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Case-based Legal Reasoning and Inductive Reasoning: Determination of the input parameters using Neutrosophic AHP

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Abstract. An Expert System as Case-Based Reasoning with the interaction of inductive reasoning is useful in predicting and evaluating domains that are difficult to formalize as is the case of the legal one. Since in this domain, a subjective environment is presented where the parties have contradictory points of view. Despite the multiple advantages of its implementation, the design of this type of system presents certain difficulties in establishing which are the predictive attributes that allow determining the similarity between a past situation and the current one, since they are mostly based on Boolean expressions. Which is in contradiction with the subjectivity of the process. To address this situation, the authors intend to establish the parameters to develop a Case-Based Legal Reasoning algorithm endorsed in a neutrosophic environment. Since the hypothesis states that its incorporation would guarantee a process where neutralities will be handled not by classical numbers but using neutrosophic numbers, which are the most natural form of measurement for human beings.

Keywords: case-based legal reasoning, subjectivity, Neutrosophy.

1. Introduction

The reasoning is a set of mental processes through which inferences are incorporated into knowledge. According to traditional philosophy, deductive and inductive methods exist to infer new information and enrich knowledge from premises and conclusions. The inference is derived from the premises, which leads to the conclusion in the inductive method. The results of induction may or may not be true, since their hypotheses have to be proved or disproved with other meanings. These hypotheses are based on personal experiences and beliefs. This leads to the key difference between deduction and induction: the first moves in the world of necessary truths and the second in probable truths [1, 2].

The validity of induction is a matter of degree and depends on the empirical support provided by the premises to reach a conclusion. Therefore, it can be said that one of the problems it faces is its justification. As a solution, it is accepted that its validity is based on the law of uniformity of nature, by which it can be assumed that the future will be similar to the past. Although it can be said that the method is a way of acquiring information through conclusions that always refer to reality, even though these are always provisional. That is, these inductive inferences are formed by empirical themes about past and future events [1].

According [3] cited in [1], for its conception the following phases are established:

1. Analysis of the elements, or the structure of the reasoning where the reasons that support the conclusion are identified.
2. Establishment of relationships between the elements determining if the relationships that may occur between the elements that make up the argument are convergent, chained, vertical, horizontal, etc. This can be achieved through answers to the following questions:
   - What are the relationships between the reasons and the conclusions?
   - How do the reasons support the conclusions?
3. Graphic representation of reasoning, both its elements and its relationships (concept maps, hierarchical diagrams, etc.). To facilitate understanding, information must be synthesized, storage and retrieval improved, and the efficiency of inference, problem-solving, and decision-making mechanisms increased.

4. Global assessment of the argument that can be carried out using in an orderly and systematic way the criteria of:
   a) degree of acceptability of the reasons that support the argument;
   b) relevance, that is, the importance of the relationships between reasons and conclusions;
   c) sufficiency, of significant reasons, whether qualitative or quantitative.

This type of reasoning is fundamental for the legal framework, where the resolution of sentences becomes a subjective process where induction is a fundamental part. According to [4]:

> Making a fair criminal decision can become an arduous task for those who administer justice to criminal offenders. This is because we are facing a subjective process, where the parties have contradictory points of view, and the one who imparts justice must be impartial before the facts, to determine the degree of guilt of the accused. Added to this is the possible lack of information on the facts, the multidisciplinary nature of the investigations, as it contains components of the natural, psychological, social, and criminal sciences. That is why an Expert System could serve as support for making this decision, although it does not replace it. (p. 1)

An Expert System as Case-Based Reasoning is useful in predicting and evaluating difficult to formalize domains as legal. In this world, casuistry is a valuable source of predictions, and therefore suitable for case-based approximations. Therefore, from a technical point of view, the main difficulty in developing this type of system lies in establishing the predictive attributes that allow determining the similarity between a past situation and the current one [5].

According to some authors, this situation can be corrected with the classification based on factors, which can be seen as an inadequate description of the reason for the decision of a case. However, from a practical point of view, a greater granularity in the factors will allow obtaining fewer arguments, which in turn will be more precise. The balance between quantity and precision can only be achieved through a set of factors established by experts in the domain being studied [6, 7].

