A Novel Evaluative Method of the Subject “Education and Society” of the Autonomous University of the Andes, Ecuador, based on Plithogenic Numbers

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Abstract. The subject “Education and Society” is part of the syllabus of the “Basic Education” program of the Autonomous University of the Andes, Ecuador. This subject has the complexity of linking different aspects of society, some of them not free from contradictions among themselves, and imprecision and uncertainties in assessing students’ performance. For this reason, the concepts and aspects to be evaluated on this subject are represented through the use of plithogenic numbers. Plithogenic sets were defined to model concepts arising from the dynamic interaction among other simpler ones, which may have contradictions with each other and include neutralities or indeterminacies. This research aims to propose a novel method for the evaluation of the subject “Education and Society” through the use of plithogenic numbers and their operators. The lecturers will be able to perform the evaluations with the use of natural language; in the same way, the results will be given using a linguistic scale, which will facilitate the understanding and representation of the evaluations. On the other hand, plithogenic numbers will allow capturing the complexity, imprecision, and uncertainty that lecturers may face in their evaluations.

Keywords: Higher education, syllabus, plithogenic number, decision-making.

1 Introduction

The subject “Education and Society” is taught in the “Basic Education” program, at the Autonomous University of the Andes (UNIANDES), Ecuador. The main goal of this subject is that the student can recognize and creatively apply the fundamental principles of Social Sciences to the process of Education, increasing the complex and multidimensional interaction of the social and cultural phenomena on education.

The course will be carried out taking into consideration the theories of the cognitive psychology of constructivist learning by discovery [1]; learning and the significant assimilation [2, 3]; learning with a humanistic approach [4], focused on the pillars of UNESCO [5]. That is to say, the “learning to learn” in its various dimensions, learning to know; to know; to know how; to live; to know to undertake, which are the theories that are the basis of the pedagogical model of this University [6-11].

Theoretical bases:

- The position of the student as an active subject and builder of his knowledge, through the process directed and guided by teachers, the experiences that are provided through study activity and autonomous work, based on their cultural background and their willingness to continue learning;
- The cognitive reproduction for interpretation, construction, consolidation, and reconstruction of knowledge;
- The application of knowledge in experiential situations of their environment and the solution of real problems of the profession, through the development of motivation, interpersonal relationships, and communication capacity;
- The evaluation and self-evaluation of the products obtained in the processes of reproduction, construction, and reconstruction of knowledge.

Most of the above basis also integrate the technological, hermeneutic, and socio-critical approach of the contemporary education and special andragogical character, which are the object of our work: the teaching-
learning process of higher education, which are reflected in the Pedagogical Model of UNIANDES, since this will contribute to the achievement of the learning outcomes set out in the profile of egress.

The approach to the subject “Education and society” through the idea of a system allows applying mathematical-logical modeling based on plithogenic sets. In addition to that, this type of combination allows the inclusion of indeterminacy [12-17].

Plithogeny is the genesis or origin, creation, formation, development, and evolution of new entities from dynamics and organic mergers of multiple contradictory and/or neutral and/or non-contradictory old entities. Plithogeny advocates for connections and unification of theories and ideas in any field. As "entities" are taken as the "knowledge" in various fields, such as the soft sciences, the hard sciences, the theories of the arts and letters, etc., [12, 18].

Plithogeny is the dynamics of many types of opposites, and/or their neutrals, and/or non-opposites and their organic fusion. Plithogeny is a generalization of dialectics (dynamics of a kind of opposites: <A> and <antiA>), the Neutrosophy (dynamics of a kind of opposites and their neutral: <A> and <antiA>, and <neutA>), <B> and <antiB> and <neutB>, etc.), and many that are not opposites (<C>, <D>, etc.) altogether. A plithogenic set is a particular application and concept derived from Plithogeny, as it is an extension of the classical set, fuzzy set, intuitionist fuzzy set, and neutrosophic set, and has many scientific applications, [12, 19-21].

