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Article
A Study of a Neutrosophic Complex Numbers and Applications

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Abstract: In this paper, we will define the definition of neutrosophic complex number, by forms cartesian and polar, and some application for it. For sum two neutrosophic complex number, product, and division. The main objective is define a power and roots of neutrosophic complex number, Also, define a neutrossophic complex functions, And conditions Cauchy-Riemann, In addition, we have given the method of denote the harmonic conjugate.

Keywords: Neutrosophic numbers, neutrosophic complex number, the exponential form of a nutrosophic complex number.

1. Introduction

The American scientist and philosopher F. Smarandache came to place the neutrosophic logic in $[5-6]$, and this logic is as a generalization of the fuzzy logic $[7]$, conceived by L. Zadeh in 1965.The neutrosophic logic is of grerat in many areas of them, including applications in image processing $[8-9]$, the field of geographic information systems $[10]$, and possible applications to database $[11-12]$, Neutrosophic logic. Neutrosophy, Neutrosophic set, Neutrosophic probabilityand alike, are recently creations of F. Smarandache, being characterized by having the indeterminacy as component of their framework,and a notable feature of neutrosophic logic is that can be considered a generaliazation of fuzzy logics, encompassing the classical logic as well $[1]$. Also. Finally F. Smarandache, presented the definition of the standard form of neutrosophic conditions for the division of two neutrosophic real numbers to exist, he defined the standard form of neutrosophic complex number in year 2011 in $[2]$.

Among the recent applications there are: neutrosophic crisp set theory in image processing [13][14], neutrosophic sets medical field [15][16][17][18][19], in information geographic systems [20] and possible applications to database [21]. Also, neutrosophic triplet group application to physics [22]. Morever Several researches have made multiple contributions to neutrosophic topological [23][24][25][26][27][28][29], Also More researches have made multiple contributions to neutrosophic analysis [30]. This paper aims to study and define the roots of neutrosophic number, and a neutrossophic complex functions, conditions Cauchy-Riemann, In addition, and the harmonic conjugate.

2. Preliminaries

In this paperwe recall some definitions which are useful in this paper.

Definition 2.1. $[\mathbf{1}]$ Neutrosophic Real Number: Suppose that $\boldsymbol{w}$ is a neutrosophic number, then it takes the following standard form: $\boldsymbol{w}=\boldsymbol{a}+\boldsymbol{b}\boldsymbol{I}$ where $\boldsymbol{a},\boldsymbol{b}$ are real coefficients, and $\boldsymbol{I}$ represent indeterminacy, such $\mathbf{0}.\boldsymbol{I}=\mathbf{0}$ and $\boldsymbol{I}^{\boldsymbol{n}}=\boldsymbol{I}$ for all positive integers $\boldsymbol{n}$.

For example:

$w = 1 + 2I, w = 3 = 3 + 0I$

Definition 2.2. [1]

Division of neutrosophic real numbers:

Suppose that w_1, w_2 are two neutrosophic number, where

$$w_1 = a_1 + b_1 I, w_2 = a_2 + b_2 I$$

Then:

$$\frac{w_1}{w_2} = \frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} I \dots\dots (1)$$

Definition 2.3. [2]

Neutrosophic Complex Number:

Suppose that z is a neutrosophic complex number, then it takes the following standard form: $z = a + bI + i(\mathrm{c} + \mathrm{d}I)$ where a, b, c, d are real coefficients, and I indeterminacy, such $i^2 = -1$ then $i = \sqrt{-1}$.

We recall $a + bI$ the real part, then it takes the following standard form $Re(z) = a + bI$.

We recall $c + dI$ the imagine part, then it takes the following standard form $Im(z) = c + dI$.

For example:

$$z = 4 + I + i(2 + 2I)$$

Note: we can say that any real number can be considered a nutrosophic number.

For example: $z = 3 = 3 + 0.I + i(0 + 0.I)$

Definition 2.4. [2]

Conjugate of a neutrosophic complex number:

Suppose that z is a neutrosophic complex number, where $z = a + bI + i(\mathrm{c} + \mathrm{d}I)$. We demote the conjugate of a neutrosophic complex number by $\bar{z}$ and define it by the following form:

$$\bar{z} = a + bI - i(c + dI)$$

Example 2.5.

$z = 4 + I + i(2 + 2I) \Longrightarrow \bar{z} = 4 + I - i(2 + 2I)$

Definition 2.6. [3]

The absolute value of a neutrosophic complex number:

Suppose that $z = a + bI + i(\mathrm{c} + \mathrm{d}I)$ is a neutrosophic complex number, the absolute value of a neutrosophic complex number defined by the following form:

$$|\mathrm{z}| = \sqrt{(a + bI)^2 + (c + dI)^2}$$

Remarks 2.7 [3]

(1). $\overline{(\bar{z})} = z$.

(2). $\bar{z} + z = 2Re(z)$

(3). $z - \bar{z} = 2Im(z)$

(4). $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

(5). $\overline{z_1 . z_2} = \bar{z}_1 . \bar{z}_2$

(6). $z.\bar{z} = |\mathrm{z}|^2$

3. The polar form of a neutrosophic complex number

Definition 3.1. [4]

We defined the exponential form of a neutrosophic complex number as follows:

$$z = re^{i(\theta+I)}$$

Where:

$$r = |z| = \sqrt{(a+bI)^2 + (c+dI)^2}$$

$$\cos(\theta+I) = \frac{x}{r} = \frac{a+bI}{r}$$

$$\sin(\theta+I) = \frac{y}{r} = \frac{c+dI}{r}$$

Then:

$$z = re^{i(\theta+I)} = r\cos(\theta+I) + i\,r\sin(\theta+I)$$

Remarks 3.2 [4]

(1). $z_1.z_2 = r_1e^{i(\theta_1+I_1)}.r_2e^{i(\theta_2+I_2)} = r_1r_2e^{i(\theta_1+\theta_2+I)}\ ;\ I_1+I_2 = I$

(2). $\frac{z_1}{z_2} = \frac{r_1e^{i(\theta_1+I_1)}}{r_2e^{i(\theta_2+I_2)}} = \frac{r_1}{r_2}e^{i(\theta_1-\theta_2+I)}\ ;\ I_1-I_2 = I$

(3). $z.\bar{z} = |z|^2 = r^2$

(4). $\bar{z} = re^{-i(\theta+I)}$

Example 3.3. Let:

$$z_1 = 2e^{i\left(\frac{2\pi}{3}+I\right)}\ , \qquad z_2 = e^{i\left(\frac{\pi}{4}+I\right)}$$

