

Research Article

Performance of a New Time-Truncated Control Chart for Weibull Distribution Under Uncertainty

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ABSTRACT

To detect indeterminacy effect in the manufacturing process, attribute control chart using neutrosophic Weibull distribution is proposed in this paper. To make the attribute control chart more efficient for persistent shifts in the industrial process, an attribute control chart using Weibull distribution has been proposed recently. In this study, a neutrosophic Weibull distribution-based attribute control chart develop for efficient monitoring of the process. The indeterminacy effect was studied with the control chart's performance using characteristics of run length. In addition, the proposed chart effectively detected shifts in uncertainty. The relative efficiency of the proposed structure is compared with the existing attribute control chart under the Weibull time-truncated life test. The relative analysis reveals that the proposed time-truncated control chart for Weibull distribution under uncertainty design performance more efficiently than the existing counterparts. From the comparison, the proposed chart provides smaller values for the out-of-control average run length as compared to the existing attribute control chart. An illustrative application related to automobile manufacturing is also incorporated to demonstrate the proposal.

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1. INTRODUCTION

A leading tool in the manufacturing process is the control chart that is applied to watch manufacturing shifts. A slight change in the quality target may cause big losses for a company. The variable control charts are designed to monitor the process using measurement data. Nonconforming items can be monitored using an attribute control chart. According to [1], “the traditional Shewhart np control charts are the statistical control scheme most commonly used for monitoring the number of non-conforming items.” For more details, the reader may refer to [1–4].

Usually, control charts work when data is derived from a normal process. When data moves away from normality, the chart designed with normal distribution cannot monitor the process. Control charts designed with non-normal distributions present a good alternative. According to [5], “In most life testing, to reduce the test time of the experiment, a failure-censored (type-II) scheme, or time-censored (type-I) scheme is usually adopted.” Therefore, the control chart designed with a time-truncated control can be used to save monitoring time. In the operational procedure of time-truncated charts, the experiment and watching times are fixed in advance, where an item is labeled as defective if its failure/service time is less than the specified time. The noted numbers of defects are plotted on the control chart to decide the state of the process. Previous studies [6] have proposed an attribute chart for Weibull

distribution. Specifically, some studies [7] designed control charts using censored data. Others [8] have studied time-truncated charts for exponentiated half logistic distribution and Dagum distribution, respectively. Further, [9–12] show further control chart applications for various statistical distributions.

The Shewhart control charts given in the literature are unable to apply if some observations in the production data are uncertain or fuzzy. Fuzzy-based control charts are applied when uncertainty is presented in observations and parameters. According to [13], “fuzzy control charts are more sensitive than traditional ones; hence, they provide better quality products.” Furthermore, Darestani *et al.* [14] proposed a u-chart using a fuzzy approach, whereas [15] proposed a chart using fuzzy logic and [16] proposed a c-chart using fuzzy logic [17]. More work can be seen in [17–21].

The major drawback of a fuzzy-based control chart is that it is less informative than other charts. As such, Smarandache [22] mentioned that the generalized form of fuzzy logic is known as neutrosophic logic. Moreover, Smarandache and Khalid [23] stated that neutrosophic logic is more efficient than fuzzy logic and interval-based analysis. Other researchers [24] have discussed neutrosophic logic applications. Neutrosophic statistics is a branch of statistics that analyzes neutrosophic data, and many studies [25] have presented methods to analyze the indeterminacy data. Other works [26] have introduced indeterminacy in an attribute chart and proposed attribute charts for neutrosophic statistics [27]. Moreover, Khan *et al.* [28] proposed an S-chart using neutrosophic statistics.

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Lastly, Aslam *et al.* [29] proposed an efficient attribute chart. More information in neutrosophic statistics can be seen in [30,31].

The attribute control charts under neutrosophic statistics are presented in the literature. No previous work has designed a time-truncated attribute control chart for neutrosophic Weibull distribution using neutrosophic statistics. In this paper, we designed a time-truncated attribute control chart using neutrosophic statistics. The effect of the indeterminacy parameter was studied on the average run length (ARL). The advantages of the proposed chart are also discussed. In next section, we describe the design of the proposed chart. In Section 3, the comparison of the proposed chart with the counterpart chart and simulation study is described. The Illustrative example is described in Section 4. In the final section, the conclusion and future recommendations are displayed.

