A New Methodology for Neutrosophic Multi-attribute Decision-making with Unknown Weight Information

Pranab Biswas¹, Surapati Pramanik²*, and Bibhas C. Giri³

¹ Department of Mathematics, Jadavpur University, Kolkata,700032, India. E-mail: paldam2010@gmail.com
² Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, P.O.-Narayanpur, 743126, India. Email: sura_pati@yahoo.co.in
³ Department of Mathematics, Jadavpur University, Kolkata,700032, India. Email: bcrijumath@gmail.com
*Corresponding author’s email: sura_pati@yahoo.co.in

Abstract. In this paper, we present multi-attribute decision-making problem with neutrosophic assessment. We assume that the information about attribute weights is incompletely known or completely unknown. The ratings of alternatives with respect to each attributes are considered as single-valued neutrosophic set to catch up imprecise or vague information. Neutrosophic set is characterized by three independent degrees namely truth-membership degree (T), indeterminacy-membership degree (I), and falsity-membership degree (F). The modified grey relational analysis method is proposed to find out the best alternative for multi-attribute decision-making problem under neutrosophic environment. We establish a deviation based optimization model based on the ideal alternative to determine attribute weight in which the information about attribute weights is incompletely known. Again, we solve an optimization model with the help of Lagrange functions to find out the completely unknown attributes weight. By using these attributes weight we calculate the grey relational coefficient of each alternative from ideal alternative for ranking the alternatives. Finally, an illustrative example is provided in order to demonstrate its applicability and effectiveness of the proposed approach.

Keywords: Neutrosophic set; Single-valued neutrosophic set; Grey relational analysis; Multi-attribute decision making; Unknown weight information.

1 Introduction

In the real world problem, we often encounter different type of uncertainties that cannot be handled with classical mathematics. In order to deal different types of uncertainty, Fuzzy set due to Zadeh [1] is very useful and effective. It deals with a kind of uncertainty known as ‘fuzziness’. Each real value of [0,1] represents the membership degree of an element of a fuzzy set i.e partial belongingness is considered. If \( \mu_A(x) \in [0,1] \) is the membership degree of an element \( x \) of a fuzzy set \( A \) then \((1-\mu_A(x)) \) is assumed as the non-membership degree of that element. This is not generally hold for an element with incomplete information. In 1986, Atanassov [2] developed the idea of intuitionistic fuzzy set (IFS). An element of intuitionistic fuzzy set \( A \) characterized by the membership degree \( \mu_A(x) \in [0,1] \) as well as non-membership degree \( v_A(x) \in [0,1] \) with some restriction \( 0 \leq \mu_A(x) + v_A(x) \leq 1 \). Therefore certain amount of indeterminacy or incomplete information \( 1-(\mu_A(x) + v_A(x)) \) remains by default. However, one may also consider the possibility \( \mu_A(x) + v_A(x) > 1 \), so that inconsistent beliefs are also allowed. In this case, an element may be regarded as both member and non-member at the same time. A set connected with this features is called Para-consistent Set [3]. Smarandache [3-5] introduced the concept of neutrosophic set (NS) which is actually generalization of different type of FSs and IFSs. Consider an example, if \( \mu_A(x) \in [0,1] \) is a membership degree, \( v_A(x) \in [0,1] \) is a non-membership degree of an element \( x \) of a set \( A \), then fuzzy set can be expressed as \( A= \{x/(\mu_A(x),0,1-\mu_A(x))\} \) and IFS can be represented as \( A= \{x/(\mu_A(x),1-\mu_A(x)-v_A(x),v_A(x))\} \) with \( 0 \leq \mu_A(x) + v_A(x) \leq 1 \). The main feature of neutrosophic set is that every element of the universe has not only a certain degree of truth (T) but also a falsity degree (F) and indeterminacy degree (I). These three degrees have to consider independently from each other. Another interesting feature of neutrosophic set is that we do not even assume that the incompleteness or indeterminacy degree is always given by \( 1-(\mu_A(x) + v_A(x)) \).

Multiple attribute decision-making (MADM) problem in the area of operation research, management science, economics, systemic optimization, urban planning and many other fields has gained very much attention to the researchers during the last several decades. These problems generally consist of choosing the most desirable alternative that has the highest degree of satisfaction from a set of alternatives with respect to their attributes. In this approach the decision makers have to provide qualitative and/or quantitative assessments for determining the performance of each alternative with respect to each attribute, and the relative importance of evaluation attribute.