Then:

$$\bar{z_1} = 2e^{-i\left(\frac{2\pi}{3}+I\right)}\ , \qquad \bar{z_2} = e^{-i\left(\frac{\pi}{4}+I\right)}$$

$$z_1.z_2 = 2e^{i\left(\frac{2\pi}{3}+I\right)}e^{i\left(\frac{\pi}{4}+I\right)} = 2e^{i\left(\frac{11\pi}{12}+I\right)}$$

$$\frac{z_1}{z_2} = \frac{2e^{i\left(\frac{2\pi}{3}+I\right)}}{e^{i\left(\frac{\pi}{4}+I\right)}} = 2e^{i\left(\frac{5\pi}{12}+I\right)}$$

4. The Power of a neutrosophic complex number.

Definition 4.1. Suppose that $z = re^{i(\theta+I)}$ is a neutrosophic complex number, the power of a neutrosophic complex number defined by the following form:

$$z^n = \left(re^{i(\theta+I)}\right)^n = r^ne^{in(\theta+I)} = r^ne^{i(\theta n+nI)}$$

Then:

$$z^n = r^ne^{i(n\theta+I)} = r^n\cos(n\theta+nI) + i\,r^n\sin(n\theta+nI) \ldots\ldots (2)$$

Example 4.2. Let $z = Ie^{i\left(\frac{\pi}{4}+I\right)}$ find z^2, z^8.

Solution.

$$z^2 = I^2 e^{i\left(\frac{2\pi}{4}+2I\right)} = I e^{i\left(\frac{\pi}{2}+2I\right)} = I\cos\left(\frac{\pi}{2}+2I\right) + i\, I\sin\left(\frac{\pi}{2}+2I\right)$$

$$z^8 = I^8 e^{i\left(\frac{8\pi}{4}+8I\right)} = I e^{i(2\pi+8I)} = I\cos(2\pi+8I) + i\, I\sin(2\pi+8I)$$

5. The Roots of a neutrosophic complex number.

Definition 5.1.

Suppose that $z = re^{i(\theta+I)}$ is a neutrosophic complex number, a a neutrosophic complex number $w = \check{r}e^{i(\varphi+I)} = \alpha + \beta I + i(\gamma + \delta I)$, and it satisfay relation $z = w^n$ is call the root by a neutrosophic complex number z, we have:

$$w = \sqrt[n]{z} = z^{\frac{1}{n}}$$

Then:

$$w = |w|e^{i(\varphi+I)} \Rightarrow |z| = |w|^n e^{in(\varphi+I)} = re^{i(\theta+I)}.e^{2\pi k} \Rightarrow |w^n| = r \;, n(\varphi+I) = (\theta+I) + 2\pi k$$

$$\Rightarrow |w| = \sqrt[n]{r} \;, \varphi + I = \frac{(\theta+I)+2\pi k}{n} \Rightarrow w_k = \sqrt[n]{z} = \sqrt[n]{r}e^{i(\varphi+I)} = \sqrt[n]{r}e^{i\left(\frac{(\theta+I)+2\pi k}{n}\right)}$$

$$\Rightarrow w_k = \sqrt[n]{z} = \sqrt[n]{r}\cos\left(\frac{(\theta+I)+2\pi k}{n}\right) + i\,\sqrt[n]{r}\sin\left(\frac{(\theta+I)+2\pi k}{n}\right);\; k = 0,1,2,\ldots\ldots,n-1 \ldots\ldots(3)$$

Example 5.2. Let $z = e^{i\left(\frac{-\pi}{2}+I\right)}$ find $\sqrt[3]{z}$.

Solution.

$$w_k = \sqrt[n]{z} = \sqrt[n]{r}\cos\left(\frac{(\theta+I)+2\pi k}{n}\right) + i\,\sqrt[n]{r}\sin\left(\frac{(\theta+I)+2\pi k}{n}\right)$$

$$\Rightarrow w_k = \sqrt[n]{z} = \cos\left(\frac{\frac{-\pi}{2}+I+2\pi k}{3}\right) + i\,\sin\left(\frac{\frac{-\pi}{2}+I+2\pi k}{3}\right)$$

$$k = 0 \Rightarrow w_0 = \cos\left(\frac{\frac{-\pi}{2}+I}{3}\right) + i\,\sin\left(\frac{\frac{-\pi}{2}+I}{3}\right)$$

$$\Rightarrow w_0 = \cos\left(\frac{-\pi+2I}{6}\right) + i\,\sin\left(\frac{-\pi+2I}{6}\right)$$

By using (1) we have:

$$\frac{-\pi+2I}{6} = \frac{-\pi}{6} + \frac{1}{3}I$$

$$\Rightarrow w_0 = \cos\left(\frac{-\pi}{6}+\frac{1}{3}I\right) + i\,\sin\left(\frac{-\pi}{6}+\frac{1}{3}I\right)$$

$$k = 1 \Rightarrow w_1 = \cos\left(\frac{\frac{-\pi}{2}+I+2\pi}{3}\right) + i\,\sin\left(\frac{\frac{-\pi}{2}+I+2\pi}{3}\right)$$

$$\Rightarrow w_1 = \cos\left(\frac{\frac{3\pi}{2}+I}{3}\right) + i\ \sin\left(\frac{\frac{3\pi}{2}+I}{3}\right)$$

$$\Rightarrow w_1 = \cos\left(\frac{3\pi+2I}{6}\right) + i\ \sin\left(\frac{3\pi+2I}{6}\right)$$

By using (1) we have:

$$\frac{3\pi+2I}{6} = \frac{\pi}{2} + \frac{1}{3}I$$

$$\Rightarrow w_1 = \cos\left(\frac{\pi}{2}+\frac{1}{3}I\right) + i\ \sin\left(\frac{\pi}{2}+\frac{1}{3}I\right)$$

$$k = 2 \Rightarrow w_2 = \cos\left(\frac{\frac{-\pi}{2}+I+4\pi}{3}\right) + i\ \sin\left(\frac{\frac{-\pi}{2}+I+4\pi}{3}\right)$$

$$\Rightarrow w_2 = \cos\left(\frac{\frac{7\pi}{2}+I}{3}\right) + i\ \sin\left(\frac{\frac{7\pi}{2}+I}{3}\right)$$

$$\Rightarrow w_2 = \cos\left(\frac{7\pi+2I}{6}\right) + i\ \sin\left(\frac{7\pi+2I}{6}\right)$$

By using (1) we have:

$$\left(\frac{7\pi+2I}{6}\right) = \frac{7\pi}{6} + \frac{1}{3}I$$

$$\Rightarrow w_2 = \cos\left(\frac{7\pi}{6}+\frac{1}{3}I\right) + i\ \sin\left(\frac{7\pi}{6}+\frac{1}{3}I\right)$$

6. A neutrosophic complex Function.

Definition 6.1

Let $z = (x+I) + i(y+I), w = (u+I) + i(v+I)$, Then we call the function:

$$w = f(z) \Rightarrow w = (u+I) + i(v+I) = f\big((x+I) + i(y+I)\big)$$

Is a neutrosophic complex Function.

