **Plithogenic Crisp Hyper Soft matrix** $\boldsymbol{B} = \left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]$ $\mathbf{1}$for $\boldsymbol{k} = \mathbf{1}, \boldsymbol{j} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}$ and $\boldsymbol{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}$ i.e. $\boldsymbol{\alpha}$ Combination (first level sheet) and $\boldsymbol{k} = \mathbf{2}, \boldsymbol{j} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}$ and $\boldsymbol{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}$ i.e.$\boldsymbol{\beta}$ Combination (second level sheet) will be represented as $\boldsymbol{B} = \left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]$

$$\boldsymbol{B} = \begin{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{bmatrix} \qquad (14)$$

In Crisp Hyper Soft matrix $\boldsymbol{B}$ the first level sheet for $\boldsymbol{\alpha}$ Combination of attribute values all memberships are one means the members fulfill the first level requirements fully but for second-level sheet i.e.$\boldsymbol{\beta}$ combination members may fulfill the requirements partially, but the crisp universe don't deal with partial belongingness, so all memberships are zero.

Step 3. Plithogenic Crisp Subjective Hyper-Super-Soft Matrix:

The Plithogenic Crisp Hyper Soft matrix $\boldsymbol{B}_{\mathbf{S}_t} = \left[\left[\boldsymbol{\mu}_{A_j^k}(\boldsymbol{x_i})\right]\left[\boldsymbol{\Omega}^{\boldsymbol{t}}_{\mathbf{A}^k}(\boldsymbol{x_i})\right]\right]$ by using eq. $\mathbf{5}$ for $= \mathbf{1}, \mathbf{2}, \mathbf{3}$, $\boldsymbol{j} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}$ and $\boldsymbol{k} = \mathbf{1}$, i.e., .$\boldsymbol{\alpha}$ Combination (first level sheet), $\boldsymbol{k} = \mathbf{2}$, $\boldsymbol{\beta}$ Combination (second level sheet) $\boldsymbol{t} = \mathbf{1}$ (disjunction operator) first set of two-level sheets, $\boldsymbol{t} = \mathbf{2}$ (conjunction operator) second Set of two-level sheets will be represented as,

$$\boldsymbol{B}_{\mathbf{S}_t} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \end{bmatrix} \qquad (15)$$

It is obvious from this HSS-Matrix that in Crisp environment $\boldsymbol{B}_{\boldsymbol{St}}$ representing trivial cases where the subjective categorization is not visible because all values in the last column are the same, so we in future Classifications we will use Plithogenic Fuzzy subjective Hyper-soft matrices for subjective categorization. Further, it is observed that the crisp universe case is trivial, where the categorization is not possible. So next steps would not proceed.

Note: In the next levels, we will consider only a fuzzy environment to achieve transparent classification.

Application of Subjective ranking model in Fuzzy environment

Step1. Construction of Universe: To formulate Plithogenic Fuzzy Hyper Soft Matrix Step 1 and Step 2 are the same as already described in the crisp universe case.

Step 2. Plithogenic Fuzzy Hyper-Super-Soft Matrix:

Memberships of $\boldsymbol{T} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}\}$ In PFHSS with respect to given attribute memberships are assigned by the decision-makers by using linguistic five point scale method,[1], [18], [34] [35]

DOI: 10.5281/zenodo.3841624

$$F(A_1^1, A_2^1, A_3^1, A_4^1) = F(\alpha) = \{x_1(0.3, 0.6, 0.4, 0.5), x_2(0.4, 0.5, 0.3, 0.1), x_3(0.6, 0.3, 0.3, 0.7)\}$$

$$G(A_1^2, A_2^2, A_3^2, A_4^2) = G(\beta) = \{x_1(0.5, 0.3, 0.2, 0.6), x_2(0.6, 0.7, 0.8, 0.4), x_3(0.7, 0.7, 0.5, 0.9)\}$$

The Plithogenic Fuzzy Hyper Soft matrix $B = \left[\mu_{A_j^k}(x_i)\right]$ by using eq.1 for $i = 1, 2, 3$,

$j = 1, 2, 3, 4$ and $k = 1$, i.e.α Combination (first level sheet) , $k = 2$, β Combination (second level sheet) will be represented as,

$$B = \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.2 & 0.6 \\ 0.6 & 0.7 & 0.8 & 0.4 \\ 0.7 & 0.7 & 0.5 & 0.9 \end{bmatrix} \end{bmatrix} \quad (16)$$

Step 3. Plithogenic Fuzzy Subjective Hyper Super Soft Matrix:

The Plithogenic Fuzzy Hyper Super Soft matrix $B_{S_t} = \left[\left[\mu_{A_j^k}(x_i)\right]\left[\Omega_{A^k}^t(x_i)\right]\right]$ by using eq. 5, 11, 12, and 13 for $i = 1, 2, 3$, $j = 1, 2, 3, 4$ and $k = 1$, i.e., α combination (first level sheet), $k = 2$, β combination (second level sheet), $t = 1$, (disjunction operator) first set of two-level sheets, $t = 2$, (conjunction operator) second set of two-level sheets, $t = 3$ (averaging operator) a third set of two-level sheets will be represented as,

$$B_{S_t} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \begin{bmatrix} 0.6 \\ 0.5 \\ 0.7 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.2 & 0.6 \\ 0.6 & 0.7 & 0.8 & 0.4 \\ 0.7 & 0.7 & 0.5 & 0.9 \end{bmatrix} \begin{bmatrix} 0.6 \\ 0.8 \\ 0.9 \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0.1 \\ 0.3 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.2 & 0.6 \\ 0.6 & 0.7 & 0.8 & 0.4 \\ 0.7 & 0.7 & 0.5 & 0.9 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.4 \\ 0.5 \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \begin{bmatrix} 0.45 \\ 0.35 \\ 0.475 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.2 & 0.6 \\ 0.6 & 0.7 & 0.8 & 0.4 \\ 0.7 & 0.7 & 0.5 & 0.9 \end{bmatrix} \begin{bmatrix} 0.4 \\ 0.625 \\ 0.7 \end{bmatrix} \end{bmatrix} \end{bmatrix} \quad (17)$$

Step 4. Local Subjective Ranking:

$B_{S_{1\alpha}}$ In eq. 17 provides the local ordering of subjects for α Combination of attributes or α universe (first level sheet for $k = 1$) by using the first operator ($t = 1$) given in eq. $\mathbf{11}$ as

$$x_3 \succ x_1 \succ x_2 \quad (18)$$

DOI: 10.5281/zenodo.3841624

$\boldsymbol{B_{S_{2\alpha}}}$in eq. 17 provides the local ordering of subjects for $\boldsymbol{\alpha}$ Combination of attributes or $\boldsymbol{\alpha}$ universe (first level sheet for $\boldsymbol{k = 1}$) by using second operator ($\boldsymbol{t = 2}$) given in eq. $\mathbf{12}$ as

$$\boldsymbol{x_3 = x_1 \succ x_2} \qquad (19)$$

which shows a tie between $\boldsymbol{x_1}$ and $\boldsymbol{x_3}$ which can be removed in the final Global Ranking of subjects by using frequency matrix $\boldsymbol{F_{ij}}$

$\boldsymbol{B_{S_{3\alpha}}}$ In eq. 17 provides the local ordering of subjects for $\boldsymbol{\alpha}$ Combination of attributes ($\boldsymbol{\alpha}$ universe) by using a third operator ($\boldsymbol{t = 3}$) given in eq. 13 as

$$\boldsymbol{x_3 \succ x_1 \succ x_2} \qquad (20)$$

Similarly

$\boldsymbol{B_{S_{1\beta}}}$ in eq. $\mathbf{17}$ provides the local ordering of subjects for $\boldsymbol{\beta}$ Combination of attributes

($\boldsymbol{\beta}$ universe) by using first operator ($\boldsymbol{t = 1}$) given in eq. 11 as

$$\boldsymbol{x_3 \succ x_2 \succ x_1} \qquad (21)$$

$\boldsymbol{B_{S_{2\beta}}}$ in eq.. 17provides the local ordering of subjects for $\boldsymbol{\beta}$ Combination of attributes

($\boldsymbol{\beta}$ universe) by using the second operator ($\boldsymbol{t = 2}$) given in eq. 12as

$$\boldsymbol{x_3 \succ x_2 \succ x_1} \qquad (22)$$

$\boldsymbol{B_{S_{3\beta}}}$ in eq.. 17 provides the local ordering of subjects for $\boldsymbol{\beta}$ Combination of attributes

($\boldsymbol{\beta}$ universe) by using the third operator ($\boldsymbol{t = 3}$) given in eq. 13 as

$$\boldsymbol{x_3 \succ x_2 \succ x_1} \qquad (23)$$

We can observe that the three local ordering of subjects under consideration in $\boldsymbol{\alpha}$ Combination can be provided by using three operators described in eq., 11, 13, 12, to find cumulative memberships for $\boldsymbol{\alpha}$ Combination first, then for $\boldsymbol{\beta}$ Combination and then writing in descending order.

Step 5. Global Subjective Ranking :

Final global ordering of subjects can be provided by using frequency matrix $\boldsymbol{F_{ij}}$ described in eq.. 6, $\mathbf{18}$, 19, and 20

In the frequency matrix $\boldsymbol{F_{ij}^{\alpha}}$ which is a square matrix of frequencies positions for first level sheet $\boldsymbol{\alpha}$ Combination the $\boldsymbol{colF_{ij}^{\alpha}}$ represents frequencies of

positions, i.e., the entries of the first column represents the frequencies of attaining first position by given subjects while $\boldsymbol{rowF_{ij}}$ represents subjects.

DOI: 10.5281/zenodo.3841624

$$\boldsymbol{F_{ij}^{\alpha}} = \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Global Ranking of subjects from $\boldsymbol{F_{ij}^{\alpha}}$ is $\boldsymbol{x_3 \succ x_1 \succ x_2}$ (24)

The frequency matrix for $\boldsymbol{\beta}$ universe is $\boldsymbol{F_{ij}^{\beta}}$

$$\boldsymbol{F_{ij}^{\beta}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{3} \\ \mathbf{0} & \mathbf{3} & \mathbf{0} \\ \mathbf{3} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Global Ranking of subjects from $\boldsymbol{F_{ij}^{\beta}}$ is $\boldsymbol{x_3 \succ x_2 \succ x_1}$ (25)

We can observe that in both universes $\boldsymbol{x_3}$ is the superior subject which means that in both Combination of attributes and sub-attributes $\boldsymbol{x_3}$ is reflecting the best.

Step 6. Authenticity measure of final Ranking:

Percentage authenticity for first level $\boldsymbol{\alpha}$ universe

By using eq. 7 :

Percentage authenticity of first position for $\boldsymbol{x_3 = 75}$%

Percentage authenticity of first position for $\boldsymbol{x_2 = 66.67}$%

Percentage authenticity of first position for $\boldsymbol{x_1 = 100}$%

Percentage authenticity for first level $\boldsymbol{\beta}$ universe

By using eq. 7 :

Percentage authenticity of first position for $\boldsymbol{x_3 = 100}$%

Percentage authenticity of first position for $\boldsymbol{x_2 = 100}$%

Percentage authenticity of first position for $\boldsymbol{x_1 = 100}$%

Step 7. Final Universal Ranking:

The final ordering of universes provided as.

Maximum Universal Memberships of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ universes:

taking $\boldsymbol{k = 1, 2}$ for $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ universes and fixing $\boldsymbol{t = 1}$ as described in eq. 8, we can provide maximum universal memberships of all given subjects with respect to attributes,

$$\boldsymbol{\Omega^1_{A^1}(X) = 0.7,\ \Omega^1_{A^2}(X) = 0.9} \qquad (26)$$

Where $\boldsymbol{X}$ representing merged subjects, we can see by using operator $\boldsymbol{t = 1}$, $\boldsymbol{\beta}$ universe is better than $\boldsymbol{\alpha}$ universe.

DOI: 10.5281/zenodo.3841624

Minimum Universal Memberships of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ universes:

Taking $\boldsymbol{k=1,2}$ for $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ universes and fixing $\boldsymbol{t=2}$ as described in eq.. 9 we can provide minimum universal memberships of all given subjects with respect to attributes,

$$\boldsymbol{\Omega^2_{A^1}(X)=0.1,\ \Omega^2_{A^2}(X)=0.2} \qquad (27)$$

We can see by using operator $\boldsymbol{t=2}$, $\boldsymbol{\beta}$ universe is better than $\boldsymbol{\alpha}$ universe.

Average Universal Memberships of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ universes:

similarly, taking $\boldsymbol{k=1,2}$ for $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ universes and fixing $\boldsymbol{t=3}$ as described in eq.. 10, we can provide average universal memberships of all given subjects with respect to attributes,

$$\boldsymbol{\Omega^3_{A^1}(X)=0.425,\ \Omega^3_{A^2}(X)=0.575} \qquad (28)$$

we can see by using operator $\boldsymbol{t=3}$, $\boldsymbol{\beta}$ universe is better than $\boldsymbol{\alpha}$ universe

5. Conclusions

1. We can see from expressions 18, 19, 20 the final order in eq. 24 is the most frequently observed order in all these ranking orders which is also observed same in local ordering of β universe (Eq. 21, 22, 23) and is again same in final global ordering of β universe 25, which shows the final global Ranking is transparent and authentic.

2. expressions 26, 27, 28 provides highest, lowest, and average cumulative memberships of universes.

3. In Universal ordering, it is observed that on the Global Universal level, β universe is better than α universe.

1. Local ordering: we can observe local ordering by using the novel plithogenic hyper-super-soft-matrix and local operators. We can judge the performance of some fixed subjects in a particular universe, i.e., one combination or level sheet. This is the case of the inner classification of the universe.