It is important to remark that this issue underlies the problem of the recovery (location) of the information stored in databases. Which is subject to the procedure used for this purpose by combining keywords using Boolean expressions (True or False, [0; 1]). In the legal domain, users - mostly lawyers - have difficulties satisfactorily expressing their needs in Boolean terms; which usually leads to queries containing many references or irrelevant material. To make effective use of the database, this weakness must be overcome by finding a method that translates the information needs into a query expressed in technical terms that can distort the semantics of the requirements [6, 7], which is defined as the problem situation to analyze.

Therefore, it is proposed as a problem: how to develop a Case-Based Legal Reasoning (CBLR) algorithm according to inductive reasoning that provides users of the legal domain with a recovery method superior to Boolean expressions.

A bibliographic search made it possible to determine that developing this algorithm in a neutrosophic environment would suit the problem. This can be affirmed since Neutrosophy is the branch of philosophy that studies the origin, nature, and scope of neutralities, which goes beyond Boolean expressions. Therefore, its incorporation would guarantee that the uncertainty of decision-making is taken into account, including neutralities through the neutrosophic single-valued numbers, which constitutes the most natural form of measurement in human beings [8-14].

Then, the main objective of this paper is *to establish the input parameters to develop an algorithm of Case-Based Legal Reasoning endorsed in a neutrosophic environment that favors inductive reasoning*. For which it will work as follows:

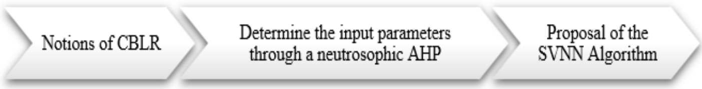

Figure 1: Specific objectives derived from the resolution of the problem raised.

2 Case-Based Legal Reasoning System

2.1 Case-Based Reasoning

Building smart systems somehow simulate the way humans solve problems. Within Artificial Intelligence there is a discipline called Knowledge Engineering that provides the methods and techniques to build computational systems called Knowledge-Based Systems [15]. These systems differ from others in their handling of large volumes of domain knowledge. Case-Based Reasoning is a set of techniques for the development of knowledge-based systems that recovers and reuses solutions from past experiences to solve similar problems and thus obtain the best results [5, 16-23].

A case-based reasoning system is a reasoning model that allows solving problems, understanding situations, and learning. These systems start from a problem already solved (case) hosted in a library of cases. These tasks are what a lawyer usually performs in everyday life, etc. A lawyer appeals to legal precedents to defend a cause, then it is said that he is using reasoning based on cases since it is a way of reasoning by making analogies [15].

According to [3], these reasoning models allow solving problems, understanding situations, and learning using memorization mechanisms, overlapping problems, and optimality criteria. They are based on three basic principles:

- Overlapping Troubleshooting - Applies to cases that use minor resolved cases.
- Bellman's optimality principle: memorize the best solution, after a selection process.
- Memorization: memorize the solutions obtained in the case library for later use.

Where its essential parts are the case base (also called the case library) and the similarity engine [5].

In general, it can be said that they have some advantages compared to traditional systems such as [3]:

- Acquisition of knowledge: The acquisition of knowledge is carried out from the previous experience stored in the case library.
- Knowledge maintenance: This allows the library to increase new cases without the intervention of the expert, making the maintenance process of the knowledge base unnecessary, lowering the cost.
- Efficiency in problem-solving: Reusability is a basic principle of computing that supports that similar cases can be solved without having to redo the knowledge base.
- Solution quality: by applying the optimality principle, it is guaranteed to memorize the best solution or what has happened in a given context.
- User acceptance: Using solutions based on cases that have already been used and tested gives confidence and acceptance to the user, which does not happen in solutions such as neural networks and case-based reasoning systems, since they can be incomprehensible to users.

Restrictions:

1. The domain of application of the cases must be regular, that is, it must not be changeable. What is true today must also be true tomorrow.
2. The problems must be recurring, that is, they must occur regularly; otherwise, it will not be necessary to memorize a case.

Solving such a system requires the following steps [15]:

Figure 2: Phases of the design of a case-based reasoning system. Source: Adapted from[15]

In cases description, the "case" itself is the main element of a case library. This allows organizing the cases of situations in a structured way. The organization must allow: first, the recovery of a subset of cases that can be applied to the problem posed, and then apply similarity measures to select from among the set of cases, the one that is closest to the problem posed. The simplest realization of the case library is through a flat memory (list or arrangement), although it can also be implemented using hierarchical memory (graph or trees).