Example 6.2. Let $w = f(z) = |z|^2$ find the real part and imagine part.

Solution.

Let $\boldsymbol{z = (x+I) + i(y+I), w = (u+I) + i(v+I)}$, then:

$$w = (u+I) + i(v+I) = \left(\sqrt{(x+I)^2 + (y+I)^2}\right)^2$$

$$\Rightarrow w = (u+I) + i(v+I) = x^2 + y^2 + (2x+2y+1)I$$

$$\Rightarrow \boldsymbol{(u+I) = x^2 + y^2 + (2x+2y+1)I\ , (v+I) = 0 + 0I}$$

Definition 6. 3. Cauchy-Riemann conditions.

Cartesian:

Suppose that $\boldsymbol{w} = \boldsymbol{f}(\boldsymbol{z})$ is a neutrosophic complex Function, where $\boldsymbol{z} = (\boldsymbol{x}+\boldsymbol{I}) + \boldsymbol{i}(\boldsymbol{y}+\boldsymbol{I})$, $w = (u+I) + i(v+I)$, Cauchy-Riemann conditions by Cartesian defined by the following form:

$$\begin{cases} \dfrac{\partial(\boldsymbol{u}+\boldsymbol{I})}{\partial(\boldsymbol{x}+\boldsymbol{I})} = \dfrac{\partial(\boldsymbol{v}+\boldsymbol{I})}{\partial(\boldsymbol{y}+\boldsymbol{I})} \\ \dfrac{\partial(\boldsymbol{v}+\boldsymbol{I})}{\partial(\boldsymbol{x}+\boldsymbol{I})} = -\dfrac{\partial(\boldsymbol{u}+\boldsymbol{I})}{\partial(\boldsymbol{y}+\boldsymbol{I})} \end{cases} \dots\dots(\mathbf{4})$$

And derivate for function $\boldsymbol{w} = \boldsymbol{f}(\boldsymbol{z})$ defined by the following form:

$$\acute{\boldsymbol{f}}(\boldsymbol{z}) = \frac{\partial(\boldsymbol{u}+\boldsymbol{I})}{\partial(\boldsymbol{x}+\boldsymbol{I})} + \boldsymbol{i}\frac{\partial(\boldsymbol{v}+\boldsymbol{I})}{\partial(\boldsymbol{x}+\boldsymbol{I})} \; \boldsymbol{or} \; \acute{\boldsymbol{f}}(\boldsymbol{z}) = \frac{\partial(\boldsymbol{u}+\boldsymbol{I})}{\partial(\boldsymbol{y}+\boldsymbol{I})} - \boldsymbol{i}\frac{\partial(\boldsymbol{v}+\boldsymbol{I})}{\partial(\boldsymbol{y}+\boldsymbol{I})} \dots\dots(\mathbf{5})$$

Example 6.4. Let $\boldsymbol{f}(\boldsymbol{z}) = \mathbf{z}^{\mathbf{2}}$, prove $\acute{\boldsymbol{f}}(\boldsymbol{z}) = \mathbf{2}\boldsymbol{z}$.

Solution.

Let $\boldsymbol{z} = (\boldsymbol{x}+\boldsymbol{I}) + \boldsymbol{i}(\boldsymbol{y}+\boldsymbol{I}), \boldsymbol{w} = (\boldsymbol{u}+\boldsymbol{I}) + \boldsymbol{i}(\boldsymbol{v}+\boldsymbol{I})$, then:

$(u+I) + i(v+I) = (x^2 - y^2 + 2(x-y)I + I) + i\,2(x+I)(y+I)$

$\Rightarrow (u+I) = (x^2 - y^2 + 2(x-y)I + I)$

$\Rightarrow (v+I) = 2(x+I)(y+I)$

Then:

$$\frac{\partial(u+I)}{\partial(x+I)} = 2x + 2I\,, \frac{\partial(u+I)}{\partial(y+I)} = -2y - 2I$$

$$\frac{\partial(v+I)}{\partial(x+I)} = 2y + 2I\,, \frac{\partial(v+I)}{\partial(y+I)} = 2x + 2I$$

$$\Rightarrow \begin{cases} \dfrac{\partial(u+I)}{\partial(x+I)} = \dfrac{\partial(v+I)}{\partial(y+I)} \\ \dfrac{\partial(v+I)}{\partial(x+I)} = -\dfrac{\partial(u+I)}{\partial(y+I)} \end{cases}$$

Cauchy-Riemann conditions is satisfytion. Then we have:

$$\acute{\boldsymbol{f}}(\boldsymbol{z}) = \frac{\partial(\boldsymbol{u}+\boldsymbol{I})}{\partial(\boldsymbol{x}+\boldsymbol{I})} + \boldsymbol{i}\frac{\partial(\boldsymbol{v}+\boldsymbol{I})}{\partial(\boldsymbol{x}+\boldsymbol{I})} \Rightarrow \acute{\boldsymbol{f}}(\boldsymbol{z}) = \mathbf{2}\boldsymbol{x} + \mathbf{2}\boldsymbol{I} + \boldsymbol{i}(\mathbf{2}\boldsymbol{y} + \mathbf{2}\boldsymbol{I}) = \mathbf{2}\big((\boldsymbol{x}+\boldsymbol{I}) + \boldsymbol{i}(\boldsymbol{y}+\boldsymbol{I})\big) = \mathbf{2}\boldsymbol{z}$$

$$\Rightarrow \acute{\boldsymbol{f}}(\boldsymbol{z}) = \mathbf{2}\boldsymbol{z}$$

Polar:

Suppose that $\boldsymbol{w} = \boldsymbol{f}(\boldsymbol{z})$ is a neutrosophic complex Function, where $\boldsymbol{z} = \boldsymbol{r}\boldsymbol{e}^{\boldsymbol{i}(\boldsymbol{\theta}+\boldsymbol{I})}, \boldsymbol{w} = (\boldsymbol{u}+\boldsymbol{I}) + \boldsymbol{i}(\boldsymbol{v}+\boldsymbol{I})$

