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Article

n-Cyclic Refined Neutrosophic Vector Spaces and Matrices

Malath. F. Aswad [1,*]

[1] Faculty of science,, PhD,of mathematices,GaziAntep University, Turkey; Malaz.aswad@yahoo.com

* Correspondence: Malaz.aswad@yahoo.com

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Abstract: This paper is dedicated to study for the first time the concept of n-cyclic refined neutrosophic vector space as a direct application of n-cyclic refined neutrosophic sets. Also, It presents some elementary properties of these spaces such as homomorphisms and subspaces. On the other hand, this work defines n-cyclic refined neutrosophic real matrices, and illustrates some examples to clarify these structures.

Keywords: n-cyclic refined neutrosophic vector space, n-cyclic refined neutrosophic ring, n-cyclic refined neutrosophic matrix

1. Introduction

Neutrosophy is a new branch of philosophy which concerns with the indeterminacy in real life actions and sciences. The Neutrosophic is a new view of Modeling , designed to effectively deal underlying doubts in the real world, as it came to replace binary logic that recognized right and wrong by introducing a third neutral case which could be interpreted as non-specific or uncertain. Founded by Florentin Smarandache [6], he presented it in 1999 as a generalization of fuzzy logic. As an extension of this, A. A. Salama introduced the Neutrosophic crisp sets Theory as a generalization of crisp sets theory [53] and developed, inserted and formulated new concepts in the fields of mathematics, statistics, computer science and information systems through neutrosophics [53-56]. In the literature, neutrosophy has got many applications in pure mathematics areas such as space theory [1,2], module theory [4,5], matrix theory [31,32,42], and number theory [3,35]. Also, it plays an important role in applied mathematics such as equations [30], special elements [41], and topology [27,29].

n-cyclic refined neutrosophic sets were defined in [39], and used in the study of some related rings and modules. These sets are considered as a new kind of n-refined neutrosophic sets [12], with a similar structure and different operations. In this work, we define the concept of n-cyclic refined neutrosophic vector spaces and n-cyclic refined neutrosophic matrices. Also, we illustrate many examples to clarify the validity of these concepts, and we list some of related open questions.

2. n-Cyclic Refined neutrosophic vector space.

Definition 2.1 [39]

Let $(\boldsymbol{R},+,\times)$ be a ring and $\boldsymbol{I_k}; \boldsymbol{1 \leq k \leq n}$ be $\boldsymbol{n}$ indeterminacies. We define $\boldsymbol{R_n(I)} = \{\boldsymbol{a_0 + a_1 I_1 + a_2 I_2 + \cdots + a_n I_n; a_i \in R}\}$ to be n-cyclic refined neutrosophic ring.

Operations on $\boldsymbol{R_n(I)}$ are defined as:

$\boldsymbol{\sum_{i=0}^{n} x_i I_i + \sum_{i=0}^{n} y_i I_i = \sum_{i=0}^{n}(x_i + y_i) I_i}$, $\boldsymbol{\sum_{i=0}^{n} x_i I_i \times \sum_{i=0}^{n} y_i I_i = \sum_{i,j=0}^{n}(x_i \times y_i) I_i\, I_j = \sum_{i,j=0}^{n}(x_i \times y_i) I_{(i+j mod n)}}$

Where $\times$ is the multiplication on the ring $\boldsymbol{R}$, and $\boldsymbol{xI_0 = x}$, for all $\boldsymbol{x \in R}$.

Definition 2.2 [39]

Let $(K,+,\times)$ be a field, we say that $K_n(I) = K + KI_1 + \cdots + KI_n = \{a_0 + a_1 I_1 + a_2 I_2 + \cdots + a_n I_n; a_i \in K\}$ is a n-cyclic refined neutrosophic field.

Definition 2.3

Let $(V,+,\times)$ be any vector space over a field K. Then we say that $V_n(I) = V + VI_1 + \cdots + VI_n = \{x_0 + x_1 I_1 + \cdots + x_n I_n; x_i \in V\}$is a weak n-cyclic refined neutrosophic vector space over the field K.Elements of $V_n(I)$are called n-cyclic refined neutrosophic vectors, elements of $K$are called scalars.

