Neutrosophic Knowledge, Vol. 2, 2021
University of New Mexico

Article
A Study of the Neutrosophic Bimatrix

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Received: February 2021; *Accepted:* April 2021

Abstract: In this paper, the definition of neutrosophic bimatrix. The main objective is to define a neutrosophic bipolynomial. And to find neutrosophic biinverse for neutrosophic bimatrix. And finding the biinverse of the square neutrossophic bimatrix using the Kayley-Hamilton theorem, Also defining the eigenvalues of the neutrosophic bimatrix and diagonalizing the neutrosophic bimatrix.

Keywords: Neutrosophic Bimatrices, Neutrosophic Bivectores, Bimatrices integers.

1. Introduction

Neutrosophic logic. Neutrosophy, Neutrosophic set, Neutrosophic probability and a like, are recently creations of Smarandache F., being characterized by having the indeterminacy as component of their framework,and a notable feature of neutrosophic logic is that can be considered a generaliazation of fuzzy logics, encompassing the classical logic as well [1]. Also. F. Smarandache, has defined the bimatrix integers in year 2005 in [1], and laplace equation for bimatrix [1]. Moreover, he has defined the special types of bimatrix in [3]. Finally he introduced introduction to neutrosophic bimatrix in [2].

Among the recent applications there are: neutrosophic crisp set theory in image processing [6][7], neutrosophic sets medical field [8][9][10][11][12], in information geographic systems [13] and possible applications to database [14]. Also, neutrosophic triplet group application to physics [15]. Morever Several researches have made multiple contributions to neutrosophic topological [16][17][18][19][20][21][22], Finally More researches have made multiple contributions to neutrosophic analysis [23].

2. Preliminaries

In this paper $A_B = A_1 \cup A_2$ is called a bimatrix integers or a Neutrosophic bimatrix . Now, we recall some definitions which are useful in this paper.

Definition 2.1. [**1**][**2**]

A bimatrix $A_B$is defined as the union of two rectangular away of numbers A_1 and A_2 arranged into rows and columns. It is written as follows $A_B = A_1 \cup A_2$where $A_1 \neq A_2$with:

$$A_1 = \begin{bmatrix} a_{11}^1 & \cdots & a_{1n}^1 \\ \vdots & \ddots & \vdots \\ a_{m1}^1 & \cdots & a_{mn}^1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} a_{11}^2 & \cdots & a_{1n}^2 \\ \vdots & \ddots & \vdots \\ a_{m1}^2 & \cdots & a_{mn}^2 \end{bmatrix}$$

'∪' is just the notational convenience (symbol) only.

The above array is called a m by n bimatrix.

Definition 2.2. [1]

Let $A_B = A_1 \cup A_2$ be a bimatrix, If both A_1 and A_2 are $m \times n$ rectangular matrices then the bimatrix A_B is called the rectangular $m \times n$ bimatrix.

Definition 2.3. [1]

Let $A_B = A_1 \cup A_2$ be a bimatrix, If both A_1 and A_2 are square matrices then A_B is called the square bimatrix.

Definition 2.4. [2]

Let $A_B = A_1 \cup A_2$ be a bimatrix, If one of matrices in the bimatrix $A_B = A_1 \cup A_2$ is square and other is rectangular or if both A_1 and A_2 are rectangular matrices say $m_1 \times n_1$ and $m_2 \times n_2$ with $m_1 \neq m_2$ or $n_1 \neq n_2$ then we say A_B is a mixed bimatrix.

Example 2.5. [2] Let:

$$\boldsymbol{A_B = A_1 \cup A_2} = \begin{bmatrix} \mathbf{3} & \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{2} & \mathbf{1} \end{bmatrix} \cup \begin{bmatrix} \mathbf{0} & \mathbf{2} & \mathbf{-1} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \end{bmatrix}$$

Is a ectangular $\mathbf{2 \times 3}$ bimatrix.

Example 2.6. [2] Let:

$$\boldsymbol{A_B = A_1 \cup A_2} = \begin{bmatrix} \mathbf{3} & \mathbf{1} & \mathbf{1} \\ \mathbf{2} & \mathbf{-1} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} & \mathbf{0} \end{bmatrix} \cup \begin{bmatrix} \mathbf{0} & \mathbf{2} & \mathbf{0} \\ \mathbf{-1} & \mathbf{1} & \mathbf{0} \end{bmatrix}$$

Is a mixed bimatrix.

Example 2.7. [2] Let:

$$\boldsymbol{A_B = A_1 \cup A_2} = \begin{bmatrix} \mathbf{3} & \mathbf{0} & \mathbf{1} \\ \mathbf{2} & \mathbf{1} & \mathbf{1} \\ \mathbf{-1} & \mathbf{1} & \mathbf{0} \end{bmatrix} \cup \begin{bmatrix} \mathbf{4} & \mathbf{1} & \mathbf{1} \\ \mathbf{2} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

Is a square $\mathbf{3 \times 3}$ bimatrix.