2. DESIGN OF THE PROPOSED CHART

First, we introduce the neutrosophic Weibull distribution. Then, we present the design of the current chart.

Suppose that $x_N \in [x_L, x_U]$ be a neutrosophic random variable having neutrosophic Weibull distribution, where x_L and x_U are the lower and upper values of neutrosophic random variable. The Weibull distribution neutrosophic probability density function (NPDF) and neutrosophic cumulative distribution function (NCDF) are defined as follows

$$f(x_N) = \left\{ \left(\frac{\beta}{\alpha} \right) \left(\frac{x_N}{\alpha} \right)^{\beta-1} e^{-\left(\frac{x_N}{\alpha} \right)^\beta} \right\} + \left\{ \left(\frac{\beta}{\alpha} \right) \left(\frac{x_N}{\alpha} \right)^{\beta-1} e^{-\left(\frac{x_N}{\alpha} \right)^\beta} \right\} I_N; \quad I_N \in [I_L, I_U], x_N \in [x_L, x_U] \quad (1)$$

$$F(x_N) = 1 - \left\{ e^{-\left(\frac{x_N}{\alpha} \right)^\beta} (1 + I_N) \right\} + I_N; \quad I_N \in [I_L, I_U], x_N \in [x_L, x_U] \quad (2)$$

where β is the shape parameter, α is the scale parameter, and I_N shows the neutrosophic interval. The neutrosophic mean, say μ_N of the neutrosophic Weibull distribution is given by

$$\mu_N = \alpha \Gamma(1 + 1/\beta) (1 + I_N) \quad (3)$$

where $\Gamma(\cdot)$ denotes the gamma function. Let μ_{N0} target the product's lifetime and x_{N0} be a truncated time. The probability p_N that an item fails by time x_{N0} is given as

$$p_N = 1 - \left\{ e^{-\left(\frac{x_N}{\alpha} \right)^\beta} (1 + I_N) \right\} + I_N; \quad I_N \in [I_L, I_U], x_N \in [x_L, x_U] \quad (4)$$

Let $x_{N0} = a\mu_{N0}$ be the termination time, where a is constant and we express the unknown scale parameter α in term of μ_N using Eq. (3).

Then, Eq. (4) can be rewritten as

$$p_N = 1 - \left\{ e^{-a^\beta \left(\frac{\mu_N}{\mu_{N0}} \right)^{-\beta} \left(\Gamma(1+1/\beta)(1+I_N) \right)^\beta} (1 + I_N) \right\} + I_N; \quad I_N \in [I_L, I_U], x_N \in [x_L, x_U] \quad (5)$$

When $\mu_N = \mu_{N0}$, then the probability in Eq. (5) reduces to

$$p_{N0} = 1 - \left\{ e^{-a^\beta \left(\Gamma(1+1/\beta)(1+I_N) \right)^\beta} (1 + I_N) \right\} + I_N; \quad I_N \in [I_L, I_U], x_N \in [x_L, x_U] \quad (6)$$

The np attribute control chart having lower control limit (LCL) and upper control limit (UCL) for the neutrosophic Weibull distribution is explained in the following steps:

Step 1. Count defective items of the products (D) from the selected sample.

Step 2. If $D > UCL$ or $D < LCL$, the process is considered out of control. The process is considered in control if $LCL \leq D \leq UCL$.

Note that D follows a neutrosophic binomial distribution with parameters n and in-control probability p_{N0} . Therefore, the control limits of the proposed np control chart are as follows:

$$UCL = np_{N0} + k\sqrt{np_{N0}(1 - p_{N0})} \quad (7a)$$

$$LCL = \max \left[0, np_{N0} - k\sqrt{np_{N0}(1 - p_{N0})} \right] \quad (7b)$$

where k is the coefficient or control constant of the control limits to be resolute. The p_{N0} values are normally unknown; therefore, the averages of failed items ($D =$ defective items) such as \bar{D} are taken. Thus, the neutrosophic control limits for practical use are

$$UCL_N = \bar{D} + k\sqrt{\bar{D} \left(1 - \bar{D}/n \right)} \quad (8a)$$

$$LCL_N = \max \left[0, \bar{D} - k\sqrt{\bar{D} \left(1 - \frac{\bar{D}}{n} \right)} \right] \quad (8b)$$