A plithogenic set P is a set whose elements are characterized by one or more attributes, and each attribute can have many values. Until now, these have mainly been applied in mathematical models of decision-making, [15, 22, 23]. This new theory fits the purpose of this paper since this is a tool to represent complex, systemic, and dynamic concepts, with degrees of indeterminacy, where different simpler concepts interact with each other.

Specifically, in our investigation, the plithogenic numbers are obtained from the evaluation of specialists, which are lectures on the subject “Education and Society” in UNIANDES. We propose a new method to assess the skills and learning of students of this subject. Moreover, lectures do not directly use plithogenic numbers in their evaluations, they evaluate by using linguistic terms, which are associated with plithogenic numbers. Other applications of Plithogeny can be read in [24, 25]. Some papers where Neutrosophy is applied to pedagogy and education can be read in [26-29].

This paper is divided into the following sections: section 2 is devoted to exposing the main concepts of Plithogeny. Section 3 contains the details of the design of the method. And in the last section, we explain the conclusions.

2 Plithogenic sets

Let U be a universe of discourse, and P a non-empty set of elements, P ⊆ U. Let A be a non-empty set of unidimensional attributes A = {α1, α2, ..., αm}, m ≥ 1, and x ∈ A is a given attribute whose spectrum of all the possible values (or states) is the non-empty set S, where S can be a set of finite discrete, S = {s1, s2, ..., sn}, 1 ≤ l ≤ ∞, or infinitely denumerable set S = {s1, s2, ..., sn}, or an infinitely uncountable set (continuous), S = ]a, b[, a < b, where ] is any open, semi-open or a closed interval set of real numbers or of another set [13, 14, 22-28, 30-51].

Let V be a non-empty subset of S, where V is the range of all attribute values needed by experts for the application. Each element x ∈ P is characterized by the values of all attributes in V = {v1, v2, ..., vn}, for n ≥ 1. In the set of values of the attribute V, in general, there is a dominant attribute value, which is determined by experts in its application. The dominant attribute value means the most important attribute that experts are interested in.

Each attribute value v ∈ V has a corresponding degree of membership d(x,v) of the element x, to the set P, regarding some given criteria.

The degree of membership can be either a fuzzy degree of membership, or an intuitionist fuzzy degree of membership, or a neutrosophic degree of membership to the plithogenic set.

Thus, the attribute value membership degree function is:

∀x ∈ P, d: P × V → P ([0,1]*)

Such that d(x,v) is a subset of [0,1]^z, where P([0,1]^z) is the power set of [0,1]^z, where z = 1 (fuzzy degree of membership), z = 2 (intuitionistic fuzzy degree of membership), or z = 3 (neutrosophic degree of membership).

Let |V| ≥ 1 be the cardinal. Let c: V × V → [0,1] be the attribute value contradiction degree function between any two attribute values v1 and v2, denoted by c(v1, v2), and satisfying the following axioms:

1. c(v1, v1) = 0, the degree of contradiction between the same attribute values is zero;
2. c(v1, v2) = c(v2, v1), commutativity.

We can define a fuzzy attribute value contradiction degree function (c as before, we denote by $c_{IF}$ to distinguish it from the following two), an intuitionistic fuzzy attribute value contradiction degree function ($c_{IF} : V \times V \rightarrow [0,1]^2$), or more generally, a neutrosophic attribute value contradiction degree function ($c_{N} : V \times V \rightarrow [0,1]^3$), the latter one can be used to increase the complexity of the calculation, but also to increase the accuracy.

We mainly calculate the degree of contradiction between the values of uni-dimensional attributes. For multidimensional attribute values, we divide them into their corresponding one-dimensional attribute values.

The attribute value contradiction degree function helps the plithogenic aggregation operators and the plithogenic inclusion (partial order) to obtain a more accurate result.

The attribute value contradiction degree function is designed in each field where a plithogenic set is used according to the application to be solved. If ignored, aggregations still work, but the result may lose accuracy.

