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Article

A Multi-objective Transportation Data Problems and their Based on Fuzzy Random Variables

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Abstract: This paper is specialized with fuzzy random multi-objective unbalanced transportation data problems by using of fuzzy programming technique when the sources and destination parameters are fuzzy random variables in inequality type of constraints. In this paper, we focus on the solution procedure of the specified transportation data problems where the objective functions are minimization type and supplies and demands are replaced by the fuzzy random variables. We first convert the fuzzy random multi-objective unbalanced transportation problem into deterministic problem by using fuzzy random chance constraints approach. By introducing the concept of linear membership function of fuzzy programming, multi-objective deterministic transportation problem is converted into single objective deterministic problem and then we solve it and we obtain the optimal compromise solution. Lastly a numerical example is provided for illustration the methodology.

Keywords: Fuzzy Programming, Fuzzy Random chance constrained programming, Stochastic Transportation Data Problems, Multi-objective Decision Making.

1. Introduction

A linear programming problem is said to be probabilistic linear programming (PLP) [23] problem if one or more of the parameters is known only by its probability distribution. These problems can be solved by one of the following principal approaches: (i) Expected value model (EVM), which optimizes expected objective function subject to some expected constraints (see, Sengpta [24] ; Liu [18]), (ii) Linear programming under uncertainty which, in some special cases, is called two stages programming under uncertainty. The two-stage approach was inutility presented Dantizing [10], and (iii) Chance-constrained programming (CCP) developed by Charnce and Cooper [7, 8, 9], (CCP) offers a powerful means of modeling stochastic decision systems with the assumption that the stochastic constraints will hold at least the $100(\alpha)\%$ of time.

Stancu-Minasian and Wets [25] discussed different stochastic multi objective programming problems; the chebyshev's problem, the stochastic goal programming problem, the fractional programming problem and the multiple minimum-risk problem. Armstrong and Balintfly [2] studied the stochastic linear VOP by using the disjoint chance-constrained approach to solve the problem in case of the left hand side parameters are independent random variables normally distributed. Stancu-Minasian [26] solved a stochastic linear multi objective programming problem with random parameters in the objectives. A general review of stochastic multi objective programming problems could be found in references [16, 23]. The fuzzy mathematical programming can be classified into three categories in view of the kinds of uncertainties treated in the method. The fuzzy mathematical programming in the first category was initially developed by Bellman and zadeh [3]. Tanaka and Asai [24] and Zimmerman [29, 30]. It treats decision making problem under fuzzy goals and constraints. The fuzzy goals and constraints represent the flexibility of the target values of objective functions and the elasticity of constraints.

The second category in fuzzy mathematical programming treats ambiguous coefficients of objective functions and constraints. Dubois and Prad [11] treated systems of linear equations with ambiguous coefficients suggesting the possible application to fuzzy mathematical programming for the first time. This kind of programming is called possibilistic programming that has been approached by many authors in the literature such as Dubois [11,12, 13], Buckley [6], and others.

The last type of fuzzy mathematical programming treats ambiguous coefficient as well as vague decision maker's For optimization problems with fuzzy random information, we need fuzzy random programming to model them. Some fuzzy random linear programming with single objective has been discussed by several researchers, see, e.g., Wang and Qiao [28]. They incorporated fuzzy random variable coefficients in linear programming, within the "here and now" and the "wait and see" philosophies. Luhandjula and Gupta [20] described an approach for solving a linear program with fuzzy random. Liu [18,19] presented a new concept of chance of fuzzy random events and then constructs a general framework of fuzzy random chance-constrained program (CCP). Abo El-Kheir [1] presented a new concept of fuzzy random chance constrained linear programming when fuzzy number is symmetric and random variables are normal (by using LR fuzzy number).

