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Article

On Neutrosophic Graph

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Abstract: This paper is devoted for presenting new neutrosophic similarity measures between neutrosophic graphs. We propose two ways to determine the neutrosophic distance between neutrosophic vertex graphs. The two neutrosophic distances are based on the Haussdorff distance, and a robust modified variant of the Haussdorff distance, moreover we show that they both satisfy the metric distance measure axioms. Furthermore, a similarity measure between neutrosophic edge graphs, that is based on a probabilistic variant of Haussdorff distance, is introduced. The aim is to use those measures for the purpose of matching neutrosophic graphs whose structure can be described in the neutrosophic domain.

Keywords: Neutrosophic Graphs, Haussdorff Distance, Graph Matching.

1. Introduction

Graphs are essential for encoding information, which may serve in several fields ranging from computational biology to computer vision. The notion of graph theory was first introduced by Euler in 1736, given a graph where vertices and edges represent pairwise interactions between entities [2, 5]. The past years have witnessed a high development in the areas of the applications of graphs of pattern recognition and computer vision, where graphs are the most powerful and handy tool used in representing both objects and concepts. The invariance properties, as well as the fact that graphs are well suited to model objects in terms of parts and their relations, make them very attractive for various applications. Hence, the theory of graph became an extremely useful tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, operations research, optimization and computer science [1]. In 1975, a fuzzy graph theory as a generalization of Euler's graph theory was introduced by Rosenfeld [7], based on the concepts of fuzzy set theory proposed by Zadeh in 1965 [19].

In a world full of indeterminacy, traditional crisp set with its boundaries of truth and false has not infused itself with the ability of reflecting the reality. Therefore, neutrosophic found its place into contemporary research as an alternative representation of the real world. Established by Florentin Smarandache [16], Neutrosophy was presented as the study of "the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra". The main idea was to consider an entity ″A″ in relation to its opposite ″Non-A″, and to that which is neither "A″ nor″ Non-A″, denoted by "Neut-A″. From then on, Neutrosophy became the basis of Neutrosophic Logic, Neutrosophic Probability, Neutrosophic Set Theory, and Neutrosophic Statistics. According to this theory every idea ″A″ tends to be neutralized and balanced by ″neut-A″ and ″non- A″ ideas - as a state of equilibrium. In a classical way "A", "neut-A", "anti-A" are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise or sorties, it is possible that "A", "neut-A" and "anti-A" have common parts two by two, or even all three of them as well. In [16, 17], Smarandache introduced the fundamental concepts of neutrosophic set, that had led Salama and Smarandache [15], to provide a mathematical treatment for the neutrosophic phenomena which already existed in our real world. Moreover the work of Salama and Smarandache[15,16,17] formed a starting point to construct new branches of neutrosophic mathematics. Hence, Neutrosophic set theory turned out to be a generalization of both the classical and fuzzy counterparts.

In [6,11,12,13], the authors gave a new dimension for the graph theory using the concept of neutrosophy, some study for different types of neutrosophic graphs were presented and some of their properties were investigated. The aim of this paper is to compute the dissimilarity between two graphs, our methodology is based on the Haussdorff distance, which is invariant to rotation. Whereas several neutrosophic distances where introduced in [4, 14], the authors constructed the neutrosophic distance between neutrosophic sets. The remaining of the paper is structured as follows: definitions of neutrosophic sets and graphs are presented in §2 and §3. Whereas, §4 introduces the idea behind the Haussdorff distance between two crisp sets. In §5.2 and §5.3, we propose two new neutrosophic dissimilarity measures between neutrosophic vertex graphs based on the classical and the modified Haussdorff distances. Furthermore, we investigate the metric axioms for the obtained distances. A neutrosophic similarity measure between neutrosophic edge graphs, based on a probabilistic variant of Haussdorff distance, is introduced in §5.3.

2. Neutrosophic Sets

let X be a space of points (objects), with a generic element in X denoted by x, a neutrosophic set A in X is characterized by a truth-membership function T, a indeterminacy-membership function I and a falsity-membership function F [15, 18], That is: T, I, F: x →] -0, 1+ [.

Where T (x), I(x) and F (x) are real standard or non-standard subsets of] -0, 1+ [.

In general if there is no restriction on the sum of T (x), I(x) and F (x), so $0^{-} \leq$ T(x) + I (x) + F (x) $\leq 3^{+}$. T, I, F are called neutrosophic components. In this paper we will restrict our work to use the standard unit interval [0, 1].