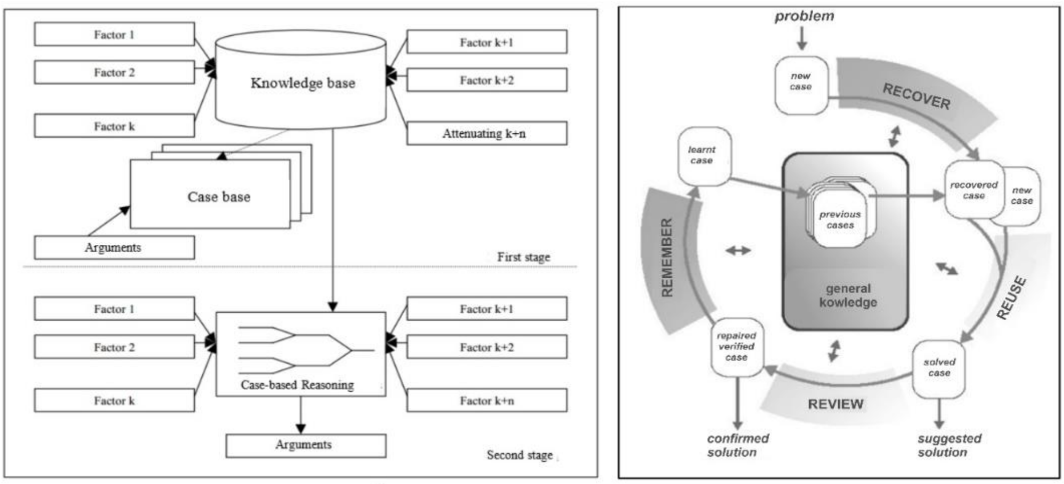

Figure 3: Case-based reasoning. Source: [5, 6]

2.2 Inductive legal reasoning in case-based reasoning

A case can be defined as a particular set of empirical circumstances that constitute a problem that needs a decision, solution, or classification. It has the particularity of presenting the circumstances and situation of a discreet episode, action, person, or thing. In practically all systems that use artificial intelligence, a case is represented by a particular name, a set of empirical circumstances or facts, and an output represented by a decision, solution, or classification given to it [6].

Experience can be referred to as the set of all instances, which means: cases that have occurred in the past and may form the basis for predicting the output of a new case. Precedent is understood to be a legal decision made in a previous case. Precedents are a subset of cases, which are a subset of experience. Based on what has been established, the term "precedence or precedent-based reasoning" indicates a form of explicit legal reasoning, where the precedent determines the outcome of a case. Through the use of factors, cases are indexed based on concepts or legal issues, rather than using keywords. This approach helps users express their information needs more consistently with their thoughts [6, 7].

3 Methods

3.1 Neutrosophy

Definition 1. Be X a universe of discourse. A *Neutrosophic Set* (NS) is characterized by three membership functions, $u_A(x), r_A(x), v_A(x) : X \rightarrow]^-0, 1^+[$, which satisfy the condition $^-0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3^+$ for all $x \in X$. $u_A(x), r_A(x)$ and $v_A(x)$ denote the membership functions of true, indeterminate, and false of x in A, respectively, and their images are standard or non-standard subsets of $]^-0, 1^+[$.

Definition 2. Be X a universe of discourse. A Single Value Neutrosophic Set of (SVNS) A over X is an object of the form:

$$A = \{\langle x, u_A(x), r_A(x), v_A(x)\rangle : x \in X\} \quad (1)$$

Where $u_A, r_A, v_A : X \rightarrow [0,1]$, satisfy the condition $0 \leq u_A(x) + r_A(x) + v_A(x) \leq 3$ for all $x \in X$. $u_A(x), r_A(x)$ and $v_A(x$ denotes the membership functions of true, indeterminate, and false of x in A, respectively. For convenience, a Single Value Neutrosophic Number (SVNN) will be expressed as A = (a, b, c), where a, b, c $\in$ [0,1] and satisfies $0 \leq a + b + c \leq 3$.

