**ORIGINAL ARTICLE**

# A new goodness of fit test in the presence of uncertain parameters

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## Abstract

The Weibull distribution has been widely used in the areas of quality and reliability. The Anderson–Darling test has been popularly used either the data in hand follow the Weibull distribution or not. The existing Anderson–Darling test under classical statistics is applied when all the observations in quality and reliability work are determined, précised, and exact. In the areas of reliability and quality, the data may indeterminate, in-interval and fuzzy. In this case, the existing Anderson–Darling test cannot be applied for testing the assumption of the Weibull distribution. In this paper, we present the Anderson–Darling test under neutrosophic statistics. We present the methodology to fit the neutrosophic Weibull distribution on the data. We discuss the testing procedure with the help of reliability data. We present the comparisons of the proposed test with the existing Anderson–Darling the goodness of fit test under classical statistics. From the comparison, it is concluded that the proposed test is more informative than the existing Anderson–Darling test under an indeterminate environment. In addition, the proposed test gives information about the measure of indeterminacy.

**Keywords** Neutrosophy · Neutrosophic numbers · Reliability · Weibull distribution · Classical statistics

## Introduction

The derivation of statistical methods is based on the assumption that a random variable or the data follow some specific distribution. According to Romeu [1] "when we assume that our data follow a specific distribution, we take a serious risk. If our assumption is wrong, then the results obtained may invalid". For example, before testing a hypothesis, the suitable test statistic is chosen according to the nature of the data in hand. The tests based on normal distribution are chosen when the assumption of the normality is met; otherwise, the non-parametric tests are applied for testing the hypothesis. Two approaches have been widely used to checking the assumption of any distribution. An approach in which the assumption of the data is checked using the graphical properties is called the empirical procedure. Another approach which provides the more formal, a quantifiable, and reliable result is called the goodness of fit test. The goodness of fit tests is based on the cumulative distribution function (cdf) or the probability density function (pdf) of the underlying distribution. Arshad et al. [2] applied the Anderson–Darling

test for testing the assumption of generalized Pareto distribution. Marsaglia and Marsaglia [3] and Razali and Wah [4] presented a study of the performance evaluation of this test. Jäntschi and Bolboacă [5] worked on the computational probabilities of Anderson–Darling test. Formenti et al. [6] applied the Anderson–Darling test in risk assessment. Islam [7] worked on the ranking of skewed distribution using this test. Jäntschi [8] and Jäntschi [9] worked on detecting outliers for continuous distributions. More information can be read et al., in Rahman [10], Anderson [11], Li et al. [12] and Wijekularathna et al. [13].

The statistical test and models have been widely used for the testing of energy generating devices. The choice of the better statistical model will lead to the best estimation and forecasting of the lifetime of energy produced items. Zhang and Lee [14] worked on the health monitoring of batteries. He et al. [15] and Nuhic et al. [16] used the Bayesian approach and data-driven approach for the batteries' data, respectively. Hu et al. [17] worked on the capacity estimation of the batteries. Ng et al. [18] applied the Bayes model for the life prediction of the batteries. Barré et al. [19] presented a statistical study for batteries used in vehicles. Chiodo et al. [20] worked on an accelerated test using batteries' data. Chiodo et al. [20] presented statistical analysis of lithium-ion battery recycling processes. Mathis et al. [21] presented statistical

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work on the consumption due to the energy heating system. Pramanik et al. [21] provided a review on energy equipment. For more applications of statistical models, the reader may refer to Shim et al. [22], Andre et al. [23], Xing et al. [24], and Harris et al. [25].