If we take scalars from the n-cyclic refined neutrosophic field $K_n(I)$, we say that $V_n(I)$ is a strong n-cyclic refined neutrosophic vector space over thea n-cyclic refined neutrosophic field $K_n(I)$. Elements of $K_n(I)$n-cyclic refined neutrosophic scalars.

Remark 2.1. Multiplication by an n-cyclic refined neutrosophic scalar $m = \sum_{i=0}^{n} m_i I_i \in k_n(I)$is defined as:

$$\left(\sum_{i=0}^{n} m_i I_i\right) \times \left(\sum_{i=0}^{n} a_i I_i\right) = \sum_{i,j=0}^{n} (m_i a_j) I_i I_j$$

Where $a_i \in V,\ m_i \in K,\ I_i I_j = \boldsymbol{I_{(i+j mod n)}}$.

Definition 2.5

Let $V_n(I)$be a weak n-cyclic refined neutrosophic vector space over the n-cyclic refined neutrosophic field K; a nonempty $W_n(I)$ is called a weak n-cyclic refined neutrosophic vector subspaceof $V_n(I)$ if $W_n(I)$ is a subspace of $V_n(I)$ itself.

Definition 2.6

Let $V_n(I)$ be a strong n-cyclic refined neutrosophic vector space over then-cyclic refined neutrosophic field $K_n(I)$. A nonempty subset $W_n(I)$ is called a strong n-cyclic refined neutrosophic vector submodule of $V_n(I)$ if $W_n(I)$ is a submodule of $V_n(I)$ itself.

Theorem 2.1

Let $V_n(I)$ be a weak n-cyclic refined neutrosophic vector space over the n-cyclic refined neutrosophic field$K$, $W_n(I)$ be a nonempty subset of $V_n(I)$. Then $W_n(I)$ is a weak n-cyclic refined neutrosophic subspace if only if:

$x + y \in W_n(I), m \times x \in W_n(I)$ for all $x, y \in W_n(I),\ m \in K$.

proof:

it holds directly from the condition of subspace.

Definition 2.7

Let $V_n(I)$ be a weak n-cyclic refined neutrosophic vector space over the field K, x be an arbitrary element of $V_n(I)$, we say that x is a linear combination of $\{x_1, x_2, \dots, x_m\} \subseteq V_n(I)$ if $x = (a_1 \times x_1) + (a_2 \times x_2) + \cdots + (a_m \times x_m)$: $a_i \in K(I), x_i \in V_n(I)$.

Definition 2.8

Let $V_n(I)$ be a strong n-cyclic refined neutrosophic vector space over the n-cyclic refined neutrosophic field $K_n(I)$, x be an arbitrary element of $V_n(I)$, we say that x is a linear combination of $\{x_1, x_2, \dots, x_m\} \subseteq V_n(I)$if $x = (a_1 \times x_1) + (a_2 \times x_2) + \cdots + (a_m \times x_m)$: $a_i \in K_n(I), x_i \in V_n(I)$.

Definition 2.9

Let $X = \{x_1, x_2, \dots, x_m\}$ be a subset of a weak n-cyclic refined neutrosophic vector space $V_n(I)$ over the field K, X is a weak linearly independent set if $\sum_{i=0}^{n} a_i \times x_i = 0$ implies $a_i = 0$; $a_i \in K$.

Definition 2.10

Let $X = \{x_1, x_2, \dots, x_m\}$ be a subset of a strong n-cyclic refined neutrosophic vector space $V_n(I)$ over the n-cyclic refined neutrosophic field $K_n(I)$, X is a weak linearly independent set if $\sum_{i=0}^{n} a_i \times x_i = 0$ implies $a_i = 0$; $a_i \in K_n(I)$.

Definition 2.11

Let $V_n(I), W_n(I)$ be two strong n-cyclic refined neutrosophic vector space over the n-cyclic refined neutrosophic field $K_n(I)$, let $f: V_n(I) \to U_n(I)$ be a well defined map. It is called a strong n-cyclic refined neutrosophic homomorphism if:

$f((a \times x) + (b \times y)) = a \times f(x) + b \times f(y)$ for all $x, y \in V_n(I), a, b \in K_n(I)$.

A weak n-cyclic refined neutrosophic homomorphism can be defined as the same.

