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Article

Some Topics Related Neutrosophic Fuzzy Ideal Bitopological Spaces

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Abstract: The aim of this paper is to introduce and study some new neutrosophic fuzzy pairwise notian via neutrosophic fuzzy ideals. In addition to generalize the concept of NFPL-continuity and NFPL-open functions. Relationships between the above new neutrosophic fuzzy pairwise notions and there other relevant neutrosophic fuzzy classes are investigated.

Keywords: Neutrosophic set; Neutrosophic topology; Neutrosophic ideal; Neutrosophic ideal open set; Neutrosophic closed set.

Introduction

The neutrosophic set was introduced by Smarandache in [3,4,7] and Salama introduced the neutrosophic crisp set neutrosophic topological spaces and many applications in computer science and system [5,6,8], information systems. The fundamental concepts of neutrosophic set, introduced by Smarandache in 2002 [3, 4] and Salama in [5-14], provides a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics. neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts [1, 2, 3,4,5,7], such as a neutrosophic set theory, in this paper is to introduce and study some new neutrosophic fuzzy pairwise notion via neutrosophic fuzzy pairwise ideals. We, also generalize the notion of crisp PL-open sets due to Abd El-Monsef, et. al [1,2]. In addition to generalize the concept of crisp PL-closed sets, NPL-continuity and NPL-open functions. A neutrosophic fuzzy quasi-pairwise L-openness and neutrosophic fuzzy quasi-pairwise L-continuity are considered as a generalization of a fuzzy PL-openness and fuzzy PL-continuity which are known before.

1. Terminologies

We recollect some relevant basic preliminaries, in the following references [5-14].

2. Neutrosophic Fuzzy pairwise L-continuous Functions.

By utilizing the notion of NFPL-open sets, we establish in this article a class of neutrosophic fuzzy NFPL-continuous function which contained in the class of neutrosophic fuzzy pairwise pre-continuous function.

Each of neutrosophic fuzzy PL-continuous and neutrosophic fuzzy pairwise continuous function are independent concepts. Many characterizations and properties of this concept are investigated.

Definition 2.1: A Neutrosophic fuzzy pairwise function $f:(X,\tau_i)\longrightarrow(Y,T), i\in\{1,2\}$ with neutrosophic fuzzy ideal L on X is said to be neutrosophic fuzzy PL-continuous if for every $<\xi,\rho,\theta>$ in T, $f^{-1}(<\xi,\rho,\theta>)$ in NPLO(X).

Remark 2.1: Every NFPL-continuity is neutrosophic fuzzy pairwise precontinuity but the converse may be not true in general as seen by the following example.

Example 2.1: Let X=Y={x}, $\tau_i, i\in\{1,2\}$ may be neutrosophic fuzzy pairwise indiscrete bitopological , σ may be neutrosophic fuzzy pairwise discrete bitopological and L=$\{0_N,,<\mu,\sigma,u>\}\vee\{x_{\varepsilon=<\alpha,\gamma,\beta>}:\varepsilon\le X_{<0.2,0.5,0.8>}\}$ $<\mu,\sigma,u>(x)=<0.2,0.5,0.8>$. The neutrosophic fuzzy pairwise identity function $f:(X,\tau_i)\longrightarrow(Y,T), i\in\{1,2\}$ may be neutrosophic fuzzy pairwise precontinuous but not neutrosophic fuzzy NPL-continuous, since $<\mu,\sigma,u>$ in T while $f^{-1}(<\mu,\sigma,u>)\notin$ NPLO(X).

Theorem 2.1: For a function $f:(X,\tau_i)\longrightarrow(Y,T), i\in\{1,2\}$ with neutrosophic fuzzy ideal L on X the following are equivalent

(i.) f may be neutrosophic fuzzy NFPL-continuous.

(ii.) For $x_{\varepsilon=<\alpha,\gamma,\beta>}$ in X and each $<\xi,\rho,\theta>$ inT containing $f(x_{\varepsilon=<\alpha,\gamma,\beta>})$, there exists $<\mu,\sigma,u>$ in NPLO(X) containing $x_{\varepsilon=<\alpha,\gamma,\beta>}$ such that $(<\mu,\sigma,u>)\le$ T.

(iii.) For each neutrosophic fuzzy pairwise point $x_{\varepsilon=<\alpha,\gamma,\beta>}$ in X and $<\xi,\rho,\theta>$ in T containing $f(x_{\varepsilon=<\alpha,\gamma,\beta>})$, $(f^{-1}(\zeta))^*$ may be neutrosophic fuzzy pairwise npbd of x_ε.

(iv.) The inverse image of each neutrosophic fuzzy pairwise closed set in Y may be neutrosophic fuzzy NPL-closed.

