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Sampling Plan Using Process Loss Index Using Multiple Dependent State Sampling Under Neutrosophic Statistics

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ABSTRACT This paper presents the designing of a sampling plan using the process loss consideration for the multiple dependent state sampling under the neutrosophic statistics. The operating characteristics under the neutrosophic statistical interval method (NSIM) are developed to find the neutrosophic plan parameters of the proposed sampling plan. A non-linear optimization under NSIM is used to find the optimal neutrosophic plan parameters under the given conditions. The advantages of the proposed sampling plan are discussed over the existing sampling plans. A real example having some uncertain observations is given for the illustration purpose.

INDEX TERMS Classical statistics, producer's risk, consumer's risk, neutrosophic plan parameters, inspection.

I. INTRODUCTION

The service companies and the manufacturing industries are in a race to enhance the quality level of their product. Therefore, the producers are very careful in inspection at each stage of the manufacturing of the product. The inspection of the product from the raw material to the final product plays a significant role to maintain the high quality of the product. At the final stage, it may not possible to inspect all items in a lot of the product. Therefore, the inspection is done using the acceptance sampling plans. The acceptance sampling plans minimize the inspection cost and time. The plan parameters of the sampling plan are determined at the given producer's risk and consumer's risk. The acceptance sampling plans have many applications in the variety of fields. [1] used the inspection plan for the ocean data. Reference [2] used the Bayesian approach to design a sampling plan. Reference [3] applied the Weibull distribution to the sampling plan. Reference [4] proposed the coefficient of variation based sampling plan.

As mentioned by [5] "Because the sampling cannot guarantee that every defective item in a lot will be inspected, the sampling involves risks of not adequately reflecting the quality levels of the lot. Such risk is even more significant

as the rapid advancement of the manufacturing technology and stringent customers demand is enforced". Therefore, reduction in the defective items and produced the product according to the given target are important aims of the industries. The process loss index is applied to design the product according to the given specifications. Reference [5] introduced this index in the sampling plan. Reference [6] presented the improved form of [5] plan. Reference [7] proposed the variable plan the process loss. The multiple dependent state (MDS) sampling is the extension of single sampling. The MDS sampling is applied the experimenter is in-decision state at the sample information. Therefore, to make a decision, the previous information is also considered. Reference [8] proposed the repetitive sampling plan using this index. Reference [9] designed the MDS plan using the process loss index.

When the industrial engineer is not sure about the proportion defective of the product, the fuzzy approach is applied to design the sampling plans for this situation. Several authors contributed to designing the sampling plan using the fuzzy logic. Reference [10] and [11] proposed the attribute plans using the fuzzy logic. Reference [12] designed plan assuming plan parameters are fuzzy. Reference [13] applied the fuzzy Poisson distribution. Reference [14] studied the behaviors of fuzzy OC curves. Reference [15] proposed two-stage fuzzy

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plan. Reference [16] proposed the fuzzy sequential plan. Reference [17] worked on the plan using fuzzy gamma distribution. Reference [18] proposed the fuzzy mathematical analysis for sampling plan. Reference [19] worked on fuzzy MDS sampling plan.

Reference [20] claimed that the neutrosophic logic is the extension of the fuzzy logic. The neutrosophic logic considers the measure of indeterminacy which fuzzy logic does not take into account. Reference [21] introduced the neutrosophic statistics (NS) using the neutrosophic numbers (NN). The NS is the generalized form of the NS and applied when the sample is selected from a population having unclear, fuzzy and uncertain observations. In this situation, the data that is taken or the parameters are in indeterminacy interval and can be analyzed using the NS. Reference [22] and [23] worked on the NN. Reference [24] introduced the area of neutrosophic statistical quality control. Reference [25] proposed the plan for the inspection of product coming from various lines. Reference [26] presented the plan for the exponential using the NS. Reference [27] proposed the NS sudden death plan. Reference [28] worked on the attribute plan using the neutrosophic binomial distribution. Reference [29] worked on a plan under the NS and having measurement error.

By exploring the literature traditional sampling plans and fuzzy plans, we could not find the work on the sampling plan for the process loss index using the MDS under the NS. In this paper, we will present the designing of the plan using MDS under the neutrosophic statistical interval method (NSIM). We will derive the neutrosophic operating characteristic (NOC) for the MDS using NSIM. The plan parameters will be determined using NOC and used to explain the data that is taken from the industry. We expect that the proposed sampling plan will perform better in uncertainty than the existing plan in sample size.

II. DESIGN OF THE PROPOSED PLAN

In this section, we will first introduce the process loss function under NSIM. The proposed plan using the neutrosophic process index (NPI) will also be given.