The transportation problem is an earliest application of linear programming problem. Hitchcock [15] was first developed the basis concept of transportation problem and later discussed in detailed by Koopmans [17]. In 1973, Appa [3] considered variants of the transportation in which all constraints involving the supply and demand are of inequality type. However, he has not considered the supply and demand constraints are of mixed type. Brigden [5] extended the concept of Appa [3] and considered the mixed type constraints. Then the original problem is converted into a related transportation problem with equality type of constraints by augmenting the original problem with the addition of two sources and two destinations. He obtained the optimal solution of the original problem from the optimal solution of transformed transportation problem. Mahaptra, Roy and Biswal [21] considered the fuzzy programming technique to stochastic multi-objective unbalanced transportation problem when the sources and the destination parameters are random variables.

1.1 Multi-objective Transportation Problem (MOTP).

Consider m origin (or supply) O_i $(i = 1,2,3,\dots,m)$ and n destination (or demand) $D_j(j = 1,2,3,\dots,m)$. The sources may be production facilities and they are characterized by available supplies $a_1, a_2, a_3, \dots, a_m$ The destination may be public destination center and they are characterized by demand level $b_1, b_2, b_3, \dots, b_n$. A penalty c_{ij} is transportation cost or time cost, associated from origin i to the destination j and the variables $x_{ij}, i = 1,2,3,\dots,m, j = 1,2,3,\dots,n$ are represented the unknown quantity goods to be transported from origin O_i to destination D_j .

The single objective transportation problem can be extended to multi-

Objective transportation problem by considering the k-th (k= 1, 2, 3,…,K) cost coefficients c_{ij}^k (k= 1, 2, 3,…,K) in the objective functions. Then the mathematical model of multi-objective transportation problem can be represented as follows:

Model: $$min\, z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij} \qquad \text{k= 1, 2, 3,…,K} \tag{1}$$

$$\sum_{j=1}^{n} x_{ij} = a_i, i = 1,2,3,..., m \quad (2)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1,2,3,..., n \quad (3)$$

$$x_{ij} \geq 0, i = 1,2,3,..., m, j = 1,2,3,..., n \quad (4)$$

The balanced transportation problem is defined when the total availability at supply point is equal to the total requirement at demand point with an equilibrium condition ($\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$) for the existence of a feasible solution.

2. Mathematical Model Involving Fuzzy Random Multi-Objective Transportation Problem (FRMOTP).

Here, we have presented the mathematical model of fuzzy random multi-objective transportation problem (FRMOTP) as follows:

$$\textbf{Mode2:} \quad \min z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} \qquad k= 1, 2, 3,\ldots,K \quad (5)$$

$$\sum_{j=1}^{n} x_{ij} = a_i(\omega), i = 1,2,3,..., m \quad (6)$$

$$\sum_{i=1}^{m} x_{ij} = b_j(\omega), j = 1,2,3,..., n \quad (7)$$

$$x_{ij} \quad , i = 1,2,3,..., m \quad , j = 1,2,3,..., n \quad (8)$$

Where: $a_i(\omega)$ is an m- vector of fuzzy random coefficients,

$b_j(\omega)$ is an n- vector of fuzzy random coefficients.

2.1 LR Fuzzy Random Variables.

$b_{LR} = (m, l, r,)_{LR}$ (see [1]) is said to be fuzzy random variables if (m, l, r) are random variables, and this is rather convenient representation to model "numbers approximately to random variables.

(i) The fuzzy random vector, $b_j(\omega),\ j = 1,..., n$ in (7) can be represented as follows:

$$b_j(\omega) = m_j + r_j b_{jR}(\alpha) - l_j b_{jL}(\alpha), \qquad j = 1,\ldots,n, \quad (9)$$

where

$$b_{jL}(\alpha) = \frac{1}{L_j(\alpha)}, \qquad j = 1,\ldots,n \quad (10)$$

$$b_{jR}(\alpha) = \frac{1}{R_j(\alpha)}, \qquad j = 1,\ldots,n$$

The linearity of the expectation leads to

$$E(\ b_j(\omega)) = m_j + r_j b_{jR}(\alpha) - l_j b_{jL}(\alpha), \quad j = 1,\ldots,n$$

