Interval Neutrosophic Sets and Topology

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Abstract: On 2005, F. Smarandache generalized the Atanassov's intuitionistic fuzzy setsto neutrosophic sets. Also, this author and some co-workers introduced the notion of interval neutrosophic set, which is an instance of neutrosophic set and studied various properties. The notion of neutro- sophic topology on the non-standard interval is also due to Smarandache. We study in this paper relations between interval neutrosophic sets and topology.

Key–Words: Logic, Set-Theory, Topology, Atanassov's intuitionistic fuzzy sets Typing manuscripts, LaTeX

1 Introduction

In various recent papers, F. Smarandache generalizes intuitionistic fuzzy sets (IFSs) and other kinds of sets to neutrosophic sets (NSs)..

The notion of intuitionistic fuzzy set defined by K.T. Atanassov (1983, 1986) has been applied by Çoker (1997) for study intuitionistic fuzzy topological spaces. This concept has been developed by many authors (specially Çoker and co-workers, 1997; Hanafy; Hur, Kim and Ryou; Lee and Lee; and Lupiáñez).

F. Smarandache also defined the notion of neutrosophic topology on the non-standard interval (Sm1).

One can expect some relation between the intuitionistic fuzzy topology on an IFS and the neutrosophic topology. We show in (Lupiáñez 2008) that this is false. Indeed, an intuitionistic fuzzy topology is not necessarily a neutrosophic topology.

Also, (Wang, Smarandache, Zhang, and Sunderraman, 2005) introduced the notion of interval neutrosophic set, which is an instance of neutrosophic set and studied various properties. We study in this paper relations between interval neutrosophic sets and topology.

2 Basic Definitions

First, we present some basic definitions. For definitions on non-standard Analysis, see (Robinson, 1996):

Definition 1 *Let X be a non-empty set. An **intuitionistic fuzzy set** (IFS for short) A, is an object having the form $A = \{< x, \mu_A, \gamma_A > /x \in X\}$ where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x)+\gamma_A(x) \leq 1$ for each $x \in X$. (Atanassov, 1983).*

Definition 2 *Let X be a non-empty set. An **intuitionistic fuzzy set** (IFS for short) A, is an object having the form $A = \{< x, \mu_A, \gamma_A > /x \in X\}$ where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x)+\gamma_A(x) \leq 1$ for each $x \in X$. (Atanassov, 1983).*

Definition 3 *Let X be a non-empty set, and the IFSs $A = \{< x, \mu_A, \gamma_A > |x \in X\}$, $B = \{< x, \mu_B, \gamma_B > |x \in X\}$. Let*

$\overline{A} = \{< x, \gamma_A, \mu_A > |x \in X\}$

$A \cap B = \{< x, \mu_A \wedge \mu_B, \gamma_A \vee \gamma_B > |x \in X\}$

$A \cup B = \{< x, \mu_A \vee \mu_B, \gamma_A \wedge \gamma_B > |x \in X\}$.*(Atanassov, 1988).*

Definition 4 *Let X be a non-empty set. Let $0_\sim = \{< x, 0, 1 > |x \in X\}$ and $1_\sim = \{< x, 1, 0 > |x \in X\}$.(Çoker, 1997).*

Definition 5 *An **intuitionistic fuzzy topology** (IFT for short) on a non-empty set X is a family τ of IFSs in X satisfying:*

(a) $0_\sim$,$1_\sim \in \tau$,

(b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(c) $\cup G_j \in \tau$ for any family $\{G_j | j \in J\} \subset \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS for short) and any

IFS in τ is called an intuitionistic fuzzy open set (IFOS for short) in X. (Çoker, 1997).

Definition 6 *Let T, I,F be real standard or non-standard subsets of the non-standard unit interval $]^{-}0, 1^{+}[$, with*

$\sup T = t_{\sup}$, $\inf T = t_{\inf}$

$\sup I = i_{\sup}$, $\inf I = i_{\inf}$

$\sup F = f_{\sup}$, $\inf F = f_{\inf}$ *and* $n_{\sup} = t_{\sup} + i_{\sup} + f_{\sup}$ $n_{\inf} = t_{\inf} + i_{\inf} + f_{\inf}$,

*T, I,F are called **neutrosophic components**. Let U be an universe of discourse, and M a set included in U. An element x from U is noted with respect to the set M as $x(T, I, F)$ and belongs to M in the folloing way: it is $t\%$ true in the set, $i\%$ indeterminate (unknown if it is) in the set, and $f\%$ false, where t varies in T, i varies in I, f varies in F. The set M is called a **neutrosophic set** (NS). (Smarandache, 2005).*

Remark. *All IFS is a NS.*

Definition 7 *Let X be a space of points (objects) with generic elements in X denoted by x. An interval neutrosophic set (INS) A in X is characterized by **thuth-membership function** T_A, **indeteminacy-membership function** I_A and **falsity-membership function** F_A. For each point x in X, we have that $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. (Wang, Smarandache, Zhang, and Sunderraman, 2005).*

Remark. *All INS is clearly a NS.*

Definition 8 *a) An interval neutrosophic set A is **empty** if* $\inf T_A(x) = \sup T_A(x) = 0$, $\inf I_A(x) = \sup I_A(x) = 1$, $\inf F_A(x) = \sup F_A(x) = 0$ *for all x in X.*

b) Let $\underline{0} =< 0, 1, 1 >$ and $\underline{1} =< 1, 0, 0 >$.(Wang, Smarandache, Zhang, and Sunderraman, 2005).

