



2-Additive Choquet Similarity Measures For Multi-Period Medical Diagnosis in Single-Valued Neutrosophic Set Setting

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Abstract. Medical diagnosis is a disease identification process that matches symptoms with diseases based on the symptoms of target patient. In this process, it is necessary to establish a similarity relation between symptoms and diseases so as to determine the correct diagnosis. Similarity measure theory is a beneficial way that is used to model this relationship mathematically under vary environment. In the literature, various similarity measures have been constructed in single-valued neutrosophic set setting. However, these similarity measures ignores the interaction between symptoms. To overcome this deficiency, we propose four new similarity measures by using the Choquet integral under single-valued neutrosophic environment that take into account both period and the interaction between symptoms. Moreover, we take advantage of the concept of 2-additivity to reduce the computational effort to obtain multi-period medical diagnosis results. We implement them to a multi-period medical diagnosis example existing in the literature. We also compare our results with some previous ones and we analyze the consistency of the results via some statistical methods.

Keywords: Single-valued neutrosophic set; similarity measure; Choquet integral; medical diagnosis

1. Introduction

Neutrosophic set theory was proposed by Smarandache [18] from a philosophical perspective as a generalisation of the concept of fuzzy set (FS) and intuitionistic fuzzy set (IFS). A neutrosophic set (NS) is characterized by a truth membership function T , an indeterminacy membership function I and a falsity membership function F and each membership degree is a real standard or non-standard subset of the non-standard unit interval $]^{-0, 1^{+}[$. Besides, there

is no restraint on the sum of the membership functions. The concept has various generalizations such as single-valued neutrosophic set (SVNS) [24], interval neutrosophic set (INS) [25], neutrosophic cubic set [9] and single-valued neutrosophic linguistic set [30].

In this paper, we focus on two multi-period medical diagnosis (MPMD) applications in SVNS setting. MPMD is a process of decision making on a disease which evaluates the effect of symptoms on the target patients according to several different periods. The most important factor that discriminates this process from other medical diagnosis processes is the presentation of the solution algorithm by paying attention to the period variable. The symptoms of the target patients or the effects of the symptoms on the target patients may change, as period progresses. A medical diagnosis includes a lot of incoherent and incomplete data because of the patient's imprecise data and the indeterminate information of the symptoms of the diseases. To solve the medical diagnosis problems in case of uncertainty, some solution methods have been proposed in the literature [1, 4, 11, 15, 17, 26]. One of these methods is the determination of the disease of the target patient with the help of the similarity measures.

A similarity measure plays an important role to specify the degree of similarity between two sets such as FSs, IFSs and NSs. Similarity measures are frequently used to figure out medical diagnosis problems under neutrosophic environment [3, 28, 29]. The target patients and possible diseases are represented by SVNSs according to symptoms and the most accurate diagnosis is obtained by establishing a similarity between the target patients and the symptoms of the possible diseases. In a MPMD problem, period variable is also added to the problem. For example, Ye and Fu [27] proposed tangent similarity measures for SVNSs and apply them to a MPMD problem. Later, Chou et al. [7] introduced new similarity measures for SVNSs and applied them to the same MPMD problem. However, these similarity measures ignore the interaction between symptoms. A symptom may occur as a result of another symptom. In such a case, an interaction is valid between these symptoms. To overcome this deficiency, we propose new similarity measures for SVNSs based on the Choquet integral that considers the interaction between symptoms.

The concept of Choquet integral [6] was presented by Gustave Choquet in 1953 as a non-linear continuous aggregation operator. A Choquet integral is characterized with a fuzzy measure [19] which is able to model interaction between elements of a set or between criteria in real life problems. Actually, the concept is an enlargement of the Lebesgue integral and a non-additive extension of the weighted arithmetic mean. Although, a fuzzy integral has more complicated structure due to the lack of additivity in contrast to the additive integrals such as Lebesgue integral, use of a fuzzy measure and a fuzzy integral is more effective in the aggregation. In [16], it is shown that the Choquet integral performs substantially more orders than the weighted arithmetic mean and that the difference gets larger when the number of the

elements of the set gets larger. Moreover, it has been proved in [12] that when the number of the element of the finite set increases, the probability of getting more optimal ranking in the Choquet integral increases compared to the weighted arithmetic mean. Actually, fuzzy measures and fuzzy integrals let us to take the preferences into account that are not contained in the weights in the weighted arithmetic mean [23]. The notion of fuzzy measure is defined on the power set. As a result, the process of fuzzy measure identification is complicated for a set with a large number of elements due to the exponential increase in the number of the subsets. To facilitate this situation, researchers have proposed various methods and many authors studied various fuzzy measure identification methods (see, e.g., [10, 20, 21]). One of these methods is the concept of k -additive fuzzy measure proposed by Grabisch [8]. Whenever a fuzzy measure is k -additive, the notion of Choquet integral is expressed with the help of Möbius transform of the fuzzy measure. Moreover, the Möbius transform of a fuzzy measure corresponds to the correlation coefficients that indicate the direction and strength of the linear relationship between two or more random variables in probability theory and statistics. Thus, thanks to the k -additivity, the effort of fuzzy measure calculation can be reduced.

In this paper, we focus on constructing four new similarity measures based on the Choquet integral with respect to a 2-additive fuzzy measure under single-valued neutrosophic environment. Then, we give a MPMD method. We apply this method to a MPMD problem to demonstrate the effectiveness of the proposed method. The remainder of this paper is set out as follows. In Section 2, we recall the concept of SVN. Then, we recall the MPMD methods of Ye and Fu [27] and Chou et al. [7]. In Section 3, the concepts of fuzzy measure, Möbius transform of a fuzzy measure and the concept of 2-additive fuzzy measure, the concept of Choquet integral with respect to a 2-additive fuzzy measure are recalled. In Section 4, we propose four similarity measures based on Choquet integral with respect to 2-additive fuzzy measure for SVN. Then, we propose the promised MPMD method. In Section 5, to indicate the effectiveness of the proposed method, we apply it to a MPMD problem from the literature. Then, the results of the problem are compared with some previous ones. Moreover, we give a consistency analysis of the results with Spearman's rank correlation coefficients. In Section 6, we give a conclusion.

2. The Concept of SVN and Some Existing MPMD Methods

The concept of NS is a helpful mathematical tool that models uncertainty and inconsistent data. However, the set theoretical operators such as intersection, union and inclusion cannot be defined on the non-standard unit interval. Therefore, it is not easy to perform the applications of NS. To come through this hassle, Wang et al. [24] presented the notion of SVN.

Definition 2.1. [24] Let X be a universal set. A SVN \tilde{A}_1 of X is given with

$$\tilde{A}_1 = \left\{ \left\langle \xi, T_{\tilde{A}_1}(\xi), I_{\tilde{A}_1}(\xi), F_{\tilde{A}_1}(\xi) \right\rangle : \xi \in X \right\} \tag{1}$$

where $T_{\tilde{A}_1}$, $I_{\tilde{A}_1}$ and $F_{\tilde{A}_1}$ are functions from X to closed interval $[0, 1]$. The values $T_{\tilde{A}_1}(\xi)$, $I_{\tilde{A}_1}(\xi)$ and $F_{\tilde{A}_1}(\xi)$ indicate the truth, the indeterminacy and the falsity membership degrees of the element ξ to the set \tilde{A}_1 , respectively. Clearly, the sum of the three values satisfies the condition $0 \leq T_{\tilde{A}_1}(\xi) + I_{\tilde{A}_1}(\xi) + F_{\tilde{A}_1}(\xi) \leq 3$. Moreover, the triplet $\left\langle T_{\tilde{A}_1}(\xi), I_{\tilde{A}_1}(\xi), F_{\tilde{A}_1}(\xi) \right\rangle$ is called a single-valued neutrosophic value (SVNV).