Proof: (i.)⟶(ii.). Since $<\xi,\rho,\theta>$ in T containing $f(x_{\varepsilon=<\alpha,\gamma,\beta>})$, then by (i), $f^{-1}(<\xi,\rho,\theta>)$ in NPLO(X), by putting $<\mu,\sigma,u>=f^{-1}(<\xi,\rho,\theta>)$ which containing $x_{\varepsilon=<\alpha,\gamma,\beta>}$, we have $f(<\mu,\sigma,u>)$ in T (ii.) ⟶(iii.). Let $<\xi,\rho,\theta>$ in T containing $f(x_{\varepsilon=<\alpha,\gamma,\beta>})$. Then by (ii) there exists $<\mu,\sigma,u>$ in NFPLO(X) containing $x_{\varepsilon=<\alpha,\gamma,\beta>}$ such that $(<\mu,\sigma,u>)\le\sigma$, so $x_{\varepsilon=<\alpha,\gamma,\beta>}$ in $<\mu,\sigma,u>\le \mathrm{Nint}<\mu,\sigma,u>^*\le \mathrm{Nint}(f^{-1}(<\xi,\rho,\theta>))^*\le (f^{-1}(<\xi,\rho,\theta>))^*$. Hence $(f^{-1}(<\xi,\rho,\theta>))^*$ may be neutrosophic fuzzy npbd of $x_{\varepsilon=<\alpha,\gamma,\beta>}$.

(iii.)→(i.) Let $<\xi,\rho,\theta>$ in T, since $(f^{-1}(<\xi,\rho,\theta>))$ may be neutrosophic fuzzy pairwise npbd of any point $f^{-1}(<\xi,\rho,\theta>)$, every point $x_{\varepsilon=<\alpha,\gamma,\beta>}$ in $(f^{-1}(<\xi,\rho,\theta>))^*$ may be a neutrosophic fuzzy pairwise interior point of $f^{-1}(<\xi,\rho,\theta>)^*$ .Then $f^{-1}(<\xi,\rho,\theta>)\le \mathrm{Nint}(f^{-1}(\zeta<\xi,\rho,\theta>))^*$ and hence f may be neutrosophic fuzzy pairwise NPL-continuous.

(i.) →(iv.) Let $<\xi,\rho,\theta>$ in T be a neutrosophic fuzzy pairwise closed set. Then $<\xi,\rho,\theta>^c$ may be neutrosophic fuzzy pairwise open set, by $f^{-1}(<\xi,\rho,\theta>^c)=f^{-1}(<\xi,\rho,\theta>)^c$ in NPLO(X). Thus $f^{-1}(<\xi,\rho,\theta>)$ may be neutrosophic fuzzy pairwise NFPL-closed set.

The following theorem establish the relationship between neutrosophic fuzzy pairwise NPL-continuous and neutrosophic fuzzy pairwise continuous by using the previous neutrosophic fuzzy pairwise notions.

Theorem 2.2: Given$f:(X,\tau_i)\longrightarrow(Y,T), i\in\{1,2\}$ may be a function with neutrosophic fuzzy ideal L on X then we have. If f may be neutrosophic fuzzy pairwise NPL-continuous of each neutrosophic fuzzy pairwise*-perfect set in X, then f may be neutrosophic fuzzy pairwise continuous.

Proof: Obvious.

Corollary 2.1: Given a function $f:(X,\tau_i)\longrightarrow(Y,T), i\in\{1,2\}$ and each member of X may be neutrosophic fuzzy pairwise*-dense-in-itself.

Then we have

(i.) Every neutrosophic fuzzy pairwise continuous function may be neutrosophic fuzzy pairwise NFPL-continuous.

(ii.) Each of neutrosophic fuzzy pairwise precontinuous function and neutrosophic fuzzy pairwise NFPL-continuous are equivalent.

Proof: It's clear.

3. Neutrosopic Fuzzy quasi pairwise NFPL-open and Neutrosophic Fuzzy quasi pairwise NPL-continuity.

Definition 3.1: In a nfbts $(X,\tau_i), i\epsilon\{1,2\}$ with neutrosophic fuzzy ideal L on X, μ,σ,u in I^X is said to be neutrosophic fuzzy quasi pairwise NPL-open if $<\mu,\sigma,u>\leq Ncl(Nint(<\mu,\sigma,u>^*))$, $<\mu,\sigma,u>^c$ may be called neutrosophic fuzzy quasi pairwise NPL-closed set. The collection of all neutrosophic fuzzy quasi pairwise NFPL-open sets of $(X,\tau_i), i\epsilon\{1,2\}$ will denoted by $NFQPLO(X,\tau_i), i\epsilon\{1,2\}$.

The connection between neutrosophic fuzzy quasi pairwise NFPL-openness with some other corresponding types have been given throughout the following implication

NPL-open ⟶ N quasi NPL-open

Proposition 3.1: Arbitrary union of neutrosophic fuzzy quasi pairwise NPL-open sets may be neutrosophic fuzzy quasi pairwise NPL-open.

