Improved Fusion Machine Based on T-norm Operators for Robot Perception

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Abstract

Map reconstruction for autonomous mobile robots navigation needs to deal with uncertainties, imprecisions and even imperfections due to the limited sensors quality and knowledge acquisition. An improved fusion machine is proposed by replacing the classical conjunctive operator with T-norm operator in Dezert-Smarandache Theory (DSmT) framework for building grid map using noisy sonar measurements. An experiment using a Pioneer II mobile robot with 16 sonar detectors onboard is done in a small indoor environment, and a 2D Map is built online with our self-developing software platform. A quantitative comparison of the results of our new method for map reconstruction with respect to the classical fusion machine is proposed. We show how the new approach developed in this work outperforms the classical one.

Keywords: DSmT; Fuzzy sets theory; Information fusion; Robot perception.

1. Introduction

Since few years, there is a growing number of interests for intelligent mobile robots in several different fields of applications. For example, robot rovers have been developed (and are still under development and improvement) for planetary and space exploration, autonomous submersibles for submarines prospecting and surveying, mobile robots for automation purposes or for operation in hazardous mining environments, nuclear facilities and eliminating-explosion spot, and so on. All these applications require powerful and flexible perception systems, where the robot needs to perceive and even interacts coherently within its surrounding world using its onboard sensors. Therefore, map building in unknown environments is one of the principal issues in the field of intelligent mobile robot aside the robot command and control. However, information acquired in map building presents characteristics of uncertainty, imprecision and even high conflict, especially when building grid map using sonar sensors.

There exists already several methods for grid-map building based on different theories, such as the Probability Theory [3], the Fuzzy Sets Theory [7], DST (Dempster-Shafer Theory) [13], the Gray System Theory [15] and DSmT [4]. All these aforementioned methods have their own drawbacks and limitations for practical uses specially in real large and crowded environments. In this paper, we propose a new one based on a Fuzzy extension of the Improved Fusion Machine (denoted FIFM) using T-norm operators introduced in [14].

This paper is organized as follows: in section 2, FIFM is presented which the ESMS (Evidence Supporting Measure of Similarity) information filter and the fuzzy extension of the PCR5 (Proportional Conflict Redistribution rule no. 5) fusion rule [10]. In section 3, an experiment in a small indoor environment with a Pioneer II mobile robot is performed to show the advantage of our approach compared from the classical one by a quantitative analysis. Concluding remarks are then given in section 4.

2 Fuzzy-extended Improved Fusion Machine

2.1 General principle of FIFM

The general principle of the FIFM we propose consists in considering $s$ sources of evidences (i.e. the inputs) which are managed in the following way:

- at first, one selects a number $n \leq s$ of consistent sources among the $s$ sources through the ESMS information filter, i.e. only the sources being in a low conflict are selected for their combination,
- then, for each cell of the grid, one combines the selected sources at each measurement time-step by a given fusion rule (here the fuzzy extension of PCR5 rule) in order to get a generalized basic belief assignment (gbba) over a propositional space generated by
elements of a frame of discernment \( \Theta \) defined by

\[ \Theta = \{ \theta_1 = \text{cell empty}, \theta_2 = \text{cell occupied} \}. \]

- finally, from the result of the fusion obtained for each cell of the grid, one estimates the level of occupancy (or emptiness) of each cell of the grid in order to build online the grid-map to perceive and reconstruct the unknown environment with its obstacles.

In this work, we use the fuzzy T-norm operator combined with the efficient PCR5 fusion rule developed in DSmT framework. As soon as the combination of uncertain information is concerned, there exists a given amount of degree of conflict between sources, say \( K \), which needs to be managed efficiently by the fusion rule. The fusion machine includes the fusion rule itself but also some preprocessing tasks, like selection of consistent sources etc.

The way the conflict (total or partial) is managed is the key of the fusion and does really make the difference between the fusion machines. The fusion can be performed globally/optimally by combining the sources in one derivation step altogether, or sequentially (one source after another as in Fig.1). The sequential fusion processing (well adapted for temporal fusion) is more natural and simple than the global fusion in our application, but in general it remains only suboptimal if the fusion rule chosen is not associative, which is the case for most of fusion rules, but Dempster’s rule.

