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Radar data analysis in the presence of uncertainty

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ABSTRACT

This paper presents the designing of the V-test test for testing the randomness of angles under the neutrosophic statistics. The proposed test applies when the decision-maker is uncertain about the sample size or in the measurement of angles. The operational procedure for testing the randomness of the radar angles under an indeterminate environment is given. An example of radar data is chosen for illustration purpose. From the radar example, it is concluded that the proposed test is suitable to apply for testing the radar angles randomness when uncertainty is present. In addition, the proposed test is more efficient, flexible, and informative than the existing test under classical statistics.

Introduction

The statistical tests are applied for data measured on a linear scale mostly. For the data collected on the circular, it is common to test whether the observed angles are cluster or random around the given angle. The circular data are measured in degree or in radian. This types of data have periodic nature and obtained from various scientific research fields including ecology, data on wind and ocean directions, sociology, space science, medical sciences and education, for more details, the reader may refer to (Mardia et al., 2007), (Rutishauser et al., 2010) (Rivest et al., 2016), (Puglisi et al., 2017) and (Warren et al., 2017). (Landler et al., 2018) discussed the statistical analysis for circular data in biological study. The circular data need special treatment as the usual statistical techniques cannot be applied for testing it. According to (Landler et al., 2018) "Circular data need special treatment in data analysis: consider that an angle of 355° is much nearer to an angle of 5° than it is to an angle of 330°, and so simple arithmetic mean for example, can be quite misleading". (Fisher, 1995) presented a book on the applications of circular tests. (Kanji, 2006) discussed V-test using radar angles data. (Yedlapalli et al., 2016) presented the circular Weibull distribution. (Cremers & Klugkist, 2018) presented the tutorial for circular data using R. (Landler et al., 2018) used the statistical method on circular data from biology. (Landler et al., 2019) presented the circular statistical test for non-continuous data. (Landler et al., 2020) compared traditional and circular tests.

The sample size selection is a critical issue in applying the statistical test. The statistical test for circular cannot be applied when uncertainty in sample size, measurement of angles, and other parameters are involved in the tests. In such cases, the statistical tests designed under fuzzy logic can be applied. L. Chen et al. (2013) applied the test for flood data. Kesemen et al. (2016) discussed the application of fuzzy logic for directional data. Lubiano et al. (2016) discussed the testing procedure under fuzzy logic. Pewsey and García-Portugués (2020) studied the development of circular data. More information can be seen in Yang and Pan (1997), Pewsey et al. (2013) and Benjamin et al. (2019).

The fuzzy logic is a special case of neutrosophic logic proposed by (Smarandache, 1998). The neutrosophic logic consists of measures of truth, false, and indeterminacy. The neutrosophic logic reduces to fuzzy logic when no measure of indeterminacy is found. The applications of neutrosophic logic can be seen in Abdel-Basset et al. (2019), Nabeeh et al. (2019), Pratihar, Kumar, Dey et al. (2020) and Pratihar, Kumar, Edalatpanah et al. (2020). Smarandache and Khalid (2015) showed that neutrosophic logic is more efficient than fuzzy logic and interval-based analysis. Based on the idea of neutrosophic logic (Smarandache, 2014) introduced descriptive neutrosophic statistics. The neutrosophic statistics which is the extension of classical statistics deals with the data having Neutrosophy, inexact values, unclear observations, and interval values. Like the neutrosophic logic, neutrosophic statistics give additional information about the measure of indeterminacy which classical statistics do not provide. Chen, Ye and Du (2017) and Chen, Ye, Du et al. (2017) discussed the methods to analyze the neutrosophic data. Aslam (2019a, 2019b, and 2020) presented statistical tests under neutrosophic statistics.

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Several tests under classical statistics and fuzzy logic to analyzing the circular data are available in the literature. The existing tests are unable to give information about the measure of indeterminacy under the uncertainty. By exploring the literature, the author did not see any work on the V-test under neutrosophic statistics. In this paper, we will introduce the neutrosophic statistical test for circular data originally and for the first time. The necessary measures in the implementation of the proposed test are given under the neutrosophic statistics. The application of the proposed test is given using the angular data measured from radar. The simulation and comparative studies are given using radar data.