Definition 3. A Single Value Triangular Neutrosophic Number (SVTNN), which is denoted by: $\tilde{a} = \langle (a_1, a_2. a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is a NS on $\mathbb{R}$, whose membership functions of truthfulness, indeterminacy, and falsehood are defined below:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \leq x \leq a_2 \\ \alpha_{\tilde{a}}, & x = a_2 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), & a_2 < x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{\left(a_2 - x + \beta_{\tilde{a}}(x - a_1)\right)}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \beta_{\tilde{a}}, & x = a_2 \\ \frac{\left(x - a_2 + \beta_{\tilde{a}}(a_3 - x)\right)}{a_3 - a_2}, & a_2 < x \leq a_3 \\ 1, & \text{otherwise} \end{cases} \quad (3)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{\left(a_2 - x + \gamma_{\tilde{a}}(x - a_1)\right)}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \gamma_{\tilde{a}}, & x = a_2 \\ \frac{\left(x - a_2 + \gamma_{\tilde{a}}(a_3 - x)\right)}{a_3 - a_2}, & a_2 < x \leq a_3 \\ 1, & \text{otherwise} \end{cases} \quad (4)$$

Where, $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0,1]$ $a_1, a_2, a_3, a_4 \in \mathbb{R} a_1 \leq a_2 \leq a_3 \leq a_4$

Definition 4: ([24-27]) given $\tilde{a} = \langle (a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3, b_4); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$ two single-valued trapezoidal neutrosophic numbers and λ any non-null number in the real line. Then, the following operations are defined:

Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$ (5)

Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$ (6)

Inversion: $\tilde{a}^{-1} = \langle ({a_4}^{-1}, {a_3}^{-1}, {a_2}^{-1}, {a_1}^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$ where $a_1, a_2, a_3, a_4 \neq 0$ (7)

Definitions 3 and 4 refer to single-valued triangular neutrosophic number when the condition $a_2 = a_3$, [28-30]. For simplicity, we use the linguistic scale of triangular neutrosophic numbers, see Table 1 and also compare it with the scale defined in [31].

3.2 Neutrosophic Analytic Hierarchy Process (NAHP)

The analytic hierarchy process was proposed by Thomas Saaty in 1980 [8]. This technique models the problem that leads to the formation of a hierarchy representative of the associated decision-making scheme [9, 10]. The formulation of the decision-making problem in a hierarchical structure is the first and main stage. This stage is where the decision-maker must break down the problem into its relevant components [11-13]. The hierarchy is constructed so that the elements are of the same order of magnitude and can be related to some of the next levels. In a typical hierarchy, the highest level locates the problem of decision-making. The elements that affect decision-making are represented at the intermediate level, the criteria occupying the intermediate levels. At the lowest level, the decision options are placed [14]. The levels of importance or weighting of the criteria are estimated through paired comparisons between them. This comparison is carried out using a scale, as expressed in equation (6)[32].

$$S = \left\{\frac{1}{9}, \frac{1}{7}, \frac{1}{5}, \frac{1}{3}, 1,3,5,7,9\right\} \tag{9}$$

We can find in [31, 33-43] the theory of the AHP technique in a neutrosophic framework. Thus, we can model the indeterminacy of decision-making by applying neutrosophic AHP, or NAHP for short. Equation 10 contains a generic neutrosophic pair-wise comparison matrix for NAHP.

$$\tilde{A} = \begin{bmatrix} \tilde{1} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \vdots & & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \tilde{1} \end{bmatrix} \tag{10}$$

The matrix must satisfy the condition based on the inversion operator of Definition 4. $\tilde{A}$ $\tilde{a}_{ji} = \tilde{a}_{ij}^{-1}$

To convert neutrosophic triangular numbers into crisp numbers, there are two indexes defined in [31], are the so-called score and accuracy indexes, respectively, see Equations 11 and 12:

$$S(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} - \gamma_{\tilde{a}}) \tag{11}$$

$$A(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}}) \tag{12}$$

Saaty's scale	Definition	Neutrosophic Triangular Scale
1	Equally influential	$\tilde{1} = \langle (1,1,1); 0.50, 0.50, 0.50 \rangle$
3	Slightly influential	$\tilde{3} = \langle (2,3,4); 0.30, 0.75, 0.70 \rangle$
5	Strongly influential	$\tilde{5} = \langle (4,5,6); 0.80, 0.15, 0.20 \rangle$
7	Very strongly influential	$\tilde{7} = \langle (6,7,8); 0.90, 0.10, 0.10 \rangle$
9	Absolutely influential	$\tilde{9} = \langle (9,9,9); 1.00, 1.00, 1.00 \rangle$
2, 4, 6, 8	Sporadic values between two close scales	$\tilde{2} = \langle (1,2,3); 0.40, 0.65, 0.60 \rangle$
		$\tilde{4} = \langle (3,4,5); 0.60, 0.35, 0.40 \rangle$

	$\tilde{6} = \langle(5,6,7);0.70,0.25,0.30\rangle$
	$\tilde{8} = \langle(7,8,9);0.85,0.10,0.15\rangle$

Table 1: Saaty's scale translated to a neutrosophic triangular scale.