In the traditional tests under classical statistics, it is assumed that all observations are crisp in the population or the sample. But, the data obtained from the complex system may not be determined, exact, and certain. To test this type of data, the statistical tests based on the fuzzy approach are applied. Arnold [26] discussed the fuzzy test and power function of the test. Przemysław Grzegorzewski [27] and Jamkhaneh and Ghara [28] discussed the application of the statistical test for vague data. Montenegro et al. [29] presented a fuzzy-based test for two populations. Taheri and Behboodian [30] used the Bayesian approach to develop a test under fuzzy logic. Wu [31] presented a test for more than two populations using fuzzy logic. Przemyslaw et al. [32] and Noughabi and Akbari [33] presented the testing of hypothesis procedure for fuzzy logic. Momeni et al. [34] presented Kolomogorov-Smirnov for testing the normality of the fuzzy data. More applications of fuzzy-based tests can be seen in Van Cutsem and Gath [29, 35], Mohanty and AnnanNaidu [36], Moradnezhadi [37], Moewes et al. [38] and Choi et al. [39].

A generalization of fuzzy logic is called the neutrosophic logic was introduced by Smarandache [40]. The neutrosophic provides information about the measure of indeterminacy, the measure of truthiness, and measure of falseness. Smarandache and Khalid [41] proved the efficiency of the neutrosophic logic over the fuzzy logic and interval-based analysis. The applications of neutrosophic logic can be seen in Hanafy et al. [42], Broumi and Smarandache [43], Guo and Sengur [44], Guo and Sengur [45], Guo et al. [46], Patro and Smarandache [47], Broumi et al. [48], Peng and Dai [49], Abdel-Baset et al. [50], Abdel-Baset et al. [51], Abdel-Baset et al. [52], Nabeeh et al. [53], Pratihar et al. [54] and Pratihar et al. [55]. Smarandache [56] introduced the neutrosophic statistics as the extension of classical statistics. The neutrosophic statistics can be applied when the data have indeterminacy. Chen et al. [57] and Chen et al. [58] presented the idea of analyzing the neutrosophic numbers. Aslam [59] and Aslam [60] proposed a statistical test to test normality using the neutrosophic statistics. For more applications of neutrosophic statistics, the reader may refer to Aslam and Albassam [61] and Aslam [62].

Our literature search shows that there is no work on the Anderson–Darling test in the presence of indeterminacy. The existing Anderson–Darling test cannot be applied when the data are given in neutrosophic numbers. In this paper, we will present the Anderson–Darling test under neutrosophic statistics. We will present the methodology to fit the neutrosophic Weibull distribution on the batteries' data. We will discuss the testing procedure with the help of batteries' reliability

data. From the comparison, it is concluded that the proposed test is more informative than the existing Anderson–Darling test. We expect that the proposed test will help the energy experts in the selection of appropriate statistical distribution for better estimation of energy produced devices.

## Preliminaries

Let $I_N \in [I_L, I_U]$ be an indeterminacy interval. Suppose that $X_N = X_L + X_U I_N$; $I_N \in [I_L, I_U]$ denotes the lifetime follows the neutrosophic Weibull distribution with neutrosophic scale parameter $\theta_N = \theta_L + \theta_U I_N$; $I_N \in [I_L, I_U]$ and neutrosophic shape parameter $\beta_N = \beta_L + \beta_U I_N$; $I_N \in [I_L, I_U]$. The neutrosophic cumulative distribution function (ncdf) is defined as

$$F_N(x_N) = 1 - \exp\left\{ -\left( \frac{x_N}{\theta_N} \right)^{\beta_N} \right\}; \; \theta_N$$
$$\in [\theta_L, \theta_U], \; \beta_N \in [\beta_L, \beta_U], \; I_N \in [I_L, I_U] \qquad (1)$$

Note here that the neutrosophic Weibull distribution is given in Eq. (1) is the generalization of the Weibull distribution under classical statistics. The neutrosophic Weibull distribution reduces to the Weibull distribution under classical statistics if $I_L = 0$.