The probability for the in-control process (i.e., P_{Nin}^0) is given by

$$P_{Nin}^0 = P(LCL \leq D \leq UCL | p_{N0}) = \sum_{d=LCL+1}^{UCL} \binom{n}{d} p_{N0}^d (1 - p_{N0})^{n-d} \quad (9)$$

If $LCL = 0$, d should be 0 when applied. Let μ_{N1} be the shifted average and the probability at the new mean is evaluated by

$$p_{N1} = 1 - \left\{ e^{-a^\beta \left(\frac{\mu_{N1}}{\mu_{N0}} \right)^{-\beta} \left(\Gamma(1+1/\beta)(1+I_N) \right)^\beta} (1 + I_N) \right\} + I_N; \quad I_N \in [I_L, I_U], x_N \in [x_L, x_U] \quad (10)$$

If the shifted mean is $\mu_{N1} = f\mu_{N0}$ for a constant f , then Eq. (10) can be rewritten as

$$p_{N1} = 1 - \left\{ e^{-a^\beta(f)^{-\beta}(\Gamma(1+1/\beta)(1+I_N))^\beta} (1 + I_N) \right\} + I_N; I_N \in [I_L, I_U], x_N \in [x_L, x_U] \tag{11}$$

The in-process probability for the shifted mean can be written as

$$P_{Nim}^1 = P(LCL \leq D \leq UCL | p_{N1}) = \sum_{d=LCL+1}^{UCL} \binom{n}{d} p_{N1}^d (1 - p_{N1})^{n-d} \tag{12}$$

The in-control ARL can be calculated as

$$ARL_0 = \frac{1}{1 - P_{Nim}^0} \tag{13}$$

The ARL for the shifted process can be calculated as

$$ARL_1 = \frac{1}{1 - P_{Nim}^1} \tag{14}$$

ARL values for various values (e.g., I_N , f , and β) are shown in Tables 1–3, where the following behavior can be noted.

1. When the indeterminacy parameter I_N increases, the decreasing trend in ARL_1 is noted.
2. For the same values, the values of ARL_1 decreases as β increases.

The values of k , ARL_0 , and ARL_1 can be obtained through the following steps:

Step 1: Fix the values of β , n , and I_N , thus determining the values of k via the grid search method.

Step 2: Several values of k are noted during simulation, where $ARL_0 \geq r_0$ is a specified ARL value.

Step 3: Choose the k value when ARL_0 is close to r_0 .

Step 4: Determine ARL_1 values for various values of f .

3. COMPARATIVE STUDIES

Our proposed control chart becomes the same chart proposed in [6] when $I_N = 0$. Table 4 presents both control charts when $I_N = 0.1$ and $I_N = 0.5$. Table 4 shows that the proposed control chart provides smaller ARLs values when compared to the chart proposed by [6]. For example, when $f = 0.9$, $\beta = 1.1$, $n = 30$, and $I_N = 0.1$, the proposed control chart detects the first out-of-control value at the 151st sample, whereas the existing chart proposed by [6] detects the first out-of-control value at the 177th sample. Similarly, when $f = 0.9$, $\beta = 1.1$, $n = 30$, and $I_N = 0.5$, the proposed control chart detects the first out-of-control value at the 131st sample, whereas the existing chart proposed by [6] detects the first out-of-control value at the 177th sample. From this study, it can be seen that the proposed chart provides a quick indication about the shift in the process when compared to the control chart proposed by [6]. This study thus proved the efficiency of the proposed chart.

3.1. Simulation Study

The performance of the proposed control chart was compared with the existing chart proposed by [6] using simulated data. The data was generated when $a = 0.1548$, $n = 30$, $\beta = 1$, and $I_N = 0.1$. Among 50 observations, the first 20 values were generated when the process was in-control (IC) state when $\mu_0 = 1$. Further, the next 30 observations were generated from the shifted process when $f = 0.8$ by using the R software. The values of the number of defective items (D) under the proposed scheme are plotted in Figure 1 and the existing control chart displayed in Figure 2. Figure 1 indicates the shift at the 31st sample; the existing chart proposed by [6] indicates no shift. By comparing both figures, we can conclude that the proposed control chart indicates an issue in the existing control chart. Therefore, the application of the proposed control chart under uncertainty is helpful for minimizing the number of non-conforming items and got conforming products in less time.