Step 1 Select a group of experts.

Step 2 Structure the neutrosophic pair-wise comparison matrix of factors, sub-factors, and strategies, through the linguistic terms shown in Table 1.

The neutrosophic scale is attained according to expert opinions [44]. The neutrosophic pair-wise comparison matrix of factors, sub-factors, and strategies are as described in Equation 10.

Step 3 Check the consistency of experts' judgments.

If the pair-wise comparison matrix has a transitive relation, ie, aik = aijajk for all i, j, and k, then the comparison matrix is consistent, focusing only on the lower, median, and upper values of the triangular neutrosophic number of the comparison matrix.

Step 4 Calculate the weight of the factors from the neutrosophic pair-wise comparison matrix, by transforming it to a deterministic matrix using Equations 13 and 14. To get the score and the accuracy degree of the following equations are used:$\tilde{a}_{ji}$

$$S(\tilde{a}_{ji}) = 1/S(\tilde{a}_{ij}) \tag{13}$$

$$A(\tilde{a}_{ji}) = 1/A(\tilde{a}_{ij}) \tag{14}$$

With compensation by accuracy degree of each triangular neutrosophic number in the neutrosophic pair-wise comparison matrix, we derive the following deterministic matrix:

$$A = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ \vdots & & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & 1 \end{bmatrix} \tag{15}$$

Determine the ranking of priorities, namely the Eigen Vector X, from the previous matrix:

1. Normalize the column entries by dividing each entry by the sum of the column.
2. Take the total of the row averages.

Note that Step 3 refers to consider the use of the calculus of the Consistency Index (CI) when applying this technique, which is a function depending on λ_{max}, the maximum eigenvalue of the matrix. Saaty establishes that consistency of the evaluations can be determined by the equation:

$$CI = \frac{\lambda_{max} - n}{n-1} \text{ [45]}, \tag{16}$$

where n is the order of the matrix. In addition, the Consistency Ratio (CR) is defined by equation:

$$CR = \frac{CI}{RI} \tag{17}$$

RI is given in table 2.

Order (n)	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

Table 2: RI associated with every order.

If CR≤0.1 we may consider that experts' evaluation is sufficiently consistent and hence we can proceed to use NAHP. We apply this procedure to matrix "A" in Equation 17.

Other useful neutrosophic insights for the document:

Linguistic term	SVN
Extremely mild (EM)	(1,0,0)
Very very mild (VVM)	(0.9, 0.1, 0.1)
Very mild (VM)	(0.8, 0.15, 0.20)
Mild (M)	(0.70, 0.25, 0.30)
Medium mild (MDM)	(0.60, 0.35, 0.40)
Medium (MD)	(0.50, 0.50, 0.50)
Medium severe (MDS)	(0.40, 0.65, 0.60)
Grave (G)	(0.30, 0.75, 0.70)
Very grave (MG)	(0.20, 0.85, 0.80)

Very very grave (MMG)	(0.10, 0.90, 0.90)
Extremely grave (EG)	(0; 1; 1)

Table 3: Neutrosophic Unique Value Numbers. Source: [13].

Similarity function Si between n NNVU, (i = 1, 2,…, m) (j = 1, 2,…, n) and a vector of values. $A_{ij} = \langle a_{ij}, b_{ij}, c_{ij}\rangle B_j^* = \langle a_j^*, b_j^*, c_j^*\rangle$

$$S_i = 1 - \left(\left(\frac{1}{3}\sum_{j=1}^{n}\left\{\left(a_{ij}-a_j^*\right)^2+\left(b_{ij}-b_j^*\right)^2+\left(c_{ij}-c_j^*\right)^2\right\}\right)^{\frac{1}{2}}\right) \tag{18}$$

4 Results

4.1 Determine the input parameters to the case library using NAHP

To start the design of the legal reasoning system based on cases from an inductive perspective, it is proposed to structure the information in a library of cases. For the elaboration of this base library of the system, a structure must be entered as a form. 8 experts (legal professionals) were consulted. The method was only applied to determine the fields referring to the legal parameters of interest for the information structure and to be able to design the user interface. Not so to the regulatory fields for the identification of cases: such as id, date (yyyy /mm/dd). The information processing was divided into two phases:

Phase 1: Details of the defendant

ID	Field	Description example
A1	Name	Juan Perez
A2	Age	Under 18, between 19-29, between 30-40
A3	Race/ethnicity	White, black, Indian
A4	Sex	M / F / other
TO 5	Educational level	None, Primary, Secondary
A6	Recidivist	Otherwise
A7	Municipality	Canton, Province
A8	Guilty plea	Whether or not you plead guilty in the first instance

Table 4: Defendant's data entry parameters to the library of cases and examples.