There are many MADM methods available in the literature in crisp environment such as TOPSIS (Hwang & Yoon [6]), PROMETHEE (Brans et al. [7]), VIKOR (Opricovic [8-9]), and ELECTRE (Roy [10]) etc. However it is not always...

In IFS the sum of membership degree and non-membership degree of a vague parameter is less than unity. Therefore, a certain amount of incomplete information or indeterminacy arises in an intuitionistic fuzzy set. It cannot handle all types of uncertainties successfully in different real physical problems. Hence further generalizations of fuzzy set as well as intuitionistic fuzzy sets are required.

Neutrosophic set information is helpful to handling MADM for the most general ambiguity cases, including paradox. The assessment of attribute values by the decision maker takes the form of single-valued neutrosophic set (SVNS) which is defined by Wang et al. [16]. Ye [17] studied multi-criteria decision-making problem under SVNS environment. He proposed a method for ranking of alternatives by using weighted correlation coefficient. Ye [18] also discussed single-valued neutrosophic cross entropy for multi-criteria decision-making problems. He used similarity measure for interval valued neutrosophic set for solving multi-criteria decision-making problems. Grey relational analysis (GRA) is widely used for MADM problems. Deng [19-20] developed the GRA method that is applied in various areas, such as economics, marketing, personal selection and agriculture. Zhang et al. [21] discussed GRA method for multi attribute decision-making with interval numbers. An improved GRA method proposed by Rao & Singh [22] is applied for making a decision in manufacturing situations. Wei [23] studied the GRA method for intuitionistic fuzzy multi-criteria decision-making. Biswas et al. [24] developed an entropy based grey relational analysis method for multi-attribute decision-making problem under single valued neutrosophic assessments.

The objective of this paper is to study neutrosophic MADM with unknown weight information using GRA. The rest of the paper is organized as follows. Section 2 briefly presents some preliminaries relating to neutrosophic set and single-valued neutrosophic set. In Section 3, Hamming distance between two single-valued neutrosophic sets is defined. Section 4 is devoted to represent the new model of MADM with SVNSs based on modified GRA. In section 5, an illustrative example is provided to show the effectiveness of the proposed model. Finally, section 6 presents the concluding remarks.

2 Preliminaries of Neutrosophic sets and Single valued neutrosophic set

In this section, we provide some basic definition about neutrosophic set due to Smranda [3], which will be used to develop the paper.

**Definition 1** Let \( X \) be a space of points (objects) with generic element in \( X \) denoted by \( x \). Then a neutrosophic set \( A \) in \( X \) is characterized by a truth membership function \( T_A \), an indeterminacy membership function \( I_A \) and a falsity membership function \( F_A \). The functions \( T_A, I_A \) and \( F_A \) are real standard or non-standard subsets of \( [0, 1] \) that is \( T_A: X \rightarrow [0, 1] \), \( I_A: X \rightarrow [0, 1] \), \( F_A: X \rightarrow [0, 1] \). It should be noted that there is no restriction on the sum of \( T_A(x), I_A(x), F_A(x) \) i.e. \( 0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3 \).

**Definition 2** The complement of a neutrosophic set \( A \) is denoted by \( \Lambda^A \) and is defined by

\[
T_{\Lambda^A}(x) = \{1^+ - T_A(x) \} ; \quad I_{\Lambda^A}(x) = \{1^+ - I_A(x) \} ; \\
F_{\Lambda^A}(x) = \{1^+ - F_A(x) \}
\]

**Definition 3** A neutrosophic set \( A \) is contained in the other neutrosophic set \( B \), \( A \subset B \) if and only if the following result holds.

\[
\inf T_A(x) \leq \inf T_B(x) , \quad \sup T_A(x) \leq \sup T_B(x) \quad (1) \\
\inf I_A(x) \leq \inf I_B(x) , \quad \sup I_A(x) \leq \sup I_B(x) \quad (2) \\
\inf F_A(x) \leq \inf F_B(x) , \quad \sup F_A(x) \leq \sup F_B(x) \quad (3)
\]

for all \( x \) in \( X \).

3 Some basics of single valued neutrosophic sets (SVNSs)

In this section we provide some definitions, operations and properties about single valued neutrosophic sets due to Wang et al. [17]. It will be required to develop the rest of the paper.