4. ILLUSTRATIVE EXAMPLE

The proposed control chart was applied in an automobile manufacturing company located in South Korea. The company was interested in monitoring the service time in months for specific subsystems [6]. The same service data was applied by [6], who showed

Table 1 | ARLs of the proposed chart when $r_0 = 370$.

f	$\beta = 1, n = 30$			
	$k = 3.26994,$ $I_N = 0.1, a = 0.1545$	$k = 3.12394,$ $I_N = 0.2, a = 0.1289$	$k = 3.23313,$ $I_N = 0.4, a = 0.1082$	$k = 3.22172,$ $I_N = 0.5, a = 0.09385$
ARL				
1.0	374.93	373.97	373.41	370.43
0.9	165.38	163.88	154.50	152.48
0.8	70.06	68.93	61.44	60.33
0.7	28.65	27.97	23.68	23.13
0.6	11.47	11.11	9.03	8.79
0.5	4.65	4.48	3.59	3.48
0.4	2.07	1.99	1.65	1.61
0.3	1.18	1.15	1.06	1.05
0.2	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00

Table 2 | ARLs of the proposed chart when $r_0 = 370$.

f	$\beta = 1.1, n = 30$			
	$k = 3.13684, I_N = 0.1, a = 0.1883$	$k = 3.19848, I_N = 0.2, a = 0.1584$	$k = 3.1863, I_N = 0.4, a = 0.1332$	$k = 3.21308, I_N = 0.5, a = 0.1310$
	ARL			
1.0	371.62	371.68	371.37	374.92
0.9	151.50	150.41	141.13	131.00
0.8	59.51	58.61	51.77	46.61
0.7	22.75	22.22	18.61	16.04
0.6	8.67	8.40	6.78	5.68
0.5	3.47	3.35	2.69	2.27
0.4	1.64	1.50	1.34	1.21
0.3	1.07	1.00	1.01	1.00
0.2	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00

Table 3 | ARLs of the proposed chart when $r_0 = 370$.

f	$\beta = 2, n = 30$			
	$k = 3.58595, I_N = 0.1, a = 0.2167$	$k = 3.58132, I_N = 0.2, a = 0.2224$	$k = 2.89913, I_N = 0.4, a = 0.2472$	$k = 3.3222, I_N = 0.5, a = 0.2428$
	ARL			
1.0	372.83	370.03	371.83	370.58
0.9	136.70	120.64	88.27	79.58
0.8	47.76	37.90	21.24	17.70
0.7	16.20	11.85	5.64	4.53
0.6	5.58	3.96	1.94	1.62
0.5	2.17	1.64	1.09	1.03
0.4	1.17	1.05	1.00	1.00
0.3	1.00	1.00	1.00	1.001
0.2	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00

Table 4 | Comparison of the proposed chart with the existing chart in ARLs.

f	[6] chart ($I_N = 0$)		The proposed chart			
	$\beta = 1$	$\beta = 1.1$	$I_N = 0.1$		$I_N = 0.5$	
	$k = 3.40009, a = 0.1344$	$k = 3.33227, a = 0.1394$	$k = 3.26994, I_N = 0.1, a = 0.1545$	$k = 3.13684, \beta = 1.1, a = 0.1883$	$k = 3.22172, \beta = 1, a = 0.09385$	$k = 3.21308, \beta = 1.1, a = 0.1310$
ARL						
1.0	370.32	370.80	374.93	371.62	370.43	374.92
0.9	180.12	177.71	165.38	151.50	152.48	131.00
0.8	83.88	81.21	70.06	59.51	60.33	46.61
0.7	37.40	35.50	28.65	22.75	23.13	16.04
0.6	16.05	14.94	11.47	8.67	8.79	5.68
0.5	6.74	6.18	4.65	3.47	3.48	2.27
0.4	2.91	2.66	2.07	1.64	1.61	1.21
0.3	1.45	1.35	1.18	1.07	1.05	1.00
0.2	1.03	1.01	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00	1.00	1.00

that the data followed the Weibull distribution with $\beta = 1$. For this study, let $a = 0.1883, k = 3.1368, n = 30, p_0 = 0.1722$, and $I_N = 0.1$, which leads to the value of $\beta = 1(1 + 0.1) = 1.1$ for the neutrosophic Weibull distribution. Let the value of $\mu_{N0} = 60$, which leads to time $x_{N0} = a\mu_{N0} = 0.1883 * 60 = 11.29$. From 400

observations, the statistic D is made by counting the observation as a failure if it is smaller than 11.29 months. The numbers of defectives D are shown in Figures 3 and 4, and also listed as follows:

3, 1, 4, 4, 3, 3, 3, 6, 4, 3, 5, 2, 3, 3, 4, 4, 4, 4, 6, 1, 4, 5, 5, 6, 7, 7, 2, 1, 7, 8, 3, 3, 3, 9, 4, 5, 10, 5, 5, 7.

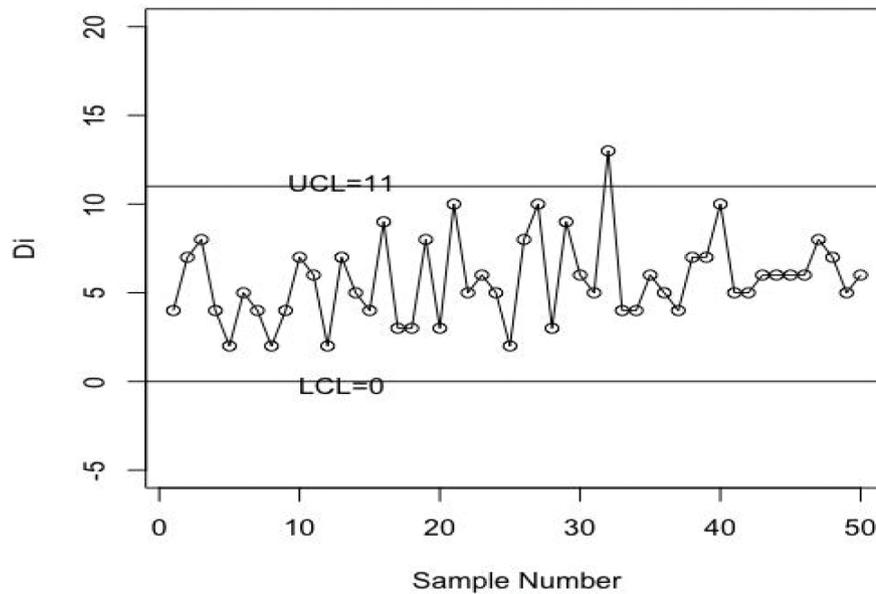


Figure 1 | The proposed chart for simulation data.

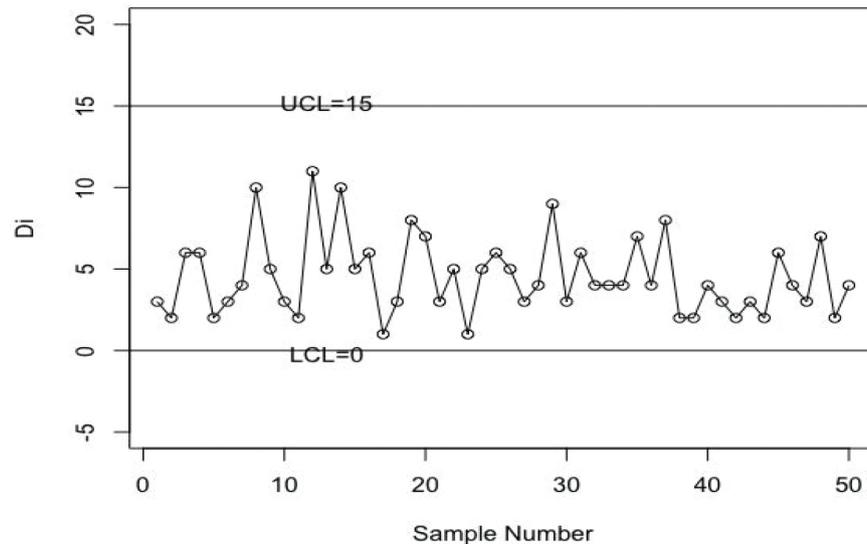


Figure 2 | The [6] chart for simulation data.

