Neutrosophic and Plithogenic Statistical Analysis in Educational Development
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Abstract. Education in the history of humanity briefly establishes the general characteristics of education over time. Throughout life, the student goes through more than 18 years in the classrooms, because education has rigorously changed throughout history, as a result of the evolution of some universities during the processes carried out until changing categories and an improvement in the evaluated indicators. On the other hand, governments and leading entities of education speak of the application of standards of educational quality, the efficiency of the educational system, and sustainability of financing and quality of spending to ensure that students develop knowledge, skills, and attitudes in specific situations, in different contexts for solving problems and interacting with the main dimensions, with their factors or sub-dimensions, limitations or critical nodes and challenges of education. This study analyzes the effects that affect educational development through the neutrosophic statistical study with the application of plithogenic sets.

Keywords: education, neutrosophic statistics, plithogenic sets

1 Introduction

This research collects the perspectives and experiences of teachers and analyzes the Ecuadorian educational system to determine how it influences the educational reality of the country. The educational changes made in recent years have not achieved the objectives set by the Ministry of Education and the Ministry of Higher Education, Science and Technology. Education is a right of people throughout their lives and an inescapable and inexcusable duty of the State. It constitutes a priority area of public policy and state investment, a guarantee of equality and social inclusion, and an essential condition for good living. Individuals, families, and society have the right and responsibility to participate in the educational process [1-8].

Education contributes to the achievement of great transformations of humanity and in turn, it is the educational transformations supported by philosophy that give meaning to human existence. Thus, it is not without reason that education has historically been the motor that drives all social processes. Hence we can say that it is crucial to address this issue [9-12].
investment in education has tripled, quality has declined [13]. Furthermore, dropouts and reprobation in education were higher, so redistribution of the State's budget invests in education, quality spending policies, and an accountability system are actual goals to be achieved. At present, a revision of the standards and the curriculum is being carried out to strengthen the quality of education, the more poverty decreases, the quality of life improves because basic needs are met [14, 15].

Although the quality of education in Ecuador has improved, it lacks highly trained professionals and technicians, and that is why on many occasions, there is a need for millionaire hiring of foreigners to take those positions that need a certain degree of knowledge. In addition, this shortage decreases the development of our economy and all national life, because to a large extent, the main problem of national education is the little budget that is assigned to education in the country [16-18].

The actual society demands to have a higher quality education, an imperative of the demanding world in which we are immersed, which has created the urgent need that the work of man is much more effective, for which greater preparation is required. This is an active task of a psycho-physical-social order that allows understanding the new realities according to the level of inner maturation of the subjects. It is a reality that belongs to the space-time becoming of all people [19, 20]. The Ecuadorian educational system has progressed in recent times, but it has some obstacles and critical nodes to resolve. To think about the foundations of education in history and the transformations in current Ecuadorian education implies determining its entelechy, understanding the limitations, achievements, and challenges that the educational problem brings along [21, 22]. For the analysis of the development of education, this study defines:

- **Problematic Situation:** effects on educational development
- **Main objective:** define the main factors that affect development in education
- **Specific objectives:**
  - Determine the dimensions of the analyzed variable
  - Analyze the effects at each stage of the process
  - Carry out the neutrosophic statistical measurement and modeling of the variable
  - Determine the interrelation of each dimension in the plithogenic set of education
  - Present potential solutions to mitigate the impact of the factors on the variable

![Figure 2: Stages of the study of educational development](image)

### 2 Materials and methods

Neutrosophic probabilities and statistics are a generalization of classical and imprecise probabilities and statistics. The Neutrosophic Probability of an event E is the probability that event E will occur [23], the probability that event E does not occur, and the probability of indeterminacy (not knowing whether event E occurs or not). In classical probability $\mathbb{P}(\mathbb{E}) \leq 1$, while in neutrosophic probability $\mathbb{N}(\mathbb{E}) \leq 3$. The function that models the neutrosophic probability of a random variable $x$ is called the neutrosophic distribution: $NP(x) = (T(x), I(x), F(x))$, where $T(x)$ represents the probability that the value $x$ occurs, $F(x)$ represents the probability that the value $x$ does not occur, and $I(x)$ represents the indeterminate or unknown probability of the value $x$.