Ye and Fu [27] proposed similarity measures T_1 and T_2 between SVNNS based on arithmetic mean and applied the similarity measure T_2 to a MPMD problem. Let $X = \{\xi_1, \dots, \xi_m\}$ be a set of symptoms, let $T = \{t_1, \dots, t_q\}$ be a set of periods and let $D = \{D_1, \dots, D_n\}$ be a set of diseases. For a patient P_s with assorted symptoms, $C_j(t_k)$ denotes the SVNV between a patient and j th symptom ξ_j for $j = 1, \dots, m$ in the k th period t_k for $k = 1, \dots, q$ (see, Table 2 in [27]). It is represented as $C_j(t_k) = \langle T_j(t_k), I_j(t_k), F_j(t_k) \rangle$ in the form of a SVNV. Apparently, if $q = 1$, the MPMD problem is generally a single period medical diagnosis problem. Moreover, C_{ij} denotes the SVNV between the j th symptom ξ_j for $j = 1, \dots, m$ and the i th noted disease D_i for $i = 1, \dots, n$ (see, Table 3 in [27]). It is represented as $C_{ij} = \langle T_{ij}, I_{ij}, F_{ij} \rangle$ in the form of a SVNV.

Let weights of the symptoms be $0 \leq w_1, \dots, w_m \leq 1$ with $\sum_{j=1}^m w_j = 1$ and the weights of the periods be $0 \leq \omega(t_1), \dots, \omega(t_q) \leq 1$ with $\sum_{k=1}^q \omega(t_k) = 1$. The MPMD method is constructed as follows: Firstly, the similarity measure between a patient P_s and the noted disease D_i for $i = 1, \dots, n$ in each period t_k for $k = 1, \dots, q$ is calculated with the help of the weighted version of similarity T_2 with the following:

$$T_{w_i}(P_s, t_k) := 1 - \sum_{j=1}^m w_j \tan \left[\frac{\pi}{12} (|T_j(t_k) - T_{ij}| + |I_j(t_k) - I_{ij}| + |F_j(t_k) - F_{ij}|) \right]. \tag{2}$$

Then, the weighted aggregation value $M(P_s, D_i)$ for $i = 1, \dots, n$ is obtained with the following:

$$M(P_s, D_i) := \sum_{k=1}^q \omega(t_k) T_{w_i}(P_s, t_k). \tag{3}$$

Finally, the weighted values with respect to D_i for $i = 1, \dots, n$ are put in order and the highest value is determined as the most appropriate choice.

Chou et al. [7] constructed a MPMD method for SVNNS by motivating from Ye and Fu's working [27]. They proposed two weighted similarity measures M_{w1} and M_{w2} . Then, this two

Murat Olgun, Ezgi Türkarslan, Mehmet Ünver, Jun Ye, 2-Additive Choquet Similarity Measures For Multi-Period Medical Diagnosis in Single-Valued Neutrosophic Set Setting

similarity measures are used in the same MPMD with the help of the same algorithm of Ye and Fu [27].

In this study, our aims are to express new similarity measures, motivating by [7] and [27] with the help of the 2-additive Choquet integral that pays attention to the interaction between the symptoms and to propose a MPMD method.

3. Some Basic Concepts of Fuzzy Measure Theory and Choquet Integral

The basis of Choquet integral is inherently fuzzy measure. Therefore, we recall the concept of fuzzy measure.

Definition 3.1. Let X be a non-empty set and let $P(X)$ be the power set of X . If

i) $\sigma(\emptyset) = 0$,

ii) $\sigma(X) = 1$,

iii) $A_1 \subseteq A_2$ implies $\sigma(A_1) \leq \sigma(A_2)$ (monotonicity),

then the set function $\sigma : P(X) \rightarrow [0, 1]$ is called a fuzzy measure on X [6].

There exist $2^n = \sum_{k=0}^n \binom{n}{k}$ coefficients to be determined on the power set of a set with n elements. For this reason, the process of determining a fuzzy measure over a set with excess number of elements is quite difficult. Thus, Grabisch introduced a crucial kind of fuzzy measure which is named k -additive fuzzy measure to facilitate the process of determining a fuzzy measure on set with large elements [8]. For instance, if $k = 2$, it is enough to determine the fuzzy measure of $n(n-1)/2$ subsets so as to specify the whole fuzzy measure (see, [8]).

Definition 3.2. The Möbius transform of a set function σ on X is a set function $m : P(X) \rightarrow \mathbb{R}$ defined by

$$m(A_1) := \sum_{A_2 \subset A_1} (-1)^{|A_1 \setminus A_2|} \sigma(A_2). \quad (4)$$

A fuzzy measure σ is expressed as:

$$\sigma(A_1) = \sum_{A_2 \subset A_1} m(A_2) \quad (5)$$

for all $A_1 \in P(X)$ [5] whenever its Möbius transform m is given. As a result, the Möbius transform over singletons is equal to the fuzzy measure itself.

Definition 3.3. Let X be a finite set and let σ be a fuzzy measure on X . σ is said to be 2-additive if its Möbius transform m satisfies $m(A_1) = 0$ for all $A_1 \subset X$ such that $|A_1| > 2$ and there exist at least one subset $A_1 \subset X$ with $|A_1| = 2$ such that $m(A_1) \neq 0$ [8].

The following important theorem gives some properties of the Möbius transform corresponding to a fuzzy measure and they can be used in the fuzzy measure identification process.

Theorem 3.4. [5] *Let X be a finite set and let $\sigma : P(X) \rightarrow \mathbb{R}$ be a function. σ is a fuzzy measure on X if and only if its Möbius transform m satisfies*

- i) $m(\emptyset) = 0$,
- ii) $\sum_{A_1 \subset X} m(A_1) = 1$,
- iii) $\sum_{\xi \in A_2 \subset A_1} m(A_2) \geq 0$, for all $A_1 \subset X$ and for all $\xi \in A_1$.

Another crucial notion that is related to the fuzzy measure theory is the concept of interaction index (see, e.g., [8]). The following theorem gives the relationship between the interaction index and the Möbius transform of a fuzzy measure.

Theorem 3.5. [8] *Let X be a finite set and let m be the Möbius transform of a fuzzy measure on X . The interaction index I satisfies*

$$I(A_1) = \sum_{k=0}^{|X \setminus A_1|} \frac{1}{k+1} \sum_{\substack{A_2 \subset X \setminus A_1 \\ |A_2|=k}} m(A_1 \cup A_2) \quad (6)$$

for any $A_1 \subset X$.

From Theorem 3.5, we have

$$I(A_1) = \begin{cases} m(A_1), & |A_1| = 2 \\ 0, & |A_1| > 2 \end{cases} \quad (7)$$

whenever the fuzzy measure is 2-additive [8]. Interaction between at most two criteria can exist whenever the fuzzy measure is 2-additive. That is, there is no interaction between more than two criteria when σ is 2-additive.

Let $X = \{\xi_1, \dots, \xi_n\}$ be a finite set and let $I_{ij} := I(\{\xi_i, \xi_j\})$.

- (1) If $I_{ij} > 0$, then there is a positive interaction between the criteria ξ_i and ξ_j , and when they come together, their severity increases.
- (2) If $I_{ij} < 0$, then there is negative interaction between the criteria ξ_i and ξ_j , and one of the criteria is more redundant. When these two criteria come together, their severity decreases.
- (3) If $I_{ij} = 0$, then there is no interaction between the criteria ξ_i and ξ_j and they are independent from each other.