The control limits for the proposed chart are $UCL = 8$ and $LCL = 0$ when the values of parameters values are used from Table 2. The proposed chart is displayed in Figure 3 and the existing chart is shown in Figures 4, respectively. Figure 3 shows the shift in service time means the shift values detect out of control at 31st sample. The existing chart indicates that no action is needed for service time because all values are in control. By comparing both charts, it can be seen that the proposed control chart indicates an issue in monthly service time for the specific subsystem of the car while the existing chart does not show any issue in monthly service time for the specific subsystem of the car.

5. CONCLUSIONS

The time-truncated attribute control chart was herein presented using the neutrosophic Weibull distribution. The measurement's

indeterminacy effect was studied on the performance of the proposed control chart with the counterpart control chart. The proposed chart was the generalized version of the attribute control chart under a neutrosophic environment. From the results presented in Tables 1-4, we observed that the indeterminacy parameter significantly affected ARL values. The out-of-control ARL values reduced when indeterminacy increased. The comparative study proved the current chart's efficiency. The proposed control chart was proven to monitor service time in the automobile industry. The proposed control chart has limitations, i.e., it can only be applied when the life/service time follows the neutrosophic Weibull distribution. Secondly, the proposed control chart cannot be applied for variable data. The proposed control chart can be applied in the automobile industry, aircraft industry, and mobile industry for monitoring the defective items. In the future, the proposed control chart should be used for other neutrosophic statistical dis-

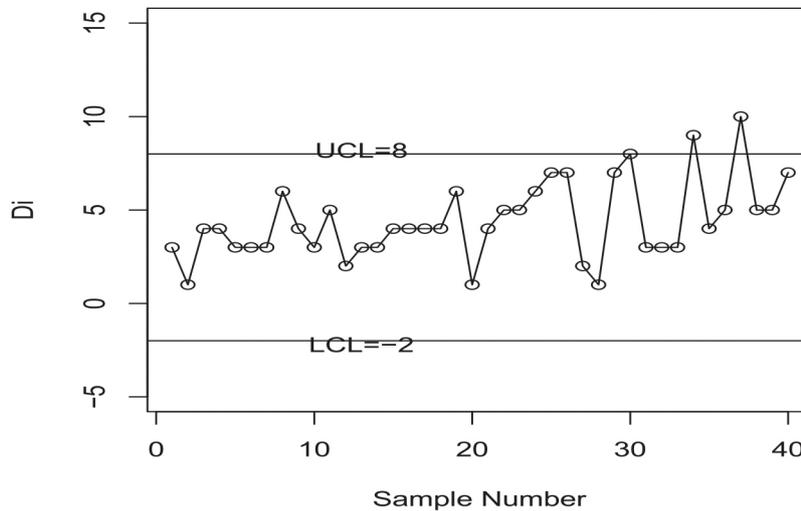


Figure 3 | The proposed chart for real data.

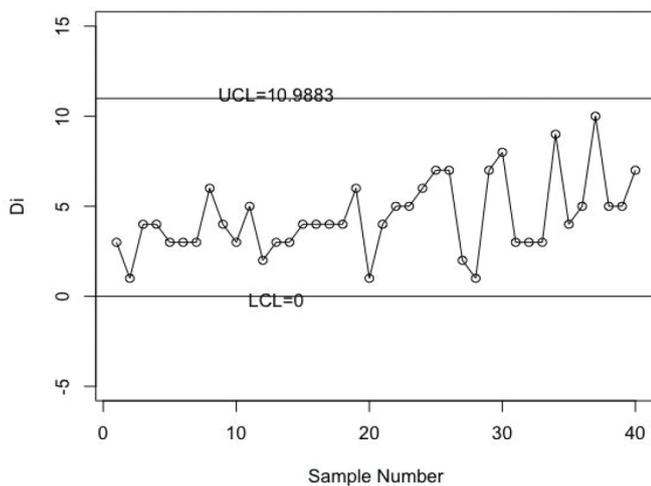


Figure 4 | The existing chart for real data.

tributions. Following [7], the proposed control chart used an exponentially weighted moving average (EWMA) statistic and cumulative sum (CUSUM) statistic, both of which should be considered in future research.

CONFLICTS OF INTEREST

The authors declare no conflict of interest.

AUTHORS' CONTRIBUTIONS

A.H.A.M, A.S, M.A and A.A wrote the paper.

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