The proposed test under NS

The existing V-test under classical statistics is applied to test whether the measured angles are cluster around a specified angle or random around it. The existing test is applied under the assumption that angles or sample size should be determined. In practice, the selection of an appropriate sample size is always a challenge for decision-makers. In addition, the measurement of angles is not exact always. When such uncertainties are presented, the use of the existing test under classical statistics may mislead the decision-maker. In this section, the design of the V-test will be presented under neutrosophic statistics. The proposed test will be applied for testing the randomness of the angles in the presence of uncertainty. The methodology of the proposed test is stated as follows.

Suppose that $n_N = n_L + n_U I_{Nn}; I_{Nn}\varepsilon[I_{Ln}, I_{Un}]$ be a neutrosophic random sample of the size $n_N\varepsilon[n_L, n_U]$ of neutrosophic angular values $\Phi_{iN} = \Phi_{iL} + \Phi_{iU}I_{N\Phi}; I_{N\Phi}\varepsilon[I_{L\Phi}, I_{U\Phi}]$. Note that the first values n_L and Φ_{iL} denote classical statistics and the second values $n_U I_{Nn}$ and $I_{N\Phi}$ denote the indeterminate part of the neutrosophic forms. Where $I_{Nn}\varepsilon[I_{Ln}, I_{Un}]$ and $I_{N\Phi}\varepsilon[I_{L\Phi}, I_{U\Phi}]$ are indeterminate intervals associated with sample size and angular values. Note here that the above mentioned neutrosophic forms reduce to classical statistics when $I_{Ln} = 0$ and $I_{L\Phi} = 0$. Suppose that $\Phi_{0N}\varepsilon[\Phi_{0L}, \Phi_{0U}]$ be a specified neutrosophic angle and $r_N\varepsilon[r_L, r_U]$ be the length of the neutrosophic mean vector. The test statistic of the proposed test is given by

$$V_N = (2n_N)^{\frac{1}{2}}\vartheta_N; \ V_N\varepsilon[V_L, V_U], \ \vartheta_N\varepsilon[\vartheta_L, \vartheta_U] \quad (1)$$

where $\vartheta_N = r_N \cos(\bar{\Phi}_N - \theta_{0N})$ and $r_N\varepsilon[r_L, r_U]$ is given by

$$\vartheta_N = r_N \cos(\bar{\Phi}_N - \theta_{0N}) \ and \ r_N\varepsilon[r_L, r_U] \quad (2)$$

where $\bar{X}_N\varepsilon[\bar{X}_L, \bar{X}_U]$ and $\bar{Y}_N\varepsilon[\bar{Y}_L, \bar{Y}_U]$ are defined by

$$\bar{X}_N = \frac{1}{n_N}[\cos\Phi_1, \ldots, \cos\Phi_2; \cos\Phi_3, \ldots, \cos\Phi_{n_N}]$$

$$\bar{Y}_N = \frac{1}{n_N}[\sin\Phi_1, \ldots, \sin\Phi_2; \sin\Phi_3, \ldots, \sin\Phi_{n_N}]$$

The values of $\bar{\Phi}_N\varepsilon[\bar{\Phi}_L, \bar{\Phi}_U]$ can be calculated using the following formula:

$$\bar{\Phi}_N = \begin{cases} \arctan\left(\frac{\bar{Y}_N}{\bar{X}_N}\right) if \bar{X}_N > 0 \\ 180^o + \arctan\left(\frac{\bar{Y}_N}{\bar{X}_N}\right) if \bar{X}_N < 0 \end{cases}; \quad (3)$$

$$\bar{X}_N\varepsilon[\bar{X}_L, \bar{X}_U], \ \bar{Y}_N\varepsilon[\bar{Y}_L, \bar{Y}_U], \ \bar{\Phi}_N\varepsilon[\bar{\Phi}_L, \bar{\Phi}_U]$$

The statistic $V_N\varepsilon[V_L, V_U]$ in neutrosophic form can be expressed as follows:

$$V_N = V_L + V_U I_{vN}; I_{vN}\varepsilon[I_{vL}, I_{vU}] \quad (4)$$

The neutrosophic form of the statistic $V_N\varepsilon[V_L, V_U]$ consists of two parts, the first part V_L and the second part $V_U I_{vN}$. The first part is known as the determined part and the second part is known as the indeterminate part of the neutrosophic form. The proposed statistic is the generalization of the V-test under classical statistics. The proposed test reduces to test under classical statistics when $V_L = 0$.