Definition 2.12

Let $f: V_n(I) \to U_n(I)$ be a weak/strong n-cyclic refined neutrosophic homomorphism, we define:

(a) $Ker(f) = \{x \in V_n(I); f(x) = 0\}$.

(b) $Im(f) = \{y \in U_n(I);\ \exists x \in V_n(I) \text{and } y = f(x)\}$.

Theorem 2.2

Let $f: V_n(I) \to U_n(I)$ be a weak n-cyclic refined neutrosophic homomorphism. Then

(a) $Ker(f)$ is a weak n-cyclic refined neutrosophic subspace of V_n(I).

(b) $Im(f)$ is a weak nn-cyclic refined neutrosophic subspace of $U_n(I)$.

Proof:

(a) f is a vector space homomorphism since $V_n(I), U_n(I)$ are vector spaces, hence $Ker(f)$ is a subspace of the vector space $V_n(I)$, thus $Ker(f)$ is a weak n- cyclic refined neutrosophic subspace of V_n(I).

(b) It hold by similar argument.

Theorem 2.3

Let $\boldsymbol{f}: \boldsymbol{V_n}(\boldsymbol{I}) \to \boldsymbol{U_n}(\boldsymbol{I})$ be a strong n-cyclic refined neutrosophic homomorphism. Then

(a) $\boldsymbol{Ker}(\boldsymbol{f})$ is a strong n- cyclic refined neutrosophic subspace of $\boldsymbol{V_n}$(I).

(b) $\boldsymbol{Im}(\boldsymbol{f})$ is a strong n- cyclic refined neutrosophic subspace of $\boldsymbol{U_n}(\boldsymbol{I})$.

Proof:

$(\boldsymbol{a})\ \boldsymbol{f}$ is a module homomorphism since $\boldsymbol{V_n}(\boldsymbol{I}), \boldsymbol{U_n}(\boldsymbol{I})$ are modules over the n-cyclic refined neutrosophic field $\boldsymbol{K_n}(\boldsymbol{I})$, hence $\boldsymbol{Ker}(\boldsymbol{f})$ is a submodule of the module $\boldsymbol{V_n}(\boldsymbol{I})$, thus $\boldsymbol{Ker}(\boldsymbol{f})$is a strong n- cyclic refined neutrosophic subspace of $\boldsymbol{V_n}$(I).

(b) Holds by similar argument.

Definition 2.13 n-cyclic refined neutrosophic matrix

Let $\boldsymbol{A_{m\times n}} = \{(\boldsymbol{a_{ij}}) : \boldsymbol{a_{ij}} \in \boldsymbol{K_n}(\boldsymbol{I})\}$, where $\boldsymbol{K_n}(\boldsymbol{I})$ is a n-cyclic refined neutrosophic field. We call to be the n-cyclic refined neutrosophic matrix.

Definition 2.14 n-cyclic refined neutrosophic square matrix

Let $\boldsymbol{A_{m\times n}}$ is a neutrosophic matrix. We call to be the n-cyclic refinedneutrosophic square matrix if $\boldsymbol{m} = \boldsymbol{n}$.

Example 2.1

Let $\boldsymbol{n} = \boldsymbol{3}$, then

$$\boldsymbol{A} = \begin{pmatrix} 1 + I_1 + 2I_2 - I_3 & 2 - I_1 - 2I_3 & -1 + I_1 + I_2 + I_3 \\ I_2 + I_3 & 3 + I_1 + 2I_2 & I_1 + I_2 + I_3 \\ 1 - I_1 - I_3 & 4 - I_1 + I_2 - I_3 & -2 - I_1 + 3I_2 - I_3 \end{pmatrix}$$

$\boldsymbol{A}$ is a 3-cyclic refined neutrosophic square matrix.