Neutrosophic Statistics is the analysis of neutrosophic events and deals with neutrosophic numbers, the neutrosophic probability distribution [24], neutrosophic estimation, neutrosophic regression, etc. It refers to a set of data, which is formed totally or partially by data with some degree of indeterminacy and the methods to analyze them. Neutrosophic statistical methods allow the interpretation and organization of neutrosophic data (data that can be ambiguous, vague, imprecise, incomplete, or even unknown) to reveal the underlying patterns [25]. In short, the Neutrosophic Logic [26, 27], Neutrosophic Sets, and Neutrosophic Probabilities and Statistics have a wide application in various research fields and constitute a new reference of study in full development [28-49].
The Neutrosophic Descriptive Statistics includes all the techniques to summarize and describe the characteristics of the neutrosophic numerical data [50]. Neutrosophic Numbers are numbers of the form where a and b are real or complex numbers [51], while "I" is the indeterminacy part of the neutrosophic number N.

\[ N = a + bI. \]

The study of neutrosophic statistics refers to a neutrosophic random variable where \( X \) and \( X \) represents the corresponding lower and upper level that the studied variable can reach in an indeterminate interval \([I, I]\). Following the neutrosophic mean of the variable when formulating:

\[ X_N = X + X_{\mu}I_N; \quad I_N \in [I, I] \]

Where \( \bar{x}_a = \frac{1}{n^N} \sum_{i=1}^{n^N} X_i \bar{x}_b = \frac{1}{n^N} \sum_{i=1}^{n^N} X_i n_i \in [n, n] \]

is a neutrosophic random sample. However, for the calculation of neutral frames (NNS) it can be calculated as follows:

\[ \Sigma_{i=1}^{n^N} (x_i - \bar{x}_a)^2 = \Sigma_{i=1}^{n^N} \left[ \frac{\min \left( (a_i + b_i) (a_i + b_i), (a_i + b_i) (a_i + b_i) \right)}{\min \left( (a_i + b_i) (a_i + b_i), (a_i + b_i) (a_i + b_i) \right)} \right] \left[ I [I, I] \right] \]

Where \( a_i = X_i b_i = X_i \). The variance of the neutrosophic sample can be calculated by

\[ S_N^2 = \frac{\Sigma_{i=1}^{n^N} (x_i - \bar{x}_a)^2}{n}; \quad S_N^2 \in [S^2_k, S^2_k] \]

The neutrosophic coefficient (NCV) measures the consistency of the variable. The lower the NCV value, the more consistent the factor’s performance is. NCV can be calculated as follows [52].

\[ CV_N = \frac{S_N}{x_N} \times 100; \quad CV_N \in [CV_L, CV_U] \]

**Mathematical modeling through neutrosophic logic to plithogenic logic**

Neutrosophic sets were introduced in the literature by F. Smarandache since fuzzy intuitionistic sets could only handle incomplete information, but not the indeterminate and inconsistent information, which commonly exists in fuzzy systems. The term neutrosophy means knowledge of neutral thought and this neutrality represents the main distinction between fuzzy logic and fuzzy intuitionist [27]. In neutrosophic sets, the indeterminacy is explicitly quantified through a new parameter I. True membership (t), indeterminate membership (I), and false membership (F) are independent of each other and the sum between them satisfies the inequalities 0 ≤ t + I + F ≤ 3. In fuzzy intuitionistic sets, the uncertainty depends on the degree of membership and the degree of nonmembership [26]. In neutrosophic sets, the indeterminacy factor (I) is independent of the true and false values. There are no restrictions between the degree of truth, the degree of indeterminacy, and falsehood [24, 53-55].

If U is a universe of discourse, a Neutrosophic Set (NS) is characterized by three membership functions \( uA(x), rA(x), vA(x) : X \to [0, 1 + I] \), which satisfy the condition \( 0 \leq \inf uA(x) + \inf rA(x) \leq \sup uA(x) + \sup rA(x) \leq 3 + I \) for all \( x \in X \). \( uA(x), rA(x) \), and \( vA(x) \) are the membership functions of truth, indeterminacy, and falsehood of x in A, respectively y your images are standard or non-standard subsets of \( 0 \leq \inf uA(x) \leq \sup uA(x) \leq 3 + I \).

When approaching the perspective of indeterminacy and contradiction, as is the case with Gödel's incompleteness theorem, he states that any proposition in a mathematical axiom system will present a degree of truth (T), falsehood (F), and indeterminacy (I). Neutrosophy, therefore, establishes a unique solution for the existence of paradoxes in philosophy [50]. Plithogenic advocates for the connections and unification of theories and ideas in varied fields of science [56].

