Original article

Analyzing alloy melting points data using a new Mann-Whitney test under indeterminacy

Muhammad Aslam *, Mansour Sattam Aldosari

Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia

ABSTRACT

Objective: The Mann-Whitney test is applied for testing the means of two non-normal populations. The Mann-Whitney test in the presence of Neutrosophy present in this paper. The design and implantation of the proposed test under neutrosophic statistic are given.
Method: Two methods to apply this test will be given. The application of the proposed test is given with the aid of melting points of alloy data.
Results: The comparison of the proposed test with the Mann-Whitney test under classical statistics is given.
Conclusion: The comparison shows the effeteness, efficiency, and flexibility of the proposed test over the Mann-Whitney test under classical statistics.

© 2020 The Author(s). Published by Elsevier B.V. on behalf of King Saud University. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

1. Introduction

The *t*-test, which is applied in various fields for the testing of the mean of a single population or equality of two means for two populations. This test is implemented under the assumption that the samples are independent and drawn from the normal distribution. In practice, it is not necessary that the population under study is always normal. In this case, the *t*-test cannot be applied for the testing of the means of the populations. The non-parametric test which is called the Mann-Whitney (MW) test is alternative to the *t*-test and applied for testing of the means when population under-investigated is independent of any underlying distribution. The MW test is applied to test either the independent samples are selected from the same population or not (de Winter and Dodou, 2010; Hollander et al., 2013) pointed out the "MW test does not have considerably lower power than the *t*-test even when all the assumptions of the *t*-test are met" (Perme and Manevski, 2019) studied the properties of the MW test (Newcombe, 2006a; Newcombe, 2006b) presented work on confidence interval and dis-

cussed the effect of size on it. (Qin and Hotilovac, 2008; Feng et al., 2017) presented the comparison among the non-parametric tests (Vermeulen et al., 2015) worked on the power of the MW test (Fong and Huang, 2019) presented some modifications in the MW test (Happ et al., 2019) discussed finding the optimal sample size for the MW test. More information about the application of the MW test can be seen in Haynam and Govindarajulu (1966), Kacprzyk et al. (2017), Shin et al. (2019), Mollan et al. (2019).

The existing WM tests are applied when the observations in the samples or population are determined and precise. However, in the real-life the data may be imprecise, fuzzy and indeterminate such as in measuring the lifetime of the virus, water level, and alloy melting points, see (Kacprzyk et al., 2017; Taheri and Hesamian, 2017). The fuzzy logic is an alternative approach to model the imprecise data (Kahraman et al., 2004) used the fuzzy logic to study parametric and non-parametric tests (Taheri and Hesamian, 2017) worked on the MW test using the fuzzy approach. More details about the MW tests for fuzzy logic can be seen in Dubois and Prade (1983), Grzegorzewski (1998), Denœux et al. (2005), Grzegorzewski (2009), Taheri and Arefi (2009), Taheri and Hesamian (2013), Kacprzyk et al. (2017), Taheri and Hesamian (2017).

Smarandache (1998) introduced the neutrosophic logic which considered the measure of indeterminacy in addition to measures of falseness and truthiness. The neutrosophic logic is more efficient than the fuzzy logic and data in the interval approach, see (Wang et al., 2005). More details on the applications of neutrosophic logic can be seen in Broumi and Smarandache (2013), Abdel-Basset et al.

* Corresponding author.
   E-mail address: aslam_ravian@hotmail.com (M. Aslam).
Peer review under responsibility of King Saud University.

Production and hosting by Elsevier

(2018), Peng and Dai (2018), Shahin et al. (2018), Broumi et al. (2018), Abdel-Basset et al. (2019a,b), Nabeeh et al. (2019). The neutrosophic statistics (NS) which is the extension of classical statistics (CS) used for analyzing the data having indeterminacy (Smarandache, 2014). Chen et al. (2017a,b) introduced the idea to deal with the neutrosophic numbers. Aslam (2018) introduced NS in statistical quality control. More applications of NS can be seen in Aslam and Albassam (2019), Aslam (2019).

Although the MW tests under the CS and fuzzy approach are available in the literature. For the testing of data in indeterminate form, the neutrosophic statistics is the best alternative of previous tests. By exploring the literature and best of our information, there is no work on the design of MW test under the neutrosophic statistics. In this paper, we will propose the design and implementation of the neutrosophic MW (NMW) test. We will present a case study of alloy melting points data. We will compare the efficiency of the proposed test over the existing MW test under CS. We expect that the proposed MW test will be more adequate to be applied when the data is fuzzy, in interval and indeterminate.