Definition 2.8. [3]

Let $A_B = A_1 \cup A_2$ and $C_B = C_1 \cup C_2$ be any two $m \times n$ bimatrix. The sum D_B of the bimatrices A_B and C_B is defined as:

$$D_B = A_B + C_B = (A_1 \cup A_2) + (C_1 \cup C_2) = (A_1 + C_1) \cup (A_2 + C_2)$$

Definition 2.9. [4]

Let $A_B = A_1 \cup A_2$ and $C_B = C_1 \cup C_2$ be any two $m \times n$ bimatrix. Subtraction D_B of the bimatrices A_B and C_B is defined as:

$$D_B = A_B - C_B = A_B + (-C_B) = (A_1 \cup A_2) + (-C_1 \cup -C_2) = (A_1 - C_1) \cup (A_2 - C_2)$$

Definition 2.10. [5]

Let $A_B = A_1 \cup A_2$ and $C_B = C_1 \cup C_2$ be two $m \times n$ a square bimatrix. A product $D_B = A_B . C_B = (A_1 . C_1) \cup (A_2 . C_2)$ is bimatrix.

Example 2.11. [5] Let:

$$A_B = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 0 & 2 \end{bmatrix} \cup \begin{bmatrix} 4 & 0 & -1 \\ 0 & 1 & -2 \end{bmatrix} , C_B = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 2 & -1 \end{bmatrix} \cup \begin{bmatrix} 3 & 3 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Then:

$$A_B + C_B = \left\{\begin{bmatrix} 3 & 1 & 1 \\ -1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 2 & 2 & -1 \end{bmatrix}\right\} \cup \left\{\begin{bmatrix} 4 & 0 & -1 \\ 0 & 1 & -2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & 1 \\ 0 & 2 & -1 \end{bmatrix}\right\} = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix} \cup \begin{bmatrix} 7 & 3 & 0 \\ 0 & 3 & -3 \end{bmatrix}$$

Example 2.12. Let:

$$A_B = \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 3 \end{bmatrix} \cup \begin{bmatrix} 5 & -2 \\ 1 & 1 \\ 3 & -2 \end{bmatrix}, C_B = \begin{bmatrix} 8 & -1 \\ 4 & 2 \\ -1 & 3 \end{bmatrix} \cup \begin{bmatrix} 9 & 2 \\ 2 & 9 \\ -1 & 1 \end{bmatrix}$$

Then:

$$A_B - B_B = A_B + (-B_B) = \left\{\begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 3 \end{bmatrix} \cup \begin{bmatrix} 5 & -2 \\ 1 & 1 \\ 3 & -2 \end{bmatrix}\right\} + \left\{-\begin{bmatrix} 8 & -1 \\ 4 & 2 \\ -1 & 3 \end{bmatrix} \cup -\begin{bmatrix} 9 & 2 \\ 2 & 9 \\ -1 & 1 \end{bmatrix}\right\} = \begin{bmatrix} -5 & 2 \\ -5 & 0 \\ 1 & 0 \end{bmatrix} \cup \begin{bmatrix} -4 & -4 \\ -1 & -8 \\ 4 & -3 \end{bmatrix}$$

3. A Neutrosophic Bimatrix

Previous definitions can be reformulated by adding an indeterminacy.

Example 3.1. Let:

$$\boldsymbol{A_B = A_1 \cup A_2 = \begin{bmatrix} 3+I & 0 & 1 \\ -1 & 2I & I \end{bmatrix} \cup \begin{bmatrix} 0 & 2-I & -1 \\ 1+I & 1 & 0 \end{bmatrix}}$$

Is a neutrosophic rectangular $\mathbf{2 \times 3}$ bimatrix.

Example 3.2. Let:

$$\boldsymbol{A_B = A_1 \cup A_2 = \begin{bmatrix} 3I & 1 & 1 \\ 2 & I-1 & 1 \\ -1 & 0 & 0 \end{bmatrix} \cup \begin{bmatrix} 0 & 2I & 0 \\ -1 & I+1 & 0 \end{bmatrix}}$$

Is a neutrosophic mixed bimatrix.

Example 3.3. Let:

$$\boldsymbol{A_B = A_1 \cup A_2 = \begin{bmatrix} 3 & 0 & 1 \\ 2 & I & 1 \\ -1 & 1+I & 0 \end{bmatrix} \cup \begin{bmatrix} 4I & 1 & 1-I \\ 2 & 1 & 0 \\ 0 & 0 & I \end{bmatrix}}$$

Is a square $\mathbf{3 \times 3}$ neutrosophic bimatrix.