So $(P, a, V, d, c)$ is called a plithogenic set. \cite{12, 52}:

1. Where "P" is a set, "a" is an attribute (multi-dimensional in general), "V" is the range of values of the attribute, "d" is the degree of membership of the attribute value of each element x to the set P regarding some given criteria (x $\in$ P), and "d" means $d_P$ or $d_{IP}$ or $d_N$, when it is a degree of fuzzy membership, an intuitionistic fuzzy membership, or a degree of neutrosophic membership, respectively, of an element x to the plithogenic set P.

2. "c" means $c_{IF}$ or $c_{N}^T$ or $c_{N}^P$, when it is a fuzzy attribute value contradiction degree function, intuitionistic fuzzy attribute value contradiction degree function, or neutrosophic attribute value contradiction degree function, respectively.

Functions $d(\cdot, \cdot)$ and $c(\cdot, \cdot)$ are defined according to the applications that experts need to solve.

Then, the following notation is used:

\[ d(x, V), \text{ where } d(x, V) = d(x, v), \forall v \in V, \forall x \in P. \]

The attribute value contradiction degree function is calculated between each attribute value with respect to the dominant attribute value (denoted by $v_D$) in particular, and other attribute values as well.

The attribute value contradiction degree function $c$ evaluated between the values of two attributes is used in the definition of plithogenic aggregation operators (intersection (AND), union (OR), implication ($\Rightarrow$), equivalence ($\Leftrightarrow$), inclusion (partial order), and other plithogenic aggregation operators that combine two or more degrees of values of the attribute based on a t-norm and a t-conorm, \cite{53}).

Most plithogenic aggregation operators are linear combinations of one fuzzy t-norm (denoted by $\Lambda_P$) with one fuzzy t-conorm (denoted by $\Lambda_F$), but nonlinear combinations can also be constructed.

If the t-norm is applied over the dominant attribute value denoted by $v_D$, and the contradiction between $v_D$ and $v_2$ is $c(v_D, v_2)$, then $v_2$ is applied over the attribute value as follows:

$$[1 - c(v_D, v_2)] \cdot \text{norm}(v_D, v_2) + c(v_D, v_2) \cdot \text{conorm}(v_D, v_2) \quad (2).$$

Or, by using symbols:

$$[1 - c(v_D, v_2)] \cdot \Lambda_P(v_D, v_2) + c(v_D, v_2) \cdot \Lambda_F(v_D, v_2) \quad (3).$$

Similarly, if the t-conorm is applied on the dominant attribute value denoted by $v_D$, and the contradiction between $v_D$ and $v_2$ is $c(v_D, v_2)$, then on the attribute value $v_2$ it is applied:

$$[1 - c(v_D, v_2)] \cdot \text{conorm}(v_D, v_2) + c(v_D, v_2) \cdot \text{norm}(v_D, v_2) \quad (4).$$

Or, by using symbols:

$$[1 - c(v_D, v_2)] \cdot \Lambda_F(v_D, v_2) + c(v_D, v_2) \cdot \Lambda_P(v_D, v_2) \quad (5).$$

The Plithogenic Neutrosophic Intersection is defined as:

$$(a_1, a_2, a_3) \Lambda_P(b_1, b_2, b_3) = \left( a_1 \Lambda_P b_1, \frac{1}{2}[(a_2 \Lambda_P b_2) + (a_2 \Lambda_F b_2)], a_3 \Lambda_F b_3 \right) \quad (6).$$

The Plithogenic Neutrosophic Union is defined as:

$$(a_1, a_2, a_3) \Lambda_F(b_1, b_2, b_3) = \left( a_1 \Lambda_F b_1, \frac{1}{2}[(a_2 \Lambda_F b_2) + (a_2 \Lambda_P b_2)], a_3 \Lambda_P b_3 \right) \quad (7).$$

In other words, what applies to membership, the opposite applies to non-membership, while in indeterminacy the average between them applies.