2. The NMW test

In this section, we will present the proposed NMW test when midpoints of indeterminacy intervals are assigned ranks and when the interval values are assigned ranks. The objective of the proposed test is to see either two independent samples came from populations having the same means. The proposed NMW test is used to test the null hypothesis that samples came from the same populations versus the alternative hypothesis that samples came from the different populations.

2.1. Method-I

Suppose that $X_{1N} = a_1 + b_1 I_{1N}; I_{1N} \epsilon [I_{1L}, I_{1U}]$, $X_{2N} = a_2 + b_2 I_{2N}$; $I_{2N} \epsilon [I_{2L}, I_{2U}], \ldots, X_{nN} = a_n + b_n I_{nN}; I_{nN} \epsilon [I_{nL}, I_{nU}]$ be the first neutrosophic sample. Let $Y_{1N} = a_1 + b_1 I_{1N}; I_{1N} \epsilon [I_{1L}, I_{1U}]$, $Y_{2N} = a_2 + b_2 I_{2N}$; $I_{2N} \epsilon [I_{2L}, I_{2U}], \ldots, Y_{nN} = a_n + b_n I_{nN}; I_{nN} \epsilon [I_{nL}, I_{nU}]$ be the second neutrosophic variable. Note here that both neutrosophic random variables have determinate parts $a_1, \ldots, a_n$ and indeterminate parts $b_1 I_{1N}, \ldots, b_n I_{nN}$. The neutrosophic random variables reduce to the random variable under CS when no indeterminacy in the populations. In general, let $X_N \epsilon [X_L, X_U]$ and $Y_N \epsilon [Y_L, Y_U]$ be two neutrosophic random variables. Suppose that $n_N \epsilon [n_L, n_U]$ be the neutrosophic sample size and $N_N \epsilon [N_L, N_U]$ be the size of combined samples. As mentioned earlier, the main objective of the proposed NMW tests to see the either neutrosophic samples are selected from the populations having the same means. Therefore, we present methodology of the NMW test using the midpoints of indeterminacy intervals. The methodology of the NMW test using the midpoints of indeterminacy intervals is given in the following steps.

Step-1: Draw two random samples $X_N \epsilon [X_L, X_U]$ and $Y_N \epsilon [Y_L, Y_U]$ from the neutrosophic populations.

Step-2: Calculate the average of each indeterminacy interval as $X_L + X_U/2$ and $Y_L + Y_U/2$.

Step-3: Combined and arrange the midpoints of indeterminacy interval in ascending order and assign them the ranks.

Step-4: Find the sum of ranks for $X_N \epsilon [X_L, X_U]$ and $Y_N \epsilon [Y_L, Y_U]$. Note the rank of the smaller sample size, say $R_N \epsilon [R_L, R_U]$ and compute the quantity $R_N^1 = n_N(N_N + 1) - R_N$; $R_N \epsilon [R_L, R_U]$, $n_N \epsilon [n_L, n_U]$, $N_N \epsilon [N_L, N_U]$.

Step-5: Accept the null hypothesis of equal means of two neutrosophic populations if $R_N \epsilon [R_L, R_U]$ or $R_N^1 \epsilon \left[R_L^1, R_U^1 \right]$ is smaller than the tabulated value at the level of significance α, where α is the probability of rejecting the null hypothesis when it is true.

Note here that, according to Kanji (2006), if the sample size is the same for variables $X_N \epsilon [X_L, X_U]$ and $Y_N \epsilon [Y_L, Y_U]$, the statistic $R_N \epsilon [R_L, R_U]$ will be applied for testing the hypothesis.