Definition 3.6. Let $X = \{\xi_1, \dots, \xi_n\}$ be a finite set and let σ be a fuzzy measure on X . The Choquet integral [6] of a function $f : X \rightarrow [0, 1]$ with respect to σ is defined by

Murat Olgun, Ezgi Türkarlan, Mehmet Ünver, Jun Ye, 2-Additive Choquet Similarity Measures For Multi-Period Medical Diagnosis in Single-Valued Neutrosophic Set Setting

$$(C) \int_X f d\sigma := \sum_{k=1}^n (f(\xi_{(k)}) - f(\xi_{(k-1)})) \sigma(E_{(k)}), \tag{8}$$

where the sequence $\{\xi_{(k)}\}_{k=0}^n$ is the permutation of the sequence $\{\xi_k\}_{k=0}^n$ such that $0 := f(\xi_{(0)}) \leq f(\xi_{(1)}) \leq f(\xi_{(2)}) \leq \dots \leq f(\xi_{(n)})$ and $E_{(k)} := \{\xi_{(k)}, \xi_{(k+1)}, \dots, \xi_{(n)}\}$.

If the fuzzy measure is 2-additive Definition 3.6 is equivalent to following expressions:

$$(C_{2-add.}) \int_X f d\sigma =: \sum_{\xi_i \in X} m_i f(\xi_i) + \sum_{\{\xi_i, \xi_j\} \subseteq X} m_{ij} \min(f(\xi_i), f(\xi_j)) \tag{9}$$

where m is the Möbius transform of a 2-additive fuzzy measure σ on X and $m_i := m(\{\xi_i\})$, $m_{ij} := m(\{\xi_i, \xi_j\})$ [13, 14].

From (7) and (9), we see that interaction indices are enough to calculate the Choquet integral with respect to a 2-additive fuzzy measure. Therefore, in Section 5 we use interaction indices.

4. 2-ADDITIVE CHOQUET SIMILARITY MEASURES FOR SVN_Ss

In this section, we propose four new similarity measures for SVN_Ss by using a 2-additive Choquet integral and we give some propositions associated with these similarity measures. Moreover, the proposed similarity measures are integrated into a MPMD method with the help of the Choquet integral. Motivating by [7] and [27], now we define the following similarity measures.

Definition 4.1. Let $X = \{\xi_1, \dots, \xi_n\}$ be a finite set, let \tilde{A}_1 and \tilde{A}_2 be two SVN_Ss of X and let σ be a 2-additive fuzzy measure on X . Two 2-additive Choquet similarity measures are given with

$$W_{T_1}^{(C_{2-add.,\sigma})}(\tilde{A}_1, \tilde{A}_2) := 1 - (C_{2-add.}) \int_X f_{\tilde{A}_1, \tilde{A}_2}^{(1)} d\sigma \tag{10}$$

$$W_{T_2}^{(C_{2-add.,\sigma})}(\tilde{A}_1, \tilde{A}_2) := 1 - (C_{2-add.}) \int_X f_{\tilde{A}_1, \tilde{A}_2}^{(2)} d\sigma \tag{11}$$

where

$$f_{\tilde{A}_1, \tilde{A}_2}^{(1)}(\xi_j) := \max \left(\left| T_{\tilde{A}_1}(\xi_j) - T_{\tilde{A}_2}(\xi_j) \right|, \left| I_{\tilde{A}_1}(\xi_j) - I_{\tilde{A}_2}(\xi_j) \right|, \left| F_{\tilde{A}_1}(\xi_j) - F_{\tilde{A}_2}(\xi_j) \right| \right), \tag{12}$$

$$f_{\tilde{A}_1, \tilde{A}_2}^{(2)}(\xi_j) := \frac{\left| T_{\tilde{A}_1}(\xi_j) - T_{\tilde{A}_2}(\xi_j) \right| + \left| I_{\tilde{A}_1}(\xi_j) - I_{\tilde{A}_2}(\xi_j) \right| + \left| F_{\tilde{A}_1}(\xi_j) - F_{\tilde{A}_2}(\xi_j) \right|}{3}, \tag{13}$$

for $j = 1, \dots, n$.

Definition 4.2. Let $X = \{\xi_1, \dots, \xi_n\}$ be a finite set, let \tilde{A}_1 and \tilde{A}_2 be two SVNNS of X and let σ be a 2-additive fuzzy measure on X . Two 2-additive Choquet similarity measures are given with

$$W_{T_3}^{(C_2\text{-add.}, \sigma)}(\tilde{A}_1, \tilde{A}_2) := 1 - (C_2\text{-add.}) \int_X f_{\tilde{A}_1, \tilde{A}_2}^{(3)} d\sigma \quad (14)$$

$$W_{T_4}^{(C_2\text{-add.}, \sigma)}(\tilde{A}_1, \tilde{A}_2) := 1 - (C_2\text{-add.}) \int_X f_{\tilde{A}_1, \tilde{A}_2}^{(4)} d\sigma \quad (15)$$

where

$$f_{\tilde{A}_1, \tilde{A}_2}^{(3)}(\xi_j) := \tan \left[\frac{\pi}{4} \max \left(\left| T_{\tilde{A}_1}(\xi_j) - T_{\tilde{A}_2}(\xi_j) \right|, \left| I_{\tilde{A}_1}(\xi_j) - I_{\tilde{A}_2}(\xi_j) \right|, \left| F_{\tilde{A}_1}(\xi_j) - F_{\tilde{A}_2}(\xi_j) \right| \right) \right], \quad (16)$$

and

$$f_{\tilde{A}_1, \tilde{A}_2}^{(4)}(\xi_j) := \tan \left[\frac{\pi}{12} \left(\left| T_{\tilde{A}_1}(\xi_j) - T_{\tilde{A}_2}(\xi_j) \right| + \left| I_{\tilde{A}_1}(\xi_j) - I_{\tilde{A}_2}(\xi_j) \right| + \left| F_{\tilde{A}_1}(\xi_j) - F_{\tilde{A}_2}(\xi_j) \right| \right) \right], \quad (17)$$

for $j = 1, \dots, n$.

Note here that, if we consider additive measures, then we obtain the similarity measures of [7] and [27].

Proposition 4.3. Let X be a finite set and let \tilde{A}_1 and \tilde{A}_2 be two SVNNS in X . The 2-additive Choquet similarity measure $W_{T_i}^{(C_2\text{-add.}, \sigma)}$ for $i = 1, 2, 3, 4$ satisfies the following properties:

$$(\mathbf{P}_1) \quad 0 \leq W_{T_i}^{(C_2\text{-add.}, \sigma)}(\tilde{A}_1, \tilde{A}_2) \leq 1;$$

$$(\mathbf{P}_2) \quad W_{T_i}^{(C_2\text{-add.}, \sigma)}(\tilde{A}_1, \tilde{A}_2) = W_{T_i}^{(C_2\text{-add.}, \sigma)}(\tilde{A}_2, \tilde{A}_1);$$

$$(\mathbf{P}_3) \quad \tilde{A}_1 = \tilde{A}_2 \text{ if and only if } W_{T_i}^{(C_2\text{-add.}, \sigma)}(\tilde{A}_1, \tilde{A}_2) = 1,$$

$$(\mathbf{P}_4) \quad \text{If } \tilde{A}_3 \text{ is a SVNNS on } X \text{ and } \tilde{A}_1 \subseteq \tilde{A}_2 \subseteq \tilde{A}_3, \text{ then}$$

$$W_{T_i}^{(C_2\text{-add.}, \sigma)}(\tilde{A}_1, \tilde{A}_3) \leq W_{T_i}^{(C_2\text{-add.}, \sigma)}(\tilde{A}_1, \tilde{A}_2)$$

and

$$W_{T_i}^{(C_2\text{-add.}, \sigma)}(\tilde{A}_1, \tilde{A}_3) \leq W_{T_i}^{(C_2\text{-add.}, \sigma)}(\tilde{A}_2, \tilde{A}_3).$$