2. Global ordering: We can provide global ordering by considering the performance of some fixed subjects in multiple universes, i.e., different combinations of sub-attribute or level sheets are combined by using the frequency matrix. This is the case of combining several angles of visions in the final decision.

3. Universal ordering: We can compare these universes by combining the cumulative memberships of the last column for each universe. The universe having the largest cumulative average membership is the better and then can write them by descending order.

4. **Extreme Universal Memberships:** We can also find out extreme values of these universes and can judge these subjects in a grand universe which is made by multiple smaller parallel universes so we can choose the best subject from all universes that is the one who is best in most universes or we can select one reality out of multiple parallel realities these facts are useful in the field of artificial intelligence.

References

[1] Smarandache, F. "A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability: Neutrsophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. Infinite Study", pp. 443-458, 2005.

[2] Atanassov , K. T, "Intuitionistic fuzzy sets. Fuzzy Sets and Systems", 20(1), pp. 87-96, 1986.

[3] Atanassov, K. T, "Applications of intuitionistic fuzzy sets", In Intuitionistic Fuzzy Set, Series Studies in Fuzziness and Soft Computing", 35(1), pp. 237-238, 1999.

[4] Cagman N, Enginoglu S, Citak F," Fuzzy soft set theory and its applications", Iranian journal of fuzzy systems", 18,8(3), pp. 137-147, 2011.

[5] Molodtsov, D. "Soft set theory-First results, Computers and mathematics with applications", 37, pp.19-31, 1999.

[6] Pie, D and Miao, D, "From soft sets to informations systems", In:Proceedings of Granular computing IEEE, 2, pp. 617-621, 2005.

[7] Zadeh, L. A "Fuzzy sets", Inform and Control, 8(3), pp. 338-353, 1965.

[8] Kandasamy, W. V., & Smarandache, F, "Superbimatrices and their generalizations", Infinite Study", 2009.

[9] Kandasamy, W. V., & Smarandache, F, "DSm Super Vector Space of Refined Labels", Zip Publishing, 2011.

[10] Herrera, F., & Martínez, L, "A 2-tuple fuzzy linguistic representation model for computing with words", IEEE Transactions on fuzzy systems, 8(6), pp. 746-752, 2000.

[11] Horst, P, "Matrix algebra for social scientists", Holt, Rinehart and Winston, 1963.

[12] Onyeozili, I.A.,Gwary T.M, "A Study of The Fundamentals of Soft Set Theory", International Journal of Scientific & Technology Research, (3), pp. 2277-8616, 2014.

[13] Nye, J. F, "Physical properties of crystals: their representation by tensors and matrices", Oxford university press, 1985.

[14] Rana S, Qayyum M, Saeed M, Smarandache F, Khan BA," Plithogenic Fuzzy Whole Hypersoft Set, Construction of Operators and their Application in Frequency Matrix Multi Attribute Decision Making Technique", Neutrosophic Sets and Systems, Vol. 28, pp.34-50, 2019.

[15] Smarandache, F,"Neutrosophic logic and set, mss", 1995.

[16] Smarandache, F," Extension of soft Set to hypersoft Set, and then to plithogenic hypersoft Set", Neutrosophic Sets and Systems, Vol. 22 ,pp.168-170, 2018.

[17] Smarandache, "Neutrosophic Set is a Generalization of Intuitionistic Fuzzy Set, Inconsistent Intuitionistic Fuzzy Set (Picture Fuzzy Set, Ternary Fuzzy Set), Pythagorean Fuzzy Set, Spherical Fuzzy Set, and q-Rung Orthopair Fuzzy Set, while Neutrosophication is a Generalization of Regret Theory, Grey System Theory, and Three-Ways Decision (revisited)", Journal of New Theory, (29), pp. 1-31 , 2019

[18] de Sáa, S. D. L. R., Gil, M. Á., González-Rodríguez, G., López, M. T., & Lubiano, M. A, "Fuzzy rating scale-based questionnaires and their statistical analysis", IEEE Transactions on Fuzzy Systems, 23(1), pp.111-126, 2014.

[19] Wang, W. P, "A fuzzy linguistic computing approach to supplier evaluation", Applied Mathematical Modelling, 34(10), pp. 3130-3141, 2010.

[20] Khalid M, Khalid NA, Khalid H, Broumi S. "Multiplicative Interpretation of Neutrosophic Cubic Set on B-Algebra", International Journal of Neutrosophic Science, Vol1, issue1.pp, 64-73, 2020.

[21] Smarandache F. "NeutroAlgebra is a Generalization of Partial Algebra", International Journal of Neutrosophic Science, Volume 2 , Issue 1, pp. 08-17, 2020

[22] Prem. K , "Plithogenic set for multi-variable data analysis", International Journal of Neutrosophic Science, Vol1, issue 2, pp, 82-89, 2020

[23] Deli I, Broumi S. "Neutrosophic soft matrices and NSM-decision making", Journal of Intelligent & Fuzzy Systems, 1;28(5),pp.2233-2241, 2015.

[24] Deli I, Çağman N. "Intuitionistic fuzzy parameterized soft set theory and its decision making", Applied Soft Computing, Vol1, issue, 28, pp.109-113, 2015.

[25] Broumi S., Dey A., Talea M., Bakali A., Smarandache F., Nagarajan D., Lathamaheswari M. and Kumar R., "Shortest Path Problem using Bellman Algorithm under Neutrosophic Environment", Complex & Intelligent Systems, 5, pp.409–416, 2019. https://doi.org/10.1007/s40747-019-0101-8,

[26] Broumi S. , Talea M., Bakali A. , Smarandache F. , Nagarajan D., Lathamaheswari M. and Parimala M., "Shortest path problem in fuzzy, intuitionistic fuzzy and neutrosophic environment: an overview", Complex & Intelligent Systems, 5, pp.371–378, 2019.https://doi.org/10.1007/s40747-019-0098-z

AH-Subspaces in Neutrosophic Vector Spaces

Mohammad Abobala

Faculty of Science, Tishreen University, Lattakia, Syria

e-mail: mohammadabobala777@gmail.com

Abstract

In this paper, we introduce the concept of AH-subspace of a neutrosophic vector space and AHS-linear transformations. We study elementary properties of these concepts such as Kernel, AH-Quotient, and dimension.

Keywords: Neutrosophic vector space , AH-supspace , AHS-subspace , AH-Quotient.

1. Introduction

Neutrosophy, as a new branch of philosophy and of logic, founded by Smarandache, got its way in algebraic structure studies. Many neutrosophic algebraic structures were defined and studied such as neutrosophic groups, neutrosophic rings, neutrosophic refined rings, and neutrosophic vector spaces. See [3,4,5,6,9]. In 2019 and 2020 Smarandache [12,13,14] generalized the classical Algebraic Structures to NeutroAlgebraic Structures (or NeutroAlgebra) {whose operations and axioms are partially true, partially indeterminate, and partially false} as an extension of Partial Algebra, and to AntiAlgebraic Structures (or AntiAlgebra) {whose operations and axioms are totally false}.
AH-substructures were firstly defined in neutrosophic rings in [1], and then they have been studied in refined neutrosophic rings in [2]. These structures have had many symmetric properties that illustrate a bridge between classical algebra and neutrosophic algebra. In this paper, we try to define AH-subspace and AHS-subspace of a neutrosophic vector space and introduce some of its elementary properties. Also, some interesting concepts were defined and used in this study, such as neutrosophic AH-linear transformations, and AH-Quotient.

Motivation

This work is a continuation of works done in [1,2], that established the theory of neutrosophic AH-substructures in neutrosophic algebraic structures.

2. Preliminaries

Definition 2.3:[5] Let (V, +, $\cdot$) be a vector space over the field K then (V(I), +, $\cdot$) is called a weak neutrosophic vector space over the field K, and it is called a strong neutrosophic vector space if it is a vector space over the neutrosophic field K(I).

DOI: 10.5281/zenodo.3841628
Received: February29, 2020 Revised: April 28, 2020 Accepted: May 16, 2020

A neutrosophic field K(I) is a triple (K(I),+, ·), where K is a classical field. A neutrosophic field is not a field by classical meaning, but it is a ring.

Elements of V(I) have the form $\boldsymbol{x+yI}; \boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{V}$, i.e V(I) can be written as $\boldsymbol{V(I)=V+VI}$.

Definition 2.4: [5] Let V(I) be a strong neutrosophic vector space over the neutrosophic field K(I) and W(I) be a non empty set of V(I), then W(I) is called a strong neutrosophic subspace if W(I) itself is a strong neutrosophic vector space.

Definition 2.5:[5] Let U(I) and W(I) be two strong neutrosophic subspaces of V(I) then we say that V(I) is a direct sum of U(I) and W(I) if and only if for each element x ∈ $\boldsymbol{V(I)}$ then x can be written uniquely as x = y + z such y ∈ $\boldsymbol{U(I)}$ and z ∈ $\boldsymbol{W(I)}$

Definition 2.6:[5]

Let U(I) and W(I) be twostrong neutrosophic subspaces of V(I) and let $f: V(I) \to W(I)$, we say that f is a neutrosophic vector space homomorphism if

(a) $f(I) = I$.

(b) f is a vector space homomorphism.

We define the kernel of f by Ker f = { x∈ $V(I)$; $f(x)$ = 0 }.

Definition 2.7:[5]

Let $v_1, v_2 .. v_s \in V(I)$ and $x \in V(I)$ we say that x is a linear combination of $\{ v_i; i = 1, .., s\}$ if

x= $a_1v_1 + \cdots + a_sv_s$ such that $a_i \in K(I)$.

The set $\{ v_i; i = 1, .., s\}$ is called linearly independent if $a_1v_1 + \cdots + a_sv_s = 0$ implies $a_i = 0$ for all i.

Theorem 2.10: [5]

If $\{ v_1, \ldots, v_s \}$ is a bases of V(I) and $f: V(I) \to W(I)$ is a neutrosophic vector space homomorphism then $\{$f(v_1) $, \ldots, f(v_s)\}$ is a bases of W(I).

Definition 2.8: [1]

Let R(I) be a neutrosophic ring and $P = P_0 + P_1I = \{a_0 + a_1I;\ a_0 \in P_0, a_1 \in P_1\}$.

(a) We say that P is an AH-ideal if P_0 andP_1 are ideals in the ring R.

(b) We say that P is an AHS-ideal if $P_0 = P_1$.

Definition 2.9: [2]

Let (R(I_1, I_2), $+$, $\times$) be a refined neutrosophic ring, and P_0, P_1, P_2 be three ideals in the ring R then the set

$P = (P_0, P_1I_1, P_2I_2) = \{(a, bI_1, cI_2); a \in P_0, b \in P_1, c \in P_2\}$ is called a refined neutrosophic AH-ideal.

If $P_0 = P_1 = P_2$ then P is called a refined neutrosophic AHS-ideal.

DOI: 10.5281/zenodo.3841628

3. Main concepts and discussion

Definition 3.1:

Let V(I) = V+VI be a strong/weak neutrosophic vector space, the set

$S = P + QI = \{x + yI; x \in P, y \in Q\}$, where P and Q are subspaces of V is called an AH-subspace of V(I).

If $P = Q$ then S is called an AHS-subspace of V(I).

Example 3.2:

We have V = R^2 is a vector space, $P = <(0,1)>$, $Q = <(1,0)>$, are two subspaces of V. The set

$S = P + QI = \{(0,a) + (b,0)I; a, b \in R\}$ is an AH-subspace of V(I).

The set $L = P + PI = \{(0,a) + (0,b)I\}; a, b \in R$ is an AHS-subspace of V(I).

Theorem 3.3:

Let $V(I) = V + VI$ be a neutrosophic weak vector space, and let $S = P + QI$ be an AH-subspace of $V(I)$, i.e Q, P are supspaces of V, then S is a subspace by the classical meaning.

Proof:

Suppose that $x = a + bI, y = c + dI \in S; a, c \in P,\ b, d \in Q$, we have

$x + y = (a + c) + (b + d)I \in S$. For each scalar $m \in K$ we obtain $m.x = m.a + (m.b)I \in S$, since P and Q are subspaces; thus $S = P + QI$ is a subspace of V(I) over the field K.

Remark 3.4:

An AH-subspace S is not necessary a subspace of neutrosophic strong vector space V(I) over a neutrosophic field K(I), see Example 3.5.

Example 3.5:

Let S be the AH-subspace defined in Example 3.2, and $m = 1 + I \in R(I)$ be a neutrosophic scalar, and $x = (0,1) + (2,0)I \in S$, we have:

$m.x = (1 + I) \cdot x = (0,1) + [(2,0) + (0,1) + (2,0)]I = (0,1) + (4,1)I$, since (4,1) does not belong to Q, we find that $m \cdot x$ is not in S, thus S is not a subspace of V(I).

The following theorem shows that any AHS-subspace is a subspace of a neutrosophic strong vector space.

Theorem 3.6:

Let V(I) be a neutrosophic strong vector space over a neutrosophic field K(I), let $S = P + PI$ be an AHS-subspace. S is a subspace of V(I).

Proof:

Suppose that $x = a + bI, y = c + dI \in S; a, c, b,$ and $c \in P$, we have

$x+y=(a+c)+(b+d)I \in S$. Let $m=x+yI \in K(I)$ be a neutrosophic scalar, we find

$m \cdot x=(x.a)+(y.a+y.b+x.b)I \in S$, since $y.a+y.b+x.b \in P$, thus we get the desired result.

Definition 3.7:

(a) Let V and W be two vector spaces, $L_V: V \rightarrow W$ be a linear transformation. The AHS-linear transformation can be defined as follows:

$L: V(I) \rightarrow W(I); L(a+bI)=L_V(a)+L_V(b)I$.

(b) If $S = P+QI$ is an AH-subspace of V(I), $L(S)=L_V(P)+L_V(Q)I$.