Application

In this section, the application of the proposed test is given using the angles data measured from the radar. The decision-maker is interested to test either the angular data is a cluster around $\theta_{0N} = 265^0$ or angles are random. For testing the hypothesis, he is uncertain about the sample size. Suppose that the decision-maker is neutrosophic in the selection of sample size of angular values. The neutrosophic form of sample size is $n_N = n_L + n_U I_N; I_N\varepsilon[I_L, I_U]$. Suppose that for this study I_U=0.13 and n_L=13. Under this information, the neutrosophic form of sample size becomes $n_N = 13 + 15I_N; I_N\varepsilon[0, 0.13]$. This yields a sample size $n_N\varepsilon[13, 15]$. Under uncertainty, the decision-maker should select a random sample of angular values from 13 to 15. The data of angles measured from the radar is shown as follows

$n_L = 13$

$\Phi_1 = 250$, $\Phi_2 = 275$, $\Phi_3 = 285$, Φ_4=285, $\Phi_5 = 290, \Phi_6 = 290$,
$\Phi_7 = 295, \Phi_8 = 300, \Phi_9 = 305, \Phi_{10} = 310, \Phi_{11} = 315,$-
$\Phi_{12} = 320, \Phi_{13} = 330$,

$n_L = 15$

Φ_1=250, $\Phi_2 = 275$, $\Phi_3 = 285$, Φ_4=285, $\Phi_5 = 290, \Phi_6 = 290$,
   $\Phi_7 = 295$, $\Phi_8 = 300$, $\Phi_9 = 305$, $\Phi_{10} = 310$,
$\Phi_{11} = 315, \Phi_{12} = 320, \Phi_{13} = 330$,
   $\Phi_{14} = 330, \Phi_{15} = 5$

The values of $\bar{X}_N \varepsilon[\bar{X}_L, \bar{X}_U]$ can be computed as follows

$$\bar{X}_N = \frac{1}{[13, 15]}[\cos\Phi_1, \ldots, \cos\Phi_{13}; \cos\Phi_1, \ldots, \cos\Phi_{15}]$$
$$= [41.73, 0.4848]$$

The values of $\bar{Y}_N \varepsilon[\bar{Y}_L, \bar{Y}_U]$ can be computed as follows

$$\bar{Y}_N = \frac{1}{[13, 15]}[\sin\Phi_1, \ldots, \sin\Phi_{13}; \sin\Phi_1, \ldots, \sin\Phi_{15}]$$
$$= [-0.8445, -0.7595]$$

The length of the neutrosophic mean vector is calculated as follows

$$r_N = \left[(\bar{X}_L^2 + \bar{Y}_L^2)^{\frac{1}{2}}, (\bar{X}_U^2 + \bar{Y}_U^2)^{\frac{1}{2}} \right] = [0.9420, 0.9016]$$

The values of $\bar{\Phi}_N \varepsilon[\bar{\Phi}_L, \bar{\Phi}_U]$ can be calculated as follows

$$\bar{\Phi}_N = \arctan\left(\frac{\bar{Y}_N}{\bar{X}_N}\right) = [-63.71, -57.3]$$

The values of $\vartheta_N \varepsilon[\vartheta_L, \vartheta_U]$ can be calculated as follows

$$\vartheta_N = r_N \cos(\bar{\Phi}_N - \theta_{0N}) = [0.8049, 0.7122]$$

Based on the above calculations, the values of the statistic $V_N \varepsilon[V_L, V_U]$ can be computed as follows

$$V_N = (2n_N)^{\frac{1}{2}}\vartheta_N = [4.10, 3.90]$$

The proposed test for the example can be explained in the following steps.

Step-1: The null hypothesis H_{0N} that radar angles are cluster around $\theta_{0N} = 265^0$ and the alternative hypothesis that H_{1N} that radar angles are random around $\theta_{0N} = 265^0$.

Step-2: The level of significance α=0.05.

Step-3: The values of the statistic are $V_N \varepsilon[4.10, 3.90]$.

Step-4: The critical value from (Kanji, 2006) is [1.6474,1.6470].

Step-5: The values of $V_N > [1.6474, 1.6470]$, so we do not accept the null hypothesis and conclude that the radar angles are random around $\theta_{0N} = 265^0$.