Cauchy-Riemann conditions by Polar defined by the following form:

$$\begin{cases} \dfrac{\partial(\boldsymbol{u}+\boldsymbol{I})}{\partial(\boldsymbol{r}+\boldsymbol{I})}=\dfrac{\mathbf{1}}{\boldsymbol{r}}\dfrac{\partial(\boldsymbol{v}+\boldsymbol{I})}{\partial(\boldsymbol{\theta}+\boldsymbol{I})} \\ \dfrac{\partial(\boldsymbol{v}+\boldsymbol{I})}{\partial(\boldsymbol{r}+\boldsymbol{I})}=-\dfrac{\mathbf{1}}{\boldsymbol{r}}\dfrac{\partial(\boldsymbol{u}+\boldsymbol{I})}{\partial(\boldsymbol{\theta}+\boldsymbol{I})} \end{cases} \ldots\ldots(\mathbf{6})$$

And derivate for function $\boldsymbol{w}=\boldsymbol{f}(\boldsymbol{z})$ defined by the following form:

$$\acute{\boldsymbol{f}}(\boldsymbol{z})=\boldsymbol{e}^{-\boldsymbol{i}(\boldsymbol{\theta}+\boldsymbol{I})}\left(\frac{\partial(\boldsymbol{u}+\boldsymbol{I})}{\partial(\boldsymbol{r}+\boldsymbol{I})}+\boldsymbol{i}\frac{\partial(\boldsymbol{v}+\boldsymbol{I})}{\partial(\boldsymbol{r}+\boldsymbol{I})}\right)\ \boldsymbol{or}\ \acute{\boldsymbol{f}}(\boldsymbol{z})=\frac{\mathbf{1}}{\boldsymbol{r}}\boldsymbol{e}^{-\boldsymbol{i}(\boldsymbol{\theta}+\boldsymbol{I})}\left(\frac{\partial(\boldsymbol{v}+\boldsymbol{I})}{\partial(\boldsymbol{\theta}+\boldsymbol{I})}-\boldsymbol{i}\frac{\partial(\boldsymbol{u}+\boldsymbol{I})}{\partial(\boldsymbol{\theta}+\boldsymbol{I})}\right)\ldots\ldots(\mathbf{7})$$

Example 6.5. Let $\boldsymbol{f}(\boldsymbol{z})=\frac{\mathbf{1}}{\boldsymbol{z}}$, prove $\acute{\boldsymbol{f}}(\boldsymbol{z})=\frac{-\mathbf{1}}{\boldsymbol{z}^{\mathbf{2}}}$.

Solution.

Let $\boldsymbol{z}=\boldsymbol{r}\boldsymbol{e}^{\boldsymbol{i}(\boldsymbol{\theta}+\boldsymbol{I})},\boldsymbol{w}=(\boldsymbol{u}+\boldsymbol{I})+\boldsymbol{i}(\boldsymbol{v}+\boldsymbol{I})$, then:

$$(u+I)+i(v+I)=\frac{1}{re^{i(\theta+I)}}=\frac{1}{r}e^{-i(\theta+I)}$$

$$\Rightarrow (u+I)+i(v+I)=\frac{1}{r}\cos(\theta+I)-i\,\frac{1}{r}\sin(\theta+I)$$

$$\Rightarrow (u+I)=\frac{1}{r}\cos(\theta+I)=\frac{1}{(r+I)-I}\cos(\theta+I)$$

$$\Rightarrow (v+I)=-\frac{1}{(r+I)-I}\sin(\theta+I)$$

Then:

$$\frac{\partial(u+I)}{\partial(r+I)}=-\frac{1}{\big((r+I)-I\big)^2}\cos(\theta+I)$$

$$\frac{\partial(u+I)}{\partial(\theta+I)}=-\frac{1}{(r+I)-I}\sin(\theta+I)$$

$$\frac{\partial(v+I)}{\partial(r+I)}=-\frac{1}{\big((r+I)-I\big)^2}\sin(\theta+I)$$

$$\frac{\partial(v+I)}{\partial(\theta+I)}=-\frac{1}{(r+I)-I}\cos(\theta+I)$$

$$\Rightarrow \begin{cases} \dfrac{\partial(u+I)}{\partial(r+I)}=\dfrac{1}{r}\dfrac{\partial(v+I)}{\partial(\theta+I)} \\ \dfrac{\partial(v+I)}{\partial(r+I)}=-\dfrac{1}{r}\dfrac{\partial(u+I)}{\partial(\theta+I)} \end{cases}$$

Cauchy-Riemann conditions is satisfytion. Then we have:

$$\Rightarrow \acute{f}(z)=e^{-i(\theta+I)}\left(\frac{\partial(u+I)}{\partial(r+I)}+i\frac{\partial(v+I)}{\partial(r+I)}\right)$$

$$\acute{f}(z) = e^{-i(\theta+I)}\left(-\frac{1}{\left((r+I)-I\right)^2}\cos(\theta+I) + i\frac{1}{\left((r+I)-I\right)^2}\sin(\theta+I)\right)$$

$$\Rightarrow \acute{f}(z) = -\frac{1}{\left((r+I)-I\right)^2}e^{-i(\theta+I)}(\cos(\theta+I) - i\sin(\theta+I))$$

$$\Rightarrow \acute{f}(z) = -\frac{1}{\left((r+I)-I\right)^2}e^{-i(\theta+I)}e^{-i(\theta+I)}$$

$$\Rightarrow \acute{f}(z) = -\frac{1}{\left((r+I)-I\right)^2}e^{-2i(\theta+I)}$$

$$\Rightarrow \acute{f}(z) = -\frac{1}{(r+I)^2 - 2(r+I)I + I^2}e^{-2i(\theta+I)}$$

$$\Rightarrow \acute{f}(z) = -\frac{1}{r^2 + 2rI + I^2 - 2rI - 2I^2 + I^2}e^{-2i(\theta+I)}$$

$$\Rightarrow \acute{f}(z) = -\frac{1}{r^2 + 2rI + 2I^2 - 2rI - 2I^2}e^{-2i(\theta+I)}$$

$$\Rightarrow \acute{f}(z) = -\frac{1}{r^2}e^{-2i(\theta+I)} = -\frac{1}{r^2 e^{2i(\theta+I)}} = -\frac{1}{(re^{i(\theta+I)})^2} = \frac{-1}{z^2}$$

$$\Rightarrow \acute{f}(z) = \frac{-1}{z^2}$$

7. A neutrosophic complex Harmonic Function.

Definition 7. 1.