## Fitting of neutrosophic Weibull distribution

Now, we discuss the methodology to test the assumption either the given data having neutrosophic numbers follow the neutrosophic Weibull distribution or not. To develop the proposed test, it is assumed that the neutrosophic shape parameter and neutrosophic scale parameter of the neutrosophic Weibull distribution are unknown and estimated from the given neutrosophic data. The proposed test will be applied to test the null hypothesis that the neutrosophic data are fitted to the neutrosophic Weibull distribution versus the alternative hypothesis that the neutrosophic data do not follow the neutrosophic Weibull distribution. The goodness of fit test statistic, when the neutrosophic data do, not follows the neutrosophic statistics is given by

$$AD_N = \left[ \sum_i \frac{1-2i}{n_N} \left\{ \ln(1 - \exp(-Z_N(i))) - Z_{n_N-i+1} \right\} - n_N \right];$$
$$n_N \in [n_L, n_U], \; AD_N \in [AD_L, AD_U] \qquad (2)$$

where $n_N \in [n_L, n_U]$ be a neutrosophic random sample and $Z_N(i) = [x_N(i)/\theta_N]^{\beta_N}$. According to Romeu [1], the Anderson–Darling test can be applied for small and large samples.
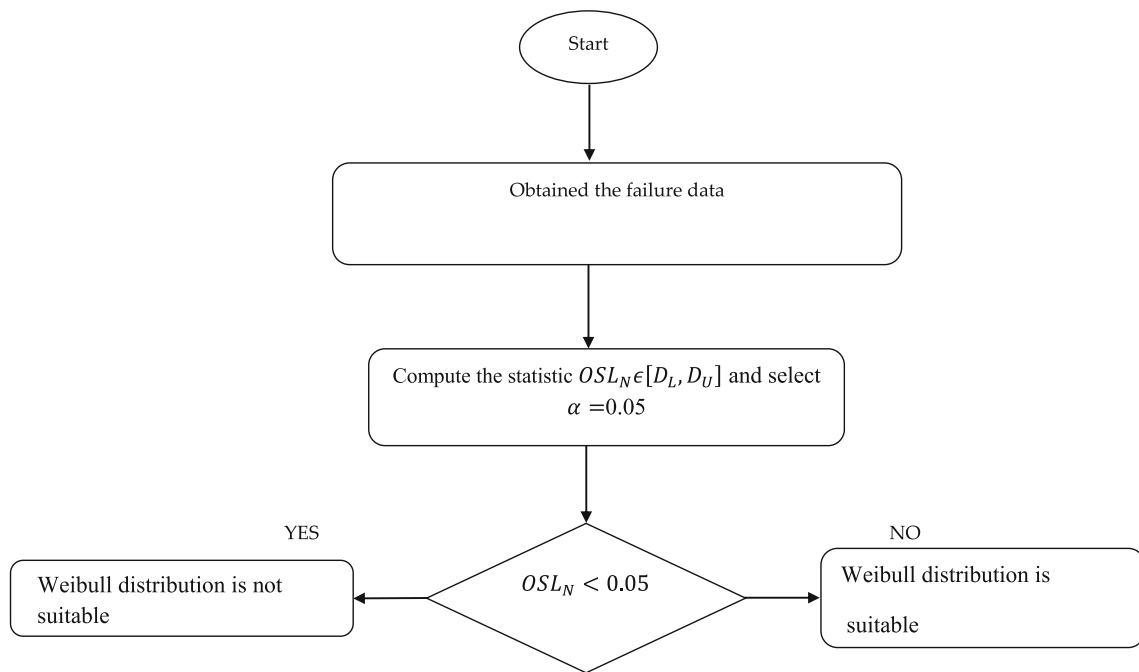
**Fig. 1** The procedure of the proposed test