Let $X_{Ni} \in \{X_L, X_U\} = i = 1, 2, 3, \dots, n$ be a neutrosophic random sample selected from the population having some vogues and imprecise observations, where X_L denotes the lower value in the indeterminacy interval and X_U denotes the larger value in the indeterminacy interval. Suppose that $\mu_N \in \{\mu_L, \mu_U\}$ and $\sigma_N \in \{\sigma_L, \sigma_U\}$ are the corresponding neutrosophic population mean and population standard of the population. Therefore, $X_{Ni} \in \{X_L, X_U\} = i = 1, 2, 3, \dots, n$ follows the neutrosophic normal distribution with $\mu_N \in \{\mu_L, \mu_U\}$ and $\sigma_N \in \{\sigma_L, \sigma_U\}$. Reference [24] defined the following form of NPI

$$L_{Ne} = \frac{\sigma_N^2}{d^2} + \frac{(\mu_N - T)^2}{d^2}; \quad \mu_N \in \{\mu_L, \mu_U\}, \sigma_N \in \{\sigma_L, \sigma_U\} \quad (1)$$

where $d = (USL - LSL) / 2$, USL=upper specification limit and LSL=lower specification limit. Usually, the values of

$\mu_N \in \{\mu_L, \mu_U\}$ and $\sigma_N \in \{\sigma_L, \sigma_U\}$ are not known in practice. The NPI based on the sample is given as

$$\hat{L}_{Ne} = \frac{S_N^2}{d^2} + \frac{(\bar{X}_N - T)^2}{d^2}; \quad \bar{X}_N \in \{\bar{X}_L, \bar{X}_U\}, S_N^2 \in \{S_L^2, S_U^2\} \quad (2)$$

where $\bar{X}_N \in \{\bar{X}_L, \bar{X}_U\}$ and S_N^2 belong to $\{S_L^2, S_U^2\}$ are the best sample estimate of $\mu_N \in \{\mu_L, \mu_U\}$ and $\sigma_N^2 \in \{\sigma_L^2, \sigma_U^2\}$, respectively and $\bar{X}_L = \sum_{i=1}^n x_i^L / n_L, \bar{X}_U = \sum_{i=1}^n x_i^U / n_U, s_L = \sqrt{\sum_{i=1}^n (x_i^L - \bar{X}_L)^2} / n_L$ and $s_U = \sqrt{\sum_{i=1}^n (x_i^U - \bar{X}_U)^2} / n_U$ Based on the given information, we proposed the following plan

Step 1: Choose a sample of size $n_N \in \{n_L, n_U\}$ from the lot of the product and compute $\hat{L}_{Ne} = \frac{S_N^2}{d^2} + \frac{(\bar{X}_N - T)^2}{d^2}; \bar{X}_N \in \{\bar{X}_L, \bar{X}_U\}, S_N^2 \in \{S_L^2, S_U^2\}$.

Step 2: A lot of product is accepted if $\hat{L}_{Ne} \leq k_{aN}$; $k_{aN} \in \{k_{aL}, k_{aU}\}$, where $k_{aN} \in \{k_{aL}, k_{aU}\}$ is the neutrosophic maximum allowed number of defective and a lot is rejected if $\hat{L}_{Ne} > k_{rN}$; $k_{rN} \in \{k_{rL}, k_{rU}\}$, where k_{rN} is the neutrosophic rejection number. If $k_{aN} < \hat{L}_{Ne} \leq k_{rN}$, then accept the current lot provided that the preceding m lots were accepted on the condition that $\hat{L}_{Ne} \leq k_{aN}$.

So, the proposed sampling plan has three parameters, $n_N \in \{n_L, n_U\}$, $k_{aN} \in \{k_{aL}, k_{aU}\}$ and $k_{rN} \in \{k_{rL}, k_{rU}\}$. Several sampling plans are the special case of the present sampling plan. The plan under the NSIM proposed by [24] is the special case when $m = 0$. The proposed plan reduces to [5] plan under classical statistics when $m = 0, k_{rL} = k_{rU}$ and $k_{aL} = k_{aU}$. The neutrosophic operating characteristic (NOC) for the proposed plan is given as follows

$$L(p)_N = P_{1N} + P_{2N} [P_{1N}]^m; \quad L(p)_N \in \{L(p)_L, L(p)_U\} \quad (3)$$

Note here that P_{1N} denotes the lot acceptance probability for the single sampling plan under NSIM and given as follows [24]

$$P_{1N} = P \left\{ \hat{L}_{Ne} \leq k_a \right\}; \quad n_N \in \{n_L, n_U\}, \bar{X}_N \in \{\bar{X}_L, \bar{X}_U\}, S_N^2 \in \{S_L^2, S_U^2\}, k_{aN} \in \{k_{aL}, k_{aU}\}, \bar{X}_N \in \{\bar{X}_L, \bar{X}_U\} \quad (4)$$