$\boldsymbol{A}$ can be written as:

$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 0 \\ 1 & 4 & -2 \end{pmatrix} + \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & -1 \end{pmatrix} I_1 + \begin{pmatrix} 2 & 0 & 1 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix} I_2 + \begin{pmatrix} -1 & -2 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & -1 \end{pmatrix} I_3$$

Example 2.2

Let $\boldsymbol{n} = \boldsymbol{4}$, then

$$\boldsymbol{A} = \begin{pmatrix} -I_1 + I_2 - I_4 & 1 + I_1 - I_3 + I_4 & 1 + 2I_1 + I_2 + I_3 - I_4 \\ 3 - I_2 + 2I_3 - 3I_4 & -2 + I_1 + I_2 + I_4 & 2 - I_1 - I_2 + I_3 \\ 1 - I_1 - I_3 - 2I_4 & 5 + 3I_1 - I_2 - I_3 + 2I_4 & I_1 - 3I_2 - I_3 + I_4 \end{pmatrix}$$

$\boldsymbol{A}$ is a 4-cyclic refined neutrosophic square matrix.

$\boldsymbol{A}$ can be written as

$$A=\begin{pmatrix}0&1&1\\3&-2&2\\1&4&-2\end{pmatrix}+\begin{pmatrix}-1&-1&1\\0&1&-1\\-1&3&1\end{pmatrix}I_1+\begin{pmatrix}1&0&1\\-1&1&-1\\0&-1&-3\end{pmatrix}I_2+\begin{pmatrix}0&-1&1\\2&0&1\\-1&-1&-1\end{pmatrix}I_3+\begin{pmatrix}-1&1&-1\\-3&1&0\\-2&2&1\end{pmatrix}I_4$$

Example 2.3: Multiplication of n-cyclic refined neutrosophic square matrix

Let $A=A_0+A_1I_1+A_2I_2+A_3I_3$, $B=B_0+B_1I_1+B_2I_2+B_3I_3$ are two 3-cyclic refined neutrosophic square matrixes, where

$$A=\begin{pmatrix}1&0\\1&1\end{pmatrix}+\begin{pmatrix}-2&1\\3&2\end{pmatrix}I_1+\begin{pmatrix}1&1\\-1&-2\end{pmatrix}I_2+\begin{pmatrix}0&2\\3&0\end{pmatrix}I_3\ ,\quad B=\begin{pmatrix}1&1\\0&1\end{pmatrix}+\begin{pmatrix}-1&-2\\3&0\end{pmatrix}I_1+\begin{pmatrix}4&2\\3&0\end{pmatrix}I_2+\begin{pmatrix}2&-2\\1&1\end{pmatrix}I_3$$

Then we have.

$$A\times B=A_0B_0+A_0B_1I_1+A_0B_2I_2+A_0B_3I_3+A_1B_0I_1+A_1B_1I_1+A_1B_2I_1I_2+A_1B_3I_1I_3+A_2B_0I_2+A_2B_1I_2I_1+A_2B_2I_2I_2+A_2B_3I_2I_3+A_3B_0I_3+A_3B_1I_3I_1+A_3B_2I_3I_2+A_3B_3I_3I_3$$

$$\begin{aligned}A\times B=&\begin{pmatrix}1&0\\1&1\end{pmatrix}\begin{pmatrix}1&1\\0&1\end{pmatrix}+\begin{pmatrix}1&0\\1&1\end{pmatrix}\begin{pmatrix}-1&-2\\3&0\end{pmatrix}I_1+\begin{pmatrix}1&0\\1&1\end{pmatrix}\begin{pmatrix}4&2\\3&0\end{pmatrix}I_2+\begin{pmatrix}1&0\\1&1\end{pmatrix}\begin{pmatrix}2&-2\\1&1\end{pmatrix}I_3\\&+\begin{pmatrix}-2&1\\3&2\end{pmatrix}\begin{pmatrix}1&1\\0&1\end{pmatrix}I_1+\begin{pmatrix}-2&1\\3&2\end{pmatrix}\begin{pmatrix}-1&-2\\3&0\end{pmatrix}I_1I_1+\begin{pmatrix}-2&1\\3&2\end{pmatrix}\begin{pmatrix}4&2\\3&0\end{pmatrix}I_1I_2\\&+\begin{pmatrix}-2&1\\3&2\end{pmatrix}\begin{pmatrix}2&-2\\1&1\end{pmatrix}I_1I_3+\begin{pmatrix}1&1\\-1&-2\end{pmatrix}\begin{pmatrix}1&1\\0&1\end{pmatrix}I_2+\begin{pmatrix}1&1\\-1&-2\end{pmatrix}\begin{pmatrix}-1&-2\\3&0\end{pmatrix}I_2I_1\\&+\begin{pmatrix}1&1\\-1&-2\end{pmatrix}\begin{pmatrix}4&2\\3&0\end{pmatrix}I_2I_2+\begin{pmatrix}1&1\\-1&-2\end{pmatrix}\begin{pmatrix}2&-2\\1&1\end{pmatrix}I_2I_3+\begin{pmatrix}0&2\\3&0\end{pmatrix}\begin{pmatrix}1&1\\0&1\end{pmatrix}I_3\\&+\begin{pmatrix}0&2\\3&0\end{pmatrix}\begin{pmatrix}-1&-2\\3&0\end{pmatrix}I_3I_1+\begin{pmatrix}0&2\\3&0\end{pmatrix}\begin{pmatrix}4&2\\3&0\end{pmatrix}I_3I_2+\begin{pmatrix}0&2\\3&0\end{pmatrix}\begin{pmatrix}2&-2\\1&1\end{pmatrix}I_3I_3\end{aligned}$$