The neutrosophic form of statistic $AD_N \in [AD_L, AD_U]$ can be expressed as follows

$$AD_N = AD_L + AD_U I_N; \quad I_{ND} \in [I_{LD}, I_{UD}] \quad (3)$$

Note here that the proposed neutrosophic statistic $AD_N \in [AD_L, AD_U]$ has two parts. The first part $AD_L$ is determined part and $AD_U I_{ND}$ is an indeterminate part of the test and $I_{ND} \in [I_{LD}, I_{UD}]$ shows the indeterminate interval. The proposed statistic is the generalization of the goodness of fit test statistic under classical statistic discussed by Romeu and Grethlein [63], Amsc & CMPS [64] and Romeu [1]. The proposed test statistic reduces to the existing test statistic when $I_L = 0$. The modified statistic $AD_N^* \in [AD_L^*, AD_U^*]$ is given by

$$AD_N^* = \left(1 + 0.2/\sqrt{n_N}\right) AD_N; \quad n_N$$
$$\in [n_L, n_U], \ AD_N \in [AD_L, \ AD_U] \quad (4)$$

The neutrosophic form of statistic $AD_N^* \in [AD_L^*, AD_U^*]$ can be expressed as follows

$$AD_N^* = AD_L^* + AD_U^* I_N; \quad I_{ND^*} \in [I_{LD^*}, \ I_{UD^*}] \quad (5)$$

The statistic $AD_N^* \in [AD_L^*, AD_U^*]$ reduces to statistic under classical statistics when no Neutrosophy is recorded. Note here that the first part of the neutrosophic form presents the statistic under classical statistics and $D_U^* I_N; \ I_{ND^*} \in [I_{LD^*}, I_{UD^*}]$ shows the indeterminate part, where $I_{ND^*} \in$

$[I_{LD^*}, I_{UD^*}]$ denotes the indeterminate interval associated with the statistic $D_N^* \in [AD_L^*, \ AD_U^*]$.

The $p$ value of the test will be calculated using neutrosophic observed significance level (NOSL) which is given by

$$OSL_N = \frac{1}{\{1 + \exp[-0.1 + 1.24 \ln(AD_N^*) + 4.48(AD_N^*)]\}};$$
$$OSL_N \in [OSL_L, OSL_U] \quad (6)$$

Based on the proposed test, if $OSL_N < 0.05$, the null hypothesis that the failure time having neutrosophic numbers follow the neutrosophic Weibull distribution is rejected and this error committed is less than 0.05. Based on this study, it is concluded that the given neutrosophic data do not fit the neutrosophic Weibull distribution if the calculated values of the statistic $OSL_N$ is less than 0.05, otherwise, the neutrosophic data follow the neutrosophic Weibull distribution. The operational process of the proposed test is shown in Fig. 1.

## Application of the proposed test

To discuss the application of the proposed test, we consider a life test experiment where 23 batteries are put on the test. A tested battery is labeled as a failed item if at least one of its parts fails to meet the given specification limits. According to Khoolenjani and Shahsanaie [65] "tested the battery

**Table 1** The necessary computations for the proposed test

| Row | $X_N$ | $Z_N(i)$ | $F_N(X_N)$ |
|---|---|---|---|
| 1 | [2.9, 3.99] | [0.0488, 0.04716] | [0.0476, 0.0460] |
| 2 | [5.24, 7.2] | [0.1158, 0.1129] | [0.1094, 0.1068] |
| 3 | [6.56, 9.02] | [0.1608, 0.1577] | [0.1486, 0.1458] |
| 4 | [7.14, 9.82] | [0.1820, 0.1788] | [0.1664, 0.1637] |
| 5 | [11.6, 15.96] | [0.3697, 0.3669] | [0.3090, 0.3071] |
| 6 | [12.14, 16.69] | [0.3951, 0.3920] | [0.3264, 0.3243] |
| 7 | [12.65, 17.4] | [0.4196, 0.4170] | [0.3427, 0.3409] |
| 8 | [13.24, 18.21] | [0.4485, 0.4460] | [0.3614, 0.3598] |
| 9 | [13.67, 18.79] | [0.4699, 0.4672] | [0.3749, 0.3732] |
| 10 | [13.88, 19.09] | [0.4805, 0.4783] | [0.3815, 0.3801] |
| 11 | [15.64, 21.51] | [0.572, 0.5707] | [0.4356, 0.4348] |
| 12 | [17.05, 23.45] | [0.6488, 0.6485] | [0.4773, 0.4771] |
| 13 | [17.4, 23.93] | [0.6683, 0.6682] | [0.4874, 0.4874] |
| 14 | [17.8, 24.48] | [0.6909, 0.6911] | [0.4988, 0.4990] |
| 15 | [19.01, 26.14] | [0.7605, 0.7616] | [0.5325, 0.5330] |
| 16 | [19.34, 26.59] | [0.7798, 0.7811] | [0.5415, 0.5421] |
| 17 | [23.13, 31.81] | [1.0126, 1.0184] | [0.6367, 0.6388] |
| 18 | [23.34, 32.09] | [1.0262, 1.0317] | [0.6416, 0.6436] |
| 19 | [26.07, 35.84] | [1.2060, 1.2150] | [0.7006, 0.7033] |
| 20 | [30.29, 41.65] | [1.5014, 1.5176] | [0.7771, 0.7807] |
| 21 | [43.97,60.46] | [2.5871, 2.6345] | [0.9247, 0.9282] |
| 22 | [48.09, 66.13] | [2.9485, 3.0083] | [0.9475, 0.9506] |
| 23 | [73.48, 98.04] | [5.4754, 5.3878] | [0.9958, 0.9954] |