Proof. (P₁) Since $T, I, F : X \rightarrow [0, 1]$, we have $\left|T_{\tilde{A}_1}(\xi_j) - T_{\tilde{A}_2}(\xi_j)\right|$, $\left|I_{\tilde{A}_1}(\xi_j) - I_{\tilde{A}_2}(\xi_j)\right|$, $\left|F_{\tilde{A}_1}(\xi_j) - F_{\tilde{A}_2}(\xi_j)\right| \in [0, 1]$. So, we obtain $f_{\tilde{A}_1, \tilde{A}_2}^{(1)}(\xi_j), f_{\tilde{A}_1, \tilde{A}_2}^{(2)}(\xi_j) \in [0, 1]$, for $j = 1, \dots, n$. Moreover, since the value of the tangent function is within $[0, 1]$ when $\xi \in [0, \pi/4]$, we obtain $f_{\tilde{A}_1, \tilde{A}_2}^{(3)}(\xi_j), f_{\tilde{A}_1, \tilde{A}_2}^{(4)}(\xi_j) \in [0, 1]$ for $j = 1, \dots, n$. As the Choquet integral is monotone, we have $0 \leq W_{T_i}^{(C_2-add., \sigma)}(\tilde{A}_1, \tilde{A}_2) \leq 1$, for $i = 1, 2, 3, 4$.

(P₂) Since $f_{\tilde{A}_1, \tilde{A}_2}^{(k)}(\xi_j) = f_{\tilde{A}_2, \tilde{A}_1}^{(k)}(\xi_j)$ for any $j = 1, \dots, n$ and $k = 1, 2, 3, 4$, the proof is trivial.

(P₃) If $\tilde{A}_1 = \tilde{A}_2$, then $T_{\tilde{A}_1}(\xi_j) = T_{\tilde{A}_2}(\xi_j)$, $I_{\tilde{A}_1}(\xi_j) = I_{\tilde{A}_2}(\xi_j)$ and $F_{\tilde{A}_1}(\xi_j) = F_{\tilde{A}_2}(\xi_j)$ for $j = 1, \dots, n$. Then, we have $f_{\tilde{A}_1, \tilde{A}_2}^{(k)}(\xi_j) = 0$ for $k = 1, 2, 3, 4$. Therefore, we obtain that $W_{T_i}^{(C_2-add., \sigma)}(\tilde{A}_1, \tilde{A}_2) = 1$ for $i = 1, 2, 3, 4$. Conversely, assume that $W_{T_i}^{(C_2-add., \sigma)}(\tilde{A}_1, \tilde{A}_2) = 1$, for $i = 1, 2, 3, 4$. This implies $f_{\tilde{A}_1, \tilde{A}_2}^{(k)}(\xi_j) = 0$, for $k = 1, 2, 3, 4$. Thus, we obtain $\left|T_{\tilde{A}_1}(\xi_i) - T_{\tilde{A}_2}(\xi_i)\right| = 0$, $\left|I_{\tilde{A}_1}(\xi_i) - I_{\tilde{A}_2}(\xi_i)\right| = 0$ and $\left|F_{\tilde{A}_1}(\xi_i) - F_{\tilde{A}_2}(\xi_i)\right| = 0$, and $\tan 0 = 0$. Therefore, we have $T_{\tilde{A}_1}(\xi_j) = T_{\tilde{A}_2}(\xi_j)$, $I_{\tilde{A}_1}(\xi_j) = I_{\tilde{A}_2}(\xi_j)$ and $F_{\tilde{A}_1}(\xi_j) = F_{\tilde{A}_2}(\xi_j)$, for $j = 1, \dots, n$. Hence, $\tilde{A}_1 = \tilde{A}_2$.

(P₄) If $\tilde{A}_1 \subseteq \tilde{A}_2 \subseteq \tilde{A}_3$ then $T_{\tilde{A}_1}(\xi_j) \leq T_{\tilde{A}_2}(\xi_j) \leq T_{\tilde{A}_3}(\xi_j)$, $I_{\tilde{A}_1}(\xi_j) \geq I_{\tilde{A}_2}(\xi_j) \geq I_{\tilde{A}_3}(\xi_j)$ and $F_{\tilde{A}_1}(\xi_j) \geq F_{\tilde{A}_2}(\xi_j) \geq F_{\tilde{A}_3}(\xi_j)$, for all $j = 1, \dots, n$. Thus, we have

$$\begin{aligned} \left|T_{\tilde{A}_1}(\xi_j) - T_{\tilde{A}_2}(\xi_j)\right| &\leq \left|T_{\tilde{A}_1}(\xi_j) - T_{\tilde{A}_3}(\xi_j)\right|, \left|T_{\tilde{A}_2}(\xi_j) - T_{\tilde{A}_3}(\xi_j)\right| \leq \left|T_{\tilde{A}_1}(\xi_j) - T_{\tilde{A}_3}(\xi_j)\right|, \\ \left|I_{\tilde{A}_1}(\xi_j) - I_{\tilde{A}_2}(\xi_j)\right| &\leq \left|I_{\tilde{A}_1}(\xi_j) - I_{\tilde{A}_3}(\xi_j)\right|, \left|I_{\tilde{A}_2}(\xi_j) - I_{\tilde{A}_3}(\xi_j)\right| \leq \left|I_{\tilde{A}_1}(\xi_j) - I_{\tilde{A}_3}(\xi_j)\right|, \\ \left|F_{\tilde{A}_1}(\xi_j) - F_{\tilde{A}_2}(\xi_j)\right| &\leq \left|F_{\tilde{A}_1}(\xi_j) - F_{\tilde{A}_3}(\xi_j)\right|, \left|F_{\tilde{A}_2}(\xi_j) - F_{\tilde{A}_3}(\xi_j)\right| \leq \left|F_{\tilde{A}_1}(\xi_j) - F_{\tilde{A}_3}(\xi_j)\right|. \end{aligned}$$

So, we obtain $f_{\tilde{A}_1, \tilde{A}_2}^{(k)}(\xi_j) \leq f_{\tilde{A}_1, \tilde{A}_3}^{(k)}(\xi_j)$ and $f_{\tilde{A}_2, \tilde{A}_3}^{(k)}(\xi_j) \leq f_{\tilde{A}_1, \tilde{A}_3}^{(k)}(\xi_j)$, for $k = 1, 2$. Moreover, since the tangent function is increasing within the interval $[0, \pi/4]$, we obtain $f_{\tilde{A}_1, \tilde{A}_2}^{(k)}(\xi_j) \leq f_{\tilde{A}_1, \tilde{A}_3}^{(k)}(\xi_j)$ and $f_{\tilde{A}_1, \tilde{A}_2}^{(k)}(\xi_i) \leq f_{\tilde{A}_1, \tilde{A}_3}^{(k)}(\xi_i)$, for $k = 3, 4$. Therefore, from monotonicity of the Choquet integral and definition of proposed similarity measures, we have $W_{T_i}^{(C_2-add., \sigma)}(\tilde{A}_1, \tilde{A}_3) \leq W_{T_i}^{(C_2-add., \sigma)}(\tilde{A}_1, \tilde{A}_2)$ and $W_{T_i}^{(C_2-add., \sigma)}(\tilde{A}_1, \tilde{A}_3) \leq W_{T_i}^{(C_2-add., \sigma)}(\tilde{A}_2, \tilde{A}_3)$, for $i = 1, 2, 3, 4$. Hence, the proof is completed. \square

Remark 4.4. In the proof of Proposition 4.3 we assume that $0 < \sigma(A) < 1$ where $A \neq \emptyset, X$, which is consistent with the nature of the decision making.