Note 3.4. $I^2 = I$, $1.I = I.1 = I$, $0.I = I.0 = 0$, where I is indeterminacy.

Example 3.5. Let:

$$A_B = \begin{bmatrix} -I & I & I-1 \\ -1 & 0 & 2I \end{bmatrix} \cup \begin{bmatrix} I+1 & 1 & 1 \\ I & I-1 & -I+2 \end{bmatrix}, C_B = \begin{bmatrix} -1 & 0 & I \\ 0 & 0 & -I \end{bmatrix} \cup \begin{bmatrix} I-3 & 3I & -1 \\ 1 & 0 & -I-1 \end{bmatrix}$$

Then:

$$A_B + C_B = \begin{bmatrix} -I & I & I-1 \\ -1 & 0 & 2I \end{bmatrix} + \begin{bmatrix} -1 & 0 & I \\ 0 & 0 & -I \end{bmatrix} \cup \begin{bmatrix} I+1 & 1 & 1 \\ I & I-1 & -I+2 \end{bmatrix} + \begin{bmatrix} I-3 & 3I & -1 \\ 1 & 0 & -I-1 \end{bmatrix}$$

$$= \begin{bmatrix} -I-1 & I & -1 \\ -1 & 0 & I \end{bmatrix} \cup \begin{bmatrix} 2I-2 & 3I+1 & 0 \\ I+1 & I-1 & -2I+1 \end{bmatrix}$$

Example 3.6. Let:

$$A_B = \begin{bmatrix} I & I-1 \\ 2I & -1 \\ 3 & 0 \end{bmatrix} \cup \begin{bmatrix} -5I & I-2 \\ I & -I \\ -I & I-2 \end{bmatrix}, C_B = \begin{bmatrix} 2I & I-1 \\ 3I-3 & -2I \\ I+2 & 0 \end{bmatrix} \cup \begin{bmatrix} 4I & -2I \\ 0 & 3I \\ -I & 0 \end{bmatrix}$$

Then:

$$A_B - B_B = \left\{\begin{bmatrix} I & I-1 \\ 2I & -1 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 2I & I-1 \\ 3I-3 & -2I \\ I+2 & 0 \end{bmatrix}\right\} \cup \left\{\begin{bmatrix} -5I & I-2 \\ I & -I \\ -I & I-2 \end{bmatrix} - \begin{bmatrix} 4I & -2I \\ 0 & 3I \\ -I & 0 \end{bmatrix}\right\}$$
$$= \begin{bmatrix} -I & 0 \\ -I & -2I-1 \\ -I+1 & 0 \end{bmatrix} \cup \begin{bmatrix} -9I & 3I-2 \\ I & -5I \\ 0 & I-2 \end{bmatrix}$$

Definition 3.7. Let $A_B = A_1 \cup A_2$ a neutrosophic bimatrix. The neutrosophic transpose of a bimatrix A_B is defined as:

$$A_B^T = (A_1 \cup A_2)^T = A_1^T \cup A_2^T$$

Example 3.8. Let:

$$A_B = \begin{bmatrix} -3+I & 0 & 1 \\ 1 & -I & 0 \end{bmatrix} \cup \begin{bmatrix} 1 & I & 1 \\ 1+I & 0 & -2 \end{bmatrix}$$

Then:

$$A_B^T = \begin{bmatrix} -3+I & 1 \\ 0 & -I \\ 1 & 0 \end{bmatrix} \cup \begin{bmatrix} 1 & 1+I \\ I & 0 \\ 1 & -2 \end{bmatrix}$$

Note 3.9. $(A_B + C_B)^T = A_B^T + C_B^T$.

Example 3.10. Let:

$$A_B = \begin{bmatrix} 3I & 1 & 1 \\ -1 & 0 & 2+I \end{bmatrix} \cup \begin{bmatrix} 4 & 1 & -I \\ I & 1 & 2 \end{bmatrix}$$

$$C_B = \begin{bmatrix} -I & 0 & 1-I \\ 2 & 2 & I-1 \end{bmatrix} \cup \begin{bmatrix} 3I & 0 & 1 \\ 0 & 2 & I-1 \end{bmatrix}$$