If m, r, l are independent then from theorem 2(see Abo El-Kheir [1]) we have

$$V(b_j(\omega)) = V(m_j) + \|R\|_2^2 V(r_j) - \|L\|_2^2 V(l_j).$$

For a symmetric triangular fuzzy number suppose the following

$$L_j(x) = 1 - x, \quad x > 0, \quad j = 1, \ldots, n, \tag{11}$$
$$R_j(x) = 1 - x, \quad x < 0, \quad j = 1, \ldots, n,$$

The α - Level set of $L_j(x)$ and $R_j(x)$ can be written as follows:

$$L_j(x) = R_j(x) = 1 - x \quad \geq \alpha, \qquad 0 \leq \alpha \quad < 1. \tag{12}$$

For equation (9) (m_j, r_j, l_j) $j = 1, \ldots, m$ are independent normal distributions, see Abo El-Kheir [1].

ii) The fuzzy random vector $a_i(\omega)$, $i = 1, \ldots, m$ can be represented as follows:

$$a_i(\omega) = w_i + q_i a_{iR}(\alpha) - s a_{iL}(\alpha), \qquad i = 1, \ldots, m, \tag{13}$$

where

$$a_{iL}(\alpha) = \frac{1}{L_i(\alpha)}, \qquad i = 1, \ldots, m, \tag{14}$$
$$a_{iR}(\alpha) = \frac{1}{R_i(\alpha)}, \qquad i = 1, \ldots, m.$$

For a symmetric triangular fuzzy number suppose the following

$$L_i(x) = 1 - x, \quad x > 0, \qquad i = 1, \ldots, m,$$
$$R_i(x) = 1 - x, \quad x < 0, \qquad i = 1, \ldots, m.$$

The α - Level set of $L_i(x)$ and $R_i(x)$ can be written as follows:

$$L_i(x) = R_i(x) = 1 - x \quad \geq \alpha, \qquad 0 \leq \alpha \quad < 1. \tag{15}$$

For equation(13) $(w_i, q_i, s_i), i = 1, \ldots, m$ are independent normal distributions.

In our study we will focus on this case of fuzzy random vectors $b_j(\omega), a_i(\omega)$ when (m_j, r_j, L_j) and (w_i, q_i, s_i) are independent and have normal distributions as follows (see Abo El-Kheir [1]):

$$\textbf{(i)} \ r_j = l_j = m_j \sim N(\mu_j, \sigma_j^2), \qquad j = 1, \ldots, n \tag{16}$$

$$E(b_j(\omega)) = \mu_j + \frac{1}{1-\alpha}\mu_j - \frac{1}{1-\alpha}\mu_j = \mu_j, \qquad j = 1, \ldots, n \tag{17}$$

$$V(b_j(\omega)) = \sigma_j^2 + \frac{1}{6}\sigma_j^2 + \frac{1}{6}\sigma_j^2 = \frac{4}{3}\sigma_j^2, \qquad j = 1, \ldots, n \tag{18}$$

$$F(u_j) = \frac{1}{\sqrt{2\pi}\,\frac{2}{\sqrt{3}}\sigma} \int_{-\infty}^{u} \exp\left(-\frac{1}{2}\frac{(b_s(\omega) - \mu_s)^2}{\frac{4}{3}\sigma_s^2}\right) d\, b_s(\omega), \qquad j = 1, \ldots, n$$

$$\textbf{(ii)} \ w_i = q_i = s_i \sim N(v_i, \eta_i^2), \qquad i = 1, \ldots, m \tag{19}$$