Note that, the proposed similarity measures take into account the interaction between symptoms thanks to Choquet integral. If we consider an additive measure instead of a fuzzy measure, then the similarity measures proposed in Definition 4.1 and 4.2 reduced to the weighted similarity measures in [7, 27].

Now, we construct a MPMD method by using proposed 2-additive Choquet similarity measures.

Step1: Let $X = \{\xi_1, \dots, \xi_m\}$ be a set of symptoms. In this step, we will use the Möbius transform to construct a 2-additive fuzzy measure. We assume that the weights of the criteria are given and each weight is considered as the fuzzy measure of the corresponding singleton (criteria). The fuzzy measure σ will be constructed with the help of interaction indices (see Subsection 5.1).

Step2: Let $T = \{t_1, \dots, t_q\}$ be a set of periods and let $D = \{D_1, \dots, D_n\}$ be the set of diseases. For a patient P_s with various symptoms, SVN between a patient and j th symptom ξ_j for $j = 1, \dots, m$ in the k th period t_k for $k = 1, \dots, q$ is indicated with $\langle T_s^{(t_k)}(\xi_j), I_s^{(t_k)}(\xi_j), F_s^{(t_k)}(\xi_j) \rangle$. Moreover, the SVN between the j th symptom ξ_j for $j = 1, \dots, m$ and the i th noted disease D_i for $i = 1, \dots, n$ is indicated $\langle T_{D_i}(\xi_j), I_{D_i}(\xi_j), F_{D_i}(\xi_j) \rangle$. The similarity measures between a patient P_s and the noted disease D_i for $i = 1, \dots, n$ in each period t_k for $k = 1, \dots, q$ are calculated by using following formulas:

$$W_{T_i}^{(C_2\text{-add.}, \sigma)}(t_k) := 1 - (C_2\text{-add.}) \int_X f_{P_s, D_i}^{(l)}(\xi_j) d\sigma \quad (18)$$

for $l = 1, 2, 3, 4$ where

$$f_{P_s, D_i}^{(1)}(\xi_j) := \max \left(\left| T_s^{(t_k)}(\xi_j) - T_{D_i}(\xi_j) \right|, \left| I_s^{(t_k)}(\xi_j) - I_{D_i}(\xi_j) \right|, \left| F_s^{(t_k)}(\xi_j) - F_{D_i}(\xi_j) \right| \right), \quad (19)$$

$$f_{P_s, D_i}^{(2)}(\xi_j) := \frac{\left| T_s^{(t_k)}(\xi_j) - T_{D_i}(\xi_j) \right| + \left| I_s^{(t_k)}(\xi_j) - I_{D_i}(\xi_j) \right| + \left| F_s^{(t_k)}(\xi_j) - F_{D_i}(\xi_j) \right|}{3}, \quad (20)$$

$$f_{P_s, D_i}^{(3)}(\xi_j) := \tan \left[\frac{\pi}{4} \max \left(\left| T_s^{(t_k)}(\xi_j) - T_{D_i}(\xi_j) \right|, \left| I_s^{(t_k)}(\xi_j) - I_{D_i}(\xi_j) \right|, \left| F_s^{(t_k)}(\xi_j) - F_{D_i}(\xi_j) \right| \right) \right], \quad (21)$$

$$f_{P_s, D_i}^{(4)}(\xi_j) := \tan \left[\frac{\pi}{12} \left(\left| T_s^{(t_k)}(\xi_j) - T_{D_i}(\xi_j) \right| + \left| I_s^{(t_k)}(\xi_j) - I_{D_i}(\xi_j) \right| + \left| F_s^{(t_k)}(\xi_j) - F_{D_i}(\xi_j) \right| \right) \right]. \quad (22)$$

Step3: We assume that a fuzzy measure η is given on the set of periods. We aggregate similarities obtained in Step 2 with respect to periods by using Choquet integral. We obtain the aggregated value $M_{f_i}^{(C, \eta)}(P_s, D_i)$ for each $l = 1, 2, 3, 4$ by the following formula:

$$M_{f_i}^{(C, \eta)}(P_s, D_i) := (C) \int_X W_{T_i}^{(C_2\text{-add.}, \sigma)} d\eta := \sum_{k=1}^q \left(W_{T_i}^{(C_2\text{-add.}, \sigma)}(t_{(k)}) - W_{T_i}^{(C_2\text{-add.}, \sigma)}(t_{(k-1)}) \right) \eta(E_{(k)}), \quad (23)$$

where the sequence $\{t_{(k)}\}_{k=0}^q$ is a new permutation of the sequence $\{t_k\}_{k=0}^q$ such that $0 := W_{T_i}^{(C_2-add.,\sigma)}(t_{(0)}) \leq W_{T_i}^{(C_2-add.,\sigma)}(t_{(1)}) \leq \dots \leq W_{T_i}^{(C_2-add.,\sigma)}(t_{(q)})$ and $E_{(k)} := \{t_{(k)}, t_{(k+1)}, \dots, t_{(q)}\}$.

Step4: We rank all the weighted measures of $M_{f_i}^{(C,\eta)}(P_s, D_i)$ for $i = 1, \dots, n$ in a descending order and give a proper diagnosis relative to the maximum weighted measure value.

5. MULTI-PERIOD MEDICAL DIAGNOSIS PROBLEM

In this section, we implement the proposed method to a MPMD problem from the literature. Then we compare our results with those obtained by Chou et al. [7] and Ye and Fu [27].

5.1. Illustrative Example

Example 5.1. Let us consider the set of symptoms and diagnoses as follows, respectively:

$$S = \left\{ \xi_1(\text{Temperature}), \xi_2(\text{Headache}), \xi_3(\text{Stomach pain}), \xi_4(\text{Cough}), \xi_5(\text{Chest pain}) \right\}$$

$$D = \left\{ D_1(\text{Viral fever}), D_2(\text{Malaria}), D_3(\text{Typhoid}), D_4(\text{Gastritis}), D_5(\text{Stenocardia}) \right\}.$$

Each diagnosis $D_i, i = 1, 2, 3, 4, 5$, is given as a SVN (see, Table 4 of [27]) and the patients P_1, P_2, P_3 and P_4 that have all the symptoms are represented with respect to t_1, t_2 and t_3 periods as SVNV (see, Table 5 of [27]).

So as to use the proposed Choquet integral based method, let us construct a 2-additive fuzzy measure σ . For this purpose, the weight of all criteria is taken equally. We use the Möbius transform to determine the fuzzy measures of the remaining two element subsets. We also know that whenever measure is 2-additive, the Möbius transform of subsets of two elements is equal to the interaction index (see Equation 7). As the sum of the Möbius transforms (fuzzy measures) of singletons is equal to 1 we have from (ii) of Theorem 3.4 that the sum of Möbius transforms of subsets of two elements should be equal to zero.

Now considering interaction of the symptoms we assign Möbius transforms (interaction indices) to the sets of two elements (see, Table 1).

TABLE 1. Möbius Representation of σ

$m(\{\xi_1, \xi_2\}) = -0.06$	$m(\{\xi_1, \xi_3\}) = 0$	$m(\{\xi_1, \xi_4\}) = -0.12$
$m(\{\xi_1, \xi_5\}) = 0$	$m(\{\xi_2, \xi_3\}) = 0$	$m(\{\xi_2, \xi_4\}) = 0$
$m(\{\xi_2, \xi_5\}) = 0.08$	$m(\{\xi_3, \xi_4\}) = 0$	$m(\{\xi_3, \xi_5\}) = 0.09$
$m(\{\xi_4, \xi_5\}) = 0.01$		

For example, since there is a redundancy between the symptoms ξ_1, ξ_2 we assign a negative value for $I_{1,2} = m_{1,2}$.