Then:

$$A_B + C_B = \begin{bmatrix} 3I & 1 & 1 \\ -1 & 0 & 2+I \end{bmatrix} + \begin{bmatrix} -I & 0 & 1-I \\ 2 & 2 & I-1 \end{bmatrix} \cup \begin{bmatrix} 4 & 1 & -I \\ I & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3I & 0 & 1 \\ 0 & 2 & I-1 \end{bmatrix}$$

$$A_B + C_B = \begin{bmatrix} 2I & 1 & 2-I \\ 1 & 2 & 1+2I \end{bmatrix} \cup \begin{bmatrix} 4+3I & 1 & 1-I \\ I & 3 & I+1 \end{bmatrix}$$

$$(A_B + C_B)^T = \begin{bmatrix} 2I & 1 \\ 1 & 2 \\ 2-I & 1+2I \end{bmatrix} \cup \begin{bmatrix} 4+3I & I \\ 1 & 3 \\ 1-I & I+1 \end{bmatrix}$$

$$A_B^T = (A_1 \cup A_2)^T = A_1^T \cup A_2^T = \begin{bmatrix} 3I & -1 \\ 1 & 0 \\ 1 & 2+I \end{bmatrix} \cup \begin{bmatrix} 4 & I \\ 1 & 1 \\ -I & 2 \end{bmatrix}$$

$$C_B^T = (C_1 \cup C_2)^T = C_1^T \cup C_2^T = \begin{bmatrix} -I & 2 \\ 0 & 2 \\ 1-I & I-1 \end{bmatrix} \cup \begin{bmatrix} 3I & 0 \\ 0 & 2 \\ 1 & I-1 \end{bmatrix}$$

$$A_B^T + C_B^T = \begin{bmatrix} 3I & -1 \\ 1 & 0 \\ 1 & 2+I \end{bmatrix} + \begin{bmatrix} -I & 2 \\ 0 & 2 \\ 1-I & I-1 \end{bmatrix} \cup \begin{bmatrix} 4 & I \\ 1 & 1 \\ -I & 2 \end{bmatrix} + \begin{bmatrix} 3I & 0 \\ 0 & 2 \\ 1 & I-1 \end{bmatrix} = \begin{bmatrix} 2I & 1 \\ 1 & 2 \\ 2-I & 1+2I \end{bmatrix} \cup \begin{bmatrix} 4+3I & I \\ 1 & 3 \\ 1-I & I+1 \end{bmatrix}$$

Then: $(A_B + C_B)^T = A_B^T + C_B^T$.

Definition 3.11. Let $A_B = A_1 \cup A_2$ be a neutrosophic square bimatrix. The bideterminant of a square bimatrix A_B is an ordered pair (d_1, d_2) where $d_1 = |A_1|$ and $d_2 = |A_2|$. $|A_B| = (d_1, d_2)$ where d_1 and d_2 are reals integers or reals neutrosophic may be positive or negative or even zero.

Example 3.12. Let:

$$A_B = A_1 \cup A_2 = \begin{bmatrix} 3I & 0 & 0 \\ 2 & I & 1 \\ 0 & 1 & -1 \end{bmatrix} \cup \begin{bmatrix} 4+I & 5 \\ -2I & 0 \end{bmatrix}$$

Then:

$d_1 = |A_1| = 3I\begin{vmatrix} I & 1 \\ 1 & -1 \end{vmatrix} - (0)\begin{vmatrix} 2 & 1 \\ 0 & -1 \end{vmatrix} + (0)\begin{vmatrix} 2 & I \\ 0 & 1 \end{vmatrix} = 3I(-I-1) = -3I^2 - 3I = -3I - 3I = -6I.$

$d_2 = |A_2| = \begin{vmatrix} 4+I & 5 \\ -2I & 0 \end{vmatrix} = (4+I)(0) - 5(-2I) = 10I.$

$$|A_B| = (d_1, d_2) = (-6I, 10I)$$

Note 3.13. $|A_B + C_B| \neq |A_B| + |C_B|$.

Example 3.14. Let:

$$A_B = A_1 \cup A_2 = \begin{bmatrix} I & 0 \\ -I & 2-I \end{bmatrix} \cup \begin{bmatrix} I-1 & I \\ I & 0 \end{bmatrix}$$

$$C_B = C_1 \cup C_2 = \begin{bmatrix} I+1 & 1 \\ -I & -I \end{bmatrix} \cup \begin{bmatrix} 3I & -I \\ 2I-5 & 0 \end{bmatrix}$$

Then:

$A_B + C_B = \begin{bmatrix} I & 0 \\ -I & 2-I \end{bmatrix} + \begin{bmatrix} I+1 & 1 \\ -I & -I \end{bmatrix} \cup \begin{bmatrix} I-1 & I \\ I & 0 \end{bmatrix} + \begin{bmatrix} 3I & -I \\ 2I-5 & 0 \end{bmatrix} = \begin{bmatrix} 2I+1 & 1 \\ -2I & 2-2I \end{bmatrix} \cup \begin{bmatrix} 4I-1 & 0 \\ I-5 & 0 \end{bmatrix}$

$d_1 = \begin{vmatrix} 2I+1 & 1 \\ -2I & 2-2I \end{vmatrix} = (2I+1)(2-2I) - 1(2I) = 4I - 4I^2 + 2 - 2I - 2I - 3I$