As mentioned by [5] and [24], \hat{L}_{Ne} follows the neutrosophic Chi-square χ_{nN}^2 distribution with quantity $L_{Ne} \chi_{nN}^2 / n_N$. Therefore, the Eq. (1) can be written as

$$P_{1N} = P \left\{ \chi_{nN}^2 \leq (n_N k_{aN} / L_{Ne}) \right\}; \quad n_N \in \{n_L, n_U\}, k_{aN} \in \{k_{aL}, k_{aU}\} \quad (5)$$

Similarly, the probability of \hat{L}_{Ne} lying within $k_{aN} \in \{k_{aL}, k_{aU}\}$ and $k_{rN} \in \{k_{rL}, k_{rU}\}$ is derived as follows:

$$P_{2N} = P \left\{ k_{aN} < \hat{L}_{Ne} \leq k_{rN} \right\}; \quad n_N \in \{n_L, n_U\}, k_{aN} \in \{k_{aL}, k_{aU}\} \text{ and } k_{rN} \in \{k_{rL}, k_{rU}\} \quad (6)$$

TABLE 1. The plan parameters of the proposed plan when m = 1.

p_1	p_2	n_N	k_{rN}	k_{aN}	$L(C_{AQL})$	$L(C_{LQL})$
0.001	0.002	[33,40]	[0.065,0.044]	[0.001,0.001]	[0.961,0.996]	[0.082,0.049]
	0.003	[13,16]	[0.219,0.275]	[0.001,0.001]	[0.958,0.997]	[0.095,0.058]
	0.004	[9,11]	[0.08,0.024]	[0.001,0.001]	[0.957,1]	[0.097,0.068]
	0.006	[6,8]	[0.002,0.009]	[0.001,0.001]	[0.951,1]	[0.083,0.043]
	0.008	[6,6]	[0.372,0.032]	[0.001,0.001]	[0.971,0.998]	[0.083,0.064]
	0.01	[5,7]	[0.111,0.003]	[0.001,0.001]	[0.967,0.961]	[0.079,0.011]
	0.15	[4,4]	[0.011,0.003]	[0.002,0.001]	[0.99,0.952]	[0.095,0.045]
	0.02	[4,4]	[0.114,0.21]	[0.001,0.001]	[0.983,0.96]	[0.096,0.059]
0.0025	0.005	[29,41]	[0.061,0.004]	[0.003,0.003]	[0.952,0.972]	[0.091,0.033]
	0.01	[9,11]	[0.005,0.032]	[0.004,0.003]	[0.962,0.999]	[0.095,0.05]
	0.15	[7,9]	[0.341,0.005]	[0.003,0.003]	[0.953,0.955]	[0.034,0.016]
	0.2	[5,6]	[0.01,0.016]	[0.003,0.003]	[0.975,0.959]	[0.068,0.051]
	0.25	[5,5]	[0.015,0.008]	[0.004,0.003]	[0.988,1]	[0.098,0.044]
	0.3	[4,5]	[0.009,0.073]	[0.003,0.003]	[0.968,1]	[0.079,0.049]
	0.5	[4,4]	[0.016,0.011]	[0.005,0.003]	[0.998,0.975]	[0.059,0.031]
0.005	0.001	[29,40]	[0.136,0.035]	[0.006,0.006]	[0.953,0.995]	[0.078,0.049]
	0.015	[13,16]	[0.086,0.11]	[0.006,0.006]	[0.954,1]	[0.092,0.052]
	0.02	[9,10]	[0.078,0.016]	[0.006,0.006]	[0.956,0.995]	[0.089,0.051]
	0.03	[6,8]	[0.031,0.057]	[0.006,0.008]	[0.961,0.994]	[0.099,0.083]
	0.04	[5,7]	[0.011,0.036]	[0.008,0.007]	[0.963,1]	[0.077,0.042]
	0.05	[4,5]	[0.016,0.022]	[0.006,0.007]	[0.956,0.998]	[0.089,0.063]
	0.1	[4,4]	[0.027,0.321]	[0.012,0.007]	[0.999,1]	[0.067,0.065]
0.01	0.02	[27,37]	[0.023,0.17]	[0.012,0.012]	[0.951,0.983]	[0.086,0.07]
	0.03	[13,20]	[0.353,0.19]	[0.012,0.012]	[0.957,0.958]	[0.063,0.031]
	0.04	[8,9]	[0.029,0.265]	[0.012,0.012]	[0.955,0.987]	[0.094,0.092]
	0.05	[7,9]	[0.024,0.095]	[0.014,0.012]	[0.973,1]	[0.09,0.047]
	0.1	[4,5]	[0.03,0.058]	[0.013,0.013]	[0.969,0.983]	[0.095,0.058]
	0.15	[4,4]	[0.059,0.106]	[0.016,0.012]	[0.99,1]	[0.088,0.071]
	0.2	[3,4]	[0.024,0.127]	[0.013,0.011]	[0.957,1]	[0.055,0.039]
0.03	0.06	[28,38]	[0.344,0.155]	[0.035,0.036]	[0.951,0.987]	[0.086,0.058]
	0.09	[13,18]	[0.298,0.102]	[0.036,0.037]	[0.953,0.961]	[0.062,0.037]
	0.12	[9,11]	[0.377,0.055]	[0.036,0.039]	[0.957,0.975]	[0.081,0.039]
	0.15	[7,9]	[0.119,0.168]	[0.039,0.037]	[0.972,1]	[0.086,0.046]
	0.3	[4,5]	[0.097,0.076]	[0.036,0.041]	[0.96,1]	[0.066,0.05]
0.05	0.1	[27,37]	[0.097,0.087]	[0.059,0.058]	[0.961,0.965]	[0.083,0.036]
	0.15	[13,15]	[0.134,0.124]	[0.063,0.061]	[0.968,1]	[0.098,0.05]
	0.2	[8,10]	[0.14,0.153]	[0.063,0.063]	[0.964,0.978]	[0.098,0.057]
	0.25	[7,8]	[0.828,0.149]	[0.059,0.06]	[0.95,0.955]	[0.079,0.046]
	0.5	[4,5]	[0.135,0.231]	[0.061,0.082]	[0.958,1]	[0.085,0.08]