Definition 9 *Let C_N denote a **neutrosophic complement** of A.*

Then C_N is a function $C_N : N \to N$ and C_N must satisfy at least the following three axiomatic requirements:

1. C_N $(\underline{0}) = \underline{1}$ and C_N $(\underline{1}) = \underline{0}$ (boundary conditions).

2. Let A and B be two interval neutrosophic sets defined on X, if $A(x) \leq B(x)$, then $C_N\,(A(x)) \geq C_N\,(B(x))$, for all x in X. (monotonicity).

3. Let A be an interval neutrosophic set defined on X, then $C_N\,(C_N\,(A(x))) = A(x)$, for all x in X. (involutivity).(Wang, Smarandache, Zhang, and Sunderraman, 2005).

Definition 10 *Let I_N denote a **neutrosophic intersection** of two interval neutrosophic sets A and B. Then I_N is a function $I_N : N \times N \to N$ and I_N must satisfy at least the following four axiomatic requirements:*

1. $I_N\,(A(x), \underline{1}) = A(x)$, for all x in X. (boundary condition).

2. $B(x) \leq C(x)$ implies $I_N\,(A(x), B(x)) \leq I_N(A(x), C(x))$, for all x in X. (monotonicity).

3. $I_N\,(A(x), B(x)) = I_N(B(x), A(x))$, for all x in X. (commutativity).

4. $I_N\,(A(x), I_N(B(x), C(x))) = I_N(I_N(A(x), B(x)), C(x))$, for all x in X. (associativity).(Wang, Smarandache, Zhang, and Sunderraman, 2005).

Definition 11 *Let U_N denote a **neutrosophic union** of two interval neutrosophic sets A and B. Then U_N is a function $U_N : N \times N \to N$*

and U_N must satisfy at least the following four axiomatic requirements:

1. $U_N(A(x), \underline{0}) = A(x)$, for all x in X. (boundary condition).

2. $B(x) \leq C(x)$ implies $U_N(A(x), B(x)) \leq U_N(A(x), C(x))$, for all x in X. (monotonicity).

3. $U_N(A(x), B(x)) = U_N(B(x), A(x))$, for all x in X. (commutativity).

4. $U_N(A(x), U_N(B(x), C(x))) = U_N(U_N(A(x), B(x)), C(x))$, for all x in X. (associativity). (Wang, Smarandache, Zhang, and Sunderraman, 2005).

3 Results

Theorem *Let A be an IFS in X, and $j(A)$ be the corresponding INS. We have that the complement of $j(A)$ is not necessarily $j(\overline{A})$.*

Proof. If $A =< x, \mu_A, \gamma_A >$ is $j(A) =< \mu_A, 0, \gamma_A >$.

Then ,

for $0_\sim =< x, 0, 1 >$ is $j(0_\sim) = j(< x, 0, 1 >) =< 0, 0, 1 > \neq \underline{0} =< 0, 1, 1 >$

for $1_\sim =< x, 1, 0 >$ is $j(1_\sim) = j(< x, 1, 0 >) =< 1, 0, 0 >= \underline{1}$

Thus, $1_\sim = \overline{0_\sim}$ and $j(\ 1_\sim) = \underline{1} \neq C_N(j(0_\sim))$ because $C_N\,(\underline{1}) = \underline{0} \neq j(0_\sim)$.

Definition 12 *Let's construct a **neutrosophic topology** on $NT =]^{-}0, 1^{+}[$, considering the associated family of standard or non-standard subsets included in NT, and the empty set which is closed under set union and finite intersection neutrosophic. The interval NT endowed with this topology forms a neutrosophic topological space. (Smarandache, 2002).*

Theorem 13 *Let (X, τ) be an intuitionistic fuzzy topological space. Then, the family of INSs $\{j(U) | U \in \tau\}$ is not necessarily a neutrosophic topology.*

Proof. Let $\tau = \{1_\sim, 0_\sim, A\}$ where $A =< x, 1/2, 1/2 >$ then $j(1_\sim) = \underline{1}$, $j(0_\sim) =< 0, 0, 1 > \neq \varnothing$ and $j(A) =< 1/2, 0, 1/2 >$. Thus $\{j(1_\sim), j(0_\sim), j(A)\}$ is not a neutrosophic topology, because the empty INS is not in this family.

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