3. Neutrosophic Graphs

In [6], the authors defined the neutrosophic graph, to be a graph G < V, E > combined with six mappings, written in the form $G_N = < V, E, T_e, I_e, F_e, T_v, I_v, F_v >$, where

T_v :V→ [0,1] , I_v :V→ [0,1], F_v :V→ [0,1] denoting the degree of membership ,degree of indeterminacy and non- membership of the element vi ∈V respectively and 0 ≤ T_v (vi) + I_v (vi) + F_v (vi) ≤ 3 for every vi ∈ V, (i = 1, 2, ….. , n) , and
T_e :V × V →[0,1], I_e : V × V → [0,1] and F_e : V × V →[0,1] are such that $T_e(v_i, v_j) \leq \min(T_v(v_i), T_v(v_j))$, $I_e(v_i, v_j) \leq \min(I_v(v_i), I_v(v_j))$ and $F_e(v_i, v_j) \leq \min(F_v(v_i), F_v(v_j))$ and $0 \leq T_e(v_i, v_j) + I_e(v_i, v_j) + F_e(v_i, v_j) \leq 3$ for every $(v_i, v_j) \in$ E (i, j = 1, 2, 3, …., n).

The concept of neutrosophic graph was used by several authors; nevertheless they took different points of view when describing the interpretation of graph neutrosophy. We constructed the following structure depending on the one given in [6, 12].

3.1. Neutrosophic Edge Graphs

A neutrosophic graph is defined as a graph combined with three mappings, written as $G = (V, E, T_e, I_e, F_e)$, where T_e:V× V →[0,1], I_e: V× V → [0,1] and F_e: V× V →[0,1] are such that $T_e(v_i, v_j) \leq \min(T_v(v_i), T_v(v_j))$, $I_e(v_i, v_j) \leq \min(I_v(v_i), I_v(v_j))$ and $F_e(v_i, v_j) \leq \min(F_v(v_i), F_v(v_j))$ and $0 \leq T_e(v_i, v_j) + I_e(v_i, v_j) + F_e(v_i, v_j) \leq 3$ for every $(v_i, v_j) \in$ E (i , j= 1, 2, 3, …., n).

3.2. Neutrosophic Vertex Graphs

The term neutrosophic vertex graph was used to define a graph of the form: $G = (V\ ,E\ ,T_v,\ I_v\ ,F_v\ )$ combined with three mappings, written as T_v:V→ [0,1], I_v:V→ [0,1], F_v:V→ [0,1] denoting the degree of membership, degree of indeterminacy and non- membership of the element vi ∈ V respectively and $0 \le T_v$ (vi) + I_v (vi) + F_v (vi) ≤ 3 for every vi ∈ V, (i = 1, 2, ….. , n).

4. Haussdorff distance

Since first introduced by Haussdorff in 1914 [8], the Haussdorff distance has been used in several areas including matching and recognition problems. It provides a means of computing the distance between sets of unordered observations when the correspondences between the individual items are unknown. In its most general setting, the Haussdorff distance measures how far two subsets of a metric space are from each other. It turns the set of non-empty compact subsets of a metric space into a metric space in its own right. Given two such sets, the closest point in the second set for each point in the first set is considered. Hence, the Haussdorff distance is the maximum over all these values. More formally, the classical Haussdorff distance (H D) [4, 10], between two finite point sets A and B is given by:

$$\mathrm{H(A,B)} = \max\big(h(A,B), h(B,A)\big)$$

Where the directed Haussdorff distance from A to B is defined to be:

$$h(A,B) = \max_{a\in A} \min_{b\in B} \| a - b \|$$

And $\|.\|$ is some underlying norm on the points of A and B (e.g., the L_2 or Euclidean norm). Regardless of the norm, the Haussdorff metric captures the notion of the worst match between two objects. The computed value is the largest distance between a point in one set and a point in the other one. Several variants of the Haussdorff distance have been proposed as alternatives to the maximum of the minimum approach in the classical one, like Haussdorff fraction, Haussdorff quintile [10] and Spatially Coherent Matching [3].

A robust modified Haussdorff distance (MHD) based on the average distance value instead of the maximum value was proposed by Dubuisson and Jain [7], in this sense they defined the directed distance of the MHD as:

$$MH(A,B) = \frac{1}{N_A} \sum_{a\in \mathrm{A}} \min_{\mathrm{b}\in\mathrm{B}} \|\mathrm{a} - \mathrm{b}\|$$

5. Neutrosophic Graph Similarity Measures

In this section, we introduce neutrosophic graph similarity measures, based on the concept of Haussdorff distance and some of its variants.