Plithogenic is the dynamics of various types of opposites, and/or their neutrals, and/or non-opposites and their organic fusion. Plithogenic is a generalization of dialectics (dynamics of a type of opposites: \(<A>\) and \(<antiA>\)), neutrosophy (dynamics of a type of opposites and their neutrals: \(<A>\) and \(<antiA>\) and \(<neutA>\)), since Plithogenic...
studies the dynamics of many types of opposites and their neutrals and non-opposites (<A> and <antiA> and <neutA>, <B> and <antiB> and <neutB>, etc.), and not opposites (<C>, <D>, etc.) all together. As an application and particular case derived from Plithogeny, the plithogenic set is an extension of the classical set, fuzzy set, fuzzy intuitionist set, and neutrosophic set, and has multiple scientific applications [56].

So, \((P, a, V, d, c)\) is called a plithogenic set

1. Where "P" is a set, "a" is an attribute (multi-dimensional in general), "V" is the range of values of the attribute, "d" is the degree of membership of the attribute value of each element \(x\) to the set \(P\) with respect to some given criteria \((x \in P)\), and "d" means "\(dP\)" or "\(dF\)" or "\(d\)", when it is a degree of fuzzy membership, an intuitionistic fuzzy membership, or a degree of neutrosophic membership, respectively, of an element \(x\) to the plithogenic set \(P\).

2. "c" means "\(cP\)" or "\(cF\)" or "\(c\)" when it is a fuzzy attribute value contradiction degree function, intuitionistic fuzzy attribute value contradiction degree function, or neutrosophic attribute value contradiction degree function, respectively.

3. Functions \(d(\cdot;\cdot)\) and \(c(\cdot;\cdot)\) are defined according to the applications that experts need to solve.

4. Then, the following notation is used:

5. \(x(d(x, V))\), where \(d(x, V) = (d(x, v), \text{for all } v \in V, \forall x \in P)\).

6. The attribute value contradiction degree function is calculated between each attribute value regarding the dominant attribute value (denoted by \(v_0\)) in particular, and with regard to other attribute values as well.

7. The attribute value contradiction degree function \(c\) evaluated between the values of two attributes is used in the definition of plithogenic aggregation operators (intersection (AND), union (OR), implication (\(\Rightarrow\)), equivalence (\(\Leftrightarrow\)), inclusion (partial order), and other plithogenic aggregation operators that combine two or more degrees of values of the attribute based on a t-norm and a t-conorm. Most plithogenic aggregation operators are linear combinations of a fuzzy t-norm (indicated by) with a fuzzy t-conorm (indicated by), but nonlinear combinations can also be constructed. AD and VD [57].

If the t-norm is applied on the value of the dominant attribute denoted by, and the contradiction between and is, then it is applied on the value of the attribute as follows:

\[
[1 - c(v_D, v_2)] \cdot t_{\text{norm}}(v_D, v_2) + c(v_D, v_2) \cdot t_{\text{conorm}}(v_D, v_2).
\]

Or, using symbols:

\[
[1 - c(v_D, v_2)] \cdot (v_D \wedge v_2) + c(v_D, v_2) \cdot (v_D \vee v_2),
\]

Similarly, if the t-conorm applies to the value of the dominant attribute denoted by \(v_D\), and the contradiction between \(v_D\) and \(v_2\) is \(c(v_D, v_2)\), then \(v_2\) applies to the value of the attribute as follows

\[
[1 - c(v_D, v_2)] \cdot t_{\text{conorm}}(v_D, v_2) + c(v_D, v_2) \cdot t_{\text{norm}}(v_D, v_2),
\]

Or, using symbols:

\[
[1 - c(v_D, v_2)] \cdot (v_D \vee v_2) + c(v_D, v_2) \cdot (v_D \wedge v_2).
\]

The plithogenic neutrosophic intersection is defined as:

\[
(a_1, a_2, a_3) \wedge_F (b_1, b_2, b_3) = \left( a_1 \wedge_F b_1, a_2 \wedge_F b_2, a_3 \wedge_F b_3 \right).
\]

The plithogenic neutrosophic union is defined as:

\[
(a_1, a_2, a_3) \vee_F (b_1, b_2, b_3) = \left( a_1 \vee_F b_1, a_2 \vee_F b_2, a_3 \vee_F b_3 \right).
\]

In other words, if something applies to membership, the opposite applies to non-membership, while in indeterminacy the average between them is what applies. Plithogenic neutrosophic inclusion is defined as follows:

Since the degrees of contradiction are:

\[
c(a_1, a_3) = c(a_2, a_3) = c(b_1, b_2) = c(b_2, b_3) = 0.5.
\]

We apply

\[
a_2 \geq 1 - c(a_1, a_2)b_2 \text{ or } a_2 \geq (1 - 0.5)b_2 \text{ or } a_2 \geq 0.5b_2, \text{ while } c(a_3, a_3) = c(b_3, b_3) = 1
\]

So, the opposite is true for \(a_1 \leq b_1\) if and only if

\[
a_3 \geq b_3, \text{ therefore } (a_1, a_2, a_3) \leq_F (b_1, b_2, b_3) \text{ if and only if } a_1 \leq b_1, a_2 \geq 0.5b_2, \text{ or } a_3 \geq b_3.
\]

Next, an algorithm for the resolution of this research is presented where Plithogeny will be merged with the algorithm of Neutrosophy. From this moment on, expressions 2 to 8 must be applied to execute the operations of the classical algorithm with plithogenic numbers. For the elaboration of a single decision matrix, the median of

the plithogenic numbers is calculated for each combination, for all specialists. The median is calculated using the following formula:

$$\text{median}^n_{i=1}\{\text{PN}_i\} = (\text{median}^n_{i=1}\{\text{T(\text{PN}_i)}\}, \text{median}^n_{i=1}\{\text{I(\text{PN}_i)}\}, \text{median}^n_{i=1}\{\text{F(\text{PN}_i)}\}).$$

(12)

Where $\text{PN}_i$ are plithogenic numbers, $\text{T(\text{PN}_i)}$ are their true components, $\text{I(\text{PN}_i)}$ are their indeterminate components and $\text{F(\text{PN}_i)}$ are their false components. In other words, Equation 8 means that the median of a set of plithogenic numbers is defined as the plithogenic number of the medians of its components. To compare the relationships between the quadrants, the following formula is used to blur a neutrosophic number [58]:

$$\mathcal{S}[\{\text{T}, \text{I}, \text{F}\}] = \frac{2 + \text{T} - \text{I} - \text{F}}{3}$$

(13)

- Determine for each line of the pairwise comparison matrix, a weighted sum based on the sum of the product of each cell by the priority of each alternative or corresponding criterion.
- For each line, divide its weighted sum by the priority of its corresponding alternative or criterion.
- Determine the $\hat{\phi}_{\text{max}}$ mean of the result of the previous stage.
- Calculate the consistency index (CI) for each alternative or criterion:

$$\text{CI} = \frac{\hat{\phi}_{\text{max}} - m}{m-1}$$

(14)

Where $m$ is the number of alternatives.

- Determine the Random Index (RI) from table 2.
- Determine the consistency ratio index (the ratio between the consistency index and the random index).

3 Results

3.1 Data

Development in education has been one of the challenges for governments to achieve the professional level demanded. For the modeling of this work, a group of experts was created to analyze the data collected in the information collection stage. As a consensus, the experts determined for the modeling:

- **Set:** Development, $\forall \text{F}_\text{V}_n \{\text{F}_{\text{V}_1}, \text{F}_{\text{V}_2}, \text{F}_{\text{V}_3}\}$
  - System efficiency, $\forall \text{F}_{\text{V}_{1n}} \{\text{F}_{\text{V}_{11}}, \text{F}_{\text{V}_{12}}, \text{F}_{\text{V}_{13}}\}$
  - Educational quality, $\forall \text{F}_{\text{V}_{2n}} \{\text{F}_{\text{V}_{21}}, \text{F}_{\text{V}_{22}}\}$
  - Sustainability of financing and quality of spending, $\forall \text{F}_{\text{V}_{3n}} \{\text{F}_{\text{V}_{31}}, \text{F}_{\text{V}_{32}}\}$

- **Variable:** Development in education. Code (E)
- **Factors (F):** Impacts on the development of education (Figure 3)
- **Sample:** 145 days
- **Scale:** [0, 6] (See table 1)