**Definition 4** (Single-valued neutrosophic set). Let \( X \) be a universal space of points (objects), with a generic element of \( X \) denoted by \( x \). A single-valued neutrosophic set \( A \subset X \) is characterized by a true membership function \( T_A(x) \), a falsity membership function \( F_A(x) \) and an
indeterminacy function $I_{\tilde{c}}(x)$ with $T_{\tilde{c}}(x)$, $I_{\tilde{c}}(x)$, $F_{\tilde{c}}(x)$ in $[0, 1]$ for all $x$ in $X$.

When $X$ is continuous a SVNSs, $\tilde{N}$ can be written as

$$\tilde{N} = \{ \frac{1}{x} (T_{\tilde{c}}(x), I_{\tilde{c}}(x), F_{\tilde{c}}(x)) \mid x, \quad \forall x \in X. $$

and when $X$ is discrete a SVNSs $\tilde{N}$ can be written as

$$\tilde{N} = \{ \sum_{x=1}^{X} (T_{\tilde{c}}(x), I_{\tilde{c}}(x), F_{\tilde{c}}(x)) \mid x, \quad \forall x \in X. $$

Actually, SVNS is an instance of neutrosophic set that can be used in real life situations like decision making, scientific and engineering applications. In case of SVNS, the degree of the truth membership $x(\tilde{T})$, indeterminacy-membership $x(\tilde{I})$, and falsity membership $x(\tilde{F})$ values belong to $[0, 1]$ instead of non standard unit interval $0^\leftrightarrow, 1^\leftrightarrow$ as in the case of ordinary neutrosophic sets.

It should be noted that for a SVNS $\tilde{N}$,

$$0 \leq \sup T_{\tilde{c}}(x) + \sup I_{\tilde{c}}(x) + \sup F_{\tilde{c}}(x) \leq 3, \quad \forall x \in X. \quad (4)$$

and for a neutrosophic set, the following relation holds $0 \leq \sup T_{\tilde{c}}(x) + \sup I_{\tilde{c}}(x) + \sup F_{\tilde{c}}(x) \leq 3^\leftrightarrow, \quad \forall x \in X. \quad (5)$

**Definition 5** The complement of a neutrosophic set $\tilde{N}$ is denoted by $\tilde{N}_c$ and is defined as

$$T_{\tilde{c}}(x) = F_{\tilde{c}}(x); \quad I_{\tilde{c}}(x) = 1 - I_{\tilde{c}}(x); \quad F_{\tilde{c}}(x) = T_{\tilde{c}}(x).$$

**Definition 6** A SVNS $\tilde{N}_A$ is contained in the other SVNS $\tilde{N}_B$, denoted as $\tilde{N}_A \subseteq \tilde{N}_B$, if and only if $T_{\tilde{A}}(x) \leq T_{\tilde{B}}(x)$, $I_{\tilde{A}}(x) \geq I_{\tilde{B}}(x)$, $F_{\tilde{A}}(x) \geq F_{\tilde{B}}(x)$, $\forall x \in X.$

**Definition 7** Two SVNSs $\tilde{N}_A$ and $\tilde{N}_B$ are equal, i.e. $\tilde{N}_A = \tilde{N}_B$, if and only if $\tilde{N}_A \subseteq \tilde{N}_B$ and $\tilde{N}_B \subseteq \tilde{N}_A.$

**Definition 8** (Union) The union of two SVNSs $\tilde{N}_A$ and $\tilde{N}_B$ is a SVNS $\tilde{N}_C$, written as $\tilde{N}_C = \tilde{N}_A \cup \tilde{N}_B.$ Its truth membership, indeterminacy-membership and falsity membership functions are related to those of $\tilde{N}_A$ and $\tilde{N}_B$ by

$$T_{\tilde{C}}(x) = \max (T_{\tilde{A}}(x), T_{\tilde{B}}(x));$$

$$I_{\tilde{C}}(x) = \max (I_{\tilde{A}}(x), I_{\tilde{B}}(x));$$

$$F_{\tilde{C}}(x) = \min (F_{\tilde{A}}(x), F_{\tilde{B}}(x)), \text{ for all } x \in X.$$  

