Image and Inverse Image of Neutrosophic Cubic Sets in UP-Algebras under UP-Homomorphisms

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Abstract

The concept of a neutrosophic cubic set in a UP-algebra was introduced by Songsaeng and Iampan [Neutrosophic cubic set theory applied to UP-algebras, 2019]. In this paper, we define the image and inverse image of a neutrosophic cubic set in a non-empty set under any function and study the image and inverse image of a neutrosophic cubic UP-subalgebra (resp., neutrosophic cubic near UP-filter, neutrosophic cubic UP-filter, neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal) of a UP-algebra under some UP-homomorphisms.

Keywords: UP-algebra, UP-homomorphism, neutrosophic cubic UP-subalgebra, neutrosophic cubic near UP-filter, neutrosophic cubic UP-filter, neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal

1 Introduction


From literature review, we will study the image and inverse image of neutrosophic cubic UP-subalgebras (resp., neutrosophic cubic near UP-filters, neutrosophic cubic UP-filters, neutrosophic cubic UP-ideals, neutrosophic cubic strong UP-ideals) under some UP-homomorphisms.

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2 Basic concepts and preliminary notes on a UP-algebra

Before the study, we will review the definition of a UP-algebra.

Definition 2.1. An algebra $X = (X, \circ, 0)$ of type $(2, 0)$ is said to be a UP-algebra, where $X$ is a non-empty set, $\circ$ is a binary operation on $X$, and $0$ is a fixed element of $X$ if it holds the followings:

(UP-1) (for all $x, y, z \in X$), $(y \circ z) \circ ((x \circ y) \circ (x \circ z)) = 0$,

(UP-2) (for all $x \in X$), $(0 \circ x = x)$,

(UP-3) (for all $x \in X$), $(x \circ 0 = 0)$, and

(UP-4) (for all $x, y \in X$), $(x \circ y = 0, y \circ x = 0 \Rightarrow x = y)$.

From we already know that the concept of a UP-algebra is a generalization of a KU-algebra (see).

Example 2.2. Let $Y$ be a universal set and let $\Omega \in \mathcal{P}(Y)$, where $\mathcal{P}(Y)$ means the power set of $Y$. Let $\mathcal{P}_\Omega(Y) = \{ A \in \mathcal{P}(Y) \mid \Omega \subseteq A \}$. Define a binary operation $\circ$ on $\mathcal{P}_\Omega(Y)$ by putting $A \circ B = B \cap (A^C \cup \Omega)$ for all $A, B \in \mathcal{P}_\Omega(Y)$, where $A^C$ means the complement of a subset $A$. Then $(\mathcal{P}_\Omega(Y), \circ, \Omega)$ is a UP-algebra. Let $\mathcal{P}^\Omega(Y) = \{ A \in \mathcal{P}(Y) \mid A \subseteq \Omega \}$. Define a binary operation $\bullet$ on $\mathcal{P}^\Omega(Y)$ by putting $A \bullet B = B \cup (A^C \cap \Omega)$ for all $A, B \in \mathcal{P}^\Omega(Y)$. Then $(\mathcal{P}^\Omega(Y), \bullet, \Omega)$ is a UP-algebra. In particular, $(\mathcal{P}(Y), \circ, \emptyset)$ and $(\mathcal{P}(Y), \bullet, X)$ are UP-algebras.

Example 2.3. Let $\mathbb{N}_0$ be the set of all natural numbers with zero. Define two binary operations $\cdot$ and $*$ on $\mathbb{N}_0$ by

$$(\text{for all } m, n \in \mathbb{N}_0) \begin{cases} m \cdot n = \begin{cases} n & \text{if } m < n, \\
0 & \text{otherwise} \end{cases} \\
0 = \begin{cases} n & \text{if } m > n \text{ or } m = 0, \\
0 & \text{otherwise} \end{cases} \end{cases}$$

and

$$(\text{for all } m, n \in \mathbb{N}_0) \begin{cases} m \ast n = \begin{cases} n & \text{if } m > n \text{ or } m = 0, \\
0 & \text{otherwise} \end{cases} \end{cases}.$$ Then $(\mathbb{N}_0, \cdot, 0)$ and $(\mathbb{N}_0, *, 0)$ are UP-algebras.

For more examples of a UP-algebra, see.

In a UP-algebra $X = (X, \circ, 0)$, the followings are valid (see).

$$\begin{align*}
(\text{for all } x \in X)(x \circ x = 0), \quad & (2.1) \\
(\text{for all } x, y, z \in X)(x \circ y = 0, y \circ z = 0 \Rightarrow x \circ z = 0), \quad & (2.2) \\
(\text{for all } x, y, z \in X)(x \circ y = 0 \Rightarrow (z \circ x) \circ (z \circ y) = 0), \quad & (2.3) \\
(\text{for all } x, y, z \in X)(x \circ y = 0 \Rightarrow (y \circ z) \circ (x \circ z) = 0), \quad & (2.4) \\
(\text{for all } x \in X)(x \circ (y \circ x) = 0), \quad & (2.5) \\
(\text{for all } x, y \in X)(x \circ (y \circ x) = 0), \quad & (2.6) \\
(\text{for all } a, x, y, z \in X)((x \circ (y \circ z)) \circ (x \circ ((a \circ y) \circ (a \circ z))) = 0), \quad & (2.7) \\
(\text{for all } a, x, y, z \in X)((a \circ x) \circ ((a \circ y) \circ z) \circ (x \circ (y \circ z) = 0), \quad & (2.8) \\
(\text{for all } a, x, y, z \in X)((x \circ y) \circ z) \circ (y \circ z) = 0), \quad & (2.9) \\
(\text{for all } a, x, y, z \in X)((x \circ y) \circ z) \circ (x \circ (y \circ z) = 0), \quad & (2.10) \\
(\text{for all } a, x, y, z \in X)((x \circ y) \circ z) \circ (x \circ (y \circ z) = 0), \quad & (2.11) \\
(\text{for all } a, x, y, z \in X)((x \circ y) \circ z) \circ (x \circ (y \circ z) = 0), \quad & (2.12) \\
(\text{for all } a, x, y, z \in X)((x \circ y) \circ z) \circ (x \circ (y \circ z) = 0). \quad & (2.13)
\end{align*}$$

From the binary relation $\leq$ on a UP-algebra $X = (X, \circ, 0)$ is defined as follows:

$$(\text{for all } x, y \in X)(x \leq y \Leftrightarrow x \circ y = 0).$$

In a UP-algebra, 5 types of special subsets are defined as follows.
Definition 2.4. A non-empty subset $A$ of a UP-algebra $X = (X, \circ, 0)$ is said to be
(1) a **UP-subalgebra** of $X$ if (for all $x, y \in A)(x \circ y \in A)$.
(2) a **near UP-filter** of $X$ if
   (i) the constant 0 of $X$ is in $A$, and
   (ii) (for all $x, y \in X)(y \in A \Rightarrow x \circ y \in A)$.
(3) a **UP-filter** of $X$ if
   (i) the constant 0 of $X$ is in $A$, and
   (ii) (for all $x, y, z \in X)(x \circ (y \circ z) \in A, y \in A \Rightarrow x \circ z \in A)$.
(4) a **UP-ideal** of $X$ if
   (i) the constant 0 of $X$ is in $A$, and
   (ii) (for all $x, y, z \in X)((z \circ y) \circ (z \circ x) \in A, y \in A \Rightarrow x \in A)$.

Guntasov et al.\textsuperscript{9} and Lampa\textsuperscript{2} proved that the concept of a UP-subalgebra is a generalization of a near UP-filter, a near UP-filter is a generalization of a UP-filter, a UP-filter is a generalization of a UP-ideal, and a UP-ideal is a generalization of a strong UP-ideal. Moreover, they proved that the only strong UP-ideal of a UP-algebra $X$ is $X$.

Definition 2.5. Let $(X, \circ, 0_X)$ and $(Y, \bullet, 0_Y)$ be two UP-algebras. A function $f$ from $X$ to $Y$ is said to be a **UP-homomorphism** if
\[(\forall x, y \in X)(f(x \circ y) = f(x) \bullet f(y)).\]
A UP-homomorphism $f : X \rightarrow Y$ is said to be a **UP-epimorphism** if $f$ is surjective, a **UP-monomorphism** if $f$ is injective, and a UP-epimorphism if $f$ is bijective.

Theorem 2.6. Let $X$ and $Y$ be two UP-algebras with fixed elements of $0_X$ and $0_Y$, respectively, and let $f : X \rightarrow Y$ be a UP-homomorphism. Then the followings hold:
(1) $f(0_X) = 0_Y$, and
(2) (for all $x_1, x_2 \in X)(x_1 \leq x_2 \Rightarrow f(x_1) \leq f(x_2))$.

In 1965, the concept of a fuzzy set in a non-empty set was introduced by Zadeh\textsuperscript{10} with the following definition.

Definition 2.7. A **fuzzy set** (briefly, FS) in a non-empty set $X$ (or a fuzzy subset of $X$) is defined to be a function $\lambda : X \rightarrow [0, 1]$, where $[0, 1]$ is the unit segment of the real line. Denote by $[0, 1]^X$ the collection of all FSs in $X$. Define a binary relation $\leq$ on $[0, 1]^X$ as follows:
\[(\forall \lambda, \mu \in [0, 1]^X)(\lambda \leq \mu \Leftrightarrow (\forall x \in X)(\lambda(x) \leq \mu(x))).\] (2.14)

Definition 2.8. Let $\lambda$ be a FS in a non-empty set $X$. The **complement** of $\lambda$, denoted by $\lambda^c$, is defined by
\[(\forall x \in X)(\lambda^c(x) = 1 - \lambda(x)).\] (2.15)

Definition 2.9. Let $\{\lambda_j \mid j \in J\}$ be a family of FSs in a non-empty set $X$. We define the **join** and the **meet** of $\{\lambda_j \mid j \in J\}$, denoted by $\vee_{j \in J}\lambda_j$ and $\wedge_{j \in J}\lambda_j$, respectively, as follows:
\[(\forall x \in X)((\vee_{j \in J}\lambda_j)(x) = \sup_{j \in J}\{\lambda_j(x)\}),\] (2.16)
\[(\forall x \in X)((\wedge_{j \in J}\lambda_j)(x) = \inf_{j \in J}\{\lambda_j(x)\}).\] (2.17)
In particular, if $\lambda$ and $\mu$ be FSs in $X$, we have the join and meet of $\lambda$ and $\mu$ as follows:
\[(\forall x \in X)((\lambda \vee \mu)(x) = \max\{\lambda(x), \mu(x)\}),\] (2.18)
\[(\forall x \in X)((\lambda \wedge \mu)(x) = \min\{\lambda(x), \mu(x)\}),\] (2.19)
respectively.
An interval number we mean a close subinterval \( \hat{a} = [a^-, a^+] \) of \([0, 1]\), where \( 0 \leq a^- \leq a^+ \leq 1 \). The interval number \( \tilde{a} = [a^-, a^+] \) with \( a^- = a^+ \) is denoted by \( a \). Denote by \( \text{int}[0, 1] \) the set of all interval numbers.