| $\text{Exp}(Z_N(i))$ | $\ln(1 - \text{Exp}(Z_N(i)))$ | $Z_{n_N-i+1}$ | $i\text{th} - \text{term}$ |
|---|---|---|---|
| [0.9523, 0.9539] | [3.0432, 3.0776] | [5.4754,5.3878] | [1.4, 1.41] |
| [0.8905, 0.8931] | [2.2125, 2.2365] | [2.9485, 3.0083] | [2.5, 2.62] |
| [0.8513, 0.8541] | [1.9064, 1.92486] | [2.5871, 2.6345] | [3.7, 3.79] |
| [0.8335, 0.8362] | [1.7930, 1.80936] | [1.5014, 1.5176] | [3.8, 3.88] |
| [0.6909, 0.6928] | [1.1740, 1.18037] | [1.2060, 1.2150] | [3.5, 3.59] |
| [0.6735, 0.6756] | [1.1195, 1.12594] | [1.0262, 1.0317] | [3.9, 3.95] |
| [0.6572, 0.6590] | [1.0708, 1.07591] | [1.0127, 1.0184] | [4.5, 4.53] |
| [0.6385, 0.6401] | [1.0177, 1.02205] | [0.7798, 0.7811] | [4.4, 4.50] |
| [0.6250, 0.6267] | [0.9809, 0.98544] | [0.7605, 0.7616] | [4.9, 4.95] |
| [0.6184, 0.6198] | [0.9635, 0.9671] | [0.6909, 0.6911] | [5.2, 5.25] |
| [0.5643, 0.5651] | [0.8310, 0.83265] | [0.6683, 0.6682] | [5.2, 5.25] |