**Definition 9** (Intersection) The intersection of two SVNSs $\tilde{N}_A$ and $\tilde{N}_B$ is a SVNS $\tilde{N}_C$ written as $\tilde{N}_C = \tilde{N}_A \cap \tilde{N}_B,$ whose truth membership, indeterminacy-membership and falsity membership functions are related to those of $\tilde{N}_A$ and $\tilde{N}_B$ by

$$T_{\tilde{C}}(x) = \min (T_{\tilde{A}}(x), T_{\tilde{B}}(x));$$

$$I_{\tilde{C}}(x) = \min (I_{\tilde{A}}(x), I_{\tilde{B}}(x));$$

$$F_{\tilde{C}}(x) = \max (F_{\tilde{A}}(x), F_{\tilde{B}}(x)) \text{ for all } x \in X.$$  

**4 Distance between two neutrosophic sets.**

Similar to fuzzy or intuitionistic fuzzy set, the general SVNS having the following pattern

$$\tilde{N} = \{ (\frac{x}{(T_{\tilde{c}}(x), I_{\tilde{c}}(x), F_{\tilde{c}}(x))) \mid x \in X. \} \text{ For finite SVNSs can be represented by the ordered tetrads:}$$

$$\tilde{N} = \{ (\frac{x}{(T_{\tilde{c}}(x), I_{\tilde{c}}(x), F_{\tilde{c}}(x))) \mid x \in X. \}$$

**Definition 10** Let

$$\tilde{N}_A = \{ (\frac{x}{(T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x))) \mid x \in X. \} \text{ and } \tilde{N}_B = \{ (\frac{x}{(T_{\tilde{B}}(x), I_{\tilde{B}}(x), F_{\tilde{B}}(x))) \mid x \in X. \}$$

be two SVNSs in $X = \{ x_1, x_2, \ldots, x_m \}$. Then the Hamming distance between two SVNSs $\tilde{N}_A$ and $\tilde{N}_B$ is defined as follows:

$$d(x, \tilde{N}_A, \tilde{N}_B) = \sum_{i=1}^m \left[ T_{\tilde{A}}(x_i) - T_{\tilde{B}}(x_i) \right] + \left[ I_{\tilde{A}}(x_i) - I_{\tilde{B}}(x_i) \right] + \left[ F_{\tilde{A}}(x_i) - F_{\tilde{B}}(x_i) \right]$$

and normalized Hamming distance between two SVNSs $\tilde{N}_A$ and $\tilde{N}_B$ is defined as follows:

$$\frac{d(x, \tilde{N}_A, \tilde{N}_B)}{m} = \frac{1}{m} \sum_{i=1}^m \left[ T_{\tilde{A}}(x_i) - T_{\tilde{B}}(x_i) \right] + \left[ I_{\tilde{A}}(x_i) - I_{\tilde{B}}(x_i) \right] + \left[ F_{\tilde{A}}(x_i) - F_{\tilde{B}}(x_i) \right]$$

with the following two properties

1. $0 \leq d(x, \tilde{N}_A, \tilde{N}_B) \leq 3n$ \hspace{1cm} (9)

2. $0 \leq \frac{d(x, \tilde{N}_A, \tilde{N}_B)}{m} \leq 1$ \hspace{1cm} (10)

**5 GRA based single valued neutrosophic multiple attribute decision-making problems with incomplete weight information.**

Consider a multi-attribute decision-making problem with $m$ alternatives and $n$ attributes. Let $A_1, A_2, \ldots, A_m$ be a discrete set of alternatives, and $C_1, C_2, \ldots, C_n$ be the set of attributes. The rating provided by the decision maker, describes the performance of alternative $A_i$ against attribute $C_j$. The values associated with the alternatives for MADM problems can be presented in the following decision matrix.

---

Pranab Biswas, Surapati Pramanik, Bibhas C. Giri, A New Methodology for Neutrosophic Multiple Attribute Decision-making with Unknown Weight Information.
Table 1. Decision matrix of attribute values

<table>
<thead>
<tr>
<th></th>
<th>C₁</th>
<th>C₂</th>
<th>...</th>
<th>Cₙ</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>d₁₁</td>
<td>d₁₂</td>
<td>...</td>
<td>d₁ₙ</td>
</tr>
<tr>
<td>A₂</td>
<td>d₂₁</td>
<td>d₂₂</td>
<td>...</td>
<td>d₂ₙ</td>
</tr>
<tr>
<td>Am</td>
<td>dₘ₁</td>
<td>dₘ₂</td>
<td>...</td>
<td>dₘₙ</td>
</tr>
</tbody>
</table>

\[ D = \{d_{ij}\}_{mn} \]  (11)

The weight \( w_j \in [0,1] \) (\( j = 1, 2, ..., n \)) reflects the relative importance of attribute \( C_j \) (\( j = 1, 2, ..., m \)) to the decision-making process such that \( \sum_{j=1}^{n} w_j = 1 \). \( S \) is a set of known weight information that can be represented by the following forms due to Park et al. [25], Park and Kim [26], Kim et al. [27], Kim and Ahn [28], and Park [29].