or

$$P_{2N} = P \left\{ \chi_{nN}^2 \leq (n_N k_{rN} / L_{Ne}) \right\} - P \left\{ \chi_{nN}^2 < (n_N k_{aN} / L_{Ne}) \right\} \quad (7)$$

Finally, NOC is given by

$$L(p)_N = P \left\{ \chi_{nN}^2 \leq (n_N k_{aN} / L_{Ne}) \right\} + \left[P \left\{ \chi_{nN}^2 \leq (n_N k_{rN} / L_{Ne}) \right\} \right]$$

TABLE 2. The plan parameters of the proposed plan when $m = 2$.

p_1	p_2	n_N	k_{r_N}	k_{a_N}	$L(C_{AQL})$	$L(C_{LQL})$
0.001	0.002	[27,38]	[0.02,0.062]	[0.001,0.001]	[0.966,1]	[0.067,0.063]
	0.003	[12,18]	[0.046,0.002]	[0.001,0.002]	[0.951,1]	[0.073,0.06]
	0.004	[8,12]	[0.519,0.021]	[0.001,0.001]	[0.956,1]	[0.075,0.042]
	0.006	[6,8]	[0.019,0.003]	[0.001,0.002]	[0.974,1]	[0.091,0.083]
	0.008	[5,7]	[0.813,0.149]	[0.001,0.001]	[0.97,0.953]	[0.08,0.021]
	0.01	[4,5]	[0.589,0.002]	[0.001,0.002]	[0.963,1]	[0.076,0.071]
	0.15	[4,4]	[0.999,0.006]	[0.002,0.002]	[0.979,0.957]	[0.086,0.066]
	0.02	[3,4]	[0.123,0.112]	[0.001,0.001]	[0.951,0.997]	[0.047,0.037]
0.0025	0.005	[25,35]	[0.211,0.049]	[0.003,0.003]	[0.951,1]	[0.06,0.05]
	0.01	[8,12]	[0.352,0.01]	[0.003,0.003]	[0.963,1]	[0.065,0.028]
	0.15	[6,7]	[0.743,0.061]	[0.004,0.003]	[0.974,1]	[0.061,0.047]
	0.2	[5,6]	[0.048,0.006]	[0.004,0.005]	[0.974,0.971]	[0.092,0.073]
	0.25	[4,5]	[0.213,0.041]	[0.003,0.004]	[0.959,1]	[0.074,0.062]
	0.3	[4,5]	[0.298,0.007]	[0.004,0.004]	[0.977,0.99]	[0.092,0.04]
	0.5	[3,3]	[0.007,0.517]	[0.003,0.003]	[0.954,0.998]	[0.096,0.094]
0.005	0.001	[26,32]	[0.61,0.024]	[0.006,0.006]	[0.955,0.989]	[0.093,0.054]
	0.015	[12,15]	[0.487,0.013]	[0.007,0.007]	[0.966,1]	[0.087,0.049]
	0.02	[8,11]	[0.806,0.032]	[0.007,0.007]	[0.963,1]	[0.088,0.042]
	0.03	[6,7]	[0.956,0.012]	[0.007,0.007]	[0.973,1]	[0.072,0.048]
	0.04	[5,5]	[0.048,0.545]	[0.008,0.007]	[0.985,0.96]	[0.083,0.083]
	0.05	[4,5]	[0.857,0.14]	[0.007,0.007]	[0.964,0.961]	[0.061,0.053]
	0.1	[3,4]	[0.026,0.062]	[0.006,0.007]	[0.95,1]	[0.045,0.037]
0.01	0.02	[26,28]	[0.096,0.049]	[0.012,0.012]	[0.957,1]	[0.096,0.081]
	0.03	[11,13]	[0.475,0.049]	[0.013,0.013]	[0.95,0.969]	[0.093,0.067]
	0.04	[8,10]	[0.285,0.212]	[0.014,0.014]	[0.962,1]	[0.086,0.061]
	0.05	[7,8]	[0.901,0.167]	[0.013,0.013]	[0.952,1]	[0.059,0.05]
	0.1	[4,5]	[0.12,0.032]	[0.014,0.015]	[0.966,1]	[0.06,0.055]
	0.15	[4,4]	[0.832,0.039]	[0.019,0.017]	[0.995,0.956]	[0.096,0.07]
	0.2	[3,3]	[0.044,0.035]	[0.013,0.013]	[0.956,0.982]	[0.1,0.096]
0.03	0.06	[25,28]	[0.207,0.078]	[0.037,0.037]	[0.951,0.998]	[0.094,0.074]
	0.09	[11,15]	[0.081,0.165]	[0.039,0.04]	[0.954,0.965]	[0.07,0.055]
	0.12	[8,8]	[0.289,0.123]	[0.04,0.041]	[0.957,1]	[0.099,0.096]
	0.15	[7,8]	[0.412,0.1]	[0.045,0.04]	[0.98,0.997]	[0.096,0.05]
	0.3	[4,5]	[0.451,0.063]	[0.04,0.047]	[0.96,0.984]	[0.097,0.06]
0.05	0.1	[25,33]	[0.438,0.295]	[0.061,0.061]	[0.95,0.992]	[0.067,0.052]
	0.15	[12,14]	[0.755,0.218]	[0.068,0.064]	[0.967,1]	[0.072,0.056]
	0.2	[8,9]	[0.82,0.266]	[0.068,0.07]	[0.961,1]	[0.082,0.081]
	0.25	[7,9]	[0.529,0.277]	[0.073,0.069]	[0.974,0.999]	[0.069,0.038]
	0.5	[4,4]	[0.149,0.351]	[0.068,0.065]	[0.957,1]	[0.097,0.089]