Proof: let $(X,\tau_i), i\epsilon\{1,2\}$ a nfbts with neutrosophic fuzzy ideal L on X and $<\mu,\sigma,u>_J$ in $NQPLO(X)$ this means that for each for $i\in N$, $<\mu,\sigma,u>_J\leq Ncl\big(Nint(<\mu,\sigma,u>^*)\big)$ and so, $\bigvee_{J\in HN}<\mu,\sigma,u>_J\leq$

$$\bigvee_{J\in HN} Ncl\left(Nint\left(<\mu,\sigma,u>^*_J\right)\right)\leq Ncl\left(\vee\left(Nint(<\mu,\sigma,u>^*)\right)\right)\leq Ncl(Nint(\vee<\mu,\sigma,u>)^*).$$

Hence $\bigvee_{J\in HN}<\mu,\sigma,u>_J\in NQPLO(X)$.

Above two results are useful to obtained the following theorem.

Theorem 3.1: For a NFTS $(X,\tau_i), i\epsilon\{1,2\}$ with neutrosophic fuzzy ideal L_n, the class NFQPLO(X) forms a neutrosophic fuzzy pairwise suprabitopological.

Proof: Follows by the fact $0^*_N=0_N$ and both of the fact $1_{N=<\eta,\delta,\theta>}=1^*_{N=<\eta,\delta,\theta>}$, and proposition 3.1.

Remark 3.1: A finite neutrosophic fuzzy intersection pairwise of neutrosophic fuzzy quasi pairwise NPL-open may be neutrosophic fuzzy quasi pairwise NPL-open.

Theorem 3.2: $NFQPLO(X,\tau_i), i\epsilon\{1,2\}$ from a neutrosophic fuzzy bitopological.

Proof: Follows directly from Theorem 3.1 and Remark 3.1.

Theorem 3.3: For a NFTS $(X,\tau_i), i\epsilon\{1,2\}$ with neutrosophic fuzzy ideal L on X. The following statements are verified.

(i) If L=$\{0_{N=<\eta,\delta,\theta>}\}$ then $NFQPLO(X,\tau_i)=NP\beta O(X,\tau_i), i\epsilon\{1,2\}$.

(ii) If L=$\{I^X\}$ then $NQPLO(X,\tau_i)=NPLO(X,\tau_i), i\epsilon\{1,2\}$.

(iii) If L neutrosophic fuzzy ideal on X, each neutrosophic fuzzy quasi pairwise NPL-open(resp. neutrosophic fuzzy semi pairwise open) which it is neutrosophic fuzzy pairwise-closed (resp.PN*F-dense – in itself) may be neutrosophic fuzzy semi pairwise open (resp. neutrosophic fuzzy quasi pairwise NFPL-open).

Proof: Obvious.

Theorem 3.4: In a NFTS $(X, \tau_i), i\epsilon\{1,2\}$ with neutrosophic fuzzy ideal L_n on X, if $< \mu, \sigma, u >$ in $NQPLO(X, \tau_i), i\epsilon\{1,2\}$, then it may be neutrosophic fuzzy semi pairwise open.

Hence we can deduce that an neutrosophic fuzzy quasi pairwise NPL-open set which is neutrosophic fuzzy pairwise*-closed for any $(X,\tau_i), i\epsilon\{1,2\}$ with neutrosophic fuzzy ideal L may be equivalent with the neutrosophic fuzzy quasi pairwise NPL-openness in $(X, \tau_i), i\epsilon\{1,2\}$ with neutrosophic fuzzy ideal L_n which may be useful to obtain the following.

Proposition 3.2: In a nfbts $(X,\tau_i), i\epsilon\{1,2\}$ with neutrosophic fuzzy ideal L on X, any neutrosophic fuzzy pairwise preclosed set μ, σ, u in $I^{x=\eta,\delta,\theta}$ is neutrosophic fuzzy pairwise regular closed if one of the following hold:

(i) $< \mu, \sigma, u >$ may be PN*F-closed and neutrosophic fuzzy quasi pairwise NFPL-open.

(ii) $< \mu, \sigma, u >$ in $NQPLO(X, \tau_i), i\epsilon\{1,2\}$ with neutrosophic fuzzy ideal L_n.

Definition 3.2: A function $f:(X, \tau_i) \longrightarrow (Y, T), i \in \{1,2\}$ with neutrosophic fuzzy ideal Lon X may be called neutrosophic fuzzy quasi pairwise NPL-continuous if for every μ, σ, u in T, $f^{-1}(< \mu, \sigma, u >)$in $NFQPLO(X, \tau_i), i \in \{1,2\}$.

The relationships between this new class of functions and some types of known continuous ones are obtained as follows.