2.2. Method-II

We present the second method to perform the proposed NMW test same as the test under the CS. Suppose that $X_{1N} = a_1 + b_1 I_{1N}; I_{1N} \epsilon [I_{1L}, I_{1U}]$, $X_{21N} = a_2 + b_2 I_{2N}; I_{2N} \epsilon [I_{21L}, I_{2U}], \ldots, X_{nN} = a_n + b_n I_{nN}; I_{nN} \epsilon [I_{nL}, I_{nU}]$ be the first neutrosophic sample. Let $Y_{1N} = a_1 + b_1 I_{1N}; I_{1N} \epsilon [I_{1L}, I_{1U}]$, $Y_{21N} = a_2 + b_2 I_{2N}; I_{2N} \epsilon [I_{21L}, I_{2U}], \ldots, Y_{nN} = a_n + b_n I_{nN}; I_{nN} \epsilon [I_{nL}, I_{nU}]$ be the second neutrosophic variable. Assume that $X_N \epsilon [X_L, X_U]$ and $Y_N \epsilon [Y_L, Y_U]$ be two neutrosophic random variables. The methodology of NMW test is given as below

Step-1: Draw two random samples $X_N \epsilon [X_L, X_U]$ and $Y_N \epsilon [Y_L, Y_U]$ from the neutrosophic populations.

Step-2: combine and arrange the lower values of indeterminacy interval in ascending order and assign ranks.

Step-3: combine and arrange the upper values of indeterminacy interval in ascending order and assign ranks.

Step-4: Find the sum of ranks for both values of indeterminacy intervals. Note the rank of the smaller sample size; say R_L from lower values of indeterminacy intervals and R_U from the upper values of intermediacy interval. Based on these ranks compute the quantity $R_N^1 = n_N(N_N + 1) - R_N$; $R_N \epsilon [R_L, R_U]$, $n_N \epsilon [n_L, n_U]$, $N_N \epsilon [N_L, N_U]$.

Step-5: Accept the null hypothesis of equal means of two neutrosophic populations if $R_N \epsilon [R_L, R_U]$ or $R_N^1 \epsilon \left[R_L^1, R_U^1 \right]$ is smaller than the tabulated value.

Note here that, according to Kanji (2006), if the sample size is the same for variables $X_N \epsilon [X_L, X_U]$ and $Y_N \epsilon [Y_L, Y_U]$, the statistic $R_N \epsilon [R_L, R_U]$ will be applied for testing the hypothesis.

3. Case study

The alloy is a metal made by combining more than two metals. The alloy is useful in increasing the strength of the material and helpful in resisting the corrosion. Suppose that an industrial engineer working in metal industry is interested to see either the alloy melting points data is came from the population having the same average of melting points. The measuring the melting points is not an easy task, therefore, the observations are not determined values and expressed in intervals. The data of sample size 34 is taken from Kacprzyk et al. (2017). The measurement of melting points of two alloys are given below

$X_N \epsilon [X_L, X_U]$ For measurement of the first alloy

[563.3, 545.5], [529.4, 511.6], [523.1, 503.5], [470.1, 449.2], [506.7, 489.0],

[495.6, 479.1], [495.3, 467.9], [520.9, 495.6], [496.9, 472.8], [542.9, 519.1],

[505.4, 484.0], [550.7, 525.9], [517.7, 500.9], [499.2, 483.0], [500.6, 480.0],

[516.8, 499.6], [535.0, 515.1], [489.3, 464.4]

$Y_N \epsilon [Y_L, Y_U]$ For measurement of the second alloy

[444.1, 426.1], [430.5, 406.7], [407.2, 387.3], [475.8, 450.9], [458.5, 440.2],

[507.7, 490.6], [496.8, 480.2], [520.9, 503.8], [503.6, 482.8], [458.2, 432.8],

[480.5, 453.3], [473.8, 446.9], [468.4, 451.2], [496.2, 478.1], [477.0, 459.4],

[496.3, 479.4]

The testing procedure of the alloy melting points using method-I is given in Tables 1 and 2. The sum of rank for the melting points of the first sample and the second samples are 411 and 184, respectively. According to the proposed test procedure, we will

Table 1
The ranks of first alloy melting points using method-I.

Data	[563.3, 545.5]	[529.4, 511.6]	[523.1, 503.5]	[470.1, 449.2]	[506.7, 489.0]	[495.6, 479.1]
Mid Points	**554.4**	**520.5**	**513.3**	**459.65**	**497.85**	**487.35**
Ranks	**34**	**30**	**29**	**5**	**23**	**16**
Data	[495.3, 467.9]	[520.9, 495.6]	[496.9, 472.8]	[542.9, 519.1]	[505.4, 484.0]	[550.7, 525.9]
Mid Points	**481.6**	**508.25**	**484.85**	**531.0**	**494.7**	**538.3**
Ranks	**13**	**26**	**14**	**32**	**22**	**33**
Data	[517.7, 500.9]	[499.2, 483.0]	[500.6, 480.0]	[516.8, 499.6]	[535.0, 515.1]	[489.3, 464.4]
Mid Points	**509.3**	**491.1**	**490.3**	**508.2**	**525.05**	**476.85**
Ranks	**27**	**20**	**19**	**25**	**31**	**12**

Table 2
The ranks of second alloy melting points using method-I.