(c) If $S = P+QI$ is an AH-subspace of W(I), $L^{-1}(S)=L_W^{-1}(P)+L_W^{-1}(Q)I$.

(d) $AH-Ker\ L = KerL_V+KerL_V\ I=\{x+yI; x,y \in KerL_V\}$.

Theorem 3.8:

Let W(I) and V(I) be two neutrosophic strong/weak vector spaces, and $L: V(I) \rightarrow W(I)$ be an AHS-linear transformation, we have:

(a) $AH-Ker\ L$ is an AHS-subspace of V(I).

(b) If $S = P+QI$ is an AH-subspace of V(I), $L(S)$ is an AH-subspace of W(I).

(c) If $S = P+QI$ is an AH-subspace of W(I), $L^{-1}(S)$ is an AH-subspace of V(I).

Proof:

(a) Since $KerL_V$ is a subspace of V, we find that $AH-Ker\ L = KerL_V+KerL_V\ I$ is an AHS-subspace of V(I).

(b) We have $L(S)=L_V(P)+L_V(Q)I$; thus L(S) is an AH-subspace of W(I), since $L_V(P), L_V(Q)$are subspaces of W.

(c) By regarding $L^{-1}(S)=L_W^{-1}(P)+L_W^{-1}(Q)I$, $L_W^{-1}(P)$ and $L_W^{-1}(Q)$ are subspaces of V, we obtain that $L^{-1}(S)$ is an AH-subspace of V(I).

Theorem 3.9:

Let W(I) and V(I) be two neutrosophic strong vector spaces over a neutrosophic field K(I), and $L: V(I) \rightarrow W(I)$ be an AHS-linear transformation, we have:

$L(x+y)=L(x)+L(y), L(m.x)=m.L(x)$, for all $x,y \in V(I), m \in K(I)$.

Proof:

Suppose $x=a+bI, y=c+dI; a,b,c,d \in V, and\ m=s+tI \in K(I)$, we have

$$L(x+y)=L([a+c]+[b+d]I)=L_V(a+c)+L_V(b+d)I=$$

$$[L_V(a)+L_V(b)I]+[L_V(c)+L_V(d)I]=L(x)+L(y).$$

$m.x=(s.a)+(s.b+t.a+t.b)I$, $L(m.x)=L_V(s.a)+L_V(s.b+t.a+t.b)I$

DOI: 10.5281/zenodo.3841628

= $s.L_V(a) + [s.L_V(b) + t.L_V(a) + t.L_V(b)]I$ = $(s + tI).(L_V(a) + L_V(b)I) = m.L(x)$.

Theorem 3.10:

Let $S = P + QI$ be an AH-subspace of a neutrosophic weak vector space V(I) over a field K, suppose that

$X = \{x_i; 1 \le i \le n\}$ is a bases of P and $Y = \{y_j; 1 \le j \le m\}$ is a bases of Q then $X \cup YI$ is a bases of S.

Proof:

Let $z = x + yI$ be an arbitrary element in S; $x \in P, y \in Q$. Since P and Q are subspaces of V we can write

$x = a_1x_1 + a_2x_2 + \cdots + a_nx_n;\ a_i \in K\ and\ x_i \in X,\ y = b_1y_1 + b_2y_2 + \cdots + b_my_m;\ b_i \in K, y_i \in Y.$

Now we obtain $z = (a_1x_1 + \cdots + a_nx_n) + (b_1y_1I + \cdots + b_my_mI)$; thus $X \cup YI$ generates the subspace S.

$X \cup YI$ is linearly independent set. Assume that $(a_1x_1 + \cdots + a_nx_n) + (b_1y_1I + \cdots + b_my_mI) = 0$, this implies

$a_1x_1 + a_2x_2 + \cdots + a_nx_n = 0$ and $(b_1y_1 + b_2y_2 + \cdots + b_my_m)I = 0$. Since X and Y are linearly independent sets over K, we get $a_i = b_j = 0\ for\ all\ i, j$ and $X \cup YI$ is linearly independent then it is a bases of S.

Result 3.11:

Let $S = P + QI$ be an AH-subspace of a neutrosophic weak vector space V(I) with finite dimension over a field K, from Theorem 3.10 and the fact that $X \cap YI = \emptyset$, we find $\dim(S) = \dim(P) + \dim(Q)$.

Example 3.12:

Let V =R^3, P=<(0,0,1)>, Q=<(0,1,0)> be two subspaces of V,

(a) $S = P + QI = \{(0,0,m) + (0,n,0)I;\ m, n \in R\ \}$is an AH-subspace of V(I).

(b) The set $\{(0,0,1), (0,1,0)I\}$ is a basis of S, $\dim(S) = \dim(P) + \dim(Q) = 1 + 1 = 2$.

(c) $L_V: V \to V;\ L_V(x, y, z) = (x + y, y, z) for\ all\ x, y, z \in R$ is a linear transformation, the corresponding AHS-linear transformation is $L: V(I) \to V(I); L[(x, y, z) + (a, b, c)I] = L_V(x, y, z) + L_V(a, b, c)I$ =

$(x + y, y, z) + (a + b, b, c)I.$

(d) $L(S) = L_V(P) + L_V(Q) = L_V\{(0,0,m)\} + L_V\{(0,n,0)\}I = \{(0,0,m) + (n,n,0)I\};\ m, n \in R$, which is an AH-subspace of V(I).

Example 3.13 :

Let V=R^2, W=R^3, $L_V: V \to W;\ L_V(x, y) = (x + y, x + y, x + y)$ is a linear transformation. The corresponding AHS-linear transformation is

$L: V(I) \to W(I); L[(x, y) + (a, b)I] = (x + y, x + y, x + y) + (a + b, a + b, a + b)I.$

$KerL_V =< (1, -1) >, AH - KerL = KerL_V + KerL_V\ I =< (1, -1) > + < (1, -1) > I$ =

$\{(a, -a) + (b, -b)I; a, b \in R\}$ which is an AHS-subspace of V(I).

It is clear that $\dim(KerL) = 1 + 1 = 2$ according to Theorem 3.10.

DOI: 10.5281/zenodo.3841628

Definition 3.14:

Let V(I) be a neutrosophic strong/weak vector space, $S = P + QI$ be an AH-subspace of V(I), we define

AH-Quotient as:

$V(I)/S = V/P + (V/Q)\,I = (x + P) + (y + Q)I; x, y \in V$.

Theorem 3.15:

Let V(I) be a neutrosophic weak vector space over a field K, and $S = P + QI$ be an AH-subspace of V(I). The AH-Quotient $V(I)/S$ is a vector space over the field K with respect to the following operations:

Addition: $[(x + P) + (y + Q)I] + [(a + P) + (b + Q)]I = (x + a + P) + (y + b + Q)I; x, y, a, b \in V$.

Multiplication by a scalar: $(m).[(x + P) + (y + Q)I] = (m.x + P) + (m.y + Q)I$;

$x, y \in V$ and $m \in K$.

Proof:

It is easy to check the operations completely are well defined, and $(V(I)/S\,, +)$ is abelian group.

Let $z = [(x + P) + (y + Q)I] \in V(I)/S$, we have $1.z = z$.

Assume that $m, n \in K$, we have $m.(n.z) = m.[(n.x + P) + (n.y + Q)I] = (m.n.x + P) + (m.n.y + Q)I = (m.n).z$.

$(m + n).z = [(m + n).x + P] + [(m + n).y + Q]I = m.z + n.z$.

Let $h = [(a + P) + (b + Q)I] \in V(I)/S$, $z + h = (x + a + P) + (y + b + Q)I$,

$m.(z + h) = (m.x + m.a + P) + (m.y + m.b + Q)m.z + m.h$.

Example 3.16:

We have V = R^2 is a vector space over the field R, $P =< (0,1) >, Q =< (1, 0) >$ are two subspaces of V,

$S = P + QI = \{(0, a) + (b, 0)I; a, b \in R\}$ is an AH-subspace of V(I).

The AH-Quotient is $V(I)/S = \{[(x, y) + P] + [(a, b) + Q]I; x, y, a, b \in V\}$.

We clarify operations on $V(I)/S$ as follows:

$x = [(2,1) + P] + [(1,3) + Q]I, y = [(2,5) + P] + [(1,1) + Q]I$ are two elements in $V(I)/S\,, m = 3\ is\ a\ scalar\ in\ R$.

$x + y = [(4,6) + P] + [(2,4) + Q]I$, $3.x = [(6,3) + P] + [(3,9) + Q]I$.

Remark 3.17:

If $S = P + PI$ is an AHS-subspace of a neutrosophic weak vector space V(I) over the field K, then AH-Quotient $V(I)/S = V/P + V/P\ I$ is a weak neutrosophic vector space, since V/P is a vector space.

DOI: 10.5281/zenodo.3841628

5. Conclusion

In this article, we have defined the concepts of AH-subspace, AHS-subspace, and AHS-linear transformation in neutrosophic vector spaces. Also, we have studied some basic properties of these concepts.

References:

[1] Abobala, M., "On Some Special Substructures of Neutrosophic Rings and Their Properties", International Journal of Neutrosophic Science", Vol 4, pp.72-81, 2020.

[2] Abobala, M., "On Some Special Substructures of Refined Neutrosophic Rings", International Journal of Neutrosophic Science, Vol 5, pp.59-66, 2020.

[3] Abobala, M,. "Classical Homomorphisms Between Refined Neutrosophic Rings and Neutrosophic Rings", International Journal of Neutrosophic Science, Vol 5, pp.72-75, 2020.

[4] Adeleke, E.O., Agboola, A.A.A., and Smarandache, F., "Refined Neutrosophic Rings I", International Journal of Neutrosophic Science, Vol 2, pp. 77-81, 2020.

[5] Agboola, A.A.A,. and Akinleye, S.A,. "Neutrosophic Vector Spaces", Neutrosophic Sets and Systems, Vol 4 , pp 9-17, 2014.

[6] Agboola, A.A.A,. Akwu, A.D,. and Oyebo, Y.T,. "Neutrosophic Groups and Subgroups", International .J .Math.Combin, Vol 3, pp. 1-9, 2012.

[7] Agboola, A.A.A., Akinola, A.D., and Oyebola, O.Y.," NeutrosophicRings I ", International J.Mathcombin, vol(4), pp 1-14, 2011.

[8] Bera, T and Mahapatra, N. K., "On Neutrosophic Soft Field", IJMTT, 56(7), pp.472-494, 2018.

[9] Bera, T and Mahapatra, N. K., "On Neutrosophic Soft Linear Spaces", Fuzzy Information and Engineering., 9, pp.299-324, 2017.

[10] Injrou, S., "Linear Algebra 2 ", Tishreen University Press, pp.180-230, 2015.

[11] Kandasamy, V.W.B,. and Smarandache, F., "Some Neutrosophic Algebraic Structures and Neutrosophic N-Algebraic Structures", Hexis, Phonex, Arizona 2006.

[12] Smarandache, F.,"NeutroAlgebra is a Generalization of Partial Algebra", International Journal of Neutrosophic Science (IJNS), Vol. 2, No. 1, pp. 08-17, 2020.

[13] Smarandache, F., "Introduction to NeutroAlgebraic Structures and AntiAlgebraic Structures, in Advances of Standard and Nonstandard Neutrosophic Theories," Pons Publishing House Brussels, Belgium, Ch. 6, pp. 240-265, 2019.

[14] Smarandache, F.,"Introduction to NeutroAlgebraic Structures and AntiAlgebraic Structures (revisited)", Neutrosophic Sets and Systems, vol. 31, pp. 1-16, 2020. DOI: 10.5281/zenodo.3638232.

DOI: 10.5281/zenodo.3841628

A Review on Superluminal Physics and Superluminal Communication in light of the Neutrosophic Logic perspective

Victor Christianto[1*] & Florentin Smarandache[2]

[1] Malang Institute of Agriculture (IPM), Malang – INDONESIA, email: victorchristianto@gmail.com

[2] Dept. Mathematics & Sciences, University of New Mexico, Gallup, USA. Email: smarand@unm.edu

*Corresponding author: victorchristianto@gmail.com

Abstract

In a recent paper, we describe a model of quantum communication based on combining consciousness experiment and entanglement, which can serve as impetus to stop 5G-network-caused diseases. Therefore, in this paper we consider superluminal physics and superluminal communication as a bridge or intermediate way between subluminal physics and action-at-a-distance (AAAD) physics, especially from neutrosophic logic perspective. Although several ways have been proposed to bring such a superluminal communication into reality, such as Telluric wave or Telepathy analog of Horejev and Baburin, here we also review two possibilities: quaternion communication and also quantum communication based on quantum noise. Further research is recommended in the direction outlined herein. *Aim of this paper:* We discuss possibilities to go beyond 4G and 5G network, and avoid the unnecessary numerous health/diseases problems caused by massive 5G network. *Contribution*: We consider quaternionic communication and quantum communication based on quantum noise, which are largely unnoticed in literature. *Limitation:* We don't provide scheme for operationalization, except what we have provided in other paper.

Keywords: quantum entanglement, quantum communication, consciousness, superluminal communication, action at a distance.

PACS 2010: 02, 03, 41, 98

Quote: "The Hertz wave theory of wireless transmission may be kept up for a while, but I do not hesitate to say that in a short time it will be recognized as one of the most remarkable and inexplicable aberrations of the scientific mind which has ever been recorded in history." —Nikola Tesla, *The True Wireless,* 1919.

DOI: 10.5281/zenodo.3841630

Received: February 09, 2020 Revised: April 27, 2020 Accepted: May 19, 2020

1. Introduction

In line with the rapid development of new branch of foundational mathematics, i.e. Neutrosophic Logic, here we discuss potential application of NL theory in the field of telecommunication. See for recent papers on NL: [31-35]. It is known that nowadays telecommunication systems are predominated by RF systems, including numerous wireless systems, such as 4G Wi-Fi, 4G network etc. And the world is now in transition stage towards 5G network deployment.