**Definition 2.10.** Let \( \{\hat{a}_j \mid j \in J\} \) be a family of interval numbers. We define the refined infimum and the refined supremum of \( \{\hat{a}_j \mid j \in J\} \), denoted by \( \text{rinf}_{j \in J}\hat{a}_j \) and \( \text{rsup}_{j \in J}\hat{a}_j \), respectively, as follows:

\[
\text{rinf}_{j \in J}\hat{a}_j = \left\{ \inf_{j \in J}\{a^-_j\}, \inf_{j \in J}\{a^+_j\}\right\},
\]

\[
\text{rsup}_{j \in J}\hat{a}_j = \left\{ \sup_{j \in J}\{a^-_j\}, \sup_{j \in J}\{a^+_j\}\right\}.
\]

In particular, if \( \hat{a}_1, \hat{a}_2 \in \text{int}[0, 1] \), we define the refined minimum and the refined maximum of \( \hat{a}_1 \) and \( \hat{a}_2 \), denoted by \( \text{rmin}\{\hat{a}_1, \hat{a}_2\} \) and \( \text{rmax}\{\hat{a}_1, \hat{a}_2\} \), respectively, as follows:

\[
\text{rmin}\{\hat{a}_1, \hat{a}_2\} = [\text{min}\{a_1^-, a_2^-\}, \text{min}\{a_1^+, a_2^+\}],
\]

\[
\text{rmax}\{\hat{a}_1, \hat{a}_2\} = [\text{max}\{a_1^-, a_2^-\}, \text{max}\{a_1^+, a_2^+\}].
\]

**Definition 2.11.** Let \( \hat{a}_1, \hat{a}_2 \in \text{int}[0, 1] \). We define the symbols “\( \preceq \)” , “\( \succeq \)” , “\( \preccurlyeq \)” in case of \( \hat{a}_1 \) and \( \hat{a}_2 \) as follows:

\[
\hat{a}_1 \preceq \hat{a}_2 \Leftrightarrow a^-_1 \geq a^-_2 \text{ and } a^+_1 \geq a^+_2,
\]

and similarly we may have \( \hat{a}_1 \preceq \hat{a}_2 \) and \( \hat{a}_1 \succeq \hat{a}_2 \). To say \( \hat{a}_1 \succeq \hat{a}_2 \) (resp., \( \hat{a}_1 \preceq \hat{a}_2 \)) we mean \( \hat{a}_1 \succeq \hat{a}_2 \) and \( \hat{a}_1 \preceq \hat{a}_2 \) (resp., \( \hat{a}_1 \succeq \hat{a}_2 \) and \( \hat{a}_1 \preceq \hat{a}_2 \)).

**Definition 2.12.** Let \( \hat{a} \in \text{int}[0, 1] \). The complement of \( \hat{a} \), denoted by \( \hat{a}^C \), is defined by the interval number

\[
\hat{a}^C = [1 - a^+, 1 - a^-].
\]

In the int[0, 1], the followings are valid (see (2.25)):

\[
\text{for all } \hat{a} \in \text{int}[0, 1] (\hat{a} \succeq \tilde{a}),
\]

\[
\text{for all } \hat{a} \in \text{int}[0, 1] ((\hat{a}^C)^C = \tilde{a}),
\]

\[
\text{for all } \hat{a} \in \text{int}[0, 1] (\text{rmax}\{\hat{a}, \tilde{a}\} = \tilde{a} \text{ and } \text{rmin}\{\hat{a}, \tilde{a}\} = \hat{a}),
\]

\[
\text{for all } \hat{a}_1, \hat{a}_2 \in \text{int}[0, 1] (\text{rmax}\{\hat{a}_1, \hat{a}_2\} = \text{max}\{\hat{a}_1, \hat{a}_2\} \text{ and } \text{rmin}\{\hat{a}_1, \hat{a}_2\} = \text{min}\{\hat{a}_1, \hat{a}_2\}),
\]

\[
\text{for all } \hat{a}_1, \hat{a}_2 \in \text{int}[0, 1] (\text{rmax}\{\hat{a}_1, \hat{a}_2\} \preceq \tilde{a}_1 \text{ and } \tilde{a}_2 \preceq \text{rmin}\{\hat{a}_1, \hat{a}_2\}),
\]

\[
\text{for all } \hat{a}_1, \hat{a}_2 \in \text{int}[0, 1] (\text{rmax}\{\hat{a}_1, \hat{a}_2\} \preceq \text{rmax}\{\tilde{a}_1, \tilde{a}_2\}),
\]

\[
\text{for all } \hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4 \in \text{int}[0, 1] (\hat{a}_1 \preceq \hat{a}_2 \text{ and } \hat{a}_3 \preceq \hat{a}_4 \Rightarrow \text{rmin}\{\hat{a}_1, \hat{a}_2\} \preceq \text{rmin}\{\hat{a}_3, \hat{a}_4\}),
\]

\[
\text{for all } \hat{a}_1, \hat{a}_2, \hat{a}_3 \in \text{int}[0, 1] (\hat{a}_1 \preceq \hat{a}_2 \preceq \hat{a}_3 \Rightarrow \text{rmin}\{\hat{a}_1, \hat{a}_2\} \preceq \text{rmin}\{\hat{a}_1, \hat{a}_3\}),
\]

\[
\text{for all } \hat{a}_1, \hat{a}_2, \hat{a}_3 \in \text{int}[0, 1] (\hat{a}_1 \preceq \hat{a}_2 \preceq \hat{a}_3 \Rightarrow \text{rmin}\{\hat{a}_1, \hat{a}_2\} \preceq \text{rmin}\{\hat{a}_1, \hat{a}_3\}),
\]

\[
\text{for all } \hat{a}_1, \hat{a}_2, \hat{a}_3 \in \text{int}[0, 1] (\hat{a}_1 \preceq \hat{a}_2 \preceq \hat{a}_3 \Rightarrow \text{rmin}\{\hat{a}_1, \hat{a}_2\} \preceq \text{rmin}\{\hat{a}_1, \hat{a}_3\}),
\]

In 1975, the concept of an interval-valued fuzzy set in a non-empty set was first introduced by Zadeh with the following definition.

**Definition 2.13.** An interval-valued fuzzy set (briefly, IVFS) in a non-empty set \( X \) is an arbitrary function \( A : X \to \text{int}[0, 1] \). Let \( IVFS(X) \) stands for the set of all IVFS in \( X \). For every \( A \in IVFS(X) \) and \( x \in X \), \( A(x) = [A^-(x), A^+(x)] \) is said to be the degree of membership of an element \( x \) to \( A \), where \( A^- \), \( A^+ \) are FSs in \( X \) which are called a lower fuzzy set and an upper fuzzy set in \( X \), respectively. For simplicity, we denote \( A = [A^-, A^+] \).
Definition 2.14. Let $A$ and $B$ be IVFSs in a non-empty set $X$. We define the symbols $\subseteq$, $\supseteq$, $=$ in case of $A$ and $B$ as follows:

$$A \subseteq B \iff \forall x \in X (A(x) \leq B(x)),$$

and similarly we may have $A \supseteq B$ and $A = B$.

Definition 2.15. Let $A$ be an IVFS in a non-empty set $X$. The complement of $A$, denoted by $A^C$, is defined as follows: $A^C(x) = A(x)^C$ for all $x \in X$, that is,

$$(\forall x \in X)(A^C(x) = [1 - A^+(x), 1 - A^-(x)]).$$

We note that $A^{C-}(x) = 1 - A^+(x)$ and $A^{C+}(x) = 1 - A^-(x)$ for all $x \in X$.

Definition 2.16. Let $\{A_j \mid j \in J\}$ be a family of IVFSs in a non-empty set $X$. We define the intersection and the union of $\{A_j \mid j \in J\}$, denoted by $\cap_{j \in J}A_j$ and $\cup_{j \in J}A_j$, respectively, as follows:

$$\forall x \in X, (\cap_{j \in J}A_j)^-(x) = \inf_{j \in J} A_j^-(x)),$$

and

$$\forall x \in X, (\cup_{j \in J}A_j)^-(x) = \sup_{j \in J} A_j^-(x)),$$

Similarly,

$$\forall x \in X, (\cap_{j \in J}A_j)^+(x) = \inf_{j \in J} A_j^+(x)),$$

and

$$\forall x \in X, (\cup_{j \in J}A_j)^+(x) = \sup_{j \in J} A_j^+(x)),$$

In particular, if $A_1$ and $A_2$ are IVFSs in $X$, we have the intersection and the union of $A_1$ and $A_2$ as follows:

$$\forall x \in X, ((A_1 \cap A_2)(x) = \min\{A_1(x), A_2(x))\},$$

and

$$\forall x \in X, ((A_1 \cup A_2)(x) = \max\{A_1(x), A_2(x))\}.$$}

In 1999, the concept of a neutrosophic set in a non-empty set was introduced by Smarandache with the following definition.

Definition 2.17. A neutrosophic set (briefly, NS) in a non-empty set $X$ is a structure of the form:

$$\Lambda = \{(x, \lambda_T(x), \lambda_I(x), \lambda_F(x)) \mid x \in X\},$$

where $\lambda_T : X \rightarrow [0, 1]$ is a truth membership function, $\lambda_I : X \rightarrow [0, 1]$ is an indeterminate membership function, and $\lambda_F : X \rightarrow [0, 1]$ is a false membership function. For our convenience, we will denote a NS as $\Lambda = (X, \lambda_T, \lambda_I, \lambda_F) = (X, \lambda_{T,I,F}) = \{(x, \lambda_T(x), \lambda_I(x), \lambda_F(x)) \mid x \in X\}$.

Definition 2.18. Let $\Lambda$ be a NS in a non-empty set $X$. The NS $\Lambda^{C} = (X, \Lambda_T^C, \Lambda_I^C, \Lambda_F^C)$ in $X$ is said to be the complement of $\Lambda$ in $X$.