$= 4I - 4I + 2 - 2I - 2I - 3I = -I + 2.$

$d_2 = \begin{vmatrix} 4I-1 & 0 \\ I-5 & 0 \end{vmatrix} = (4I-1)(0) - (0)(I-5) = 0.$

$$|A_B + C_B| = (d_1, d_2) = (-I + 2, 0)$$

Now we have:

$|A_1| = \begin{vmatrix} I & 0 \\ -I & 2-I \end{vmatrix} = I(2-I) - (0)(-I) = 2I - I^2 = 2I - I = I.$

$|A_2| = \begin{vmatrix} I-1 & I \\ I & 0 \end{vmatrix} = (I-1)(0) - I(I) = 0 - I^2 = -I.$

$$|A_B| = (I, -I)$$

$|C_1| = \begin{vmatrix} I+1 & 1 \\ -I & -I \end{vmatrix} = (I+1)(-I) - (1)(-I) = -I^2 - I + I = -I^2 = -I.$

$|C_2| = \begin{vmatrix} 3I & -I \\ 2I-5 & 0 \end{vmatrix} = 3I(0) - I(2I-5) = 0 - 2I^2 + 5I = -2I + 5I = 3I.$

$$|C_B| = (-I, 3I)$$

Then:

$$|A_B| + |c_B| = (I, -I) + (-I, 3I) = (0, 2I)$$

We note: $(-I + 2, 0) \neq (0, 2I)$.

Then:

$$|A_B + C_B| \neq |A_B| + |C_B|$$

Definition 3.15. Let $A_B = A_1 \cup A_2$ a neutrosophic square bimatrix. The biinverse of A_B is written as:

$$A_B^{-1} = (A_1 \cup A_2)^{-1} = A_1^{-1} \cup A_2^{-1}$$

Note 3.16. $A_B^{-1}.A_B = A_B.A_B^{-1} = I_B$, And $A_B^{-1} = \frac{I}{A_B}$ or $A_B^{-1} = \frac{1}{A_B}$.

Note 3.17. $(A_B^{-1})^{-1} = A_B$.

Example 3.18. Let:

$$A_B = A_1 \cup A_2 = \begin{bmatrix} -3I & I \\ 2 & I \end{bmatrix} \cup \begin{bmatrix} 3I & 1 \\ 7 & 5I \end{bmatrix}$$

Then:

$$A_1^{-1} = \frac{1}{|A_1|}\begin{bmatrix} I & -I \\ -2 & -3I \end{bmatrix} = \frac{1}{-5I}\begin{bmatrix} I & -I \\ -2 & -3I \end{bmatrix} = \begin{bmatrix} \frac{I}{-5I} & \frac{-I}{-5I} \\ \frac{-2}{-5I} & \frac{-3I}{-5I} \end{bmatrix}$$

$$A_2^{-1} = \frac{1}{|A_2|}\begin{bmatrix} 5I & -1 \\ -7 & 3I \end{bmatrix} = \frac{1}{15I-7}\begin{bmatrix} 5I & -1 \\ -7 & 3I \end{bmatrix} = \begin{bmatrix} \frac{5I}{15I-7} & \frac{-1}{15I-7} \\ \frac{-7}{15I-7} & \frac{3I}{15I-7} \end{bmatrix}$$

Then:

$$A_B^{-1} = (A_1 \cup A_2)^{-1} = A_1^{-1} \cup A_2^{-1} = \begin{bmatrix} \frac{I}{-5I} & \frac{-I}{-5I} \\ \frac{-2}{-5I} & \frac{-3I}{-5I} \end{bmatrix} \cup \begin{bmatrix} \frac{5I}{15I-7} & \frac{-1}{15I-7} \\ \frac{-7}{15I-7} & \frac{3I}{15I-7} \end{bmatrix}$$

4. Neutrosophic bipolynomial characteristic of neutrosophic bimatrix.

In this section is given the definition of the neutrosophic bipolynomial characteristic of neutrosophic bimatrix, as well the operation of integration over it.

Defintion 4.1. Let $A_B = A_1 \cup A_2$ a square mixed bimatrix neutrosophic. Then we define the neutrosophic bipolynomial characteristic of neutrosophic bimatrix A_B such as:

$$\varphi(x) = \left(\varphi_1(x) \cup \varphi_2(x)\right) = (|xE_1 - A_1| \cup |xE_2 - A_2|)$$

Where E_1, E_2 are a neutrosophic Unitary matrixes

Example 4.2. Let

$$A_B = A_1 \cup A_2 = \begin{bmatrix} I & 2 & I-1 \\ 1 & 0 & 1 \\ 4 & -4 & 5I \end{bmatrix} \cup \begin{bmatrix} 1 & 2 \\ 3I & I-4 \end{bmatrix}$$

Then:

$$\varphi(x) = \left(\varphi_1(x) \cup \varphi_2(x)\right) = (|xE_1 - A_1| \cup |xE_2 - A_2|)$$