In this paper, the sequential fusion based on the fuzzy extension of PCR5 rule \([12]\) has been chosen because this rule has shown good performances in all works where it has already been tested so far \([4]\) and because the sequential fusion is much more simple to implement and to test. In the next section, we present the fuzzy extension of PCR5 rule and the implementation of the FIFM.

\[ \text{Figure 1. sequential FIFM} \]

### 2.2 Fuzzy extension of DSmT rules

Smarandache, Dezert and Tchamova did propose a fuzzy extension of DSmT fusion rules with T-norms \([10, 14]\). Since there are many functions which satisfy the four conditions of a T-norm, we just focus our works on the most common ones given in \([8]\) and listed below:

1. The Algebraic Product T-norm:
   \[ T_n-\text{Algebraic}(x, y) = x \cdot y. \]

2. The Bounded Product T-norm:
   \[ T_n-\text{Bounded}(x, y) = \max\{0, x + y - 1\}. \]

3. The Einstein Product T-norm:
   \[ T_n-\text{Einstein}(x, y) = \frac{xy}{1+(1-a)(1-b)}. \]

4. The Classical Fuzzy Product T-norm:
   \[ T_n-\text{Min}(x, y) = \min(x, y). \]

1. Fuzzy extension of DSm classic rule (DSmC) \([9]\)
   It is defined \( \forall A \in G^\Theta \setminus \{ \emptyset \} \) as follows:
   \[ m_{\text{fuzzDSmC}}(A) = \Gamma \{ \sum_{X,Y \in G^\Theta \atop X \cap Y = A} T_n(m_1(X)m_2(Y)) \} \]
   where, \( \Gamma \{ \cdot \} \) is the renormalization function, \( T_n(\cdot) \) is a T-norm operator aforementioned i.e. Algebraic Product, Bounded Product, Einstein Product, Classical Fuzzy Product. \( G^\Theta \) is the generic notation for the set on which the belief masses are defined (the power-set, hyper-power set or the super-power set) \([11]\).

2. Fuzzy extension of PCR5 rule \([11]\)
   It is defined by \( m_{\text{fuzzPCR5}}(\emptyset) = 0 \) and for all \( X \in G^\Theta \setminus \{ \emptyset \} \) by:
   \[ m_{\text{fuzzPCR5}}(X) = \Gamma \{ m_{12}(X) + \sum_{Y \in G^\Theta \setminus \{ \emptyset \} \atop X \cap Y = \emptyset} \frac{T_n(T_n(m_1(X), m_1(Y)), m_2(Y))}{m_1(X) + m_2(Y)} + \frac{T_n(T_n(m_2(X), m_2(Y)), m_1(Y))}{m_2(X) + m_1(Y)} \} \]
   where, \( m_{12}(\cdot) \) is the fuzzy extension of the conjunctive rule, i.e. \( m_{12}(\cdot) = \sum_{X,Y \in G^\Theta} T_n(m_1(X)m_2(Y)) \).

In DSmT, all propositions of \( G^\Theta \) are expressed in their canonical form, i.e. the disjunctive normal form which is a disjunction of conjunctions and is unique in Boolean algebra and simplest.
2.3 Implementation of the FIFM

The main steps of the algorithm for FIFM are:

1) Initialization: the number of sources of evidence \( s \) is set to zero and the number of evidence sources in the filter window is \( n = 0 \).

2) Include\(^1\) a source of evidence \( \hat{S}_s \) and then test if the number of sources \( s \) is less than 2. If \( s \geq 2 \), then go to next step, otherwise include/take into account another source of evidence \( \hat{S}_2 \).

3) Based on the barycenter of gbba’s of the front \( n \leq 10 \) evidence sources, the degree of similarity of the source \( n \) is computed according to the ESMS formula given in [5], and compared with a prior tuned threshold. If it is larger than the threshold, then one tests the next sensor \( (n = n + 1) \). Otherwise, we keep this new source of evidence \( \hat{S}_{s+1} \) for the fusion.

4) If \( n = 1 \), the current source, say \( S \), is not involved in the fusion. If \( n = 2 \), the fusion applies between \( S \) and \( \hat{S}_2 \) with the fuzzy DSmC rule (1). Then we use the fuzzy PCR5 rule (2) to redistribute the partial conflicts only to the sets involved in them. We commit the same index \( S \) to the combined source. If \( 2 < n \leq 10 \), after the current evidence source \( \hat{S}_s \) accepted by the filter is combined with the last combined results \( S \), a new evidence source is obtained and reassigned to \( S \). Whenever \( n \leq 10 \), go back to step 2). Otherwise, if the current evidence source \( \hat{S}_s \) under test is accepted by the ESMS filter, \( \hat{S}_i \) is assigned to \( \hat{S}_{i-1}, i \in [2, \cdots, 10] \), and \( \hat{S}_s \) is assigned to \( \hat{S}_{10} \). Then, \( \hat{S}_{10} \) is combined with the last source \( S \), the combined result is reassigned to \( S^2 \), and then, go back to step 2).

5) Stop test: if the robot receives new sonar’s data then introduce a new source of evidence \( \hat{S}_{s+1} \), otherwise stop and exit.

3. Experiment

The experiment is performed with a Pioneer II mobile robot carrying 16 sonar detectors and a laser range finder in our (indoor) laboratory environment. The size of the environment is \( 4550mm \times 3750mm \) and the grid map method is adopted. The environment is divided into \( 91 \times 75 \) rectangular cells having the same size. The robot starts to move from the location \( (1m, 0.6m) \), which faces towards 0 degree. We take the corner of left bottom as the global coordinate origin of the map. Some objects/obstacles in the rectangular grid map are shown in Fig. 2.