In 2005, the concept of an interval neutrosophic set in a non-empty set was introduced by Wang et al. with the following definition.

Definition 2.19. An interval-valued neutrosophic set (briefly, IVNS) in a non-empty set $X$ is a structure of the form:

$$A := \{(x, A_T(x), A_I(x), A_F(x)) \mid x \in X\},$$

where $A_T$, $A_I$ and $A_F$ are IVFSs in $X$, which are called an interval truth membership function, an interval indeterminacy membership function and an interval falsity membership function, respectively. For our convenience, we will denote a IVNS as $A = (X, A_T, A_I, A_F) = (X, A_{T,I,F}) = \{(x, A_T(x), A_I(x), A_F(x)) \mid x \in X\}$.

Definition 2.20. Let $A = (X, A_T, A_I, A_F)$ be an IVNS in a non-empty set $X$. The IVNS $A^C = (X, A_T^C, A_I^C, A_F^C)$ in $X$ is said to be the complement of $A$ in $X$. 

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In 2012, the concept of a cubic set in a non-empty set was introduced by Jun et al.\cite{2} with the following definition.

**Definition 2.21.** A cubic set (briefly, CS) in a non-empty set $X$ is a structure of the form:

$$C = \{(x, A(x), \lambda(x)) \mid x \in X\},$$

where $A$ is an IVFS in $X$ and $\lambda$ is a FS in $X$. For our convenience, we will denote a CS as $C = (X, A, \lambda) = \{(x, A(x), \lambda(x)) \mid x \in X\}$.

In 2017, Jun et al.\cite{3} introduced the concept of a neutrosophic cubic set with the following definition.

**Definition 2.22.** A neutrosophic cubic set (briefly, NCS) in a non-empty set $X$ is a pair $\mathcal{C} = (A, \Lambda)$, where $A = (X, A_T, A_I, A_F)$ is an IVNS and $\Lambda = (X, \lambda_T, \lambda_I, \lambda_F)$ is a neutrosophic set in $X$. For simplicity, we denote $\mathcal{C} = (A, \Lambda)$.

A NCS $\mathcal{C} = (A, \Lambda)$ in a non-empty set $X$ is said to be constant if $A_T, A_I, A_F, \lambda_T, \lambda_I, \lambda_F$ are constant functions. The complement of a NCS $\mathcal{C} = (A, \Lambda)$ is defined to be the NCS $\mathcal{C}^C = (A^C, \Lambda^C)$.

In 2020, the concepts of a neutrosophic cubic UP-subalgebra, a neutrosophic cubic near UP-filter, a neutrosophic cubic UP-ideal, and a neutrosophic cubic strong UP-ideal of a UP-algebra were introduced by Songsaeng and Iampan\cite{4} with the following definition.

**Definition 2.23.** A NCS $\mathcal{C} = (A, \Lambda)$ in a UP-algebra $X = (X, \circ, 0)$ is said to be

1. a neutrosophic cubic UP-subalgebra of $X$ if

   \begin{align}
   &\text{(for all } x, y \in X) & A_T(x \circ y) \geq \min\{A_T(x), A_T(y)\} \\
   & & A_I(x \circ y) \leq \max\{A_I(x), A_I(y)\} \\
   & & A_F(x \circ y) \geq \min\{A_F(x), A_F(y)\} \tag{2.53}
   \end{align}

2. a neutrosophic cubic near UP-filter of $X$ if

   \begin{align}
   &\text{(for all } x \in X) & A_T(0) \geq A_T(x) \\
   & & A_I(0) \leq A_I(x) \\
   & & A_F(0) \geq A_F(x) \tag{2.55}
   \end{align}

   \begin{align}
   &\text{(for all } x \in X) & \lambda_T(0) \leq \lambda_T(x) \\
   & & \lambda_I(0) \geq \lambda_I(x) \\
   & & \lambda_F(0) \leq \lambda_F(x) \tag{2.56}
   \end{align}

   \begin{align}
   &\text{(for all } x, y \in X) & A_T(x \circ y) \geq A_T(y) \\
   & & A_I(x \circ y) \leq A_I(y) \\
   & & A_F(x \circ y) \geq A_F(y) \tag{2.57}
   \end{align}

   \begin{align}
   &\text{(for all } x, y \in X) & \lambda_T(x \circ y) \leq \lambda_T(y) \\
   & & \lambda_I(x \circ y) \geq \lambda_I(y) \\
   & & \lambda_F(x \circ y) \leq \lambda_F(y) \tag{2.58}
   \end{align}

3. a neutrosophic cubic UP-filter of $X$ if it holds the followings: (2.55), (2.56), and

   \begin{align}
   &\text{(for all } x, y \in X) & A_T(y) \geq \min\{A_T(x \circ y), A_T(x)\} \\
   & & A_I(y) \leq \max\{A_I(x \circ y), A_I(x)\} \\
   & & A_F(y) \geq \min\{A_F(x \circ y), A_F(x)\} \tag{2.59}
   \end{align}

   \begin{align}
   &\text{(for all } x, y \in X) & \lambda_T(y) \leq \max\{\lambda_T(x \circ y), \lambda_T(x)\} \\
   & & \lambda_I(y) \geq \min\{\lambda_I(x \circ y), \lambda_I(x)\} \\
   & & \lambda_F(y) \leq \max\{\lambda_F(x \circ y), \lambda_F(x)\} \tag{2.60}
   \end{align}
(4) a neutrosophic cubic UP-ideal of \( X \) if it holds the followings: \((2.55), (2.56)\), and
\[
\begin{align*}
\text{(for all } x, y, z \in X) & \quad \begin{cases} 
A_T(x \circ z) \geq \min \{A_T((z \circ y) \circ (z \circ x)), A_T(y)\} \\
A_T(x \circ z) \leq \max \{A_T((z \circ y) \circ (z \circ x)), A_T(y)\} \\
A_P(x \circ z) \geq \min \{A_P((z \circ y) \circ (z \circ x)), A_P(y)\} \\
A_P(x \circ z) \leq \max \{A_P((z \circ y) \circ (z \circ x)), A_P(y)\}
\end{cases}, \\
\lambda_T(x \circ z) \leq \max \{\lambda_T((z \circ y) \circ (z \circ x)), \lambda_T(y)\} \\
\lambda_T(x \circ z) \geq \min \{\lambda_T((z \circ y) \circ (z \circ x)), \lambda_T(y)\} \\
\lambda_P(x \circ z) \leq \max \{\lambda_P((z \circ y) \circ (z \circ x)), \lambda_P(y)\}
\end{align*}
\]
(2.61)

(5) a neutrosophic cubic strong UP-ideal of \( X \) if it holds the followings: \((2.55), (2.56)\), and
\[
\begin{align*}
\text{(for all } x, y, z \in X) & \quad \begin{cases} 
A_T(x) \geq \min \{A_T((z \circ y) \circ (z \circ x)), A_T(y)\} \\
A_T(x) \leq \max \{A_T((z \circ y) \circ (z \circ x)), A_T(y)\} \\
A_P(x) \geq \min \{A_P((z \circ y) \circ (z \circ x)), A_P(y)\} \\
A_P(x) \leq \max \{A_P((z \circ y) \circ (z \circ x)), A_P(y)\}
\end{cases}, \\
\lambda_T(x) \leq \max \{\lambda_T((z \circ y) \circ (z \circ x)), \lambda_T(y)\} \\
\lambda_T(x) \geq \min \{\lambda_T((z \circ y) \circ (z \circ x)), \lambda_T(y)\} \\
\lambda_P(x) \leq \max \{\lambda_P((z \circ y) \circ (z \circ x)), \lambda_P(y)\}
\end{align*}
\]
(2.64)

Songsaeng and Iampan\(^{[10]}\) proved that the concept of a neutrosophic cubic UP-subalgebra is a generalization of a neutrosophic cubic near UP-filter, a neutrosophic cubic near UP-filter is a generalization of a neutrosophic cubic UP-algebra, and a neutrosophic cubic UP-ideal is a generalization of a neutrosophic cubic strong UP-ideal. Moreover, they proved that a neutrosophic cubic strong UP-ideal and a constant NCS coincide.

3 Homomorphic properties of a NCSs in a UP-algebra

In this section, the image and inverse image of a NCS are defined and some results are studied.

**Definition 3.1.** Let \( f \) be a function from a non-empty set \( X \) into a non-empty set \( Y \) and \( \mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F}) \) be a NCS in \( X \). Then the image of \( \mathcal{A} \) under \( f \) is defined as a NCS \( f(\mathcal{A}) = (f(A)_{T,I,F}, f(\lambda)_{T,I,F}) \) in \( Y \), where
\[
\begin{align*}
&f(A)_T(y) = \begin{cases} 
\text{rsup}_{x \in f^{-1}(y)} \{A_T(x)\} & \text{if } f^{-1}(y) \text{ is non-empty,} \\
[0, 0] & \text{otherwise,}
\end{cases} \\
&f(A)_I(y) = \begin{cases} 
\text{rinf}_{x \in f^{-1}(y)} \{A_I(x)\} & \text{if } f^{-1}(y) \text{ is non-empty,} \\
[1, 1] & \text{otherwise,}
\end{cases} \\
&f(A)_F(y) = \begin{cases} 
\text{rsup}_{x \in f^{-1}(y)} \{A_F(x)\} & \text{if } f^{-1}(y) \text{ is non-empty,} \\
[0, 0] & \text{otherwise,}
\end{cases} \\
&f(\lambda)_T(y) = \begin{cases} 
\text{inf}_{x \in f^{-1}(y)} \{\lambda_T(x)\} & \text{if } f^{-1}(y) \text{ is non-empty,} \\
1 & \text{otherwise,}
\end{cases} \\
&f(\lambda)_I(y) = \begin{cases} 
\text{sup}_{x \in f^{-1}(y)} \{\lambda_I(x)\} & \text{if } f^{-1}(y) \text{ is non-empty,} \\
0 & \text{otherwise,}
\end{cases} \\
&f(\lambda)_F(y) = \begin{cases} 
\text{inf}_{x \in f^{-1}(y)} \{\lambda_F(x)\} & \text{if } f^{-1}(y) \text{ is non-empty,} \\
1 & \text{otherwise.}
\end{cases}
\end{align*}
\]