$$\begin{aligned}
 & - P \left\{ \chi_{n_N}^2 < (n_N k_{a_N} / L_{Ne}) \right\} \\
 & \times \left[P \left\{ \chi_{n_N}^2 \leq (n_N k_{a_N} / L_{Ne}) \right\} \right]^m ; \\
 & L(p)_N \in \{L(p)_L, L(p)_U\} \tag{8}
 \end{aligned}$$

The neutrosophic operating characteristics curve (NOCC) which passes through two points $(p_1, 1 - \alpha)$ and (p_2, β) is considered as the most idea curve. Note here that α and β denote the producer’s risk and consumer’s risk, and

TABLE 3. The plan parameters of the proposed plan when $m = 3$.

p_1	p_2	n_N	k_{rN}	k_{aN}	$L(C_{AQL})$	$L(C_{LQL})$
0.001	0.002	[27,35]	[0.394,0.003]	[0.001,0.001]	[0.952,0.994]	[0.092,0.051]
	0.003	[12,19]	[0.015,0.003]	[0.001,0.001]	[0.954,0.998]	[0.058,0.036]
	0.004	[9,10]	[0.058,0.012]	[0.001,0.002]	[0.97,0.987]	[0.094,0.092]
	0.006	[6,8]	[0.041,0.004]	[0.001,0.001]	[0.964,1]	[0.083,0.037]
	0.008	[5,6]	[0.008,0.042]	[0.002,0.002]	[0.972,0.965]	[0.082,0.08]
	0.01	[4,5]	[0.983,0.494]	[0.001,0.002]	[0.956,1]	[0.09,0.075]
	0.15	[4,4]	[0.014,0.307]	[0.001,0.002]	[0.954,1]	[0.071,0.063]
	0.02	[3,4]	[0.761,0.19]	[0.001,0.002]	[0.956,1]	[0.087,0.041]
0.0025	0.005	[26,34]	[0.053,0.007]	[0.003,0.003]	[0.95,0.952]	[0.098,0.052]
	0.01	[8,12]	[0.117,0.139]	[0.004,0.004]	[0.956,0.999]	[0.082,0.058]
	0.15	[6,6]	[0.557,0.117]	[0.004,0.004]	[0.955,1]	[0.098,0.079]
	0.2	[5,6]	[0.279,0.533]	[0.004,0.004]	[0.982,1]	[0.085,0.043]
	0.25	[4,5]	[0.103,0.063]	[0.003,0.004]	[0.952,1]	[0.061,0.06]
	0.3	[4,4]	[0.247,0.012]	[0.004,0.004]	[0.977,0.993]	[0.1,0.083]
	0.5	[3,4]	[0.037,0.068]	[0.003,0.004]	[0.958,1]	[0.076,0.037]
0.005	0.001	[26,35]	[0.31,0.008]	[0.006,0.006]	[0.95,0.963]	[0.091,0.058]
	0.015	[12,16]	[0.417,0.068]	[0.007,0.007]	[0.954,0.999]	[0.09,0.045]
	0.02	[8,11]	[0.121,0.012]	[0.007,0.008]	[0.953,0.998]	[0.084,0.075]