| [0.5226, 0.5228] | [0.7395, 0.73982] | [0.6488, 0.6485] | [5.3, 5.32] |
| [0.5125, 0.5125] | [0.7185, 0.71863] | [0.572, 0.5707] | [5.3, 5.37] |
| [0.5011, 0.5009] | [0.6953, 0.69514] | [0.4805, 0.4783] | [5.2, 5.28] |
| [0.4674, 0.4669] | [0.6300, 0.62905] | [0.4699, 0.4672] | [5.3, 5.29] |
| [0.4584, 0.4578] | [0.6133, 0.61229] | [0.4485, 0.4460] | [5.4, 5.46] |
| [0.3632, 0.3611] | [0.4513, 0.44811] | [0.4196, 0.4170] | [4.7, 4.75] |
| [0.3583, 0.3563] | [0.4437, 0.44067] | [0.3951, 0.3920] | [4.8, 4.85] |
| [0.2993, 0.2966] | [0.3557, 0.35196] | [0.3697, 0.3669] | [4.4, 4.43] |
| [0.2228, 0.2192] | [0.2520, 0.24748] | [0.1820, 0.1788] | [2.8, 2.77] |
| [0.0752, 0.0717] | [0.0782, 0.07445] | [0.1608, 0.1577] | [1.6, 1.58] |
| [0.0524, 0.0493] | [0.0538, 0.05063] | [0.1158, 0.1129] | [1.2, 1.17] |
| [0.0041, 0.0045] | [− 0.0042, 0.00458] | [0.0488, 0.0471] | [0.39,0.38] |
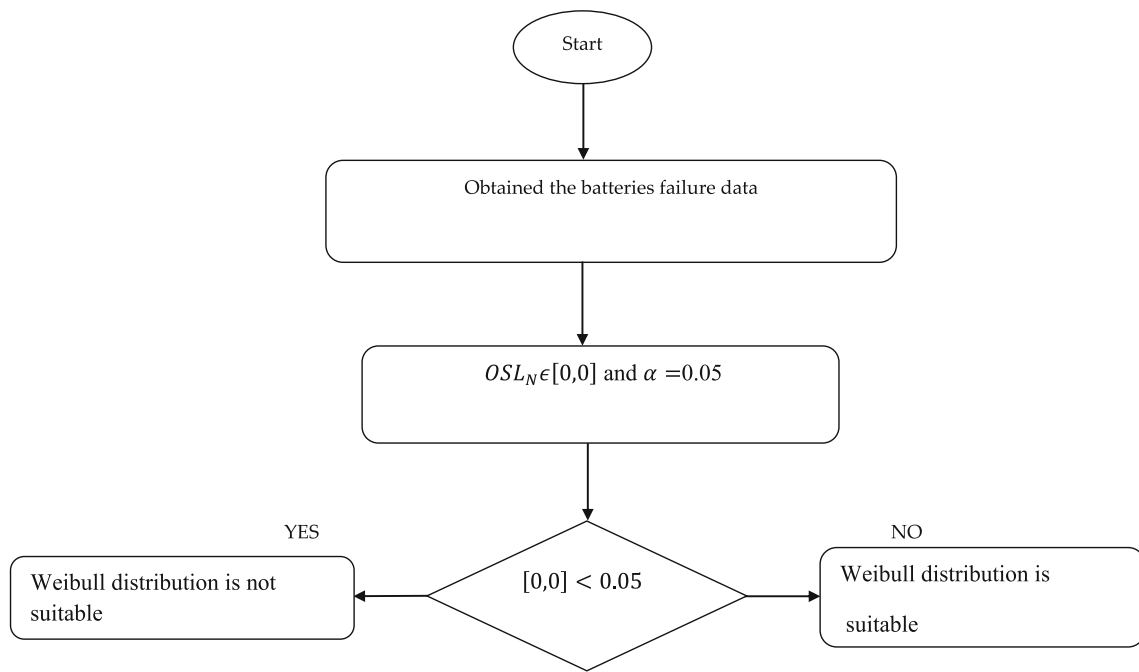
**Fig. 2** The procedure of the proposed test for batteries' data