A growing number of individuals are coming together in many countries - to attempt to block or stop the current telecoms roll-out of 5G electromagnetic microwave radiation which has proved to be extremely harmful to all sentient life forms, including plant life.[1]

It would prove invaluable if some methods could be found to render the 5G millimetre wave transmissions ineffective. In other words, to block or dissolve their ability to irradiate surrounding matter/life. Our concern is not to find a way to 'to protect the individual' but to prevent whole areas from being affected by microwaves via the tens of thousands of transmission bases that 5G requires - and from satellite sources.

In literature, there are known proposals or experiments which were purported to suggest possible ways to develop superluminal communication, to name a few: Telluric wave and also Telepathy analog way. For instance, in biofield site, it is written, which can be paraphrased as follows:

> "Torsion fields is one of the names given to the more unpretentious parts of the biofield. Torsion fields have additionally been alluded to as orgone, od, tachyon, aether, Tesla waves, scalar waves, the zero point field and then some. There is no settled upon logical agreement on these increasingly inconspicuous parts of the biofield. Torsion fields are guessed to the moderating mode for separation recuperating, which happens immediately and which research has been demonstrated that to be difficult to be transmitted through exemplary electromagnetic frequencies."[2]

See also Horejev and Baburin's paper [27]. Besides, there are also other suggestions of telepathic analog communications [28-30].

From Neutrosophic Logic perspective, we need to distinguish the subluminal communication from superluminal communication. In fact, Smarandache's Hypothesis states that there is no speed limit of anything, including light and "particles [16]. One of us (FS) also wrote in this regards:

> "In a similar way as passing from Euclidean Geometry to Non-Euclidean Geometry, we can pass from Subluminal Physics to Superluminal Physics, and further to Instantaneous Physics (instantaneous traveling). In the lights of two consecutive successful CERN experiments with superluminal particles in the Fall of 2011, we believe these two new fields of research should begin developing. A physical law has a form in Newtonian physics, another form in the Relativity Theory, and different form at Superluminal theory, or at Instantaneous (infinite) speeds – according to the S-Denying Theory spectrum. First, we extend physical laws and formulas

[1] A brief explanation here: https://www.5gspaceappeal.org/the-appeal
[2] https://www.biofieldlab.com/whatisthebiofield

DOI: 10.5281/zenodo.3841630

> to superluminal traveling and to instantaneous traveling. Afterwards, we should extend existing classical physical theories from subluminal to superluminal and instantaneous traveling.”[3]

While the idea behind Smarandache hypothesis is quite simple and based on known hypothesis of quantum mechanics, called Einstein-Podolski-Rosen paradox, in reality such a *superluminal physics* seems still hard to accept by majority of physicists.

The background idea and our motivation for suggesting to go beyond RF/subluminal communication towards superluminal communication are two previous papers: (a) there will be more than 720! (factorial) types of new diseases which may arise, if the 5G network is massively implemented – and the present covid-19 pandemic may be just the beginning; (b) in a recent paper [14], we describe a model of quantum communication based on combining consciousness experiment and entanglement, which can serve as impetus to stop 5G-network-caused diseases, and (c) another recent paper that we presented in CTPNP 2019, where we discuss a realistic interpretation of wave mechanics, based on a derivation of Maxwell equations from quaternionic Dirac equation [36]. From these previous papers, we come to conclusion to superluminal communication is not only possible, but it is indeed embedded in quaternionic Maxwell equations, which are close to their original idea by Prof. James Clerk Maxwell.

Therefore, in this paper we consider superluminal physics and superluminal communication as a bridge or intermediate way between subluminal physics and action-at-a-distance (AAAD) physics, especially from *Neutrosophic Logic* perspective. Although several ways have been proposed to bring such a superluminal communication into reality, such as Telluric wave or Telepathy analog of Horejev and Baburin, here we also review two possibilities: quaternion communication and also quantum communication based on quantum noise.

These two choices will be discussed in section 4. But first of all we will discuss what is the difference between subluminal communication and superluminal communication.

2. What is the difference between subluminal communication and superluminal communication?

According to Belrose, which can be paraphrased as follows:

> “The extremely plausibility of remote correspondences was established on the investigations of James Clerk Maxwell, and his conditions structure the premise of computational electromagnetics. Their accuracy was built up by Heinrich Hertz, when in 1887 he found EM radiation at UHF frequencies as anticipated by Maxwell. Since the spearheading work of Maxwell starting in the center 1850s, and of his adherents, a little gathering that got known as Maxwellians, which incorporated UK's Poynting and Heaviside, Maxwell's conditions have been read for longer than a century, and have demonstrated to be one of the best speculations in the historical backdrop of radioscience. For instance, when Einstein saw that Newtonian elements had as changed to be good with his exceptional hypothesis of relativity, he found that Maxwell's conditions were at

[3] See Florentin Smarandache. url: http://fs.unm.edu/SuperluminalPhysics.htm

DOI: 10.5281/zenodo.3841630

> that point relativistically right. EM field impacts are created by the increasing speed of charges, thus Maxwell had naturally incorporated relativity with his conditions." [25]

History also told us that around a century ago, there was proponents of wireless telegraphy, including Marconi, and opponents of it, including Tesla. And there were records of conflict between Tesla and Marconi. History also told us that around the same months after wireless telegraphy networks were installed everywhere, there was a flu pandemic, just like we observed now. It is also known, that wireless telegraphy was based on RF technology, which is actually a subluminal physics, while Tesla preferred a superluminal technology beyond Radio Frequency (sometimes he referred to as *non-Hertzian* waves). See also [18-26].

For instance, Paul Brenner wrote:

> "Marconi's work is based on copies of patents of many other inventors without their permission. His so called original "*two-circuit*" device, a spark-gap transmitter plus a coherer-receiver, was similar to those used by Oliver Lodge in a series of worldwide reported tests in 1894. Tesla disputed that Marconi was able to signal for greater distances than anyone else when using the spark-gap and coherer combination. In 1900 Alexander Stepanovich Popov declared to the Congress of Russian Electrical Engineers: "[...] *the emission and reception of signals by Marconi by means of electric oscillations [was] nothing new. In America, the famous engineer Nikola Tesla carried the same experiments in 1893*." [20]

It is also known from history books, that in the last century, the understanding of the nature of electromagnetic phenomena was proceeding with a constant rivalry between two concepts of interaction: namely, Newton instantaneous *action at a distance* (IAAAD) and Faraday-Maxwell *short-range interaction*. Finally, the discovery of Faraday's law of induction (explicit time dependence of electromagnetic phenomena) and the experimental observation of electromagnetic waves seemed to confirm the short-range interaction. Nevertheless, the idea of IAAAD still has many supporters. Among the physicists who have developed some theories based, in any case, on this concept, we can find names such as Tetrode and Fokker, Frenkel and Dirac, Wheeler and Feynman, and Hoyle and Narlikar. This interest in the concept of IAAAD is explained by the fact that classical theory of electromagnetism is an unsatisfactory theory all by itself, and so there have been many attempts to modify either the Maxwell equations or the principal ideas of electromagnetism.

As Augusto Garrido wrote in his review to Chubykalo et al's book:

> "On the other hand, the famous article "*Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?*" by Einstein, Rosen and Podolsky published in Physical Review in 1935 revived this discussion in a new panorama. In this article Einstein made public his position against the Copenhagen interpretation of the quantum mechanics. The controversy unleashed since then made this article a very popular one for its implications in our physical and philosophical understanding of the physical reality. The main objective of this article was to demonstrate that the quantum mechanics, the same way the Newtonian mechanics was for the relativistic mechanics, is an incomplete theory, and therefore, transitory of reality. For that reason Einstein made evident what is now known as the EPR paradox. According to EPR quantum mechanics is no local theory, that is to say, it permits action at a distance and, that is forbidden by the relativity theory, instantaneous action at a distance.
> Unfortunately for Einstein, and for common sense the experiment performed by Aspect seems to indicate that the IAAAD following from quantum mechanics exists. As a consequence of this confusion, physicists

DOI: 10.5281/zenodo.3841630

> are divided in two big groups according their position about IAAAD. These disputants are the quantum physicists and the relativists, who, almost after a century, have not been able to answer the old question whether the subject of their studies is a complete and integrated Universe – a physical Universe in its own right – or simply a assemblage of locally interacting parts."[15]

Therefore, to summarize the above paragraph, it seems we can distinguish among few technologies for communication:

- Subluminal RF physics → subluminal wave → subluminal communication
- Close to light speed physics → relativistic wave
- Superluminal physics → superluminal wave → superluminal communication
- Action at a distance physics → AAAD/quantum communication

From *Neutrosophic Logic* perspective, we can see that superluminal physics and superluminal communication are an intermediate way between the subluminal physics and AAAD physics, because in NL theory there is always a possibility to find a third way or intermediate state.

Summarizing:

Standpoint A (subluminal) → Intermediate (*superluminal*) → Standpoint B (AAAD)

In the next sections, we will discuss shortly quantum entanglement and how it can be used in developing new telecommunication technologies.

3. What is quantum entanglement?

In its simplest form the quantum theory's features can be reduced to: (a) wave function description of microscopic entities and (b) entanglement. Entanglement is a key property that makes quantum information theory di□erent from its classical counterpart.[14]

According to Scolarici and Solombrino [5]:

> The essential difference in the concept of state in classical and quantum mechanics is clearly pointed out by the phenomenon of entanglement, which may occur whenever the product states of a compound quantum system are superposed. Entangled states play a key role in all controversial features of QM; moreover, the recent developments in quantum information theory have shown that entanglement can be considered a concrete physical resource that it is important to identify, quantify and classify.

Nonetheless, they concluded: "our research has pointed out a puzzling situation, in which the same state of a physical system is entangled in CQM, while it seems to be separable in QQM."

While entanglement is usually considered as purely quantum effect, it by no means excludes possibility to describe it in a classical way.

DOI: 10.5281/zenodo.3841630

In this regards, from history of QM we learn that there were many efforts to describe QM features in more or less classical picture. For example: Einstein in 1927 presented his version of hidden variable theory of QM, starting from Schrödinger's picture, which seems to influence his later insistence that "*God does not play dice*" philosophy.[6][7]

Efforts have also been made to extend QM to QQM (quaternionic Quantum Mechanics), for instance by Stephen Adler from IAS.[8]

But in recent decades, another route began to appear, what may be called as Maxwell-Dirac isomorphism route, where it can be shown that there is close link between Maxwell equations of classical electromagnetism and Dirac equations of electron. Intuitively, this may suggest that there is one-to-one correspondence between electromagnetic wave and quantum wave function.

4. Two possible ways of superluminal communications

4.a. Maxwell-Dirac isomorphism through Quaternion algebra

Textbook quantum theory is based on complex numbers of the form $a_0+a_1 i$, with i the imaginary unit $i^2 = -1$. It has long been known that an alternative quantum mechanics can be based on the quaternion or hyper-complex numbers of the form $a_0 + a_1 i + a_2 j + a_3 k$, with i,j,k three non-commuting imaginary units.[8]

On the other hand, recognizing that the Maxwell's equations were originally formulated in terms of quaternionic language, some authors investigate formal correspondence between Maxwell and Dirac equations. To name a few who worked on this problem: Kravchenko and Arbab. These authors have arrived to a similar conclusion, although with a different procedures based on Gersten decomposition of Dirac equation.[4]

This MD isomorphism can also be extended further to classical description of boson mass which was usually called Higgs boson[3], so it may be a simpler option compared to scale symmetry theory.

4.b. Quaternionic QM and Entanglement

Having convinced ourselves that Maxwell-Dirac isomorphism has sufficient reasoning to consider seriously in order to come up with realistic interpretation of quantum wave function, now let us discuss QQM and entanglement.

Singh & Prabakaran are motivated to examine the geometry of a two qubit quantum state using the formalism of the Hopf map. However, when addressing multiple qubit states, one needs to carefully consider the issue of quantum entanglement. The "quaternions" again come in handy in studying the two qubit state. [10]

In his exposition of Quaternionic Quantum Mechanics, J.P. Singh concluded that [9]:

> "Having established the compatibility of the Hopf fibration representation with the conventional theory for unentangled states, let us, now, address the issue of measurability of entanglement in this formalism. In the

DOI: 10.5281/zenodo.3841630

context, "Wootters' Concurrence" and the related "Entanglement of Formation" constitute well accepted measures of entanglement, particularly so, for pure states. …

It follows that any real linear combination of the "magic basis" would result in a fully entangled state with unit concurrence. Conversely, any completely entangled state can be written as a linear combination in the "magic basis" with real components, up to an overall phase factor. In fact, these properties are not unique to a state description in the "magic basis" and hold in any other basis that is obtained from the "magic basis" by an orthogonal transformation…"

Singh & Prabakaran also suggest that this quaternionic QM may be useful for exploring quaternionic computing.[10]

Therefore, it shall be clear that entanglement and quantum communication have a sound theoretical basis.

4.c. Basic principles of quantum communication based on quantum noise

Our proposed communication method can provide an infinite number of infinite bandwidth communications channels for each user. See our recent paper describing one of plausible way to do quantum communication [14].

Communication using this method travels much faster than light. It does not use radio waves and does not need wires. It cannot be monitored nor tracked nor interfered with. It cannot be regulated due to the infinities involved, and due to the fact that it is unmonitorable. Each user benefits personally from the perfect information security provided by quantum communications.