The Plithogenic Neutrosophic Inclusion is defined as follows:

Since the degrees of contradiction are $c(a_1, a_2) = c(a_2, a_3) = c(b_1, b_2) = c(b_2, b_3) = 0.5$, applies: $a_2 \geq [1 - c(a_1, a_2)]b_2$ or $a_2 \geq (1 - 0.5)b_2$ or $a_2 \geq 0.5b_2$ while $c(a_1, a_3) = c(b_1, b_3) = 1$.

Having $a_1 \leq b_1$, the opposite is done for $a_3 \geq b_3$, hence $(a_1, a_2, a_3) \leq_P (b_1, b_2, b_3)$ if and only if $a_1 \leq b_1$, $a_2 \geq 0.5b_2$, and $a_3 \geq b_3$.

### 3 Evaluative model of the subject “Education and Society”

To facilitate the evaluation of the aspects to be measured, two tables relate linguistic terms with plithogenic...
numbers. These plithogenic numbers consist of a vector of three components \((T, I, F)\) such that \(T\) represents the degree of truthfulness of what is claimed, \(I\) represents the degree of its indeterminacy and \(F\) represents the degree of its falsehood. \(T, I, F\) are elements of the interval \([0, 1]\). Decision-makers make assessments using linguistic terms, which better express what they want to say, and then numerical calculations are carried out with the support of the associated plithogenic numbers. That is why this paper deals with two tables, Table 1 that associates linguistic evaluation terms with plithogenic numbers, and Table 2 that associates linguistic terms of weight or importance with plithogenic numbers.

<table>
<thead>
<tr>
<th>Language expression</th>
<th>Plithogenic number ((T, I, F))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very poor (VP)</td>
<td>((0.10, 0.75, 0.85))</td>
</tr>
<tr>
<td>Poor (P)</td>
<td>((0.25, 0.60, 0.80))</td>
</tr>
<tr>
<td>Medium poor (MP)</td>
<td>((0.40, 0.70, 0.50))</td>
</tr>
<tr>
<td>Medium (M)</td>
<td>((0.50, 0.40, 0.60))</td>
</tr>
<tr>
<td>Medium good (MG)</td>
<td>((0.65, 0.30, 0.45))</td>
</tr>
<tr>
<td>Good (G)</td>
<td>((0.80, 0.10, 0.30))</td>
</tr>
<tr>
<td>Very good (VG)</td>
<td>((0.95, 0.05, 0.05))</td>
</tr>
</tbody>
</table>

*Table 1*: Linguistic values associated with plithogenic numbers for the evaluation of students. Source: [54].

<table>
<thead>
<tr>
<th>Language expression</th>
<th>Plithogenic number ((T, I, F))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low significance (LS)</td>
<td>((0.10, 0.70, 0.80))</td>
</tr>
<tr>
<td>Equal significance (ES)</td>
<td>((0.30, 0.40, 0.80))</td>
</tr>
<tr>
<td>Robust significance (RS)</td>
<td>((0.50, 0.40, 0.60))</td>
</tr>
<tr>
<td>Very robust significance (VRS)</td>
<td>((0.70, 0.30, 0.10))</td>
</tr>
<tr>
<td>Absolutely significance (AS)</td>
<td>((0.90, 0.10, 0.10))</td>
</tr>
</tbody>
</table>

*Table 2*: Linguistic values associated with plithogenic numbers for the evaluation of the weight of the criteria. Source: [54].

Plithogenic numbers can be converted into crisp values using formula 8.

\[
S((T, I, F)) = \frac{2 + T - I - F}{3} \tag{8}
\]

For evaluating, the following criteria are used for each type of scholar grade as is summarized below:

1. Oral/written exams (OWE):
   - OWEC1. Content mastery: demonstrates mastery of the required content. The information is relevant, accurate, and written (spoken) consistently.
   - OWEC2. Clear ideas and logical expression: the writing (speech) is clearly written (spoken), no ambiguities are presented in the written (spoken) expression.
   - OWEC3. Analysis: carries out a thorough analysis of technical-scientific development to fulfill competencies in the field of action.
   - OWEC4. Knowledge received: demonstrates correspondence very adequately with learning theories.
   - OWEC5. Identification of characteristics of topics and sub-topics.
   - OWEC6. Argumentation: the argumentations are complete, precise, demonstrate a total understanding of the contents, and identify all the elements beyond what is expected.
   - OWEC7. Vocabulary: full use of the appropriate lexicon. With identification, consistent and relevant terms that exceed the expected level.