Form 1. A weak ranking: \( w_j \geq w_i \), for \( i \neq j \);
Form 2. A strict ranking: \( w_i - w_j \geq \varphi_i, \varphi_j > 0 \), for \( i \neq j \);
Form 3. A ranking of differences: \( w_i - w_j \geq w_k - w_l \), for \( j \neq k \neq i \);
Form 4. A ranking with multiples: \( w_i \geq \beta_j w_j, \beta_j \in [0,1] \), for \( i \neq j \);
Form 5. An interval form: \( \alpha_i \leq w_i \leq \alpha_i + \varepsilon_i \), \( 0 \leq \alpha_i < \alpha_i + \varepsilon_i \leq 1 \).

GRA is one of the derived evaluation methods for MADM based on the concept of grey relational space. The first step of GRA method is to create a comparable sequence of the performance of all alternatives. This step is known as data pre-processing. A reference sequence (ideal target sequence) is defined from these sequences. Then, the grey relational coefficient between all comparability sequences is calculated. Finally, based on these grey relational coefficients, the best alternative can be selected.

Step 2. Construct the decision matrix with SVNSs

Assume that a multiple attribute decision making problem have \( m \) alternatives and \( n \) attributes. The general form of decision matrix as shown in Table 1 can be presented after data pre-processing. The original GRA method can effectively deal with quantitative attributes. However, there exist some difficulties in the case of qualitative attributes. In the case of a qualitative attribute, an assessment value may be taken as SVNSs. In this paper we assume that the ratings of alternatives \( A_i \) (\( i = 1, 2, ..., m \)) with respect to attributes \( C_j \) (\( j = 1, 2, ..., n \)) are SVNSs. Thus the neutrosophic values associated with the alternatives for MADM problems can be represented in the following decision matrix:

Table 2. Decision matrix with SVNS

<table>
<thead>
<tr>
<th></th>
<th>C₁</th>
<th>C₂</th>
<th>...</th>
<th>Cₙ</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>{T₁₁, I₁₁, F₁₁}</td>
<td>{T₁₂, I₁₂, F₁₂}</td>
<td>...</td>
<td>{T₁ₙ, I₁ₙ, F₁ₙ}</td>
</tr>
<tr>
<td>A₂</td>
<td>{T₂₁, I₂₁, F₂₁}</td>
<td>{T₂₂, I₂₂, F₂₂}</td>
<td>...</td>
<td>{T₂ₙ, I₂ₙ, F₂ₙ}</td>
</tr>
<tr>
<td>Am</td>
<td>{Tₘ₁, Iₘ₁, Fₘ₁}</td>
<td>{Tₘ₂, Iₘ₂, Fₘ₂}</td>
<td>...</td>
<td>{Tₘₙ, Iₘₙ, Fₘₙ}</td>
</tr>
</tbody>
</table>

In this matrix \( D = \{T_{ij}, I_{ij}, F_{ij}\}_{mn} \), \( T_{ij}, I_{ij} \) and \( F_{ij} \) denote the degrees of truth membership, degree of indeterminacy and degree of falsity membership of the alternative \( A_i \) with respect to attribute \( C_j \). These three degrees for SVNS satisfying the following properties:

1. \( 0 \leq T_{ij} \leq 1 \), \( 0 \leq I_{ij} \leq 1 \), \( 0 \leq F_{ij} \leq 1 \)
2. \( 0 \leq T_{ij} + I_{ij} + F_{ij} \leq 3 \).

Step 3. Determine the ideal neutrosophic estimates reliability solution (INERS) and the ideal neutrosophic estimates un-reliability solution (INEURS) for neutrosophic decision matrix.

The ideal reliability estimation can be easily determined due to Biswas et al. [24].

Definition 11 The ideal neutrosophic estimates reliability solution (INERS) \( Q^- = \{q^-_{A₁}, q^-_{A₂}, ..., q^-_{Am}\} \) is a solution in which every component \( q^-_{A_i} = \{T^-_{ij}, I^-_{ij}, F^-_{ij}\} \), where \( T^- = \max(T_{ij}) \), \( I^- = \min(I_{ij}) \) and \( F^- = \min(F_{ij}) \) in the neutrosophic decision matrix \( D^- = \{T^-_{ij}, I^-_{ij}, F^-_{ij}\}_{mn} \).