Firstly, we propose two new neutrosophic dissimilarity measures based on the classical and the modified Haussdorff distances [4, 6, 14]. Basically the neutrosophic dissimilarity measure is a triple: the first part is a dissimilarity measure of the true value of the neutrosophic object, the second part is a dissimilarity measure of the indeterminate value of the neutrosophic object, and the third part is a dissimilarity measure of the false value of the neutrosophic object; that is the opposite of the neutrosophic object. Secondly, we propose a new neutrosophic similarity measure based on the probabilistic Haussdorff distance [9]. With a similar structure, the neutrosophic similarity measure is also a triple as the explained in the neutrosophic dissimilarity measure. Obviously, if the indeterminate part does not exist (its measure is zero) and if the measure of the opposite object is ignored the suggested neutrosophic dissimilarity measure is reduced to the concept of Haussdorff distance in the fuzzy sense.

5.1 Neutrosophic Haussdorff Distance

To commence, we consider two neutrosophic vertex graphs $G_1 = (V_1 \ , E_1 \ , T_{v1}, \ I_{v1} \ , F_{v1} \ )\ and\ G_2 = (V_2 \ , E_2 \ , T_{v2}, \ I_{v2} \ , F_{v2} \ )$, where V_i, i = 1, 2 are the sets of nodes, E_i, where i= 1, 2 are the sets of edges and T_{vi}, I_{vi}, F_{vi}, where i = 1, 2 are the matrices whose elements are the true, indeterminate and false values defined for each element of V_i, i = 1, 2, respectively. We can now write the distances between the two neutrosophic vertex graphs G_1, G_2 as follows:

$$NGD\ (G_1 \ , G_2) = (T_{NGD}\ (G_1, G_2), I_{NGD}\ (G_1, G_2), F_{NGD}(G_1, G_2))$$

Where,

$$T_{NGD}\ (G_1, G_2) = \max(T_{NGd}(G_1, G_2), T_{NGd}(G_2, G_1))$$
$$I_{NGD}\ (G_1, G_2) = \max(I_{NGd}(G_1, G_2), I_{NGd}(G_2, G_1))$$
$$F_{\mathrm{NGD}}\ (\mathrm{G}_1, \mathrm{G}_2) = \max(F_{NGd}(\mathrm{G}_1, \mathrm{G}_2), \mathrm{F}_{\mathrm{NGd}}(\mathrm{G}_2, \mathrm{G}_1))$$

And

$$T_{\mathrm{NGd}}(G_1, \mathrm{G}_2) = \max_{\mathrm{i}\epsilon \mathrm{V}_1} \max_{\mathrm{j}\epsilon \mathrm{V}_1} \min_{\mathrm{I}\epsilon \mathrm{V}_2} \min_{\mathrm{J}\epsilon \mathrm{V}_2} \left\| \mathrm{T}_{v_2}(\mathrm{I}, \mathrm{J}) - \mathrm{T}_{v_1}(\mathrm{i}, \mathrm{j}) \right\|$$
$$I_{\mathrm{NGd}}(\mathrm{G}_1, \mathrm{G}_2) = \max_{\mathrm{i}\epsilon \mathrm{V}_1} \max_{\mathrm{j}\epsilon \mathrm{V}_1} \frac{1}{|\mathrm{V}_2 \ | \times |\mathrm{V}_2|} \sum_{\mathrm{I}\epsilon \mathrm{V}_2} \sum_{\mathrm{J}\epsilon \mathrm{V}_2} \left\| \mathrm{I}_{v_2}(\mathrm{I}, \mathrm{J}) - \mathrm{I}_{v_1}(\mathrm{i}, \mathrm{j}) \right\|$$
$$F_{\mathrm{NGd}}(\mathrm{G}_1, \mathrm{G}_2) = \min_{\mathrm{i}\epsilon \mathrm{V}_1} \min_{\mathrm{j}\epsilon \mathrm{V}_1} \max_{\mathrm{I}\epsilon \mathrm{V}_2} \max_{\mathrm{J}\epsilon \mathrm{V}_2} \left\| \mathrm{F}_{v_2}(\mathrm{I}, \mathrm{J}) - \mathrm{F}_{v_1}(\mathrm{i}, \mathrm{j}) \right\|$$

$NGd(G_2, G_1)$ can be computed in a similar way.