Now we have:

$$xE_1 - A_1 = x\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} I & 2 & I-1 \\ 1 & 0 & 1 \\ 4 & -4 & 5I \end{bmatrix} = \begin{bmatrix} x-I & -2 & -I+1 \\ -1 & x & -1 \\ -4 & 4 & x-5I \end{bmatrix}$$

$$\varphi_1(x) = |xE_1 - A_1| = \begin{vmatrix} x-I & -2 & -I+1 \\ -1 & x & -1 \\ -4 & 4 & x-5I \end{vmatrix} = (x-I)\begin{vmatrix} x & -1 \\ 4 & x-5I \end{vmatrix} + 2\begin{vmatrix} -1 & -1 \\ -4 & x-5I \end{vmatrix} + (-I+1)\begin{vmatrix} -1 & x \\ -4 & 4 \end{vmatrix}$$

$\varphi_1(x) = (x-I)(x^2 - 5Ix + 4) + 2(-x + 5I - 4) + (-I+1)(-4 + 4x)$

$\varphi_1(x) = x^3 - 5Ix^2 + 4x - Ix^2 + 5I^2x - 4I - 2x + 10I - 8 + 4I - 4Ix - 4 + 4x$

$\varphi_1(x) = x^3 - 6Ix^2 + (I+6)x + 10I - 12$

Now:

$$xE_2 - A_2 = x\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3I & I-4 \end{bmatrix} = \begin{vmatrix} x-1 & 2 \\ -3I & x-I+4 \end{vmatrix}$$

$$\varphi_2(x) = |xE_2 - A_2| = \begin{vmatrix} x-1 & 2 \\ -3I & x-I+4 \end{vmatrix} = (x-1)(x-I+4) - 2(-3I)$$

$\varphi_2(x) = x^2 - Ix + 4x - x + I - 4 - 6I = x^2 + (-I+3)x - 5I - 4$

$\varphi_2(x) = x^2 + (-I+3)x - 5I - 4$

$\varphi(x) = \big(\varphi_1(x) \cup \varphi_2(x)\big) = (x^3 - 6Ix^2 + (I+6)x + 10I - 12 \cup x^2 + (-I+3)x - 5I - 4)$

Theorem 4.3. The neutrosophic bipolynomial characteristic of neutrosophic bimatrix $A_B = A_1 \cup A_2$ is equal a neutrosophic bipolynomial characteristic of their transpose.

proof. Let $\varphi(x) = \big(\varphi_1(x) \cup \varphi_2(x)\big)$ is a neutrosophic bipolynomial characteristic of bimatrix neutrosophic $A_B = A_1 \cup A_2$, and let $\acute{\varphi}(x) = \big(\acute{\varphi}_1(x) \cup \acute{\varphi}_2(x)\big)$ is a neutrosophic bipolynomial characteristic of transpose $A_B^T = (A_1 \cup A_2)^T = A_1^T \cup A_2^T$. Then:

$$\varphi(x) = \big(\varphi_1(x) \cup \varphi_2(x)\big) = (|xE_1 - A_1| \cup |xE_2 - A_2|) = (|xE_1 - A_1| \cup |xE_2 - A_2|)^T$$

$$= (|xE_1^T - A_1^T| \cup |xE_2^T - A_2^T|) = \big(\acute{\varphi}_1(x) \cup \acute{\varphi}_2(x)\big) = \acute{\varphi}(x).$$

Theorem 4.4. Let $A_B = A_1 \cup A_2$ is a neutrosophic square bimatrix over field k. Then neutrosophic bipolynomial characteristic of neutrosophic bimatrix A_B is equal neutrosophic bipolynomial characteristic for any neutrosophic bimatrix similar of bimatrix A_B.

proof. Let $C_B = C_1 \cup C_2$ is a neutrosophic square bimatrix over field k similar to neutrosophic bimatrix $A_B = A_1 \cup A_2$. Then we have a neutrosophic square bimatrix $P_B = P_1 \cup P_2$ where:

$$C_B = C_1 \cup C_2 = [P_1^{-1} A_1 P_1] \cup [P_2^{-1} A_2 P_2]$$

Now let $\varphi(x) = \big(\varphi_1(x) \cup \varphi_2(x)\big)$ is a neutrosophic bipolynomial characteristic of a neutrosophic square bimatrix $A_B = A_1 \cup A_2$, and let $\psi(x) = \big(\psi_1(x) \cup \psi_2(x)\big)$ is a neutrosophic bipolynomial characteristic of a neutrosophic square bimatrix $C_B = C_1 \cup C_2$. Then:

$$\psi(x) = \big(\psi_1(x) \cup \psi_2(x)\big) = (|xE_1 - C_1| \cup |xE_2 - C_2|) = (|xP_1^{-1}P_1 - C_1| \cup |xP_2^{-1}P_2 - C_2|)$$
$$= (|xP_1^{-1}P_1 - P_1^{-1}A_1P_1| \cup |xP_2^{-1}P_2 - P_2^{-1}A_2P_2|)$$
$$= (|P_1^{-1}(xE_1 - A_1)P_1| \cup |P_2^{-1}(xE_2 - A_2)P_2|)$$
$$= |P_1^{-1}||xE_1 - A_1||P_1| \cup |P_2^{-1}||xE_2 - A_2||P_2| = (|xE_1 - A_1| \cup |xE_2 - A_2|)$$
$$= \big(\varphi_1(x) \cup \varphi_2(x)\big) = \varphi(x)$$