Definition 12 The ideal neutrosophic estimates un-reliability solution (INEURS) \( Q^+ = \{q^+_{A₁}, q^+_{A₂}, ..., q^+_{Am}\} \) can
be taken as a solution in the form \( q_{ij} = \left\{ T_i, I_j, F_j \right\} \), where \( T_i = \min \{ T_{ij} \} \), \( I_j = \max \{ I_{ij} \} \) and \( F_j = \max \{ F_{ij} \} \) in the neutrosophic decision matrix \( D=x\left\{ T_i, I_j, F_j \right\}_{m,n} \).

Step 4. Calculate the neutrosophic grey relational coefficient of each alternative from INERS and INEURS.

Grey relational coefficient of each alternative from INERS is defined as:

\[
\chi_{ij}^{\rho} = \frac{i \Delta_{ij} + \rho \max_{1 \leq j \leq m} \Delta_{ij}}{i \Delta_{ij} + \rho \max_{1 \leq j \leq m} \Delta_{ij}},
\]

where

\[
\Delta_{ij} = d(q_{i1}, q_{ij}), i=1, 2, \ldots, m \text{ and } j=1, 2, \ldots, n.
\]

(13)

Grey relational coefficient of each alternative from INEURS is defined as:

\[
\chi_{ij}^{\rho} = \frac{i \Delta_{ij} + \rho \max_{1 \leq j \leq m} \Delta_{ij}}{i \Delta_{ij} + \rho \max_{1 \leq j \leq m} \Delta_{ij}},
\]

where

\[
\Delta_{ij} = d(q_{i1}, q_{ij}), i=1, 2, \ldots, m \text{ and } j=1, 2, \ldots, n.
\]

(14)

\( \rho \) is the distinguishing coefficient or the identification coefficient, \( \rho \in [0,1] \). Smaller value of distinguishing coefficient will yield in large range of grey relational coefficient. Generally, \( \rho = 0.5 \) is considered for decision-making situation.

Step 5. Determine the weights of criteria.

In the decision-making process, decision maker may often feel that the importance of the attributes is not same. Due to the complexity and uncertainty of real world decision-making problems, the information about attribute weights is usually incomplete. The estimation of the attribute weights plays an important role in MADM. Therefore, we need to determine reasonable attribute weight for making a reasonable decision. Many methods are available to determine the unknown attribute weight in the literature such as maximizing deviation method (Wu and Chen [31]), entropy method (Wei and Tang [32]; Xu and Hui [33]), optimization method (Wang and Zhang [34-35]) etc. In this paper, we use optimization method to determine unknown attribute weights for neutrosophic MADM.

The basic principle of the GRA method is that the chosen alternative should have the largest degree of grey relation from the INERS. Thus, the larger grey relational coefficient determines the best alternative for the given weight vector. To obtain the grey relational coefficient, we have to calculate weight vector of attributes if the information about attribute weights is incompletely known. The grey relational coefficient between INERS and itself is \((1, 1, \ldots, 1)\), similarly, coefficient between INEURS and itself is also \((1, 1, \ldots, 1)\). So the corresponding comprehensive deviations are

\[
d_i^+(W) = \sum_{j=1}^{n} (w_j \chi_{ij}) w_j
\]

(15)

\[
d_i^-(W) = \sum_{j=1}^{n} (1 - \chi_{ij}) w_j
\]

(16)

We utilize the max-min operator developed by Zimmermann and Zysco [36] to integrate all the distances \(d_i^-(W) = \max_{j=1}^{n} (1 - \chi_{ij}) w_j \) for \( i = 1, 2, \ldots, m \) and \(d_i^+(W) = \max_{j=1}^{n} (w_j \chi_{ij}) w_j \) for \( i = 1, 2, \ldots, m \) separately.