Proposition 1: The Neutrosophic vertex graph distance NGD satisfies the metric distance measure axioms:

A1) (Symmetry): NGD (G_1, G_2) = NGD (G_2, G_1),
A2) (Non-negativity): NGD $(G_1, G_2) \geq 0$,
A3) (Coincidence): if NGD (G_1, G_2) =0 then $G_1 = G_2$,
A4) (Triangle Inequality): for any three neutrosophic vertex graphs G_1, G_2 and G_3 we have: NGD $(G_1, G_2) \leq$: NGD (G_1, G_2) + NGD (G_2, G_3).

Poof: A1 and A2 can easily be proven.

A3): When NGD(G_1, G_2) = $(T_{NGD}(G_1, G_2), I_{NGD}(G_1, G_2), F_{NGD}(G_1, G_2))$ = (0,0,0), that is every component of the triple which is the maximum of two positive values is zero, the values of $T_{NGd}(G_i, G_j)$,$I_{NGd}(G_i, G_j)$ and $F_{NGd}(G_i, G_j)$ for i, j= 1, 2 are all zeros. Namely the maximum distance among the nearest nodes in both G_1, G_2 is zero. That means that the distance between each element of V_1 and its nearest element in the set V_2 is zero. That is each element in V_1 coincides with an element in V_2 and vice versa; hence $V_1 = V_2$.

A4): Consider any three neutrosophic graphs $G_1 = (V_1 \ , E_1 \ , T_1, \ I_1 \ , F_1 \ )$, $G_2 = (V_2 \ , E_2 \ , T_2, \ I_2 \ , F_2 \ ) and\ G_3 = (V_3 \ , E_3 \ , T_3, \ I_3 \ , F_3 \ )$. For any $i_k, j_k \in V_k$, k = 1, 2, 3, we can easily see that:

$$\| T_3(i_3, j_3) - T_1(i_1, j_3)\| \leq \|T_3(i_3, j_3) - T_2(i_2, j_2)\| + \|T_2(i_2, j_2) - T_1(i_1, j_1)\|$$

where the values $T_K(i_K, j_K)$, K=1, 2, 3, lye in the interval [0, 1]. Consequently, one can show that:

$$\max_{i_1\epsilon V_1} \max_{j_1\epsilon V_1} \min_{i_3\epsilon V_3} \min_{j_3\epsilon V_3} \|T_3(i_3, j_3) - T_1(i_1, j_3)\|$$
$$\leq \max_{i_2\epsilon V_2} \max_{j_2\epsilon V_2} \min_{i_3\epsilon V_3} \min_{j_3\epsilon V_3} \|T_3(i_3, j_3) - T_2(i_2, j_2)\|$$
$$+ \max_{i_1\epsilon V_1} \max_{j_1\epsilon V_1} \min_{i_2\epsilon V_2} \min_{j_2\epsilon V_2} \|T_2(i_2, j_2) - T_1(i_1, j_1)\|$$

That is: $T_{NGd}(G_1, G_3) \leq T_{NGd}(G_2, G_3) + T_{NGd}(G_1, G_2)$ and similarly $T_{NGd}(G_3, G_1) \leq T_{NGd}(G_3, G_2) + T_{NGd}(G_2, G_1)$

Hence, max $(T_{NGd}(G_1, G_3), T_{NGd}(G_3, G_1)) \leq$ max $(T_{NGd}(G_2, G_3), T_{NGd}(G_3, G_2))$ + max $(T_{NGd}(G_1, G_2)$,$T_{NGd}(G_2, G_1))$. Then, $T_{NGD}(G_1, G_3) \leq T_{NGD}(G_1, G_2) + T_{NGD}(G_2, G_3)$.

The same procedure goes for both I_{NGD} andF_{NGD}. That leads to

NGD $(G_1, \ G_3) \ \leq$ NGD (G_1, G_2) + NGD (G_2, G_3).

5.2 Modified Neutrosophic Haussdorff Distance

Consider two neutrosophic vertex graphs $G_1 = (V_1 \ ,E_1 \ ,T_{v1},\ I_{v1} \ ,F_{v1})$ and $G_2 = (V_2 \ ,E_2 \ ,T_{v2},\ I_{v2} \ ,F_{v2})$, where V_i, i= 1, 2 are the sets of nodes, E_i, where i= 1, 2 are the sets of edges and T_{vi}, I_{vi}, F_{vi} , where i= 1, 2 are the matrices whose elements are the true, indeterminate and false values defined for each element of V_i, i = 1, 2, respectively. We can now write the distances between the two neutrosophic vertex graphs G_1, G_2 as follows:

MNGD $(G_1,G_2) = (T_{MNGD}(G_1,G_2), I_{MNGD}(G_1,G_2), F_{MNGD}(G_1,G_2))$

Where,

$T_{\text{MNGD}}(G_1,G_2) = \max(T_{MNGd}(G_1,G_2), T_{MNGd}(G_2,G_1))$

$I_{MNGD}(G_1,G_2) = \max(I_{MNGd}(G_1,G_2), I_{MNGd}(G_2,G_1))$

$F_{\text{MNGD}}(G_1,G_2) = \max(F_{MNGd}(G_1,G_2), F_{MNGd}(G_2,G_1))$

And,

$$\text{T}_{\text{MNGd}}(G_1,G_2) = \frac{1}{|V_1| \times |V_1|}\sum_{i\in v_1}\sum_{j\in v_1} \min_{i\in v_2}\min_{j\in v_2}\|T_2(I,J) - T_1(i,j)\|$$

$$\text{I}_{\text{MNGd}}(G_1,G_2) = \frac{1}{|V_1| \times |V_1|}\sum_{i\in v_1}\sum_{j\in v_1} \frac{1}{|V_2| \times |V_2|}\sum_{I\in V_2}\sum_{J\in V_2}\|T_2(I,J) - T_1(i,j)\|.$$

$$\text{F}_{\text{MNGd}}(\text{G}_1,\text{G}_2) = \frac{1}{|\text{V}_1| \times |\text{V}_1|}\sum_{\text{i}\in \text{V}_1}\sum_{j\in V_1} \max_{I\in V_2}\max_{J\in V_2}\|F_2(I,J) - F_1(i,j)\|$$

Similarly, we can find MNGd (G_2,G_1).

Proposition 2: The Modified Neutrosophic vertex graph distance MNGD satisfies the metric distance measure axioms:

AA1) (symmetry): MNGD (G_1,G_2) = MNGD (G_2,G_1),

AA2) (non-negativity): MNGD $(G_1,G_2) \geq 0$,

AA3) (coincidence): if MNGD $(G_1,G_2) = 0$ then $G_1 = G_2$,

AA4) (triangle inequality): for any three neutrosophic vertex graphs G_1,G_2 and G_3 we have:

MNGD $(G_1,G_3) \leq$ MNGD (G_1,G_2) + M NGD (G_2,G_3).

Proof: Similar to the procedure used to prove Proposition 1.

5.3 Probabilistic Neutrosophic Haussdorff Distance

To overcome the robustness of both the classical and the modified Haussdorff distance, Hue and Hancock [9] have developed a probabilistic variant of the Haussdorff distance. This measure the similarity of the set of attributes rather than using defined set based distance measures. To commence, we recall two edge graphs $G_1 = (V_1 \ ,E_1 \ ,T_{e_1},\ I_{e_1} \ ,F_{e_1})$, $G_2 = (V_2 ,E_2 ,T_{e_2},\ I_{e_2} ,F_{e_2})$ as mentioned before, the set of all nodes connected to the node I $\in G_2$ by an edge is defined as:

$C_I^2 = \{J|(I,J) \in E_2\}$, and the corresponding set of nodes connected to the node $i \in G_1$ by an edge $C_i^1 = \{j|(i,j) \in E_1\}$. A measure for the match of the graph G_2 onto G_1 is:

$PNGD\ (G_1\ ,G_2) = (T_{PNGD}\ (G_1,G_2), I_{PNGD}\ (G_1,G_2), F_{PNGD}(G_1,G_2))$

where, $T_{PNGD}\ (G_1,G_2) = \max(T_{PNGd}(G_1,G_2), T_{PNGd}(G_2,G_1))$

$I_{PNGD}\ (G_1,G_2) = \max(I_{PNGd}(G_1,G_2), I_{PNGd}(G_2,G_1))$

$F_{\text{PNGD}}\ (\text{G}_1,\text{G}_2) = \max(F_{PNGd}(\text{G}_1,\text{G}_2), \text{F}_{\text{PNGd}}(\text{G}_2,\text{G}_1))$

and, $T_{PNGd}(G_1,G_2) = \frac{1}{|V_2|\times|V_1|}\sum_{i\in V_1}\sum_{j\in C_i^1}\max_{I\in V_2}\max_{J\in C_I^2} P((i,j) \to (I,J)\ |T_{e_2}(I,J), T_{e_1}(i,j))$

$$I_{PNGd}(G_1,G_2) = \frac{1}{|V_2| \times |V_1|}\sum_{i\in V_1}\sum_{j\in C_i^1}\max_{I\in V_2}\max_{J\in C_I^2} P((i,j) \to (I,J)\ |I_{e_2}(I,J), I_{e_1}(i,j))$$

$$F_{PNGd}(G_1,G_2) = \frac{1}{|V_2| \times |V_1|}\sum_{i\in V_1}\sum_{j\in C_i^1}\min_{I\in V_2}\min_{J\in C_I^2} P((i,j) \to (I,J)\ |F_{e_2}(I,J), F_{e_1}(i,j))$$