Quantum communication does not harm any form of life, nor the environment, in any way, as quantum events are, and always have been, constantly a part of the Natural Environment. This method is not related to "Q-bits" nor "quantum teleportation" nor "quantum amplification" approaches, in any way. It is based on the Schrödinger equations of Quantum Mechanics. One of the features of the Schrödinger equations is a descriptive prediction of what is called "*quantum noise*". This is the constant "hiss" that one hears when using an FM radio, and setting the frequency selector in between active broadcast channels. The sound is called "quantum noise"

Quantum noise is observable at every location in the infinite volume universe. Quantum noise is the result of non-local Subquantum processes which cause apparently random quantum behaviors in physical systems, particularly those which involve electric, magnetic, or electromagnetic fields.

The situation is described by the quantum observable A of the system. This boils down to the fact that there is an expectation value in situations which involve quantum noise, which should normally appear as perfect randomity in the quantum system we are observing. Perfect randomity is called 3rd Order randomity and is completely unpredictable.

DOI: 10.5281/zenodo.3841630

3rd Order randomity then represents the normal behavior of our quantum system as it interacts with Subquantum entities which are interacting with the system from up to infinity away and with up to an infinite velocity. 3rd Order randomity is the quantum expectation value of all Natural systems, in all locations and at all times.

There are ways to detect and predict quantum noise and the physical changes produced by quantum noise in quantum systems (These methods will not be discussed at this time). When we detect the quantum noise, for example, in the form of "white noise" between radio stations, we expect the quantum spectrum centered on the channel of our receiver to exhibit 3rd Order randomity in both electromagnetic frequency and magnitude domains, in our selected channel. However, environmental factors such as the presence of physical or non-physical forms of Consciousness can act on the 3rd Order randomity so as to bring predictability and order to the stream of random number which our E/M detector array passes on to our discriminator system.

Related to this, it was proved by instrumented experiments in the USA and in France during the 1990s that the Attention, Intentions, and Emotional State of operators of symplectic, complex, and standard electromagnetic transmission facilities, resulted in instantaneous changes in the radiation patterns of the transmission antennas.

All of the above mentioned facts can be useful for developing a working quantum communication device, see for further exposition of our method in [14].

5. Concluding remarks

Despite its enormous practical success, many physicists and philosophers alike agree that the quantum theory is so full of contradictions and paradoxes which are difficult to solve consistently. Even after 90 years, the experts themselves still do not all agree what to make of it. In this paper, we review the most puzzling feature of QM, i.e. entanglement.

In the meantime, the problem of the dangers of 5G creates a potential to develop new solutions of telecommunications, without having to use 5G/RF technologies. Therefore, in this paper we consider superluminal physics and superluminal communication as a bridge or intermediate way between subluminal physics and action-at-a-distance (AAAD) physics, especially from Neutrosophic Logic perspective. Although several ways have been proposed to bring such a superluminal communication into reality, such as Telluric wave or Telepathy analog of Horejev and Baburin, here we also review two possibilities: quaternion communication and also quantum communication based on quantum noise.

From *Neutrosophic Logic* perspective, we discuss on superluminal physics and superluminal communication as an intermediate way between the subluminal physics and AAAD physics, because in NL theory there is always a possibility to find a third way or intermediate state.

Summarizing:

DOI: 10.5281/zenodo.3841630

Standpoint A (*subluminal*) → Intermediate (*superluminal*) → Standpoint B (*AAAD*)

This paper was inspired by an old question: Is there an alternate way to communication beyond RF method?

Further research is recommended for future implementation.

Acknowledgment

These authors gratefully appreciate four anonymous reviewers for their suggestions to improve this manuscript. Discussions with Robert N. Boyd PhD., are also appreciated.

REFERENCES

[1] Hans Sallhofer. Elementary Derivation of the Dirac equation. X. *Z. Naturforsch*. 41a, pp.468-470, 1986 [1a] See also his series of papers on classical description of hydrogen.
[2] Volodimir Simulik. Some Algebraic Properties of Maxwell-Dirac Isomorphism. *Z. Naturforsch*. 49a, 1074-1076 (1994)
[3] Bo Lehnert. Minimum mass of a composite boson. *J. Modern Physics,* 5, pp.2074-2079, 2016.
[4] Victor Christianto & Florentin Smarandache. A derivation of Maxwell equations in Quaternion Space. *Progress in Physics* vol. 2, April 2010. url: http://www.ptep-online.com
[5] G. Scolarici and L. Solombrino. COMPLEX ENTANGLEMENT AND QUATERNIONIC SEPARABILITY. In C. Garola, A. Rossi, S. Sozzo, The Foundations of Quantum Mechanics, Cesena, Italy, October 2004. New Jersey: World Scientific Publ. Co., pp.301-310, 2006.
[6] Peter Holland. WHAT'S WRONG WITH EINSTEIN'S 1927 HIDDEN-VARIABLE INTERPRETATION OF QUANTUM MECHANICS? Arxiv: quant-ph/0401017, 2004.
[7] Darrin W. Belousek. Einstein's 1927 Unpublished Hidden-Variable Theory: Its Background, Context and Significance. *Stud. Hist. Phil. Mod. Phys*., Vol. 21, No. 4, pp. 431461, 1996
[8] Stephen L. Adler. Does the Peres experiment using photons test for hyper-complex (quaternionic) quantum theories? arXiv: 1604.04950, 2016.
[9] J.P. Singh. Quantum entanglement through quaternions. *Apeiron*, Vol. 16, No. 4, 2009.
[10] J.P. Singh & S. Prabakaran. Quantum Computing Through Quaternions. EJTP 5, No. 19, pp.1–8, 2008.
[11] G. Najarbashi *et al.* Two and Three-Qubits Geometry, Quaternionic and Octonionic Conformal Maps, and Intertwining Stereographic Projection. arXiv: 1501.06013,2015
[12] Matthew E. Graydon. Quaternions and Quantum Theory. A thesis presented to the University of Waterloo, Ontario, Canada, 2011.
[13] William K. Wootters. Entanglement of Formation and Concurrence. *Quantum Information and Computation*, Vol. 1, No. 1 pp.27-44, 2001.
[14] R.N. Boyd, V. Christianto, F. Smarandache. A harmless wireless quantum alternative to cell phones based on quantum noise. *EC Neurology* 11.10,pp.942-946, 2019. url: https://www.ecronicon.com/ecne/pdf/ECNE-11-00585.pdf
[15] Augusto Espinoza Garrido. Review to Andrew E. **Chubykalo,** Viv **Pope** and Roman Smirnov-Rueda. *Instantaneous Action at a Distance in Modern Physics: Pro and Contra*. Nova Science Publishers, Inc., 1999. AFLB. url: http://aflb.ensmp.fr/AFLB-293/aflb293m179.pdf

[16] F. Smarandache. There is no speed barrier in the Universe. *Bull. Pure and Appl. Science,* vol. 17D (Physics) No. 1, vol. 61, 1998. Also in url: http://fs.unm.edu/NoSpLim.htm; see also [16a] E Weisstein, Smarandache Hypothesis, *Science World Encyclopedia*, Wolfram, http://scienceworld.wolfram.com/physics/SmarandacheHypothesis.html

[17] F. Smarandache. There Is No Speed Barrier for a Wave Phase Nor for Entangled Particles. *Prog. In Phys*. vol. 1 (2005). www.ptep-online.com

DOI: 10.5281/zenodo.3841630

[18] Tesla's world system. Museum of Science & Technology, Belgrade, 2016.

[19] Ljiljana Trajković. TESLA'S VISION OF THE WIRELESS GLOBAL COMMUNICATIONS. Communication Networks Laboratory, Simon Fraser University, http://www.ensc.sfu.ca/research/cnl

[20] Paul Brenner. TESLA AGAINST MARCONI : The Dispute for the Radio Patent Paternity. Paper TPC-8. (unknown date)

[21] Aleksandar Marincic, Zorica Civric. NIKOLA TESLA'S CONTRIBUTIONS TO RADIO DEVELOPMENTS. (unknown date)

[22] Nikola Tesla, Method of operating arc lamps, *U.S. patent*, 447,920, Mar. 10, 1891, applied on Oct.1, 1890.

[23] Nikola Tesla, Alternating electric current generator, *U.S. patent,* 447,921, Mar. 10, 1891, applied on Nov.15, 1890.

[24] Nikola Tesla: Experiments with alternate currents of very high frequency and their application to methods of artificial illumination, lecture delivered before A.I.E.E., at Columbia College, May 20, 1891; republished many times after publication in Electrical Engineer, New York, July 8, pp.25-48, 1891.

[25] John S. Belrose. ON THE BIRTH OF WIRELESS TELEPHONY, 2000.

[26] A. David Wunsch. Nikola Tesla's True Wireless: A Paradigm Missed. *Proceedings of the IEEE*, Vol. 106, No. 6, June 2018.

[27] V.I. Horejev & L.A. Baburin. *Transfer of information by means of Physical medium of Superweak (Torsion) Fields (Telepathy analog).* url: https://static1.squarespace.com/static/5e3f6f3ecac0f04ddd54be55/t/5e8eb471408bde73996dcf5d/1586410611532/Torsion.pdf

[28] Yupapin, P.P. Human ad hoc networks using telepathic conections. *J Biosens Bioelectron* 2014, 5:1 DOI: 10.4172/2155-6210.1000e129. url: https://www.hilarispublisher.com/open-access/human-ad-hoc-networks-using-telepathic-connections-2155-6210.1000e129.pdf

[29] Viviana Siddhi. Is Telepathy Allowed or Is Controled? J Complement & Alt Med. 1(3): 2019. OJCAM.MS.ID.000515. DOI: 10.33552/OJCAM.2019.01.000515. url: https://irispublishers.com/ojcam/pdf/OJCAM.MS.ID.000515.pdf

[30] Abdolkarim Afroozeh, Seyed Ebrahim Pourmand. Telepathic Communication Generation using Nano Fiber Ring. J. Optoelectronical Nanostructures, Vol. 3 no. 1, 2018. url: http://jopn.miau.ac.ir/article_2822_e87af1d61239a08a94f473c0496773be.pdf

[31] M. Al- Tahan, "Some Results on Single Valued Neutrosophic (Weak) Polygroups", International Journal of Neutrosophic Science, Volume 2 , Issue 1, pp. 38-46 , 2020.

[32] M. Khalid, Young Bae Jun, Mohammad Mohseni Takallo, Neha Andaleeb Khalid, "Magnification of MBJ-Neutrosophic Translation on G-Algebra", International Journal of Neutrosophic Science Volume 2 , Issue 1, pp.27-37 , 2020

[33] S. A. Edalatpanah, "A Direct Model for Triangular Neutrosophic Linear Programming", International Journal of Neutrosophic Science, Volume 1, Issue 1, pp.19-28 , 2020

[34] M. Parimala , M. Karthika , Florentin Smarandache , Said Broumi, "On αω-closed sets and itsconnectedness in terms of neutrosophic topological spaces", International Journal of Neutrosophic Science, Volume 2 , Issue 2, pp. 82-88 , 2020

[35] E.O. Adeleke , A.A.A. Agboola , F. Smarandache, "Refined Neutrosophic Rings II", International Journal of Neutrosophic Science, Volume 2 , Issue 2, pp. 89-94 , 2020

[36] V. Christianto, F. Smarandache, Y. Umniyati. Towards realism interpretation of wave mechanics based on Maxwell equations in quaternion space and some implications, including Smarandache's hypothesis. CTPNP 2019, published 18th May 2020. AIP Conference Proceedings 2234, 040008 (2020); https://doi.org/10.1063/5.0008139

DOI: 10.5281/zenodo.3841630
Received: February24, 2020 Revised: April 27, 2020 Accepted: May 16, 2020

A Study of the Integration of Neutrosophic Thick Function

Malath F. Alaswad [1,*]

[1]Department of Mathematics, Faculty of Science, AL- Baath University, Homs, Syria

* Correspondence: Malaz.aswad@yahoo.com

Abstract

In this paper, the definition of neutrosophic thick function and its integral are introduced. The main objective is defining a differential linear equation based on the thick function and finding solutions for this equation.

Keywords: Neutrosophic Thick Function, Neutrosophic Integration.

1.Introduction

Neutrosophic logic. Neutrosophy, Neutrosophic set, Neutrosophic probability and alike, are recent creations of F. Smarandache, being characterized by having the indeterminacy, as component of their framework and a notable feature of neutrosophic logic is that can be considered a generalization of fuzzy logic, encompassing the classical logic as well[1]. Also, in 2015, F. Smarandache has defined the concept of continuity of a neutrosophic function in [1], and neutrosophic mereo-limit [1], mereo-continuity. Moreover, in 2014, F. Smarandache has defined the concept of a neutro-oscillator differential in [3], and mereo-derivative. Finally, in 2013 F. Smarandache introduced neutrosophic integration in [2], and mereo-integral, besides the classical definitions of limit, continuity, derivative, and integral respectively.

Among the recent applications there are: neutrosophic crisp set theory in image processing[4][5], neutrosophic sets medical field [6][7][8][9][10], geographic information systems [11] and possible applications to database [12]. Also, neutrosophic triplet group application to physics[13]. Moreover, several types of research have made multiple contributions to neutrosophic topology [14][15] [16] [17] [18] [19] [20] Also more researches have made multiple contributions to the neutrosophic analysis[21]. Finally, the neutrosophic integration may be applied in calculating the area between two neutrosophic functions.

2. Preliminaries

DOi: 10.5281/zenodo.3841632

Received: February 19, 2020 Revised: April 08, 2020 Accepted: May 14, 2020

In this paper $m(x) = [m_1(x), m_2(x)]$ is called a neutrosophic thick function. Now, we recall some definitions which are useful in this paper.

Definition 2.1. [**1**]
A neutrosophic crisp function:

$f: A \to B$ is called a crisp relation if there exists an element $a \in A$ with $f(a) = b$ and $f(a) = c$ where $b, c \in B$ then $b \equiv c$. (this is the classical vertical line test).