Now, we calculate the similarity between patients and diseases with respect to given symptoms:

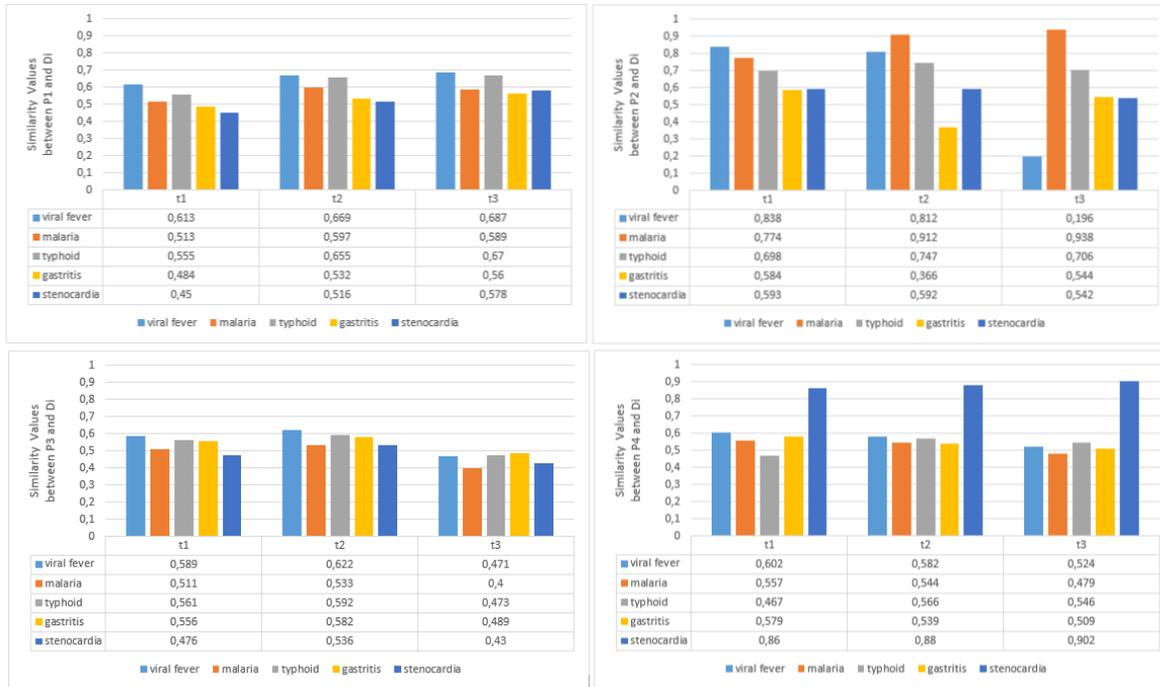


FIGURE 1. $W_{T_1}^{(C_2-add.\sigma)}$ for given patients with respect to periods

The results in Figure 1 show that for the P_1 , similarity increase with viral fever, typhoid, gastritis and stenocardia. For the P_2 patient, the similarity of the symptoms with viral fever decreases, while the similarity with malaria increases. For the P_3 , all diseases fluctuate over period. For the P_4 , similarity decrease with viral fever, malaria, gastritis while increases with stenocardia.

The results in Figure 2 show that for the P_1 , similarity increases with typhoid. Other diseases fluctuate over period For the P_2 patient, the similarity of the symptoms with viral fever and gastritis decreases, while the similarity with malaria increases. For the P_3 , similarity decrease with malaria, typhoid. Other diseases fluctuate over period. For the P_4 , similarity decrease with viral fever while fluctuating other disease over period.

The results in Figure 3 show that for the P_1 , similarity increases with typhoid. Other diseases fluctuate over period For the P_2 patient, the similarity of the symptoms with viral fever and gastritis decreases, while the similarity with malaria increases. For the P_3 , similarity

Murat Olgun, Ezgi Türkarşlan, Mehmet Ünver, Jun Ye, 2-Additive Choquet Similarity Measures For Multi-Period Medical Diagnosis in Single-Valued Neutrosophic Set Setting

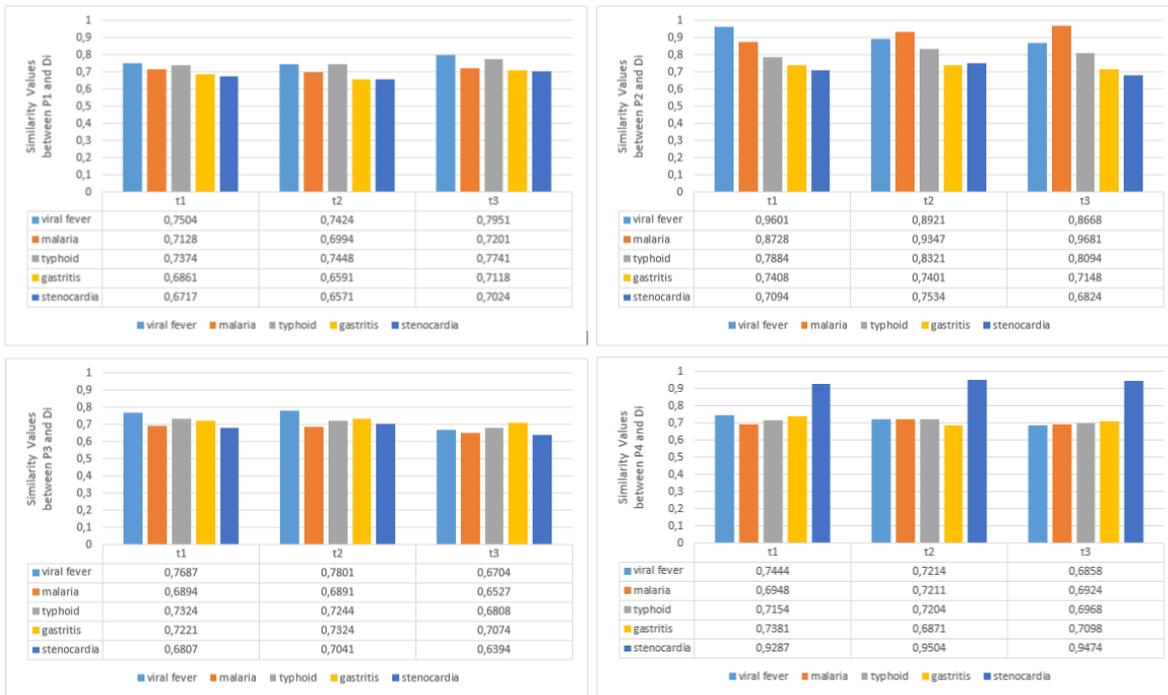


FIGURE 2. $W_{T_2}^{(C_2-add.,\sigma)}$ for given patients with respect to periods