$$E(a_i(\omega)) = v_i, \qquad i = 1, \ldots, m, \tag{20}$$

$$V(a_i(\omega)) = \frac{4}{3}\eta_i^2, \qquad i = 1, \ldots, m. \tag{21}$$

3. The Fuzzy Random Chance Constrained Multi-Objective Transportation Problem (FRMOTP).

Liu [7]define problem (5-8) as a fuzzy random chance-constrained Multi-objective transportation problem (FRMOTP). The fuzzy random chance-constrained MOTP problem (5-8) degenerates to stochastic chance-constrained MOTP and it can be written as follows:

$$\textbf{Model 3}: \quad \min z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij} \qquad \text{k= 1, 2, 3,…,K} \tag{22}$$

$$\Pr(\sum_{j=1}^{n} x_{ij} \le a_i) \ge 1-\gamma_i, i=1,2,3,...,m \tag{23}$$

$$\Pr(\sum_{i=1}^{m} x_{ij} \le b_j) \ge 1-\beta_j, j=1,2,3,...,n \tag{24}$$

$$x_{ij}, i=1,2,3,...,m, j=1,2,3,...,n \tag{25}$$

Where $0 \le \beta_j \le 1$. $j=1,\dots,n$, and $0 \le \gamma_i \le 1$, $i=1,\dots,m$.The above problem is multi-objective stochastic transportation problem where a_i $(i=1,2,3,...,m)$ and b_j $(j=1,2,3,...,n)$ are random variables with known distributions and c_{ij}^k, k= 1, 2, 3,…,K is deterministic cost coefficients for $i=1,\dots,m$ and $j=1,\dots,n$.

Probabilistic chance constraints (23-25) can be written as follows:

$$\Pr(\sum_{j=1}^{n} x_{ij} \le a_i) \ge p_i, i=1,2,3,...,m \tag{26}$$

$$\Pr(\sum_{i=1}^{m} x_{ij} \ge b_j) \ge q_j, j=1,2,3,...,n \tag{27}$$

$$x_{ij} \ge 0, i=1,2,3,...,m, j=1,2,3,...,n \tag{28}$$

$1-\beta_j = q_j$ The Probabilistic chance constraints (26-28) can be Where $1-\gamma_i = p_i$, transformed as follows:

$$\Pr(\frac{\sum_{j=1}^{n} x_{ij} - \nu_i}{\sqrt{v(a_i)}} \le \frac{a_i-\nu_i}{\sqrt{v(a_i)}}) \ge p_i, i=1,2,3,...,m \tag{29}$$

$$\Pr(\frac{\sum_{i=1}^{m} x_{ij} - \mu_j}{\sqrt{v(b_j)}} \ge \frac{b_j-\mu_j}{\sqrt{v(b_j)}}) \ge q_j, j=1,2,3,...,n \tag{30}$$

$$x_{ij}, i=1,2,3,...,m, j=1,2,3,...,n \tag{31}$$

Let $\Phi(.)$ denote the cumulative density function of the standard normal variate evaluated at z. then the constraints (29-31) can be stated as

$$1-\Phi(\frac{a_i-\nu_i}{\sqrt{\frac{4}{3}\eta_i^2}} \ge \frac{\sum_{j=1}^{n} x_{ij}-\nu_i}{\sqrt{\frac{4}{3}\eta_i^2}}) \ge p_i, i=1,2,3,...,m \tag{32}$$

$$\Phi(\frac{b_j-\mu_j}{\sqrt{\frac{4}{3}\sigma_j^2}} \le \frac{\sum_{i=1}^{m} x_{ij}-\mu_j}{\sqrt{\frac{4}{3}\sigma_j^2}}) \ge q_j, j=1,2,3,...,n \tag{33}$$

$$x_{ij} \geq 0, i = 1,2,3,..., m, j = 1,2,3,..., n \tag{34}$$

The constraints (32-34) can be stated as

$$\Phi\left(\frac{b_j - \mu_j}{\sqrt{\frac{4}{3}\sigma_j^2}} \leq \frac{\sum_{i=1}^{m} x_{ij} - \mu_j}{\sqrt{\frac{4}{3}\sigma_j^2}}\right) \geq q_j, j = 1,2,3,..., n \tag{35}$$