2. Projects (P):
   - PC1. Identify the problem: identify the problem and its cause-effect relationship.
   - PC2. Structure of the work or project: the work or project is properly structured, its planning phases can be seen, and there is an agreement between objectives, activities, and goals.
   - PC3. Relationship of objectives: there is a correct definition of the overall objective, and the specific objectives ensure the achievement of the overall objective.
   - PC4. Purpose: The overall purpose of the work or project is clear and the activities that need to be developed to achieve its solution in the short, medium, or long term are easily appreciated.
   - PC5. Goals achieved: the work or project has a 100% level of compliance achieved.

3. Class participation (discussions, presentations) (CP)
   - CPC1. Attend regularly: meet 100% of attendance.
   - CPC2. Pay attention to the teacher and peers: be attentive to the concepts issued by the teacher and peers.
CPC3. Contributes to the class discussion: makes good contributions to class discussion.
CPC4. Ask questions about the topic dealt with in class: ask interesting questions about the topic dealt with in class.
CPC5. He/she has the materials to be used in class: He/she has all the materials necessary for the work in class.

4. Reports (R):
RC1. Presentation and punctuality: delivered the report on the stipulated date and with adequate quality.
RC2. Organization and structure of the report: the arguments are linked to the main idea and are organized logically.
RC3. Information quality: the information presented in the work is clear, accurate, correct, and relevant.
RC4. Drafting: it is adequate, clear, concise, and with technical vocabulary.
RC5. Bibliography: it presents bibliographies consulted in alphabetical order, following the APA format.

5. Memories of practices/portfolio/heading (M)
MC1. Introduction and objectives: properly drafts and sets general and specific objectives.
MC2. Materials and methods: complete list of materials and equipment used according to the manual. Describe the experimental procedure correctly.
MC3. Results: collects and sorts the data obtained by presenting them in clearly identified paragraphs, tables, or graphs.
MC4. Bibliography: presents bibliographies consulted in alphabetical order, following the APA format.

6. Research/portfolio (RP)
RPC1. Creative presentation: the presentation of the portfolio is creative.
RPC2. Punctuality: the portfolio is delivered on the stipulated date.
RPC3. Growth and development: in the presentation of the works of the portfolio it can be evidenced that there was learning.
RPC4. Bibliography: presents bibliographies consulted in alphabetical order, following the APA format.

7. Tutoring report (TR)
TRC1. Attitude: he/she comes to the tutoring with enthusiasm, a positive attitude, behaves kindly, respectful, recognize mistakes, and accepts suggestions.
TRC2. Material resources: the student is presented with all materials required for tutoring, and he/she carries the bibliographic resources.
TRC3. Knowledge of the topic: master the contents with accurate and relevant information for the development of the topic.
TRC4. Organization: he/she keeps track of information and organizes it by topic.

8. Observation guide (OG):
OGC1. Guide structure: all arguments are linked to the main idea and are organized logically.
OGC2. Procedure: efficiently complies with the protocol to perform the procedure, following the procedures established in the manual practices promptly.
OGC3. Information quality: the information is related to the main topic and provides several secondary ideas and/or examples.
OGC4. Materials: he/she has the materials required for the procedure.

9. Exercise and problem resolutions (EPR):
EPRC1. Obtaining and comparing results: the results of the exercises and problems were accurate, consistent, and logical.
EPRC2. Resolution: the student understood the problem; he/she solved it and presented their information clearly and convincingly.
EPRC3. Problem analysis: the student provides knowledge oriented to solving the problem.
EPRC4. Knowledge contribution: the student provides clear and precise knowledge oriented to the analysis of the problem.
EPRC5. Problem approach: the student identifies the problem.
EPRC6. Evaluation: he/she checks the result obtained and proposes other ways to solve the problem.