Therefore, we can formulate the following programming model:

\[
\text{Min } \xi^+
\]

subject to: \( \sum_{j=1}^{n} (1 - \chi_{ij}) w_j \leq \xi^+ \text{ for } i=1,2,\ldots,m \) \( W \in S \)

(17)

\[
\text{Min } \xi^-
\]

subject to: \( \sum_{j=1}^{n} (1 - \chi_{ij}) w_j \leq \xi^- \text{ for } i=1,2,\ldots,m \) \( W \in S \)

(18)

Here \( \xi^+ = \max \left\{ \sum_{j=1}^{n} (1 - \chi_{ij}) w_j \right\} \) \( i = 1, 2, \ldots, m. \)

(19)

and \( \xi^- = \max \left\{ \sum_{j=1}^{n} (1 - \chi_{ij}) w_j \right\} \) for \( 1, 2, \ldots, m. \)

(20)

Solving these two model (M-1a) and (M-1b), we obtain the optimal solutions \( W^+ = (w_1^+, w_2^+, \ldots, w_m^+) \) and \( W^- = (w_1, w_2, \ldots, w_m) \) respectively. Combinations of these two optimal solutions will give us the weight vector of the attributes i.e. \( W = \gamma W^+ + (1 - \gamma) W^- \) for \( \gamma \in [0,1] \).

If the information about attribute weights is completely unknown, we can establish another multiple objective programming:

\[
\min d_i^+(W) = \sum_{j=1}^{n} (w_j \chi_{ij}) w_j \text{ for } i=1,\ldots,m.
\]

subject to: \( \sum_{j=1}^{n} w_j = 1 \)

(22)
Since each alternative is non-inferior, so there exists no preference relation between the alternatives. Then, we can aggregate the above multiple objective optimization models with equal weights in to the following single objective optimization model:

\[
\begin{aligned}
\min^*(W) &= \frac{w_i}{2} \sum_{j=1}^{n} (1 - \chi_{ij}) w_j^2, \\
\text{subject to: } & \sum_{j=1}^{n} w_j = 1
\end{aligned}
\]  

(23)

To solve this model, we construct the Lagrange function:

\[
L(W, \lambda) = \sum_{i=1}^{m} \left( \sum_{j=1}^{n} (1 - \chi_{ij}) w_j \right)^2 + 2\lambda \left( \sum_{j=1}^{n} w_j - 1 \right)
\]

(24)

Where \( \lambda \) is the Lagrange multiplier. Differentiating equation (24) with respect to \( w_j \) (\( j = 1, 2, \ldots, n \)) and \( \lambda \), and putting these partial derivatives equal to zero, we have the following set of equations:

\[
\frac{\partial L(w, \lambda)}{\partial w_j} = 2 \sum_{i=1}^{m} (1 - \chi_{ij}) w_j = 0
\]

(25)

\[
\frac{\partial L(w, \lambda)}{\partial \lambda} = \sum_{j=1}^{n} w_j - 1 = 0
\]

(26)

Solving equations (25) and (26), we obtain the following relation

\[
w_j = \left[ \frac{\sum_{i=1}^{m} (1 - \chi_{ij})^2}{\sum_{i=1}^{m} (1 - \chi{ij})^2} \right]^{-1} \left( \sum_{i=1}^{m} (1 - \chi_{ij}) \right)
\]

(27)

Then we get \( \chi_{ij} \) for \( i = 1, 2, \ldots, m. \)

Similarly, we can find out the attribute weight \( w_j \) taking into consideration of INERUS as:

\[
w_j = \left[ \frac{\sum_{i=1}^{m} (1 - \chi_{ij})^2}{\sum_{i=1}^{m} (1 - \chi_{ij})} \right]^{-1} \left( \sum_{i=1}^{m} (1 - \chi_{ij}) \right)
\]

(28)

Combining (27) and (28), we can determine the j-th attribute weight with the help of (21).

**Step 6. Calculate of neutrosophic grey relational coefficient (NGRC).**

The degree of neutrosophic grey relational coefficient of each alternative from ITFPIS and ITFNIS are calculated by using the following equations:

\[
\chi_i^* = \sum_{j=1}^{n} w_j \chi_{ij}
\]

(29)

and \( \chi_i^* = \sum_{j=1}^{n} w_j \chi_{ij}^* \) for \( i = 1, 2, \ldots, m. \)

**Step 7. Calculate the neutrosophic relative relational degree (NRD).**

We calculate the neutrosophic relative relational degree of each alternative from ITFPIS by employing the following equation:

\[
R_i = \frac{\chi_i^*}{\chi_i^* + \chi_i}
\]

(30)

**Step 8. Rank the alternatives.**

Based on the neutrosophic relative relational degree, the ranking order of all alternatives can be determined. The highest value of \( R_i \) presents the most desired alternatives.