	0.03	[6,7]	[0.701,0.019]	[0.008,0.009]	[0.97,0.997]	[0.087,0.08]
	0.04	[5,6]	[0.553,0.019]	[0.008,0.01]	[0.984,1]	[0.1,0.084]
	0.05	[4,5]	[0.042,0.196]	[0.007,0.008]	[0.959,0.965]	[0.1,0.057]
	0.1	[3,4]	[0.077,0.02]	[0.007,0.008]	[0.96,1]	[0.081,0.043]
0.01	0.02	[26,32]	[0.692,0.028]	[0.013,0.012]	[0.951,1]	[0.095,0.061]
	0.03	[12,18]	[0.036,0.039]	[0.014,0.014]	[0.961,1]	[0.091,0.04]
	0.04	[8,9]	[0.365,0.031]	[0.014,0.014]	[0.952,0.994]	[0.075,0.071]
	0.05	[7,8]	[0.558,0.074]	[0.015,0.014]	[0.966,1]	[0.099,0.058]
	0.1	[4,4]	[0.441,0.181]	[0.014,0.014]	[0.958,1]	[0.093,0.093]
	0.15	[4,4]	[0.511,0.021]	[0.014,0.017]	[0.96,1]	[0.093,0.073]
	0.2	[3,4]	[0.587,0.102]	[0.014,0.014]	[0.956,1]	[0.078,0.037]
0.03	0.06	[26,40]	[0.456,0.173]	[0.038,0.038]	[0.95,0.981]	[0.062,0.042]
	0.09	[12,15]	[0.285,0.124]	[0.041,0.043]	[0.956,0.999]	[0.089,0.075]
	0.12	[8,10]	[0.809,0.322]	[0.041,0.041]	[0.951,1]	[0.077,0.056]
	0.15	[7,7]	[0.117,0.069]	[0.047,0.044]	[0.977,1]	[0.087,0.083]
	0.3	[4,5]	[0.267,0.162]	[0.042,0.048]	[0.952,1]	[0.073,0.063]
0.05	0.1	[26,34]	[0.531,0.092]	[0.063,0.062]	[0.952,1]	[0.091,0.054]
	0.15	[12,14]	[0.86,0.15]	[0.068,0.066]	[0.955,0.995]	[0.095,0.061]
	0.2	[8,11]	[0.439,0.096]	[0.069,0.078]	[0.952,1]	[0.087,0.067]
	0.25	[7,7]	[0.184,0.536]	[0.071,0.07]	[0.958,1]	[0.08,0.076]
	0.5	[4,5]	[0.346,0.143]	[0.072,0.074]	[0.959,0.972]	[0.085,0.054]

p_1 and p_2 is acceptable quality level and limiting quality level, respectively. The neutrosophic plan parameters $n_N \in \{n_L, n_U\}$, $k_{aN} \in \{k_{aL}, k_{aU}\}$ and $k_{rN} \in \{k_{rL}, k_{rU}\}$ will be determined for

various values of AQL, LQL and m . The following neutrosophic non-linear optimization problem (NNOP) will be applied to select the suitable combinations of the

TABLE 4. Comparison of the proposed plan and existing plan.