Comparative studies

In this section, we will compare the efficiency of the proposed test with the existing test in terms of the measure of indeterminacy. As mentioned before that the proposed test is an extension of the test under classical statistics. The proposed test reduces to test under classical statistics if no uncertain or indeterminate observations are recorded in the data. The neutrosophic form of the statistic $V_N \varepsilon[V_L, V_U]$ is expressed as: $V_N = 4.10 - 3.90I_{\nu N}; I_{\nu N}\varepsilon[0, 0.051]$. The first value of statistic $V_N \varepsilon[V_L, V_U]$ represents the value of test statistic under classical statistics. The second part $3.90I_{\nu N}$ is an indeterminate part

of the neutrosophic form. Based on this information, it can be seen that the proposed test provides the values of the statistic $V_N \varepsilon[V_L, V_U]$ in an interval that is required when testing is done under an uncertain environment. In addition, the proposed test gives information about the measure of indeterminacy associated with the test. For the radar data, the measure of indeterminacy is 0.051. Using this information, the results of the test can be interpreted as the probability that H_{0N} is accepted is 95%; the probability of rejecting H_{0N} is 5% and the chance of uncertainty about H_{0N} is 0.051. From this study, it can be noted that the proposed test has an edge over the existing test. The proposed test provides flexible and provides more information about H_{0N} than the existing test. Therefore, the proposed test can be applied for the testing of the hypothesis when the decision-maker is uncertain about the sample size.

Simulation study

In this section, the effect of the measure of indeterminacy on the results is studied through the simulation. For this study, the neutrosophic forms of sample size and statistic $V_N \varepsilon[V_L, V_U]$ are considered. The neutrosophic sample size and the values of the statistic $V_N \varepsilon[V_L, V_U]$ for various values of I_U are shown in Table 1. From Table 1, it can be noted that the indeterminacy affects the sample size and statistic significantly. The values of the sample size increase as the measure of indeterminacy increases. On the other hand, the same trend is observed for the values of the statistic. From the study, it can be seen that the measure of indeterminacy also affects the decision about H_{0N}. From Table 1, it can be noted that the null hypothesis is accepted when the measure of indeterminacy is large than 0.60. It means the larger values

Table 1. Effect of measure of indeterminacy.

I_U	$n_N = 13 + 15I_U$	$V_N = 4.10 - 3.90I_U$	Decision
0	13	4.1	Do not accept H_{0N}
0.01	13	4.061	Do not accept H_{0N}
0.02	13	4.022	Do not accept H_{0N}
0.03	13	3.983	Do not accept H_{0N}
0.04	14	3.944	Do not accept H_{0N}
0.05	14	3.905	Do not accept H_{0N}
0.06	14	3.866	Do not accept H_{0N}
0.07	14	3.827	Do not accept H_{0N}
0.08	14	3.788	Do not accept H_{0N}
0.09	14	3.749	Do not accept H_{0N}
0.10	15	3.71	Do not accept H_{0N}
0.15	15	3.515	Do not accept H_{0N}
0.20	16	3.32	Do not accept H_{0N}
0.30	18	2.93	Do not accept H_{0N}
0.40	19	2.54	Do not accept H_{0N}
0.50	21	2.15	Do not accept H_{0N}
0.60	22	1.76	Do not accept H_{0N}
0.70	24	1.37	Accept H_{0N}
0.80	25	0.98	Accept H_{0N}
0.90	27	0.59	Accept H_{0N}
1.00	28	0.2	Accept H_{0N}

of indeterminacy may mislead the decision-maker about the null hypothesis. In addition, a more sample size is needed for the larger values of the measure of indeterminacy.

Concluding remarks

This paper presented the designing of the V-test test for testing the randomness of angles under the neutrosophic statistics. The proposed test can be applied in an uncertain environment. An example of radar data was chosen for illustration purpose. The results were presented in the neutrosophic form. From the analysis, it is concluded that the existing V-test under classical statistics is a special case of the proposed test. In addition, the proposed test provides more information about the data than the existing test. From the simulation study, it is concluded that indeterminacy affects the results significantly. In nutshell, the proposed test can be applied in a variety of fields where the data is recorded in angles. The proposed test using some other sampling schemes can be studied as future research. The application of the proposed test for big data is also a fruitful area for future research.

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Data Availability

The data is given in the paper.

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