\(^1\)Shafer’s model is assumed for the frame \( \Theta \) and gbba’s are given.

\(^2\)In this work, we use also an ESMS filter window in a sliding mode.

All the steps of the FIFM for self-localization and grid map building have been implemented in our intelligent perception and fusion system developed as a specific Toolbox under C++ 6.0 and OpenGL. When the robot moves in the environment, the server-end collects a lot of information (i.e. the location of robot, sensors measurements, velocity, etc) from the mobile robot and its sensors onboard. Through the protocol of TCP/IP, the client-end can get any information from the server-end and then fuse them. Since our environment is small, the robot moves on a short distance during a rather small period of time. Therefore, one only considers the self-localization method based on \( \delta \)-NFAM method [6] with the search from \( \theta - \delta_\theta \) to \( \theta + \delta_\theta \) (\( \delta_\theta = 5^\circ \) in our experiment). To reduce the computation burden, the restricted spreading arithmetic has been used.

The main steps of this procedure are:

1) Initialize the parameters of the robot (location, velocity, etc.).

2) Obtain the 16 sonar measurements, and the robot’s location from odometer when the robot is moving in the environment. The robot’s pose is calibrated with a self-localization method. The period of the clock is 100 ms.

3) Compute the gbba’s of the fan-form area detected by each sonar sensor [6].

4) Apply the FIFM to choose a basic consistent evidence source according to the formula [5]. Then combine the consistent sources of evidence with fuzzy DSmC rule (1) (as an intermediary set) and then use the fuzzy PCR5 rule (2) (see section 2.2).

5) Compute the total belief of cell occupancy \( Bel(\theta_2) \) of some grids according to [9], after saving them into the map matrix, and go to next step.
6) Update the map of the environment. More scans we take, more accurate the final map reconstructed is. If during this process, the robot stops to receive measurements, then one stops the fusion and exit, otherwise, one goes back to step 2).

Figure 3. Map building with FIFM based on Algebraic Product operator.

Figure 4. Map building with FIFM based on Bounded Product operator.

Figure 5. Map building with FIFM based on Einstein Product operator.

Figure 6. Map building with FIFM based on Classical Fuzzy Product operator.

Figure 7. Map building with FIFM based on Classical Product operator.

For a fair quantitative comparison of the different measured results, we picked up the inner and outer borders of maps in order to compare them with the real environment depicted in the Figure 2. Because the borders are both incomplete and discrete, it is very difficult to pick up the borders outline directly, so we have considered the Mathematical Morphology approach [2]. Then the Shape Context method [1] was adopted to compare the difference between the picked-up outline and the real one by setting different resolutions \((i \times j)\) and radius length \(r_s\), where the Hausdorff distance is computed between the pixels. The compared results of the outlines with the real one are shown in the Table 1, where the outlines (inner and outer border) are measured by different T-norm operators. Due to space limitation constraint, we omit here some details of this procedure.

<table>
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<th></th>
<th>Alg</th>
<th>Bou</th>
<th>Ein</th>
<th>Min</th>
<th>Cla</th>
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<td>(GS)</td>
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<td>2</td>
<td>3</td>
<td>1</td>
<td>5</td>
</tr>
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</table>

Table 1. Quantitative comparison from the different T-norms.

where the following notation has been used: \(O \rightarrow \text{Outer}_A\), \(I \rightarrow \text{Inner}_A\), \(N \rightarrow \text{Noises}\), \(GS \rightarrow \text{GeneralSequence}\).
From the analysis of Table 1 and in our experiment, we see that the classical fuzzy product operator (Min) has the minimum Hausdorff distance. The Classical (Cla) and Algebraic Product operators (Alg) perform worst. The noises (pixels) with different maps between outer and inner outlines picked have also been computed and listed in the Table 1. We see also that Einstein Product operator (Ein) provides the least number of pixel noises, while the Classical Product operator (Cla) performs as the worst method with respect to the robustness to noises. The global order resulting from the quantitative analysis of performances based both on Hausdorff distance and noises is:

\[
\text{Min} \succ \text{Bou} \succ \text{Ein} \succ \text{Alg} \succ \text{Cla}.
\]

where, Min has the best performance, and Cla has the worst one.

In summary, the classical fuzzy product operator (Min) performs the best among all the T-norms operators and the classical one tested in this work.

4. Conclusions

In this paper, one has applied successfully a fuzzy extension of IFM based on T-norm operators for mobile robot’s map-building in an unknown environment with the help of a self-localization approach based on the δ-NFAM method. Based on a belief model for sonar grid map, an experiment has been conducted with a Pioneer II mobile robot moving in our small laboratory environment. For this study, we have also developed a human-computer interface of mobile robot for exploring unknown environments, for path planning and navigation. A quantitative comparative analysis of the results shows clearly the advantage of this approach with respect to the classical ones.

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References