Suppose that $\boldsymbol{h = h(x+I\,, y+I)}$ is a neutrosophic real Function, we say $\boldsymbol{h(x+I\,, y+I)}$ is a a neutrosophic harmonic Function, if satisfy the Laplas equation:

$$\frac{\boldsymbol{\partial 2h}}{\boldsymbol{\partial(x+I)^2}} + \frac{\boldsymbol{\partial 2h}}{\boldsymbol{\partial(y+I)^2}} = \boldsymbol{0} \;\ldots\ldots (\boldsymbol{8})$$

Definition 7. 2. A harmonic conjugate Cartesian.

Suppose that $(\boldsymbol{u}+\boldsymbol{I}), (\boldsymbol{v}+\boldsymbol{I})$ is a neutrosophic harmonic Functions, we say $(\boldsymbol{v}+\boldsymbol{I})$ is a harmonic conjugate by $(\boldsymbol{u}+\boldsymbol{I})$, if $(\boldsymbol{u}+\boldsymbol{I}), (\boldsymbol{v}+\boldsymbol{I})$ are satisfy Cauchy- Riemann conditions.

Example 7.3. Let $\boldsymbol{f(z) = \frac{1}{z^2}}$.

1- Prove $(\boldsymbol{u}+\boldsymbol{I}), (\boldsymbol{v}+\boldsymbol{I})$ are a neutrosophic harmonic Functions.

2- Find the harmonic conjugate $(\boldsymbol{v}+\boldsymbol{I})$.

Solution.

1- Let $\boldsymbol{z = (x+I) + i(y+I), w = (u+I) + i(v+I)}$, then:

$$(u+I) + i(v+I) = (x^2 - y^2 + 2(x-y)I + I) + i\,2(x+I)(y+I)$$
$$\Rightarrow (u+I) = (x^2 - y^2 + 2(x-y)I + I)$$
$$\Rightarrow (v+I) = 2(x+I)(y+I)$$

Then:

$$\begin{cases} \frac{\partial(u+I)}{\partial(x+I)} = 2x + 2I \,, \frac{\partial(u+I)}{\partial(y+I)} = -2y - 2I \\ \frac{\partial(v+I)}{\partial(x+I)} = 2y + 2I \,, \frac{\partial(v+I)}{\partial(y+I)} = 2x + 2I \end{cases}$$

$$\Rightarrow \begin{cases} \frac{\partial 2(u+I)}{\partial(x+I)^2} = 2 \,, \frac{\partial 2(u+I)}{\partial(y+I)^2} = -2 \\ \frac{\partial 2(v+I)}{\partial(x+I)^2} = 0 \,, \frac{\partial 2(v+I)}{\partial(y+I)^2} = 0 \end{cases}$$

We have:

$$\boldsymbol{\frac{\partial 2(u+I)}{\partial(x+I)^2} + \frac{\partial 2(u+I)}{\partial(y+I)^2} = 2 - 2 = 0}$$

The function $(\boldsymbol{u}+\boldsymbol{I})$ satisfy Laplac equation, so $(\boldsymbol{u}+\boldsymbol{I})$ is a neutrosophic harmonic Functions.

Similary we have:

$$\boldsymbol{\frac{\partial 2(v+I)}{\partial(x+I)^2} + \frac{\partial 2(v+I)}{\partial(y+I)^2} = 0 + 0 = 0}$$

The function $(\boldsymbol{v}+\boldsymbol{I})$ satisfy Laplac equation, so $(\boldsymbol{v}+\boldsymbol{I})$ is a neutrosophic harmonic Functions.

2- We have:

$$\begin{cases} \boldsymbol{\frac{\partial(u+I)}{\partial(x+I)} = \frac{\partial(v+I)}{\partial(y+I)}} \\ \boldsymbol{\frac{\partial(v+I)}{\partial(x+I)} = -\frac{\partial(u+I)}{\partial(y+I)}} \end{cases}$$

Then $(\boldsymbol{u}+\boldsymbol{I}), (\boldsymbol{v}+\boldsymbol{I})$ are satisfy Cauchy Riemann conditions, forever $(\boldsymbol{v}+\boldsymbol{I})$ is a harmonic conjugate by $(\boldsymbol{u}+\boldsymbol{I})$.

Example 7.4. Let $(\boldsymbol{u}+\boldsymbol{I}) = \boldsymbol{2}(\boldsymbol{x}+\boldsymbol{I}) - \boldsymbol{2}(\boldsymbol{x}+\boldsymbol{I})(\boldsymbol{y}+\boldsymbol{I})$. Finde Find the harmonic conjugate $(\boldsymbol{v}+\boldsymbol{I})$ and write $\boldsymbol{f}(\boldsymbol{z})$ by z .

Solution.

1- We prove the function $(\boldsymbol{u}+\boldsymbol{I})$ is a neutrosophic harmonic Function.

$$\frac{\partial(u+I)}{\partial(x+I)} = 2 - 2(y+I) \Longrightarrow \frac{\partial 2(u+I)}{\partial(x+I)^2} = 0$$

$$\frac{\partial(u+I)}{\partial(y+I)} = -2(x+I) \Longrightarrow \frac{\partial 2(u+I)}{\partial(y+I)^2} = 0$$

Then:

$$\frac{\partial 2(u+I)}{\partial(x+I)^2} + \frac{\partial 2(u+I)}{\partial(y+I)^2} = 0 + 0 = 0$$

Then $(\boldsymbol{u}+\boldsymbol{I})$ is a neutrosophic harmonic Function.

2- We use the first condition of Cauchy Riemann conditions. Then:

$$\frac{\boldsymbol{\partial}(\boldsymbol{u}+\boldsymbol{I})}{\boldsymbol{\partial}(\boldsymbol{x}+\boldsymbol{I})}=\frac{\boldsymbol{\partial}(\boldsymbol{v}+\boldsymbol{I})}{\boldsymbol{\partial}(\boldsymbol{y}+\boldsymbol{I})}\Rightarrow\frac{\boldsymbol{\partial}(\boldsymbol{v}+\boldsymbol{I})}{\boldsymbol{\partial}(\boldsymbol{y}+\boldsymbol{I})}=\mathbf{2}-\mathbf{2}(\boldsymbol{y}+\boldsymbol{I})\ \dots\dots(\mathbf{9})$$

3- We integral $(\mathbf{9})$ for $(\boldsymbol{y}+\boldsymbol{I})$, we have:

$$\int\frac{\partial(v+I)}{\partial(y+I)}d(y+I)=\int\big(2-2(y+I)\big)d(y+I)+\psi(x+I)$$

$$\Rightarrow(v+I)=2(y+I)-(y+I)^2+\psi(x+I)\ \dots\dots(10)$$

Where $\boldsymbol{\psi}(\boldsymbol{x}+\boldsymbol{I})$ is a constant integral.