### Table 1: Linguistic expression for determining the level of importance of the factor on the variable

<table>
<thead>
<tr>
<th>Linguistic Expression</th>
<th>Scale</th>
<th>Phytogenic number (T, I, F)</th>
<th>$\mathcal{S}[{\text{T}, \text{I}, \text{F}}] = \frac{2 + \text{T} - \text{I} - \text{F}}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poor Importance (PI)</td>
<td>0</td>
<td>(0.12, 0.92, 0.97)</td>
<td>0.08</td>
</tr>
<tr>
<td>Fewer important (FI)</td>
<td>1</td>
<td>(0.17, 0.87, 0.92)</td>
<td>0.13</td>
</tr>
<tr>
<td>Low Importance (BI)</td>
<td>2</td>
<td>(0.42, 0.67, 0.82)</td>
<td>0.31</td>
</tr>
<tr>
<td>Medium important MDI</td>
<td>3</td>
<td>(0.67, 0.62, 0.72)</td>
<td>0.44</td>
</tr>
<tr>
<td>Important (I)</td>
<td>4</td>
<td>(0.72, 0.37, 0.52)</td>
<td>0.61</td>
</tr>
<tr>
<td>Most Important (MI)</td>
<td>5</td>
<td>(0.92, 0.27, 0.12)</td>
<td>0.84</td>
</tr>
<tr>
<td>Very important (VI)</td>
<td>6</td>
<td>(0.97, 0.07, 0.03)</td>
<td>0.96</td>
</tr>
</tbody>
</table>

3.2 Method development

Stage I. Neutrosophic statistical analysis

By modeling the variable with the use of neutrosophic statistics, the absolute frequencies are obtained to determine the level of involvement in the development of education, \( F_n = \{F_{V_{11}}, F_{V_{12}}, F_{V_{13}}, F_{V_{21}}, F_{V_{22}}, F_{V_{31}}, F_{V_{32}}\} \) in a sample of 145 days of research (Table 2). It is noted:

- For the *System Efficiency* dimension, the factor \( F_{V_{12}} \) has an incidence of 35.2%. It shows that insufficient infrastructure and equipment, inadequate and lacking cultural identity, are positioned as an element that slows down the efficiency of the educational system.
- For the educational quality dimension, the factor \( F_{V_{21}} \) has an incidence of 51.4%. It shows that the low quality of education is positioned as a factor that affects the development of education.
- For the dimension Sustainability of financing and quality of spending, the factor \( F_{V_{31}} \) has an incidence rate of 51.9%. It shows that the absence of financing strategies is positioned as a financial factor that hinders educational development.

To obtain the level of incidence of each factor in its dimension, as the measure of indeterminacy for modeling, a scale of \( 0 \leq F_n \leq 1 \) was set, so that the relative membership level within the subset is determined, as well as the neutrosophic set from the neutrosophic frequency \( F_n = \{F_{V_{11}}, F_{V_{12}}, F_{V_{13}}, F_{V_{21}}, F_{V_{22}}, F_{V_{31}}, F_{V_{32}}\} \) (Table 3).