Definition 4.5 . Let $f(x) = f_1(x) \cup f_2(x)$ is a neutrosophic bipolynomial characteristic over biring $K[x]$ where

$$f_1(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$
$$f_2(x) = b_n x^n + b_{n-1} x^{n-1} + \cdots + b_1 x + b_0$$

And let $A_B = A_1 \cup A_2$ is a neutrosophic square bimatrix over bifield k. Then:

$$f(A_B) = f_1(A_1) \cup f_2(A_2)$$
$$= \big(a_n {A_1}^n + a_{n-1} {A_1}^{n-1} + \cdots + a_1 A_1 + a_0\big) \cup \big(b_n {A_2}^n + b_{n-1} {A_2}^{n-1} + \cdots + b_1 A_2 + b_0\big)$$

We call $f(A_B)$ is a value neutrosophic bipolynomial characteristic for $x = A_B$.

Theorem 4.6.(Kayley-Hamilton theorem). Any neutrosophic square bimatrix (or mixed square) is root of this neutrosophic bipolynomial characteristic.

5. Biinverse neutrosophic of a bimatrix neutrosophic by using Kayley-Hamilton theorem.

Let $A_B = A_1 \cup A_2$ is a neutrosophic square bimatrix (or mixed square), and let $\varphi(x) = \big(\varphi_1(x) \cup \varphi_2(x)\big)$ is a neutrosophic bipolynomial characteristic of A_B.

Method of solution.

We have by theorem 4.6 :

$\varphi(A_B) = \big(\varphi_1(A_1) \cup \varphi_2(A_2)\big) = (0_1 \cup 0_2)$

$$\varphi(A_B) = \left({A_1}^n + a_{n-1}{A_1}^{n-1} + \cdots + a_1A_1 + a_0\right) \cup \left({A_2}^n + b_{n-1}{A_2}^{n-1} + \cdots + b_1A_2 + b_0\right) = (0_1 \cup 0_2)$$

Then:

$$\begin{cases} {A_1}^n + a_{n-1}{A_1}^{n-1} + \cdots + a_1A_1 + a_0 = 0 \\ {A_2}^n + b_{n-1}{A_2}^{n-1} + \cdots + b_1A_2 + b_0 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} {A_1}^n + a_{n-1}{A_1}^{n-1} + \cdots + a_1A_1 = -a_0I_1 \\ {A_2}^n + b_{n-1}{A_2}^{n-1} + \cdots + b_1A_2 = -b_0I_2 \end{cases}$$

$$\Rightarrow \begin{cases} \left[{A_1}^{n-1} + a_{n-1}{A_1}^{n-2} + \cdots + a_1\right]A_1 = -a_0I_1 \\ \left[{A_2}^{n-1} + b_{n-1}{A_2}^{n-2} + \cdots + b_1\right]A_2 = -b_0I_2 \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{-1}{a_0}\left[{A_1}^{n-1} + a_{n-1}{A_1}^{n-2} + \cdots + a_1\right] = A_1^{-1} \\ \dfrac{-1}{b_0}\left[{A_2}^{n-1} + b_{n-1}{A_2}^{n-2} + \cdots + b_1\right] = A_2^{-1} \end{cases}$$

Then:

$$A_B^{-1} = A_1^{-1} \cup A_2^{-1}$$

Example 5.1. Find neutrosophic biinverse of neutrosophic bimatrix A_B by using Kayley-Hamilton theorem where:

$$A_B = A_1 \cup A_2 = \begin{bmatrix} I & 1 & 2 \\ 3 & 1 & I \\ 2I & 3 & 1 \end{bmatrix} \cup \begin{bmatrix} I & 2 \\ 3I & I \end{bmatrix}$$

Solution.

$$\varphi(x) = \left(\varphi_1(x) \cup \varphi_2(x)\right) = (|xE_1 - A_1| \cup |xE_2 - A_2|)$$

$$\varphi(x) = \begin{vmatrix} x-I & -1 & -2 \\ -3 & x-1 & -I \\ -2I & -3 & x-1 \end{vmatrix} \cup \begin{vmatrix} x-I & -2 \\ -3I & x-I \end{vmatrix}$$

$$\varphi(x) = (x^3 + (-I-2)x^2 + (-5I-2)x + 4I - 15 \cup x^2 - 2Ix - 5I)$$

Now we have depending on theorem (4.6):

$$\varphi(A_B) = \left(\varphi_1(A_1) \cup \varphi_2(A_2)\right) = (0_1 \cup 0_2)$$