$$\Phi\left(\frac{a_i - \nu_i}{\sqrt{\frac{4}{3}\eta_i^2}} \leq \frac{\sum_{j=1}^{n} x_{ij} - \nu_i}{\sqrt{\frac{4}{3}\eta_i^2}}\right) \geq 1 - p_i, i = 1,2,3,..., m \tag{36}$$

$$x_{ij}, i = 1,2,3,..., m, j = 1,2,3,..., n \tag{37}$$

Where:

$\frac{a_i - \nu_i}{\sqrt{\frac{4}{3}\eta_i^2}}$ is standard normal variate with mean zero and unit variance 1,

$\frac{b_j - \mu_j}{\sqrt{\frac{4}{3}\sigma_j^2}}$ is standard normal variate with mean zero and unit variance 1.

The constraints (35-37) can be stated as

$$\Phi\left(\frac{\sum_{j=1}^{n} x_{ij} - \nu_i}{\sqrt{\frac{4}{3}\eta_i^2}}\right) \leq \Phi(-k_{a_i}), i = 1,2,3,..., m \tag{38}$$

$$\Phi\left(\frac{\sum_{i=1}^{m} x_{ij} - \mu_j)}{\sqrt{\frac{4}{3}\sigma_j^2}} \geq \Phi(k_{b_j}), j = 1,2,3,..., n \tag{39}$$

$$x_{ij} \geq 0 \quad , i = 1,2,3,..., m \quad , j = 1,2,3,..., n \tag{40}$$

Using the cumulative density function of the standard normal varieties the constraints (38-40) can be simplified as:

$$\frac{n\sum_{j=1}^{n} x_{ij} - \nu_i}{\sqrt{\frac{4}{3}\eta_i^2}} \leq -k_{a_i} \ , i = 1,2,3,..., m \tag{41}$$

$$\frac{\sum_{i=1}^{m} x_{ij} - \mu_j}{\sqrt{\frac{4}{3}\sigma_j^2}} \geq k_{b_j},\ j = 1,2,3,\ldots,n. \tag{42}$$

$$x_{ij} \geq 0, i = 1,2,3,\ldots,m,\ j = 1,2,3,\ldots,n \tag{43}$$

The Equivalent Multi-objective Transportation Problem.

The equivalent deterministic multi-objective transportation problem of model 3 can be written as follows:

$$\text{Model 4:} \quad \min z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}, \quad \text{k= 1, 2, 3,…,K} \tag{44}$$

s.t

$$\sum_{j=1}^{n} x_{ij} - \nu_i + k_{a_i}\sqrt{\frac{4}{3}\eta_i^{\,2}} \leq 0, i = 1,2,3,\ldots,m \tag{45}$$

$$\sum_{i=1}^{m} x_{ij} - \mu_j - k_{b_j}\sqrt{\frac{4}{3}\sigma_j^2} \geq 0,\ \ j = 1,2,3,\ldots,n \tag{46}$$

$$x_{ij} \geq 0, i = 1,2,3,\ldots,m,\ j = 1,2,3,\ldots,n \tag{47}$$

4. Fuzzy Programming Technique (Solution Procedure).

To solve the equivalent deterministic multi-objective transportation problem (44-47) we apply fuzzy Programming technique on consideration of multi-objective vector minimum problem.

Let, L_r = aspiration level of achievement for objective r,

U_r = highest acceptable level of achievement for objective r,

d_r = U_r - L_r = the degradation allowance for objective r,

when the aspiration level and the degradation allowance for each objective are specified.

Algorithm

Step 1: Pick the first objective function and solve it as a single objective transportation problem subject to the constraints (45-47). Continue this process K times for K different objective functions. If all the solutions are the same, then one of them is the optimal compromise solution and stop. Otherwise, go to step 2.