Data	[444.1, 426.1]	[430.5, 406.7]	[407.2, 387.3]	[475.8, 450.9]	[458.5, 440.2]	[507.7, 490.6]
Mid Points	**435.1**	**418.6**	**397.25**	**463.35**	**462.85**	**499.15**
Ranks	**3**	**2**	**1**	**9**	**8**	**24**
Data	[496.8, 480.2]	[520.9, 503.8]	[503.6, 482.8]	[458.2, 432.8]	[480.5, 453.3]	[473.8, 446.9]
Mid Points	**488.5**	**512.35**	**493.2**	**445.5**	**466.9**	**460.35**
Ranks	**18**	**28**	**21**	**4**	**10**	**7**
Data	[468.4, 451.2]	[496.2, 478.1]	[477.0, 459.4]	[496.3, 479.4]		
Mid Points	**459.8**	**487.15**	**468.2**	**487.85**		
Ranks	**6**	**15**	**11**	**17**		

Table 3
The ranks using method-II.

Combined Determinate Part	Ranks	Combined Indeterminate Part	Ranks
407.2	1	387.3	1
430.5	2	406.7	2
444.1	3	426.1	3
458.2	4	432.8	4
458.5	5	440.2	5
468.4	6	446.9	6
470.1	7	449.2	7
473.8	8	450.9	8
475.8	9	451.2	9
477	10	453.3	10
480.5	11	459.4	11
489.3	12	464.4	12
495.3	13	467.9	13
495.6	14	472.8	14
496.2	15	478.1	15
496.3	16	479.1	16
496.8	17	479.4	17
496.9	18	480	18
499.2	19	480.2	19
500.6	20	482.8	20
503.6	21	483	21
505.4	22	484	22
506.7	23	489	23
507.7	24	490.6	24
516.8	25	495.6	25
517.7	26	499.6	26
520.9	27	500.9	27
520.9	28	503.5	28
523.1	29	503.8	29
529.4	30	511.6	30
535	31	515.1	31
542.9	32	519.1	32
550.7	33	525.9	33
563.3	34	545.5	34

select R_N = 184. The value of R_N^1 = 376. The tabulated value which is 222 at a level of significance 0.05 from Kanji (2006). We see that $R_N < 222$, therefore, we do not reject the null hypothesis and conclude that alloy melting points came from the population having the same means. On the other hand, $R_N^1 > 222$, we reject the null hypothesis and conclude that alloy melting points came from the population having different means. In this example, samples size

is different, therefore, the conclusion obtained from the statistic R_N^1 is recommended.

We now apply the method-II for analyzing the alloy melting points data. The necessary computations for this test are shown in Table 3. The smaller values of the sum of ranks of $R_N \epsilon [180, 183]$; $I_{nN} \epsilon [0, 0.016]$. The values of $R_N \epsilon [180, 183] < 222$, therefore, we do not reject the null hypothesis and reach the same conclusion as in method-I. On the other hand, the values of $R_N^1 \epsilon [380, 377] > 222$, so we reject the null hypothesis and reach on the same decision as in method-I. In this example, samples size is different, therefore, the conclusion obtained from the statistic R_N^1 is recommended.