Definition 2.2. [1]

Neutrosophic subset or crisp function:

A neutrosophic (subset or crisp) function in general is a function that has some indeterminacy.

Definition 2.3. [2]

The neutrosophic constant thick function:

$$l: R \to P(R)\ ; for\ example: l(x) = [2,3];\ for\ any\ x \in R$$

Where $P(R)$ is the set of all subsets of R.

For example, $l(7)$, is the vertical segment of line [2,3] shown in figure 2.1.

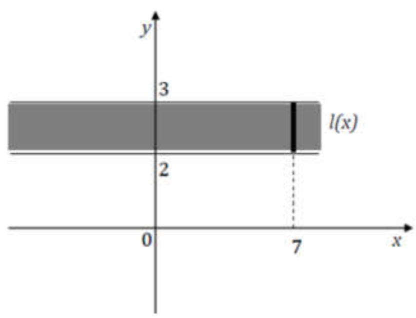

Figure 2.1.

Definition 2.4. [2]

A neutrosophic non-constant thick function:

$k: R \to P(R)$

For example, $k(x) = [2x, 2x + 1]$;

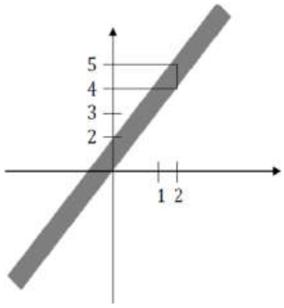

Figure 2.2

Let $k(x) = [2x, 2x + 1]$ then $k(2) = [2(2), 2(2) + 1] = [4,5]$. The respective graph is the Figure 2.2

Definition 2.5. [2]

The general neutrosophic thick function is defined as:

There is something confusing in the use of notations for intervals. For example, it is used $(m_1(x), m_2(x))$ for open interval and $]m_1(x), m_2(x)]$ and $[m_1(x), m_2(x)[$ for half closed (or semi closed) intervals.

$$m: R \to P(R)\,; m(x) = [m_1(x), m_2(x)]$$

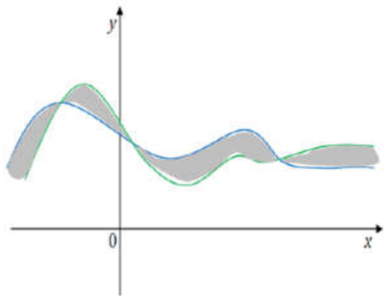

Figure 2.3

3. A Neutrosophic Integration

As in Euclidean integration, integration is the opposite of differentiation.

In other words, the anti-thesis of the derivative of the neutrosophic function $f(x)$ is also a neutrosophic function $F(x)$. That the definition of neutrosophic integral is:

$$F(x) = \int f(x, I)dx$$

Where I represents the indeterminacy and constant integration is $a + bI$.

Example 3.1.

$$f : R \to R \cup \{I\}\,; f(x) = 5x^2 + (3x + 1)I$$

Then:

$$F(x) = \int [5x^2 + (3x+1)I]dx = \int 5x^2 dx + \int (3x+1)I dx = 5\frac{x^3}{3} + \left(3\frac{x^2}{2} + x\right)I + a + bI.$$

Where $\boldsymbol{a + bI}$ is the neutrosophic integration constant.

4. Neutrosophic integration thick function

In this section is given the definition of the neutrosophic thick function, as well the operation of integration over it.

Definition 4.1. Let $m(x) = [m_1(x), m_2(x)]$ be a neutrosophic thick function. Then we define the integration of this function as:

$$\int m(x)dx = \int [m_1(x), m_2(x)]dx = \left[\int m_1(x)\,dx + c_1, \int m_2(x)\,dx + c_2\right] = [A, B]$$

Where $c_1 = a_1 + b_1 I_1$, $c_2 = a_2 + b_2 I_2$.

Example 4.1. Let $m(x) = [m_1(x), m_2(x)] = \left[xe^x, xe^{x^2}\right]$ then:

$$\int m(x)dx = \int \left[xe^x, xe^{x^2}\right]dx = \left[\int xe^x\,dx + c_1, \int xe^{x^2}dx + c_2\right] = [A, B]$$

$$A = \int xe^x\,dx + c_1 = x.e^x - e^x + c_1$$

$$B = \int xe^{x^2}dx + c_2 = \frac{1}{2}.e^{x^2} + c_2$$

$$\int m(x)dx = \left[x.e^x - e^x + c_1, \frac{1}{2}.e^{x^2} + c_2\right]$$

Where $c_1 = a_1 + b_1 I_1$, $c_2 = a_2 + b_2 I_2$.

Example 4.2. let $m(x) = [m_1(x), m_2(x)] = \left[\frac{1}{1+x^2}, \frac{x^2}{1+x^2}\right]$ then:

$$\int m(x)dx = \int \left[\frac{1}{1+x^2}, \frac{x^2}{1+x^2}\right]dx = \left[\int \frac{1}{1+x^2}\,dx + c_1, \int \frac{x^2}{1+x^2}dx + c_2\right] = [A, B]$$

$$A = \int \frac{1}{1+x^2}\,dx + c_1 = arc\ tg(x) + c_1$$

$$B = \int \frac{x^2}{1+x^2}dx + c_2 = x - arc\ tg(x) + c_2$$

$$\int m(x)dx = [arc\ tg(x) + c_1, x - arc\ tg(x) + c_2]$$

Where $c_1 = a_1 + b_1 I_1$, $c_2 = a_2 + b_2 I_2$.

5. Neutrosophic linear differential equation

In this section we define a linear differential equation based on the thick function and find solutions of this equation.

Neutrosophic identical linear differential equation

Definition. We define the Neutrosophic identical linear differential equation by a neutrosophic thick function form:

$$\acute{y} + m(x)y = 0\ ;\ m(x) = [m_1(x), m_2(x)] \quad (1)$$

Method of solution.

$$\acute{y} + m(x)y = 0$$

$$\acute{y} + [m_1(x), m_2(x)]y = 0$$

$$\acute{y} = -[m_1(x), m_2(x)]y$$

$$\frac{\acute{y}}{y} = -[m_1(x), m_2(x)]$$

$$\frac{\ln y}{c} = -\int [m_1(x), m_2(x)]\, dx = -\left[\int m_1(x)\ dx\, , \int m_2(x)\ dx\right]$$

$$y = c\left[e^{-\int m_1(x)\, dx}, e^{-\int m_2(x)\, dx}\right] \quad (2)$$

Example 5.1. Find a solution to the equation:

$$\acute{y} + \left[\frac{1}{x}, 2x\right] y = 0$$

Solution.

$$y = (a + bI)\left[e^{-\int \frac{1}{x} dx}, e^{-\int 2x\, dx}\right] = (a + bI)\left[e^{-\ln x}, e^{-x^2}\right] = (a + bI)\left[\frac{1}{x}, e^{-x^2}\right]$$

5.4 Neutrosophic non-homogeneous linear differential equation

Definition 5.5. We define the Neutrosophic non- homogeneous linear differential equation by a neutrosophic thick function which takes one of the following forms:

$$\acute{y} + [m_1(x), m_2(x)]y = q(x) \dots\dots (2)$$

$$\acute{y} + p(x)y = [f_1(x), f_2(x)] \dots\dots (3)$$

$$\acute{y} + [m_1(x), m_2(x)]y = [f_1(x), f_2(x)] \dots\dots (4)$$

Now we will derive the solution of equation (2) and solution of equations (3), (4) can be driven in the same way:

Method of solution.

1- We find the complement factor of the equation (2) as follows:

$$\mu(x) = e^{[\int m_1(x)dx, \int m_2(x)dx]} = \left[e^{\int m_1(x)dx}\ , e^{\int m_2(x)dx}\right]$$

2- We multiply equation (2) by the complement factor:

$$\mu(x)\acute{y} + \mu(x)[m_1(x), m_2(x)]y = q(x)\mu(x)$$

DOi: 10.5281/zenodo.3841632

$$\mu(x)\acute{y} + \left[m_1(x)e^{\int m_1(x)dx}, m_2(x)e^{\int m_2(x)dx}\right]y = \left[q(x)e^{\int m_1(x)dx}, q(x)e^{\int m_2(x)dx}\right]$$

3- Note that the left side is only the function derivative: $\mu(x)y$ and therefor the equation can be sequenced as:

$$\left(y\mu(x)\right)' = \left[q(x)e^{\int m_1(x)dx}, q(x)e^{\int m_2(x)dx}\right]$$

4- By integrating the latter, we obtain the general solution of equation(2):

$$y = \frac{1}{\mu(x)}\left(a + bI + \left[\int q(x)e^{\int m_1(x)dx}, \int q(x)e^{\int m_2(x)dx}\right]\right) \dots\dots (5)$$

Example 5.6. Find the general solution of the following neutrosophic non- homogeneous linear differential equation:

$$\acute{y} + \left[2x, \frac{1}{x}\right]y = x$$

Solution. The equation is of the first form:

$$\acute{y} + [m_1(x), m_2(x)]y = q(x)$$

First we find the complement factor:

$$\mu(x) = e^{[\int m_1(x)dx, \int m_2(x)dx]} = \left[e^{\int m_1(x)dx}, e^{\int m_2(x)dx}\right] = e^{\left[\int 2xdx, \int \frac{1}{x}dx\right]} = \left[e^{x^2}, e^{\ln x}\right] = \left[e^{x^2}, x\right]$$

Now by multiplying both sides of the equation by the complement factor, we find that the general solution can be written as:

$$y = \frac{1}{\mu(x)}\left(a + bI + \int x\mu(x)dx\right) = \frac{1}{\left[e^{x^2}, x\right]}\left(a + bI + \left[\int xe^{x^2}dx, \int x^2 dx\right]\right)$$

where:

$$\int xe^{x^2}dx = \frac{1}{2}e^{x^2}$$

$$\int x^2 dx = \frac{1}{3}x^3$$

Thus the general solution of the given equation is:

$$y = \frac{1}{\left[e^{x^2}, x\right]}\left(a + bI + \left[\frac{1}{2}e^{x^2}, \frac{1}{3}x^3\right]\right)$$

Example 5.7. Find the general solution for the following neutrosophic non- homogeneous linear differential equation:

$$\acute{y} + cot(x)\, y = [sin(x), cos(x)]$$

Solution. The equation is of the form:

$$\acute{y} + p(x)y = [f_1(x), f_2(x)]$$

We find the complement factor as:

$$\mu(x) = e^{\int p(x)dx} = e^{\int cot(x)dx} = e^{\ln sin(x)} = sin(x)$$

Now by multiplying both sides of the equation by the complement factor, we find that the general solution can be written as:

$$y = \frac{1}{\mu(x)}\left(a + bI + \int \mu(x)\,.[sin(x), cos(x)]dx\right) = \frac{1}{sin(x)}\left(a + bI + \int sin(x)\,.[sin(x), cos(x)]dx\right)$$
$$= \frac{1}{sin(x)}\left(a + bI + \left[\int sin^2(x)\,dx, \int sin(x).cos(x)\,dx\right]\right)$$

where:

$$\int sin^2(x)\,dx = \frac{1}{2}x - \frac{1}{4}sin(2x)$$

$$\int sin(x).cos(x)\,dx = \frac{1}{2}\int sin(2x)dx = -\frac{1}{4}cos(2x)$$

Thus the general solution of the given equation is:

$$y = \frac{1}{sin(x)}\left(a + bI + \left[\frac{1}{2}x - \frac{1}{4}sin(2x)\,, -\frac{1}{4}cos(2x)\right]\right).$$

Example 5.8. Find the general solution for the following neutrosophic non- homogeneous linear differential equation:

$$\acute{y} + \left[\frac{1}{x}, x\right]y = [x^2, x]$$

Solution: the equation is of the third form:

$$\acute{y} + [m_1(x), m_2(x)]y = [f_1(x), f_2(x)]$$

We find the complement:

$$\mu(x) = e^{[\int m_1(x)dx, \int m_2(x)dx]} = \left[e^{\int m_1(x)dx}\,, e^{\int m_2(x)dx}\right] = \left[e^{\ln x}\,, e^{\frac{1}{2}x^2}\right] = \left[x, e^{\frac{1}{2}x^2}\right].$$

Now by multiplying both sides of the equation by the complement factor, we find that the general solution is written as:

$$y = \frac{1}{\mu(x)}\left(a + bI + \int \mu(x)\,[f_1(x), f_2(x)]dx\right) = \frac{1}{\left[x, e^{\frac{1}{2}x^2}\right]}\left(a + bI + \int \left[x, e^{\frac{1}{2}x^2}\right].[x^2, x]dx\right)$$
$$= \frac{1}{\left[x, e^{\frac{1}{2}x^2}\right]}\left(a + bI + \left[\int x.x^2\,dx, \int x.e^{\frac{1}{2}x^2}\,dx\right]\right)$$
$$= \frac{1}{\left[x, e^{\frac{1}{2}x^2}\right]}\left(a + bI + \left[\int x^3\,dx, \int x.e^{\frac{1}{2}x^2}\,dx\right]\right)$$

where:

$$\int x^3\,dx = \frac{1}{4}x^4$$

DOi: 10.5281/zenodo.3841632

$$\int x.e^{\frac{1}{2}x^2}\,dx = e^{\frac{1}{2}x^2}$$

Thus the general solution of the given equation is:

$$y = \frac{1}{\left[x, e^{\frac{1}{2}x^2}\right]}\left(a + bI + \left[\frac{1}{4}x^4, e^{\frac{1}{2}x^2}\right]\right)$$

6. Conclusion

In this paper, a new type of neutrosophic integration has been defined by using the thick function, Moreover, we studied a linear differential equation based on the thick function and found solutions to this equation. In addition, solutions to other types of neutrosophic differential equations can be found depending on the thick function such as Bernoulli's equation. We will work on this in the future.