Step 2: Evaluate the kth objective function at the K optimal solutions ($k = 1, 2, \ldots, K$). For each objective function, determine its lower and upper bounds (L_r and U_r) according to the set of optimal solution .Let z_{rr} = L_r and U_r =max[$z_{1r}, z_{2r}, z_{3r}, \ldots, z_{kr}$]. For satisfy, $z_r \leq L_r$,r = 1, 2, 3,…,k and constraints (45-47).

Step 3: Construct the membership function as:

$$\mu_{ij}(x_{ij}) = \begin{cases} 1 & if\ z_r \le L_r \\ 1 - \dfrac{z_r - L_r}{U_r - L_r} & if\ L_r \le z_r \le U_r \\ 0 & if\ z_r \ge L_r \end{cases}$$

if $\mu_{z_r}(x_{ij})$ =1; then z_r is perfectly achieved,

=0; then z_r is nothing achieved,

if $0 < \mu_{z_r}(x_{ij}) < 1$; then z_r is partially achieved,

Let $\lambda_r = \dfrac{U_r - z_r}{U_r - L_r}$, r = 1, 2, 3,…,k.

Step 4: Using max-min/ min-max operator, we have *Max[min(*$\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_k$*)],*

Then we have, max λ

$$\begin{gathered} \lambda_1 \ge \lambda \\ \lambda_1 \ge \lambda \\ \lambda_1 \ge \lambda \\ \ldots \\ \ldots \\ \lambda_1 \ge \lambda \end{gathered}$$

where λ=min{ $\mu_{z_r}(x_{ij})$ }, x_{ij} $, i = 1,2,3,\ldots,m$ $, j = 1,2,3,\ldots,n$.

Finally we can obtain the mathematical model through fuzzy programming technique as follows:

$$\text{Max:} \quad \lambda \tag{48}$$

s.t

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} + \lambda(U_k - L_k) - U_k \le 0, \tag{49}$$

$$\sum_{j=1}^{n} x_{ij} - \nu_i + k_{a_i} \sqrt{\frac{4}{3}\eta_i^{\,2}} \le 0 \tag{50}$$

$$\sum_{i=1}^{m} x_{ij} - \mu_j - k_{b_j} \sqrt{\frac{4}{3}\sigma_j^2} \ge 0 \tag{51}$$

$$x_{ij} \ge 0 \quad , i = 1,2,3,\ldots,m \quad , j = 1,2,3,\ldots,n \tag{52}$$

5. Numerical Example

The Defense Communications Agency is responsible for operating and maintaining a world-wide communications system. It thinks of costs as being proportional to the ‘‘message units’’ transmitted in one direction over a particular link in the system. Hence, under normal operating conditions it faces the following minimum-cost flow problem:

$$Min\ z = \sum_i \sum_j c_{ij} x_{ij}$$

Subject to:

$\sum_j x_{ij} - \sum_k x_{ki} = b_i$, i = 1,2,…,n Flow balance

$0 \leq x_{ij} \leq u_{ij}$ Flow capacities

c_{ij} = cost per message unit over link $(i - j)$.
b_i = message units generated (or received) at station i,
u_{ij} = upper bound on number of message units that can be transmitted over link $(i - j)$.
Lij = lower bound on number of message units that can be transmitted over link $(i - j)$.
Suppose the three production sources and supply to four destination center in which all availability and demand (parameters) are fuzzy random with known means and variances defined in (3.17-3.18)and (3.20-3.21). The decision maker lays emphasis on criteria such as minimization of transportation cost, transportation time or (delivery time) and loss during transportation through a given route (i, j) where i = 1, 2, 3 & j = 1, 2, 3, 4. Here z_1, z_2, z_3 represented the total transportation cost with by Rs. Thousand respectively from each production sources to each destination center along with availability and demand are represented by the matrices in C^1, C^2, C^3 as mentioned below:

$$C^1 = \begin{bmatrix} 8 & 9 & 7 & 2 \\ 5 & 6 & 4 & 7 \\ 3 & 7 & 7 & 5 \end{bmatrix},\ C^2 = \begin{bmatrix} 2 & 9 & 8 & 1 \\ 4 & 3 & 6 & 7 \\ 5 & 2 & 8 & 2 \end{bmatrix},\ C^3 = \begin{bmatrix} 2 & 4 & 7 & 3 \\ 6 & 4 & 8 & 4 \\ 8 & 2 & 5 & 1 \end{bmatrix}$$