**Example 3.2.** Let \( X = \{0_X, 1_X, 2_X\} \) be a UP-algebra with a fixed element \( 0_X \) and a binary operation \( \circ \) defined by the following Cayley table:
\[
\begin{array}{c|ccc}
\circ & 0_X & 1_X & 2_X \\
\hline
0_X & 0_X & 1_X & 2_X \\
1_X & 0_X & 1_X & 2_X \\
2_X & 0_X & 0_X & 1_X \\
\end{array}
\]
and let $Y = \{0_Y, 1_Y, 2_Y\}$ be a UP-algebra with a fixed element $0_Y$ and a binary operation $\bullet$ defined by the following Cayley table:

<table>
<thead>
<tr>
<th></th>
<th>0_Y</th>
<th>1_Y</th>
<th>2_Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0_Y</td>
<td>0_Y</td>
<td>1_Y</td>
<td>2_Y</td>
</tr>
<tr>
<td>1_Y</td>
<td>0_Y</td>
<td>0_Y</td>
<td>2_Y</td>
</tr>
<tr>
<td>2_Y</td>
<td>0_Y</td>
<td>0_Y</td>
<td>0_Y</td>
</tr>
</tbody>
</table>

We define a function $f : X \to Y$ as follows:

$$f(0_X) = 0_Y, f(1_X) = 1_Y, \text{ and } f(2_X) = 1_Y.$$ 

We define a NCS $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ in $X$ with the tabular representation as follows:

<table>
<thead>
<tr>
<th>$X$</th>
<th>$\Lambda(x)$</th>
<th>$\lambda(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0_X$</td>
<td>$[[0.4, 0.7], [0.5, 0.7], [0.2, 0.4]]$</td>
<td>$(0.1, 0.3, 0.4)$</td>
</tr>
<tr>
<td>$1_X$</td>
<td>$[[0.1, 0.2], [0.1, 0.5], [0.4, 0.5]]$</td>
<td>$(0.3, 0.8, 0.4)$</td>
</tr>
<tr>
<td>$2_X$</td>
<td>$[[0.8, 0.9], [0.7, 0.8], [0.1, 0.6]]$</td>
<td>$(0.1, 0.5, 0.7)$</td>
</tr>
</tbody>
</table>

Then $f(\mathcal{A}) = (f(A)_{T,I,F}, f(\lambda)_{T,I,F})$ in $Y$ with the tabular representation as follows:

<table>
<thead>
<tr>
<th>$Y$</th>
<th>$\Lambda(x)$</th>
<th>$\lambda(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0_Y$</td>
<td>$[[0.4, 0.7], [0.5, 0.7], [0.2, 0.4]]$</td>
<td>$(0.1, 0.3, 0.4)$</td>
</tr>
<tr>
<td>$1_Y$</td>
<td>$[[0.8, 0.9], [0.1, 0.5], [0.4, 0.6]]$</td>
<td>$(0.1, 0.8, 0.4)$</td>
</tr>
<tr>
<td>$2_Y$</td>
<td>$[[0.0, 1.1], [0.0]]$</td>
<td>$(0, 1)$</td>
</tr>
</tbody>
</table>

Hence, $f(\mathcal{A}) = (f(A)_{T,I,F}, f(\lambda)_{T,I,F})$ is a NCS in $Y$.

**Definition 3.3.** Let $f$ be a function from a non-empty set $X$ into a non-empty set $Y$ and $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ be a NCS in $Y$. Then the inverse image of $\mathcal{A}$ is defined as a NCS $f^{-1}(\mathcal{A}) = (f^{-1}(A)_{T,I,F}, f^{-1}(\lambda)_{T,I,F})$ in $X$, where

$(\text{for all } x \in X)(f^{-1}(A)_{T,I,F}(x) = A_{T,I,F}(f(x))),$

$(\text{for all } x \in X)(f^{-1}(\lambda)_{T,I,F}(x) = \lambda_{T,I,F}(f(x))).$

**Example 3.4.** In Example 3.2 we have $(X, o, 0_X)$ and $(Y, \bullet, 0_Y)$ are two UP-algebras. We define a function $f : X \to Y$ as follows:

$$f(0_X) = 0_Y, f(1_X) = 1_Y, \text{ and } f(2_X) = 1_Y.$$ 

We define a NCS $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ in $Y$ with the tabular representation as follows:

<table>
<thead>
<tr>
<th>$Y$</th>
<th>$\Lambda(x)$</th>
<th>$\lambda(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0_Y$</td>
<td>$[[0.3, 0.7], [0.3, 0.5], [0.1, 0.4]]$</td>
<td>$(0.5, 0.4, 0.7)$</td>
</tr>
<tr>
<td>$1_Y$</td>
<td>$[[0.6, 0.7], [0.1, 0.3], [0.4, 0.5]]$</td>
<td>$(0.2, 0.7, 0.8)$</td>
</tr>
<tr>
<td>$2_Y$</td>
<td>$[[0.5, 0.9], [0.3, 0.5], [0.5, 0.8]]$</td>
<td>$(0.3, 0.5, 0.4)$</td>
</tr>
</tbody>
</table>

Then $f^{-1}(\mathcal{A}) = (f^{-1}(A)_{T,I,F}, f^{-1}(\lambda)_{T,I,F})$ in $X$ with the tabular representation as follows:

<table>
<thead>
<tr>
<th>$X$</th>
<th>$\Lambda(x)$</th>
<th>$\lambda(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0_X$</td>
<td>$[[0.3, 0.7], [0.3, 0.5], [0.1, 0.4]]$</td>
<td>$(0.5, 0.4, 0.7)$</td>
</tr>
<tr>
<td>$1_X$</td>
<td>$[[0.6, 0.7], [0.1, 0.3], [0.4, 0.5]]$</td>
<td>$(0.2, 0.7, 0.8)$</td>
</tr>
<tr>
<td>$2_X$</td>
<td>$[[0.6, 0.7], [0.1, 0.3], [0.4, 0.5]]$</td>
<td>$(0.2, 0.7, 0.8)$</td>
</tr>
</tbody>
</table>

Hence, $f^{-1}(\mathcal{A}) = (f^{-1}(A)_{T,I,F}, f^{-1}(\lambda)_{T,I,F})$ is a NCS in $X$.

**Definition 3.5.** A NCS $\mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F})$ in $X$ is said to be order preserving if

$$(\text{for all } x, y \in X) \left( x \leq y \Rightarrow \begin{cases} A_T(x) \leq A_T(y), A_I(x) \geq A_I(y), A_F(x) \leq A_F(y), \\ \lambda_T(x) \geq \lambda_T(y), \lambda_I(x) \leq \lambda_I(y), \lambda_F(x) \geq \lambda_F(y) \end{cases} \right).$$

**Lemma 3.6.** Every neutrosophic cubic UP-filter (resp., neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal) of $X$ is order preserving.
Proof. Assume that $\mathcal{A} = (A_{T, I, F}, \lambda_{T, I, F})$ is a neutrosophic cubic UP-filter of $X$. Let $x, y \in X$ be such that $x \leq y$ in $X$. Then $x \circ y = 0$. Thus $$A_T(y) \geq \min\{A_T(x \circ y), A_T(x)\} = \min\{A_T(0), A_T(x)\} = A_T(x), \quad (2.59), (2.55), (2.36)$$ $$A_I(y) \leq \max\{A_I(x \circ y), A_I(x)\} = \max\{A_I(0), A_I(x)\} = A_I(x), \quad (2.59), (2.55), (2.37)$$ $$A_F(y) \geq \min\{A_F(x \circ y), A_F(x)\} = \min\{A_F(0), A_F(x)\} = A_F(x), \quad (2.59), (2.55), (2.36)$$ $$\lambda_T(y) \leq \max\{\lambda_T(x \circ y), \lambda_T(x)\} = \max\{\lambda_T(0), \lambda_T(x)\} = \lambda_T(x), \quad (2.60), (2.56)$$ $$\lambda_I(y) \geq \min\{\lambda_I(x \circ y), \lambda_I(x)\} = \min\{\lambda_I(0), \lambda_I(x)\} = \lambda_I(x), \quad (2.60), (2.56)$$ $$\lambda_F(y) \leq \max\{\lambda_F(x \circ y), \lambda_F(x)\} = \max\{\lambda_F(0), \lambda_F(x)\} = \lambda_F(x). \quad (2.60), (2.56)$$

Hence, $\mathcal{A}$ is order preserving.

**Theorem 3.7.** Let $(X, \circ, 0_X)$ and $(Y, \bullet, 0_Y)$ be two UP-algebras, $f : X \rightarrow Y$ be a UP-homomorphism, and $\mathcal{A} = (A_{T, I, F}, \lambda_{T, I, F})$ be a NCS in $Y$. Then the followings hold:

1. If $\mathcal{A}$ is a neutrosophic cubic UP-subalgebra of $Y$, then the inverse image $f^{-1}(\mathcal{A})$ of $\mathcal{A}$ under $f$ is a neutrosophic cubic UP-subalgebra of $X$.

2. If $\mathcal{A}$ is a neutrosophic cubic near UP-filter of $Y$ which is order preserving, then the inverse image $f^{-1}(\mathcal{A})$ of $\mathcal{A}$ under $f$ is a neutrosophic cubic near UP-filter of $X$.

3. If $\mathcal{A}$ is a neutrosophic cubic UP-filter of $Y$, then the inverse image $f^{-1}(\mathcal{A})$ of $\mathcal{A}$ under $f$ is a neutrosophic cubic UP-filter of $X$.

4. If $\mathcal{A}$ is a neutrosophic cubic UP-ideal of $Y$, then the inverse image $f^{-1}(\mathcal{A})$ of $\mathcal{A}$ under $f$ is a neutrosophic cubic UP-ideal of $X$.

5. If $\mathcal{A}$ is a neutrosophic cubic strong UP-ideal of $Y$, then the inverse image $f^{-1}(\mathcal{A})$ of $\mathcal{A}$ under $f$ is a neutrosophic cubic strong UP-ideal of $X$.