<table>
<thead>
<tr>
<th>Days</th>
<th>( V_{11} )</th>
<th>( V_{12} )</th>
<th>( V_{13} )</th>
<th>( V_{21} )</th>
<th>( V_{22} )</th>
<th>( V_{31} )</th>
<th>( V_{32} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[3 ; 5]</td>
<td>[2 ; 5]</td>
<td>[2 ; 5]</td>
<td>[2 ; 3]</td>
<td>[1 ; 1]</td>
<td>[3 ; 3]</td>
<td>[1 ; 4]</td>
</tr>
<tr>
<td>2</td>
<td>[1 ; 4]</td>
<td>[0 ; 3]</td>
<td>[1 ; 1]</td>
<td>[0 ; 0]</td>
<td>[1 ; 3]</td>
<td>[1 ; 4]</td>
<td>[0 ; 3]</td>
</tr>
<tr>
<td>3</td>
<td>[3 ; 5]</td>
<td>[3 ; 6]</td>
<td>[3 ; 3]</td>
<td>[2 ; 5]</td>
<td>[0 ; 0]</td>
<td>[3 ; 3]</td>
<td>[3 ; 6]</td>
</tr>
<tr>
<td>4</td>
<td>[2 ; 5]</td>
<td>[2 ; 3]</td>
<td>[2 ; 3]</td>
<td>[0 ; 1]</td>
<td>[1 ; 4]</td>
<td>[3 ; 6]</td>
<td>[2 ; 2]</td>
</tr>
<tr>
<td>5</td>
<td>[2 ; 2]</td>
<td>[2 ; 3]</td>
<td>[1 ; 3]</td>
<td>[2 ; 3]</td>
<td>[0 ; 0]</td>
<td>[1 ; 3]</td>
<td>[2 ; 4]</td>
</tr>
<tr>
<td>6</td>
<td>[2 ; 5]</td>
<td>[0 ; 0]</td>
<td>[0 ; 1]</td>
<td>[3 ; 5]</td>
<td>[2 ; 2]</td>
<td>[3 ; 5]</td>
<td>[3 ; 4]</td>
</tr>
<tr>
<td>7</td>
<td>[2 ; 2]</td>
<td>[0 ; 0]</td>
<td>[3 ; 3]</td>
<td>[1 ; 2]</td>
<td>[1 ; 4]</td>
<td>[1 ; 3]</td>
<td>[1 ; 4]</td>
</tr>
<tr>
<td>8</td>
<td>[2 ; 3]</td>
<td>[3 ; 5]</td>
<td>[3 ; 6]</td>
<td>[1 ; 1]</td>
<td>[2 ; 2]</td>
<td>[0 ; 0]</td>
<td>[1 ; 3]</td>
</tr>
<tr>
<td>9</td>
<td>[1 ; 1]</td>
<td>[0 ; 0]</td>
<td>[3 ; 3]</td>
<td>[2 ; 3]</td>
<td>[1 ; 2]</td>
<td>[0 ; 1]</td>
<td>[2 ; 4]</td>
</tr>
<tr>
<td>10</td>
<td>[3 ; 4]</td>
<td>[0 ; 3]</td>
<td>[0 ; 2]</td>
<td>[1 ; 4]</td>
<td>[1 ; 3]</td>
<td>[1 ; 1]</td>
<td>[1 ; 3]</td>
</tr>
<tr>
<td>11</td>
<td>[3 ; 3]</td>
<td>[2 ; 5]</td>
<td>[3 ; 5]</td>
<td>[1 ; 4]</td>
<td>[0 ; 3]</td>
<td>[1 ; 3]</td>
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<tr>
<td>12</td>
<td>[2 ; 4]</td>
<td>[1 ; 4]</td>
<td>[0 ; 3]</td>
<td>[0 ; 1]</td>
<td>[2 ; 4]</td>
<td>[1 ; 3]</td>
<td>[2 ; 5]</td>
</tr>
</tbody>
</table>

To model the neutrosophic statistics of the neutrosophic set, it is suggested to code the name of the variable and its representation in graphs and tables.
Table 2. Neutrosophic frequency for each plithogenic subset in educational development

<table>
<thead>
<tr>
<th>Subset</th>
<th>Neutrosophic Frequency</th>
<th>Representativeness</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>[0; 3] [0; 1] [2; 3] [3; 4] [3; 6] [3; 3]</td>
<td>[72.01% ; 81.73%]</td>
</tr>
<tr>
<td>V2</td>
<td>[1; 2] [0; 1] [1; 3] [2; 4] [3; 6] [2; 2]</td>
<td>[70.57% ; 81.63%]</td>
</tr>
<tr>
<td>V3</td>
<td>[0; 0] [1; 2] [1; 1]</td>
<td>[64.27% ; 69.88%]</td>
</tr>
</tbody>
</table>

Of the neutrosophic frequencies observed for development in education, for a sample of 145 days analyzed, there is a total indeterminacy level of:

- Subset V1, \(V_{11} = 140, V_{12} = 178, V_{13} = 139\), with representativeness levels of \([72.01\% ; 81.73\%]\),
- Subset V2, \(V_{21} = 154, V_{22} = 153\), with representativeness levels of \([70.57\% ; 81.63\%]\),
- Subset V3, \(V_{31} = 170, V_{32} = 171\), with representativeness levels of \([64.27\% ; 69.88\%]\).

It is highlighted that the problems of insufficient infrastructure and equipment, inadequate and without cultural identity affect 81.63 in the development in education, in the days of greatest incidence.

From the results of the modeling, it is observed that the incidence relationship between each factor associated with its dimension affects the development of education (Table 3).

For the analysis of the representative mean as a function of \(\bar{x} = \{\bar{x}_L, \bar{x}_U\}\), the values of the neutrosophic means of the factors are calculated and for the study of the variations of the subsets V1, V2, V3, they are determined by the values of the neutrosophic standard deviation \(S_N \in [S_L, S_U]\), to determine in which factor there is greater coherence and precision when measuring the neutrosophic set \(CV_N \in [CV_L, CV_U]\) (Figure 4).