4- We derivate $(\mathbf{10})$ by $(\boldsymbol{x}+\boldsymbol{I})$, we have:

$$\frac{\partial(v+I)}{\partial(x+I)}=\acute{\psi}(x+I)$$

5- We use the second condition of Cauchy Riemann conditions. Then:

$$\frac{\partial(v+I)}{\partial(x+I)}=-\frac{\partial(u+I)}{\partial(y+I)}\Rightarrow\acute{\psi}(x+I)=2(x+I)$$

By integrating the latter, we obtain:

$$\int\acute{\psi}(x+I)d(x+I)=\int 2(x+I)d(x+I)$$

$$\Rightarrow\psi(x+I)=(x+I)^2+a+bI$$

6- we obtain:

$$(\boldsymbol{v}+\boldsymbol{I})=\mathbf{2}(\boldsymbol{y}+\boldsymbol{I})-(\boldsymbol{y}+\boldsymbol{I})^{\mathbf{2}}+(\boldsymbol{x}+\boldsymbol{I})^{\mathbf{2}}+\boldsymbol{a}+\boldsymbol{b}\boldsymbol{I}$$

Now:

$$f(z)=(u+I)+i(v+I)$$
$$\Rightarrow f(z)=2(x+I)-2(x+I)(y+I)+i(2(y+I)-(y+I)^2+(x+I)^2+a+bI)$$
$$\Rightarrow f(z)=2(x+I)-2(x+I)(y+I)+i2(y+I)-i(y+I)^2+i(x+I)^2+i(a+bI)$$
$$\Rightarrow f(z)=2\big((x+I)+i(y+I)\big)+i\big((x+I)^2-(y+I)^2+i2(x+I)(y+I)\big)+i(a+bI)$$
$$\Rightarrow f(z)=2z+iz^2+i(a+bI).$$

Example 7.5. Let $(\boldsymbol{u}+\boldsymbol{I})=\boldsymbol{e}^{(x+I)}\,\mathbf{cos}(\boldsymbol{y}+\boldsymbol{I})$. Finde Find the harmonic conjugate $(\boldsymbol{v}+\boldsymbol{I})$ and write $\boldsymbol{f}(\boldsymbol{z})$ by z .

Solution.

1- We prove the function $(\boldsymbol{u}+\boldsymbol{I})$ is a neutrosophic harmonic Function.

$$\frac{\partial(u+I)}{\partial(x+I)}=e^{(x+I)}\cos(y+I)\Rightarrow\frac{\partial 2(u+I)}{\partial(x+I)^2}=e^{(x+I)}\cos(y+I)$$

$$\frac{\partial(u+I)}{\partial(y+I)} = -e^{(x+I)}\sin(y+I) \Rightarrow \frac{\partial 2(u+I)}{\partial(y+I)^2} = -e^{(x+I)}\cos(y+I)$$

Then:

$$\frac{\partial 2(u+I)}{\partial(x+I)^2} + \frac{\partial 2(u+I)}{\partial(y+I)^2} = e^{(x+I)}\cos(y+I) - e^{(x+I)}\cos(y+I) = 0$$

Then $(\boldsymbol{u}+\boldsymbol{I})$ is a neutrosophic harmonic Function.

2- We use the first condition of Cauchy Riemann conditions. Then:

$$\frac{\boldsymbol{\partial(u+I)}}{\boldsymbol{\partial(x+I)}} = \frac{\boldsymbol{\partial(v+I)}}{\boldsymbol{\partial(y+I)}} \Rightarrow \frac{\boldsymbol{\partial(v+I)}}{\boldsymbol{\partial(y+I)}} = \boldsymbol{e^{(x+I)}\cos(y+I)} \ldots\ldots(\mathbf{11})$$

3- We integral $(\mathbf{11})$ for $(\boldsymbol{y}+\boldsymbol{I})$, we have:

$$\int \frac{\partial(v+I)}{\partial(y+I)} d(y+I) = \int \left(e^{(x+I)}\cos(y+I)\right) d(y+I) + \psi(x+I)$$

$$\Rightarrow (v+I) = e^{(x+I)}\sin(y+I) + \psi(x+I) \ldots\ldots(12)$$

Where $\boldsymbol{\psi(x+I)}$ is a constant integral.

4- We derivate $(\mathbf{12})$ by $(\boldsymbol{x}+\boldsymbol{I})$, we have:

$$\frac{\partial(v+I)}{\partial(x+I)} = e^{(x+I)}\sin(y+I) + \acute{\psi}(x+I)$$

5- We use the second condition of Cauchy Riemann conditions. Then:

$$\frac{\partial(v+I)}{\partial(x+I)} = -\frac{\partial(u+I)}{\partial(y+I)}$$

$$\Rightarrow -e^{(x+I)}\sin(y+I) - \acute{\psi}(x+I) = -e^{(x+I)}\sin(y+I)$$

$$\Rightarrow \acute{\psi}(x+I) = 0$$

By integrating the latter, we obtain:

$$\int \acute{\psi}(x+I) d(x+I) = \int (0) d(x+I)$$

$$(x+I) \Rightarrow \psi = a + bI$$

6- we obtain:

$$(\boldsymbol{v}+\boldsymbol{I}) = \boldsymbol{e^{(x+I)}\sin(y+I) + a + bI}$$

Now:

$$f(z) = (u+I) + i(v+I)$$

$$\Rightarrow f(z) = e^{(x+I)}\cos(y+I) + i\left(e^{(x+I)}\sin(y+I) + a + bI\right)$$

$$\Rightarrow f(z) = e^{(x+I)}\cos(y+I) + ie^{(x+I)}\sin(y+I) + i(a+bI)$$

$$\Rightarrow f(z) = e^{(x+I)}(\cos(y+I) + i\sin(y+I)) + i(a+bI)$$

$\Longrightarrow f(z) = e^{(x+I)}e^{i(y+I)} + i(a+bI)$

$\Longrightarrow f(z) = e^{(x+I)+i(y+I)} + i(a+bI)$

$\Longrightarrow f(z) = e^{z} + i(a+bI).$

Example 7.6. Let $(\boldsymbol{u}+\boldsymbol{I}) = \boldsymbol{e}^{(y+I)}\,\mathbf{cos}(\boldsymbol{x}+\boldsymbol{I})$. Finde Find the harmonic conjugate $(\boldsymbol{v}+\boldsymbol{I})$ and write $\boldsymbol{f}(\boldsymbol{z})$ by z , and find $\acute{\boldsymbol{f}}(\boldsymbol{z})$.