**Proof.** (1) Assume that $\mathcal{A}$ is a neutrosophic cubic UP-subalgebra of $Y$. Then for all $x, y \in X$,

$$f^{-1}(A_T)(x \circ y) = A_T(f(x \circ y)) \quad (3.1)$$
$$= A_T(f(x) \bullet f(y)) \quad (3.1)$$
$$\geq \min\{A_T(f(x)), A_T(f(y))\} \quad (2.53)$$
$$= \min\{f^{-1}(A_T)(x), f^{-1}(A_T)(y)\}, \quad (3.1)$$

$$f^{-1}(A_I)(x \circ y) = A_I(f(x \circ y)) \quad (3.1)$$
$$= A_I(f(x) \bullet f(y)) \quad (3.1)$$
$$\leq \max\{A_I(f(x)), A_I(f(y))\} \quad (2.53)$$
$$= \max\{f^{-1}(A_I)(x), f^{-1}(A_I)(y)\}, \quad (3.1)$$

$$f^{-1}(A_F)(x \circ y) = A_F(f(x \circ y)) \quad (3.1)$$
$$= A_F(f(x) \bullet f(y)) \quad (3.1)$$
$$\geq \min\{A_F(f(x)), A_F(f(y))\} \quad (2.53)$$
$$= \min\{f^{-1}(A_F)(x), f^{-1}(A_F)(y)\}, \quad (3.1)$$

$$f^{-1}(\lambda_T)(x \circ y) = \lambda_T(f(x \circ y)) \quad (3.2)$$
$$= \lambda_T(f(x) \bullet f(y)) \quad (3.2)$$
$$\leq \max\{\lambda_T(f(x)), \lambda_T(f(y))\} \quad (2.54)$$
$$= \max\{f^{-1}(\lambda_T)(x), f^{-1}(\lambda_T)(y)\}, \quad (3.2)$$

$$f^{-1}(\lambda_I)(x \circ y) = \lambda_I(f(x \circ y)) \quad (3.2)$$
$$= \lambda_I(f(x) \bullet f(y)) \quad (3.2)$$
$$\geq \min\{\lambda_I(f(x)), \lambda_I(f(y))\} \quad (2.54)$$
$$= \min\{f^{-1}(\lambda_I)(x), f^{-1}(\lambda_I)(y)\}, \quad (3.2)$$

Doi :10.5281/zenodo.3746022
\[ f^{-1}(\lambda)_F(x \circ y) = \lambda_F(f(x \circ y)) \]  
\[ = \lambda_F(f(x) \cdot f(y)) \]  
\[ \leq \max\{\lambda_F(f(x)), \lambda_F(f(y))\} \]  
\[ = \max\{f^{-1}(\lambda)_F(x), f^{-1}(\lambda)_F(y)\}. \]  
(3.2)

Hence, \( f^{-1}(\mathcal{A}) \) is a neutrosophic cubic UP-subalgebra of \( X \).

(2) Assume that \( \mathcal{A} \) is a neutrosophic cubic near UP-filter of \( Y \) which is order preserving. By Theorem 2.6 and (UP-3), we have for all \( x \in X \),

\[ f^{-1}(A)_T(0X) = A_T(f(0X)) \geq A_T(f(x)) = f^{-1}(A)_T(x), \]  
\[ f^{-1}(A)_1(0X) = A_1(f(0X)) \leq A_1(f(x)) = f^{-1}(A)_1(x), \]  
\[ f^{-1}(\lambda)_T(0X) = \lambda_T(f(0X)) \leq \lambda_T(f(x)) = f^{-1}(\lambda)_T(x), \]  
\[ f^{-1}(\lambda)_1(0X) = \lambda_1(f(0X)) \geq \lambda_1(f(x)) = f^{-1}(\lambda)_1(x), \]  
\[ f^{-1}(\lambda)_F(0X) = \lambda_F(f(0X)) \leq \lambda_F(f(x)) = f^{-1}(\lambda)_F(x). \]  
(2.57, 2.58)

Let \( x, y \in X \). Then

\[ f^{-1}(A)_T(x \circ y) = A_T(f(x \circ y)) = A_T(f(x) \bullet f(y)) \geq A_T(f(y)) = f^{-1}(A)_T(y), \]  
\[ f^{-1}(A)_1(x \circ y) = A_1(f(x \circ y)) = A_1(f(x) \bullet f(y)) \leq A_1(f(y)) = f^{-1}(A)_1(y), \]  
\[ f^{-1}(\lambda)_T(x \circ y) = \lambda_T(f(x \circ y)) = \lambda_T(f(x) \bullet f(y)) \leq \lambda_T(f(y)) = f^{-1}(\lambda)_T(y), \]  
\[ f^{-1}(\lambda)_1(x \circ y) = \lambda_1(f(x \circ y)) = \lambda_1(f(x) \bullet f(y)) \geq \lambda_1(f(y)) = f^{-1}(\lambda)_1(y), \]  
\[ f^{-1}(\lambda)_F(x \circ y) = \lambda_F(f(x \circ y)) = \lambda_F(f(x) \bullet f(y)) \leq \lambda_F(f(y)) = f^{-1}(\lambda)_F(y). \]  
(2.58, 2.59)

Hence, \( f^{-1}(\mathcal{A}) \) is a neutrosophic cubic near UP-filter of \( X \).

(3) Assume that \( \mathcal{A} \) is a neutrosophic cubic UP-filter of \( Y \). Then \( \mathcal{A} \) is a neutrosophic cubic near UP-filter of \( Y \). By Lemma 3.6 and the proof of (2) we have \( f^{-1}(\mathcal{A}) \) satisfies the assertions (2.55) and (2.56). Let \( x, y \in X \). Then

\[ f^{-1}(A)_T(y) = A_T(f(y)) \]  
\[ \geq \min\{A_T(f(x) \bullet f(y)), A_T(f(x))\} \]  
\[ = \min\{A_T(f(x \circ y)), A_T(f(x))\} \]  
\[ = \min\{f^{-1}(A)_T(x \circ y), f^{-1}(A)_T(x)\}, \]  
(3.1)

\[ f^{-1}(A)_1(y) = A_1(f(y)) \]  
\[ \leq \max\{A_1(f(x) \bullet f(y)), A_1(f(x))\} \]  
\[ = \max\{A_1(f(x \circ y)), A_1(f(x))\} \]  
\[ = \max\{f^{-1}(A)_1(x \circ y), f^{-1}(A)_1(x)\}, \]  
(3.1)

\[ f^{-1}(\lambda)_T(y) = \lambda_T(f(y)) \]  
\[ \leq \max\{\lambda_T(f(x) \bullet f(y)), \lambda_T(f(x))\} \]  
\[ = \max\{\lambda_T(f(x \circ y)), \lambda_T(f(x))\} \]  
\[ = \max\{f^{-1}(\lambda)_T(x \circ y), f^{-1}(\lambda)_T(x)\}, \]  
(3.2)

\[ f^{-1}(\lambda)_1(y) = \lambda_1(f(y)) \]  
\[ \geq \min\{\lambda_1(f(x) \bullet f(y)), \lambda_1(f(x))\} \]  
\[ = \min\{\lambda_1(f(x \circ y)), \lambda_1(f(x))\} \]  
\[ = \min\{f^{-1}(\lambda)_1(x \circ y), f^{-1}(\lambda)_1(x)\}. \]  
(3.2)
\[ f^{-1}(\lambda)_{F}(y) = \lambda_{F}(f(y)) \]  
\[ \leq \max\{\lambda_{F}(f(x) \cdot f(y)), \lambda_{F}(f(x))\} \]  
\[ = \max\{\lambda_{F}(f(x \circ y)), \lambda_{F}(f(x))\} \]  
\[ = \max\{f^{-1}(\lambda)_{F}(x \circ y), f^{-1}(\lambda)_{F}(x)\}. \]  

Hence, \( f^{-1}(\mathcal{A}) \) is a neutrosophic cubic UP-filter of \( X \).

(4) Assume that \( \mathcal{A} \) is a neutrosophic cubic UP-ideal of \( Y \). Then \( \mathcal{A} \) is a neutrosophic cubic UP-filter of \( Y \).

By the proof of (3) we have \( f^{-1}(\mathcal{A}) \) satisfies the assertions (2.55) and (2.56). Let \( x, y, z \in X \). Then

\[ f^{-1}(A)_{T}(x \circ z) = A_{T}(f(x \circ z)) \]  
\[ = A_{T}(f(x) \cdot f(z)) \]  
\[ \geq \min\{A_{T}(f(x) \cdot (f(y) \cdot f(z))), A_{T}(f(y))\} \]  
\[ = \min\{A_{T}(f(x) \cdot (f(y \circ z))), A_{T}(f(y))\} \]  
\[ = \min\{f^{-1}(A)_{T}(x \circ (y \circ z)), f^{-1}(A)_{T}(y)\}, \]  

(3.1)

\[ f^{-1}(A)_{I}(x \circ z) = A_{I}(f(x \circ z)) \]  
\[ = A_{I}(f(x) \cdot f(z)) \]  
\[ \geq \min\{A_{I}(f(x) \cdot (f(y) \cdot f(z))), A_{I}(f(y))\} \]  
\[ = \min\{A_{I}(f(x) \cdot (f(y \circ z))), A_{I}(f(y))\} \]  
\[ = \min\{f^{-1}(A)_{I}(x \circ (y \circ z)), f^{-1}(A)_{I}(y)\}, \]  

(3.1)

\[ f^{-1}(A)_{F}(x \circ z) = A_{F}(f(x \circ z)) \]  
\[ = A_{F}(f(x) \cdot f(z)) \]  
\[ \geq \min\{A_{F}(f(x) \cdot (f(y) \cdot f(z))), A_{F}(f(y))\} \]  
\[ = \min\{A_{F}(f(x) \cdot (f(y \circ z))), A_{F}(f(y))\} \]  
\[ = \min\{f^{-1}(A)_{F}(x \circ (y \circ z)), f^{-1}(A)_{F}(y)\}, \]  

(3.1)

\[ f^{-1}(\lambda)_{T}(x \circ z) = \lambda_{T}(f(x \circ z)) \]  
\[ = \lambda_{T}(f(x) \cdot f(z)) \]  
\[ \leq \max\{\lambda_{T}(f(x) \cdot (f(y) \cdot f(z))), \lambda_{T}(f(y))\} \]  
\[ = \max\{\lambda_{T}(f(x) \cdot (f(y \circ z))), \lambda_{T}(f(y))\} \]  
\[ = \max\{f^{-1}(\lambda)_{T}(x \circ (y \circ z)), f^{-1}(\lambda)_{T}(y)\}, \]  

(3.2)

\[ f^{-1}(\lambda)_{I}(x \circ z) = \lambda_{I}(f(x \circ z)) \]  
\[ = \lambda_{I}(f(x) \cdot f(z)) \]  
\[ \geq \min\{\lambda_{I}(f(x) \cdot (f(y) \cdot f(z))), \lambda_{I}(f(y))\} \]  
\[ = \min\{\lambda_{I}(f(x) \cdot (f(y \circ z))), \lambda_{I}(f(y))\} \]  
\[ = \min\{f^{-1}(\lambda)_{I}(x \circ (y \circ z)), f^{-1}(\lambda)_{I}(y)\}, \]  