References

[1] F. Smarandache, Neutrosophic Precalculus and Neutrosophic Calculus, University of New Mexico, Digital Repository, Albuquerque, NM, USA, 2015.

[2] F. Smarandache, Neutrosophic Measure and Neutrosophic Integral, in Neutrosophic Sets and Systems, 3 – 7, Vol. 1, 2013.

[3] A. A Salama, Smarandache F. Kroumov, Neutrosophic Closed Set and Continuous Functions, in Neutrosophic Sets and Systems, Vol.4, pp.4- 8, 2014.

[4] A. A Salama; I. M Hanafy; Hewayda Elghawalby Dabash M.S, "Neutrosophic Crisp Closed Region and Neutrosophic Crisp Continuous Functions", New Trends in Neutrosophic Theory and Applications..

[5] A. A Salama; Hewayda Elghawalby; M.S, Dabash; A.M. NASR, Retrac Neutrosophic Crisp System For Gray Scale Image, Asian Journal Of Mathematics and Computer Research, Vol 24, pp.104-117, 2018.

[6] F. smarandache. "Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, neutrosophic Logic, Set, Probability, and Statistics" University of New Mexico, Gallup, NM87301, USA 2002.

[7] M. Abdel-Basset; E. Mai. Mohamed; C. Francisco; H. Z. Abd EL-Nasser "Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases" Artificial Intelligence in Medicine Vol. 101, 101735, 2019.

[8] M. Abdel-Basset; E. Mohamed; G. Abdullah; and F. Smarandache. "A novel model for evaluation Hospital medical care systems based on plithogenic sets" Artificial Intelligence in Medicine 100 (2019), 101710.

[9] M. Abdel-Basset; G. Gunasekaran Mohamed; G. Abdullah. C. Victor, "A Novel Intelligent Medical Decision Support Model Based on soft Computing and Iot" IEEE Internet of Things Journal, Vol. 7, 2019.

[10] M. Abdel-Basset; E. Mohamed; G. Abdullah; G. Gunasekaran; L. Hooang Viet." A novel group decision making model based on neutrosophic sets for heart disease diagnosis" Multimedia Tools and Applications, 1-26, 2019.

[11] A. A Salama. Basic Structure of Some Classes of Neutrosophic Crisp Nearly Open Sets and Possible Application to GIS Topology. Neutrosophic Sets and Systems, Vol. 7, pp.18-22, 2015.

[12] A. A Salama; F. Smarandache. Neutrosophic Set Theory, Neutrosophic Sets and Systems, Vol. 5, pp.1-9, 2014.

[13] F. Smarandache, The Neutrosophic Triplet Group and its Application to Physics, Seminar Universidad National de Quilmes , Department of Science and Technology, Beunos Aires, Argentina, 20 June 2014.

[14] A. B.AL-Nafee; R.K. Al-Hamido; F.Smarandache. "Separation Axioms In Neutrosophic Crisp Topological Spaces", Neutrosophic Sets and Systems, vol. 25, pp.25-32, 2019.

DOi: 10.5281/zenodo.3841632

[15] R.K. Al-Hamido, Q. H. Imran, K. A. Alghurabi, T. Gharibah, “On Neutrosophic Crisp Semi Alpha Closed Sets”, Neutrosophic Sets and Systems”, vol. 21, pp.28-35, 2018.
[16] Q. H. Imran, F. Smarandache, R.K. Al-Hamido, R. Dhavasselan, “On Neutrosophic Semi Alpha open Sets”, Neutrosophic Sets and Systems, vol. 18, pp.37-42, 2017.
[17] R. K..Al-Hamido, “A study of multi-Topological Spaces”, PhD Theses, AlBaath university , Syria, 2019.
[18] R. K.Al-Hamido,“Neutrosophic Crisp Supra Bi-Topological Spaces”, International Journal of Neutrosophic Science, Vol. 1, pp.66-73, 2018.
[19] R.K. Al-Hamido, “Neutrosophic Crisp Bi-Topological Spaces”, Neutrosophic Sets and Systems, vol. 21, pp.66-73, 2018.
[20] R.K. Al-Hamido, T. Gharibah, S. Jafari F.Smarandache, “On Neutrosophic Crisp Topology via N-Topology”, Neutrosophic Sets and Systems, vol. 21, pp.96-109, 2018.
[21] A. Hatip, "The Special Neutrosophic Functions," International Journal of Neutrosophic Science, Volume 4 , Issue 2, pp. 104-116, 2020.

DOi: 10.5281/zenodo.3841632

Erlang Service Queueing Model with Neutrosophic Parameters

Mohamed Bisher Zeina

Department of Mathematical Statistics, Faculty of Science, University of Aleppo, Aleppo, Syria;
bisherzeina@alepuniv.edu.sy

Abstract

In this paper, we generalize the classical Erlang service queueing model, which has determined and crisp parameters to neutrosophic Erlang service queueing model which is more accurate because it opens the ability to deal with imprecise and incomplete knowledge of parameters. We have used the neutrosophic statistical interval method introduced by F. Smarandache to describe the parameters of the Erlang service queueing model and to find the neutrosophic performance measures.

Keywords: Erlang Service Queues; Neutrosophic Logic; Performance Measures; Neutrosophic Statistical Numbers.

1.Introduction

Queueing theory was developed by Erlang in 1909 to model waiting for lines and develop efficient systems that reduce customers' waiting times which makes it possible to serve more customers and increase profits to the organizations. The problem of classical queueing theory is the assumption of well and clear knowledge of the parameters of the queueing system (e.g. arrival rate, departure rate) which is often impossible [1,2].

Neutrosophic logic introduced by F. Smarandache in 1995 is a generalization of fuzzy logic and intuitionistic fuzzy logic [3,4,5,6,7,8]. This logic makes dealing with indeterminant data easier, clearer, and more realistic. So modelling queues using neutrosophic parameters makes decisions more efficient [9,10,11,12,13,14,15].

Extension of classical queueing theory to neutrosophic queueing theory means that parameters of queues can take indeterminant values that allow dealing with vagueness.

In this paper, we show the power of neutrosophic crisp sets theory [16,17] to deal with imprecise parameters of the Erlang queueing model, and we derive neutrosophic performance measures of the mentioned queue, also solved examples showing the power of this extension are presented.

2. Classical Erlang Service Model [1,2]

In this model, customers arrive at a service system with one server according to exponential interarrival times with parameter λ and served one by one according to Erlang service in k phases with parameter μ, and the performance measures are:

Expected number of customers in the system:

$$L_s = \left(\frac{k+1}{2k}\right)\left(\frac{\lambda^2}{\mu(\mu-\lambda)}\right) + \frac{\lambda}{\mu}$$

Expected number of customers in the queue:

DOI: 10.5281/zenodo.3841739

Received: February 12, 2020 Revised: March 27, 2020 Accepted: May 18, 2020

$$L_q = L_s - \frac{\lambda}{\mu} = \left(\frac{k+1}{2k}\right)\left(\frac{\lambda^2}{\mu(\mu-\lambda)}\right)$$

Expected waiting time in the system:

$$W_s = \frac{L_s}{\lambda}$$

$$W_s = \left(\frac{k+1}{2k}\right)\left(\frac{\lambda}{\mu(\mu-\lambda)}\right) + \frac{1}{\mu}$$

Expected waiting time in the queue:

$$W_q = \frac{L_q}{\lambda}$$

$$W_q = \left(\frac{k+1}{2k}\right)\left(\frac{\lambda}{\mu(\mu-\lambda)}\right)$$

Where λ is arrival rate, μ is departure (servicing) rate.

3. Neutrosophic Erlang Service Model

In this model, customers arrive at one server facility according to the Poisson process with neutrosophic (inaccurate, imprecise) parameter $N\lambda$ [17] where we are going to use neutrosophic statistical numbers presented in [9] so $N\lambda$ can be written as an interval as following $N\lambda = [\lambda^L, \lambda^U]$ where λ^L, λ^U are crisp real numbers. Customers are also served one by one according to Erlang service in k phases with neutrosophic parameter $N\mu = [\mu^L, \mu^U]$ where μ^L, μ^U are also crisp real numbers. So the neutrosophic performance measures can be driven by replacing λ by $N\lambda$ and μ by $N\mu$ as follows:

Neutrosophic expected number of customers in the system:

$$NL_s = \left(\frac{k+1}{2k}\right)\left(\frac{[\lambda^L, \lambda^U]^2}{[\mu^L, \mu^U]([\mu^L, \mu^U] - [\lambda^L, \lambda^U])}\right) + \frac{[\lambda^L, \lambda^U]}{[\mu^L, \mu^U]}$$

$$NL_s = \left(\frac{k+1}{2k}\right)\left(\left[\frac{\lambda^{L^2}}{\mu^{U^2} - \lambda^L \mu^U}, \frac{\lambda^{U^2}}{\mu^{L^2} - \lambda^U \mu^L}\right]\right) + \left[\frac{\lambda^L}{\mu^U}, \frac{\lambda^U}{\mu^L}\right] \qquad (1)$$

Neutrosophic expected number of customers in the queue:

$$NL_q = \left(\frac{k+1}{2k}\right)\left(\left[\frac{\lambda^{L^2}}{\mu^{U^2} - \lambda^L \mu^U}, \frac{\lambda^{U^2}}{\mu^{L^2} - \lambda^U \mu^L}\right]\right) \qquad (2)$$

Neutrosophic expected waiting time in the system:

$$NW_s = \frac{NL_s}{N\lambda} = \frac{NL_s}{[\lambda^L, \lambda^U]} = NL_s * \left[\frac{1}{\lambda^U}, \frac{1}{\lambda^L}\right]$$

$$NW_s = \left(\frac{k+1}{2k}\right)\left(\left[\frac{\frac{\lambda^{L^2}}{\lambda^U}}{\mu^{U^2} - \lambda^L \mu^U}, \frac{\frac{\lambda^{U^2}}{\lambda^L}}{\mu^{L^2} - \lambda^U \mu^L}\right]\right) + \left[\frac{\frac{\lambda^L}{\lambda^U}}{\mu^U}, \frac{\frac{\lambda^U}{\lambda^L}}{\mu^L}\right] \qquad (3)$$

Neutrosophic expected waiting time in the queue:

$$NW_q = \frac{NL_q}{N\lambda} = \frac{NL_q}{[\lambda^L, \lambda^U]} = NL_q * \left[\frac{1}{\lambda^U}, \frac{1}{\lambda^L}\right]$$

$$NW_q = \left(\frac{k+1}{2k}\right)\left(\left[\frac{\frac{\lambda^{L^2}}{\lambda^U}}{\mu^{U^2} - \lambda^L \mu^U}, \frac{\frac{\lambda^{U^2}}{\lambda^L}}{\mu^{L^2} - \lambda^U \mu^L}\right]\right) \qquad (4)$$

Notice that all the neutrosophic performance measures are presented as statistical numbers having lower bound and upper bound, also we can notice four cases:

Case 1: When $\lambda^L = \lambda^U$ and $\mu^L \neq \mu^U$ we have crisp arrival rate with neutrosophic departures.

DOI: 10.5281/zenodo.3841739

Case 2: When $\mu^L = \mu^U$ and $\lambda^L \neq \lambda^U$ we have crisp departures with neutrosophic arrivals.
Case 3: When $\lambda^L = \lambda^U$ and $\mu^L = \mu^U$ the neutrosophic corresponds to the classical Erlang service queue.
Case 4: When $\lambda^L \neq \lambda^U$ and $\mu^L \neq \mu^U$ we have neutrosophic arrivals and neutrosophic departures.

4. Numerical Examples

Example 1 (crisp arrival rate, neutrosophic departure rate)

Suppose that a system consists of a machine gives a service to a customer in two phases each phase time is exponentially distributed and the machine serves in each phase between 3 and 5 customers per min. Customers arrive at the machine according to the Poisson process with arrival rate equals to 15 customers/h, and suppose that we want to calculate the following:

The average number of customers in the system.

The average number of customers in the queue.

Mean waiting time in the system.

Mean waiting time in the queue.