Solution.

1- We prove the function $(\boldsymbol{u}+\boldsymbol{I})$ is a neutrosophic harmonic Function.

$$\frac{\partial(u+I)}{\partial(x+I)} = -e^{(y+I)}\sin(x+I) \Longrightarrow \frac{\partial 2(u+I)}{\partial(x+I)^2} = -e^{(y+I)}\cos(x+I)$$

$$\frac{\partial(u+I)}{\partial(y+I)} = e^{(y+I)}\cos(x+I) \Longrightarrow \frac{\partial 2(u+I)}{\partial(y+I)^2} = e^{(y+I)}\cos(x+I)$$

Then:

$$\frac{\partial 2(u+I)}{\partial(x+I)^2} + \frac{\partial 2(u+I)}{\partial(y+I)^2} = -e^{(y+I)}\cos(x+I) + e^{(y+I)}\cos(x+I) = 0$$

Then $(\boldsymbol{u}+\boldsymbol{I})$ is a neutrosophic harmonic Function.

2- We use the first condition of Cauchy Riemann conditions. Then:

$$\frac{\boldsymbol{\partial(u+I)}}{\boldsymbol{\partial(x+I)}} = \frac{\boldsymbol{\partial(v+I)}}{\boldsymbol{\partial(y+I)}} \Longrightarrow \frac{\boldsymbol{\partial(v+I)}}{\boldsymbol{\partial(y+I)}} = -\boldsymbol{e}^{(y+I)}\,\mathbf{sin}(\boldsymbol{x}+\boldsymbol{I}) \ldots\ldots(\mathbf{13})$$

3- We integral $(\mathbf{13})$ for $(\boldsymbol{y}+\boldsymbol{I})$, we have:

$$\int \frac{\partial(v+I)}{\partial(y+I)} d(y+I) = \int \left(-e^{(y+I)}\sin(x+I)\right) d(y+I) + \psi(x+I)$$

$$\Longrightarrow (v+I) = -e^{(y+I)}\sin(x+I) + \psi(x+I) \ldots\ldots(14)$$

Where $\boldsymbol{\psi}(\boldsymbol{x}+\boldsymbol{I})$ is a constant integral.

4- We derivate $(\mathbf{14})$ by $(\boldsymbol{x}+\boldsymbol{I})$, we have:

$$\frac{\partial(v+I)}{\partial(x+I)} = -e^{(y+I)}\cos(x+I) + \acute{\psi}(x+I)$$

5- We use the second condition of Cauchy Riemann conditions. Then:

$$\frac{\partial(v+I)}{\partial(x+I)} = -\frac{\partial(u+I)}{\partial(y+I)}$$

$$\Longrightarrow e^{(y+I)}\cos(x+I) - \acute{\psi}(x+I) = e^{(y+I)}\cos(x+I)$$

$$\Longrightarrow \acute{\psi}(x+I) = 0$$

By integrating the latter, we obtain:

$$\int \acute{\psi}(x+I)d(x+I) = \int (0)d(x+I)$$

$\Rightarrow \psi(x+I) = a + bI$

6- we obtain:

$(\boldsymbol{v}+\boldsymbol{I}) = -\boldsymbol{e}^{(y+I)}\,\mathbf{sin}(\boldsymbol{x}+\boldsymbol{I}) + \boldsymbol{a} + \boldsymbol{bI}$

Now:

$f(z) = (u+I) + i(v+I)$
$\Rightarrow f(z) = e^{(y+I)}\cos(x+I) + i\left(-e^{(y+I)}\sin(x+I) + a + bI\right)$
$\Rightarrow f(z) = e^{(y+I)}\cos(x+I) - ie^{(y+I)}\sin(x+I) + i(a+bI)$
$\Rightarrow f(z) = e^{(y+I)}\left(\cos(x+I) - ie^{(y+I)}\sin(x+I)\right) + i(a+bI)$
$\Rightarrow f(z) = e^{(y+I)}e^{-i(x+I)} + i(a+bI)$
$\Rightarrow f(z) = e^{(y+I)-i(x+I)} + i(a+bI)$
$\Rightarrow f(z) = e^{-i\left((x+I)+i(y+I)\right)} + i(a+bI)$
$\Rightarrow f(z) = e^{-iz} + i(a+bI).$

Now:

$$\acute{f}(z) = \frac{\partial(u+I)}{\partial(x+I)} + i\frac{\partial(v+I)}{\partial(x+I)}$$
$\Rightarrow \acute{f}(z) = -e^{(y+I)}\sin(x+I) - ie^{(y+I)}\cos(x+I)$
$$\Rightarrow \acute{f}(z) = -ie^{(y+I)}\left(\cos(x+I) + \frac{1}{i}\sin(x+I)\right)$$
$\Rightarrow \acute{f}(z) = -ie^{(y+I)}(\cos(x+I) - i\sin(x+I))$
$\Rightarrow \acute{f}(z) = -ie^{(y+I)}e^{-i(x+I)}$
$\Rightarrow \acute{f}(z) = -ie^{(y+I)}e^{-i(x+I)}$
$\Rightarrow \acute{f}(z) = -ie^{-i\left((x+I)+i(y+I)\right)}$
$\Rightarrow f(z) = -ie^{-iz}$

Example 7.7. Find the value of $\boldsymbol{\alpha},\boldsymbol{\beta}$ for the function:

$$(\boldsymbol{u}+\boldsymbol{I}) = \boldsymbol{\alpha}(\boldsymbol{x}+\boldsymbol{I})^2(\boldsymbol{y}+\boldsymbol{I}) + \boldsymbol{\beta}(\boldsymbol{y}+\boldsymbol{I})^2 - \mathbf{3}(\boldsymbol{y}+\boldsymbol{I})^3 + \mathbf{2}(\boldsymbol{x}+\boldsymbol{I})^2$$

is a harmonic function. And finde Find the harmonic conjugate $(\boldsymbol{v}+\boldsymbol{I})$ and write $\boldsymbol{f}(\boldsymbol{z})$ by z , and find $\acute{\boldsymbol{f}}(\boldsymbol{z})$.

Solution.