(3.2)

\[ f^{-1}(\lambda)_{F}(x \circ z) = \lambda_{F}(f(x \circ z)) \]  
\[ = \lambda_{F}(f(x) \cdot f(z)) \]  
\[ \leq \max\{\lambda_{F}(f(x) \cdot (f(y) \cdot f(z))), \lambda_{F}(f(y))\} \]  
\[ = \max\{\lambda_{F}(f(x) \cdot (f(y \circ z))), \lambda_{F}(f(y))\} \]  
\[ = \max\{f^{-1}(\lambda)_{F}(x \circ (y \circ z)), f^{-1}(\lambda)_{F}(y)\}. \]  

(3.2)

Hence, \( f^{-1}(\mathcal{A}) \) is a neutrosophic cubic UP-ideal of \( X \).
Let 

\[ f^{-1}(A)_T(x) = A_T(f(x)) \]

\[ \geq \min\{A_T(f(z) \bullet f(y)) \bullet (f(z) \bullet f(x)), A_f(f(y))\} \]

\[ = \min\{A_T(f(z \circ y) \bullet f(z \circ x)), A_T(f(y))\} \]

\[ = \min\{f^{-1}(A)_T((z \circ y) \circ (z \circ x)), f^{-1}(A)_T(y)\}, \]

\[ f^{-1}(A)_f(x) = A_f(f(x)) \]

\[ \geq \min\{A_f(f(z) \bullet f(y)) \bullet (f(z) \bullet f(x)), A_f(f(y))\} \]

\[ = \min\{A_f(f(z \circ y) \bullet f(z \circ x)), A_f(f(y))\} \]

\[ = \min\{f^{-1}(A)_f((z \circ y) \circ (z \circ x)), f^{-1}(A)_f(y)\}, \]

\[ f^{-1}(A)_F(x) = A_F(f(x)) \]

\[ \geq \min\{A_F(f(z) \bullet f(y)) \bullet (f(z) \bullet f(x)), A_F(f(y))\} \]

\[ = \min\{A_F(f(z \circ y) \bullet f(z \circ x)), A_F(f(y))\} \]

\[ = \min\{f^{-1}(A)_F((z \circ y) \circ (z \circ x)), f^{-1}(A)_F(y)\}, \]

\[ f^{-1}(\lambda)_T(x) = \lambda_T(f(x)) \]

\[ \leq \min\{\lambda_T(f(z) \bullet f(y)) \bullet (f(z) \bullet f(x)), \lambda_T(f(y))\} \]

\[ = \min\{\lambda_T(f(z \circ y) \bullet f(z \circ x)), \lambda_T(f(y))\} \]

\[ = \min\{f^{-1}(\lambda)_T((z \circ y) \circ (z \circ x)), f^{-1}(\lambda)_T(y)\}, \]

\[ f^{-1}(\lambda)_f(x) = \lambda_f(f(x)) \]

\[ \leq \min\{\lambda_f(f(z) \bullet f(y)) \bullet (f(z) \bullet f(x)), \lambda_f(f(y))\} \]

\[ = \min\{\lambda_f(f(z \circ y) \bullet f(z \circ x)), \lambda_f(f(y))\} \]

\[ = \min\{f^{-1}(\lambda)_f((z \circ y) \circ (z \circ x)), f^{-1}(\lambda)_f(y)\}, \]

\[ f^{-1}(\lambda)_F(x) = \lambda_F(f(x)) \]

\[ \leq \min\{\lambda_F(f(z) \bullet f(y)) \bullet (f(z) \bullet f(x)), \lambda_F(f(y))\} \]

\[ = \min\{\lambda_F(f(z \circ y) \bullet f(z \circ x)), \lambda_F(f(y))\} \]

\[ = \min\{f^{-1}(\lambda)_F((z \circ y) \circ (z \circ x)), f^{-1}(\lambda)_F(y)\}. \]

Hence, \( f^{-1}(\mathcal{A}) \) is a neutrosophic cubic strong UP-ideal of \( Y \).

**Definition 3.8.** A NCS \( \mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F}) \) in \( X \) has NCS-property if for any non-empty subset \( A \) of \( X \), there exist elements \( \alpha_{T,I,F}, \beta_{T,I,F} \in A \) (instead of \( \alpha_T, \alpha_I, \alpha_F, \beta_T, \beta_I, \beta_F \in A \)) such that

\[ A_T(\alpha_T) = \text{rsup}_{s \in A} A_T(s), A_I(\alpha_I) = \text{rinf}_{s \in A} A_I(s), A_F(\alpha_F) = \text{rsup}_{s \in A} A_F(s), \]

\[ \lambda_T(\beta_T) = \text{inf}_{s \in A} \lambda_T(s), \lambda_I(\beta_I) = \text{sup}_{s \in A} \lambda_I(s), \lambda_F(\beta_F) = \text{inf}_{s \in A} \lambda_F(s). \]

**Definition 3.9.** Let \( X \) and \( Y \) be any two non-empty sets and let \( f : X \to Y \) be any function. A NCS \( \mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F}) \) in \( X \) is said to be \( f \)-invariant if

\[ (\text{for all } x, y \in X) f(x) = f(y) \Rightarrow A_{T,I,F}(x) = A_{T,I,F}(y), \lambda_{T,I,F}(x) = \lambda_{T,I,F}(y). \]

**Lemma 3.10.** Let \( (X, \circ, 0_X) \) and \( (Y, \bullet, 0_Y) \) be two UP-algebras and let \( f : X \to Y \) be a UP-epimorphism. Let \( \mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F}) \) be an \( f \)-invariant NCS in \( X \) with NCS-property. For any \( x, y \in Y \), there exist
elements $\alpha_{T,I,F}, \gamma_{T,I,F}, \beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ such that

$$f(A)_T(x) = A_T(\alpha_T), f(A)_I(x) = A_I(\alpha_I), f(A)_F(x) = A_F(\alpha_F),$$

$$f(\lambda)_T(x) = \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F),$$

$$f(A)_T(y) = A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F),$$

$$f(\lambda)_T(y) = \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F),$$

$$f(A)_T(x \circ y) = A_T(\alpha_T \circ \beta_T), f(A)_I(x \circ y) = A_I(\alpha_I \circ \beta_I), f(A)_F(x \circ y) = A_F(\alpha_F \circ \beta_F),$$

$$f(\lambda)_T(x \circ y) = \lambda_T(\gamma_T \circ \gamma_T), f(\lambda)_I(x \circ y) = \lambda_I(\gamma_I \circ \phi_I), f(\lambda)_F(x \circ y) = \lambda_F(\gamma_F \circ \phi_F).$$

**Proof.** Let $x, y \in Y$. Since $f$ is surjective, we have $f^{-1}(x), f^{-1}(y)$, and $f^{-1}(x \circ y)$ are non-empty subsets of $X$. Since $\mathcal{A}$ has NCS-property, there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F}, \beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$, and $a_{T,I,F}, b_{T,I,F} \in f^{-1}(x \circ y)$ such that

$$f(A)_T(x) = \text{rsup}_{s \in f^{-1}(x)} \{A_T(s)\} = A_T(\alpha_T),$$

$$f(A)_I(x) = \text{rinf}_{s \in f^{-1}(x)} \{A_I(s)\} = A_I(\alpha_I),$$

$$f(A)_F(x) = \text{rsup}_{s \in f^{-1}(x)} \{A_F(s)\} = A_F(\alpha_F),$$

$$f(\lambda)_T(x) = \text{inf}_{s \in f^{-1}(x)} \{\lambda_T(s)\} = \lambda_T(\gamma_T),$$

$$f(\lambda)_I(x) = \text{inf}_{s \in f^{-1}(x)} \{\lambda_I(s)\} = \lambda_I(\gamma_I),$$

$$f(\lambda)_F(x) = \text{inf}_{s \in f^{-1}(x)} \{\lambda_F(s)\} = \lambda_F(\gamma_F),$$

$$f(A)_T(y) = \text{rsup}_{s \in f^{-1}(y)} \{A_T(s)\} = A_T(\beta_T),$$

$$f(A)_I(y) = \text{rinf}_{s \in f^{-1}(y)} \{A_I(s)\} = A_I(\beta_I),$$

$$f(A)_F(y) = \text{rsup}_{s \in f^{-1}(y)} \{A_F(s)\} = A_F(\beta_F),$$

$$f(\lambda)_T(y) = \text{inf}_{s \in f^{-1}(y)} \{\lambda_T(s)\} = \lambda_T(\phi_T),$$

$$f(\lambda)_I(y) = \text{inf}_{s \in f^{-1}(y)} \{\lambda_I(s)\} = \lambda_I(\phi_I),$$

$$f(\lambda)_F(y) = \text{inf}_{s \in f^{-1}(y)} \{\lambda_F(s)\} = \lambda_F(\phi_F),$$

and

$$f(A)_T(x \circ y) = \text{rsup}_{s \in f^{-1}(x \circ y)} \{A_T(s)\} = A_T(\alpha_T),$$

$$f(A)_I(x \circ y) = \text{rinf}_{s \in f^{-1}(x \circ y)} \{A_I(s)\} = A_I(\alpha_I),$$

$$f(A)_F(x \circ y) = \text{rsup}_{s \in f^{-1}(x \circ y)} \{A_F(s)\} = A_F(\alpha_F),$$

$$f(\lambda)_T(x \circ y) = \text{inf}_{s \in f^{-1}(x \circ y)} \{\lambda_T(s)\} = \lambda_T(\gamma_T),$$

$$f(\lambda)_I(x \circ y) = \text{inf}_{s \in f^{-1}(x \circ y)} \{\lambda_I(s)\} = \lambda_I(\gamma_I),$$

$$f(\lambda)_F(x \circ y) = \text{inf}_{s \in f^{-1}(x \circ y)} \{\lambda_F(s)\} = \lambda_F(\gamma_F).$$