The decision maker is also minimized cost per message from the i- source to the j-destination so as to satisfy all the requirement.

min $z_1 = 8x_{11} + 9x_{12} + 7x_{13} + 2x_{14} + 5x_{21} + 6x_{22} + 4x_{23} + 7x_{24} + 3x_{31} + 7x_{32} + 7x_{33} + 5x_{34}$

min $z_2 = 2x_{11} + 9x_{12} + 8x_{13} + x_{14} + 4x_{21} + 3x_{22} + 6x_{23} + 7x_{24} + 5x_{31} + 2x_{32} + 8x_{33} + 2x_{34}$

min $z_3 = 2x_{11} + 4x_{12} + 7x_{13} + 3x_{14} + 6x_{21} + 4x_{22} + 8x_{23} + 4x_{24} + 8x_{31} + 2x_{32} + 5x_{33} + x_{34}$

s.t

$$\Pr(\sum_{i=1}^{m} x_{1j} \leq a_1) \geq 1 - \gamma_1,$$

$$\Pr(\sum_{i=1}^{m} x_{2j} \leq a_2) \geq 1 - \gamma_2,$$

$$\Pr(\sum_{i=1}^{m} x_{3j} \leq a_3) \geq 1 - \gamma_3,$$

$$\Pr(\sum_{j=1}^{n} x_{i1} \leq b_1) \geq 1 - \beta_1$$

$$\Pr(\sum_{j=1}^{n} x_{i2} \leq b_2) \geq 1 - \beta_2$$

$$\Pr(\sum_{j=1}^{n} x_{i3} \leq b_3) \geq 1 - \beta_3$$

$$\Pr(\sum_{j=1}^{n} x_{i4} \leq b_4) \geq 1 - \beta_4$$

$x_{ij} \quad , i = 1,2,3,..., m \quad , j = 1,2,3,..., n$

Let: v_1 =13, $\eta_1{}^2$ = 3, predetermined confidence level γ_1=.01

v_2 =15, $\eta_2{}^2$ = 2, predetermined confidence level γ_2 =.02

v_3 =19, $\eta_3{}^2$ = 7, predetermined confidence level γ_3 =.03

μ_1 =7, σ_1^2 =5, predetermined confidence level β_1 = .04

μ_2 =5, σ_2^2 =3, predetermined confidence level β_2 = .05

μ_2 = 6, σ_3^2 =2, predetermined confidence level β_3 = .06

μ_2 = 4, σ_4^2 =1, predetermined confidence level β_4 = .07

As described in section 3, we can converted into the deterministic multi objective unbalanced transportation problem as follows:

min z_1 =8 x_{11} +9 x_{12} +7 x_{13} +2 x_{14} + 5 x_{21} +6 x_{22} +4 x_{23} +7 x_{24} +3 x_{31} +7 x_{32} +7 x_{33} +5 x_{34}

min z_2 =2 x_{11} +9 x_{12} +8 x_{13} + x_{14} + 4 x_{21} +3 x_{22} +6 x_{23} +7 x_{24} +5 x_{31} +2 x_{32} +8 x_{33} +2 x_{34}

min z_1 =2 x_{11} +4 x_{12} +7 x_{13} +3 x_{14} +6 x_{21} +4 x_{22} +8 x_{23} +4 x_{24} +8 x_{31} +2 x_{32} +5 x_{33} + x_{34}