Solution:

From the given example we find that k=2, $\boldsymbol{\lambda = 15/h}$ so $\boldsymbol{\lambda = 0.25/min}$, $\boldsymbol{N\mu} = [\mathbf{3}, \mathbf{5}]$/min

Before the foundation of neutrosophic logic, one can solve this example by assuming that $\boldsymbol{\mu}$ takes the midpoint of the range [1,2], that is $\boldsymbol{\mu} \cong \mathbf{4}$ so:

Expected number of customers in the system:

$$L_s = \left(\frac{k+1}{2k}\right)\left(\frac{\lambda^2}{\mu(\mu-\lambda)}\right) + \frac{\lambda}{\mu} = \left(\frac{2+1}{2*2}\right)\left(\frac{0.25^2}{4(4-0.25)}\right) + \frac{0.25}{4} = 0.065625\ customers$$

Expected number of customers in the queue:

$$L_q = \left(\frac{k+1}{2k}\right)\left(\frac{\lambda^2}{\mu(\mu-\lambda)}\right) = \left(\frac{2+1}{2*2}\right)\left(\frac{0.25^2}{4(4-0.25)}\right) = 0.003125\ customers$$

Expected waiting time in the system:

$$W_s = \left(\frac{k+1}{2k}\right)\left(\frac{\lambda}{\mu(\mu-\lambda)}\right) + \frac{1}{\mu} = \left(\frac{2+1}{2*2}\right)\left(\frac{0.25}{4(4-0.25)}\right) + \frac{1}{4} = 0.2625\ min$$

Expected waiting time in the queue:

$$W_q = \left(\frac{k+1}{2k}\right)\left(\frac{\lambda}{\mu(\mu-\lambda)}\right) = \left(\frac{2+1}{2*2}\right)\left(\frac{0.25}{4(4-0.25)}\right) = 0.0125\ min$$

But after derving(?) the neutrosophic solutions and using equations 1 to 4 we get:

$$\mathbf{NL_s} = \left(\frac{\mathbf{k+1}}{\mathbf{2k}}\right)\left(\left[\frac{\boldsymbol{\lambda}^{\mathbf{L}^2}}{\boldsymbol{\mu}^{\mathbf{U}^2} - \boldsymbol{\lambda}^{\mathbf{L}}\boldsymbol{\mu}^{\mathbf{U}}}, \frac{\boldsymbol{\lambda}^{\mathbf{U}^2}}{\boldsymbol{\mu}^{\mathbf{L}^2} - \boldsymbol{\lambda}^{\mathbf{U}}\boldsymbol{\mu}^{\mathbf{L}}}\right]\right) + \left[\frac{\boldsymbol{\lambda}^{\mathbf{L}}}{\boldsymbol{\mu}^{\mathbf{U}}}, \frac{\boldsymbol{\lambda}^{\mathbf{U}}}{\boldsymbol{\mu}^{\mathbf{L}}}\right]$$
$$= \left(\frac{\mathbf{2+1}}{\mathbf{2*2}}\right) * \left(\left[\frac{\mathbf{0.25^2}}{\mathbf{5^2 - 0.25*5}}, \frac{\mathbf{0.25^2}}{\mathbf{3^2 - 0.25*3}}\right]\right) + \left[\frac{\mathbf{0.25}}{\mathbf{5}}, \frac{\mathbf{0.25}}{\mathbf{3}}\right] = [\mathbf{0.051974}, \mathbf{0.089015}]$$

Which means that average number of customers in the system lies in the given range.

DOI: 10.5281/zenodo.3841739

Notice that $L_s = 0.065625 \in NL_s = [0.051974, 0.089015]$

$$NL_q = \left(\frac{k+1}{2k}\right)\left(\left[\frac{{\lambda^L}^2}{{\mu^U}^2 - \lambda^L\mu^U}, \frac{{\lambda^U}^2}{{\mu^L}^2 - \lambda^U\mu^L}\right]\right) = \left(\frac{2+1}{2*2}\right) * \left(\left[\frac{0.25^2}{25 - 0.25*5}, \frac{0.25^2}{9 - 0.25*3}\right]\right)$$

$$= [0.001974, 0.005682]$$

Which means that average number of customers in queue lies in the given range.
Also in thic case we notice that $L_q = 0.003125 \in [0.001974, 0.005682]$

$$NW_s = \left(\frac{k+1}{2k}\right)\left(\left[\frac{\frac{{\lambda^L}^2}{\lambda^U}}{{\mu^U}^2 - \lambda^L\mu^U}, \frac{\frac{{\lambda^U}^2}{\lambda^L}}{{\mu^L}^2 - \lambda^U\mu^L}\right]\right) + \left[\frac{\frac{\lambda^L}{\lambda^U}}{\mu^U}, \frac{\frac{\lambda^U}{\lambda^L}}{\mu^L}\right] = \left(\frac{2+1}{2*2}\right)\left(\left[\frac{0.25}{25 - 0.25*5}, \frac{0.25}{9 - 0.25*3}\right]\right) + \left[\frac{1}{5}, \frac{1}{3}\right]$$
$$= [0.207895, 0.356061]$$

Which means that average waiting time in the system lies between 0.207895 and 0.356061 mins that is between 12 and 21 secs approximately.

Also $W_s = 0.2625 \in [0.207895, 0.356061] = NW_s$

$$NW_q = \left(\frac{k+1}{2k}\right)\left(\left[\frac{\frac{{\lambda^L}^2}{\lambda^U}}{{\mu^U}^2 - \lambda^L\mu^U}, \frac{\frac{{\lambda^U}^2}{\lambda^L}}{{\mu^L}^2 - \lambda^U\mu^L}\right]\right) = \left(\frac{2+1}{2*2}\right)\left(\left[\frac{0.25}{5^2 - 0.25*5}, \frac{0.25}{3^2 - 0.25*3}\right]\right)$$
$$= [0.007895, 0.022727]$$

Which means that average waiting time in the queue lies between 0.007895 and 0.022727 mins that is between 0.5 and 1.36 secs approximately.

Also $W_q = 0.0125 \in [0.007895, 0.022727] = NW_q$

From all above, we can clearly see that the neutrosophic solutions are more accurate than classical solutions.

Example 2 (neutrosophic arrival rate, neutrosophic departure rate)

Suppose that a system consists of a machine gives a service to a customer in three phases each phase time is exponentially distributed and the machine serves in each phase between 3 and 5 customers per min. Customers arrive at the machine according to the Poisson process with arrival rate lies between 1 and 2 customers/min, and suppose that we want to calculate the following:

The average number of customers in the system.

The average number of customers in the queue.

Mean waiting time in the system.

Mean waiting time in the queue.

Solution:

From the given example, we find that k=3, $\boldsymbol{N\lambda = [1, 2]/min}$, $\boldsymbol{N\mu = [3, 5]}$/min

DOI: 10.5281/zenodo.3841739

Classical solutions can be derived by assuming $\lambda \cong 1.5, \mu \cong 4$ by taking midpoints of each interval, that gives:

Expected number of customers in the system:

$$L_s = \left(\frac{k+1}{2k}\right)\left(\frac{\lambda^2}{\mu(\mu-\lambda)}\right) + \frac{\lambda}{\mu} = \left(\frac{3+1}{2*3}\right)\left(\frac{1.5^2}{4(4-1.5)}\right) + \frac{1.5}{4} = 0.525\ customers$$

Expected number of customers in the queue:

$$L_q = \left(\frac{k+1}{2k}\right)\left(\frac{\lambda^2}{\mu(\mu-\lambda)}\right) = \left(\frac{3+1}{2*3}\right)\left(\frac{1.5^2}{4(4-1.5)}\right) = 0.15\ customers$$

Expected waiting time in the system:

$$W_s = \left(\frac{k+1}{2k}\right)\left(\frac{\lambda}{\mu(\mu-\lambda)}\right) + \frac{1}{\mu} = \left(\frac{3+1}{2*3}\right)\left(\frac{1.5}{4(4-1.5)}\right) + \frac{1}{4} = 0.35\ min$$

Expected waiting time in the queue:

$$W_q = \left(\frac{k+1}{2k}\right)\left(\frac{\lambda}{\mu(\mu-\lambda)}\right) = \left(\frac{3+1}{2*3}\right)\left(\frac{0.25}{4(4-0.25)}\right) = 0.1\ min$$

Neutrosophic solutions can be derived using equations 1 to 4 as follows:

$$\mathbf{NL_s} = \left(\frac{\mathbf{k+1}}{\mathbf{2k}}\right)\left(\left[\frac{\boldsymbol{\lambda^{L^2}}}{\boldsymbol{\mu^{U^2}-\lambda^L\mu^U}}, \frac{\boldsymbol{\lambda^{U^2}}}{\boldsymbol{\mu^{L^2}-\lambda^U\mu^L}}\right]\right) + \left[\frac{\boldsymbol{\lambda^L}}{\boldsymbol{\mu^U}}, \frac{\boldsymbol{\lambda^U}}{\boldsymbol{\mu^L}}\right] = \left(\frac{\mathbf{3+1}}{\mathbf{2*3}}\right) * \left(\left[\frac{\mathbf{1}}{\mathbf{25-1*5}}, \frac{\mathbf{4}}{\mathbf{9-2*3}}\right]\right) + \left[\frac{\mathbf{1}}{\mathbf{5}}, \frac{\mathbf{2}}{\mathbf{3}}\right]$$
$$= [\mathbf{0.233333, 1.555556}]$$

Which means that average number of customers in system lies in the given range.

Notice that $L_s = 0.525 \in NL_s = [0.233333, 1.555556]$

$$NL_q = \left(\frac{k+1}{2k}\right)\left(\left[\frac{\lambda^{L^2}}{\mu^{U^2}-\lambda^L\mu^U}, \frac{\lambda^{U^2}}{\mu^{L^2}-\lambda^U\mu^L}\right]\right) = \left(\frac{3+1}{2*3}\right) * \left(\left[\frac{1}{25-1*5}, \frac{4}{9-2*3}\right]\right)$$

$$= [0.03333, 0.888889]$$

Which means that average number of customers in the queue lies in the given range.
Also in thic case we notice that $L_q = 0.15 \in [0.03333, 0.888889]$

$$NW_s = \left(\frac{k+1}{2k}\right)\left(\left[\frac{\frac{\lambda^{L^2}}{\lambda^U}}{\mu^{U^2}-\lambda^L\mu^U}, \frac{\frac{\lambda^{U^2}}{\lambda^L}}{\mu^{L^2}-\lambda^U\mu^L}\right]\right) + \left[\frac{\frac{\lambda^L}{\lambda^U}}{\mu^U}, \frac{\frac{\lambda^U}{\lambda^L}}{\mu^L}\right] = \left(\frac{3+1}{2*3}\right)\left(\left[\frac{1/2}{25-1*5}, \frac{4/1}{9-2*3}\right]\right) + \left[\frac{1/2}{5}, \frac{2/1}{3}\right]$$
$$= [0.116667, 1.555556]$$

Which means that average waiting time in the system lies between 0.116667 and 1.555556 mins.

Also $W_s = 0.35 \in [0.116667, 1.555556] = NW_s$

$$NW_q = \left(\frac{k+1}{2k}\right)\left(\left[\frac{\frac{\lambda^{L^2}}{\lambda^U}}{\mu^{U^2}-\lambda^L\mu^U}, \frac{\frac{\lambda^{U^2}}{\lambda^L}}{\mu^{L^2}-\lambda^U\mu^L}\right]\right) = \left(\frac{3+1}{2*3}\right)\left(\left[\frac{1/2}{25-1*5}, \frac{4/1}{9-2*3}\right]\right) = [0.016667, 0.888889]$$

Which means that average waiting time in the queue lies between 0.016667 and 0.8888889 mins.

DOI: 10.5281/zenodo.3841739

Also $W_q = 0.1 \in [0.016667, 0.888889] = NW_q$

So we also see that neutrosophic solutions are more accurate and can be considered as an extension to classical solutions.

5. Conclusions

We conclude that neutrosophic logic can be applied to generalize the classical queueing theory and make it easy and clear to handle imprecise and incomplete data. We have chosen Erlang service models as a useful and important example of frequently used queueing systems.

The author is looking forward to studying the ability to extend other queueing models using neutrosophic logic including batch arrivals and batch services, customers behavior in single and multi-queues.

Acknowledgment: “The author is highly thankful to the Editor-in-chief and the reviewers for their valuable comments and suggestions for improving the quality of paper”.

Funding: “This research received no external funding”

Conflicts of Interest: “The authors declare no conflict of interest.”

References

[1] F Shortle J.; M Thompson J., Gross D., M Harris C., “Fundamentals of Queueing Theory”, 5th ed., Wiley: United States of America, 2018.
[2] Nick T. Thomopoulos, “Fundamentals of Queuing Systems”, 1st ed.; Springer: United States of America, 2012.
[3] Smarandache. F., “A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability”, American Research Press, Rehoboth, NM, 1999.
[4] Smarandache. F., “Neutrosophic set a generalization of the intuitionistic fuzzy sets”, Inter. J. Pure Appl. Math., 24, pp.287 – 297, 2005.
[5] Smarandache. F, “Introduction to Neutrosophic Measure, Integral, Probability”, Sitech Education publisher, 2015.
[6] Smarandache, F, “Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics”, University of New Mexico, Gallup, NM 87301, USA, 2002.
[7] Salama. A.A, Smarandache. F, and Kroumov. V, “Neutrosophic Crisp Sets & Neutrosophic Crisp Topological Spaces”, Neutrosophic Sets and Systems, Vol. 2, pp. 25-30, 2014.
[8] Salama.A.A, Smarandache. F, “Neutrosophic Crisp Set Theory”, Education Publishing, Columbus, 2015.
[9] Patro.S.K, Smarandache. F., “The Neutrosophic Statistical Distribution, More Problems, More Solutions”, Neutrosophic Sets and Systems, Vol. 12, pp. 73-79, 2016.
[10] Smarandache. F, “Neutrosophical statistics”, Sitech & Education publishing, 2014.
[11] Smarandache, F. & Pramanik, S., “ New trends in neutrosophic theory and applications”, Vol.2. Brussels: Pons Editions, 2018.
[12] Smarandache, F. & Pramanik, S., “New trends in neutrosophic theory and applications”, Brussels: Pons Editions, 2016
[13] Hatip.A., “The Special Neutrosophic Funcions”, International Journal of Neutrosophic Science, Vol. 4, Issue 2, pp. 104-116, 2020.

DOI: 10.5281/zenodo.3841739

[14] Alhabib, R., A. A. Salama, “Using Moving Averages To Pave The Neutrosophic Time Series”, International Journal of Neutrosophic Science, Vol. 3, Issue 1, pp. 14-20 , 2020.

[15] Oliveira.A., Oliveira.M.N.C, “Classical Logic as a subclass of Neutrosophic Logic”, International Journal of Neutrosophic Science, Vol. 6, , Issue 1, pp. 22-31 , 2020.

[16] Salama.A.A, Smarandache. F, “Neutrosophic Crisp Set Theory”, Education Publishing, Columbus, 2015.

[17] Alhabib.R, Ranna.M.M, Farah. H, Salama.A.A., “Some Neutrosophic Probability Distributions, Neutrosophic Sets and Systems, Vol. 22, pp. 30-38, 2018.

DOI: 10.5281/zenodo.3841739