Since

$$f(\alpha_T) = x \circ y = f(\alpha_T) \circ f(\beta_T) = f(\alpha_T \circ \beta_T),$$

$$f(\alpha_I) = x \circ y = f(\alpha_I) \circ f(\beta_I) = f(\alpha_I \circ \beta_I),$$

$$f(\alpha_F) = x \circ y = f(\alpha_F) \circ f(\beta_F) = f(\alpha_F \circ \beta_F),$$

$$f(\beta_T) = x \circ y = f(\gamma_T) \circ f(\phi_T) = f(\gamma_T \circ \phi_T),$$

$$f(\beta_I) = x \circ y = f(\gamma_I) \circ f(\phi_I) = f(\gamma_I \circ \phi_I),$$

$$f(\beta_F) = x \circ y = f(\gamma_F) \circ f(\phi_F) = f(\gamma_F \circ \phi_F),$$

and $\mathcal{A}$ is $f$-invariant, it follows that

$$f(A)_T(x \circ y) = A_T(\alpha_T) = A_T(\alpha_T \circ \beta_T),$$

$$f(A)_I(x \circ y) = A_I(\alpha_I) = A_I(\alpha_I \circ \beta_I),$$

$$f(A)_F(x \circ y) = A_F(\alpha_F) = A_F(\alpha_F \circ \beta_F),$$

$$f(\lambda)_T(x \circ y) = \lambda_T(\beta_T) = \lambda_T(\gamma_T \circ \phi_T),$$

$$f(\lambda)_I(x \circ y) = \lambda_I(\beta_I) = \lambda_I(\gamma_I \circ \phi_I),$$

$$f(\lambda)_F(x \circ y) = \lambda_F(\beta_F) = \lambda_F(\gamma_F \circ \phi_F).$$

The proof is completed. □

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Theorem 3.11. Let \((X, \circ, 0_X)\) and \((Y, \bullet, 0_Y)\) be two UP-algebras, \(f: X \to Y\) be a UP-epimorphism, and \(\mathcal{A} = (A_{T, I, F}, \lambda_{T, I, F})\) be a NCS in \(X\). Then the followings hold:

1. If \(\mathcal{A}\) is an \(f\)-invariant neutrosophic cubic UP-subalgebra of \(X\) with NCS-property, then the image \(f(\mathcal{A})\) of \(\mathcal{A}\) under \(f\) is a neutrosophic cubic UP-subalgebra of \(Y\).

2. If \(\mathcal{A}\) is an \(f\)-invariant neutrosophic cubic near UP-filter of \(X\) with NCS-property, then the image \(f(\mathcal{A})\) of \(\mathcal{A}\) under \(f\) is a neutrosophic cubic near UP-filter of \(Y\).

3. If \(\mathcal{A}\) is an \(f\)-invariant neutrosophic cubic UP-filter of \(X\) with NCS-property, then the image \(f(\mathcal{A})\) of \(\mathcal{A}\) under \(f\) is a neutrosophic cubic UP-filter of \(Y\).

4. If \(\mathcal{A}\) is an \(f\)-invariant neutrosophic cubic UP-ideal of \(X\) with NCS-property, then the image \(f(\mathcal{A})\) of \(\mathcal{A}\) under \(f\) is a neutrosophic cubic UP-ideal of \(Y\).

5. If \(\mathcal{A}\) is an \(f\)-invariant neutrosophic cubic strong UP-ideal of \(X\) with NCS-property, then the image \(f(\mathcal{A})\) of \(\mathcal{A}\) under \(f\) is a neutrosophic cubic strong UP-ideal of \(Y\).

Proof. (1) Assume that \(\mathcal{A} = (A_{T, I, F}, \lambda_{T, I, F})\) is an \(f\)-invariant neutrosophic cubic UP-subalgebra of \(X\) with NCS-property. Let \(x, y \in Y\). Since \(f\) is surjective, we have \(f^{-1}(x), f^{-1}(y), \) and \(f^{-1}(x \bullet y)\) are non-empty. By Lemma 3.10, there exist elements \(\alpha_{T, I, F}, \gamma_{T, I, F} \in f^{-1}(x)\) and \(\beta_{T, I, F}, \phi_{T, I, F} \in f^{-1}(y)\) such that

\[
\begin{align*}
f(A_T(x \bullet y)) &= A_T(\alpha_T \circ \beta_T) \succeq \min\{A_T(\alpha_T), A_T(\beta_T)\} = \min\{f(A_T(x)), f(A_T(y))\}, \\
f(A_I(x \bullet y)) &= A_I(\alpha_I \circ \beta_I) \preceq \max\{A_I(\alpha_I), A_I(\beta_I)\} = \max\{f(A_I(x)), f(A_I(y))\}, \\
f(A_F(x \bullet y)) &= A_F(\alpha_F \circ \beta_F) \succeq \min\{A_F(\alpha_F), A_F(\beta_F)\} = \min\{f(A_F(x)), f(A_F(y))\}, \\
f(\lambda_T(x \bullet y)) &= \lambda_T(\gamma_T \circ \phi_T) \preceq \max\{\lambda_T(\gamma_T), \lambda_T(\phi_T)\} = \max\{f(\lambda_T(x)), f(\lambda_T(y))\}, \\
f(\lambda_I(x \bullet y)) &= \lambda_I(\gamma_I \circ \phi_I) \preceq \max\{\alpha_I, \phi_I\} = \min\{f(\lambda_I(x)), f(\lambda_I(y))\}, \\
f(\lambda_F(x \bullet y)) &= \lambda_F(\gamma_F \circ \phi_F) \preceq \max\{\lambda_F(\gamma_F), \lambda_F(\phi_F)\} = \max\{f(\lambda_F(x)), f(\lambda_F(y))\}.
\end{align*}
\]

Hence, \(f(\mathcal{A})\) is a neutrosophic cubic UP-subalgebra of \(Y\).

(2) Assume that \(\mathcal{A} = (A_{T, I, F}, \lambda_{T, I, F})\) is an \(f\)-invariant neutrosophic cubic near UP-filter of \(X\) with NCS-property. By Theorem 2.6, we have \(0_X \in f^{-1}(0_Y)\) and so \(f^{-1}(0_Y)\) is non-empty. Thus

\[
\begin{align*}
f(A_T(0_Y)) &= \max_{s \in f^{-1}(0_Y)}\{A_T(s)\} \geq A_T(0_X), \\
f(A_I(0_Y)) &= \max_{s \in f^{-1}(0_Y)}\{A_I(s)\} \geq A_I(0_X), \\
f(A_F(0_Y)) &= \max_{s \in f^{-1}(0_Y)}\{A_F(s)\} \geq A_F(0_X), \\
f(\lambda_T(0_Y)) &= \max_{s \in f^{-1}(0_Y)}\{\lambda_T(s)\} \geq \lambda_T(0_X), \\
f(\lambda_I(0_Y)) &= \max_{s \in f^{-1}(0_Y)}\{\lambda_I(s)\} \geq \lambda_I(0_X), \\
f(\lambda_F(0_Y)) &= \max_{s \in f^{-1}(0_Y)}\{\lambda_F(s)\} \geq \lambda_F(0_X).
\end{align*}
\]

Let \(y \in Y\). Since \(f\) is surjective, we have \(f^{-1}(y)\) is non-empty. By (2.55) and (2.56), we have \(A_T(0_X) \geq A_T(s), A_I(0_X) \geq A_I(s), A_F(0_X) \geq A_F(s), \lambda_T(0_X) \geq \lambda_T(s), \lambda_I(0_X) \geq \lambda_I(s), \lambda_F(0_X) \geq \lambda_F(s)\) for all \(s \in f^{-1}(y)\). Then \(A_T(0_X)\) is an upper bound of \(\{A_T(s)\}_{s \in f^{-1}(y)}\), \(A_I(0_X)\) is a lower bound of \(\{A_I(s)\}_{s \in f^{-1}(y)}\), \(A_F(0_X)\) is an upper bound of \(\{A_F(s)\}_{s \in f^{-1}(y)}\), \(\lambda_T(0_X)\) is a lower bound of \(\{\lambda_T(s)\}_{s \in f^{-1}(y)}\), \(\lambda_I(0_X)\) is an
upper bound of \( \{ \lambda_I(s) \}_{s \in f^{-1}(y)} \) and \( \lambda_F(0_X) \) is a lower bound of \( \{ \lambda_F(s) \}_{s \in f^{-1}(y)} \). By (3.5), we have
\[
\begin{align*}
  f(A)\mathcal{T}(0_Y) &\geq A_T(0_X) \geq \sup_{s \in f^{-1}(y)}\{A_T(s)\} = f(A)\mathcal{T}(y), \\
  f(A)\mathcal{T}(0_Y) &\leq A_T(0_X) \geq \inf_{s \in f^{-1}(y)}\{A_T(s)\} = f(A)\mathcal{T}(y), \\
  f(A)\mathcal{T}(0_Y) &\geq A_T(0_X) \geq \sup_{s \in f^{-1}(y)}\{A_T(s)\} = f(A)\mathcal{T}(y), \\
  f(A)\mathcal{T}(0_Y) &\leq A_T(0_X) \leq \inf_{s \in f^{-1}(y)}\{A_T(s)\} = f(A)\mathcal{T}(y).
\end{align*}
\]
Let \( x, y \in Y \). By Lemma 3.10, there exist elements \( \alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x) \) and \( \beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y) \) such that
\[
\begin{align*}
  f(A)\mathcal{T}(x) &= A_T(\alpha_T), f(A)\mathcal{I}(x) = A_I(\alpha_I), f(A)\mathcal{F}(x) = A_F(\alpha_F), \\
  f(\lambda)_T(x) &= \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F), \\
  f(A)\mathcal{T}(y) &= A_T(\beta_T), f(A)\mathcal{I}(y) = A_I(\beta_I), f(A)\mathcal{F}(y) = A_F(\beta_F), \\
  f(\lambda)_T(y) &= \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F).
\end{align*}
\]
Then
\[
\begin{align*}
  f(A)\mathcal{T}(x \cdot y) &= A_T(\alpha_T \circ \beta_T), f(A)\mathcal{I}(x \cdot y) = A_I(\alpha_I \circ \beta_I), f(A)\mathcal{F}(x \cdot y) = A_F(\alpha_F \circ \beta_F), \\
  f(\lambda)_T(x \cdot y) &= \lambda_T(\gamma_T \circ \phi_T), f(\lambda)_I(x \cdot y) = \lambda_I(\gamma_I \circ \phi_I), f(\lambda)_F(x \cdot y) = \lambda_F(\gamma_F \circ \phi_F).
\end{align*}
\]
Hence, \( f(\mathcal{A}) \) is a neutrosophic cubic UP-filter of \( Y \).