The function $(\boldsymbol{u}+\boldsymbol{I})$ is a harmonic function is it satisfay the Laplac equation.

$$\frac{\partial 2(u+I)}{\partial(x+I)^2} + \frac{\partial 2(u+I)}{\partial(y+I)^2} = 0$$

Now we have:

$$\frac{\partial 2(u+I)}{\partial(x+I)^2} + \frac{\partial 2(u+I)}{\partial(y+I)^2} = 0$$

$$\frac{\partial(u+I)}{\partial(x+I)} = 2\alpha(x+I)(y+I) + 4(x+I)$$

$$\Rightarrow \frac{\partial 2(u+I)}{\partial(x+I)^2} = 2\alpha(y+I) + 4$$

$$\frac{\partial(u+I)}{\partial(y+I)} = \alpha(x+I)^2 + 2\beta(y+I) - 9(y+I)^2$$

$$\Rightarrow \frac{\partial 2(u+I)}{\partial(y+I)^2} = 2\beta - 18(y+I)$$

$$\frac{\partial 2(u+I)}{\partial(x+I)^2} + \frac{\partial 2(u+I)}{\partial(y+I)^2} = 0 \Rightarrow 2\alpha(y+I) + 4 + 2\beta - 18(y+I) = 0$$

$$\Rightarrow (2\alpha - 18)(y+I) + 4 + 2\beta = 0 = 0(y+I) + 0$$

Then, we have:

$$\begin{cases} \mathbf{2\alpha - 18 = 0} \\ \mathbf{4 + 2\beta = 0} \end{cases} \Rightarrow \boldsymbol{\alpha = 9\,, \beta = -2}$$

Then:

$$\boldsymbol{(u+I) = 9(x+I)^2(y+I) - 2(y+I)^2 - 3(y+I)^3 + 2(x+I)^2}$$

Now a harmonic conjugate:

$$\frac{\partial(u+I)}{\partial(x+I)} = 18(x+I)(y+I) + 4(x+I)$$

$$\frac{\partial(u+I)}{\partial(y+I)} = 9(x+I)^2 - 4(y+I) - 9(y+I)^2$$

1- We use the first condition of Cauchy Riemann conditions. Then:

$$\boldsymbol{\frac{\partial(u+I)}{\partial(x+I)} = \frac{\partial(v+I)}{\partial(y+I)} \Rightarrow \frac{\partial(v+I)}{\partial(y+I)} = 18(x+I)(y+I) + 4(x+I) \dots\dots (15)}$$

2- We integral $\mathbf{(15)}$ for $\boldsymbol{(y+I)}$, we have:

$$\int \frac{\partial(v+I)}{\partial(y+I)} d(y+I) = \int \big(18(x+I)(y+I) + 4(x+I)\big) d(y+I) + \psi(x+I)$$

$$\Rightarrow (v+I) = 9(y+I)^2(x+I) + 4(x+I)(y+I) + \psi(x+I) \dots\dots (16)$$

Where $\boldsymbol{\psi(x+I)}$ is a constant integral.

3- We derivate $\mathbf{(16)}$ by $\boldsymbol{(x+I)}$, we have:

$$\frac{\partial(v+I)}{\partial(x+I)} = -9(y+I)^2 + 4(y+I) + \acute{\psi}(x+I)$$

4- We use the second condition of Cauchy Riemann conditions. Then:

$$\frac{\partial(v+I)}{\partial(x+I)} = -\frac{\partial(u+I)}{\partial(y+I)}$$

$$\Rightarrow 9(x+I)^2 - 4(y+I) - 9(y+I)^2 = -9(y+I)^2 - 4(y+I) - \acute{\psi}(x+I)$$
$$\Rightarrow \acute{\psi}(x+I) = -9(x+I)^2$$

By integrating the latter, we obtain:

$$\int \acute{\psi}(x+I)d(x+I) = \int -9(x+I)^2 d(x+I)$$
$$\Rightarrow \psi(x+I) = -3(x+I)^3 + a + bI$$

5- we obtain:

$$\boldsymbol{(v+I) = 9(y+I)^2(x+I) + 4(x+I)(y+I) - 3(x+I)^3 + a + bI}$$

Now:

$$f(z) = (u+I) + i(v+I)$$
$$\Rightarrow f(z) = 9(x+I)^2(y+I) - 2(y+I)^2 - 3(y+I)^3 + 2(x+I)^2 + i(9(y+I)^2(x+I) + 4(x+I)(y+I) - 3(x+I)^3 + a + bI)$$
$$\Rightarrow f(z) = 9(x+I)^2(y+I) - 2(y+I)^2 - 3(y+I)^3 + 2(x+I)^2 + i9(y+I)^2(x+I) + i4(x+I)(y+I) - i3(x+I)^3 + i(a+bI)$$
$$\Rightarrow f(z) = 2\big((x+I)^2 - (y+I)^2 + i(x+I)(y+I)\big) - i3(x+I)^3 + i^2 3(y+I)^3 - i^2 9(x+I)^2(y+I) + i9(y+I)^2(x+I) + i(a+bI)$$
$$\Rightarrow f(z) = 2\big((x+I) + i(y+I)\big)^2 - 3i\big((x+I)^3 - i(y+I)^3 + 3i(x+I)^2(y+I) - 3i(y+I)^2(x+I)\big) + i(a+bI)$$
$$\Rightarrow f(z) = 2\big((x+I) + i(y+I)\big)^2 - 3i\big((x+I) + i(y+I)\big)^3 + i(a+bI)$$
$$\Rightarrow f(z) = 2z^2 - 3iz^3 + i(a+bI)$$

Now:

$$\acute{f}(z) = \frac{\partial(u+I)}{\partial(x+I)} + i\frac{\partial(v+I)}{\partial(x+I)}$$
$$\Rightarrow \acute{f}(z) = 18(x+I)(y+I) + 4(x+I) + i(9(y+I)^2 + 4(y+I) - 9(x+I)^2)$$
$$\Rightarrow \acute{f}(z) = 18(x+I)(y+I) + 4(x+I) + i9(y+I)^2 + i4(y+I) - i9(x+I)^2$$
$$\Rightarrow \acute{f}(z) = 4\big((x+I) + i(y+I)\big) - i9\big((x+I)^2 - (y+I)^2 + 2i(x+I)(y+I)\big)$$
$$\Rightarrow \acute{f}(z) = 4\big((x+I) + i(y+I)\big) - i9\big((x+I) + i(y+I)\big)^2$$
$$\Rightarrow \acute{f}(z) = 4z - 9iz^2$$

8. Conclusion

In this paper, a new type of complex functions has been defined by using the neutrosophic real number and neutrosophic complex number, Moreover, we studied a harmonic function, harmonic conjugate, and Cauchy Riemann conditions. Also solutions of other types of neutrosophic complex equations can be found depending on the complex numbers. We will work on this in the future.

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