p_1	p_2	<i>Proposed</i>			<i>Existing</i>
		$m = 1$	$m = 2$	$m = 3$	
0.001	0.002	[33,40]	[27,38]	[27,35]	[49,54]
	0.003	[13,16]	[12,18]	[12,19]	[18,38]
	0.004	[9,11]	[8,12]	[9,10]	[12,27]
	0.006	[6,8]	[6,8]	[6,8]	[7,15]
	0.008	[6,6]	[5,7]	[5,6]	[6,8]
	0.01	[5,7]	[4,5]	[4,5]	[5,10]
	0.15	[4,4]	[4,4]	[4,4]	[4,6]
	0.02	[4,4]	[3,4]	[3,4]	[4,6]
0.0025	0.005	[29,41]	[25,35]	[26,34]	[39,67]
	0.01	[9,11]	[8,12]	[8,12]	[11,16]
	0.15	[7,9]	[6,7]	[6,6]	[7,10]
	0.2	[5,6]	[5,6]	[5,6]	[6,7]
	0.25	[5,5]	[4,5]	[4,5]	[5,6]
	0.3	[4,5]	[4,5]	[4,4]	[5,8]
	0.5	[4,4]	[3,3]	[3,4]	[4,5]
0.005	0.001	[29,40]	[26,32]	[26,35]	[39,76]
	0.015	[13,16]	[12,15]	[12,16]	[16,27]
	0.02	[9,10]	[8,11]	[8,11]	[11,19]
	0.03	[6,8]	[6,7]	[6,7]	[7,11]
	0.04	[5,7]	[5,5]	[5,6]	[6,8]
	0.05	[4,5]	[4,5]	[4,5]	[5,8]
	0.1	[4,4]	[3,4]	[3,4]	[4,4]
0.01	0.02	[27,37]	[26,28]	[26,32]	[41,56]
	0.03	[13,20]	[11,13]	[12,18]	[16,25]
	0.04	[8,9]	[8,10]	[8,9]	[11,17]
	0.05	[7,9]	[7,8]	[7,8]	[9,14]
	0.1	[4,5]	[4,5]	[4,4]	[5,7]
	0.15	[4,4]	[4,4]	[4,4]	[4,6]
	0.2	[3,4]	[3,3]	[3,4]	[4,4]
0.03	0.06	[28,38]	[25,28]	[26,40]	[37,52]
	0.09	[13,18]	[11,15]	[12,15]	[16,21]
	0.12	[9,11]	[8,8]	[8,10]	[11,11]
	0.15	[7,9]	[7,8]	[7,7]	[8,10]
	0.3	[4,5]	[4,5]	[4,5]	[5,6]
0.05	0.1	[27,37]	[25,33]	[26,34]	[39,50]
	0.15	[13,15]	[12,14]	[12,14]	[16,22]
	0.2	[8,10]	[8,9]	[8,11]	[11,11]
	0.25	[7,8]	[7,9]	[7,7]	[8,11]
	0.5	[4,5]	[4,4]	[4,5]	[5,6]

neutrosophic plan parameters.

$$\text{Minimize } n_N \in \{n_L, n_U\}$$

$$\text{Subject to } P \left\{ \chi_{n_N}^2 \leq (n_N k_{aN} / L_{Ne}) \right\}$$

$$\begin{aligned}
 &+ \left[P \left\{ \chi_{n_N}^2 \leq (n_N k_{rN} / L_{Ne}) \right\} \right. \\
 &\quad \left. - P \left\{ \chi_{n_N}^2 < (n_N k_{aN} / L_{Ne}) \right\} \right] \\
 &\left[P \left\{ \chi_{n_N}^2 \leq (n_N k_{aN} / L_{Ne}) \right\} \right]^m | p_1 \geq 1 - \alpha
 \end{aligned}$$

TABLE 5. The data from amplified sensors.

observations			
[1.9422, 1.9422]	[1.9651,1.9651]	[2.0230, 2.0230]	[1.9712, 1.9712]
[1.9738, 1.9938]	[1.9541, 1.9541]	[1.9800, 2.0980]	[1.9596, 1.9596]
[2.0001, 2.0001]	[1.9659, 1.9659]	[1.9955, 1.9955]	[1.9842, 1.9842]
[1.9897, 1.9897]	[1.9836, 1.9836]	[1.9891, 1.9891]	[1.9608, 1.9608]
[2.0106, 3.000]	[1.9885, 1.9885]	[1.9704, 1.9704]	[1.9882, 1.9882]
[1.9640, 1.9640]	[2.0187, 2.0187]	[1.9616, 1.9716]	[1.9865, 1.9865]
[1.9841, 1.9841]	[1.9919, 1.9919]	[1.9737, 1.9737]	[1.9958, 1.9958]

and

$$P \left\{ \chi_{n_N}^2 \leq (n_N k_{aN} / L_{Ne}) \right\} + \left[P \left\{ \chi_{n_N}^2 \leq (n_N k_{rN} / L_{Ne}) \right\} - P \left\{ \chi_{n_N}^2 < (n_N k_{aN} / L_{Ne}) \right\} \right] \left[P \left\{ \chi_{n_N}^2 \leq (n_N k_{aN} / L_{Ne}) \right\} \right]^m$$

$$|p_2 \leq \beta$$

Using the above mentioned NNOP, the values neutrosophic plan parameters $n_N \in \{n_L, n_U\}$, $k_{aN} \in \{k_{aL}, k_{aU}\}$ and $k_{rN} \in \{k_{rL}, k_{rU}\}$ are determined for various combinations of AQL and LQL. The values of plan parameters for $m \in \{1, 1\}$, $m \in \{2, 2\}$, $m \in \{3, 3\}$ are shown in Tables 1-3, respectively. From Tables 1-3, we note the decreasing trend in interval width of parameter $n_N \in \{n_L, n_U\}$ as m incerses from $m \in \{1, 1\}$ to $m \in \{3, 3\}$. This decreasing trend shows that a smaller values of $n_N \in \{n_L, n_U\}$ if more previous subgroups are in-control for the in-decision state.