NFPL-continuity ⟶ NF quasi pairwise PL-continuity

Proposition 3.3: The following equivalents are verify

(i) $f : (X, \tau_i) \longrightarrow (Y, T), i \in \{1,2\}$ with neutrosophic fuzzy deal L= $\{0_{N=<\eta,\delta,\theta>}\}$, may be neutrosophic fuzzy quasi pairwise NFPL-continuous iff it may be neutrosophic fuzzy pairwise NFPβ-continuous.

(ii) $f:(X, \tau_i) \longrightarrow (Y, T), i \in \{1,2\}$ with neutrosophic fuzzy ideal L={I^x}, may be neutrosophic fuzzy quasi pairwise NFPL-continuous iff it may be NFPL-continuous.

Theorem 5.3: For a function $f:(X, \tau_i) \longrightarrow (Y, T), i \in \{1,2\}$ with neutrosophic fuzzy ideal L on X, the following are equivalent:

(i.) f may be neutrosophic fuzzy quasi pairwise NFPL-continuouis.

(ii.) The inverse image of each neutrosophic fuzzy pairwise closed set in (Y, T) may be neutrosophic fuzzy quasi pairwise NFPL-closed.

(iii.) For each x in X and each μ, σ, u in T containing f(x). There exists $< \lambda, \omega, \kappa >$ in $NFQPLO(X, \tau_i), i \in \{1,2\}$ containing x such tha $f(< \lambda, \omega, \kappa >) << \mu, \sigma, u >$.

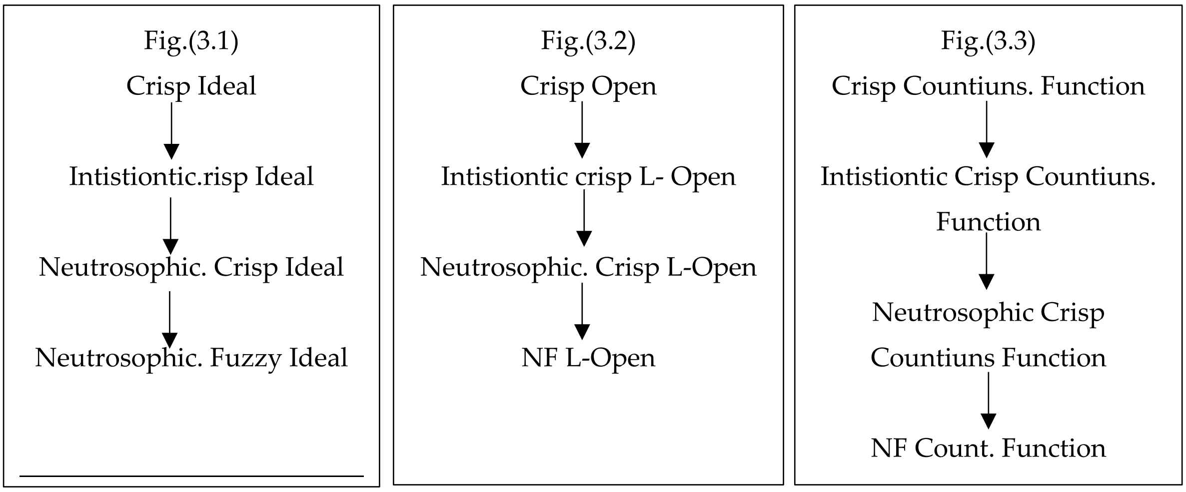
Proposition 3.4: For a function $f:(X, \tau_i) \longrightarrow (Y, T), i \in \{1,2\}$ with neutrosophic fuzzy ideal L on X, the following are true.

(i.) A neutrosophic fuzzy quasi pairwise NFP L_n - continuous function may be neutrosophic fuzzy pairwise semi continuous.

(ii) A neutrosophic fuzzy quasi pairwise NFPL-continuous (resp. Neutrosophic fuzzy pairwise semi-continuous) and for each μ, σ, u in T, $f^{-1}(< \mu, \sigma, u >)$ may be

PN*F-closed (resp.PN*dense-in-itself) then f may be neutrosophic fuzzy pairwise semi continuous (resp. Neutrosophic fuzzy pairwise NFPL-continuous).

Relations: The following Graph represent the relation between the concepts



4. Conclusions

The notions of the sets and functions in neutrosophic fuzzy bitopological spaces are highly developed and several characterizations have already been obtained. These are used extensively in many practical and engineering problems, computational bitopology for geometric design, computer-aided geometric design, engineering design research, Geographic Information System (GIS) and mathematical sciences.in this paper may turn out to be useful in quantum physics. Several characterizations of neutrosophic fuzzy sets and several generalizations of neutrosophic fuzzy continuous functions may also lead to some interesting indepth analytical study and research from the view point of neutrosophic mathematics.

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