In this formula the posteriori probability $P((i,j) \to (I,J) \to (I,J)|T_{e_2}(I,J), T_{e_1}(i,j))$ represents the true value for the match of the G_2 edge (I, J) onto the G_1 edge (i, j) provided by the corresponding pair of $T_{e_2}(I,J)$ and $T_{e_1}(i,j)$. This similarity measure works as follows, it commence with finding the maximum probability over the nodes in C_I^2 then averaging the edge compatibilities over the nodes C_i^1. Similarly we consider the maximum probability over the nodes in the graph G_2 followed by averaging over the nodes in G_1. It worth mentioned here that unlike Neutrosophic Haussdorff distance this similarity measure does not satisfy the distance axioms. Moreover, while the true components of the Neutrosophic Haussdorff distance measures the maximum distance between two sets of observations, our measures here returns the maximum similarity. Back to the rest formulae of the posteriori probability which represent the indeterminacy value and the false value for the match of the G_2 edge (I, J) onto the G_1 edge (i, j) using similar procedure to the true value. We still need to compute the probabilities $P((i,j) \to (I,J)|T_{e_2}(I,J), T_{e_1}(i,j))$,

$P((i,j) \to (I,J)|I_{e_2}(I,J), I_{e_1}(i,j))$ and $P((i,j) \to (I,J)|F_{e_2}(I,J), F_{e_1}(i,j))$. For that purpose we will use a robust weighting function:

$$P((i,j) \to (I,J)|T_{e_2}(I,J), T_{e_1}(i,j)) = \frac{\Gamma_\sigma(\|T_{e_2}(I,J), T_{e_1}(i,j)\|)}{\sum_{(I,J)\epsilon E_2} \Gamma_\sigma(\|T_{e_2}(I,J), T_{e_1}(i,j)\|)}$$

$$P((i,j) \to (I,J)|I_{e_2}(I,J), I_{e_1}(i,j)) = \frac{\Gamma_\sigma(\|I_{e_2}(I,J), I_{e_1}(i,j)\|)}{\sum_{(I,J)\epsilon E_2} \Gamma_\sigma(\|I_{e_2}(I,J), I_{e_1}(i,j)\|)}$$

$$P((i,j) \to (I,J)|TF_{e_2}(I,J), TF_{e_1}(i,j)) = \frac{\Gamma_\sigma(\|F_{e_2}(I,J), F_{e_1}(i,j)\|)}{\sum_{(I,J)\epsilon E_2} \Gamma_\sigma(\|F_{e_2}(I,J), F_{e_1}(i,j)\|)}$$

Where $\Gamma_\sigma(.)$ is a distance weighting function. There are several alternative robust weighting functions. For instance, one may consider the Gaussian of the form

$\Gamma_\sigma(p) = \exp(\frac{-\rho^2}{2\sigma^2})$ where $\rho^2 = \left(T_{e_2}(I,J) - T_{e_1}(i,j)\right)^2$ according to the true part,

$\rho^2 = \left(I_{e_2}(I,J) - I_{e_1}(i,j)\right)^2$ according to the indeterminacy part and $\rho^2 = \left(F_{e_2}(I,J) - F_{e_1}(i,j)\right)^2$ according to the false part, where σ is the standard deviation. The similarity measure can be viewed as an average pairwise attribute consistency measure.

6. Conclusion and Future Work

Graphs are the most powerful and handy tool used in representing objects and concepts. This paper is dedicated for presenting new neutrosophic similarity and dissimilarity measures between neutrosophic graphs. The proposed distance measures are based on the Haussdorff distance, a modified and a probabilistic variant of the Haussdorff distance, additionally we proved that the given Neutrosophic Haussdorff and the Neutrosophic Modified Haussdorff distances satisfy the metric distance measure axioms. The aim is to use those measures for the purpose of matching graphs whose structure is described in the neutrosophic domain. In our plan for the future we will consider using the deduced measurements in image processing applications, such as image clustering and segmentation.

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