10. Essay/Mind Map (E)
EC1. Introduction: Students explain clearly what the essay is about, specifying the parts that compose them and a broad description of each one of them.
EC2. Content: the student broadly presents all suggested points in the assigned topic.
EC3. Organization: concepts are organized so that there is a logical connection between them.
EC4. Presentation: the student presents well-defined and structured graphic supports.
EC5. Analysis: a deep and thorough personal analysis of what the student describes is evidenced.
EC6. Conclusions: includes personal opinions supported by bibliographic arguments.

11. Mind maps (MM)
- MMC1. Presentation: the work is presented with visual stimuli and correctly associates the main and secondary ideas.
- MMC2. Articulations: use keywords that maintain consistency and are organized by categories.
- MMC3. Clarity of contents: the text is understandable and allows easy understanding of the subject, a logical order is maintained.
- MMC4. Understanding the topic: Students can explain the information without any difficulties or extra help.

12. Exhibition of works (EW):
- EWC1. Knowledge and preparation of the topic: solvency and confidence are appreciated when expressing their knowledge; student exposes accurate and relevant information for the development of the topic.
- EWC2. Structure and logical order: an organized exhibition is appreciated, respecting the established times, facilitating the capture of his/her speech from the beginning to the end of his/her intervention.
- EWC3. Mechanics and body expression maintain adequate body mechanics to present the subject, its body expression encourages confidence and stimulates the participation of the audience.
- EWC4. Formal use of language: establishes a permanent contact with the public through the mastery of an adequate linguistic record, a good tone of voice, gestural code, and eye contact.

13. Laboratory Practice Evaluation (LP):
- LPC1. Presentation (regulatory uniform): the uniform is worn clean and by practice. The hair is collected with mesh and has clean short nails.
- LPC2. Observation: the student carefully observes the procedure by applying knowledge to practice.
- LPC3. Attitude: the student enters the laboratory with enthusiasm, a positive attitude, behaves kindly, is respectful, recognizes mistakes, and accepts suggestions.
- LPC4. Participation in practice: the student actively participates in practice by showing interest in it. The median is calculated by using the following formula:

\[
\text{median}^m \{\text{PN}_i\} = (\text{median}^m \{T(\text{PN}_i)\}, \text{median}^m \{I(\text{PN}_i)\}, \text{median}^m \{F(\text{PN}_i)\})
\] (9),

Where, PN\(_i\) are m plithogenic numbers, T(\text{PN}_i) the truth components, I(\text{PN}_i) the indeterminate components, and F(\text{PN}_i) the falseness components.

Let x be the student to be evaluated on the subject “Education and Society”. Then, each of the professors, lecturers, and teachers evaluate each of the exams corresponding to the 13 aspects mentioned above. Specifically for P, CP, R, M, RP, TR, OG, EPR, E, MM, EW, and LP, there are 12 reports obtained from applying Equation 9 to each of the partial assessments that are applied to the student during the school year on the scale shown in Table 1. Let us illustrate this procedure with an example to make it clearer:

**Example 1:** If 3 projects P\(_1\), P\(_2\), P\(_3\) are carried out during the course, such that the school grades for x are as follows:

<table>
<thead>
<tr>
<th>Criteria of evaluation/Project</th>
<th>P(_1)</th>
<th>P(_2)</th>
<th>P(_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC(_1)</td>
<td>MP (0.40, 0.70, 0.50)</td>
<td>MG (0.65, 0.30, 0.45)</td>
<td>G (0.80, 0.10, 0.30)</td>
</tr>
<tr>
<td>PC(_2)</td>
<td>M (0.50, 0.40, 0.60)</td>
<td>G (0.80, 0.10, 0.30)</td>
<td>G (0.80, 0.10, 0.30)</td>
</tr>
<tr>
<td>PC(_3)</td>
<td>MG (0.65, 0.30, 0.45)</td>
<td>G (0.80, 0.10, 0.30)</td>
<td>G (0.80, 0.10, 0.30)</td>
</tr>
<tr>
<td>PC(_4)</td>
<td>G (0.80, 0.10, 0.30)</td>
<td>G (0.80, 0.10, 0.30)</td>
<td>G (0.80, 0.10, 0.30)</td>
</tr>
<tr>
<td>PC(_5)</td>
<td>M (0.50, 0.40, 0.60)</td>
<td>G (0.80, 0.10, 0.30)</td>
<td>VG (0.95, 0.05, 0.05)</td>
</tr>
<tr>
<td>Aggregated evaluation by project using Equation 6</td>
<td>(0.40, 0.33125,0.60)</td>
<td>(0.65, 0.10625,0.45)</td>
<td>(0.80, 0.075,0.30)</td>
</tr>
</tbody>
</table>

Table 3: Evaluation of student x for projects P\(_1\), P\(_2\), P\(_3\) during the course for every criterion.

Equation 6 is used to obtain the evaluation of each project, aggregating assessment of the 5 criteria, see that we used \(\Lambda_p = \min\) and \(\nu_p = \max\). [53] The aggregated x’s grades for the aspect Project is obtained using formula 9, thus, \(P = \text{median}\{\text{median}^{0.40, 0.33125,0.60}, \text{median}^{0.65, 0.10625,0.45}, (0.80, 0.075,0.30)\} = (0.65, 0.10625,0.45).

For the OWT aspects the same procedure above applies, except that these exams are applied in the course on three occasions, two partial exams (with an oral part and a written part) and a final exam (with an oral part and a written part), let us denote by OT\(_1\), WT\(_1\) the results of the first partial exam of x; OT\(_2\), WT\(_2\), the results of the
second partial exam; and OT3, WT3, the results of the final exam. Where OT1, WT1, OT2, WT2, OT3, WT3, are the results of aggregating the results of the OWTC1 - OWTC7 criteria with the help of Equation 6, as it was shown in.

**Example 1.**
So, we obtain a partial grade 1 OWT1 for x with formula $\text{OWT}_1 = \text{mean}(\text{OT}_1, \text{WT}_1)$, a partial grade 2 OWT2 with formula $\text{OWT}_2 = \text{mean}(\text{OT}_2, \text{WT}_2)$, and a final evaluation OWT3 with formula $\text{OWT}_3 = \text{mean}(\text{OT}_3, \text{WT}_3)$.

Thus, the grades of x are 15, viz., in order of importance: OWT3, OWT2, OWT1, P, CP, R, M, RP, TR, OG, EPR, E, MM, EW, and LP.

The final score for x is obtained from aggregating the 15 values using the weighted mean, as follows:

1. By consensus of specialists, OWT3 has importance of “Absolutely significance (AS)” according to Table 2. Therefore, by converting plithogenic number $(0.90, 0.10, 0.10)$ to a crisp value by using Equation 8, we have an importance $w_{\text{OWT3}} = 0.9$.
2. By consensus of specialists, OWT1 and OWT2 have a “Very robust significance (VRS)” according to Table 2. Therefore, by converting their plithogenic numbers $(0.70, 0.30, 0.10)$ to crisp values by using Equation 8, they have an importance $w_{\text{OWT1}} = w_{\text{OWT2}} = 0.76667$.
3. By consensus of specialists, P, CP, R, M, RP, TR, OG, EPR, E, MM, EW, and LP have a “Robust significance (RS)” according to Table 2. Therefore, by converting its plithogenic number $(0.50, 0.40, 0.60)$ into a crisp value by utilizing Equation 8, we have an importance $w_P = w_{CP} = w_R = w_M = w_{RP} = w_{TR} = w_{OG} = w_{EPR} = w_E = w_{MM} = w_{EW} = w_{LP} = 0.5$.