s.t

$$\sum_{j=1}^{n} x_{1j} \leq 8.34,$$

$$\sum_{j=1}^{n} x_{2j} \leq 11.65,$$

$$\sum_{j=1}^{n} x_{3j} \leq 12.25,$$

$$\sum_{i=1}^{n} x_{i1} \geq 11.51$$

$$\sum_{i=1}^{m} x_{i2} \geq 8$$

$$\sum_{i=1}^{m} x_{i3} \geq 8.53$$

$$\sum_{i=1}^{m} x_{i4} \geq 5.69$$

$x_{ij} \geq 0 \quad , i = 1,2,3,..., m \quad , j = 1,2,3,..., n$

We have obtained the lower bounds of the above deterministic problem as (L_1, L_2, L_3) = (141.11,115.94,106.34), and for the same problem the upper bounds as (U_1, U_2, U_3) = (217.09,217.09,196.89).

Using problem (3.48-3.51), we formulated the following model

Max: λ

s.t

+ $\lambda \leq$141.11 8 x_{11} +9 x_{12} +7 x_{13} +2 x_{14} +5 x_{21} +6 x_{22} +4 x_{23} +7 x_{24} +3 x_{31} +7 x_{32} +7 x_{33} +5 x_{34}

2 x_{11} +9 x_{12} +8 x_{13} + x_{14} + 4 x_{21} +3 x_{22} +6 x_{23} +7 x_{24} +5 x_{31} +2 x_{32} +8 x_{33} +2 x_{34} + $\lambda \leq$217.09

2 x_{11} +4 x_{12} +7 x_{13} +3 x_{14} +6 x_{21} +4 x_{22} +8 x_{23} +4 x_{24} +8 x_{31} +2 x_{32} +5 x_{33} + x_{34} + $\lambda \leq$ 196.89

$$\sum_{i=1}^{m} x_{1j} \leq 9.79,$$

$$\sum_{i=1}^{m} x_{2j} \leq 11.65,$$

$$\sum_{i=1}^{m} x_{3j} \leq 12.25,$$

$$\sum_{j=1}^{n} x_{i1} \geq 11.51$$

$$\sum_{j=1}^{n} x_{i2} \geq 8$$

$$\sum_{j=1}^{n} x_{i3} \geq 8.53$$

$$\sum_{j=1}^{n} x_{i4} \geq 5.69$$

$$x_{ij} \geq 0 \quad , i = 1,2,3,\dots,m \quad , j = 1,2,3,\dots,n$$

The above problem is solved by the LINGO mathematical package for obtaining the optimal compromise solution of the deterministic problem. We get λ = 0 and optimal compromise solution x_{11} = 0 x_{12} = 0 x_{13} = 4.14, x_{14} = 5.65 x_{21} = 0 x_{22} = 7.26 x_{23} = 4.39 x_{24} = 0 x_{31} = 11.51 x_{32} = 0.74 x_{33} = 0 x_{34} = 0 the optimal value of each objective functions i.e., z_1, z_2, z_3 are respectively. Also we obtained the non dominated solution for each objective functions $z_1 = 141.11$, $z_2 = 115.99$ $z_3 = 106.34$ i.e., respectively

6. Conclusion

The main objective of this paper is to present a solution procedure for fuzzy random multi-objective transportation problem (FRMOTP). The transportation problem is an efficient tool to scope with many real life problems of practical importanc. Multi-objective transportation problem involve the design, modeling, and planning of many complex resource allocation systems, transportation in which demand and supply are fuzzy random in nature. After converting fuzzy random chance constraints into equivalent deterministic constraints using fuzzy random chance constraints theorem, the fuzzy programming is applied to obtain a compromise solution from the set of non dominated solution. Our technique is highly fruitful in the sense of real life problems of practical importance. A practical numerical example to provide to demonstrate the feasibility of all decision variables of the proposed method.

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