may be considered as failed, or—strictly speaking—as non-conforming, when at least one value of its parameters falls beyond specification limits. In practice, however, we do not have the possibility to measure all parameters and are not able to define precisely the moment of a failure". The lifetime in 100 h of 23 batteries are given as follows

[2.9, 3.99], [5.24,7.2], [6.56,9.02], [7.14,9.82], [11.6,15.96], [12.14,16.69], [12.65,17.4], [13.24,18.21], [13.67,18.79], [13.88,19.09], [15.64,21.51], [17.05,23.45], [17.4,23.93], [17.8,24.48], [19.01,26.14], [19.34,26.59], [23.13,31.81], [23.34,32.09], [26.07,35.84], [30.29,41.65], [43.97,60.46], [48.09,66.13], [73.48,98.04].

The industrial engineer is interested to test either the given data follow the Weibull distribution or not. It is clear that the lifetime of batteries is given in indeterminacy interval rather than the exact number. For this data, the use Anderson—Darling goodness of fit test under classical statistics is not suitable. Therefore, the alternative of the existing test is the proposed Anderson–Darling goodness of fit test under neutrosophic statistics. The necessary computation to perform the proposed test is shown in Table 1. The values of the statistic $AD_N \in [AD_L, AD_U]$ for the real data are shown as

$$AD_N = \left[ \sum_i \frac{1-2i}{[23,23]} \left\{ \ln(1 - \exp(-Z_N(i))) - Z_{n_N-i+1} \right\} - [23, 23] \right]; \quad AD_N \in [67.54, 67.47].$$

The calculation for modified statistic $AD_N^* \in [AD_L^*, AD_U^*]$ is shown as follows

$$AD_N^* = \left(1 + 0.2/\sqrt{[23, 23]}\right)[67.54, 67.47]; \quad AD_N^* \in [70.35, 70.28]; .$$

The approximate values $OSL_N \in [OSL_L, OSL_U]$ is shown as follows

$$OSL_N = \frac{1}{\{1 + \exp[-0.1 + 1.24\ln([70.35, 70.28]) + 4.48([70.35, 70.28])]\}};$$
$$OSL_N \in [0, 0]$$

We note that $OSL_N < 0.05$ for the batteries' data. Therefore, it is concluded that the lifetime of batteries does not follow the neutrosophic Weibull distribution. The operational process of the proposed test for the batteries' data are shown in Fig. 2.

## Comparative study

Now we compare the efficiency of the proposed Anderson–Darling goodness of fit test under neutrosophic statistics with the Anderson–Darling goodness of fit test under classical statistics in the measure of indeterminacy. For the fair comparison, we will fix the same values of $n_N \in [n_L, n_U]$ as used in the real example. The values of the proposed statistic $AD_N \in [67.54, 67.47]$ in neutrosophic form can be written as $AD_N = 67.54 - 67.47I_N$; $I_N \in [0, 0.001]$. The values modified statistic $AD_N^* \in [70.35, 70.28]$ can be written as $AD_N^* = 70.35 - 70.28I_N$; $I_N \in [0, 0.0009]$. Note here that the values 67.54 and 70.35 represent the values of Anderson–Darling the goodness of fit test under classical statistics when $I_N = 0$. For the real data when $\alpha = 0.05$, the chance of accepting the null hypothesis is 0.95, the chance of rejecting the null hypothesis is 0.05 and measures of indeterminacy are

0.001 and 0.0009. From this study, it can be noted that the proposed Anderson–Darling, the goodness of fit test under neutrosophic statistics provides information about the measure of indeterminacy while the existing Anderson–Darling, the goodness of fit test, does not provide any information about the measure of indeterminacy. In addition, the existing test provides the exact values of the statistics that are not required in uncertainty. Therefore, the proposed test Anderson–Darling, the goodness of fit test, under neutrosophic statistics is quite effective to be applied under uncertainty.

## Concluding remarks

In this paper, we presented the Anderson–Darling test under neutrosophic statistics. We presented the methodology to fit the neutrosophic Weibull distribution on the data. We discussed the testing procedure with the help of reliability data. We applied the proposed test for batteries' failure data and found that the data do not follow the neutrosophic Weibull distribution. From the application and comparative studies, it can be concluded that the proposed test is more informative, flexible, and effective to be applied in an uncertainty environment as compared to the existing Anderson–Darling test. The proposed test provides information about the measure of indeterminacy for testing of the null hypothesis. The proposed test has the limitation that it can be applied to test either the data follow the non-normal distribution such as neutrosophic Weibull distribution. The proposed test can be modified for other distribution accordingly. The proposed test cannot apply for testing the hypothesis for neutrosophic normal distribution. The efficiency of the proposed test in the power of the test can be studied as future research. The proposed test can be applied for big data as future research. In addition, the proposed test can be extended for other non-normal distributions as future research.

## Compliance with ethical standards

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