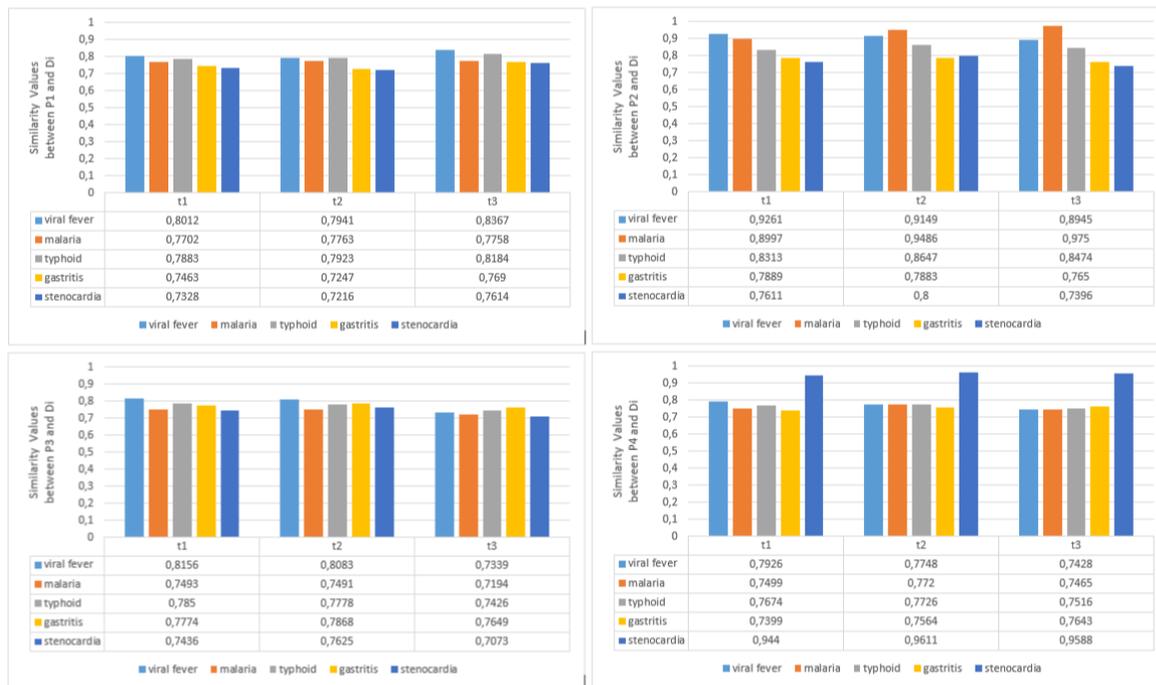


FIGURE 3. $W_{T_4}^{(C_2-add.,\sigma)}$ for given patients with respect to periods

decrease with viral fever and malaria, typhoid. Other diseases fluctuate over period. For P_4 , similarity decreases with viral fever while increases with gastritis.

For the sake of completeness, we show the calculation of the similarity between P_1 and D_1 with respect to $W_{T_4}^{(C_2-add.,\sigma)}$ and t_1 :

$$\begin{aligned} f^{(4)}(\xi_1) &= \tan \left[\frac{\pi}{12} (|0.8 - 0.4| + |0.6 - 0.6| + |0.5 - 0|) \right] = 0.2400, \\ f^{(4)}(\xi_2) &= \tan \left[\frac{\pi}{12} (|0.5 - 0.3| + |0.4 - 0.2| + |0.3 - 0.5|) \right] = 0.1583, \\ f^{(4)}(\xi_3) &= \tan \left[\frac{\pi}{12} (|0.2 - 0.1| + |0.1 - 0.3| + |0.3 - 0.7|) \right] = 0.1853, \\ f^{(4)}(\xi_4) &= \tan \left[\frac{\pi}{12} (|0.7 - 0.4| + |0.6 - 0.3| + |0.3 - 0.3|) \right] = 0.1583, \\ f^{(4)}(\xi_5) &= \tan \left[\frac{\pi}{12} (|0.4 - 0.1| + |0.3 - 0.2| + |0.2 - 0.7|) \right] = 0.2400. \end{aligned}$$

and

$$\begin{aligned} W_{T_4}^{(C_2-add.,\sigma)}(t_1) &= 1 - \left[\begin{aligned} &m(\{\xi_1\}) \times f^{(4)}(\xi_1) + m(\{\xi_2\}) \times f^{(4)}(\xi_2) + m(\{\xi_3\}) \times f^{(4)}(\xi_3) \\ &+ m(\{\xi_4\}) \times f^{(4)}(\xi_4) + m(\{\xi_5\}) \times f^{(4)}(\xi_5) + m(\{\xi_1, \xi_2\}) \times \min(f^{(4)}(\xi_1), f^{(4)}(\xi_2)) \\ &+ m(\{\xi_1, \xi_4\}) \times \min(f^{(4)}(\xi_1), f^{(4)}(\xi_4)) + m(\{\xi_2, \xi_5\}) \times \min(f^{(4)}(\xi_2), f^{(4)}(\xi_5)) \\ &+ m(\{\xi_3, \xi_5\}) \times \min(f^{(4)}(\xi_3), f^{(4)}(\xi_5)) + m(\{\xi_4, \xi_5\}) \times \min(f^{(4)}(\xi_4), f^{(4)}(\xi_5)) \end{aligned} \right] \\ &= 1 - (0.2(0.2400 + 0.1583 + 0.1853 + 0.1583 + 0.2400) - 0.06 \times \min(0.2400, 0.1583) \\ &- 0.12 \times \min(0.2400, 0.1583) + 0.08 \times \min(0.1583, 0.2400) + 0.09 \times \min(0.1853, 0.2400) \\ &+ 0.01 \times \min(0.1583, 0.2400)) = 0.8012. \end{aligned}$$

We also show the aggregation of the similarities for $W_{T_4}^{(C_2-add.,\sigma)}$ with respect to periods for P_1 and D_1 . Consider the following fuzzy measure η on $T = \{t_1, t_2, t_3\}$ given as follows: $\eta(\{t_1\}) = 0.25$, $\eta(\{t_2\}) = 0.35$, $\eta(\{t_3\}) = 0.40$, $\eta(\{t_1, t_2\}) = 0.45$, $\eta(\{t_1, t_3\}) = 0.95$, $\eta(\{t_2, t_3\}) = 0.45$, $\eta(\{t_1, t_2, t_3\}) = 1$. The fuzzy measure of singletons is taken as the weights of the singletons proposed in [7] and [27]. It is also thought that the synergy is greater between the initial period and the end period .

For D_1 disease, $W_{T_4}^{(C_2-add.,\sigma)}(t_2) \leq W_{T_4}^{(C_2-add.,\sigma)}(t_1) \leq W_{T_4}^{(C_2-add.,\sigma)}(t_3)$ and so

$$\begin{aligned} M_{f_4}^{(C,\eta)}(P_1, D_1) &= (C) \int_X W_{T_4}^{(C_2-add.,\sigma)} d\eta = \sum_{k=1}^3 \left(W_{T_4}^{(C_2-add.,\sigma)}(t_{(k)}) - W_{T_4}^{(C_2-add.,\sigma)}(t_{(k-1)}) \right) \eta(E_{(k)}) \\ &= W_{T_4}^{(C_2-add.,\sigma)}(t_2) + (W_{T_4}^{(C_2-add.,\sigma)}(t_1) - W_{T_4}^{(C_2-add.,\sigma)}(t_2))\eta(\{t_1, t_3\}) \\ &+ (W_{T_4}^{(C_2-add.,\sigma)}(t_3) - W_{T_4}^{(C_2-add.,\sigma)}(t_1))\eta(\{t_3\}) \\ &= 0.7941 + (0.8012 - 0.7941) \times 0.95 + (0.8367 - 0.8012) \times 0.40 = 0.8150. \end{aligned}$$

We can see from Table 2 that other results except for $M_{f_4}^{(C,\eta)}(P_3, D_i)$ are consistent with previous studies. This difference is due to the consideration of the interaction between symptoms in the proposed MPMD method.