A1	A2	A3	A4	TO 5	A6	A7	A8
$\langle(1,1,1); 0.50,0.50,0.50\rangle$	$\langle(6,7,8); 0.90,0.10,0.10\rangle$	$\langle(6,7,8); 0.90,0.10,0.10\rangle$	$\langle(6,7,8); 0.90,0.10,0.10\rangle$	$\langle(6,7,8); 0.90,0.10,0.10\rangle$	$\langle(4,5,6); 0.80,0.15,0.20\rangle$	$\langle(6,7,8); 0.90,0.10,0.10\rangle$	$\langle(6,7,8); 0.90,0.10,0.10\rangle$
$\frac{1}{\langle(6,7,8); 0.90,0.10,0.10\rangle}$	$\langle(1,1,1); 0.50,0.50,0.50\rangle$	$\langle(2,3,4); 0.30,0.75,0.70\rangle$	$\langle(4,5,6); 0.80,0.15,0.20\rangle$	$\langle(4,5,6); 0.80,0.15,0.20\rangle$	$\langle(4,5,6); 0.80,0.15,0.20\rangle$	$\langle(4,5,6); 0.80,0.15,0.20\rangle$	$\langle(4,5,6); 0.80,0.15,0.20\rangle$
$\frac{1}{\langle(6,7,8); 0.90,0.10,0.10\rangle}$	$\frac{1}{\langle(2,3,4); 0.30,0.75,0.70\rangle}$	$\langle(1,1,1); 0.50,0.50,0.50\rangle$	$\langle(2,3,4); 0.30,0.75,0.70\rangle$	$\langle(2,3,4); 0.30,0.75,0.70\rangle$	$\langle(4,5,6); 0.80,0.15,0.20\rangle$	$\langle(4,5,6); 0.80,0.15,0.20\rangle$	$\langle(4,5,6); 0.80,0.15,0.20\rangle$
$\frac{1}{\langle(6,7,8); 0.90,0.10,0.10\rangle}$	$\frac{1}{\langle(4,5,6); 0.80,0.15,0.20\rangle}$	$\frac{1}{\langle(2,3,4); 0.30,0.75,0.70\rangle}$	$\langle(1,1,1); 0.50,0.50,0.50\rangle$	$\langle(1,1,1); 0.50,0.50,0.50\rangle$	$\langle(2,3,4); 0.30,0.75,0.70\rangle$	$\langle(2,3,4); 0.30,0.75,0.70\rangle$	$\langle(2,3,4); 0.30,0.75,0.70\rangle$
$\frac{1}{\langle(6,7,8); 0.90,0.10,0.10\rangle}$	$\frac{1}{\langle(4,5,6); 0.80,0.15,0.20\rangle}$	$\frac{1}{\langle(2,3,4); 0.30,0.75,0.70\rangle}$	$\langle(1,1,1); 0.50,0.50,0.50\rangle$	$\langle(1,1,1); 0.50,0.50,0.50\rangle$	$\langle(1,1,1); 0.50,0.50,0.50\rangle$	$\langle(2,3,4); 0.30,0.75,0.70\rangle$	$\langle(2,3,4); 0.30,0.75,0.70\rangle$
$\frac{1}{\langle(4,5,6); 0.80,0.15,0.20\rangle}$	$\frac{1}{\langle(4,5,6); 0.80,0.15,0.20\rangle}$	$\frac{1}{\langle(4,5,6); 0.80,0.15,0.20\rangle}$	$\frac{1}{\langle(2,3,4); 0.30,0.75,0.70\rangle}$	$\langle(1,1,1); 0.50,0.50,0.50\rangle$	$\langle(1,1,1); 0.50,0.50,0.50\rangle$	$\langle(1,1,1); 0.50,0.50,0.50\rangle$	$\langle(1,1,1); 0.50,0.50,0.50\rangle$
$\frac{1}{\langle(6,7,8); 0.90,0.10,0.10\rangle}$	$\frac{1}{\langle(4,5,6); 0.80,0.15,0.20\rangle}$	$\frac{1}{\langle(4,5,6); 0.80,0.15,0.20\rangle}$	$\frac{1}{\langle(2,3,4); 0.30,0.75,0.70\rangle}$	$\frac{1}{\langle(2,3,4); 0.30,0.75,0.70\rangle}$	$\langle(1,1,1); 0.50,0.50,0.50\rangle$	$\langle(1,1,1); 0.50,0.50,0.50\rangle$	$\langle(1,1,1); 0.50,0.50,0.50\rangle$