Comparative analysis

The modeling of the neutrosophic statistics defines the associated indeterminacy factor for \(\bar{x} = \{\bar{x}_L, \bar{x}_U\}\), \(S_N \in [S_L, S_U]\) and \(CV_N \in [CV_L, CV_U]\) in the form of neutrosophic numbers (Table 3). From the results obtained, it is observed that for the \(CV_N\) values for the:

- Subset V1, from 0.493 to 0.637 with the indeterminacy measure of \([21.3\% ; 45.0\%]\),
- Subset V2, from 0.504 to 0.509 with the measure of indeterminacy from \([34.0\% ; 40.8\%]\),
- Subset V3, 0.438 to 0.492 with the measure of indeterminacy from \([31.0\% ; 35.0\%]\).

Figure 4. Neutrosophic bubble graph and its plithogenic interrelation of each factor and dimension.
<table>
<thead>
<tr>
<th>COD</th>
<th>Dimension</th>
<th>COD</th>
<th>Sub-dimension or factor</th>
<th>Scale</th>
<th>Phytogenic number (T, I, F)</th>
<th>dn (x; Vn)</th>
<th>Attribute value</th>
<th>$\bar{X}_N$</th>
<th>YN</th>
<th>CVN</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>System Efficiency</td>
<td>v11</td>
<td>Limited access to education and lack of equity</td>
<td>[0; 6]</td>
<td>(0.72, 0.37, 0.52)</td>
<td>0.3</td>
<td>I</td>
<td>1,391 + 2,664</td>
<td>[0; 0; 47.8]</td>
<td>0.686 + 2,386</td>
</tr>
<tr>
<td></td>
<td></td>
<td>v12</td>
<td>Insufficient infrastructure and equipment, inadequate and lacking cultural identity</td>
<td>[0; 6]</td>
<td>(0.92, 0.27, 0.12)</td>
<td>0.4</td>
<td>M: YES</td>
<td>1,218 + 2,836</td>
<td>[0; 0; 57.1]</td>
<td>0.82 + 2.425</td>
</tr>
<tr>
<td></td>
<td></td>
<td>v13</td>
<td>Difficulties in the governance of the sector and lack of an accountability system for all actors in the system</td>
<td>[0; 6]</td>
<td>(0.72, 0.37, 0.52)</td>
<td>0.3</td>
<td>I</td>
<td>1.3 + 2,564</td>
<td>[0; 0; 49.3]</td>
<td>0.72 + 2.22</td>
</tr>
<tr>
<td>V2</td>
<td>Educational quality</td>
<td>v21</td>
<td>Low quality of education</td>
<td>[0; 6]</td>
<td>(0.97, 0.07, 0.03)</td>
<td>0.51</td>
<td>VI</td>
<td>1,318 + 2,718</td>
<td>[0; 0; 51.5]</td>
<td>0.664 + 2.077</td>
</tr>
<tr>
<td></td>
<td></td>
<td>v23</td>
<td>Weak application of new information and communication technologies</td>
<td>[0; 6]</td>
<td>(0.67, 0.62, 0.72)</td>
<td>0.49</td>
<td>MDI</td>
<td>1,182 + 2,573</td>
<td>[0; 0; 54.1]</td>
<td>0.602 + 2.212</td>
</tr>
<tr>
<td>V3</td>
<td>Sustainability of financing and quality of spending</td>
<td>v31</td>
<td>Lack of financing strategies</td>
<td>[0; 6]</td>
<td>(0.97, 0.07, 0.03)</td>
<td>0.51</td>
<td>VI</td>
<td>1,609 + 3,155</td>
<td>[0; 0; 49.0]</td>
<td>0.705 + 2.128</td>
</tr>
<tr>
<td></td>
<td></td>
<td>v32</td>
<td>Poor quality of spending</td>
<td>[0; 6]</td>
<td>(0.67, 0.62, 0.72)</td>
<td>0.49</td>
<td>MDI</td>
<td>1,373 + 2,927</td>
<td>[0; 0; 53.1]</td>
<td>0.675 + 2.11</td>
</tr>
</tbody>
</table>

Table 3. Neutrosophic measures with levels of indeterminacy for each plithogenic subset of development in education.