TABLE 2. Evaluation Scores for SVNNSs

	Aggregation Values	D_1	D_2	D_3	D_4	D_5	Diagnosis
The results of [7]	$M_{w_1}(P_1, D_i)$	0.6730	0.5990	0.6560	0.5260	0.5270	viral fever
	$M_{w_1}(P_2, D_i)$	0.7970	0.8730	0.6780	0.5510	0.5330	malaria
	$M_{w_1}(P_3, D_i)$	0.5860	0.5040	0.5540	0.5300	0.4520	viral fever
	$M_{w_1}(P_4, D_i)$	0.5210	0.4750	0.5640	0.6020	0.8910	stenocardia
	$M_{w_2}(P_1, D_i)$	0.7770	0.7323	0.7483	0.6810	0.6510	viral fever
	$M_{w_2}(P_2, D_i)$	0.8683	0.9250	0.7897	0.6883	0.6670	malaria
	$M_{w_2}(P_3, D_i)$	0.7573	0.6983	0.6960	0.7133	0.6573	viral fever
	$M_{w_2}(P_4, D_i)$	0.6917	0.6617	0.7170	0.7577	0.9443	stenocardia
The results of [27]	$M(P_1, D_i)$	0.8183	0.7852	0.7966	0.7427	0.7167	viral fever
	$M(P_2, D_i)$	0.8985	0.9409	0.8315	0.7451	0.7220	malaria
	$M(P_3, D_i)$	0.8058	0.7554	0.7738	0.7701	0.7230	viral fever
	$M(P_4, D_i)$	0.7491	0.7214	0.7692	0.8036	0.9562	stenocardia
The results of proposed Choquet integral methods	$M_{f_1}^{(C,\eta)}(P_1, D_i)$	0.6454	0.5500	0.6060	0.5168	0.5045	viral fever
	$M_{f_1}^{(C,\eta)}(P_2, D_i)$	0.4797	0.8465	0.7159	0.5451	0.5647	malaria
	$M_{f_1}^{(C,\eta)}(P_3, D_i)$	0.5356	0.4576	0.5234	0.5282	0.4717	viral fever
	$M_{f_1}^{(C,\eta)}(P_4, D_i)$	0.5551	0.5115	0.5095	0.5325	0.8778	stenocardia
	$M_{f_2}^{(C,\eta)}(P_1, D_i)$	0.7678	0.7150	0.7524	0.6950	0.6832	viral fever
	$M_{f_2}^{(C,\eta)}(P_2, D_i)$	0.8951	0.9140	0.8057	0.7263	0.7099	malaria
	$M_{f_2}^{(C,\eta)}(P_3, D_i)$	0.7186	0.6691	0.7024	0.7176	0.6661	viral fever
	$M_{f_2}^{(C,\eta)}(P_4, D_i)$	0.7075	0.7026	0.7069	0.7157	0.9381	stenocardia
	$M_{f_4}^{(C,\eta)}(P_1, D_i)$	0.8150	0.7728	0.8005	0.7543	0.7436	viral fever
	$M_{f_4}^{(C,\eta)}(P_2, D_i)$	0.9064	0.9322	0.8446	0.7756	0.7628	malaria
	$M_{f_4}^{(C,\eta)}(P_3, D_i)$	0.7692	0.7328	0.7602	0.7738	0.7302	gastritis
	$M_{f_4}^{(C,\eta)}(P_4, D_i)$	0.7616	0.7557	0.7605	0.7504	0.9514	stenocardia

5.2. Ranking Analysis with Spearman’s Rank Correlation Coefficient

In this subsection, we use the Spearman’s correlation coefficients to analyze the ranking differences between the obtained results. The Spearman’s rank correlation coefficient, denoted by ρ , is shown below and the results of the test are presented in Table 3 and 4:

$$\rho =: 1 - \frac{6}{n(n^2 - 1)} \sum_{i=1}^n d_i^2 \tag{24}$$

where n is the number of results and d_i is difference between rankings of results obtained.

TABLE 3. Spearman’s Rank Correlations between M and $M_{f_i}^{(C,\eta)}$ for $i = 1, 2, 4$

Patients	Similarity Measures	Correlation Value	Consistency Ranking
P_1	$M_{f_1}^{(C,\eta)}$	1.0	1
	$M_{f_2}^{(C,\eta)}$	1.0	1
	$M_{f_4}^{(C,\eta)}$	1.0	1
P_2	$M_{f_2}^{(C,\eta)}$	1.0	1
	$M_{f_4}^{(C,\eta)}$	1.0	1
	$M_{f_1}^{(C,\eta)}$	0.3	2
P_3	$M_{f_2}^{(C,\eta)}$	0.9	1
	$M_{f_1}^{(C,\eta)}$	0.8	2
	$M_{f_4}^{(C,\eta)}$	0.7	3
P_4	$M_{f_2}^{(C,\eta)}$	0.9	1
	$M_{f_1}^{(C,\eta)}$	0.5	2
	$M_{f_4}^{(C,\eta)}$	0.3	3

TABLE 4. Spearman’s Rank Correlations between $M_{f_i}^{(C,\eta)}$ and M_{w_1} and M_{w_2} , for $i = 1, 2, 4$

Patients	Similarity Measures	Correlation Value	Consistency Ranking	Similarity Measures	Correlation Value	Consistency Ranking
	P_1	$M_{f_1}^{(C,\eta)}$	0.9	1	$M_{f_1}^{(C,\eta)}$	1.0
$M_{f_2}^{(C,\eta)}$		0.9	1	$M_{f_2}^{(C,\eta)}$	1.0	1
$M_{f_4}^{(C,\eta)}$		0.9	1	$M_{f_4}^{(C,\eta)}$	1.0	1
P_2	$M_{f_2}^{(C,\eta)}$	1.0	1	$M_{f_2}^{(C,\eta)}$	1.0	1
	$M_{f_4}^{(C,\eta)}$	1.0	1	$M_{f_4}^{(C,\eta)}$	1.0	1
	$M_{f_1}^{(C,\eta)}$	0.3	2	$M_{f_1}^{(C,\eta)}$	0.3	2
P_3	$M_{f_2}^{(C,\eta)}$	0.9	1	$M_{f_2}^{(C,\eta)}$	0.9	1
	$M_{f_1}^{(C,\eta)}$	0.8	2	$M_{f_4}^{(C,\eta)}$	0.8	2
	$M_{f_4}^{(C,\eta)}$	0.7	3	$M_{f_1}^{(C,\eta)}$	0.7	3
P_4	$M_{f_2}^{(C,\eta)}$	0.9	1	$M_{f_2}^{(C,\eta)}$	0.9	1
	$M_{f_1}^{(C,\eta)}$	0.5	2	$M_{f_1}^{(C,\eta)}$	0.5	2
	$M_{f_4}^{(C,\eta)}$	0.3	3	$M_{f_4}^{(C,\eta)}$	0.3	3

6. Conclusion

In this paper, we focus on increasing the sensitivity of some existing fuzzy measures by taking into account the interaction between symptoms with the help of the Choquet integral. For this purpose, we propose four new similarity measures based on the Choquet integral Murat Olgun, Ezgi Türkarşlan, Mehmet Ünver, Jun Ye, 2-Additive Choquet Similarity Measures For Multi-Period Medical Diagnosis in Single-Valued Neutrosophic Set Setting

for SVNSSs, both by providing the opportunity to evaluate more symptoms with the help of 2-additivity, and taking into account the interaction between symptoms. We adapted the proposed similarity measures to a MPMD problem that exists in the literature and we compare the results with some existing results. The most of the obtained results are consistent with past results. The consistency between these results is showed with the Spearman's correlation coefficients. Inconsistent result may occur because of the novelty of the proposed relatively sensitive method.

Conflicts of Interest: "The authors declare no conflict of interest."

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