4. Comparative study

The efficiency and adequacy of the proposed NMW test will be compared with the existing MW test under CS is given by Kanji (2006), existing test using the interval data given by Kacprzyk et al. (2017) and test under the fuzzy approach presented by Taheri and Hesamian (2017). To discuss the effectiveness of the proposed NMW test, we will fix the same values of $n_N \epsilon [n_L, n_U]$, $N_N \epsilon [N_L, N_U]$ and α. The values of the statistic from the proposed test can be expressed as $R_N = 180 + 183 I_{nN}$; $I_{nN} \epsilon [0, 0.016]$. Similarly, the neutrosophic form of $R_N^1 = 380 - 377 I_{nN}$; $I_{nN} \epsilon [0, 0.007]$. Note here that the proposed tests are the generalization of tests under CS, interval approach and fuzzy approach. For example, the proposed test reduces to test discussed by Kanji (2006) if no indeterminacy is recorded in the data in hand. In this case, the value of the test statistic will be 180 and 380 which are the determined parts of the proposed tests. The statistic $R_N \epsilon [180, 183]$ and $R_N^1 \epsilon [380, 377]$ without the indeterminacy, interval shows the values under the interval approaches. For the alloy melting points, it can be noted that when α = 0.05, the experimenters can expect that the null hypothesis will be accepted with probability 0.95, rejected with probability 0.05 and there are measures of indeterminacy which are 0.016 and 0.007. Nevertheless, on the other hand, the test under CS only provides the exact value of the test. In addition, the values under fuzzy and interval approach only provided the range of the values from 180 to 183 and 380 to 377. It means, the null hypothesis will be accepted with the probability 0.95 and rejected with the probability 0.05. By comparing the proposed

tests with the existing tests, we conclude that the proposed tests deal with the presence of a measure of indeterminacy, which other tests do not consider. Therefore, the proposed tests will be more effective and adequate to be applied when the data is recorded from the complex system in the industry.

5. Concluding remarks

In this article, we presented the design, methodology and decision criterion for the testing of the means. The test was presented using different methods of ranking. The proposed test was designed for the testing of means of the neutrosophic populations. The proposed test was the extension of several existing tests. From the comparison, we conclude that the proposed test is quite reasonable to be applied under the uncertainty. The proposed test was applied on a real data from the industry. The proposed test for big data can be studied as future research.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

References

Abdel-Basset, M., Atef, A., Smarandache, F., 2018. A hybrid Neutrosophic multiple criteria group decision making approach for project selection. Cognit. Syst. Res.

Abdel-Basset, M., Mohamed, M., Elhoseny, M., Chiclana, F., Zaied, A.-E.-N.-H., 2019a. Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases. Artif. Intell. Med. 101, 101735.

Abdel-Basset, M., Nabeeh, N.A., El-Ghareeb, H.A., Aboelfetouh, A., 2019b. Utilising neutrosophic theory to solve transition difficulties of IoT-based enterprises. Enterprise Inf. Syst., 1–21

Aslam, M., 2018. A new sampling plan using neutrosophic process loss consideration. Symmetry 10, 132.

Aslam, M., 2019. Neutrosophic analysis of variance: application to university students. Complex Intell. Syst., 1–5

Aslam, M., Albassam, M., 2019. Application of neutrosophic logic to evaluate correlation between prostate cancer mortality and dietary fat assumption. Symmetry 11, 330.

Broumi, S., Bakali, A., Talea, M., Smarandache, F., 2018. Bipolar neutrosophic minimum spanning tree. Infinite Study.

Broumi, S., Smarandache, F., 2013. Correlation coefficient of interval neutrosophic set. Appl. Mech. Mater. Trans. Tech. Publ., 511–517

Chen, J., Ye, J., Du, S., 2017a. Scale effect and anisotropy analyzed for neutrosophic numbers of rock joint roughness coefficient based on neutrosophic statistics. Symmetry 9, 208.

Chen, J., Ye, J., Du, S., Yong, R., 2017b. Expressions of rock joint roughness coefficient using neutrosophic interval statistical numbers. Symmetry 9, 123.

de Winter, J., Dodou, D., 2010. Five-point likert items: t test versus Mann-Whitney-Wilcoxon (Addendum added October 2012). Pract. Assess., Res. Eval. 15, 11.

Denœux, T., Masson, M.-H., Hébert, P.-A., 2005. Nonparametric rank-based statistics and significance tests for fuzzy data. Fuzzy Sets Syst. 153, 1–28.

Dubois, D., Prade, H., 1983. Ranking fuzzy numbers in the setting of possibility theory. Inf. Sci. 30, 183–224.

Feng, D., Cortese, G., Baumgartner, R., 2017. A comparison of confidence/credible interval methods for the area under the ROC curve for continuous diagnostic tests with small sample size. Stat. Methods Med. Res. 26, 2603–2621.

Fong, Y., Huang, Y., 2019. Modified Wilcoxon–Mann–Whitney test and power against strong null. Am. Statist. 73, 43–49.

Grzegorzewski, P., 1998. Statistical inference about the median from vague data. Control Cybern. 27, 447–464.

Grzegorzewski, P., 2009. k-Sample median test for vague data. Int. J. Intellig. Syst. 24, 529–539.