Although the need to use the lowest percentage level of indeterminacy to obtain accurate and homogeneous results influences with a greater degree and a low level of indeterminacy than the other factors in the development of education. For each determining factor in the pertaining subset and its hierarchy level in the plithogenic set as established: 

\[ CV_N \]

- Low quality of education
- Insufficient infrastructure and equipment, inadequate and lacking cultural identity
- Lack of financing strategies

### Stage II. Development of mathematical modeling through neutrosophic logic to plithogenic logic in the development of education

Plithogenic set: Development of education (table 3)

The plithogenic set is defined for three subsets \( V_1, V_2, \) and \( V_3 \)

That is why a plithogenic set is defined that consists of 7 attributes, each of these attributes contain possible values (table 3), with their respective plithogenic particularities and possible values in the linguistic expression to determine the level of importance of the factor on the variable (table 1).

The multi-attribute of dimension 3 has cardinality \( 3 \times 2 \times 2 = 12 \).

The degrees of contradiction between the values for each attribute are defined below:

\[
\begin{align*}
c_N (v_{11}, v_{12}) &= c_N (v_{12}, v_{13}) = 0.3 \\
c_N (v_{21}, v_{22}) &= 0.1 \\
c_N (v_{31}, v_{32}) &= 0.1
\end{align*}
\]

As we can see, the dominant values for each attribute are: \( v_{12}, v_{21}, \) and \( v_{31} \)

When \( v_{21}, v_{12}, \) and \( v_{31} \) are activated, all the other nodes are activated, which means that the incidence value caused by the low quality of education in the institutions is influenced by the deterioration of the infrastructure and insufficient, inadequate equipment and without identity cultural, has a negative influence by projecting itself as a dominant value within the plithogenic set of education, in such a way that it constitutes an impairment in professional and educational growth preceded by the absence of financing strategies.

To determine a level of solution, it is necessary to know which subsets to act on through the relationship and the level of importance, as follows:

- Educational quality and System Efficiency
- Educational quality and absence of financing strategies

\[
\begin{array}{ccc} \\
V_{12} & V_{21} & V_{31} \\
M: YES & VI (0.97, 0.07, 0.03) & VI (0.97, 0.07, 0.03) \\
\end{array}
\]

Low quality of education (\( v_{21} \)) and insufficient infrastructure and equipment, inadequate and lacking cultural identity (\( v_{12} \))

<table>
<thead>
<tr>
<th>Neutrosophic Plithogenic Union</th>
<th>Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (a_1, a_2, a_3) \lor_p (b_1, b_2, b_3) = (a_1 \land_D b_1, b_2, b_3) + (a_2 \lor_D b_2) \lor (a_3 \land_D b_3) )</td>
<td>0.9061</td>
</tr>
</tbody>
</table>

It is in a sublevel
Low quality of education (v21) and absence of financing strategies (v31)

\[(a_1, a_2, a_3) \lor_p (b_1, b_2, b_3) = (0.892, 0.17, 0.004)\] between I and MI

\[(a_1, a_2, a_3) \land_p (b_1, b_2, b_3) = (0.94, 0.07, 0.059)\]

Table 4. Evaluations between v12, v21, and v31

There is a stronger relationship between the subsets of educational quality and sustainability of financing and quality of spending [in its attribute (v21) and (v31)] than between educational quality and system efficiency, taking into account the most predominant factors. A relationship is obtained one degree closer to more important than very important according to the plithogenic neutrosophic union and intersection operator. So the solutions must be focused on solving the factors (v21) and (v31) that affect the development of education.

Partial solutions:

- To increase the quality of education in the different localities of the country, it is necessary the creation of an institute that is in charge of designing, experimenting, and gradually disseminating the educational innovations that are required.

- The governing bodies on educational development must promote and finance projects for the educational sector to promote professional development.

- It must be understood that education is one of the main development tools of a country, providing opportunities for the population and allowing progress towards a more equitable and egalitarian country.

Conclusions

- Ecuador has presented innumerable advances in education, however, like most countries in the region, there is still a long way to go, and one of them is the educational quality that is still deficient.

- Neutrosophic statistics revealed that among the factors that most affect educational development are the low quality of education; insufficient infrastructure and equipment, inadequate and without cultural identity, and absence of financing strategies, corresponding to the dimension it belongs.

- The analysis of the mathematical modeling through the neutrosophic logic to the plithogenic logic determined within the plithogenic set of development in education, which subsets were compromised by directly influencing educational development. The relationship between the subsets of educational quality and sustainability of financing and quality of spending was at a sub-level between MI and VI. Objectively, these nodes must be influenced to reduce their incidence and activation of the remaining nodes.

References


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