Now, we have in 3-cyclic refined neutrosophic ring

$$I_1I_1=I_1, I_2I_1=I_1I_2=I_3, I_1I_3=I_3I_1=I_1, I_2I_2=I_1,\ I_2I_3=I_3I_2=I_2,\ I_3I_3=I_3.$$

Thus.

$$\begin{aligned}A\times B=&\begin{pmatrix}1&1\\1&2\end{pmatrix}+\begin{pmatrix}-1&-2\\2&-2\end{pmatrix}I_1+\begin{pmatrix}4&2\\7&2\end{pmatrix}I_2+\begin{pmatrix}2&-2\\3&-1\end{pmatrix}I_3+\begin{pmatrix}-2&-1\\3&5\end{pmatrix}I_1+\begin{pmatrix}5&4\\3&-6\end{pmatrix}I_1\\&+\begin{pmatrix}5&-4\\18&6\end{pmatrix}I_3+\begin{pmatrix}-3&5\\8&-4\end{pmatrix}I_1+\begin{pmatrix}1&2\\-1&-3\end{pmatrix}I_2+\begin{pmatrix}2&-2\\-5&2\end{pmatrix}I_3+\begin{pmatrix}7&2\\-10&-2\end{pmatrix}I_1\\&+\begin{pmatrix}1&2\\-4&0\end{pmatrix}I_2+\begin{pmatrix}0&2\\6&-6\end{pmatrix}I_3+\begin{pmatrix}6&0\\-3&-6\end{pmatrix}I_1+\begin{pmatrix}6&0\\12&6\end{pmatrix}I_2+\begin{pmatrix}2&2\\6&-6\end{pmatrix}I_3\end{aligned}$$

$$A\times B=\begin{pmatrix}1&1\\1&2\end{pmatrix}+\begin{pmatrix}12&-1\\3&-15\end{pmatrix}I_1+\begin{pmatrix}12&6\\14&5\end{pmatrix}I_2+\begin{pmatrix}11&-4\\28&-5\end{pmatrix}I_3$$

Example 2.4: Addition on n-cyclic refined neutrosophic rings

Let $A=\begin{pmatrix}2I_1-I_2+3I_3&1+I_1+2I_2\\-3-2I_1+4I_2+I_3&I_1+I_3\end{pmatrix}$, $B=\begin{pmatrix}I_2-2I_3&-2+I_1+I_2+I_3\\1+I_1-I_3&1-I_1-4I_2+I_3\end{pmatrix}$

Hence,

$$A+B=\begin{pmatrix}2I_1-I_2+3I_3&1+I_1+2I_2\\-3-2I_1+4I_2+I_3&I_1+I_3\end{pmatrix}+\begin{pmatrix}I_2-2I_3&-2+I_1+I_2+I_3\\1+I_1-I_3&1-I_1-4I_2+I_3\end{pmatrix}$$

$$A+B=\begin{pmatrix}2I_1+3I_3&-1+2I_1+3I_2+I_3\\-2-I_1+4I_2&1-4I_2+2I_3\end{pmatrix}.$$

5. Conclusions

In this paper, we have defined for the first time the concept of n-cyclic refined neutrosophic vector space, and n-cyclic refined neutrosophic real matrices. Also, we have presented some of their elementary properties and illustrated many examples to clarify the validity of our work.

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