Happ, M., Bathke, A.C., Brunner, E., 2019. Optimal sample size planning for the Wilcoxon-Mann-Whitney test. Stat. Med. 38, 363–375.

Haynam, G.E., Govindarajulu, Z., 1966. Exact power of Mann-Whitney test for exponential and rectangular alternatives. Ann. Math. Stat., 945–953

Hollander, M., Wolfe, D.A., Chicken, E., 2013. Nonparametric statistical methods. John Wiley & Sons.

Kacprzyk, J., Szmidt, E., Zadrożny, S., Atanassov, K.T., Krawczak, M., 2017. Advances in Fuzzy Logic and Technology 2017. EUSFLAT-2017–The 10th Conference of the European Society for Fuzzy Logic and Technology, September 11–15, 2017, Warsaw, Poland IWIFSGN'2017–The Sixteenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets, September 13–15, 2017. Springer, Warsaw, Poland.

Kahraman, C., Bozdag, C.E., Ruan, D., Özok, A.F., 2004. Fuzzy sets approaches to statistical parametric and nonparametric tests. Int. J. Intell. Syst. 19, 1069–1087.

Kanji, G.K., 2006. 100 statistical tests, Sage.

Mollan, K.R., Trumble, I.M., Reifeis, S.A., Ferrer, O., Bay, C.P., Baldoni, P.L., Hudgens, M.G., 2019. Exact Power of the Rank-Sum Test for a Continuous Variable. arXiv preprint arXiv:1901.04597.

Nabeeh, N.A., Smarandache, F., Abdel-Basset, M., El-Ghareeb, H.A., Aboelfetouh, A., 2019. An integrated neutrosophic-topsis approach and its application to personnel selection: a new trend in brain processing and analysis. IEEE Access 7, 29734–29744.

Newcombe, R.G., 2006a. Confidence intervals for an effect size measure based on the Mann-Whitney statistic. Part 1: general issues and tail-area-based methods. Statist. Med. 25, 543–557.

Newcombe, R.G., 2006b. Confidence intervals for an effect size measure based on the Mann-Whitney statistic. Part 2: asymptotic methods and evaluation. Stat. Med. 25, 559–573.

Peng, X., Dai, J., 2018. Approaches to single-valued neutrosophic MADM based on MABAC, TOPSIS and new similarity measure with score function. Neural Comput. Appl. 29, 939–954.

Perme, M.P., Manevski, D., 2019. Confidence intervals for the Mann-Whitney test. Stat. Methods Med. Res. 28, 3755–3768.

Qin, G., Hotilovac, L., 2008. Comparison of non-parametric confidence intervals for the area under the ROC curve of a continuous-scale diagnostic test. Stat. Methods Med. Res. 17, 207–221.

Shahin, A., Guo, Y., Amin, K., Sharawi, A.A., 2018. A novel white blood cells segmentation algorithm based on adaptive neutrosophic similarity score. Health Inf. Sci. Syst. 6, 1.

Shin, D., Yamamoto, Y., Brady, M.P., Lee, S., Haynes, J.A., 2019. Modern data analytics approach to predict creep of high-temperature alloys. Acta Mater. 168, 321–330.

Smarandache, F., 1998. Neutrosophy. Neutrosophic probability, set, and logic, ProQuest information & learning. Ann Arbor, Michigan, USA, 105, 118–123.

Smarandache, F., 2014. Introduction to neutrosophic statistics, Infinite Study.

Taheri, S., Hesamian, G., 2013. A generalization of the Wilcoxon signed-rank test and its applications. Stat. Pap. 54, 457–470.

Taheri, S.M., Arefi, M., 2009. Testing fuzzy hypotheses based on fuzzy test statistic. Soft. Comput. 13, 617–625.

Taheri, S.M., Hesamian, G., 2017. Non-parametric statistical tests for fuzzy observations: fuzzy test statistic approach. Int. J. Fuzzy Logic Intellig. Syst. 17, 145–153.

Vermeulen, K., Thas, O., Vansteelandt, S., 2015. Increasing the power of the Mann-Whitney test in randomized experiments through flexible covariate adjustment. Stat. Med. 34, 1012–1030.

Wang, H., Smarandache, F., Sunderraman, R., Zhang, Y.-Q., 2005. interval neutrosophic sets and logic: theory and applications in computing: theory and applications in computing. Infinite Study.