Then:

$$\left({A_1}^3 + (-I-2){A_1}^2 + (-5I-2)A_1 + (4I-15)I_1 \cup {A_2}^2 - 2IA_2 - 5II_2\right) = (0_1 \cup 0_2)$$

$$\Rightarrow \begin{cases} {A_1}^3 + (-I-2){A_1}^2 + (-5I-2)A_1 + (4I-15)I_1 = 0 \\ {A_2}^2 - 2IA_2 - 5II_2 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} {A_1}^3 + (-I-2){A_1}^2 + (-5I-2)A_1 = -(4I-15)I_1 \\ {A_2}^2 - 2IA_2 = 5II_2 \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{1}{-(4I-15)}\left[{A_1}^2 + (-I-2)A_1 + (-5I-2)I_1\right] = A_1^{-1} \\ \dfrac{1}{5I}\left[A_2 - 2II_2\right] = A_2^{-1} \end{cases}$$

Now we have:

$${A_1}^2 = A_1A_1 = \begin{bmatrix} I & 1 & 2 \\ 3 & 1 & I \\ 2I & 3 & 1 \end{bmatrix}\begin{bmatrix} I & 1 & 2 \\ 3 & 1 & I \\ 2I & 3 & 1 \end{bmatrix} = \begin{bmatrix} I^2+3+4I & I+1+6 & 2I+I+2 \\ 3I+3+2I^2 & 3+1+3I & 6+I+I \\ 2I^2+9+2I & 2I+3+3 & 4I+3I+1 \end{bmatrix}$$

$$= \begin{bmatrix} 5I+3 & I+7 & 3I+2 \\ 5I+3 & 3I+4 & 2I+6 \\ 4I+9 & 2I+6 & 7I+1 \end{bmatrix}$$

$$(-I-2)A_1 = (-I-2)\begin{bmatrix} I & 1 & 2 \\ 3 & 1 & I \\ 2I & 3 & 1 \end{bmatrix} = \begin{bmatrix} -I^2-2I & -I-2 & -2I-4 \\ -3I-6 & -I-2 & -I^2-2I \\ -2I^2-4I & -3I-6 & -I-2 \end{bmatrix}$$

$$= \begin{bmatrix} -3I & -I-2 & -2I-4 \\ -3I-6 & -I-2 & -3I \\ -6I & -3I-6 & -I-2 \end{bmatrix}$$

$$(-5I-2)I_1 = (-5I-2)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5I-2 & 0 & 0 \\ 0 & -5I-2 & 0 \\ 0 & 0 & -5I-2 \end{bmatrix}$$

Then:

$$A_1^{-1} = \frac{1}{-(4I-15)}\begin{bmatrix} -3I+1 & 5 & I-2 \\ 2I-3 & -3I & -I+6 \\ -2I+9 & -I & I-3 \end{bmatrix}$$

Now we have:

$$A_2 - 2II_2 = \begin{bmatrix} I & 2 \\ 3I & I \end{bmatrix} - 2I\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} I & 2 \\ 3I & I \end{bmatrix} - \begin{bmatrix} 2I & 0 \\ 0 & 2I \end{bmatrix} = \begin{bmatrix} -I & 2 \\ 3I & -I \end{bmatrix}$$

Then:

$$A_2^{-1} = \frac{1}{5I}\begin{bmatrix} -I & 2 \\ 3I & -I \end{bmatrix}$$

Then:

$$A_B^{-1} = A_1^{-1} \cup A_2^{-1} = \frac{1}{-(4I-15)}\begin{bmatrix} -3I+1 & 5 & I-2 \\ 2I-3 & -3I & -I+6 \\ -2I+9 & -I & I-3 \end{bmatrix} \cup \frac{1}{5I}\begin{bmatrix} -I & 2 \\ 3I & -I \end{bmatrix}$$

6. Conclusion

In this paper, a new type of neutrosophic bimatrix has been defined, Moreover, we studied a neutrosophic square bimatrix and a neutrosophic rectangular bimatrix and find a addition, Subtraction, and product for tow neutrosophic square bimatrix (mixed square), Kayley-Hamilton theorem by neutrosophic. Also defintion of other types of neutrosophic bimatrix can be found such as a neutrosophic symmatric bimatrix, a neutrosophic skew symmatric bimatrix, and a neutrosophic Hermitian bimatrix, and a neutrosophic diagonatizable bimatrix transformation. We will work on this in the future.

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