The plan parameters $n_N \in \{n_L, n_U\}$, $k_{aN} \in \{k_{aL}, k_{aU}\}$ and $k_{rN} \in \{k_{rL}, k_{rU}\}$ are determined using the following algorithm.

Step-1:Specify the value of m , AQL and LQL.

Step-2:Compute the probabilities of acceptance at AQL and LQL using NNOP.

Step-3:Generate 10,000 combinations of $n_N \in \{n_L, n_U\}$, $k_{aN} \in \{k_{aL}, k_{aU}\}$ and $k_{rN} \in \{k_{rL}, k_{rU}\}$.

Step-4:Choose that combinations of $n_N \in \{n_L, n_U\}$, $k_{aN} \in \{k_{aL}, k_{aU}\}$ and $k_{rN} \in \{k_{rL}, k_{rU}\}$ where $n_N \in \{n_L, n_U\}$ is minimum.

III. COMPARISON STUDY

As the inspection cost depends on sample size. The larger the sample means, a high cost is needed to perform the inspection using the sampling plan. Therefore, in quality control theory under classical statistics, a sampling plan having the smaller sample size is known as the most efficient sampling plan. In the sampling plan under the NS, a plan is said to be an efficient if the interval width of indeterminacy interval of sample size is smaller than the other plan. Now, we compare the performance of the proposed sampling plan with the plan proposed by [24] under the NS. The neutrosophic values of the parameter $n_N \in \{n_L, n_U\}$ are placed in Table 4 for various combinations of AQL and LQL. From Table 4, it can be noted that the proposed sample size smaller width of indeterminacy interval of $n_N \in \{n_L, n_U\}$ than [24] plan for ever combinations

of AQL, LQL and m . For example, when AQL = 0.001, LQL = 0.002 and $m \in \{2, 2\}$, the indeterminacy interval from the proposed sampling plan is $n_N \in \{27, 38\}$ and from the existing sampling plan it is $n_N \in \{49, 54\}$. It means that using the proposed plan in uncertainty, sample size should be from 27 to 38. On the other hand, the existing sampling plan provides sample size should be from 49 to 54. By comparing both sampling plans, it is concluded that the proposed sampling plan needs a smaller sample size for the inspection of the product. Therefore, the proposed sampling plan can be more economical than the existing sampling under the uncertainty environment.

IV. REAL EXAMPLE

Now, the application of the proposed sampling plan will be discussed with the help of company data. This company is located in Taiwan and manufacturing the amplified pressure sensor. The company is interested to use the proposed plan using the data obtained from the failure of an amplified pressure sensor. In practice, measuring the failure time is not always exact and less or fuzzier. Therefore, the recorded data in the indeterminacy interval having lower and upper values. Suppose that company does inspection when AQL = 0.03, LQL = 0.06, $m = 1$, $\alpha = 0.05$ and $\beta = 0.10$. The neutrosophic plan parameters for these specified values are selected from Table 1 as: $n_N \in \{28, 38\}$, $k_{aN} \in \{0.035, 0.036\}$ and $k_{rN} \in \{0.344, 0.155\}$. Under uncertainty environment, the experimenter is decided to select a random sample of size 28. The neutrosophic data of amplified pressure sensor with $d = 0.1$, $T = 2.0$ V, $USL = 2.1$ V, and $LSL = 1.9$ V is shown in Table 5. The similar data is also used by [24] for the single plan. The statistics for this data are shown as follows: $\bar{X}_N \in \{1.9816, 2.0217\}$, $S_N^2 \in \{0.0003, 0.0362\}$. The value of the neutrosophic statistic is given as $\hat{L}_{Ne} = \frac{S_N^2}{d^2} + \frac{(\bar{X}_N - T)^2}{d^2}$; $\hat{L}_{Ne} \in \{0.0686, 3.6698\}$. According to the present plan, we will reject the amplified pressure sensor product as $\hat{L}_{Ne} > k_{rN}$.

V. CONCLUDING REMARKS

In this paper, the designing of a sampling plan using the process loss consideration for the multiple dependent state (MDS) sampling under the neutrosophic statistics was

presented. The NOF under the proposed plan was derived. By comparing with the existing sampling plan, it is found that the proposed plan is quite effective and efficient to perform the inspection of the product under the uncertainty environment. We recommend using of the proposed plan for the inspection of product in the industry to save time and inspection cost. The proposed plan using other sampling schemes such as resubmitted, repetitive group, multiple dependent state repetitive (MDSR) can be considered as future research.

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