5. Illustrative Examples

In this section, neutrosophic MADM is considered to demonstrate the application and the effectiveness of the proposed approach. Let us consider the decision-making problem adapted from Ye [37]. Suppose there is an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible alternatives to invest the money: (1) \( A_1 \) is a car company; (2) \( A_2 \) is a food company; (3) \( A_3 \) is a computer company, and (4) \( A_4 \) is an arms company. The investment company must take a decision based on the following three criteria: (1) \( C_1 \) is the risk analysis; (2) \( C_2 \) is the growth analysis; and (3) \( C_3 \) is the environmental impact analysis. We obtain the following single-valued neutrosophic decision matrix based on the experts’ assessment:

**Table 3. Decision matrix with SVNS**

\[
\begin{array}{cccc}
C_1 & C_2 & C_3 \\
A_1 & (0.4,0.2,0.3) & (0.4,0.2,0.3) & (0.2,0.2,0.5) \\
A_2 & (0.6,0.1,0.2) & (0.6,0.1,0.2) & (0.5,0.2,0.2) \\
A_3 & (0.3,0.2,0.3) & (0.5,0.2,0.3) & (0.5,0.3,0.2) \\
A_4 & (0.7,0.0,0.1) & (0.6,0.1,0.2) & (0.4,0.3,0.2) \\
\end{array}
\]

Information about the attribute weights is partially known. The known weight information is given as follows: \( S = \{0.30 \leq w_1 \leq 0.35, 0.36 \leq w_2 \leq 0.48, 0.26 \leq w_3 \leq 0.30\} \) such that \( w_j \geq 0 \) for \( j = 1, 2, 3 \) and \( \sum_{j=1}^{n} w_j = 1 \).

**Step 1.** Determine the ideal neutrosophic estimates reliability solution (INERS) from the given decision matrix (see Table 3) as:
Pranab Biswas, Surapati Pramanik, Bibhas C. Giri, A New Methodology for Neutrosophic Multiple Attribute Decision-making with Unknown Weight Information.
4. Calculation of NGRC and NRD of each alternative from neutrosophic estimates reliability solution

<table>
<thead>
<tr>
<th>Proposed method</th>
<th>Weight Vector</th>
<th>NGRC from INERS</th>
<th>NGRC from INEURS</th>
<th>NRD from INERS</th>
<th>Ranking Result</th>
<th>Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case-1</td>
<td>(0.30, 0.44, 0.26)</td>
<td>0.4331</td>
<td>0.9422</td>
<td>0.3149</td>
<td>R₄&gt;R₂&gt;R₃&gt;R₁</td>
<td>A₄</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.8714</td>
<td>0.4320</td>
<td>0.6686</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5594</td>
<td>0.6714</td>
<td>0.4545</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.9133</td>
<td>0.4122</td>
<td>0.6890</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case-2</td>
<td>(0.2657, 0.4385, 0.2958)</td>
<td>0.4342</td>
<td>0.9343</td>
<td>0.3173</td>
<td>R₄&gt;R₂&gt;R₃&gt;R₁</td>
<td>A₄</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.8861</td>
<td>0.4272</td>
<td>0.6747</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5758</td>
<td>0.6567</td>
<td>0.4672</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.9014</td>
<td>0.4149</td>
<td>0.6847</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step 7. From Table 4, we can easily determine the ranking order of all alternatives according to the values of neutrosophic relational degrees. For case-1, we see that A₄ i.e. Arms company is the best alternative for investment purpose. Similarly, for case-2 A₄ i.e. Arms company also is the best alternative for investment purpose.

6 Conclusion

In this paper, we introduce single-valued neutrosophic multiple attribute decision-making problem with incompletely known or completely unknown attribute weight information based on modified GRA. In order to determine the incompletely known attribute weight minimizing deviation based optimization method is used. On the other hand, we solve an optimization model to find out the completely unknown attributes weight by using Lagrange functions. Finally, an illustrative example is provided to show the feasibility of the proposed approach and to demonstrate its practicality and effectiveness. However, we hope that the concept presented here will create new avenue of research in current neutrosophic decision-making arena. The main thrust of the paper will be in the field of practical decision-making, pattern recognition, medical diagnosis and clustering analysis.

References


Received: May 25, 2014. Accepted: June 10, 2014.

Pranab Biswas, Surapati Pramanik, Bibhas C. Giri, A New Methodology for Neutrosophic Multiple Attribute Decision-making with Unknown Weight Information.