(3) Assume that \( \mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F}) \) is an \( f \)-invariant neutrosophic cubic UP-filter of \( X \) with NCS-property. Then \( \mathcal{A} \) is a neutrosophic cubic near UP-filter of \( X \). By the proof of (2), we have \( f(\mathcal{A}) \) satisfies the assertions (2.55) and (2.56). Let \( x, y \in Y \). By Lemma 3.10, there exist elements \( \alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x) \) and \( \beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y) \) such that
\[
\begin{align*}
  f(A)\mathcal{T}(x) &= A_T(\alpha_T), f(A)\mathcal{I}(x) = A_I(\alpha_I), f(A)\mathcal{F}(x) = A_F(\alpha_F), \\
  f(\lambda)_T(x) &= \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F), \\
  f(A)\mathcal{T}(y) &= A_T(\beta_T), f(A)\mathcal{I}(y) = A_I(\beta_I), f(A)\mathcal{F}(y) = A_F(\beta_F), \\
  f(\lambda)_T(y) &= \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F).
\end{align*}
\]
Then
\[
\begin{align*}
  f(A)\mathcal{T}(x \cdot y) &= A_T(\alpha_T \circ \beta_T), f(A)\mathcal{I}(x \cdot y) = A_I(\alpha_I \circ \beta_I), f(A)\mathcal{F}(x \cdot y) = A_F(\alpha_F \circ \beta_F), \\
  f(\lambda)_T(x \cdot y) &= \lambda_T(\gamma_T \circ \phi_T), f(\lambda)_I(x \cdot y) = \lambda_I(\gamma_I \circ \phi_I), f(\lambda)_F(x \cdot y) = \lambda_F(\gamma_F \circ \phi_F).
\end{align*}
\]
Hence, \( f(\mathcal{A}) \) is a neutrosophic cubic UP-filter of \( Y \).

(4) Assume that \( \mathcal{A} = (A_{T,I,F}, \lambda_{T,I,F}) \) is an \( f \)-invariant neutrosophic cubic UP-ideal of \( X \) with NCS-property. Then \( \mathcal{A} \) is a neutrosophic cubic UP-filter of \( X \). By the proof of (3), we have \( f(\mathcal{A}) \) satisfies the
assertions (2.55) and (2.56). Let $x, y, z \in Y$. By Lemma 3.10 there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$, $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ and $\psi_{T,I,F}, \omega_{T,I,F} \in f^{-1}(z)$ such that
\[
\begin{align*}
    f(A)_T(y) &= A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F), \\
    f(\lambda)_T(y) &= \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F), \\
    f(A)_T(x \cdot z) &= A_T(\alpha_T \circ \psi_T), f(A)_I(x \cdot z) = A_I(\alpha_I \circ \psi_I), f(A)_F(x \cdot z) = A_F(\alpha_F \circ \psi_F), \\
    f(\lambda)_T(x \cdot z) &= \lambda_T(\gamma_T \circ \omega_T), f(\lambda)_I(x \cdot z) = \lambda_I(\gamma_I \circ \omega_I), f(\lambda)_F(x \cdot z) = \lambda_F(\gamma_F \circ \omega_F), \\
    f(A)_T(x \cdot (y \cdot z)) &= A_T(\alpha_T \circ (\beta_T \circ \psi_T)), \\
    f(A)_I(x \cdot (y \cdot z)) &= A_I(\alpha_I \circ (\beta_I \circ \psi_I)), \\
    f(A)_F(x \cdot (y \cdot z)) &= A_F(\alpha_F \circ (\beta_F \circ \psi_F)), \\
    f(\lambda)_T(x \cdot (y \cdot z)) &= \lambda_T(\gamma_T \circ (\phi_T \circ \omega_T)), \\
    f(\lambda)_I(x \cdot (y \cdot z)) &= \lambda_I(\gamma_I \circ (\phi_I \circ \omega_I)), \\
    f(\lambda)_F(x \cdot (y \cdot z)) &= \lambda_F(\gamma_F \circ (\phi_F \circ \omega_F)).
\end{align*}
\]

Then
\[
\begin{align*}
    f(A)_T(x \cdot z) &= A_T(\alpha_T \circ \psi_T) \\
    &= \min \{ f(A)_T(x \cdot (y \cdot z)), f(A)_T(y) \} \\
    &\geq \min \{ f(A)_T(x \cdot (y \cdot z)), f(A)_T(y) \} \\
    &= \min \{ f(A)_T(x \cdot (y \cdot z)), f(A)_T(y) \} \\
    f(A)_I(x \cdot z) &= A_I(\alpha_I \circ \psi_I) \\
    &\geq \min \{ f(A)_I(x \cdot (y \cdot z)), f(A)_I(y) \} \\
    &= \min \{ f(A)_I(x \cdot (y \cdot z)), f(A)_I(y) \} \\
    f(A)_F(x \cdot z) &= A_F(\alpha_F \circ \psi_F) \\
    &\geq \min \{ f(A)_F(x \cdot (y \cdot z)), f(A)_F(y) \} \\
    &= \min \{ f(A)_F(x \cdot (y \cdot z)), f(A)_F(y) \} \\
    f(\lambda)_T(x \cdot z) &= \lambda_T(\gamma_T \circ \omega_T) \\
    &\leq \max \{ \lambda_T(\gamma_T \circ (\phi_T \circ \omega_T)), \lambda_T(\phi_T) \} \\
    &= \max \{ \lambda_T(\gamma_T \circ (\phi_T \circ \omega_T)), \lambda_T(\phi_T) \} \\
    f(\lambda)_I(x \cdot z) &= \lambda_I(\gamma_I \circ \omega_I) \\
    &\geq \min \{ \lambda_I(\gamma_I \circ (\phi_I \circ \omega_I)), \lambda_I(\phi_I) \} \\
    &= \min \{ \lambda_I(\gamma_I \circ (\phi_I \circ \omega_I)), \lambda_I(\phi_I) \} \\
    f(\lambda)_F(x \cdot z) &= \lambda_F(\gamma_F \circ \omega_F) \\
    &\leq \max \{ \lambda_F(\gamma_F \circ (\phi_F \circ \omega_F)), \lambda_F(\phi_F) \} \\
    &= \max \{ \lambda_F(\gamma_F \circ (\phi_F \circ \omega_F)), \lambda_F(\phi_F) \} \\
    &= \max \{ \lambda_F(\gamma_F \circ (\phi_F \circ \omega_F)), \lambda_F(\phi_F) \}.
\end{align*}
\]

Hence, $f(\mathcal{A})$ is a neutrosophic cubic UP-ideal of $Y$.

(5) Assume that $\mathcal{A} = (A_{T,I,F}, \gamma_{T,I,F})$ is an $f$-invariant neutrosophic cubic strong UP-ideal of $X$ with NCS-property. Then $\mathcal{A}$ is a neutrosophic cubic UP-ideal of $X$. By the proof of [4], we have $f(\mathcal{A})$ satisfies the assertions (2.55) and (2.56). Let $x, y, z \in Y$. By Lemma 3.10 there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$, $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ and $\psi_{T,I,F}, \omega_{T,I,F} \in f^{-1}(z)$ such that
\[
\begin{align*}
    f(A)_T(x) &= A_T(\alpha_T), f(A)_I(x) = A_I(\alpha_I), f(A)_F(x) = A_F(\alpha_F), \\
    f(\lambda)_T(x) &= \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F), \\
    f(A)_T(y) &= A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F), \\
    f(\lambda)_T(y) &= \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F), \\
    f(A)_T((z \cdot x) \cdot (z \cdot x)) &= A_T((\psi_T \circ \phi_T) \circ (\psi_T \circ \alpha_T)), \\
    f(A)_I((z \cdot x) \cdot (z \cdot x)) &= A_I((\psi_I \circ \phi_I) \circ (\psi_I \circ \alpha_I)), \\
    f(A)_F((z \cdot x) \cdot (z \cdot x)) &= A_F((\psi_F \circ \phi_F) \circ (\psi_F \circ \alpha_F)), \\
    f(\lambda)_T((z \cdot x) \cdot (z \cdot x)) &= \lambda_T((\omega_T \circ \phi_T) \circ (\omega_T \circ \gamma_T)), \\
    f(\lambda)_I((z \cdot x) \cdot (z \cdot x)) &= \lambda_I((\omega_I \circ \phi_I) \circ (\omega_I \circ \gamma_I)), \\
    f(\lambda)_F((z \cdot x) \cdot (z \cdot x)) &= \lambda_F((\omega_F \circ \phi_F) \circ (\omega_F \circ \gamma_F)).
\end{align*}
\]
Then

\[
f(A)_T(x) = A_T(\alpha_T) \geq \min \{ A_T((\psi_T \circ \beta_T) \circ (\psi_T \circ \alpha_T)), A_T(\beta_T) \} \quad (2.63)
\]

\[
f(A)_I(x) = A_I(\alpha_I) \leq \max \{ A_I((\psi_I \circ \beta_I) \circ (\psi_I \circ \alpha_I)), A_I(\beta_I) \} \quad (2.63)
\]

\[
f(A)_F(x) = A_F(\alpha_F) \geq \min \{ A_F((\psi_F \circ \beta_F) \circ (\psi_F \circ \alpha_F)), A_F(\beta_F) \} \quad (2.63)
\]

\[
f(\lambda)_T(x) = \lambda_T(\gamma_T) \leq \max \{ \lambda_T((\omega_T \circ \phi_T) \circ (\omega_T \circ \gamma_T)), \lambda_T(\phi_T) \} \quad (2.64)
\]

\[
f(\lambda)_I(x) = \lambda_I(\gamma_I) \geq \min \{ \lambda_I((\omega_I \circ \phi_I) \circ (\omega_I \circ \gamma_I)), \lambda_I(\phi_I) \} \quad (2.64)
\]

\[
f(\lambda)_F(x) = \lambda_F(\gamma_F) \leq \max \{ \lambda_F((\omega_F \circ \phi_F) \circ (\omega_F \circ \gamma_F)), \lambda_F(\phi_F) \} \quad (2.64)
\]

Hence, \( f(\mathcal{A}) \) is a neutrosophic cubic strong UP-ideal of \( Y \). \( \square \)

### 4 Conclusions and future work

In this paper, we have studied the image and inverse image of a neutrosophic cubic UP-subalgebra (resp., neutrosophic cubic near UP-filter, neutrosophic cubic UP-filter, neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal) of a UP-algebra under some UP-homomorphisms. The results of the study, in the case of inverse image, we noticed that only a neutrosophic cubic near UP-filter required order preserving condition. In the case of image, we noticed that all concepts of NCSs required \( f \)-invariant and NCS-property assertions and UP-epimorphism.

In our future study, we will apply this concept/results to other types of NCSs in a UP-algebra. Also, we will study the P-intersection, P-union, R-intersection, R-union of neutrosophic cubic UP-subalgebras, neutrosophic cubic near UP-filters, neutrosophic cubic UP-filters, neutrosophic cubic UP-ideals, and neutrosophic cubic strong UP-ideals of a UP-algebra.

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### References


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