1/⟨(6,7,8); 0.90,0.10,0.10⟩	1/⟨(4,5,6); 0.80,0.15,0.20⟩	1/⟨(4,5,6); 0.80,0.15,0.20⟩	1/⟨(2,3,4); 0.30,0.75,0.70⟩	1/⟨(2,3,4); 0.30,0.75,0.70⟩	⟨(1,1,1); 0.50,0.50,0.50⟩	⟨(1,1,1); 0.50,0.50,0.50⟩	⟨(1,1,1); 0.50,0.50,0.50⟩

Table: 5. Neutrosophic paired comparison matrix.

Criteria	A1	A2	A3	A4	TO 5	A6	A7	A8	Weight		
A1	0.52	0.77	0.58	0.40	0.38	0.23	0.27	0.27	**0.43**	**Eigenvalue**	
A2	0.07	0.11	0.25	0.28	0.27	0.23	0.19	0.19	**0.20**		8.83
A3	0.07	0.04	0.08	0.17	0.16	0.23	0.19	0.19	**0.14**	**IC**	0.12
A4	0.09	0.07	0.07	0.07	0.11	0.15	0.12	0.12	**0.10**	**RC**	0.08
TO 5	0.07	0.02	0.03	0.06	0.05	0.05	0.12	0.12	0.06		
A6	0.10	0.02	0.02	0.02	0.05	0.05	0.04	0.04	0.04		
A7	0.07	0.02	0.02	0.02	0.02	0.05	0.04	0.05	0.03		
A8	0.07	0.02	0.02	0.02	0.02	0.05	0.04	0.04	0.03		

Table: 6. Weighted matrix and consistency analysis.

Phase 2: Process data

ID	Field	Description example
P1	Crime	Involuntary manslaughter, Robbery, Intimidation
P2	Means of aggression	Gun, knife, none, hands
Q3	Appeal	No, Supreme Court
Q4	Mitigating causes	Mental state, a victim of abuse
P5	Damages	Death, loss of items valued at $ 1,000, psychological damage
Q6	Judgment	Deprivation of liberty for 3 years, USD 5,000 fine, Community work

Table 7. Process data input parameters to the library of cases and examples.

P1	P2	Q3	Q4	P5	Q6
⟨(1,1,1); 0.50,0.50,0.50⟩	⟨(4,5,6);	⟨(4,5,6); 0.80,0.15,0.20⟩	⟨(4,5,6); 0.80,0.15,0.20⟩	⟨(2,3,4); 0.30,0.75,0.70⟩	⟨(2,3,4); 0.30,0.75,0.70⟩
1/⟨(4,5,6); 0.80,0.15,0.20⟩	⟨(1,1,1); 0.50,0.50,0.50⟩	⟨(4,5,6); 0.80,0.15,0.20⟩	⟨(2,3,4); 0.30,0.75,0.70⟩	1/⟨(2,3,4); 0.30,0.75,0.70⟩	⟨(1,1,1); 0.50,0.50,0.50⟩
1/⟨(4,5,6); 0.80,0.15,0.20⟩	1/⟨(4,5,6); 0.80,0.15,0.20⟩	⟨(1,1,1); 0.50,0.50,0.50⟩	⟨(2,3,4); 0.30,0.75,0.70⟩	1/⟨(2,3,4); 0.30,0.75,0.70⟩	1/⟨(4,5,6); 0.80,0.15,0.20⟩
1/⟨(4,5,6); 0.80,0.15,0.20⟩	1/⟨(2,3,4); 0.30,0.75,0.70⟩	1/⟨(2,3,4); 0.30,0.75,0.70⟩	⟨(1,1,1); 0.50,0.50,0.50⟩	1/⟨(4,5,6); 0.80,0.15,0.20⟩	1/⟨(4,5,6); 0.80,0.15,0.20⟩
1/⟨(2,3,4); 0.30,0.75,0.70⟩	⟨(2,3,4); 0.30,0.75,0.70⟩	⟨(2,3,4); 0.30,0.75,0.70⟩	⟨(4,5,6); 0.80,0.15,0.20⟩	⟨(1,1,1); 0.50,0.50,0.50⟩	⟨(2,3,4); 0.30,0.75,0.70⟩
1/⟨(2,3,4); 0.30,0.75,0.70⟩	⟨(1,1,1); 0.50,0.50,0.50⟩	⟨(4,5,6); 0.80,0.15,0.20⟩	⟨(4,5,6); 0.80,0.15,0.20⟩	1/⟨(2,3,4); 0.30,0.75,0.70⟩	⟨(1,1,1); 0.50,0.50,0.50⟩