$N_x$, the final score of x is calculated by Equation 10:

$$N_x = 0.9 \text{OWT3} + 0.76667(\text{OWT2} + \text{OWT1}) + 0.5 \left( \frac{P + CP + R + M + RP + TR + OG + EPR + E + MM + EW}{LP} \right)$$

$$= 8.4333$$

4. $N_x = (T(N_x), I(N_x), F(N_x))$ is compared with each of the plithogenic numbers in Table 1, using formula 11.

$$d(N_x, TV_v) = \sqrt{(T(N_x) - T(TV_v))^2 + (I(N_x) - I(TV_v))^2 + (F(N_x) - F(TV_v))^2}$$

Where $(T(TV_v), I(TV_v), F(TV_v))$ are the plithogenic numbers in Table 1, with $v = 1, 2, ..., 7$.
5. The linguistic term is selected in Table 1, such that $d(N_x, TV_v)$ is minimal and this is the linguistic value that is associated with student x about the subject “Education and Society”.

Let us end this section with one last example:

**Example 2:**
Let us suppose the x’s scores are those summarized in Table 4, for every one of the aspects:

<table>
<thead>
<tr>
<th>Aspect to evaluate</th>
<th>Result in form of plithogenic number</th>
</tr>
</thead>
<tbody>
<tr>
<td>OWE3</td>
<td>$(0.80, 0.10, 0.30)$</td>
</tr>
<tr>
<td>OWE2</td>
<td>$(0.80, 0.10, 0.30)$</td>
</tr>
<tr>
<td>OWE1</td>
<td>$(0.65, 0.30, 0.45)$</td>
</tr>
<tr>
<td>P</td>
<td>$(0.65, 0.10625, 0.45)$</td>
</tr>
<tr>
<td>CP</td>
<td>$(0.50, 0.40, 0.60)$</td>
</tr>
<tr>
<td>R</td>
<td>$(0.65, 0.30, 0.45)$</td>
</tr>
<tr>
<td>M</td>
<td>$(0.80, 0.10, 0.30)$</td>
</tr>
<tr>
<td>RP</td>
<td>$(0.65, 0.30, 0.45)$</td>
</tr>
<tr>
<td>TR</td>
<td>$(0.65, 0.30, 0.45)$</td>
</tr>
<tr>
<td>OG</td>
<td>$(0.50, 0.40, 0.60)$</td>
</tr>
<tr>
<td>EPR</td>
<td>$(0.80, 0.10, 0.30)$</td>
</tr>
</tbody>
</table>
E  
(0.80, 0.10, 0.30)

MM  
(0.65, 0.10625, 0.45)

EW  
(0.65, 0.30, 0.45)

LP  
(0.80, 0.10, 0.30)

Table 4: School grades of student x with respect to all 15 aspects.

Thus, applying Formula 10, we have $N_x = (0.69743, 0.20193, 0.40257)$, comparing this plithogenic number with those shown in Table 1, using the distance function of Equation 11, we have that the minimum distance (equals to 0.11881) is obtained for “Medium Good (MG)”, and this is the final qualification of x in the course.

See that formulas 2 or 4 can be applied to compare the dominant criteria OWT3, with any of the other evaluations always that a value is fixed for the attribute value contradiction degree function.

Conclusion

This paper proposed a novel method for the evaluation of students in the subject “Education and Society” of the “Basic Education” program at the Autonomous University of the Andes, Ecuador. Plithogeny and plithogenic numbers were used to perform this evaluation. Plithogeny is a new theory that allows modeling complex systems obtained from the interaction, sometimes nonlinear, among simpler components, not necessarily free of contradictions, to obtain a new element. In this evaluative model, professors, lecturers, and teachers offer their qualitative assessments in the form of linguistic terms, and the final result of evaluating each student is also offered in the form of a linguistic term. This way of representing and obtaining the qualification of each student is comprehensible to both, students and teachers.

References