Table 8. Neutrosophic paired comparison matrix.

Criteria	P1	P2	Q3	Q4	P5	Q6	Weight		
P1	0.9375	5.1562	5.1562	5.1562	2.6437	2.6437	**0.334410**		
P2	0.2120	0.9375	5.1562	2.6437	0.3182	0.9375	**0.096940**	**Eigenvalue**	5.46869

Q3	0.2120	0.2120	0.9375	2.6437	0.3182	0.2120	0.050220	**IC**	0.058115
Q4	0.2120	0.3182	0.3182	0.9375	0.2120	0.2120	0.035719	**RC**	5.7863
P5	0.3182	2.6437	2.6437	5.1562	0.9375	2.6437	**0.208733**		
Q6	0.3182	0.9375	5.1562	5.1562	0.3182	0.9375	**0.127504**		

Table: 9. Weighted matrix and consistency analysis.

The results were exposed to the round of experts where it was determined that in the case of the defendant's data, it must be entered in the system interface: Name, Age, Race/ethnicity, and Sex. For the criteria of the process: Crime, Means of aggression, Damages, and Sentence.

4.2 System architecture design

For the system's design, the models exposed in [4, 6, 7, 15], to which the Neutrosophy will be inserted for the level of granularity with the SVNNs in the similarity equation. With which it will be possible to obtain in the function "Retrieve Argument" several cases that indicate not only the most similar stored, but the other less similar and so on. Of course, as long as it contains the factors specified in the rule. This way, the expert system can retrieve the foundation for the recommendations it issues. The granularity of the cases is determined by the number of factors involved in the formation of the rule.

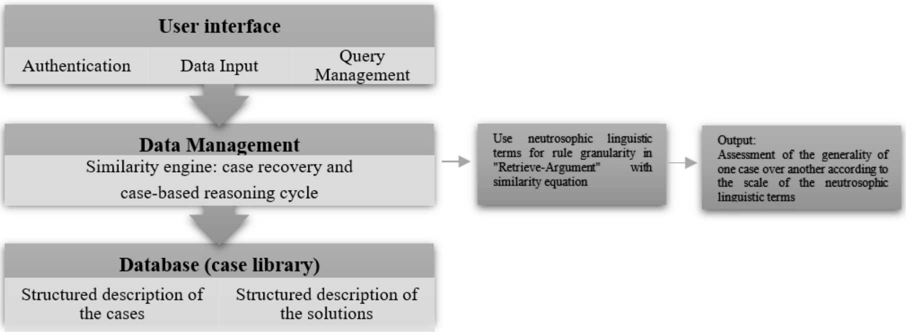

Figure 4: System design.

Conclusion

Reasoning based on precedents, as a technique to generate arguments, is easy to implement and highly useful in legal expert systems. These systems are based on the principles of overlapping problems, memorization, and the optimality principle. Likewise, it can be said that they are easily understood by the expert, since the system does not handle abstract concepts, but rather concrete situations (cases) of the domain known to the expert. With its implementation, an increase in the effectiveness of the legal expert's time management as well as of the system is evidenced, since over time it is nurtured by new cases. The advantage lies mainly in cost reduction, speed in software development, and risk reduction. However, its greatest limitation is the programming time required. Many times the translation of what is desired becomes complicated and its development and implementation take longer. But once materialized, it is widely accepted. The algorithm designed by the legal expert system will be able to apply the forward inference mechanism to recover the arguments linked to other cases and use them as the basis for the decision it generates. For this case, the use of factors instead of keywords was considered to guarantee the accuracy of the information displayed. A training action by the working group is recommended, which must be composed of legal, mathematical, and computer experts or related specialties. Explanation of recommendations is an essential feature for expert systems.

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